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THE ELEMENTS

OF GEOMETRIE

of the most auncient Philosopher

EVCLIDE

of Megara.

Faithfully (now first) translated into the Englishe toung, by H. Billingsley, *Citizen of London.*

Whereunto are annexed certaine Scholies, Annotations, and Inuentions, of the best Mathematicians, both of time past, and in this our age.

With a very fruitfull Præface made by M. I. Dee, *specifying the chiefe Mathematicall Sciẽces, what they are, and wherunto commodious: where, also, are disclosed certaine new Secrets Mathematicall and Mechanicall, vntill these our daies, greatly missed.*

Imprinted at London by *John Day*.

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Here is (gentle Reader) nothing (the word of God onely set apart) which so much beautifieth and adorneth the soule and minde of mã, as doth the knowledge of good artes and sciences: as the knowledge of naturall and morall Philosophie. The one setteth before our eyes, the creatures of God, both in the heauens aboue, and in the earth beneath: in which as in a glasse, we beholde the exceding maiestie and wisedome of God, in adorning and beautifying them as we see: in geuing vnto them such wonderfull and manifolde proprieties, and naturall workinges, and that so diuersly and in such varietie: farther in maintaining and conseruing them continually, whereby to praise and adore him, as by S. Paule we are taught. The other teacheth vs rules and preceptes of vertue, how, in common life amongest men, we ought to walke vprightly: what dueties pertaine to our selues, what pertaine to the gouernment or good order both of an housholde, and also of a citie or common wealth. The reading likewise of histories, conduceth not a litle, to the adorning of the soule & minde of man, a studie of all men cõmended: by it are seene and knowen the artes and doinges of infinite wise men gone before vs. In histories are contained infinite examples of heroicall vertues to be of vs followed, and horrible examples of vices to be of vs eschewed. Many other artes also there are which beautifie the minde of man: but of all other none do more garnishe & beautifie it, then those artes which are called Mathematicall. Vnto the knowledge of which no man can attaine, without the perfecte knowledge and instruction of the principles, groundes, and Elementes of Geometrie. But per-

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fectly to be instructed in them, requireth diligent studie and reading of olde auncient authors. Amongest which, none for a beginner is to be preferred before the most auncient Philosopher Euclide *of* Megara. *For of all others he hath in a true methode and iuste order, gathered together whatsoeuer any before him had of these Elementes written: inuenting also and adding many thinges of his owne: wherby he hath in due forme accomplished the arte: first geuing definitions, principles, & groundes, wherof he deduceth his Propositions or conclusions, in such wonderfull wise, that that which goeth before, is of necessitie required to the proufe of that which followeth. So that without the diligent studie of* Euclides *Elementes, it is impossible to attaine vnto the perfecte knowledge of Geometrie, and consequently of any of the other Mathematicall sciences. Wherefore considering the want & lacke of such good authors hitherto in our Englishe tounge, lamenting also the negligence, and lacke of zeale to their countrey in those of our nation, to whom God hath geuen both knowledge, & also abilitie to translate into our tounge, and to publishe abroad such good authors, and bookes (the chiefe instrumentes of all learninges): seing moreouer that many good wittes both of gentlemen and of others of all degrees, much desirous and studious of these artes, and seeking for them as much as they can, sparing no paines, and yet frustrate of their intent, by no meanes attaining to that which they seeke: I haue for their sakes, with some charge & great trauaile, faithfully translated into our vulgare touge, & set abroad in Print, this booke of* Euclide. *Whereunto I haue added easie and plaine declarations and examples by figures, of the definitions. In which booke also ye shall in due place finde manifolde additions, Scholies, Annotations, and Inuentions: which I haue gathered out of many of the most famous & chiefe Mathematicies, both of old time, and in our age: as by diligent reading it in course, ye shall*

well

well perceaue. The fruite and gaine which I require for these my paines and trauaile, shall be nothing els, but onely that thou gentle Reader, will gratefully accept the same: and that thou mayest thereby receaue some profite: and moreouer to excite and stirre vp others learned, to do the like, & to take paines in that behalfe. By meanes wherof, our Englishe tounge shall no lesse be enriched with good Authors, then are other straunge tounges: as the Dutch, French, Italian, and Spanishe: in which are red all good authors in a maner, found amongest the Grekes or Latines. Which is the chiefest cause, that amongest thē do florishe so many cunning and skilfull men, in the inuentions of straunge and wonderfull thinges, as in these our daies we see there do. Which fruite and gaine if I attaine vnto, it shall encourage me hereafter, in such like sort to translate, and set abroad some other good authors, both pertaining to religion (as partly I haue already done) and also pertaining to the Mathematicall Artes. Thus gentle reader farewell.

TO THE VNFAINED LOVERS of truthe, and constant Studentes of Noble *Sciences*, IOHN DEE *of London, hartily* wisheth grace from heauen, and most prosperous *succesſe in all their honeſt attemptes and* exerciſes.

Iuine *Plato*, the great Maſter of many worthy Philoſophers, and the conſtant auoucher, and pithy perſwader of *Vnum*, *Bonum*, and *Ens*: in his Schole and Academie, ſundry times (beſides his ordinary Scholers) was viſited of a certaine kinde of men, allured by the noble fame of *Plato*, and the great commendation of hys profound and profitable doctrine. But when ſuch Hearers, after long harkening to him, perceaued, that the drift of his diſcourſes iſſued out, to conclude, this *Vnum*, *Bonum*, and *Ens*, to be Spirituall, Infinite, Æternall, Omnipotent, &c. Nothyng beyng alledged or expreſſed, How, worldly goods: how, worldly dignitie: how, health, Strēgth or luſtines of body: nor yet the meanes, how a merueilous ſenſible and bodyly blyſſe and felicitie hereafter, might be atteyned: Straightway, the fantaſies of thoſe hearers, were dampt: their opinion of *Plato*, was clene chaunged: yea his doctrine was by them deſpiſed: and his ſchole, no more of them viſited. Which thing, his Scholer, *Ariſtotle*, narrowly cōſidering, founde the cauſe therof, to be, For that they had no forwarnyng and information, in generall, whereto his doctrine tended. For, ſo, might they haue had occaſion, either to haue forborne his ſchole hauntyng: (if they, then, had miſliked his Scope and purpoſe) or conſtantly to haue continued therin: to their full ſatiſfaction: if ſuch his finall ſcope & intent, had ben to their deſire. Wherfore, *Ariſtotle*, euer, after that, vſed in brief, to forewarne his owne Scholers and hearers, both of what matter, and alſo to what ende, he tooke in hand to ſpeake, or teach. While I conſider the diuerſe trades of theſe two excellent Philoſophers (and am moſt ſure, both, that *Plato* right well, otherwiſe could teach: and that *Ariſtotle* mought boldely, with his hearers, haue dealt in like ſorte as *Plato* did) I am in no little pang of perplexitie: Bycauſe, that, which I miſlike, is moſt eaſy for me to performe (and to haue *Plato* for my exāple.) And that, which I know to be moſt commendable: and (in this firſt bringyng, into common handling, the *Artes Mathematicall*) to be moſt neceſſary: is full of great difficultie and ſundry daungers. Yet, neither do I think it mete, for ſo ſtraunge matter (as now is ment to be publiſhed) and to ſo ſtraunge an audience, to be bluntly, at firſt, put forth, without a peculiar Preface: Nor (Imitatyng *Ariſtotle*) well can I hope, that accordyng to the amplenes and dignitie of the *State Mathematicall*, I am able, either playnly to preſcribe the materiall boundes: or preciſely to expreſſe the chief purpoſes, and moſt wonderfull applications therof. And though I am ſure, that ſuch as did ſhrinke from *Plato* his ſchole, after they had perceiued his finall

nall concluſion, would in theſe thinges haue ben his moſt diligent hearers) ſo infinitely mought their deſires, in fine and at length, by our *Artes Mathematicall* be ſatiſfied) yet, by this my Præface & forewarnyng, Aſwell all ſuch, may (to their great behofe) the ſoner, hither be allured: as alſo the *Pythagoricall*, and *Platonic*... perfect ſcholer, and the conſtant profound Philoſopher, with more eaſe and ſpede, may (like the Bee,) gather, hereby, both wax and hony.

Wherfore, ſeyng I finde great occaſion (for the cauſes alleged, and farder, in reſpect of my *Art Mathematike generall*) to vſe a certaine forewarnyng and Præface, whoſe content ſhalbe, that mighty, moſt pleſaunt, and frutefull *Mathematicall Tree*, with his chief armes and ſecond (grifted) braunches: Both, what euery one is, and alſo, what commodity, in generall, is to be looked for, aſwell of griff as ſtocke: And foraſmuch as this enterpriſe is ſo great, that, to this our tyme, it neuer was (to my knowledge) by any achieued: And alſo it is moſt hard, in theſe our drery dayes, to ſuch rare and ſtraunge Artes, to wyn due and common credit: Neuertheles, if, for my ſincere endeuour to ſatiſfie your honeſt expectation, you will but lend me your thākefull mynde a while: and, to ſuch matter as, for this time, my penne (with ſpede) is hable to deliuer, apply your eye or eare attentifely: perchaunce, at once, and for the firſt ſalutyng, this Preface you will finde a leſſon long enough. And either you will, for a ſecond (by this) be made much the apter: or ſhortly become, well hable your ſelues, of the lyons claw, to coniecture his royall ſymmetrie, and farder propertie. Now then, gentle, my frendes, and countrey men, Turne your eyes, and bend your myndes to that doctrine, which for our preſent purpoſe, my ſimple talent is hable to yeld you.

The intent of this Preface.

All thinges which are, & haue beyng, are found vnder a triple diuerſitie generall. For, either, they are demed Supernaturall, Naturall, or, of a third being. Thinges Supernaturall, are immateriall, ſimple, indiuiſible, incorruptible, & vnchangeable. Things Naturall, are materiall, compounded, diuiſible, corruptible, and chaungeable. Thinges Supernaturall, are, of the minde onely, comprehended: Things Naturall, of the ſenſe exterior, ar hable to be perceiued. In thinges Naturall, probabilitie and coniecture hath place: But in things Supernaturall, chief demōſtration, & moſt ſure Science is to be had. By which properties & comparaſons of theſe two, more eaſily may be deſcribed, the ſtate, condition, nature and property of thoſe thinges, which, we before termed of a third being: which, by a peculier name alſo, are called *Thynges Mathematicall*. For, theſe, beyng (in a maner) middle, betwene thinges ſupernaturall and naturall: are not ſo abſolute and excellent, as thinges ſupernatural: Nor yet ſo baſe and groſſe, as things naturall: But are thinges immateriall: and neuertheleſſe, by materiall things hable ſomewhat to be ſignified. And though their particular Images, by Art, are aggregable and diuiſible: yet the generall *Formes*, notwithſtandyng, are conſtant, vnchaungeable, vntrāſformable, and incorruptible. Neither of the ſenſe, can they, at any tyme, be perceiued or iudged. Nor yet, for all that, in the royall mynde of man, firſt conceiued. But, ſurmountyng the imperfectiō of coniecture, weenyng and opinion: and commyng ſhort of high intellectuall cōceptiō, are the Mercurial fruite of *Dianœticall* diſcourſe, in perfect imagination ſubſiſtyng. A meruaylous newtralitie haue theſe thinges *Mathematicall*: and alſo a ſtraunge participatiō betwene thinges ſupernaturall, immortall, intellectual, ſimple and indiuiſible: and thynges naturall, mortall, ſenſible, compounded and diuiſible. Probabilitie and ſenſible profe, may well ſerue in thinges naturall: and is commendable: In Mathematicall reaſoninges, a probable Argument, is nothyng regarded: nor yet the teſtimony of ſenſe, any whit credited: But onely a perfect demonſtration, of truthes certaine, neceſſary, and inuincible: vniuerſally and neceſſaryly concluded:

cluded: is allowed as ſufficient for an Argument exactly and purely Mathematical. ,,

Of *Mathematicall* thinges, are two principall kindes: namely, *Number*, and *Magnitude*. *Number*. *Number*, we define, to be, a certayne Mathematicall Sūme, of *Vnits*. And, an *Vnit*, is that thing Mathematicall, Indiuiſible, by participation of ſome likenes of whoſe property, any thing, which is in deede, or is counted One, may reſonably be called One. We account an *Vnit*, a thing *Mathematicall*, though it be no *Number*, and alſo indiuiſible: becauſe, of it, materially, Number doth conſiſt: which, principally, is a thing *Mathematicall*. *Magnitude* is a thing *Mathematicall*, by participation of ſome likenes of whoſe nature, any thing is iudged long, broade, or thicke. A thicke *Magnitude* we call a *Solide*, or a *Body*. What *Magnitude* ſo euer, is Solide or Thicke, is alſo broade, & long. A broade magnitude, we call a *Superficies* or a Plaine. Euery playne magnitude, hath alſo length. A long magnitude, we terme a *Line*. A *Line* is neither thicke nor broade, but onely long: Euery certayne Line, hath two endes: The endes of a line, are *Pointes* called. A *Point*, is a thing *Mathematicall*, indiuiſible, which may haue a certayne determined ſituation. If a Poynt moue from a determined ſituation, the way wherein it moued, is alſo a *Line*: mathematically produced. whereupon, of the auncient Mathematiciens, a *Line* is called the race or courſe of a *Point*. A Poynt we define, by the name of a thing Mathematicall: though it be no Magnitude, and indiuiſible: becauſe it is the propre ende, and bound of a *Line*: which is a true *Magnitude*. And *Magnitude* we may define to be that thing *Mathematicall*, which is diuiſible for euer, in partes diuiſible, long, broade or thicke. Therefore though a Poynt be no *Magnitude*, yet *Terminatiuely* we recken it a thing *Mathematicall* (as I ſayd) by reaſon it is properly the end, and bound of a line.

Note the worde, Vnit, to expreſſe the Greke Monas, & not Vnitie: as we haue all, commonly, till now, vſed.

Magnitude.

,,

A point.

,,

A Line.

Magnitude.

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Neither *Number*, nor *Magnitude*, haue any Materialitie. Firſt, we will conſider of *Number*, and of the Science *Mathematicall*, to it appropriate, called *Arithmetike*: and afterward of *Magnitude*, and his Science, called *Geometrie*. But that name contenteth me not: whereof a word or two hereafter ſhall be ſayd. How Immateriall and free from all matter, *Number* is, who doth not perceaue? yea, who doth not wonderfully wōder at it? For, neither pure *Element*, nor *Ariſtoteles*, *Quinta Eſſentia*, is hable to ſerue for Number, as his propre matter. Nor yet the puritie and ſimplenes of Subſtance Spirituall or Angelicall, will be found propre enough thereto. And therefore the great & godly Philoſopher *Anitius Boetius*, ſayd: *Omnia quæcunq́, a primæua rerum natura conſtructa ſunt, Numerorum videntur ratione formata. Hoc enim fuit principale in animo Conditoris Exemplar.* That is: *All thinges (which from the very firſt originall being of thinges, haue bene framed and made) do appeare to be Formed by the reaſon of Numbers. For this was the principall example or patterne in the minde of the Creator.* O comfortable allurement, O rauiſhing perſwaſion, to deale with a Science, whoſe Subiect, is ſo Auncient, ſo pure, ſo excellent, ſo ſurmounting all creatures, ſo vſed of the Almighty and incomprehenſible wiſdome of the Creator, in the diſtinct creation of all creatures: in all their diſtinct partes, properties, natures, and vertues, by order, and moſt abſolute number, brought, from *Nothing*, to the *Formalitie* of their being and ſtate. By *Numbers* propertie therefore, of vs, by all poſſible meanes, (to the perfection of the Science) learned, we may both winde and draw our ſelues into the inward and deepe ſearch and view, of all creatures diſtinct vertues, natures, properties, and *Formes*: And alſo, farder, ariſe, clime, aſcend, and mount vp (with Speculatiue winges) in ſpirit, to behold in the Glas of Creation, the *Forme* of *Formes*, the *Exemplar Number* of all thinges *Numerable*: both viſible and inuiſible: mortall and

immortall, Corporall and Spirituall. Part of this profound and diuine Science, had *Ioachim* the Propheſier atteyned vnto: by *Numbers Formall, Naturall,* and *Ratiocinall,* forſeyng, concludyng, and forſhewyng great particular euents, long before their comming. His bookes yet remainyng, hereof, are good profe: And the noble Earle of *Mirandula,* (beſides that,) a ſufficient witneſſe: that *Ioachim, in his propheſies, proceded by no other way, then by Numbers Formall.* And this Earle hym ſelfe, in Rome, *ſet vp 900. Concluſions, in all kinde of Sciences, openly to be diſputed of: and among the reſt, in his Concluſions *Mathematicall,* (in the eleuenth Concluſion) hath in Latin, this Engliſh ſentence. *By Numbers, a way is had, to the ſearchyng out, and vnderſtandyng of euery thyng, hable to be knowen. For the verifying of which Concluſion, I promiſe to aunſwere to the 74. Quæſtions, vnder written, by the way of Numbers.* Which Cõcluſions, I omit here to rehearſe: aſwell auoidyng ſuperfluous prolixitie: as, by-cauſe *Ioannes Picus, workes,* are commonly had. But, in any caſe, I would wiſh that thoſe Concluſions were red diligently, and perceiued of ſuch, as are earneſt Obſeruers and Conſiderers of the conſtant law of nũbers: which is planted in thyngs Naturall and Supernaturall: and is preſcribed to all Creatures, inuiolably to be kept. For, ſo, beſides many other thinges, in thoſe Concluſions to be marked, it would apeare, how ſincerely, & within my boundes, I diſcloſe the wonderfull myſteries, by numbers, to be atteyned vnto.

Ano. 1488.

Of my former wordes, eaſy it is to be gathered, that *Number* hath a treble ſtate: One, in the Creator: an other in euery Creature (in reſpect of his complete conſtitution:) aud the third, in Spirituall and Angelicall Myndes, and in the Soule of mã. In the firſt and third ſtate, *Number*, is termed *Number Numbryng*. But in all Creatures, otherwiſe, *Number*, is termed *Nũber Numbred*. And in our Soule, Nũber beareth ſuch a ſwaye, and hath ſuch an affinitie therwith: that ſome of the old *Philoſophers* taught, *Mans Soule, to be a Number mouyng it ſelfe*. And in dede, in vs, though it be a very Accident: yet ſuch an Accident it is, that before all Creatures it had perfect beyng, in the Creator, Sempiternally. *Number Numbryng* therfore, is the diſcretion diſcerning, and diſtincting of thinges. But in God the Creator, This diſcretion, in the beginnyng, produced orderly and diſtinctly all thinges. For his *Numbryng*, then, was his Creatyng of all thinges. And his Continuall *Numbryng*, of all thinges, is the Conſeruation of them in being: And, where and when he will lacke an *Vnit*: there and then, that particular thyng ſhalbe *Diſcreated*. Here I ſtay. But our Seuerallyng, diſtinctyng, and *Numbryng*, createth nothyng: but of Multitude conſidered, maketh certaine and diſtinct determination. And albeit theſe thynges be waighty and truthes of great importance, yet (by the infinite goodnes of the Almighty *Ternarie*,) Artificiall Methods and eaſy wayes are made, by which the zelous Philoſopher, may wyn nere this Riueriſh *Ida*, this Mountayne of Contemplation: and more then Contemplation. And alſo, though *Number*, be a thyng ſo Immateriall, ſo diuine, and æternall: yet by degrees, by litle and litle, ſtretchyng forth, and applying ſome likenes of it, as firſt, to thinges Spirituall: and then, bryngyng it lower, to thynges ſenſibly perceiued: as of a momentanye ſounde iterated: then to the leaſt thynges that may be ſeen, numerable: And at length, (moſt groſſely,) to a multitude of any corporall thynges ſeen, or felt: and ſo, of theſe groſſe and ſenſible thynges, we are trayned to learne a certaine Image or likenes of numbers: and to vſe Arte in them to our pleaſure and proffit. So groſſe is our conuerſation, and dull is our apprehenſion: while mortall Senſe, in vs, ruleth the common wealth of our litle world. Hereby we ſay, Three Lyons, are three: or a *Ternarie*. Three Egles, are three, or a *Ternarie*. Which* *Ternaries*, are eche, the *Vnion, knot,* and *Vniformitie,* of three diſcrete and diſtinct *Vnits*. That is, we may in eche *Ternarie*, thriſe, ſeuerally pointe, and ſhew a part, *One, One,* and *One*. Where, in Numbryng, we ſay One, two, Three,

Three. But how farre, these visible Ones, do differre from our Indiuisible Vnits (in pure *Arithmetike*, principally considered) no man is ignorant. Yet from these grosse and materiall thynges, may we be led vpward, by degrees, so, informyng our rude Imagination, toward the cõceiuyng of *Numbers*, absolutely (: Not supposing, nor admityng any thyng created, Corporall or Spirituall, to support, conteyne, or represent those *Numbers* imagined:) that at length, we may be hable, to finde the number of our owne name, gloriously exemplified and registred in the booke of the *Trinitie* most blessed and æternall.

But farder vnderstand, that vulgar Practisers, haue Numbers, otherwise, in sundry Considerations: and extend their name farder, then to Numbers, whose least part is an *Vnit*. For the common Logist, Reckenmaster, or Arithmeticien, in hys vsing of Numbers: of an Vnit, imagineth lesse partes: and calleth them *Fractions*. As of an *Vnit*, he maketh an halfe, and thus noteth it, $\frac{1}{2}$. and so of other, (infinitely diuerse) partes of an *Vnit*. Yea and farder, hath, *Fractions of Fractions*. *&c*. And, forasmuch, as, *Addition*, *Substraction*, *Multiplication*, *Diuision* and *Extraction of Rotes*, are the chief, and sufficient partes of *Arithmetike*: which is, the *Science that demonstrateth the properties, of Numbers, and all operatiõs, in numbers to be performed*: How often, therfore, these fiue sundry sortes of Operations, do, for the most part, of their execution, differre from the fiue operations of like generall property and name, in our Whole numbers practisable, So often, (for a more distinct doctrine) we, vulgarly account and name it, an other kynde of *Arithmetike*. And by this reason: the Consideration, doctrine, and working, in whole numbers onely: where, of an *Vnit*, is no lesse part to be allowed: is named (as it were) an *Arithmetike* by it selfe. And so of the *Arithmetike of Fractions*. In lyke sorte, the necessary, wonderfull and Secret doctrine of Proportion, and proportionalytie hath purchased vnto it selfe a peculier maner of handlyng and workyng: and so may seme an other forme of *Arithmetike*. Moreouer, the *Astronomers*, for spede and more commodious calculation, haue deuised a peculier maner of orderyng nũbers, about theyr circular motions, by Sexagenes, and Sexagesimes. By Signes, Degrees and Minutes &c. which commonly is called the *Arithmetike* of *Astronomical* or *Phisicall Fractions*. That, haue I briefly noted, by the name of *Arithmetike Circular*. By cause it is also vsed in circles, not *Astronomicall*. *&c*. Practise hath led *Numbers* farder, and hath framed them, to take vpon them, the shew of *Magnitudes* propertie: Which is *Incommensurabilitie* and *Irrationalitie*. (For in pure *Arithmetike*, an *Vnit*, is the common Measure of all Numbers.) And, here, Nũbers are become, as Lynes, Playnes and Solides: some tymes *Rationall*, some tymes *Irrationall*. And haue propre and peculier characters, (as $\sqrt{3}$. $\sqrt{c}$. and so of other. Which is to signifie *Rote Square*, *Rote Cubik*: *and so forth*:) & propre and peculier fashions in the fiue principall partes: Wherfore the practiser, estemeth this, a diuerse *Arithmetike* from the other. Practise bryngeth in, here, diuerse compoundyng of Numbers: as some tyme, two, three, foure (or more) *Radicall* nũbers, diuersly knit, by signes, of More & Lesse: as thus $\sqrt{3}$ 12 + $\sqrt{c}$ 15. Or thus $\sqrt{3}\,\sqrt{3}$ 19 + $\sqrt{c}$ 12 − $\sqrt{3}$ 2. &c. And some tyme with whole numbers, or fractions of whole Number, amõg them: as 20 + $\sqrt{3}$ 24. $\sqrt{c}$ 16 + 33 − $\sqrt{3}$ 10. $\sqrt{3}\,\sqrt{3}$ 44 + 12 $\frac{1}{2}$ + $\sqrt{c}$ 9. And so, infinitely, may hap the varietie. After this: Both the one and the other hath fractions incident: and so is this *Arithmetike* greately enlarged, by diuerse exhibityng and vse of Compositions and mixtynges. Consider how, I (beyng desirous to deliuer the student from error and Cauillation) do giue to this *Practise*, the name of the *Arithmetike of Radicall numbers*: Not, of *Irrationall* or *Surd Numbers*: which other while, are Rationall: though they haue the Signe of a Rote before them,

Arithmetike.

Note.

1.

2.

3.

4.

them, which, *Arithmetike* of whole Numbers moſt vſuall, would ſay they had no ſuch Roote: and ſo account them *Surd Numbers*: which generally ſpoke, is vnſ[illegible]ue: as *Euclides* tenth booke may teach you. Therfore to call them, generally, *Radicall Numbers*, (by reaſon of the ſigne √. prefixed,) is a ſure way: and a ſufficient generall diſtinction from all other ordryng and vſing of Numbers: And yet (beſide all this) Conſider: the infinite deſire of knowledge, and incredible power of mans Search and Capacitye: how, they, ioyntly haue waded farder (by mixtyng of ſpeculation and practiſe) and haue found out, and atteyned to the very chief perfection (almoſt) of *Numbers* Practicall vſe. Which thing, is well to be perceiued in that great Arithmeticall Arte of *Æquation*: commonly called the *Rule of Coſſ*. or *Algebra*. The Latines termed it, *Regulam Rei & Cenſus*, that is, the *Rule of the thyng and his value*. With an apt name: comprehendyng the firſt and laſt pointes of the worke. And the vulgar names, both in Italian, Frenche and Spaniſh, depend (in namyng it,) vpon the ſignification of the Latin word, *Res*: *A thing*: vnleaſt they vſe the name of *Algebra*. And therin (commonly) is a dubble error. The one, of them, which thinke it to be of *Geber* his inuentyng: the other of ſuch as call it *Algebra*. For, firſt, though *Geber* for his great ſkill in Numbers, Geometry, Aſtronomy, and other maruailous Artes, mought haue ſemed hable to haue firſt deuiſed the ſayd Rule: and alſo the name carryeth with it a very nere likenes of *Geber* his name: yet true it is, that a *Greke* Philoſopher and Mathematicien, named *Diophantus*, before *Geber* his tyme, wrote 13. bookes therof (of which, ſix are yet extant: and I had them to * vſe, of the famous Mathematicien, and my great frende, *Petrus Montaureis*:) And ſecondly, the very name, is *Algiebar*, and not *Algebra*: as by the Arabien *Auicen*, may be proued: who hath theſe preciſe wordes in Latine, by *Andreas Alpagis* (moſt perfect in the Arabik tung) ſo tranſlated. *Scientia faciendi Algiebar & Almachabel*. i. *Scientia inueniendi numerum ignotum, per additionem Numeri, & diuiſionem & æquationem*. Which is to ſay: *The Science of workyng Algiebar and Almachabel*, that is, the *Science of findyng an vnknowen number, by Addyng of a Number, & Diuiſion & æquation*. Here haue you the name: and alſo the principall partes of the Rule, touched. To name it, *The rule, or Art of Æquation*, doth ſignifie the middle part and the State of the Rule. This Rule, hath his peculier Characters: and the principal partes of *Arithmetike*, to it appertayning, do differre from the other *Arithmeticall operations*. This *Arithmetike, hath Nũbers* Simple, Cõpound, Mixt: and Fractions, accordingly. This Rule, and *Arithmetike* of *Algiebar*, is ſo profound, ſo generall and ſo (in maner) conteyneth the whole power of Numbers Application practicall: that mans witt, can deale with nothyng, more proffitable about numbers: nor match, with a thyng, more mete for the diuine force of the Soule, (in humane Studies, affaires, or exerciſes) to be tryed in. Perchaunce you looked for, (long ere now,) to haue had ſome particular profe, or euident teſtimony of the vſe, proffit and Commodity of Arithmetike vulgar, in the Common lyfe and trade of men. Therto, then, I will now frame my ſelfe: But herein great care I haue, leaſt length of ſundry profes, might make you deme, that either I did miſdoute your zelous mynde to vertues ſchole: or els miſtruſt your hable witts, by ſome, to geſſe much more. A profe then, foure, fiue, or ſix, ſuch, will I bryng, as any reaſonable man, therwith may be perſuaded, to loue & honor, yea learne and exerciſe the excellent Science of *Arithmetike*.

*Anno. 1550.

5.

And firſt: who, nerer at hand, can be a better witneſſe of the frute receiued by *Arithmetike*, then all kynde of Marchants? Though not all, alike, either nede it, or vſe it. How could they forbeare the vſe and helpe of the Rule, called the Golden

Rule?

Rule: Simple and Compounde: both forward and backward? How might they misse *Arithmeticall* helpe in the Rules of Felowshyp: either without tyme, or with tyme, and betwene the Marchant & his Factor? The Rules of Bartering in wares onely: or part in wares, and part in money, would they gladly want? Our Marchant venturers, and Trauaylers ouer Sea, how could they order their doynges iustly and without losse, vnleast certaine and generall Rules for Exchaũge of money, and Rechaunge, were, for their vse, deuised? The Rule of Alligation, in how sundry cases, doth it conclude for them, such precise verities, as neither by naturall witt, nor other experience, they, were hable, els, to know? And (with the Marchant then to make an end) how ample & wonderfull is the Rule of False positions? especially as it is now, by two excellent Mathematiciens (of my familer acquayntance in their life time) enlarged? I meane *Gemma Frisius*, and *Simon Iacob*. Who can either in brief conclude, the generall and Capitall Rules? or who can Imagine the Myriades of sundry Cases, and particular examples, in Act and earnest, continually wrought, tried and concluded by the forenamed Rules, onely? How sundry other *Arithmeticall practises*, are commonly in Marchantes handes, and knowledge: They them selues, can, at large, testifie.

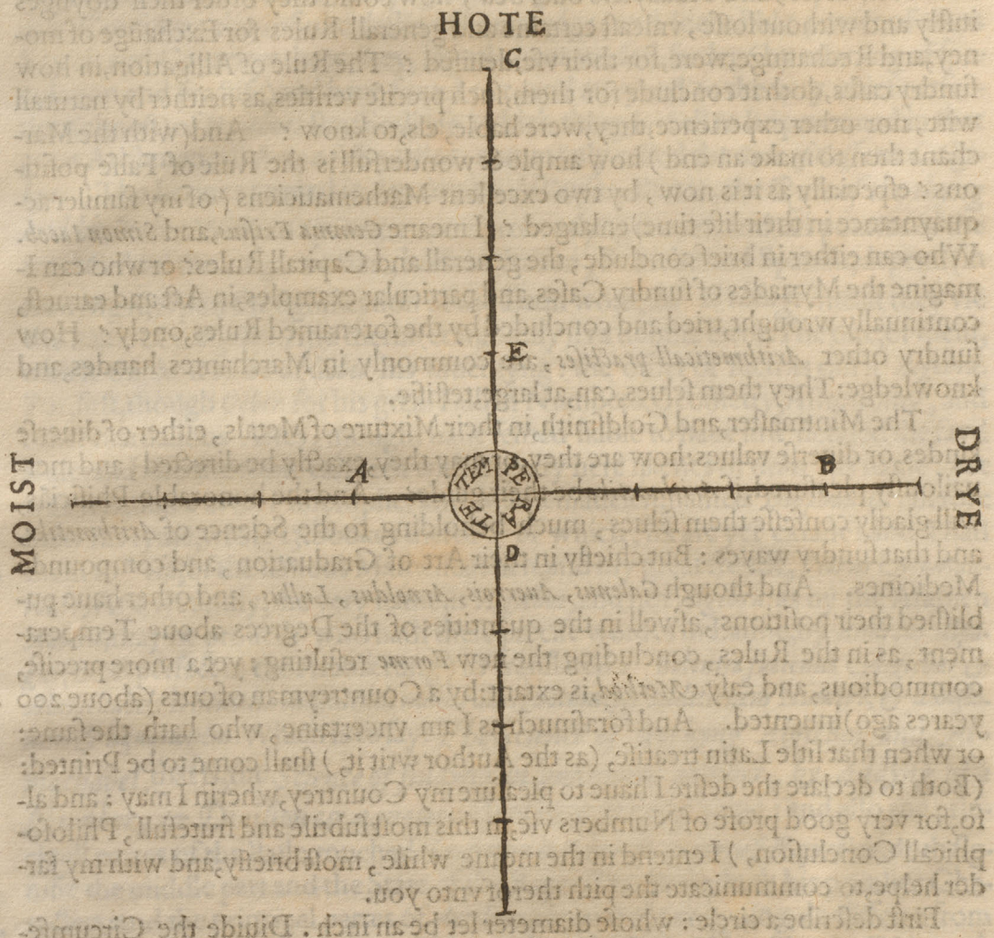
The Mintmaster, and Goldsmith, in their Mixture of Metals, either of diuerse kindes, or diuerse values: how are they, or may they, exactly be directed, and meruailously pleasured, if *Arithmetike* be their guide? And the honorable Phisiciãs, will gladly confesse them selues, much beholding to the Science of *Arithmetike*, and that sundry wayes: But chiefly in their Art of Graduation, and compounde Medicines. And though *Galenus*, *Auerrois*, *Arnoldus*, *Lullus*, and other haue publisshed their positions, aswell in the quantities of the Degrees aboue Temperament, as in the Rules, concluding the new *Forme* resulting: yet a more precise, commodious, and easy *Method*, is extant: by a Countreyman of ours (aboue 200 yeares ago) inuented. And forasmuch as I am vncertaine, who hath the same: or when that litle Latin treatise, (as the Author writ it,) shall come to be Printed: (Both to declare the desire I haue to pleasure my Countrey, wherin I may: and also, for very good profe of Numbers vse, in this most subtile and frutefull, Philosophicall Conclusion,) I entend in the meane while, most briefly, and with my farder helpe, to communicate the pith therof vnto you. *R. B.*

First describe a circle: whose diameter let be an inch. Diuide the Circumference into foure equall partes. Frõ the Center, by those 4. sections, extend 4. right lines: eche of 4. inches and a halfe long: or of as many as you liste, aboue 4. without the circumference of the circle: So that they shall be of 4. inches long (at the least) without the Circle. Make good euident markes, at euery inches end. If you list, you may subdiuide the inches againe into 10. or 12. smaller partes, equall. At the endes of the lines, write the names of the 4. principall elementall Qualities. *Hote* and *Colde*, one against the other. And likewise *Moyst* and *Dry*, one against the other. And in the Circle write *Temperate*. Which *Temperature* hath a good Latitude: as appeareth by the Complexion of man. And therefore we haue allowed vnto it, the foresayd Circle: and not a point Mathematicall or Physicall.

Now, when you haue two thinges Miscible, whose degrees are * truely knowen: Of necessitie, either they are of one Quantitie and waight, or of diuerse. If they be of one Quantitie and waight: whether their formes, be Contrary Qualities, or of one kinde (but of diuerse intentions and degrees) or a *Temperate*, and a Contrary, *The forme resulting of their Mixture, is in the Middle betwene the degrees of* the

Take some part of Lullus counsayle in his booke de Q. Essentia.

the formes mixt. As for example, let *A*, be *Moist* in the first degree: and *B*, *Dry* in the third degree. Adde 1. and 3. that maketh 4: the halfe or middle of 4. is 2.
Note. This 2. is the middle, equally distant from *A* and *B* (for the* *Temperament* is counted none. And for it, you must put a Ciphre, if at any time, it be in mixture).



Counting then from *B*, 2. degrees, toward *A*: you finde it to be *Dry* in the first degree: So is the *Forme resulting* of the Mixture of *A*, and *B*, in our example. I will geue you an other example. Suppose, you haue two thinges, as *C*, and *D*: and of *C*, the Heate to be in the 4. degree: and of *D*, the Colde, to be remisse, euen vnto the *Temperament*. Now, for *C*, you take 4: and for *D*, you take a Ciphre: which, added vnto 4, yeldeth onely 4. The middle, or halfe, whereof, is 2. Wherefore the *Forme resulting* of *C*, and *D*, is Hote in the second degree: for, 2. degrees, accounted from *C*, toward *D*, ende iuste in the 2. degree of heate. Of the third maner, I will geue also an example: which let be this: I haue a liquid Medicine whose
Note. Qualitie of heate is in the 4. degree exalted: as was *C*, in the example foregoing: and an other liquid Medicine I haue: whose Qualitie, is heate, in the first degree. Of eche of these, I mixt a like quantitie: Subtract here, the lesse frō the more: and the residue diuide into two equall partes: whereof, the one part, either added to the lesse, or subtracted from the higher degree, doth produce the degree of the

Forme

Forme reſulting, by this mixture of *C*, and *E*. As, if from 4. ye abate 1. there reſteth 3. the halfe of 3. is $1\frac{1}{2}$: Adde to 1. this $1\frac{1}{2}$: you haue $2\frac{1}{2}$. Or ſubtract from 4. this $1\frac{1}{2}$: you haue likewiſe $2\frac{1}{2}$ remayning. Which declareth, the *Forme reſulting*, to be *Heate*, in the middle of the third degree.

But if the Quantities of two thinges Commixt, be diuerſe, and the Intenſions (of their Formes Miſcible) be in diuerſe degrees, and heigthes. (Whether thoſe Formes be of one kinde, or of Contrary kindes, or of a Temperate and a Contrary, *What proportion is of the leſſe quantitie to the greater, the ſame ſhall be of the difference, which is betwene the degree of the Forme reſulting, and the degree of the greater quantitie of the thing miſcible, to the difference, which is betwene the ſame degree of the Forme reſulting, and the degree of the leſſe quantitie*. As for example. Let two pound of Liquor be geuen, hote in the 4. degree: & one pound of Liquor be geuen, hote in the third degree. I would gladly know the Forme reſulting, in the Mixture of theſe two Liquors. Set downe your nũbers in order, thus. *The Second Rule.*

℔. 2.	*Hote.* 4.
℔. 1.	*Hote.* 3.

Now by the rule of Algiebar, haue I deuiſed a very eaſie, briefe, and generall maner of working in this caſe. Let vs firſt, ſuppoſe that *Middle Forme reſulting*, to be 1ze: as that Rule teacheth. And becauſe (by our Rule, here geuen) as the waight of 1. is to 2: So is the difference betwene 4. (the degree of the greater quantitie) and 1ze: to the difference betwene 1ze and 3: (the degree of the thing, in leſſe quãtitie. And with all, 1ze, being alwayes in a certaine middell, betwene the two heigthes or degrees). For the firſt difference, I ſet $4-1\text{ze}$: and for the ſecond, I ſet $1\text{ze}-3$. And, now againe, I ſay, as 1. is to 2. ſo is $4-1\text{ze}$ to $1\text{ze}-3$. Wherfore, of theſe foure proportionall numbers, the firſt and the fourth Multiplied, one by the other, do make as much, as the ſecond and the third Multiplied the one by the other. Let theſe Multiplications be made accordingly. And of the firſt and the fourth, we haue $1\text{ze}-3$. and of the ſecond & the third, $8-2\text{ze}$. Wherfore, our Æquation is betwene $1\text{ze}-3$: and $8-2\text{ze}$. Which may be reduced, according to the Arte of Algiebar: as, here, adding 3. to eche part, geueth the Æquation, thus, $1\text{ze}=11-2\text{ze}$. And yet againe, contracting, or Reducing it: Adde to eche part, 2ze: Then haue you 3ze æquall to 11: thus repreſented $3\text{ze}=11$. Wherefore, diuiding 11. by 3: the Quotient is $3\frac{2}{3}$: the *Valew* of our 1ze, *Coſs*, or *Thing*, firſt ſuppoſed. And that is the heigth, or Intenſion of the *Forme reſulting*: which is, *Heate*, in two thirdes of the fourth degree: And here I ſet the ſhew of the worke in concluſion, thus. The proufe hereof is eaſie by ſubtracting 3. from $3\frac{2}{3}$, reſteth $\frac{2}{3}$. Subtracte the ſame heigth of the Forme reſulting, (which is $3\frac{2}{3}$) frõ 4. then reſteth $\frac{1}{3}$: You ſee, that $\frac{2}{3}$ is double to $\frac{1}{3}$: as 2. ℔. is double to 1. ℔. So ſhould it be: by the rule here geuen. Note. As you added to eche part of the Æquation, 3: ſo if ye firſt added to eche part 2ze, it would ſtand, $3\text{ze}-3=8$. And now adding to eche part 3: you haue (as afore) $3\text{ze}=11$.

℔. 2.	*Hote.* 4.
℔. 1.	*Hote.* 3.

$3\frac{2}{3}$ *The forme reſulting.*

And though I, here, ſpeake onely of two thyngs Miſcible: and moſt commonly, mo then three, foure, fiue or ſix, (&c.) are to be Mixed: (and in one Compound

to be reduced: & the Forme reſultyng of the ſame, to ſerue the turne) yet theſe Ru-
Note. les are ſufficient: duely repeated and iterated. In procedyng firſt, with any two: and then, with the Forme Reſulting, and an other: & ſo forth: For, the laſt worke, concludeth the Forme reſultyng of them all: I nede nothing to ſpeake, of the Mixture (here ſuppoſed) what it is. Common Philoſophie hath defined it, ſaying, *Mixtio eſt miſcibilium, alteratorum, per minima coniunctorum, Vnio.* Euery word in the definition, is of great importance. I nede not alſo ſpend any time, to ſhew, how, the other manner of diſtributing of degrees, doth agree to theſe Rules. Neither nede I of the farder vſe belonging to the Croſſe of Graduation (before deſcribed) in this place declare, vnto ſuch as are capable of that, which I haue all ready ſayd. Neither yet with examples ſpecifie the Manifold varieties, by the foreſayd two generall Rules, to be ordered. The witty and Studious, here, haue ſufficient: And they which are not hable to atteine to this, without liuely teaching, and more in particular: would haue larger diſcourſing, then is mete in this place to be dealt withall: And other (perchaunce) with a proude ſnuffe will diſdaine this litle: and would be vnthankefull for much more. I, therfore conclude: and wiſh ſuch as haue modeſt and earneſt Philoſophicall mindes, to laude God highly for this: and to Meruayle, that the profoundeſt and ſubtileſt point, concerning *Mixture of Formes and Qualities Naturall*, is ſo Matcht and maryed with the moſt ſimple, eaſie, and ſhort way of the noble Rule of *Algiebar*. Who can remaine, therfore vnperſuaded, to loue, allow, and honor the excellent Science of *Arithmetike*? For, here, you may perceiue that the litle finger of *Arithmetike*, is of more might and contriuing, then a hunderd thouſand mens wittes, of the middle ſorte, are hable to perfourme, or truely to conclude, with out helpe thereof.

Now will we farder, by the wiſe and valiant Capitaine, be certified, what helpe he hath, by the Rules of *Arithmetike*: in one of the Artes to him appertaining: And
Ταχλιχὴ. „ of the Grekes named Ταχλιχὴ. That is, the Skill of Ordring Souldiers in Battell ray
„ after the beſt maner to all purpoſes. This Art ſo much dependeth vppon Numbers vſe, and the Mathematicals, that *Ælianus* (the beſt writer therof,) in his worke, to the *Emperour Hadrianus*, by his perfection, in the Mathematicals, (beyng greater, then other before him had,) thinketh his booke to paſſe all other the excellent workes, written of that Art, vnto his dayes. For, of it, had written *Æneas: Cyneas* of *Theſſaly: Pyrrhus Epirota:* and *Alexander* his ſonne: *Clearchus: Pauſanias: Euangelus: Polybius*, familier frende to *Scipio: Eupolemus: Iphicrates, Poſſidonius:* and very many other worthy Capitaines, Philoſophers and Princes of Immortall fame and memory: Whoſe fayreſt floure of their garland (in this feat) was *Arithmetike*: and a litle perceiuerance, in *Geometricall* Figures. But in many other caſes doth *Arithmetike* ſtand the Capitaine in great ſtede. As in proportionyng of vittayles, for the Army, either remaining at a ſtay: or ſuddenly to be encreaſed with a certaine number of Souldiers: and for a certain tyme. Or by good Art to diminiſh his company, to make the victuals, longer to ſerue the remanent, & for a certaine determined tyme: if nede ſo require. And ſo in ſundry his other accountes, Reckeninges, Meaſurynges, and proportionynges, the wiſe, expert, and Circumſpect Capitaine will affirme the Science of *Arithmetike*, to be one of his chief Counſaylors, directers and aiders. Which thing (by good meanes) was euident to the Noble,
☞ the Couragious, the loyall, and Curteous *Iohn*, late Earle of Warwicke. Who was a yong Gentleman, throughly knowne to very few. Albeit his luſty valiantnes, force, and Skill in Chiualrous feates and exerciſes: his humblenes, and frendelynes to all men, were thinges, openly, of the world perceiued. But what rotes (otherwiſe,) vertue had faſtened in his breſt, what Rules of godly and honorable

life

life he had framed to him selfe: what vices, (in some then liuing) notable, he tooke great care to eschew: what manly vertues, in other noble men, (florishing before his eyes,) he Sythingly aspired after: what prowesses he purposed and ment to achieue: with what feats and Artes, he began to furnish and fraught him selfe, for the better seruice of his Kyng and Countrey, both in peace & warre. These (I say) his Heroicall Meditations, forecastinges and determinations, no twayne, (I thinke) beside my selfe, can so perfectly, and truely report. And therfore, in Conscience, I count it my part, for the honor, preferment, & procuring of vertue (thus, briefly) to haue put his Name, in the Register of *Fame Immortall*.

To our purpose. This *Iohn*, by one of his actes (besides many other: both in England and Fraunce, by me, in him noted.) did disclose his harty loue to vertuous Sciences: and his noble intent, to excell in Martiall prowesse: When he, with humble request, and instant Solliciting: got the best Rules (either in time past by Greke or Romaine, or in our time vsed: and new Stratagemes therin deuised) for ordring of all Companies, summes and Numbers of mē, (Many, or few) with one kinde of weapon, or mo, appointed: with Artillery, or without: on horsebacke, or on fote: to giue, or take onset: to seem many, being few: to seem few, being many. To marche in battaile or Iornay: with many such feates, to Foughten field, Skarmoush, or Ambushe appartaining: And of all these, liuely designementes (most curiously) to be in velame parchement described: with Notes & peculier markes, as the Arte requireth: and all these Rules, and descriptions Arithmeticall, inclosed in a riche Case of Gold, he vsed to weare about his necke: as his Iuell most precious, and Counsaylour most trusty. Thus, *Arithmetike*, of him, was shryned in gold: Of *Numbers* frute, he had good hope. Now, Numbers therfore innumerable, in *Numbers* prayse, his shryne shall finde.

This noble Earle, dyed Anno. 1554. skarse of 24. yeares of age: hauing no issue by his wife: Daughter to the Duke of Somerset.

What nede I, (for farder profe to you) of the Scholemasters of Iustice, to require testimony: how nedefull, how frutefull, how skillfull a thing *Arithmetike* is? I meane, the Lawyers of all sortes. Vndoubtedly, the Ciuilians, can meruaylously declare: how, neither the Auncient Romaine lawes, without good knowledge of *Numbers art*, can be perceiued: Nor (Iustice in infinite Cases) without due proportion, (narrowly considered,) is hable to be executed. How Iustly, & with great knowsledge of Arte, did *Papinianus* institute a law of partition, and allowance, betwene man and wife after a diuorce? But how *Accursius*, *Baldus*, *Bartolus*, *Iason*, *Alexander*, and finally *Alciatus*, (being otherwise, notably well learned) do iumble, gesse, and erre, from the æquity, art and Intent of the lawmaker: *Arithmetike* can detect, and conuince: and clerely, make the truth to shine. Good *Bartolus*, tyred in the examining & proportioning of the matter: and with *Accursius* Glosse, much cumbred: burst out, and sayd: *Nulla est in toto libro, hac glossa difficilior: Cuius computationem nec Scholastici nec Doctores intelligunt. &c.* That is: *In the whole booke, there is no Glosse harder then this: Whose accoumpt or reckenyng, neither the Scholers, nor the Doctours vnderstand. &c.* What can they say of *Iulianus* law, *Si ita Scriptum. &c.* Of the Testators will iustly performing, betwene the wife, Sonne and daughter? How can they perceiue the æquitie of *Aphricanus*, *Arithmeticall* Reckening, where he treateth of *Lex Falcidia*? How can they deliuer him, from his Reprouers: and their maintainers: as *Ioannes*, *Accursius Hypolitus* and *Alciatus*? How Iustly and artificially, was *Africanus* reckening made? Proportionating to the Sommes bequeathed, the Contributions of eche part? Namely, for the hundred presently receiued, 17 $\frac{1}{7}$. And for the hundred, receiued after ten monethes, 12 $\frac{6}{7}$: which make the 30: which were to be cōtributed by the legataries to the heire.

For, what proportion, 100 hath to 75: the ſame hath 17 1/2 to 12 1/2: Which is Seſquitertia: that is, as 4, to 3. which make 7. Wonderfull many places, in the Ciuile law, require an expert *Arithmeticien*, to vnderſtand the deepe Iudgemēt, & Iuſt determinatiō of the Auncient Romaine Lawmakers. But much more expert ought he to be, who ſhould be hable, to decide with æquitie, the infinite varietie of Caſes, which do, or may happen, vnder euery one of thoſe lawes and ordinances Ciuile. Hereby, eaſely, ye may now coniecture: that in the Canon law: and in the lawes of the Realme (which with vs, beare the chief Authoritie), Iuſtice and equity might be greately preferred, and ſkilfully executed, through due ſkill of Arithmetike, and proportions appertainyng. The worthy Philoſophers, and prudent lawmakers (who haue written many bookes *De Republica*: How the beſt ſtate of Common wealthes might be procured and mainteined,) haue very well determined of Iuſtice: (which, not onely, is the Baſe and foundacion of Common weales: but alſo the totall perfection of all our workes, words, and thoughtes:) de-

Iuſtice. „ fining it, to be that vertue, by which, to euery one, is rendred, that to him appertai-

„ neth. God challengeth this at our handes, to be honored as God: to be loued, as a father: to be feared as a Lord & maſter. Our neighbours proportiō, is alſo preſcribed of the Almighty lawmaker: which is, to do to other, euen as we would be done vnto. Theſe proportions, are in Iuſtice neceſſary: in duety, commendable: and of Common wealthes, the life, ſtrength, ſtay and floriſhing. *Ariſtotle* in his *Ethikes* (to fatch the ſede of Iuſtice, and light of direction, to vſe and execute the ſame) was fayne to fly to the perfection, and power of Numbers: for proportions Arithmeticall and Geometricall. *Plato* in his booke called *Epinomis* (which boke, is the Threaſury of all his doctrine) where, his purpoſe is, to ſeke a Science, which, when a man had it, perfectly: he might ſeme, and ſo be, in dede, *Wiſe*. He, briefly, of other Sciences diſcourſing, findeth them, not hable to bring it to paſſe: But of the Science of Numbers, he ſayth. *Illa, quæ numerum mortalium generi dedit, id profectò efficiet. Deum autem aliquem, magis quàm fortunam, ad ſalutem noſtram, hoc munus nobis arbitror contuliſſe. &c. Nam ipſum bonorum omnium Authorem, cur non maximi boni, Prudentiæ dico, cauſam arbitramur? That Science, verely, which hath taught mankynde number, ſhall be able to bryng it to paſſe. And, I thinke, a certaine God, rather then fortune, to haue giuen vs this gift, for our bliſſe. For, why ſhould we not Iudge him, who is the Author of all good things, to be alſo the cauſe of the greateſt good thyng, namely, Wiſedome?* There, at length, he proueth *Wiſedome* to be atteyned, by good Skill of *Numbers*. With which great Teſtimony, and the manifold proſes, and reaſons, before expreſſed, you may be ſufficiently and fully perſuaded: of the perfect Science of *Arithmetike*, to make this accounte: That of all Sciences, next to *Theologie*, it is moſt diuine, moſt pure, moſt ample and generall, moſt profounde, moſt ſubtile, moſt commodious and moſt neceſſary. Whoſe next Siſter, is the Abſolute Science of *Magnitudes*: of which (by the Direction and aide of him, whoſe *Magnitude* is Infinite, and of vs Incomprehenſible) I now entend, ſo to write, that both with the *Multitude*, and alſo with the *Magnitude* of Meruaylous and frutefull verities, you (my frendes and Countreymen) may be ſtird vp, and awaked, to behold what certaine Artes and Sciences, (to our vnſpeakable behofe) our heauenly father, hath for vs prepared, and reuealed, by ſundry *Philoſophers* and *Mathematiciens*.

BOth, *Number* and *Magnitude*, haue a certaine Originall ſede, (as it were) of an incredible property: and of man, neuer hable, Fully, to be declared. Of *Number*, an Vnit, and of *Magnitude*, a Poynte, doo ſeeme to be much like Originall

nall cauſes: But the diuerſitie neuertheleſſe, is great. We defined an *Vnit*, to be a thing Mathematicall Indiuiſible: A Point, likewiſe, we ſayd to be a Mathematicall thing Indiuiſible. And farder, that a Point may haue a certaine determined Situation: that is, that we may aſſigne, and preſcribe a Point, to be here, there, yonder. &c. Herein, (behold) our Vnit is free, and can abyde no bondage, or to be tyed to any place, or ſeat: diuiſible or indiuiſible. Agayne, by reaſon, a Point may haue a Situation limited to him: a certaine motion, therfore (to a place, and from a place) is to a Point incident and appertainyng. But an *Vnit*, can not be imagined to haue any motion. A Point, by his motion, produceth, Mathematically, a line: (as we ſayd before) which is the firſt kinde of Magnitudes, and moſt ſimple: An *Vnit*, can not produce any number. A Line, though it be produced of a Point moued, yet, it doth not conſiſt of pointes: Number, though it be not produced of an *Vnit*, yet doth it Conſiſt of vnits, as a materiall cauſe. But formally, Number, is the Vnion, and Vnitie of Vnits. Which vnyting and knitting, is the workemanſhip of our minde: which, of diſtinct and diſcrete Vnits, maketh a Number: by vniformitie, reſulting of a certaine multitude of Vnits. And ſo, euery number, may haue his leaſt part, giuen: namely, an Vnit: But not of a Magnitude, (no, not of a Lyne,) the leaſt part can be giuē: by cauſe, infinitly, diuiſion therof, may be conceiued. All Magnitude, is either a Line, a Plaine, or a Solid. Which Line, Plaine, or Solid, of no Senſe, can be perceiued, nor exactly by hād (any way) repreſented: nor of Nature produced: But, as (by degrees) Number did come to our perceiuerance: So, by viſible formes, we are holpen to imagine, what our Line Mathematicall, is. What our Point, is. So preciſe, are our Magnitudes, that one Line is no broader then an other: for they haue no bredth: Nor our Plaines haue any thicknes. Nor yet our Bodies, any weight: be they neuer ſo large of dimenſiō. Our Bodyes, we can haue Smaller, then either Arte or Nature can produce any: and Greater alſo, then all the world can comprehend. Our leaſt Magnitudes, can be diuided into ſo many partes, as the greateſt. As, a Line of an inch long, (with vs) may be diuided into as many partes, as may the diameter of the whole world, from Eaſt to Weſt: or any way extended: What priuiledges, aboue all manual Arte, and Natures might, haue our two Sciences Mathematicall? to exhibite, and to deale with thinges of ſuch power, liberty, ſimplicity, puritie, and perfection? And in them, ſo certainly, ſo orderly, ſo preciſely to procede: as, excellent is that workemā Mechanicall Iudged, who nereſt can approche to the repreſenting of workes, Mathematically demonſtrated? And our two Sciences, remaining pure, and abſolute, in their proper termes, and in their owne Matter: to haue, and allowe, onely ſuch Demonſtrations, as are plaine, certaine, vniuerſall, and of an æternall veritye? This Science of *Magnitude*, his properties, conditions, and appertenances: commonly, now is, and from the beginnyng, hath of all Philoſophers, ben called *Geometrie*. But, veryly, with a name to baſe and ſcant, for a Science of ſuch dignitie and amplenes. And, perchaunce, that name, by cōmon and ſecret conſent, of all wiſemen, hitherto hath ben ſuffred to remayne: that it might carry with it a perpetuall memorye, of the firſt and notableſt benefite, by that Science, to common people ſhewed: Which was, when Boundes and meres of land and ground were loſt, and confounded (as in *Egypt*, yearely, with the ouerflowyng of *Nilus*, the greateſt and longeſt riuer in the world) or, that ground bequeathed, were to be aſſigned: or, ground ſold, were to be layd out: or (when diſorder preuailed) that Commōs were diſtributed into ſeueralties. For, where, vpon theſe & ſuch like occaſiōs, Some by ignorāce, ſome by negligēce, Some by fraude, and ſome by violence, did wrongfully limite, meaſure, encroach, or challenge (by

Number.

Geometrie.

pretence of iuſt content, and meaſure) thoſe landes and groundes: great loſſe, diſquietnes, murder, and warre did (full oft) enſue: Till, by Gods mercy, and mans Induſtrie, The perfect Science of Lines, Plaines, and Solides (like a diuine Iuſticier,) gaue vnto euery man, his owne. The people then, by this art pleaſured, and greatly relieued, in their landes iuſt meaſuring: & other Philoſophers, writing Rules for land meaſuring: betwene them both, thus, confirmed the name of *Geometria*, that is, (according to the very etimologie of the word) Land meaſuring. Wherin, the people knew no farder, of Magnitudes vſe, but in Plaines: and the Philoſophers, of thē, had no feet hearers, or Scholers: farder to diſcloſe vnto, then of flat, plaine *Geometrie*. And though, theſe Philoſophers, knew of farder vſe, and beſt vnderſtode the etymologye of the worde, yet this name *Geometria*, was of them applyed generally to all ſortes of Magnitudes: vnleaſt, otherwhile, of *Plato*, and *Pythagoras*: When they would preciſely declare their owne doctrine. Then, was * *Geometria*, with them, *Studium quod circa planum verſatur*. But, well you may perceiue by *Euclides Elementes*, that more ample is our Science, then to meaſure Plaines: and nothyng leſſe therin is tought (of purpoſe) then how to meaſure Land. An other name, therfore, muſt nedes be had, for our Mathematicall Science of Magnitudes: which regardeth neither clod, nor turff: neither hill, nor dale: neither earth nor heauen: but is abſolute *Megethologia*: not creping on ground, and daſſeling the eye, with pole perche, rod or lyne: but liftyng the hart aboue the heauens, by inuiſible lines, and immortall beames: meteth with the reflexions, of the light incomprehenſible: and ſo procureth Ioye, and perfection vnſpeakable. Of which true vſe of our *Megethica*, or *Megethologia*, *Diuine Plato* ſeemed to haue good taſte, and iudgement: and (by the name of *Geometrie*) ſo noted it: and warned his Scholers therof: as, in hys ſeuenth *Dialog*, of the Common wealth, may euidently be ſene. Where (in Latin) thus it is: right well tranſlated: *Profecto, nobis hoc non negabunt, Quicunq; vel paululum quid Geometriæ guſtârunt, quin hæc Scientia, contrà, omnino ſe habeat, quàm de ea loquuntur, qui in ipſa verſantur*. In Engliſh, thus. *Verely* (ſayth *Plato*) *whoſoeuer haue, (but euen very litle) taſted of Geometrie, will not denye vnto vs, this: but that this Science, is of an other condicion, quite contrary to that, which they that are exerciſed in it, do ſpeake of it.* And there it followeth, of our *Geometrie*, *Quòd quæritur cognoſcendi illius gratia, quod ſemper eſt, non & eius quod oritur quandoq; & interit. Geometria, eius quod eſt ſemper, Cognitio eſt. Attollet igitur (ô Generoſe vir) ad Veritatem, animum: atq; ita, ad Philoſophandum preparabit cogitationem, vt ad ſupera conuertamus: quæ, nunc, contra quàm decet, ad inferiora deïcimus. &c. Quàm maximè igitur præcipiendum eſt, vt qui præclariſsimam hanc habitāt Ciuitatem, nullo modo, Geometriam ſpernant. Nam & quæ præter ipſius propoſitum, quodam modo eſſe videntur, haud exigua ſunt. &c.* It muſt nedes be confeſſed (ſaith *Plato*) *That* [*Geometrie*] *is learned, for the knowyng of that, which is euer: and not of that, which, in tyme, both is bred and is brought to an ende. &c. Geometrie is the knowledge of that which is euerlaſtyng. It will lift vp therfore (O Gentle Syr) our mynde to the Veritie: and by that meanes, it will prepare the Thought, to the Philoſophicall loue of wiſdome: that we may turne or conuert, toward heauenly thinges* [*both mynde and thought*] *which now, otherwiſe then becommeth vs, we caſt down on baſe or inferior things. &c. Chiefly, therfore, Commaundement muſt be giuen, that ſuch as do inhabit this moſt honorable Citie, by no meanes, deſpiſe Geometrie. For euen thoſe thinges* [*done by it*] *which, in manner, ſeame to be, beſide the purpoſe of Geometrie: are of*

*Plato. 7. de Rep.

no

no ſmall importance. &c. And beſides the manifold vſes of *Geometrie*, in matters appertainyng to warre, he addeth more, of ſecond vnpurpoſed frute, and commoditye, arriſing by *Geometrie*: ſaying: *Scimus quin etiam, ad Diſciplinas omnes facilius per diſcendas, intereſſe omnino, attigerit ne Geometriam aliquis, an non. &c. Hanc ergo Doctrinam, ſecundo loco diſcendam Iuuenibus ſtatuamus.* That is. *But, alſo, we know, that for the more eaſy learnyng of all Artes, it importeth much, whether one haue any knowledge in Geometrie, or no. &c. Let vs therfore make an ordinance or decree, that this Science, of young men ſhall be learned in the ſecond place.* This was *Diuine Plato* his Iudgement, both of the purpoſed, chief, and perfect vſe of *Geometrie*: and of his ſecond, dependyng, deriuatiue commodities. And for vs, Chriſten men, a thouſand thouſand mo occaſions are, to haue nede of the helpe of* *Megethologicall* Contemplations: wherby, to trayne our Imaginations and Myndes, by litle and litle, to forſake and abandon, the groſſe and corruptible Obiectes, of our vtward ſenſes: and to apprehend, by ſure doctrine demonſtratiue, Things Mathematicall. And by them, readily to be holpen and conducted to conceiue, diſcourſe, and conclude of things Intellectual, Spirituall, æternall, and ſuch as concerne our Bliſſe euerlaſting: which, otherwiſe (without Speciall priuiledge of Illumination, or Reuelation frō heauen) No mortall mans wyt (naturally) is hable to reach vnto, or to Compaſſe. And, veryly, by my ſmall Talent (from aboue) I am hable to proue and teſtifie, that the litterall Text, and order of our diuine Law, Oracles, and Myſteries, require more ſkill in Numbers, and Magnitudes: then (commonly) the expoſitors haue vttered: but rather onely (at the moſt) ſo warned: & ſhewed their own want therin. (To name any, is nedeles: and to note the places, is, here, no place: But if I be duely aſked, my anſwere is ready.) And without the litterall, Grammaticall, Mathematicall or Naturall verities of ſuch places, by good and certaine Arte, perceiued, no Spirituall ſenſe (propre to thoſe places, by Abſolute *Theologie*) will thereon depend. No man, therfore, can doute, but toward the atteyning of knowledge incomparable, and Heauenly Wiſedome: Mathematicall Speculations, both of Numbers and Magnitudes: are meanes, aydes, and guides: ready, certaine, and neceſſary. From henceforth, in this my Preface, will I frame my talke, to *Plato* his fugitiue Scholers: or, rather, to ſuch, who well can, (and alſo wil,) vſe their vtward ſenſes, to the glory of God, the benefite of their Countrey, and their owne ſecret contentation, or honeſt preferment, on this earthly Scaffold. To them, I will orderly recite, deſcribe & declare a great Number of Artes, from our two Mathematicall fountaines, deriued into the fieldes of *Nature*. Wherby, ſuch Sedes, and Rotes, as lye depe hyd in the groūd of *Nature*, are refreſhed, quickened, and prouoked to grow, ſhote vp, floure, and giue frute, infinite, and incredible. And theſe Artes, ſhalbe ſuch, as vpon Magnitudes properties do depende, more, then vpon Number. And by good reaſon we may call them Artes, and Artes Mathematicall Deriuatiue: for (at this tyme) I Define An Arte, to be a Methodicall cōplete Doctrine, hauing abundancy of ſufficient, and peculier matter to deale with, by the allowance of the Metaphiſicall Philoſopher: the knowledge whereof, to humaine ſtate is neceſſarye. And that I account, An Art Mathematicall deriuatiue, which by Mathematicall demonſtratiue Method, in Nūbers, or Magnitudes, ordreth and confirmeth his doctrine, as much & as perfectly, as the matter ſubiect will admit. And for that,

I. D.

* *Herein, I would gladly ſhake of, the earthly name, of Geometrie.*

„ „ „ „

An Arte.

Art Mathematicall Deriuatiue.

A Mechanitien. I entend to vſe the name and propertie of a *Mechanicien*, otherwiſe, then (hitherto) it hath ben vſed, I thinke it good, (for diſtinction ſake) to giue you alſo a briefe deſcription, what I meane therby. A Mechanicien, or a Mechanicall workman is he, whoſe ſkill is, without knowledge of Mathematicall demonſtration, perfectly to worke and finiſhe any ſenſible worke, by the Mathematicien principall or deriuatiue, demonſtrated or demonſtrable. Full well I know, that he which inuenteth, or maketh theſe demonſtrations, is generally called *A ſpeculatiue Mechanicien*: which differreth nothyng from a *Mechanicall Mathematicien*. So, in reſpect of diuerſe actions, one man may haue the name of ſundry artes: as, ſome tyme, of a Logicien, ſome tymes (in the ſame matter otherwiſe handled) of a Rethoricien. Of theſe trifles, I make, (as now, in reſpect of my Preface,) ſmall account: to fyle thē for the fine handlyng of ſubtile curious diſputers. In other places, they may commaunde me, to giue good reaſon: and yet, here, I will not be vnreaſonable.

Firſt, then, from the puritie, abſolutenes, and Immaterialitie of Principall *Geometrie*, is that kinde of *Geometrie* deriued, which vulgarly is counted *Geometrie*: and is the Arte of Meaſuring ſenſible magnitudes, their iuſt quātities and contentes. This, teacheth to meaſure, either at hand: and the practiſer, to be by the thing Meaſured: and ſo, by due applying of Cumpaſe, Rule, Squire, Yarde, Ell, Perch, Pole, Line, Gaging rod, (or ſuch like inſtrument) to the Length, Plaine, or Solide meaſured, *to be certified, either of the length, perimetry, or diſtance lineall: and this is called, *Mecometrie*. Or *to be certified of the content of any plaine Superficies: whether it be in ground Surueyed, Borde, or Glaſſe meaſured, or ſuch like thing: which meaſuring, is named *Embadometrie*. *Or els to vnderſtand the Soliditie, and content of any bodily thing: as of Tymber and Stone, or the content of Pits, Pondes, Wells, Veſſels, ſmall & great, of all faſhions. Where, of Wine, Oyle, Beere, or Ale veſſells, &c, the Meaſuring, commonly, hath a peculier name: and is called *Gaging*. And the generall name of theſe Solide meaſures, is *Stereometrie*. Or els, this *vulgar Geometrie*, hath conſideration to teach the practiſer, how to meaſure things, with good diſtance betwene him and the thing meaſured: and to vnderſtand thereby, either *how Farre, a thing ſeene (on land or water) is from the meaſurer: and this may be called *Apomecometrie*: Or, how High or depe, aboue or vnder the leuel of the meaſurers ſtāding, any thing is, which is ſene on land or water, called *Hypſometrie*. *Or, it informeth the meaſurer, how Broad any thing is, which is in the meaſurers vew: ſo it be on Land or Water, ſituated: and may be called *Platometrie*. Though I vſe here to condition, the thing meaſured, to be on Land, or Water Situated: yet, know for certaine, that the ſundry heigthe of Cloudes, blaſing Starres, and of the Mone, may (by theſe meanes) haue their diſtances from the earth: and, of the blaſing Starres and Mone, the Soliditie (aſwell as diſtances) to be meaſured: But becauſe, neither theſe things are vulgarly taught: nor of a common practiſer ſo ready to be executed: I, rather, let ſuch meaſures be reckened incident to ſome of our other Artes, dealing with thinges on high, more purpoſely, then this vulgar Land meaſuring Geometrie doth: as in *Perſpectiue* and *Aſtronomie, &c*.

1. *Geometrie vulgar.*
1. 2. 3. 2. 1. 2. 3.
Note.

OF theſe Feates (farther applied) is Sprong the Feate of *Geodeſie*, or Land Meaſuring: more cunningly to meaſure & Suruey Land, Woods, and Waters, a farre of. More cunningly, I ſay: But God knoweth (hitherto) in theſe Realmes of England and Ireland (whether through ignorance or fraude, I can not tell, in euery particular) how great wrong and iniurie hath (in my time) bene committed by

Note.

by vntrue measuring and surueying of Land or Woods, any way: And, this I am sure, that the Value of the difference, betwene the truth and such Surueyes, would haue bene hable to haue found (for euer) in eche of our two Vniuersities, an excellent Mathematicall Reader: to eche, allowing (yearly) a hundred Markes of lawfull money of this realme: which, in dede, would seme requisit, here, to be had (though by other wayes prouided for) as well, as, the famous Vniuersitie of Paris, hath two Mathematicall Readers: and eche, two hundreth French Crownes yearly, of the French Kinges magnificent liberalitie onely. Now, againe, to our purpose returning: Moreouer, of the former knowledge Geometricall, are growen the Skills of *Geographie*, *Chorographie*, *Hydrographie*, and *Stratarithmetrie*.

Geographie teacheth wayes, by which, in sundry formes, (as *Spherike*, *Plaine* or other), the Situation of Cities, Townes, Villages, Fortes, Castells, Mountaines, Woods, Hauens, Riuers, Crekes, & such other things, vpō the outface of the earthly Globe (either in the whole, or in some principall mēber and portion therof cō-tayned) may be described and designed, in cōmensurations Analogicall to Nature and veritie: and most aptly to our vew, may be represented. Of this Arte how great pleasure, and how manifolde commodities do come vnto vs, daily and hourely: of most men, is perceaued. While, some, to beautifie their Halls, Parlers, Chambers, Galeries, Studies, or Libraries with: other some, for thinges past, as battels fought, earthquakes, heauenly fyringes, & such occurentes, in histories mentioned: therby liuely, as it were, to vewe the place, the region adioyning, the distance from vs: and such other circumstances. Some other, presently to vewe the large dominion of the Turke: the wide Empire of the Moschouite: and the litle morsell of ground, where Christendome (by profession) is certainly knowen. Litle, I say, in respecte of the rest. &c. Some, either for their owne iorneyes directing into farre landes: or to vnderstand of other mens trauailes. To conclude, some, for one purpose: and some, for an other, liketh, loueth, getteth, and vseth, Mappes, Chartes, & Geographicall Globes. Of whose vse, to speake sufficiently, would require a booke peculier.

Chorographie seemeth to be an vnderling, and a twig, of *Geographie*: and yet neuertheless, is in practise manifolde, and in vse very ample. This teacheth Analogically to describe a small portion or circuite of ground, with the contentes: not regarding what commensuration it hath to the whole, or any parcell, without it, contained. But in the territory or parcell of ground which it taketh in hand to make description of, it leaueth out (or vndescribed) no notable, or odde thing, aboue the ground visible. Yea and sometimes, of thinges vnder ground, geueth some peculier marke: or warning: as of Mettall mines, Cole pittes, Stone quarries, &c. Thus, a Dukedome, a Shiere, a Lordship, or lesse, may be described distinctly. But marueilous pleasant, and profitable it is, in the exhibiting to our eye, and commensuration, the plat of a Citie, Towne, Forte, or Pallace, in true Symmetry: not approching to any of them: and out of Gunne shot. &c. Hereby, the *Architect* may furnishe him selfe, with store of what patterns he liketh: to his great instruction: euen in those thinges which outwardly are proportioned: either simply in them selues: or respectiuely, to Hilles, Riuers, Hauens, and Woods adioyning. Some also, terme this particular description of places, *Topographie*.

Hydrographie, deliuereth to our knowledge, on Globe or in Plaine, the perfect Analogicall description of the Ocean Sea coastes, through the whole world: or in the chiefe and principall partes thercof: with the Iles and chiefe

particular places of daungers, conteyned within the boundes, and Sea coastes described: as, of Quicksandes, Bankes, Pittes, Rockes, Races, Countertides, Whorlepooles. &c. This, dealeth with the Element of the water chiefly: as *Geographie* did principally take the Element of the Earthes description (with his appertenances) to taske. And besides thys, *Hydrographie*, requireth a particular Regiſter of certaine Landmarkes (where markes may be had) from the sea, well hable to be skried, in what point of the Seacumpase they appeare, and what apparent forme, Situation, and bignes they haue, in respecte of any daungerous place in the sea, or nere vnto it, assigned: And in all Coastes, what Mone, maketh full Sea: and what way, the Tides and Ebbes, come and go, the *Hydrographer* ought to recorde. The Soundinges likewise: and the Chanels wayes: their number, and depthes ordinarily, at ebbe and flud, ought the *Hydrographer*, by obseruation and diligence of *Measuring*, to haue certainly knowen. And many other pointes, are belonging to perfecte *Hydrographie*, and for to make a *Rutter*, by: of which, I nede not here speake: as of the describing, in any place, vpon Globe or Plaine, the 32. pointes of the Compase, truely: (wherof, scarsly foure, in England, haue right knowledge: bycause, the lines therof, are no straight lines, nor Circles.) Of making due proiection of a Sphere in plaine. Of the Variacion of the Compas, from true Northe: And such like matters (of great importance, all) I leaue to speake of, in this place: bycause, I may seame (al ready) to haue enlarged the boundes, and duety of an Hydographer, much more, then any man (to this day) hath noted, or prescribed. Yet am I well hable to proue, all these thinges, to appertaine, and also to be proper to the Hydrographer. The chief vse and ende of this Art, is the Art of Nauigation: but it hath other diuerse vses: euen by them to be enioyed, that neuer lacke sight of land.

Stratarithmetrie, is the Skill, (appertainyng to the warre,) by which a man can set in figure, analogicall to any *Geometricall* figure appointed, any certaine number or summe of men: of such a figure capable: (by reason of the vsuall spaces betwene Souldiers allowed: and for that, of men, can be made no Fractions. Yet, neuertheles, he can order the giuen summe of men, for the greatest such figure, that of them, cã be ordred) and certifie, of the ouerplus: (if any be) and of the next certaine summe, which, with the ouerplus, will admit a figure exactly proportionall to the figure assigned. By which Skill, also, of any army or company of men: (the figure & sides of whose orderly standing, or array, is knowen) he is able to expresse the iust number of men, within that figure conteined: or (orderly) able to be conteined. * And this figure, and sides therof, he is hable to know: either beyng by, and at hand: or a farre of. Thus farre, stretcheth the description and property of *Stratarithmetrie*: sufficient for this tyme and place. It differreth from the Feate *Tacticall, De aciebus instruendis*. bycause, there, is necessary the wisedome and foresight, to what purpose he so ordreth the men: and Skillfull hability, also, for any occasion, or purpose, to deuise and vse the aptest and most necessary order, array and figure of his Company and Summe of men. By figure, I meane: as, either of a *Perfect Square, Triangle, Circle, Ouale, long square*, (of the Grekes it is called *Eteromekes*) *Rhombe, Rhomboid, Lunular, Ryng, Serpentine*, and such other Geometricall figures: Which, in warres, haue ben, and are to be vsed: for commodiousnes, necessity, and auauntage &c. And no small skill ought he to haue, that should make true report, or nere the truth, of the numbers and Summes, of footemen or horsemen, in the Enemyes ordring. A farre of, to make an estimate, betwene nere termes of More and Lesse, is not a thyng very rife, among those that gladly would

Note.

The difference betwene Stratarithmetrie and Tacticie.

do

do it. Great pollicy may be vſed of the Capitaines, (at tymes fete, and in places conuenient) as to vſe Figures, which make greateſt ſhew, of ſo many as he hath: and vſing the aduauntage of the three kindes of vſuall ſpaces: (betwene footemen or horſemen) to take the largeſt: or when he would ſeme to haue few, (beyng many:) contrarywiſe, in Figure, and ſpace. The Herald, Purſeuant, Sergeant Royall, Capitaine, or who ſoeuer is carefull to come nere the truth herein, beſides the Iudgement of his expert eye, his ſkill of Ordering *Tacticall*, the helpe of his Geometricall inſtrument: Ring, or Staffe Aſtronomicall: (commodiouſly framed for cariage and vſe) He may wonderfully helpe him ſelfe, by perſpectiue Glaſſes. In which, (I truſt) our poſterity will proue more ſkillfull and expert, and to greater purpoſes, then in theſe dayes, can (almoſt) be credited to be poſſible.

I.D. Frende, you will finde it hard, to performe my deſcription of this Feate. But by Chorographie, you may helpe your ſelfe ſome what: where the Figures knowne (in Sides and Angles) are not Regular: And where, Reſolution into Triangles can ſerue, &c. And yet you will finde it ſtrange to deale thus generally with Arithmeticall figures: and, that for Batayle ray. Their contentes, differ ſo much from like Geometricall Figures.

Thus haue I lightly paſſed ouer the Artificiall Feates, chiefly dependyng vpon vulgar *Geometrie*: & commonly and generally reckened vnder the name of *Geometrie*. But there are other (very many) *Methodicall Artes*, which, declyning from the purity, ſimplicitie, and Immateriality, of our Principall Science of *Magnitudes*: do yet neuertheles vſe the great ayde, direction, and Method of the ſayd principall Science, and haue propre names, and diſtinct: both from the Science of *Geometrie*, (from which they are deriued) and one from the other. As Perſpectiue, Aſtronomie, Muſike, Coſmographie, Aſtrologie, Statike, Anthropographie, Trochilike, Helicoſophie, Pneumatithmie, Menadrie, Hypogeiodie, Hydragogie, Horometrie, Zographie, Architecture, Nauigation, Thaumaturgike and Archemaſtrie. I thinke it neceſſary, orderly, of theſe to giue ſome peculier deſcriptions: and withall, to touch ſome of their commodious vſes, and ſo to make this Preface, to be a little ſwete, pleaſant Noſegaye for you: to comfort your Spirites, beyng almoſt out of courage, and in deſpayre, (through brutiſh brute) Weenyng that *Geometrie*, had but ſerued for buildyng of an houſe, or a curious bridge, or the roufe of Weſtminſter hall, or ſome witty pretty deuiſe, or engyn, appropriate to a Carpenter, or a Ioyner &c. That the thing is farre otherwiſe, then the world, (commonly) to this day, hath demed, by worde and worke, good profe wilbe made.

Among theſe Artes, by good reaſon, Perſpectiue ought to be had, ere of *Aſtronomicall Apparences*, perfect knowledge can be atteyned. And bycauſe of the prerogatiue of *Light*, beyng the firſt of *Gods Creatures*: and the eye, the light of our body, and his Senſe moſt mighty, and his organ moſt Artificiall and *Geometricall*: At *Perſpectiue*, we will begyn therfore. Perſpectiue, is an Art Mathematicall, which demonſtrateth the maner, and properties, of all Radiations Direct, Broken, and Reflected. This Deſcription, or Notation, is brief: but it reacheth ſo farre, as the world is wyde. It concerneth all Creatures, all Actions, and paſſions, by Emanation of beames perfourmed. Beames, or naturall lines, (here) I meane, not of light onely, or of colour (though they, to eye, giue ſhew, witnes, and profe, wherby to ground the Arte vpon) but alſo of other *Formes*, both *Subſtantiall*, and *Accidentall*, the certaine and determined actiue Radiall emanations. By this Art (omitting to ſpeake of the higheſt pointes) we may vſe our eyes, and the light, with greater pleaſure: and perfecter Iudgement: both of things, in light ſeen, & of other: which by like order of Lightes Radiations, worke and produce their effectes. We may be aſhamed to be ignorant of the cauſe, why ſo ſundry wayes our eye is deceiued, and abuſed: as, while the eye weeneth a roūd Globe or Sphere (beyng farre of) to be a flat and plaine Circle, and ſo likewiſe iud-

geth a plaine Square, to be roũd: ſuppoſeth walles parallels, to approche, a farre of: roſe and floure parallels, the one to bend downward, the other to riſe vpward, at a little diſtance from you. Againe, of thinges being in like ſwiftnes of mouing, to thinke the nerer, to moue faſter: and the farder, much ſlower. Nay, of two thinges, wherof the one (incomparably) doth moue ſwifter then the other, to deme the ſlower to moue very ſwift, & the other to ſtand: what an error is this, of our eye? Of the Raynbow, both of his Colours, of the order of the colours, of the bignes of it, the place and heith of it, (&c) to know the cauſes demonſtratiue, is it not pleaſant, is it not neceſſary? of two or three Sonnes appearing: of Blaſing Sterres: and ſuch like thinges: by naturall cauſes, brought to paſſe, (and yet neuertheles, of farder matter, Significatiue) is it not commodious for man to know the very true cauſe, & occaſion Naturall? Yea, rather, is it not, greatly, againſt the Souerainty of Mans nature, to be ſo ouerſhot and abuſed, with thinges (at hand) before his eyes? as with a Pecockes tayle, and a Doues necke: or a whole ore, in water, holden, to ſeme broken. Thynges, farre of, to ſeeme nere: and nere, to ſeme farre of. Small thinges, to ſeme great: and great, to ſeme ſmall. One man, to ſeme an Army. Or a man to be curſtly affrayed of his owne ſhaddow. Yea, ſo much, to feare, that, if you, being (alone) nere a certaine glaſſe, and proffer, with dagger or ſword, to foyne at the glaſſe, you ſhall ſuddenly be moued to giue backe (in maner) by reaſon of an Image, appearing in the ayre, betwene you & the glaſſe, with like hand, ſword or dagger, & with like quicknes, foyning at your very eye, likewiſe as you do at the Glaſſe. Straunge, this is, to heare of: but more meruailous to behold, then theſe my wordes can ſignifie. And neuertheleſſe by demonſtration Opticall, the order and cauſe therof, is certified: euen ſo, as the effect is conſequent. Yea, thus much more, dare I take vpon me, toward the ſatiſfying of the noble courrage, that longeth ardently for the wiſedome of Cauſes Naturall: as to let him vnderſtand, that, in London, he may with his owne eyes, haue proſe of that, which I haue ſayd herein. A Gentleman, (which, for his good ſeruice, done to his Countrey, is famous and honorable: and for ſkill in the Mathematicall Sciences, and Languages, is the Od man of this land. &c.) euen he, is hable: and (I am ſure) will, very willingly, let the Glaſſe, and proſe be ſene: and ſo I (here) requeſt him: for the encreaſe of wiſedome, in the honorable: and for the ſtopping of the mouthes malicious: and repreſſing the arrogancy of the ignorant. Ye may eaſily geſſe, what I meane. This Art of *Perſpectiue*, is of that excellency, and may be led, to the certifying, and executing of ſuch thinges, as no man would eaſily beleue: without Actuall proſe perceiued. I ſpeake nothing of *Naturall Philoſophie*, which, without *Perſpectiue*, can not be fully vnderſtanded, nor perfectly atteined vnto. Nor, of *Aſtronomie*: which, without *Perſpectiue*, can not well be grounded: Nor *Aſtrologie*, naturally Verified, and auouched. That part hereof, which dealeth with Glaſſes (which name, Glaſſe, is a generall name, in this Arte, for any thing, from which, a Beame reboundeth) is called *Catoptrike*: and hath ſo many vſes, both merueilous, and proffitable: that, both, it would hold me to long, to note therin the principall concluſions, all ready knowne: And alſo (perchaunce) ſome thinges, might lacke due credite with you: And I, therby, to leeſe my labor: and you, to ſlip into light Iudgement*, Before you haue learned ſufficiently the powre of Nature and Arte.

A meruelous Glaſſe. ☞

S.W.P.

☞

NOw, to procede: Aſtronomie, is an Arte Mathematicall, which demonſtrateth the diſtance, magnitudes, and all naturall motions, apparences, and paſsions propre to the Planets and fixed Sterres: for

any

any time past, present and to come: in respect of a certaine Horizon, or without respect of any Horizon. By this Arte we are certified of the distance of the Starry Skye, and of eche *Planete* from the Centre of the Earth: and of the greatnes of any Fixed starre sene, or *Planete*, in respect of the Earthes greatnes. As, we are sure (by this Arte) that the Solidity, Massines and Body of the *Sonne*, conteineth the quantitie of the whole Earth and Sea, a hundred thre score and two times, lesse by $\frac{1}{8}$ one eight parte of the earth. But the Body of the whole earthly globe and Sea, is bigger then the body of the Mone, three and forty times lesse by $\frac{1}{8}$ of the Mone. Wherfore the *Sonne* is bigger then the ☽*Mone*, 7000 times, lesse, by $59\frac{52}{64}$ that is, precisely $6940\frac{25}{64}$ bigger then the ☽*Mone*. And yet the vnskillfull man, would iudge them a like bigge. Wherfore, of Necessity, the one is much farder from vs, then the other. The *Sonne*, when he is fardest from the earth (which, now, in our age, is, when he is in the 8. degree, of Cancer) is, 1179 Semidiameters of the Earth, distante. And the ☽*Mone* when she is fardest from the earth, is 68 Semidiameters of the earth and $\frac{1}{3}$ The nerest, that the ☽*Mone* commeth to the earth, is Semidiameters $52\frac{1}{4}$ The distance of the Starry Skye is, frō vs, in Semidiameters of the earth $20081\frac{1}{2}$ Twenty thousand foure score, one, and almost a halfe. Subtract from this, the ☽*Mones* nerest distance, from the Earth: and therof remaineth Semidiameters of the earth $20029\frac{1}{4}$ Twenty thousand nine and twenty and a quarter. So thicke is the heauenly Palace, that the *Planetes* haue all their exercise in, and most meruailously perfourme the Commaūdement and Charge to them giuen by the omnipotent Maiestie of the king of kings. This is that, which in *Genesis* is called *Ha Rakia*. Consider it well. The Semidiameter of the earth, cōteineth of our common miles $3436\frac{4}{11}$ three thousand, foure hundred thirty six and foure eleuenth partes of one myle: Such as the whole earth and Sea, round about, is 21600. One and twenty thousand six hundred of our myles. Allowyng for euery degree of the greatest circle, thre score myles. Now if you way well with your selfe but this litle parcell of frute *Astronomicall*, as concerning the bignesse, Distances of *Sonne*, *Mone*, *Sterry Sky*, and the huge massines of *Ha Rakia*, will you not finde your Consciences moued, with the kingly Prophet, to sing the confession of Gods Glory, and say, *The Heauens declare the glory of God, and the Firmament* [*Ha Rakia*] *sheweth forth the workes of his handes.* And so forth, for those fiue first staues, of that kingly Psalme. Well, well, It is time for some to lay hold on wisedome, and to Iudge truly of thinges: and not so to expound the Holy word, all by Allegories: as to Neglect the wisedome, powre and Goodnes of God, in, and by his Creatures, and Creation to be seen and learned. By parables and Analogies of whose natures and properties, the course of the Holy Scripture, also, declareth to vs very many Mysteries. The whole Frame of Gods Creatures, (which is the whole world,) is to vs, a bright glasse: from which, by reflexion, rebounded to our knowledge and perceiuerance, Beames, and Radiations: representing the Image of his Infinite goodnes, Omnipotēcy, and wisedome. And we therby, are taught and persuaded to Glorifie our Creator, as God: and be thankefull therfore. Could the Heatheniștes finde these vses, of these most pure, beawtifull, and Mighty Corporall Creatures: and shall we, after that the true *Sonne* of right wisenesse is risen aboue the *Horizon*, of our temporall *Hemisphærie*, and hath so abundantly streamed into our hartes, the direct beames of his goodnes, mercy, and grace: Whose heat All Creatures feele: Spirituall and Corporall: Visible and

Note.

Inuisible. Shall we (I say) looke vpon the *Heauen, Sterres*, and *Planets*, as an Oxe, and an Asse doth: no furder carefull or inquisitiue, what they are: why were they Created, How do they execute that they were Created for? Seing, All Creatures, were for our sake created: and both we, and they, Created, chiefly to glorifie the Almighty Creator: and that, by all meanes, to vs possible. *Nolite ignorare* (saith *Plato in Epinomis*) *Astronomiam, Sapientißimũ quiddam esse.* *Be ye not ignorant, Astronomie to be a thyng of excellent wisedome.* *Astronomie*, was to vs, from the beginning commended, and in maner commaunded by God him selfe. In asmuch as he made the *Sonne, Mone*, and *Sterres*, to be to vs, for *Signes*, and knowledge of Seasons, and for Distinctions of Dayes, and yeares. Many wordes nede not. But I wish, euery man should way this word, *Signes*. And besides that, conferre it also with the tenth Chapter of *Hieremie*. And though Some thinke, that there, they haue found a rod: Yet Modest Reason, will be indifferent Iudge, who ought to be beaten therwith, in respect of our purpose. Leauing that: I pray you vnderstand this: that without great diligence of Obseruation, examination and Calculation, their periods and courses (wherby *Distinction* of Seasons, yeares, and New Mones might precisely be knowne) could not exactely be certified. Which thing to performe, is that *Art*, which we here haue Defined to be *Astronomie*. Wherby, we may haue the distinct Course of Times, dayes, yeares, and Ages: aswell for Consideratiõ of Sacred Propheties, accomplished in due time, foretold: as for high Mysticall Solemnities holding: And for all other humaine affaires, Conditions, and couenantes, vpon certaine time, betwene man and man: with many other great vses: Wherin, (verely), would be great incertainty, Confusion, vntruth, and brutish Barbarousnes: without the wonderfull diligence and skill of this Arte: continually learning, and determining Times, and periodes of Time, by the Record of the heauenly booke, wherin all times are written: and to be read with an *Astronomicall staffe*, in stede of a festue.

Musike, of Motion, hath his Originall cause: Therfore, after the motions most swift, and most Slow, which are in the Firmament, of Nature performed: and vnder the *Astronomers Consideration*: now I will Speake of an other kinde of *Motion*, producing sound, audible, and of Man numerable. *Musike* I call here that *Science*, which of the Grekes is called *Harmonice*. Not medling with the Controuersie betwene the auncient *Harmonistes*, and *Canonistes*. Musike is a Mathematicall Science, which teacheth, by sense and reason, perfectly to iudge, and order the diuersities of soundes, hye and low. *Astronomie* and *Musike* are Sisters, saith *Plato*. As, for *Astronomie*, the eyes: So, for *Harmonious Motion*, the eares were made. But as *Astronomie* hath a more diuine Contemplation, and cõmodity, then mortall eye can perceiue: So, is *Musike* to be considered, that the
1. * Minde may be preferred, before the eare. And from audible sound, we ought to ascende, to the examination: which numbers are *Harmonious*, and which not. And why, either, the one are: or the other are not. I could at large, in the heauenly
2. * motions and distances, describe a meruailous Harmonie, of *Pythagoras* Harpe
4. 3. with eight stringes. Also, somwhat might be sayd of *Mercurius* * two Harpes, eche of foure Stringes Elementall. And very straunge matter, might be alledged
5. of the *Harmonie*, to our * Spirituall part appropriate. As in *Ptolomæus* third boke, in the fourth and sixth Chapters may appeare. * And what is the cause of the apt
6. bonde, and frendly felowship, of the Intellectuall and Mentall part of vs, with our grosse & corruptible body: but a certaine Meane, and *Harmonious Spiritualitie, with*

both

both participatyng, & of both (in a maner) resultyng: In the * *Tune of Mans voyce, and also* * *the sound of Instrument*, what might be sayd, of *Harmonie:* No common Musicien would lightly beleue. But of the sundry Mixture (as I may terme it) and concurse, diuerse collation, and Application of these *Harmonies:* as of thre, foure, fiue, or mo: Maruailous haue the effectes ben: and yet may be founde, and produced the like: with some proportionall consideration for our time, and being: in respect of the State, of the thinges then: in which, and by which, the wondrous effectes were wrought. *Democritus* and *Theophrastus* affirmed, that, by *Musike*, griefes and diseases of the Minde, and body might be cured, or inferred. And we finde in Recorde, that *Terpander*, *Arion*, *Ismenias*, *Orpheus*, *Amphion*, *Dauid*, *Pythagoras*, *Empedocles*, *Asclepiades* and *Timotheus*, by *Harmonicall* Consonãcy, haue done, and brought to pas, thinges, more then meruailous, to here of. Of them then, making no farder discourse, in this place: Sure I am, that Common *Musike*, commonly vsed, is found to the *Musiciens* and Hearers, to be so Commodious and pleasant, That if I would say and dispute, but thus much: That it were to be otherwise vsed, then it is, I should finde more repreeuers, then I could finde priuy, or skilfull of my meaning. In thinges therfore euident, and better knowen, then I can expresse: and so allowed and liked of, (as I would wish, some other thinges, had the like hap) I will spare to enlarge my lines any farder, but consequently follow my purpose.

7.
8.
I. D.
Read in Aristotle his 8. booke of Politikes: the 5, 6, and 7. chapters. Where you shall haue some occasion farder to thinke of Musike, than commonly is thought.

Of Cosmographie, I appointed briefly in this place, to geue you some intelligence. Cosmographie, is the whole and perfect description of the heauenly, and also elementall parte of the world, and their homologall application, and mutuall collation necessarie. This Art, requireth *Astronomie*, *Geographie*, *Hydrographie* and *Musike*. Therfore, it is no small Arte, nor so simple, as in common practise, it is (slightly) considered. This matcheth Heauen, and the Earth, in one frame, and aptly applieth parts Correspõdent: So, as, the Heauenly Globe, may (in practise) be duely described vpon the Geographicall, and Hydrographicall Globe. And there, for vs to consider an *Æquinoctiall Circle*, *an Ecliptike line*, *Colures*, *Poles*, *Sterres* in their true Longitudes, Latitudes, Declinations, and Verticalitie: also Climes, and Parallels: and by an *Horizon* annexed, and reuolution of the earthly Globe (as the Heauen, is, by the *Primouant*, caried about in 24. æquall Houres) to learne the Risinges and Settinges of Sterres (of *Virgill* in his *Georgikes*: of *Hesiod*: of *Hippocrates* in his *Medicinall Sphære*, to Perdicca King of the Macedonians: of *Diocles*, to King *Antigonus*, and of other famous *Philosophers* prescribed) a thing necessary, for due manuring of the earth, for *Nauigation*, for the Alteration of mans body: being, whole, Sicke, wounded, or brused. By the Reuolution, also, or mouing of the Globe Cosmographicall, the Rising and Setting of the Sonne: the Lengthes, of dayes and nightes: the Houres and times (both night and day) are knowne: with very many other pleasant and necessary vses: Wherof, some are knowne: but better remaine, for such to know and vse: who of a sparke of true fire, can make a wonderfull bonfire, by applying of due matter, duely.

Of Astrologie, here I make an Arte, seuerall from *Astronomie:* not by new deuise, but by good reason and authoritie: for, Astrologie, is an Arte Mathematicall, which reasonably demonstrateth the operations and effectes, of the naturall beames, of light, and secrete influence: of the Sterres and Planets: in euery element and elementall body:

at all times, in any Horizon assigned. This Arte is furnished with many other great Artes and experiences: As with perfecte *Perspectiue, Astronomie, Cosmographie, Naturall Philosophie* of the 4. Elementes, the Arte of Graduation, and some good vnderstãding in *Musike*: and yet moreouer, with an other great Arte, hereafter following, though I, here, set this before, for some considerations me mouing. Sufficient (you see) is the stuffe, to make this rare and secrete Arte, of: and hard enough to frame to the Conclusion Syllogisticall. Yet both the manifolde and continuall trauailes of the most auncient and wise Philosophers, for the atteyning of this Arte: and by examples of effectes, to confirme the same: hath left vnto vs sufficient proufe and witnesse: and we, also, daily may perceaue, That mans body, and all other Elementall bodies, are altered, disposed, ordred, pleasured, and displeasured, by the Influentiall working of the *Sunne, Mone*, and the other Starres and Planets. And therfore, sayth *Aristotle*, in the first of his *Meteorologicall* bookes, in the second Chapter: *Est autem necessariò Mundus iste, supernis lationibus ferè continuus. Vt, inde, vis eius vniuersa regatur. Ea siquidem Causa prima putanda omnibus est, vnde motus principium existit.* That is: *This* [Elementall] *World is of necessitie, almost, next adioyning, to the heauenly motions: That, from thence, all his vertue or force may be gouerned. For, that is to be thought the first Cause vnto all: from which, the beginning of motion, is.* And againe, in the tenth Chapter. *Oportet igitur & horum principia sumamus, & causas omnium similiter. Principium igitur vt mouens, præcipuumq; & omnium primum, Circulus ille est, in quo manifeste Solis latio, &c.* And so forth. His *Meteorologicall* bookes, are full of argumentes, and effectuall demonstrations, of the vertue, operation, and power of the heauenly bodies, in and vpon the fower Elementes, and other bodies, of them (either perfectly, or vnperfectly) composed. And in his second booke, *De Generatione & Corruptione*, in the tenth Chapter. *Quocirca & prima latio, Ortus & Interitus causa non est: Sed obliqui Circuli latio: ea namq; & continua est, & duobus motibus fit:* In Englishe, thus. *Wherefore the vppermost motion, is not the cause of Generation and Corruption, but the motion of the Zodiake: for, that, both, is continuall, and is caused of two mouinges.* And in his second booke, and second Chapter of hys *Physikes. Homo namq; generat hominem, atq; Sol.* *For Man* (sayth he) *and the Sonne, are cause of mans generation.* Authorities may be brought, very many: both of 1000. 2000. yea and 3000. yeares Antiquitie: of great *Philosophers, Expert, Wise*, and godly men, for that Conclusion: which, daily and hourely, we men, may discerne and perceaue by sense and reason: All beastes do feele, and simply shew, by their actions and passions, outward and inward: All Plants, Herbes, Trees, Flowers, and Fruites. And finally, the Elementes, and all thinges of the Elementes composed, do geue Testimonie (as *Aristotle* sayd) that theyr *Whole Dispositions, vertues, and naturall motions, depend of the Actiuitie of the heauenly motions and Influences. Whereby, beside the specificall order and forme, due to euery seede: and beside the Nature, propre to the Indiuiduall Matrix, of the thing produced: What shall be the heauenly Impression, the perfect and circumspecte Astrologien hath to Conclude.* Not onely (by *Apoteleſmes*) τὸ ὅτι, but by Naturall and Mathematicall demonstration τὸ διότι. Whereunto, what Sciences are requisite (without exception) I partly haue here warned: And in my *Propædeumes* (besides other matter there disclosed) I haue Mathematically furnished vp the whole Method: To this our age, not so carefully handled by any, that

euer

euer I saw, or heard of. I was, (for * 21. yeares ago) by certaine earnest disputations, of the Learned *Gerardus Mercator*, and *Antonius Gogana*, (and other,) therto so prouoked: and (by my constant and inuincible zeale to the veritie) in obseruations of Heauenly Influencies (to the Minute of time,) than, so diligent: And chiefly by the Supernaturall influence, from the Starre of Iacob, so directed: That any Modest and Sober Student, carefully and diligently seking for the Truth, will both finde & cõfesse, therin, to be the Veritie, of these my wordes: And also become a Reasonable Reformer, of three Sortes of people: about these Influentiall Operations, greatly erring from the truth. Wherof, the one, is Light Beleuers, the other, Light Despisers, and the third Light Practisers. The first, & most cõmon Sort, thinke the Heauen and Sterres, to be answerable to any their doutes or desires: which is not so: and, in dede, they, to much, ouer reache. The Second sorte thinke no Influentiall vertue (frõ the heauenly bodies) to beare any Sway in Generation and Corruption, in this Elementall world. And to the *Sunne*, *Mone* and *Sterres* (being so many, so pure, so bright, so wonderfull bigge, so farre in distance, so manifold in their motions, so constant in their periodes. &c.) they assigne a sleight, simple office or two, and so allow vnto thẽ (according to their capacities) as much vertue, and power Influentiall, as to the Signe of the *Sunne*, *Mone*, and seuen Sterres, hanged vp (for Signes) in London, for distinction of houses, & such grosse helpes, in our wordly affaires: And they vnderstand not (or will not vnderstand) of the other workinges, and vertues of the Heauenly *Sunne*, *Mone*, and *Sterres*: not so much, as the Mariner, or Husband man: no, not so much, as the *Elephant* doth, as the *Cynocephalus*, as the Porpentine doth: nor will allow these perfect, and incorruptible mighty bodies, so much vertuall Radiation, & Force, as they see in a litle peece of a *Magnes Stone:* which, at great distance, sheweth his operation. And perchaunce they thinke, the Sea & Riuers (as the Thames) to be some quicke thing, and so to ebbe, and flow, run in and out, of them selues, at their owne fantasies. God helpe, God helpe. Surely, these men, come to short: and either are to dull: or willfully blind: or, perhaps, to malicious. The third man, is the common and vulgare *Astrologien*, or Practiser: who, being not duely, artificially, and perfectly furnished: yet, either for vaine glory, or gayne: or like a simple dolt, & blinde Bayard, both in matter and maner, erreth: to the discredit of the *Wary*, and modest *Astrologien:* and to the robbing of those most noble corporall Creatures, of their Naturall Vertue: being most mighty: most beneficiall to all elementall Generation, Corruption and the appartenances: and most Harmonious in their Monarchie: For which thinges, being knowen, and modestly vsed: we might highly, and continually glorifie God, with the princely Prophet, saying. *The Heauens declare the Glorie of God: who made the Heauẽs in his wisedome: who made the Sonne, for to haue dominion of the day: the Mone and Sterres to haue dominion of the nyght: whereby, Day to day vttereth talke: and night, to night declareth knowledge. Prayse him, all ye Sterres, and Light. Amen.*

* *Anno.* 1548 *and* 1549. *in Louayn.*

1.

2.

3.

IN order, now foloweth, of Statike, somewhat to say, what we meane by that name: and what commodity, doth, on such Art, depend. Statike, is an Arte Mathematicall, which demonstrateth the causes of heauynes, and lightnes of all thynges: and of motions and properties, to heauynes and lightnes, belonging. And for asmuch as, by the Bilanx, or Balance (as the chief sensible Instrument,) Experience of these demonstrations may

be had: we call this Art, *Statike*: that is, *the Experimentes of the Balance*. Oh, that men wist, what proffit, (all maner of wayes) by this Arte might grow, to the hable Examiner, and diligent practiser. Thou onely, knoweſt all thinges preciſely (O God) who haſt made weight and Balance, thy Iudgement: who haſt created all thinges in *Number, Waight, and Meaſure*: and haſt wayed the mountaines and hils in a Balance: who haſt peyſed in thy hand, both Heauen and earth. We therfore warned by the Sacred word, to Conſider thy Creatures: and by that conſideration, to wynne a glyms (as it were,) or ſhaddow of perceiuerance, that thy wiſedome, might, and goodnes is infinite, and vnſpeakable, in thy Creatures declared: And being farder aduertiſed, by thy mercifull goodnes, that, three principall wayes, were, of the, vſed in Creation of all thy Creatures, namely, *Number*, *Waight* and *Meaſure*, And for as much as, of *Number* and *Meaſure*, the two Artes (auncient, famous, and to humaine vſes moſt neceſſary,) are, all ready, ſufficiently knowen and extant: This third key, we beſeche thee (through thy accuſtomed goodnes,) that it may come to the nedefull and ſufficient knowledge, of ſuch thy Seruauntes, as in thy workemanſhip, would gladly finde, thy true occaſions (purpoſely of the vſed) whereby we ſhould glorifie thy name, and ſhew forth (to the weaklinges in faith) thy wondrous wiſedome and Goodnes. Amen.

Meruaile nothing at this pang (godly frend, you Gentle and zelous Student.) An other day, perchaunce, you will perceiue, what occaſion moued me. Here, as now, I will giue you ſome ground, and withall ſome ſhew, of certaine commodities, by this Arte ariſing. And bycauſe this Arte is rare, my wordes and practiſes might be to darke: vnleaſt you had ſome light, holden before the matter: and that, beſt will be, in giuing you, out of *Archimedes* demonſtrations, a few principal Concluſions, as foloweth.

1.

The Superficies of euery Liquor, by it ſelfe conſiſtyng, and in quyet, is Sphæricall: the centre whereof, is the ſame, which is the centre of the Earth.

2.

If Solide Magnitudes, being of the ſame bignes, or quãtitie, that any Liquor is, and hauyng alſo the ſame Waight: be let downe into the ſame Liquor, they will ſettle downeward, ſo, that no parte of them, ſhall be aboue the Superficies of the Liquor: and yet neuertheles, they will not ſinke vtterly downe, or drowne.

3.

If any Solide Magnitude beyng Lighter then a Liquor, be let downe into the ſame Liquor, it will ſettle downe, ſo farre into the ſame Liquor, that ſo great a quantitie of that Liquor, as is the parte of the Solid Magnitude, ſettled downe into the ſame Liquor: is in Waight, æquall, to the waight of the whole Solid Magnitude.

4.

Any Solide Magnitude, Lighter then a Liquor, forced downe

into

into the ſame Liquor, will moue vpward, with ſo great a power, by how much, the Liquor hauyng æquall quantitie to the whole Magnitude, is heauyer then the ſame Magnitude.

5.

Any Solid Magnitude, heauyer then a Liquor, beyng let downe into the ſame Liquor, will ſinke downe vtterly: And wilbe in that Liquor, Lighter by ſo much, as is the waight or heauynes of the Liquor, hauing bygnes or quantitie, æquall to the Solid Magnitude.

6.

I.D. The Cutting of a Sphære according to any proportion aſſigned, may by this propoſition be done Mechanically by tempering Liquor to a certayne waight in reſpect of the waight of the Sphære therein Swymming.

If any Solide Magnitude, Lighter then a Liquor, be let downe into the ſame Liquor, the waight of the ſame Magnitude, will be, to the Waight of the Liquor. (Which is æquall in quantitie to the whole Magnitude,) in that proportion, that the parte, of the Magnitude ſettled downe, is to the whole Magnitude.

BY theſe verities, great Errors may be reformed, in Opinion of the Naturall Motion of thinges, Light and Heauy. Which errors, are in Naturall Philoſophie (almoſt) of all mẽ allowed: to much truſting to Authority: and falſe Suppoſitions. As, Of any two bodyes, the heauyer, to moue downward faſter then the lighter. This error, is not firſt by me, Noted: but by one *Iohn Baptiſt de Benedictis*. The chief of his propoſitions, is this: which ſeemeth a Paradox.

A common error, noted.

If there be two bodyes of one forme, and of one kynde, æquall in quantitie or vnæquall, they will moue by æquall ſpace, in æquall tyme: So that both theyr mouynges be in ayre, or both in water: or in any one Middle.

A paradox.

N. T. The wonderfull vſe of theſe Propoſitions.

Hereupon, in the feate of Gunnyng, certaine good diſcourſes (otherwiſe) may receiue great amendement, and furderance. In the entended purpoſe, alſo, allowing ſomwhat to the imperfection of Nature: not aunſwerable to the preciſenes of demonſtration. Moreouer, by the foreſaid propoſitions (wiſely vſed.) The Ayre, the water, the Earth, the Fire, may be nerely, knowen, how light or heauy they are (Naturally) in their aſſigned partes: or in the whole. And then, to thinges Elementall, turning your practiſe: you may deale for the proportion of the Elementes, in the thinges Compounded. Then, to the proportions of the Humours in Man: their waightes: and the waight of his bones, and fleſh. &c. Than, by waight, to haue conſideration of the Force of man, any maner of way: in whole or in part. Then, may you, of Ships water drawing, diuerſly, in the Sea and in freſh water, haue pleaſant conſideration: and of waying vp of any thing, ſonken in Sea or in freſh water. &c. And (to lift vp your head a loft:) by waight, you may, as preciſely, as by any inſtrument els, meaſure the Diameters of *Sonne* and *Mone*. *&c.* Frende, I pray you, way theſe thinges, with the iuſt Balance of Reaſon. And you will finde Meruailes vpon Meruailes: And eſteme one Drop of Truth (yea in Naturall Philoſophie) more worth, then whole Libraries of Opinions, vndemonſtrated: or not aunſwering to Natures Law, and your experience. Leauing theſe

 thinges,

thinges, thus: I will giue you two or three, light practises, to great purpose: and so finish my Annotation *Staticall*. In Mathematicall matters, by the Mechanicians ayde, we will behold, here, the Commodity of waight. Make a Cube, of any one Vniforme: and through like heauy stuffe: of the same Stuffe, make a Sphære or Globe, precisely, of a Diameter æquall to the Radicall side of the Cube. Your stuffe, may be wood, Copper, Tinne, Lead, Siluer. &c. (being, as I sayd, of like nature, condition, and like waight throughout.) And you may, by Say Balance, haue prepared a great number of the smallest waightes: which, by those Balance can be discerned or tryed: and so, haue proceded to make you a perfect Pyle, company & Number of waightes: to the waight of six, eight, or twelue pound waight: most diligently tryed, all. And of euery one, the Content knowen, in your least waight, that is wayable. [They that can not haue these waightes of precisenes: may, by Sand, Vniforme, and well dusted, make them a number of waightes, somewhat nere precisenes: by halfing euer the Sand: they shall, at length, come to a least common waight. Therein, I leaue the farder matter, to their discretion, whom nede shall pinche.] The *Venetians* consideration of waight, may seme precise enough: by eight descentes progressionall,* halfing, from a grayne. Your Cube, Sphære, apt Balance, and conuenient waightes, being ready: fall to worke. 1. First, way your Cube. Note the Number of the waight. Way, after that, your Sphære. Note likewise, the Nūber of the waight. If you now find the waight of your Cube, to be to the waight of the Sphære, as 21. is to 11: Then you see, how the Mechanicien and *Experimenter*, without Geometrie and Demonstration, are (as nerely in effect) tought the proportion of the Cube to the Sphere: as I haue demonstrated it, in the end of the twelfth boke of *Euclide*. Often, try with the same Cube and Sphære. Then, chaunge, your Sphære and Cube, to an other matter: or to an other bignes: till you haue made a perfect vniuersall Experience of it. Posible it is, that you shall wynne to nerer termes, in the proportion.

The practise Staticall, to know the proportion, betwene the Cube, and the Sphære.

I. D.

**For, so, haue you. 256. partes of a Graine.*

When you haue found this one certaine Drop of Naturall veritie, procede on, to Inferre, and duely to make assay, of matter depending. As, bycause it is well demonstrated, that a Cylinder, whose heith, and Diameter of his base, is æquall to the Diameter of the Sphære, is Sesquialter to the same Sphære (that is, as 3. to 2:) To the number of the waight of the Sphære, adde halfe so much, as it is: and so haue you the number of the waight of that Cylinder. Which is also Comprehended of our former Cube: So, that the base of that Cylinder, is a Circle described in the Square, which is the base of our Cube. But the Cube and the Cylinder, being both of one heith, haue their Bases in the same proportion, in the which, they are, one to an other, in their Massines or Soliditie. But, before, we haue two numbers, expressing their Massines, Solidities, and Quantities, by waight: wherfore, we haue * the proportion of the Square, to the Circle, inscribed in the same Square. And so are we fallen into the knowledge sensible, and Experimentall of *Archimedes* great Secret: of him, by great trauaile of minde, sought and found. Wherfore, to any Circle giuen, you can giue a Square æquall: * as I haue taught, in my Annotation, vpon the first proposition of the twelfth boke, And likewise, to any Square giuen, you may giue a Circle æquall: *If you describe a Circle, which shall be in that proportion, to your Circle inscribed, as the Square is to the same Circle: This, you may do, by my Annotations, vpon the second proposition of the twelfth boke of *Euclide*, in my third Probleme there. Your diligence may come to a proportion, of the Square to the Circle inscribed, nerer the truth, then is the proportion of 14. to 11. And consider, that you may begyn at the Circle and Square, and so come to conclude of the Sphære, & the Cube, what

The proportion of the Square to the Circle inscribed.

**The Squaring of the Circle, Mechanically.*

**To any Square geuen, to geue a Circle, equall.*

their

their proportion is: as now, you came from the Sphære to the Circle. For, of Siluer, or Gold, or Latton Lamyns or plates (thorough one hole drawē, as the maner is) if you make a Square figure: & way it: and then, deſcribing theron, the Circle inſcribed: & cut of, & file away, preciſely (to the Circle) the ouerplus of the Square: you ſhall then, waying your Circle, ſee, whether the waight of the Square, be to your Circle, as 14. to 11. As I haue Noted, in the beginning of *Euclides* twelfth boke. &c. after this reſort to my laſt propoſition, vpon the laſt of the twelfth. And there, helpe your ſelfe, to the end. And, here, Note this, by the way. That we may Square the Circle, without hauing knowledge of the proportion, of the Circumference to the Diameter: as you haue here perceiued. And otherwayes alſo, I can demonſtrate it. So that, many haue cumberd them ſelues ſuperfluouſly, by trauailing in that point firſt, which was not of neceſsitie, firſt: and alſo very intricate. And eaſily, you may, (and that diuerſly) come to the knowledge of the Circumference: the Circles Quantitie, being firſt knowen. Which thing, I leaue to your conſideration: making haſt to deſpatch an other Magiſtrall Probleme: and to bring it, nerer to your knowledge, and readier dealing with, then the world (before this day,) had it for you, that I can tell of. And that is, *A Mechanicall Dubblyng of the Cube: &c.* Which may, thus, be done: Make of Copper plates, or Tyn plates, a fourſquare vpright Pyramis, or a Cone: perfectly faſhioned in the holow, within. Wherin, let great diligence be vſed, to approche (as nere as may be) to the Mathematicall perfection of thoſe figures. At their baſes, let them be all open: euery where, els, moſt cloſe, and iuſt to. From the vertex, to the Circumference of the baſe of the Cone: & to the ſides of the baſe of the Pyramis: Let 4. ſtraight lines be drawen, in the inſide of the Cone and Pyramis: makyng at their fall, on the perimeters of the baſes, equall angles on both ſides them ſelues, with the ſayd perimeters. Theſe 4. lines (in the Pyramis: and as many, in the Cone) diuide: one, in 12. æquall partes: and an other, in 24. an other, in 60, and an other, in 100. (reckenyng vp from the vertex.) Or vſe other numbers of diuiſion, as experience ſhall teach you. Then,* ſet your Cone or Pyramis, with the vertex downward, perpendicularly, in reſpect of the Baſe. (Though it be otherwayes, it hindreth nothyng.) So let thē moſt ſtedily be ſtayed. Now, if there be a Cube, which you wold haue Dubbled. Make you a prety Cube of Copper, Siluer, Lead, Tynne, Wood, Stone, or Bone. Or els make a hollow Cube, or Cubik coffen, of Copper, Siluer, Tynne, or Wood &c. Theſe, you may ſo proportiō in reſpect of your Pyramis or Cone, that the Pyramis or Cone, will be hable to conteine the waight of them, in water, 3. or 4. times: at the leaſt: what ſtuff ſo euer they be made of. Let not your Solid angle, at the vertex, be to ſharpe: but that the water may come with eaſe, to the very vertex, of your hollow Cone or Pyramis. Put one of your Solid Cubes in a Balance apt: take the waight therof exactly in water. Powre that water, (without loſſe) into the hollow Pyramis or Cone, quietly. Marke in your lines, what numbers the water Cutteth: Take the waight of the ſame Cube againe: in the ſame kinde of water, which you had before: put that* alſo, into the Pyramis or Cone, where you did put the firſt. Marke now againe, in what number or place of the lines, the water Cutteth them. Two

Note Squaring of the Circle without knowledge of the proportion betwene Circumference and Diameter.

To Dubble the Cube redily: by Art Mechanicall: depending vpon Demonſtration Mathematicall.

I. D. The 4. ſides of this Pyramis muſt be 4. Iſoſceles Triangles alike and æquall.

*I. D. * In all workinges with this Pyramis or Cone, Let their Situations be in all Pointes and Conditions, a like, or all one: while you are about one worke. Els you will erre.*

*I. D. * Conſider well whan you muſt put your waters togyther: and whan, you muſt empty your firſt water, out of your Pyramis or Cone. Els you will erre.*

wayes you may conclude your purpoſe: it is to wete, either by numbers or lines. By numbers: as, if you diuide the ſide of your Fundamentall Cube into ſo many æquall partes, as it is capable of, conueniently, with your eaſe, and preciſenes of the diuiſion. For, as the number of your firſt and leſſe line (in your hollow Pyramis or Cone,) is to the ſecond or greater (both being counted from the vertex) ſo ſhall the number of the ſide of your Fundamentall Cube, be to the nũber belonging to the Radicall ſide, of the Cube, dubble to your Fundamentall Cube: Which being multiplied Cubik wiſe, will ſone ſhew it ſelfe, whether it be dubble or no, to the Cubik number of your Fundamentall Cube. By lines, thus: As your leſſe and firſt line, (in your hollow Pyramis or Cone,) is to the ſecond or greater, ſo let the Radical ſide of your Fundamẽtall Cube, be to a fourth proportionall line, by the 12. propoſition, of the ſixth boke of *Euclide*. Which fourth line, ſhall be the Rote Cubik, or Radicall ſide of the Cube, dubble to your Fundamentall Cube: which is the thing we deſired. For this, may I (with ioy) ſay, EYPHKA, EYPHKA, EYPHKA: thanking the holy and glorious Trinity: hauing greater cauſe therto, then* *Archimedes* had (for finding the fraude vſed in the Kinges Crowne, of Gold): as all men may eaſily Iudge: by the diuerſitie of the frute following of the one, and the other. Where I ſpake before, of a hollow Cubik Coffen: the like vſe, is of it: and without waight. Thus. Fill it with water, preciſely full, and poure that water into your Pyramis or Cone. And here note the lines cutting in your Pyramis or Cone. Againe, fill your coffen, like as you did before. Put that Water, alſo, to the firſt. Marke the ſecond cutting of your lines. Now, as you proceded before, ſo muſt you here procede.* And if the Cube, which you ſhould Double, be neuer ſo great: you haue, thus, the proportion (in ſmall) betwene your two litle Cubes: And then, the ſide, of that great Cube (to be doubled) being the third, will haue the fourth, found, to it proportionall: by the 12. of the ſixth of Euclide.

* *Vitruuius. Lib. 9. Cap. 3.*

☞ *God be thanked for this Inuention, & the fruite enſuing.*

* *Note.*

Note, that all this while, I forget not my firſt Propoſition Staticall, here rehearſed: that, the Superficies of the water, is Sphæricall. Wherein, vſe your diſcretion: to the firſt line, adding a ſmall heare breadth, more: and to the ſecond, halfe a heare breadth more, to his length. For, you will eaſily perceaue, that the difference can be no greater, in any Pyramis or Cone, of you to be handled. Which you ſhall thus trye. *For finding the ſwelling of the water aboue leuell.* Square the Semidiameter, from the Centre of the earth, to your firſt Waters Superficies. Square then, halfe the Subtendent of that watry Superficies (which Subtendent muſt haue the equall partes of his meaſure, all one, with thoſe of the Semidiameter of the earth to your watry Superficies): Subtracte this ſquare, from the firſt: Of the reſidue, take the Rote Square. That Rote, Subtracte from your firſt Semidiameter of the earth to your watry Superficies: that, which remaineth, is the heith of the water, in the middle, aboue the leuell. Which, you will finde, to be a thing inſenſible. And though it were greatly ſenſible,* yet, by helpe of my ſixt Theoreme vpon the laſt Propoſition of Euclides twelfth booke, noted: you may reduce all, to a true Leuell. But, farther diligence, of you is to be vſed, againſt accidentall cauſes of the waters ſwelling: as by hauing (ſomwhat) with a moyſt Sponge, before, made moyſt your hollow Pyramis or Cone, will preuent an accidentall cauſe of Swelling, &c. Experience will teach you abundantly: with great eaſe, pleaſure, and cõmoditie.

Note, as concerning the Sphæricall Superficies of the Water.

☞ „ „ „ „ „ „ „

* *Note.*

Thus, may you Double the Cube Mechanically, Treble it, and ſo forth, in any proportion. Now will I Abridge your paine, coſt, and Care herein. Without all preparing of your Fundamentall Cubes: you may (alike) worke this Concluſion. For, that, was rather a kinde of Experimentall demõſtration, then the ſhorteſt way:

Note this Abridgement of Dubbling the Cube. &c.

and

and all, vpon one Mathematicall Demonſtration depending. Take water (as muĉh as conueniently will ſerue your turne: as I warned before of your Fundamentall Cubes bignes) Way it preciſely. Put that water, into your Pyramis or Cone. Of the ſame kinde of water, then take againe, the ſame waight you had before: put that likewiſe into the Pyramis or Cone. For, in eche time, your marking of the lines, how the Water doth cut them, ſhall geue you the proportion betwen the Radicall ſides, of any two Cubes, wherof the one is Double to the other: working as before I haue taught you: *ſauing that for you Fundamentall Cube his Radicall ſide: here, you may take a right line, at pleaſure.

Note. ☞

Yet farther proceding with our droppe of Naturall truth: you may (now) geue Cubes, one to the other, in any proportiõ geuẽ: Rationall or Irrationall: on this maner. Make a hollow Parallelipipedon of Copper or Tinne: with one Baſe wãting, or open: as in our Cubike Coffen. Frõ the bottome of that Parallelipipedon, raiſe vp, many perpendiculars, in euery of his fower ſides. Now if any proportion be aſſigned you, in right lines: Cut one of your perpendiculars (or a line equall to it, or leſſe then it) likewiſe: by the 10. of the ſixth of Euclide. And thoſe two partes, ſet in two ſundry lines of thoſe perpendiculars (or you may ſet them both, in one line) making their beginninges, to be, at the baſe: and ſo their lengthes to extend vpward. Now, ſet your hollow Parallelipipedon, vpright, perpendicularly, ſteadie. Poure in water, handſomly, to the heith of your ſhorter line. Poure that water, into the hollow Pyramis or Cone. Marke the place of the riſing. Settle your hollow Parallelipipedon againe. Poure water into it: vnto the heith of the ſecond line, exactly. Poure that water* duely into the hollow Pyramis or Cone: Marke now againe, where the water cutteth the ſame line which you marked before. For, there, as the firſt marked line, is to the ſecond: So ſhall the two Radicall ſides be, one to the other, of any two Cubes: which, in their Soliditie, ſhall haue the ſame proportion, which was at the firſt aſſigned: were it Rationall or Irrationall.

To giue Cubes one to the other in any proportion, Rationall or Irrationall.

** Emptying the firſt.*

Thus, in ſundry waies you may furniſhe your ſelfe with ſuch ſtraunge and profitable matter: which, long hath bene wiſhed for. And though it be Naturally done and Mechanically: yet hath it a good Demonſtration Mathematicall. Which is this: Alwaies, you haue two Like Pyramids: or two Like Cones, in the proportions aſſigned: and like Pyramids or Cones, are in proportion, one to the other, in the proportion of their Homologall ſides (or lines) tripled. Wherefore, if to the firſt, and ſecond lines, found in your hollow Pyramis or Cone, you ioyne a third and a fourth, in continuall proportion: that fourth line, ſhall be to the firſt, as the greater Pyramis or Cone, is to the leſſe: by the 33. of the eleuenth of Euclide. If Pyramis to Pyramis, or Cone to Cone, be double, then ſhall * Line to Line, be alſo double, &c. But, as our firſt line, is to the ſecond, ſo is the Radicall ſide of our Fundamentall Cube, to the Radicall ſide of the Cube to be made, or to be doubled: and therefore, to thoſe twaine alſo, a third and a fourth line, in continuall proportion, ioyned: will geue the fourth line in that proportion to the firſt, as our fourth Pyramidall, or Conike line, was to his firſt: but that was double, or treble, &c. as the Pyramids or Cones were, one to an other (as we haue proued) therfore, this fourth, ſhalbe alſo double or treble to the firſt, as the Pyramids or Cones were one to an other: But our made Cube, is deſcribed of the ſecond in proportion, of the fower proportionall lines: therfore * as the fourth line, is to the firſt, ſo is that Cube, to the firſt Cube: and we haue proued the fourth line, to be to the firſt, as the Pyramis or Cone, is to the Pyramis or Cone: Wherefore the Cube is

The demonſtrations of this Dubbling of the Cube, and of the reſt.

I.D.

**Hereby, helpe your ſelf to become a preciſe practiſer. And ſo conſider, how, nothing at all, you are hindred (ſenſibly) by the Conuexitie of the water.*

**By the 33. of the eleuenth booke of Euclide.*

I.D. *And your diligence in practiſe, can ſo (in waight of water) performe it: Therefore, now, you are able to geue good reaſon of your whole doing.

to the Cube, as Pyramis is to Pyramis, or Cone is to Cone. But we* Suppoſe Pyramis to Pyramis, or Cone to Cone, to be double or treble. &c. Therfore Cube, is to Cube, double, or treble, &c. Which was to be demonſtrated. And of the Parallelipipedō, it is euidēt, that the water Solide Parallelipipedons, are one to the other, as their heithes are, ſeing they haue one baſe. Wherfore the Pyramids or Cones, made of thoſe water Parallelipipedons, are one to the other, as the lines are (one to the other) betwene which, our proportion was aſſigned. But the Cubes made of lines, after the proportiō of the Pyramidal or Conik *homologall* lines, are one to the other, as the Pyramides or Cones are, one to the other (as we before did proue) therfore, the Cubes made, ſhalbe one to the other, as the lines aſſigned, are one to the other: Which was to be demonſtrated. Note. *This, my Demonſtratiō is more generall, then onely in Square Pyramis or Cone: Conſider well. Thus, haue I, both Mathematically and Mechanically, ben very long in wordes: yet (I truſt) nothing tedious to them, who, to theſe thinges, are well affected. And verily I am forced (auoiding prolixitie) to omit ſundry ſuch things, eaſie to be practiſed: which to the Mathematicien, would be a great Threaſure: and to the Mechanicien, no ſmall gaine. *Now may you, Betwene two lines giuen, finde two middle proportionals, in Continuall proportion: by the hollow Parallelipipedon, and the hollow Pyramis, or Cone. Now, any Parallelipipedon rectangle being giuen: thre right lines may be found, proportionall in any proportion aſſigned, of which, ſhal be produced a Parallelipipedon, æquall to the Parallelipipedon giuen. Hereof, I noted ſomwhat, vpon the 36. propoſition, of the 11. boke of *Euclide*. Now, all thoſe thinges, which *Vitruuius* in his Architecture, ſpecified hable to be done, by dubbling of the Cube. Or, by finding of two middle proportionall lines, betwene two lines giuen, may eaſely be performed. Now, that Probleme, which I noted vnto you, in the end of my Addition, vpon the 34. of the 11. boke of *Euclide*, is proued poſsible. Now, may any regular body, be Tranſformed into an other, &c. Now, any regular body: any Sphere, yea any Mixt Solid: and (that more is) Irregular Solides, may be made (in any proportiō aſſigned) like vnto the body, firſt giuen. Thus, of a *Manneken*, (as the *Dutch* Painters terme it) in the ſame *Symmetrie*, may a Giant be made: and that, with any geſture, by the Manneken vſed: and contrarywiſe. Now, may you, of any Mould, or Modell of a Ship, make one, of the ſame Mould (in any aſſigned proportion) bigger or leſſer. Now, may you, of any* Gunne, or little peece of ordinaūce, make an other, with the ſame *Symmetrie* (in all pointes) as great, and as little, as you will. Marke that: and thinke on it. Infinitely, may you apply this, ſo long ſought for, and now ſo eaſily concluded: and withall, ſo willingly and frankly communicated to ſuch, as faithfully deale with vertuous ſtudies. Thus, can the Mathematicall minde, deale Speculatiuely in his own Arte: and by good meanes, Mount aboue the cloudes and ſterres: And thirdly, he can, by order, Deſcend, to frame Naturall thinges, to wonderfull vſes: and when he liſt, retire home into his owne Centre: and there, prepare more Meanes, to Aſcend or Deſcend by: and, all, to the glory of God, and our honeſt delectation in earth.

*Note this Corollary.

*The great Commodities following of theſe new Inuentions.

☞*

Such is the Fruite of the Mathematicall Sciences and Artes.

Although, the Printer, hath looked for this Præface, a day or two, yet could I not bring my pen from the paper, before I had giuen you comfortable warning, and brief inſtructions, of ſome of the Commodities, by *Statike*, hable to be reaped: In the reſt, I will therfore, be as brief, as it is poſsible: and with all, deſcribing them, ſomwhat accordingly. And that, you ſhall perceiue, by this, which in order commeth

meth next. For, wheras, it is so ample and wonderfull, that, an whole yeare long, one might finde fruitfull matter therin, to speake of: and also in practise, is a Threasure endeles: yet will I glanse ouer it, with wordes very few.

THis do I call Anthropographie. Which is an Art restored, and of my preferment to your Seruice. I pray you, thinke of it, as of one of the chief pointes, of Humane knowledge. Although it be, but now, first Cõfirmed, with this new name: yet the matter, hath from the beginning, ben in consideration of all perfect Philosophers. Anthropographie, is the description of the Number, Measure, Waight, figure, Situation, and colour of euery diuerse thing, conteyned in the perfect body of MAN: with certain knowledge of the Symmetrie, figure, waight, Characterization, and due locall motion, of any parcell of the sayd body, assigned: and of Nũbers, to the sayd parcell appertainyng. This, is the one part of the Definition, mete for this place: Sufficient to notifie, the particularitie, and excellency of the Arte: and why it is, here, ascribed to the Mathematicals. Yf the description of the heauenly part of the world, had a peculier Art, called *Astronomie*: If the description of the earthly Globe, hath his peculier arte, called *Geographie*. If the Matching of both, hath his peculier Arte, called *Cosmographie*: Which is the Descriptiõ of the whole, and vniuersall frame of the world: Why should not the description of him, who is the Lesse world: and, frõ the beginning, called *Microcosmus* (that is. *The Lesse World.*) And for whose sake, and seruice, all bodily creatures els, were created: Who, also, participateth with Spirites, and Angels: and is made to the Image and similitude of *God*: haue his peculier Art? and be called the *Arte of Artes*: rather, then, either to want a name, or to haue to base and impropre a name? You must of sundry professions, borow or challenge home, peculier partes hereof: and farder procede: as, God, Nature, Reason and Experience shall informe you. The Anatomistes will restore to you, some part: The Physiognomistes, some: The Chyromantistes some. The Metaposcopistes, some: The excellent, *Albert Durer*, a good part: the Arte of Perspectiue, will somwhat, for the Eye, helpe forward: *Pythagoras, Hipocrates, Plato, Galenus, Meletius*, & many other (in certaine thinges) will be Contributaries. And farder, the Heauen, the Earth, and all other Creatures, will eche shew, and offer their Harmonious seruice, to fill vp, that, which wanteth hereof: and with your own Experience, concluding: you may Methodically register the whole, for the posteritie: Whereby, good profe will be had, of our Harmonious, and Microcosmicall constitution. The outward Image, and vew hereof: to the Art of *Zographie* and Painting, to Sculpture, and Architecture: (for Church, House, Fort, or Ship) is most necessary and profitable: for that, it is the chiefe base and foundation of them. Looke in * *Vitruuius*, whether I deale sincerely for your behouse, or no. Looke in *Albertus Durerus, De Symmetria humani Corporis*. Looke in the 27. and 28. Chapters, of the second booke, *De occulta Philosophia*. Consider the *Arke* of *Noe*. And by that, wade farther. Remember the *Delphicall Oracle* NOSCE TEIPSVM (*Knowe thy selfe*) so long agoe pronounced: of so many a Philosopher repeated: and of the *Wisest* attempted: And then, you will perceaue, how long agoe, you haue bene called to the Schole, where this Arte might be learned. Well. I am nothing affrayde, of the disdayne of some such, as thinke Sciences and Artes, to be but Seuen. Perhaps, those Such, may, with ignorance, and shame enough, come short of them Seuen also: and yet neuertheless

MAN is the Lesse World.

Microcosmus.

**Lib. 3. Cap. 1.*

they can not prescribe a certaine number of Artes: and in eche, certaine vnpassable boundes, to God, Nature, and mans Industrie. New Artes, dayly rise vp: and there was no such order taken, that, All Artes, should in one age, or in one land, or of one man, be made knowen to the world. Let vs embrace the giftes of God, and wayes to wisedome, in this time of grace, from aboue, continually bestowed on them, who thankefully will receiue them: *Et bonis Omnia Cooperabuntur in bonum.*

Trochilike, is that Art Mathematicall, which demonstrateth the properties of all Circular motions, Simple and Compounde. And bycause the frute hereof, vulgarly receiued, is in Wheles, it hath the name of *Trochilike:* as a man would say, *Whele Art.* By this art, a Whele may be geuen which shall moue ones about, in any tyme assigned. Two Wheles may be giuen, whose turnynges about in one and the same tyme, (or equall tymes), shall haue, one to the other, any proportion appointed. By Wheles, may a straight line be described: Likewise, a Spirall line in plaine, Conicall Section lines, and other Irregular lines, at pleasure, may be drawen. These, and such like, are principall Conclusions of this Arte: and helpe forward many pleasant and profitable Mechanicall workes: As Milles, to Saw great and very long Deale bordes, no man being by. Such haue I seene in Germany: and in the Citie of Prage: in the kingdome of Bohemia: Coyning Milles, Hand Milles for Corne grinding: And all maner of Milles, and Whele worke: By Winde, Smoke, Water, Waight, Spring, Man or Beast, moued. Take in your hand, *Agricola De re Metallica:* and then shall you (in all Mines) perceaue, how great nede is, of Whele worke. By Wheles, straunge workes and incredible, are done: as will, in other Artes hereafter, appeare. A wonderfull example of farther possibilitie, and present commoditie, was sene in my time, in a certaine Instrument: which by the Inuenter and Artificer (before) was solde for xx. Talentes of Golde: and then had (by misfortune) receaued some iniurie and hurt: And one *Ianellus* of *Cremona* did mend the same, and presented it vnto the Emperour *Charles* the fifth. *Hieronymus Cardanus*, can be my witnesse, that therein, was one Whele, which moued, and that, in such rate, that, in 7000. yeares onely, his owne periode should be finished. A thing almost incredible: But how farre, I keepe me within my boundes: very many men (yet aliue) can tell.

Saw Milles.

Helicosophie, is nere Sister to *Trochilike:* and is, An Arte Mathematicall, which demonstrateth the designing of all Spirall lines in Plaine, on Cylinder, Cone, Sphære, Conoid, and Sphæroid, and their properties appertayning. The vse hereof, in *Architecture*, and diuerse Instrumentes and Engines, is most necessary. For, in many thinges, the Skrue worketh the feate, which, els, could not be performed. By helpe hereof, it is * recorded, that, where all the power of the Citie of Syracusa, was not hable to moue a certaine Ship (being on ground) mightie *Archimedes*, setting to, his Skruish Engine, caused *Hiero* the king, by him self, at ease, to remoue her, as he would. Wherat, the King wondring: Ἀπὸ τάυτης τῆς ἡμέρας, πεῖρ πάντος, Ἀρχιμήδη λέγοντι πιστεύεσθαι. *From this day, forward* (said the King) *Credit ought to be giuen to Archimedes, what soeuer he sayth.*

**. Atheneus Lib. 5. cap. 8.*

Proclus. Pag. 18.

Pneumatithmie demonstrateth by close hollow Geometricall Figures, (regular and irregular) the straunge properties (in motion or stay) of the Water, Ayre, Smoke, and Fire, in theyr cõtinuitie,

and

and as they are ioyned to the Elementes next them. This Arte, to the Naturall Philoſopher, is very proffitable: to proue, that *Vacuum*, or *Emptines* is not in the world. And that, all Nature, abhorreth it ſo much: that, contrary to ordinary law, the Elementes will moue or ſtand. As, Water to aſcend: rather then betwene him and Ayre, Space or place ſhould be left, more then (naturally) that quãtitie of Ayre requireth, or can fill. Againe, Water to hang, and not deſcend: rather then by deſcending, to leaue Emptines at his backe. The like, is of Fire and Ayre: they will deſcend: when, either, their Cõtinuitie ſhould be diſſolued: or their next Element forced from them. And as they will not be extended, to diſcontinuitie: So, will they not, nor yet of mans force, can be preſt or pent, in ſpace, not ſufficient and aunſwerable to their bodily ſubſtance. Great force and violence will they vſe, to enioy their naturall right and libertie. Hereupon, two or three men together, by keping Ayre vnder a great Cauldron, and forcyng the ſame downe, orderly, may without harme deſcend to the Sea bottome: and continue there a tyme &c. Where, Note, how the thicker Element (as the Water) giueth place to the thynner (as, is the ayre:) and receiueth violence of the thinner, in maner. &c. Pumps and all maner of Bellowes, haue their ground of this Art: and many other ſtraunge deuiſes. As, *Hydraulica*, Organes goyng by water. &c. Of this Feat, (called commonly *Pneumatica*,) goodly workes are extant, both in Greke, and Latin. With old and learned Schole men, it is called *Scientia de pleno & vacuo.*

To go to the bottom of the Sea without daunger.

Menadrie, is an Arte Mathematicall, which demonſtrateth, how, aboue Natures vertue and power ſimple: Vertue and force may be multiplied: and ſo, to direct, to lift, to pull to, and to put or caſt fro, any multiplied or ſimple, determined Vertue, Waight or Force: naturally, not, ſo, directible or moueable. Very much is this Art furdred by other Artes: as, in ſome pointes, by *Perſpectiue:* in ſome, by *Statike*: in ſome, by *Trochilike*: and in other, by *Helicoſophie*: and *Pneumatithmie*. By this Art, all Cranes, Gybbettes, & Ingines to lift vp, or to force any thing, any maner way, are ordred: and the certaine cauſe of their force, is knowne: As, the force which one man hath with the Duche waghen Racke: therwith, to ſet vp agayne, a mighty waghen laden, being ouerthrowne. The force of the Croſſebow Racke, is certainly, here, demonſtrated. The reaſon, why one mã, doth with a leauer, lift that, which Sixe men, with their handes onely, could not, ſo eaſily do. By this Arte, in our common Cranes in London, where powre is to Crane vp, the waight of 2000. pound: by two Wheles more (by good order added) Arte concludeth, that there may be Craned vp 200000. pound waight &c. So well knew *Archimedes* this Arte: that he alone, with his deuiſes and engynes, (twiſe or thriſe) ſpoyled and diſcomfited the whole Army and Hoſte of the Romaines, beſieging *Syracuſa*, *Marcus Marcellus the Conſul*, being their Generall Capitaine. Such huge Stones, ſo many, with ſuch force, and ſo farre, did he with his engynes hayle among them, out of the Citie. And by Sea likewiſe: though their Ships might come to the walls of *Syracuſa*, yet hee vtterly confounded the Romaine Nauye. What with his mighty Stones hurlyng: what with Pikes of* 18 fote long, made like ſhaftes: which he forced almoſt a quarter of a myle. What, with his catchyng hold of their Shyps, and hoyſing them vp aboue the water, and ſuddenly letting them fall into the Sea againe: what with his* Burning Glaſſes: by which he fired their other Shippes a farof: what, with his other pollicies, deuiſes, and engines, he ſo manfully acquit him ſelfe: that all the Force, courage, and pollicie of the Romaines (for a great ſeaſon)

Plutarchus in Marco Marcello.
Syneſius in Epiſtolis.
Polybius.
Plinius.
Quintilianus.
T. Liuius.
**Athenæus.*

* *Galenus.*
Anthemius.

could

could nothing preuaile, for the winning of Syracusa. Wherupon, the Romanes named *Archimedes*, *Briareus*, and *Centimanus*. *Zonaras* maketh mention of one *Proclus*, who so well had perceiued *Archimedes* Arte of *Menadrie*, and had so well inuented of his owne, that with his Burning Glasses, being placed vpon the walles of Bysance, he multiplied so the heate of the Sunne, and directed the beames of the same against his enemies Nauie with such force, and so sodeinly (like lightening) that he burned and destroyed both man and ship. And *Dion* specifieth of *Priscus*, a *Geometricien* in Bysance, who inuented and vsed sondry Engins, of Force multiplied: Which was cause, that the *Emperour Seuerus* pardoned him, his life, after he had wonne Bysance: Bycause he honored the Arte, wytt, and rare industrie of *Priscus*. But nothing inferior to the inuention of these engines of Force, was the inuention of Gunnes. Which, from an English man, had the occasion and order of first inuenting: though in an other land, and by other men, it was first executed. And they that should see the record, where the occasion and order generall, of Gunning, is first discoursed of, would thinke: that, small thinges, slight, and cõmon: comming to wise mens consideration, and industrious mens handling, may grow to be of force incredible.

Burning Glasses.

Gunnes.

Hypogeiodie, is an Arte Mathematicall, demonstratyng, how, vnder the Sphæricall Superficies of the earth, at any depth, to any perpendicular line assigned (whose distance from the perpendicular of the entrance: and the Azimuth, likewise, in respect of the said entrance, is knowen) certaine way may be præscribed and gone: And how, any way aboue the Superficies of the earth designed, may vnder earth, at any depth limited, be kept: goyng alwayes, perpendicularly, vnder the way, on earth designed: And, contrarywise, Any way, (straight or croked,) vnder the earth, beyng giuen: vppon the vtface, or Superficies of the earth, to Lyne out the same: So, as, from the Centre of the earth, perpendiculars drawen to the Sphæricall Superficies of the earth, shall precisely fall in the Correspondent pointes of those two wayes. This, with all other Cases and circumstances herein, and appertenances, this Arte demonstrateth. This Arte, is very ample in varietie of Conclusions: and very profitable sundry wayes to the Common Wealth. The occasion of my Inuenting this Arte, was at the request of two Gentlemen, who had a certaine worke (of gaine) vnder ground: and their groundes did ioyne ouer the worke: and by reason of the crokednes, diuers depthes, and heithes of the way vnder ground, they were in doubt, and at controuersie, vnder whose ground, as then, the worke was. The name onely (before this) was of me published, *De Itinere Subterraneo*: The rest, be at Gods will. For Pioners, Miners, Diggers for Mettalls, Stone, Cole, and for secrete passages vnder ground, betwene place and place (as this land hath diuerse) and for other purposes, any man may easily perceaue, both the great fruite of this Arte, and also in this Arte, the great aide of Geometrie.

Hydragogie, demonstrateth the possible leading of Water, by Natures lawe, and by artificiall helpe, from any head (being a Spring, standing, or running Water) to any other place assigned.

Long

Long, hath this Arte bene in vſe: and much thereof written: and very marueilous workes therein, performed: as may yet appeare, in Italy: by the Ruynes remaining of the Aqueductes. In other places, of Riuers leading through the Maine land, Nauigable many a Mile. And in other places, of the marueilous forcinges of Water to Aſcend. which all, declare the great ſkill, to be required of him, who ſhould in this Arte be perfecte, for all occaſions of waters poſſible leading. To ſpeake of the allowance of the Fall, for euery hundred foote: or of the Ventills (if the waters labour be farre, and great) I neede not: Seing, at hand (about vs) many expert men can ſufficiently teſtifie, in effecte, the order: though the Demonſtration of the Neceſſitie thereof, they know not: Nor yet, if they ſhould be led, vp and downe, and about Mountaines, from the head of the Spring: and then, a place being aſſigned: and of them, to be demaunded, how low or high, that laſt place is, in reſpecte of the head, from which (ſo crokedly, and vp and downe) they be come: Perhaps, they would not, or could not, very redily, or nerely aſſoyle that queſtion. *Geometrie* therefore, is neceſſary to *Hydragogie*. Of the ſundry wayes to force water to aſcend, eyther by *Tympane*, *Kettell mills*, *Skrue*, *Cteſibike*, or ſuch like: in *Vitruuius*, *Agricola*, (and other,) fully, the maner may appeare. And ſo, thereby, alſo be moſt euident, how the Artes, of *Pneumatithmie*, *Helicoſophie*, *Statike*, *Trochilike*, and *Menadrie*, come to the furniture of this, in Speculation, and to the Commoditie of the Common Wealth, in practiſe.

Horometrie, is an Arte Mathematicall, which demõſtrateth, how, at all times appointed, the preciſe vſuall denominatiõ of time, may be knowen, for any place aſſigned. Theſe wordes, are ſmoth and plaine eaſie Engliſhe, but the reach of their meaning, is farther, then you woulde lightly imagine. Some part of this Arte, was called in olde time, *Gnomonice:* and of late, *Horologiographia:* and in Engliſhe, may be termed, *Dialling*. Auncient is the vſe, and more auncient, is the Inuention. The vſe, doth well appeare to haue bene (at the leaſt) aboue two thouſand and three hundred yeare agoe: in * King *Achaz* Diall, then, by the Sunne, ſhewing the diſtinction of time. By Sunne, Mone, and Sterres, this Dialling may be performed, and the preciſe Time of day or night knowen. But the demonſtratiue delineation of theſe Dialls, of all ſortes, requireth good ſkill, both of *Aſtronomie*, and *Geometrie* Elementall, Sphæricall, Phænomenall, and Conikall. Then, to vſe the groundes of the Arte, for any regular Superficies, in any place offred: and (in any poſſible apt poſition therof) theron, to deſcribe (all maner of wayes) how, vſuall howers, may be (by the *Sunnes* ſhadow) truely determined: will be found no ſleight Painters worke. So to Paint, and preſcribe the Sunnes Motion, to the breadth of a heare. In this Feate (in my youth) I Inuented a way, How in any Horizontall, Murall, or Æquinoctiall Diall, &c. At all howers (the Sunne ſhining) the Signe and Degree aſcendent, may be knowen. Which is a thing very neceſſary for the Riſing of thoſe fixed Sterres: whoſe Operation in the Ayre, is of great might, euidently. I ſpeake no further, of the vſe hereof. But foraſmuch as, Mans affaires require knowledge of Times & Momentes, when, neither Sunne, Mone, or Sterre, can be ſene: Therefore, by Induſtrie Mechanicall, was inuented, firſt, how, by Water, running orderly, the Time and howers might be knowen: whereof, the famous *Cteſibius*, was Inuentor: a man, of *Vitruuius*, to the Skie (iuſtly) extolled. Then, after that, by Sand running, were howers meaſured: Then, by *Trochilike* with waight: And of late time, by *Trochilike* with Spring: without waight. All theſe,

4. *Reg.* 20.

by Sunne or Sterres direction (in certaine time) require ouerſight and reformation, according to the heauenly Æquinoctiall Motion: beſides the inæqualitie of their owne Operation. There remayneth (without parabolicall meaning herein) among the Philoſophers, a more excellent, more commodious, and more maruelous way, then all theſe: of hauing the motion of the Primouant (or firſt æquinoctiall motion,) by Nature and Arte, Imitated: which you ſhall (by furder ſearch in waighter ſtudyes) hereafter, vnderſtand more of. And ſo, it is tyme to finiſh this Annotation, of Tymes diſtinction, vſed in our common, and priuate affaires: The commoditie wherof, no man would want, that can tell, how to beſtow his tyme.

A perpetuall Motion.

Zographie, is an Arte Mathematicall, which teacheth and demonſtrateth, how, the Interſection of all viſuall Pyramides, made by any playne aſsigned, (the Centre, diſtance, and lightes, beyng determined) may be, by lynes, and due propre colours, repreſented. A notable Arte, is this: and would require a whole Volume, to declare the property thereof: and the Commodities enſuyng. Great ſkill of *Geometrie*, *Arithmetike*, *Perſpectiue*, and *Anthropographie*, with many other particular Artes, hath the *Zographer*, nede of, for his perfection. For, the moſt excellent Painter, (who is but the propre Mechanicien, & Imitator ſenſible, of the Zographer) hath atteined to ſuch perfection, that Senſe of Man and beaſt, haue iudged thinges painted, to be things naturall, and not artificiall: aliue, and not dead. This Mechanicall Zographer (commonly called the Painter) is meruailous in his ſkill: and ſeemeth to haue a certaine diuine power: As, of frendes abſent, to make a frendly, preſent comfort: yea, and of frendes dead, to giue a continuall, ſilent preſence: not onely with vs, but with our poſteritie, for many Ages. And ſo procedyng, Conſider, How, in Winter, he can ſhew you, the liuely vew of Sommers Ioy, and riches: and in Sommer, exhibite the countenance of Winters dolefull State, and nakednes. Cities, Townes, Fortes, Woodes, Armyes, yea whole Kingdomes (be they neuer ſo farre, or greate) can he, with eaſe, bring with him, home (to any mans Iudgement) as Paternes liuely, of the thinges rehearſed. In one little houſe, can he, encloſe (with great pleaſure of the beholders,) the portrayture liuely, of all viſible Creatures, either on earth, or in the earth, liuing: or in the waters lying, Creping, ſlyding, or ſwimming: or of any foule, or fly, in the ayre flying. Nay, in reſpect of the Starres, the Skie, the Cloudes: yea, in the ſhew of the very light it ſelfe (that Diuine Creature) can he match our eyes Iudgement, moſt nerely. What a thing is this? thinges not yet being, he can repreſent ſo, as, at their being, the Picture ſhall ſeame (in maner) to haue Created them. To what Artificer, is not Picture, a great pleaſure and Commoditie? Which of them all, will refuſe the Direction and ayde of Picture? The Architect, the Goldſmith, and the Arras Weauer: of Picture, make great account. Our liuely Herbals, our portraitures of birdes, beaſtes, and fiſhes: and our curious Anatomies, which way, are they moſt perfectly made, or with moſt pleaſure, of vs beholden? Is it not, by Picture onely? And if Picture, by the Induſtry of the Painter, be thus commodious and meruailous: what ſhall be thought of *Zographie*, the Scholemaſter of Picture, and chief gouernor? Though I mencion not *Sculpture*, in my Table of Artes Mathematicall: yet may all men perceiue, How, that *Picture* and *Sculpture*, are Siſters germaine: and both, right profitable, in a Commō wealth. and of *Sculpture*, aſwell as of Picture, excellent Artificers haue written great bokes in commendation. Witneſſe I take, of *Georgio Vaſari*, *Pittore Aretino*: of *Pomponius Gauricus*: and other. To theſe two Artes, (with other,) is a certaine od Arte, called *Althalmaſat*, much beholdyng: more, then the common *Sculptor*, *Entayler*, *Keruer*, *Cutter*, *Grauer*, *Founder*,

der, or *Paynter* (*&c*) know their Arte, to be commodious.

Architecture, to many may ſeme not worthy, or not mete, to be reckned among the *Artes Mathematicall*. To whom, I thinke good, to giue ſome account of my ſo doyng. Not worthy, (will they ſay,) bycauſe it is but for building, of a houſe, Pallace, Church, Forte, or ſuch like, groſſe workes. And you, alſo, defined the *Artes Mathematicall*, to be ſuch, as dealed with no Materiall or corruptible thing: and alſo did demonſtratiuely procede in their faculty, by Number or Magnitude. Firſt, you ſee, that I count, here, *Architecture*, among thoſe *Artes Mathematicall*, which are Deriued from the Principals: and you know, that ſuch, may deale with Naturall thinges, and ſenſible matter. Of which, ſome draw nerer, to the Simple and abſolute Mathematicall Speculation, then other do. And though, the *Architect* procureth, enformeth, & directeth, the *Mechanicien*, to handworke, & the building actuall, of houſe, Caſtell, or Pallace, and is chief Iudge of the ſame: yet, with him ſelfe (as chief *Maſter* and *Architect*,) remaineth the Demonſtratiue reaſon and cauſe, of the Mechaniciens worke: in Lyne, plaine, and Solid: by *Geometricall*, *Arithmeticall*, *Opticall*, *Muſicall*, *Aſtronomicall*, *Coſmographicall* (& to be brief) by all the former Deriued *Artes Mathematicall*, and other Naturall Artes, hable to be confirmed and ſtabliſhed. If this be ſo: then, may you thinke, that *Architecture*, hath good and due allowance, in this honeſt Company of *Artes Mathematicall* Deriuatiue. I will, herein, craue Iudgement of two moſt perfect *Architectes*: the one, being *Vitruuius*, the Romaine: who did write ten bookes thereof, to the Emperour *Auguſtus* (in whoſe daies our Heauenly Archemaſter, was borne): and the other, *Leo Baptiſta Albertus*, a Florentine: who alſo publiſhed ten bookes therof. *Architectura* (ſayth *Vitruuius*) *eſt Scientia pluribus diſciplinis & varijs eruditionibus ornata: cuius Iudicio probantur omnia, quæ ab cæteris Artificibus perficiuntur opera*. That is. Architecture, is a Science garniſhed with many doctrines & diuerſe inſtructions: by whoſe Iudgement, all workes, by other workmen finiſhed, are Iudged. It followeth. *Ea naſcitur ex Fabrica, & Ratiocinatione. &c. Ratiocinatio autem eſt, quæ, res fabricatas, Solertia ac ratione proportionis, demonſtrare atq; explicare poteſt*. *Architecture, groweth of Framing, and Reaſoning. &c. Reaſoning, is that, which of thinges framed, with forecaſt, and proportion: can make demonſtration, and manifeſt declaration*. Againe. *Cùm, in omnibus enim rebus, tùm maximè etiam in Architectura, hæc duo inſunt: quod ſignificatur, & quod ſignificat. Significatur propoſita res, de qua dicitur: hanc autem Stgnificat Demonſtratio, rationibus doctrinarum explicata*. *Foraſmuch as, in all thinges: therefore chiefly in Architecture, theſe two thinges are: the thing ſignified: and that which ſignifieth. The thing propounded, whereof we ſpeake, is the thing Signified. But Demonſtration, expreſſed with the reaſons of diuerſe doctrines, doth ſignifie the ſame thing*. After that. *Vt literatus ſit, peritus Graphidos, eruditus Geometriæ, & Optices non ignarus: inſtructus Arithmetica: hiſtorias complures nouerit, Philoſophos diligenter audiuerit: Muſicam ſciuerit: Medicinæ non ſit ignarus, reſponſa Iuriſperitorũ nouerit: Aſtrologiam, Cæliq; rationes cognitas habeat*. *An Architect* (ſayth he) *ought to vnderſtand Languages, to be ſkilfull of Painting, well inſtructed in Geometrie, not ignorant of Perſpectiue, furniſhed with Arithmetike, haue knowledge of many hiſtories, and diligently haue heard Philoſophers, haue ſkill of Muſike, not ignorant of Phyſike, know the aunſweres of Lawyers, and haue Aſtro-*

An obiection.

The Anſwer.

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 nomie,

nomie, and the courses Cælestiall, in good knowledge. He geueth reason, orderly, wherefore all these Artes, Doctrines, and Instructions, are requisite in an excellent *Architect*. And (for breuitie) omitting the Latin text, thus he hath. *Secondly, it is behofefull for an Architect to haue the knowledge of Painting: that he may the more easilie fashion out, in patternes painted, the forme of what worke he liketh. And Geometrie, geueth to Architecture many helpes: and first teacheth the Vse of the Rule, and the Cumpasse: wherby (chiefly and easilie) the descriptions of Buildinges, are despatched in Groundplats: and the directions of Squires, Leuells, and Lines. Likewise, by Perspectiue, the Lightes of the heauen, are well led, in the buildinges: from certaine quarters of the world. By Arithmetike, the charges of Buildinges are summed together: the measures are expressed, and the hard questions of Symmetries, are by Geometricall Meanes and Methods discoursed on. &c. Besides this, of the Nature of thinges (which in Greke is called φυσιολογία) Philosophie doth make declaration. Which, it is necessary, for an Architect, with diligence to haue learned: because it hath many and diuers naturall questions: as specially, in Aqueductes. For in their courses, leadinges about, in the leuell ground, and in the mountinges, the naturall Spirites or breathes are ingendred diuers wayes: The hindrances, which they cause, no man can helpe, but he, which out of Philosophie, hath learned the originall causes of thinges. Likewise, whosoeuer shall read Ctesibius, or Archimedes bookes, (and of others, who haue written such Rules) can not thinke, as they do: vnlesse he shall haue receaued of Philosophers, instructions in these thinges. And Musike he must nedes know: that he may haue vnderstanding, both of Regular and Mathematicall Musike: that he may temper well his Balistes, Catapultes, and Scorpions. &c. Moreouer, the Brasen Vessels, which in Theatres, are placed by Mathematicall order, in ambries, vnder the steppes: and the diuersities of the soundes (which ỹ Grecians call ἠχεῖα) are ordred according to Musicall Symphonies & Harmonies: being distributed in ỹ Circuites, by Diatessaron, Diapente, and Diapason. That the conuenient voyce, of the players sound, whẽ it came to these preparations, made in order, there being increased: with ỹ increasing, might come more cleare & pleasant, to ỹ eares of the lokers on. &c. And of Astronomie, is knowẽ ỹ East, West, South, and North. The fashion of the heauen, the Æquinox, the Solsticie, and the course of the sterres. Which thinges, vnleast one know: he can not perceiue, any thyng at all, the reason of Horologies. Seyng therfore this ample Science, is garnished, beautified and stored, with so many and sundry / kils and knowledges: I thinke, that none can iustly account them selues Architectes, of the sndde yne. But they onely, who from their childes yeares, ascendyng by these degrees of knowledges, beyng fostered vp with the atteynyng of many Languages and Artes, haue wonne to the high Tabernacle of Architecture. &c. And to whom Nature hath giuen such quicke Circumspection, sharpnes of witt, and Memorie, that they may be very absolutely skillfull in Geometrie, Astronomie, Musike, and the rest of the Artes Mathematicall:*

call: Such, ſurmount and paſſe the callyng, and ſtate, of Architectes: and are become Mathematiciens. &c. And they are found, ſeldome. As, in tymes paſt, was Ariſtarchus Samius: Philolaus, and Archytas, Tarentynes: Apollonius Pergeus: Eratoſthenes Cyreneus: Archimedes, and Scopas, Syracuſians. Who alſo, left to theyr poſteritie, many Engines and Gnomonicall workes: by numbers and naturall meanes, inuented and declared. *A Mathematicien.*

Thus much, and the ſame wordes (in ſenſe) in one onely Chapter of this Incõparable *Architect Vitruuius*, ſhall you finde. And if you ſhould, but take his boke in your hand, and ſlightly loke thorough it, you would ſay ſtraight way: This is *Geometrie, Arithmetike, Aſtronomie, Muſike, Anthropographie, Hydragogie, Horometrie. &c.* and (to cõclude) the Storehouſe of all workmãſhip. Now, let vs liſten to our other Iudge, our Florentine, *Leo Baptiſta:* and narrowly conſider, how he doth determine of *Architecture. Sed anteq; vltra progrediar. &c. But before I procede any further* (ſayth he) *I thinke, that I ought to expreſſe, what man I would haue to bee allowed an Architect. For, I will not bryng in place a Carpenter: as though you might Compare him to the Chief Maſters of other Artes. For the hand of the Carpenter, is the Architectes Inſtrument. But I will appoint the Architect to be that man, who hath the ſkill, (by a certaine and meruailous meanes and way,) both in minde and Imagination to determine: and alſo in worke to finiſh: what workes ſo euer, by motion of waight, and cuppling and framyng together of bodyes, may moſt aptly be Commodious for the worthieſt Vſes of Man. And that he may be able to performe theſe thinges, he hath nede of atteynyng and knowledge of the beſt, and moſt worthy thynges. &c. The whole Feate of Architecture in buildyng, conſiſteth in Lineamentes, and in Framyng. And the whole power and ſkill of Lineamentes, tendeth to this: that the right and abſolute way may be had, of Coaptyng and ioyning Lines and angles: by which, the face of the buildyng or frame, may be comprehended and concluded. And it is the property of Lineamentes, to preſcribe vnto buildynges, and euery part of them, an apt place, & certaine nũber: a worthy maner, and a ſemely order: that, ſo, ẏ whole forme and figure of the buildyng, may reſt in the very Lineamentes. &c. And we may preſcribe in mynde and imagination the whole formes, * all materiall ſtuffe beyng ſecluded. Which point we ſhall atteyne, by Notyng and forepointyng the angles, and lines, by a ſure and certaine direction and connexion. Seyng then, theſe thinges, are thus: Lineamente, ſhalbe the certaine and conſtant preſcribyng, conceiued in mynde: made in lines and angles: and finiſhed with a learned minde and wyt.* We thanke you Maſter *Baptiſt*, that you haue ſo aptly brought your Arte, and phraſe therof, to haue ſome Mathematicall perfection: by certaine order, nũber, forme, figure, and *Symmetrie* mentall: all naturall & ſenſible ſtuffe ſet a part. Now, then, it is euident, (Gentle reader) how aptely and worthely, I haue preferred *Architecture*, to be bred and foſtered vp in the Dominion of the peerles *Princeſſe, Mathematica:* and to be a naturall Subiect of hers. And the name of *Architecture*, is of the principalitie, which this Science hath, aboue all other Artes. And *Plato* affirmeth, the *Architect* to be *Maſter* ouer all, that make any worke. Wherupon, he is neither Smith, nor Builder: nor, ſeparately, any Artificer: but the

Vitruuius.

Who is an Architect.

** The Immaterialitie of perfect Architecture.*

What, Lineament is.

Note.

Hed, the Prouoſt, the Director, and Iudge of all Artificiall workes, and all Artificers. For, the true *Architect*, is hable to teach, Demonſtrate, diſtribute, deſcribe, and Iudge all workes wrought. And he, onely, ſearcheth out the cauſes and reaſons of all Artificiall thynges. Thus excellent, is *Architecture*: though few (in our dayes) atteyne thereto: yet may not the Arte, be otherwiſe thought on, then in very dede it is worthy. Nor we may not, of auncient Artes, make new and imperfect Definitions in our dayes: for ſcarſitie of Artificers: No more, than we may pynche in, the Definitions of *Wiſedome*, or *Honeſtie*, or of *Frendeſhyp* or of *Iuſtice*. No more will I conſent, to Diminiſh any whit, of the perfection and dignitie, (by iuſt cauſe) allowed to abſolute *Architecture*. Vnder the Direction of this Arte, are thre principall, neceſſary *Mechanicall Artes*. Namely, *Howſing*, *Fortification*, and *Naupegie*. *Howſing*, I vnderſtand, both for Diuine Seruice, and Mans common vſage: publike, and priuate. Of *Fortification* and *Naupegie*, ſtraunge matter might be told you: But perchaunce, ſome will be tyred, with this Bederoll, all ready rehearſed: and other ſome, will nycely nip my groſſe and homely diſcourſing with you: made in poſt haſt: for feare you ſhould wante this true and frendly warnyng, and taſt giuyng, of the *Power Mathematicall*. Lyfe is ſhort, and vncertaine: Tymes are perilouſe: &c. And ſtill the Printer awayting, for my pen ſtaying: All theſe thinges, with farder matter of Ingratefulnes, giue me occaſion to paſſe away, to the other Artes remainyng, with all ſpede poſsible.

THe Arte of Nauigation, demonſtrateth how, by the ſhorteſt good way, by the apteſt Directiō, & in the ſhorteſt time, a ſufficient Ship, betwene any two places (in paſſage Nauigable,) aſsigned: may be cōducted: and in all ſtormes, & naturall diſturbances chauncyng, how, to vſe the beſt poſsible meanes, whereby to recouer the place firſt aſsigned. What nede, the *Maſter Pilote*, hath of other Artes, here before recited, it is eaſie to know: as, of *Hydrographie*, *Aſtronomie*, *Aſtrologie*, and *Horometrie*. Preſuppoſing continually, the common Baſe, and foundacion of all: namely *Arithmetike* and *Geometrie*. So that, he be hable to vnderſtand, and Iudge his own neceſſary Inſtrumentes, and furniture Neceſſary: Whether they be perfectly made or no: and alſo can, (if nede be) make them, hym ſelfe. As Quadrantes, The Aſtronomers Ryng, The Aſtronomers ſtaffe, The Aſtrolabe vniuerſall. An Hydrographicall Globe. Charts Hydrographicall, true, (not with parallell Meridians). The Common Sea Compas: The Compas of variacion: The Proportionall, and Paradoxall Compaſſes (of me Inuented, for our two Moſcouy Maſter Pilotes, at the requeſt of the Company) Clockes with ſpryng: houre, halfe houre, and three houre Sandglaſſes: & ſundry other Inſtrumētes: And alſo, be hable, on Globe, or Playne to deſcribe the Paradoxall Compaſſe: and duely to vſe the ſame, to all maner of purpoſes, whereto it was inuented. And alſo, be hable to Calculate the Planetes places for all tymes. *Anno.* 1559.

Moreouer, with Sonne Mone or Sterre (or without) be hable to define the Longitude & Latitude of the place, which he is in: So that, the Longitude & Latitude of the place, from which he ſayled, be giuen: or by him, be knowne. whereto, appertayneth expert meanes, to be certified euer, of the Ships way. &c. And by foreſeing the Riſing, Settyng, Noneſtedyng, or Midnightyng of certaine tempeſtuous fixed Sterres: or their Coniunctions, and Anglynges with the Planetes, &c. he ought to haue expert coniecture of Stormes, Tempeſtes, and Spoutes: and ſuch lyke Meteorologicall effectes, daungerous on Sea. For (as *Plato* ſayth,) *Mutationes*,

oppor-

opportunitatesq́, temporum presentire, non minus rei militari, quàm Agriculturæ, Nauigationiq́, conuenit. *To foresee the alterations and opportunities of tymes, is conuenient, no leſse to the Art of Warre, then to Huſbandry and Nauigation.* And beſides ſuch cunnyng meanes, more euident tokens in Sonne and Mone, ought of hym to be knowen: ſuch as (the Philoſophicall Poëte) *Virgilius* teacheth, in hys *Georgikes*. Where he ſayth,

Sol quoq́, & exoriens & quum ſe condet in vndas,
Signa dabit, Solem certiſsima ſigna ſequuntur. &c.
——— Nam ſæpe videmus,
Ipſius in vultu varios errare colores. Georgic. 1.
Cæruleus, pluuiam denunciat, igneus Euros.
Sin maculæ incipient rutilo immiſcerier igni,
Omnia tum pariter vento, nimbiſq́, videbis
Feruere: non illa quiſquam me nocte per altum
Ire, neq́, a terra moueat conuellere funem. &c.
Sol tibi ſigna dabit. Solem quis dicere falſum
Audeat? ——— &c.

And ſo of Mone, Sterres, Water, Ayre, Fire, Wood, Stones, Birdes, and Beaſtes, and of many thynges els, a certaine Sympathicall forewarnyng may be had: ſometymes to great pleaſure and proffit, both on Sea and Land. Sufficiently, for my preſent purpoſe, it doth appeare, by the premiſſes, how *Mathematicall*, the *Arte* of *Nauigation*, is: and how it nedeth and alſo vſeth other *Mathematicall Artes*: And now, if I would go about to ſpeake of the manifold Commodities, commyng to this Land, and others, by Shypps and *Nauigation*, you might thinke, that I catch at occaſions, to vſe many wordes, where no nede is.

Yet, this one thyng may I, (iuſtly) ſay. In *Nauigation*, none ought to haue greater care, to be ſkillfull, then our Engliſh Pylotes. And perchaunce, Some, would more attempt: And other Some, more willingly would be aydyng, if they wiſt certainely, What Priuiledge, God had endued this Iland with, by reaſon of Situation, moſt commodious for *Nauigation*, to Places moſt Famous & Riche. And though, (of* Late) a young Gentleman, a Courragious Capitaine, was in a great readynes, with good hope, and great cauſes of perſuaſion, to haue ventured, for a Diſcouerye, (either *Weſterly*, by *Cape de Paramantia*: or *Eſterly*, aboue *Noua Zemla*, and the *Cyremiſſes*) and was, at the very nere tyme of Attemptyng, called and employed otherwiſe (both then, and ſince,) in great good ſeruice to his Countrey, as the Iriſh Rebels haue* taſted: Yet, I ſay, (though the ſame Gentleman, doo not hereafter, deale therewith) Some one, or other, ſhould liſten to the Matter: and by good aduiſe, and diſcrete Circumſpection, by little, and little, wynne to the ſufficient knowledge of that Trade and Voyage: Which, now, I would be ſory, (through Careleſneſſe, want of Skill, and Courrage,) ſhould remayne Vnknowne and vnheard of. Seyng, alſo, we are herein, halfe Challenged, by the learned, by halfe requeſt, publiſhed. Therof, verely, might grow Commoditye, to this Land chiefly, and to the reſt of the Chriſten Common wealth, farre paſſing all riches and worldly Threaſure.

*Anno. 1567 S. H. G.

*Anno. 1569

Thaumaturgike, is that Art Mathematicall, which giueth certaine order to make ſtraunge workes, of the ſenſe to be perceiued, and of men greatly to be wondred at. By ſundry meanes, this *Wonderworke* is wrought. Some, by *Pneumatithmie*. As the workes of *Cteſibius* and *Hero*,

Some by waight, wherof *Timæus* speaketh. Some, by Stringes strayned, or Springs, therwith Imitating liuely Motions. Some, by other meanes, as the Images of Mercurie: and the brasen hed, made by *Albertus Magnus*, which dyd seme to speake. *Boethius* was excellent in these feates. To whom, *Cassiodorus* writyng, sayth. *Your purpose is to know profound thynges: and to shew meruayles. By the disposition of your Arte, Metals do low: Diomedes of brasse, doth blow a Trumpet loude: a brasen Serpent hisseth: byrdes made, sing swetely. Small thynges we rehearse of you, who can Imitate the heauen. &c.* Of the straunge Selfmouyng, which, at Saint Denys, by Paris, * I saw, ones or twise (*Orontius* beyng then with me, in Company) it were to straunge to tell. But some haue written it. And yet, (I hope) it is there, of other to be sene. And by *Perspectiue* also straunge thinges, are done. As partly (before) I gaue you to vnderstand in *Perspectiue*. As, to see in the Ayre, a loft, the lyuely Image of an other man, either walkyng to and fro: or standyng still. Likewise, to come into an house, and there to see the liuely shew of Gold, Siluer or precious stones: and commyng to take them in your hand, to finde nought but Ayre. Hereby, haue some men (in all other matters counted wise) fouly ouershot thē selues: misdeaming of the meanes. Therfore sayd *Claudius Cælestinus. Hodie magnæ literaturæ viros & magnæ reputationis videmus, opera quedam quasi miranda, supra Naturā putare: de quibus in Perspectiua doctus causam faciliter reddidisset.* That is. *Now a dayes, we see some men, yea of great learnyng and reputation, to Iudge certain workes as meruaylous, aboue the power of Nature: Of which workes, one that were skillfull in Perspectiue might easely haue giuen the Cause.* Of *Archimedes Sphære*, *Cicero* witnesseth. Which is very straunge to thinke on. *For when Archimedes* (sayth he) *did fasten in a Sphære, the mouynges of the Sonne, Mone, and of the fiue other Planets, he did, as the God, which (in Timæus of Plato) did make the world. That, one turnyng, should rule motions most vnlike in slownes, and swiftnes.* But a greater cause of meruayling we haue by *Claudianus* report hereof. Who affirmeth this *Archimedes worke*, to haue ben of Glasse. And discourseth of it more at large: which I omit. The Doue of wood, which the *Mathematicien Archytas* did make to flye, is by *Agellius* spoken of. Of *Dædalus* straunge Images, *Plato* reporteth. *Homere* of *Vulcans Selfmouers*, (by secret wheles) leaueth in writyng. *Aristotle*, in hys *Politikes*, of both, maketh mention. Meruaylous was the workemanshyp, of late dayes, performed by good skill of *Trochilike. &c.* For in Noremberge, A flye of Iern, beyng let out of the Artificers hand, did (as it were) fly about by the gestes, at the table, and at length, as though it were weary, retourne to his masters hand agayne. Moreouer, an Artificiall Egle, was ordred, to fly out of the same Towne, a mighty way, and that a loft in the Ayre, toward the Emperour comming thether: and followed hym, beyng come to the gate of the towne. * Thus, you see, what, Arte Mathematicall can performe, when Skill, will, Industry, and Habilitie, are duely applyed to profe.

*Anno. 1551

De his quæ Mundo mirabiliter eueniunt. cap. 8.

Tusc. 1.

☞ *

A Digression. Apologeticall.

AND for these, and such like marueilous Actes and Feates, Naturally, Mathematically, and Mechanically, wrought and contriued: ought any honest Student, and Modest Christian Philosopher, be counted, & called a Coniurer? Shall the folly of Idiotes, and the Mallice of the Scornfull, so much preuaile, that He, who seeketh no worldly gaine or glory at their handes: But onely, of God, the threasor of heauenly wisedome, & knowledge of pure veritie: Shall he (I say) in the meane space,

ſpace, be robbed and ſpoiled of his honeſt name and fame? He that ſeketh (by S. Paules aduertiſement) in the Creatures Properties, and wonderfull vertues, to finde iuſte cauſe, to glorifie the Æternall, and Almightie Creator by: Shall that man, be (in hugger mugger) condemned, as a Companion of the Helhoundes, and a Caller, and Coniurer of wicked and damned Spirites? He that bewaileth his great want of time, ſufficient (to his contentation) for learning of Godly wiſdome, and Godly Verities in: and onely therin ſetteth all his delight: Will that mã leeſe and abuſe his time, in dealing with the Chiefe enemie of Chriſt our Redemer: the deadly foe of all mankinde: the ſubtile and impudent peruerter of Godly Veritie: the Hypocriticall Crocodile: the Enuious Baſiliſke, continually deſirous, in the twinke of an eye, to deſtroy all Mankinde, both in Body and Soule, æternally? Surely (for my part, ſomewhat to ſay herein) I haue not learned to make ſo brutiſh, and ſo wicked a Bargaine. Should I, for my xx. or xxv. yeares Studie: for two or three thouſand Markes ſpending: ſeuen or eight thouſand Miles going and trauailing, onely for good learninges ſake: And that, in all maner of wethers: in all maner of waies and paſſages: both early and late: in daunger of violence by man: in daunger of deſtruction by wilde beaſtes: in hunger: in thirſt: in perilous heates by day, with toyle on foote: in daungerous dampes of colde, by night, almoſt bereuing life: (as God knoweth): with lodginges, oft times, to ſmall eaſe: and ſomtime to leſſe ſecuritie. And for much more (then all this) done & ſuffred, for Learning and attaining of Wiſedome: Should I (I pray you) for all this, no otherwiſe, nor more warily: or (by Gods mercifulnes) no more luckily, haue fiſhed, with ſo large, and coſtly, a Nette, ſo long time in drawing (and that with the helpe and aduiſe of Lady Philoſophie, & Queene Theologie): but at length, to haue catched, and drawen vp, * a Frog? Nay, a Deuill? For, ſo, doth the Common peuiſh Pratler Imagine and Iangle: And, ſo, doth the Malicious ſkorner, ſecretly wiſhe, & brauely and boldly face down, behinde my backe. Ah, what a miſerable thing, is this kinde of Men? How great is the blindnes & boldnes, of the Multitude, in thinges aboue their Capacitie? What a Land: what a People: what Maners: what Times are theſe? Are they become Deuils, them ſelues: and, by falſe witneſſe bearing againſt their Neighbour, would they alſo, become Murderers? Doth God, ſo long geue them reſpite, to reclaime them ſelues in, from this horrible ſlaundering of the giltleſſe: contrary to their owne Conſciences: and yet will they not ceaſe? Doth the Innocent, forbeare the calling of them, Iuridically to aunſwere him, according to the rigour of the Lawes: and will they deſpiſe his Charitable pacience? As they, againſt him, by name, do forge, fable, rage, and raiſe ſlaunder, by Worde & Print: Will they prouoke him, by worde and Print, likewiſe, to Note their Names to the World: with their particular deuiſes, fables, beaſtly Imaginations, and vnchriſten-like ſlaunders? Well: Well. O (you ſuch) my vnkinde Countrey men. O vnnaturall Countrey men. O vnthankfull Countrey men. O Brainſicke, Raſhe, Spitefull, and Diſdainfull Countrey men. Why oppreſſe you me, thus violently, with your ſlaundering of me: Contrary to Veritie: and contrary to your owne Conſciences? And I, to this hower, neither by worde, deede, or thought, haue bene, any way, hurtfull, damageable, or iniurious to you, or yours? Haue I, ſo long, ſo dearly, ſo farre, ſo carefully, ſo painfully, ſo daungerouſly ſought & trauailed for the learning of Wiſedome, & atteyning of Vertue: And in the end (in your iudgemẽt) am I become, worſe, then when I begã? Worſe, thẽ a Mad man? A dangerous Member in the Common Wealth: and no Member of the Church of Chriſt? Call you this, to be Learned? Call you this, to be a Philoſopher? and a louer of Wiſedome? To forſake the ſtraight heauenly way: and to wallow in the broad way of

* *A prouerb. Fayre fiſht, and caught a Frog.*

damnation? To forsake the light of heauenly Wisedome: and to lurke in the dungeon of the Prince of darkenesse? To forsake the Veritie of God,& his Creatures: and to fawne vpon the Impudent,Craftie,Obstinate Lier, and continuall disgracer of Gods Veritie, to the vttermost of his power? To forsake the Life & Blisse Æternall: and to cleaue vnto the Author of Death euerlasting? that Murderous Tyrant, most gredily awaiting the Pray of Mans Soule? Well: I thanke God and our Lorde Iesus Christ, for the Comfort which I haue by the Examples of other men, before my time: To whom, neither in godlines of life, nor in perfection of learning, I am worthy to be compared: and yet, they sustained the very like Iniuries, that I do: or rather, greater. Pacient *Socrates*, his *Apologie* will testifie: *Apuleius* his *Apologies*, will declare the Brutishnesse of the Multitude. *Ioannes Picus*, Earle of Mirandula, his *Apologie* will teach you, of the Raging slaunder of the Malicious Ignorant against him. *Ioannes Trithemius*, his *Apologie* will specifie, how he had occasion to make publike Protestation: as well by reason of the Rude Simple: as also,in respect of such,as were counted to be of the wisest sort of men. Many could I recite: But I deferre the precise and determined handling of this matter: being loth to detect the Folly & Mallice of my Natiue Countrey men.*Who, so hardly, can disgest or like any extraordinary course of Philosophicall Studies: not falling within the Cumpasse of their Capacitie: or where they are not made priuie of the true and secrete cause, of such wonderfull Philosophicall Feates. These men, are of fower sortes, chiefly. The first, I may name, *Vaine pratling busie bodies*: The second, *Fond Frendes*: The third, *Imperfectly zelous*: and the fourth, *Malicious Ignorant*. To eche of these (briefly,and in charitie) I will say a word or two, and so returne to my Præface.

1. *Vaine pratling busie bodies*, vse your idle assemblies,and conferences, otherwise, then in talke of matter, either aboue your Capacities, for hardnesse: or contrary to your Consciences, in Veritie.

2. *Fonde Frendes*, leaue of,so to commend your vnacquainted frend,vpon blinde affection: As, because he knoweth more, then the common Student: that, therfore, he must needes be skilfull, and a doer, in such matter and maner, as you terme *Coniuring*. Weening, thereby, you aduaunce his fame: and that you make other men, great marueilers of your hap, to haue such a learned frend. Cease to ascribe Impietie, where you pretend Amitie. For,if your tounges were true, then were that your frend, *Vntrue*, both to God,and his Soueraigne. Such *Frendes* and *Fondlinges*, I shake of, and renounce you: Shake you of, your Folly.

3. *Imperfectly zelous*, to you, do I say: that (perhaps) well, do you Meane: But farre you misse the Marke: If a Lambe you will kill, to feede the flocke with his bloud. Sheepe, with Lambes bloud, haue no naturall sustenaunce: No more, is Christes flocke, with horrible slaunders, duely ædified. Nor your faire pretense, by such rashe ragged Rhetorike, any whit, well graced. But such, as so vse me, will finde a fowle Cracke in their Credite. Speake that you know: And know, as you ought: Know not, by Heare say, when life lieth in daunger. Search to the quicke,& let Charitie be your guide.

4. *Malicious Ignorant*, what shall I say to thee? *Prohibe linguam tuam a malo. A detractione parcite linguæ. Cause thy toung to refraine frō euill. Refraine your toung from slaunder*. Though your tounges be sharpned, Serpent like, & Adders poyson lye in your lippes: yet take heede,and thinke,betimes, with your selfe, *Vir linguosus non stabilietur in terra. Virum violentum venabitur malum, donec præcipitetur.* For,sure I am, *Quia faciet Dominus Iudicium afflicti: & vindictam pauperum.*

*Psal.*140.

Thus, I require you, my assured frendes, and Countrey men (you Mathematiciens, Mechaniciens,and Philosophers, Charitable and discrete) to deale in my behalfe,

behalf, with the light & vntrue tounged, my enuious Aduerſaries, or Fond frends. And farther, I would wiſhe, that at leyſor, you would conſider, how *Baſilius Magnus*, layeth *Moſes* and *Daniel*, before the eyes of thoſe, which count all ſuch Studies Philoſophicall (as mine hath bene) to be vngodly, or vnprofitable. Waye well *S. Stephen* his witneſſe of *Moſes*. *Eruditus eſt Moſes omni Sapientia Ægyptiorũ: & erat potens in verbis & operibus ſuis.* *Moſes was inſtructed in all maner of wiſedome of the Ægyptians: and he was of power both in his wordes, and workes.* Act. 7. C. You ſee this Philoſophicall Power & Wiſedome, which *Moſes* had, to be nothing miſliked of the Holy Ghoſt. Yet *Plinius* hath recorded, *Moſes* to be a wicked *Magicien*. And that (of force) muſt be, either for this Philoſophicall wiſedome, learned, before his calling to the leading of the Children of *Iſrael*: or for thoſe his wonders, wrought before King *Pharao*, after he had the conducting of the *Iſraelites*. As concerning the firſt, you perceaue, how *S. Stephen*, at his Martyrdome (being full of the Holy Ghoſt) in his Recapitulation of the olde Teſtament, hath made mention of *Moſes* Philoſophie: with good liking of it: And *Baſilius Magnus* alſo, auoucheth it, to haue bene to *Moſes* profitable (and therefore, I ſay, to the Church of God, neceſſary). But as cõcerning *Moſes* wonders, done before King *Pharao*: God, him ſelfe, ſayd: *Vide vt omnia oſtenta, quæ poſui in manu tua, facias coram Pharaone.* *See that thou do all thoſe wonders before Pharao, which I haue put in thy hand.* Thus, you euidently perceaue, how raſhly, *Plinius* hath ſlaundered *Moſes*, of vayne fraudulent *Magike*, ſaying: *Eſt & alia Magices Factio, a Moſe, Iamne, & Iotape, Iudæis pendens: ſed multis millibus annorum poſt Zoroaſtrem. &c.* Let all ſuch, therefore, who, in Iudgement and Skill of Philoſophie, are farre Inferior to *Plinie*, take good heede, leaſt they ouerſhoote them ſelues raſhly, in Iudging of *Philoſophers ſtraunge Actes*: and the Meanes, how they are done. But, much more, ought they to beware of forging, deuiſing, and imagining monſtrous feates, and wonderfull workes, when and where, no ſuch were done: no, not any ſparke or likelihode, of ſuch, as they, without all ſhame, do report. And (to conclude) moſt of all, let them be aſhamed of Man, and afraide of the dreadfull and Iuſte Iudge: both Foliſhly or Maliciouſly to deuiſe: and then, deuiliſhly to father their new fond Monſters on me: Innocent, in hand and hart: for treſpacing either againſt the lawe of God, or Man, in any my Studies or Exerciſes, Philoſophicall, or Mathematicall: As in due time, I hope, will be more manifeſt.

Lib. 30. Cap. 1.

1.

2.

3.

Now end I, with **Archemaſtrie**. Which name, is not ſo new, as this Arte is rare. For an other Arte, vnder this, a degree (for ſkill and power) hath bene indued with this Engliſh name before. And yet, this, may ſerue for our purpoſe, ſufficiently, at this preſent. This Arte, teacheth to bryng to actuall experience ſenſible, all worthy concluſions by all the Artes Mathematicall purpoſed, & by true Naturall Philoſophie concluded: & both addeth to them a farder ſcope, in the termes of the ſame Artes, & alſo by hys propre Method, and in peculier termes, procedeth, with helpe of the foreſayd Artes, to the performance of complet Experiẽces, which of no particular Art, are hable (Formally) to be challenged. If you remember, how we conſidered *Architecture*, in reſpect of all common handworkes: ſome light may you haue, therby, to vnderſtand the Souerainty and propertie of this Science. *Science* I may call it, rather, then an Arte: for the excellency and Maſterſhyp it hath, ouer ſo many, and ſo mighty Artes and

Sciences. And bycauſe it procedeth by *Experiences*, and ſearcheth forth the cauſes of Concluſions, by *Experiences*: and alſo putteth the Concluſions them ſelues, in *Experience*, it is named of ſome, *Scientia Experimentalis*. The *Experimentall Science*. *Nicolaus Cuſanus* termeth it ſo, in hys *Experimentes Statikall*, And an other
R. B. *Philoſopher*, of this land Natiue (the floure of whoſe worthy fame, can neuer dye nor wither) did write therof largely, at the requeſt of *Clement the ſixt*. The Arte carrieth with it, a wonderfull Credit: By reaſon, it certefieth, ſenſibly, fully, and completely to the vtmoſt power of Nature, and Arte. This Arte, certifieth by *Experience* complete and abſolute: and other Artes, with their Argumentes, and Demonſtrations, perſuade: and in wordes, proue very well their Concluſions. * But
☞ wordes, and Argumentes, are no ſenſible certifying: nor the full and finall frute of Sciences practiſable. And though ſome Artes, haue in them, *Experiences*, yet they are not complete, and brought to the vttermoſt, they may be ſtretched vnto, and applyed ſenſibly. As for example: the Naturall Philoſopher diſputeth and maketh goodly ſhew of reaſon: And the Aſtronomer, and the Opticall Mechanicien, put ſome thynges in *Experience:* but neither, all, that they may: nor yet ſufficiently, and to the vtmoſt, thoſe, which they do, There, then, the *Archemaſter* ſteppeth in, and leadeth forth on, the *Experiences*, by order of his doctrine *Experimentall*, to the chief and finall power of Naturall and Mathematicall Artes. Of two or three men, in whom, this Deſcription of *Archemaſtry* was *Experimentally*, verified, I haue read and hard: and good record, is of their ſuch perfection. So that, this Art, is no fantaſticall Imagination: as ſome Sophiſter, might, *Cum ſuis Inſolubilibus*, make a floriſh: and daſſell your Imagination: and daſh your honeſt deſire and Courage, from beleuing theſe thinges, ſo vnheard of, ſo meruaylous, & of ſuch Importance. Well: as you will. I haue forewarned you. I haue done the part of a frende: I haue diſcharged my Duety toward God: for my ſmall Talent, at hys moſt mercyfull handes receiued. To this Science, doth the *Science Alnirangiat*, great Seruice. Muſe nothyng of this name. I chaunge not the name, ſo vſed, and in Print publiſhed by other: beyng a name, propre to the Science. Vnder this, commeth *Ars Sintrillia*, by *Artephius*, briefly written. But the chief Science, of the Archemaſter, (in this world) as yet knowen, is an other (as it were) OPTICAL Science: wherof, the name ſhall be told (God willyng) when I ſhall haue ſome, (more iuſt) occaſion, therof, to Diſcourſe.

Here, I muſt end, thus abruptly (Gentle frende, and vnfayned louer of honeſt and neceſſary verities.) For, they, who haue (for your ſake, and vertues cauſe) requeſted me, (an old forworne Mathematicien) to take pen in hand: (through the confidence they repoſed in my long experience: and tryed ſincerity) for the declaryng and reportyng ſomewhat, of the frute and commodity, by the Artes Mathematicall, to be atteyned vnto: euen they, Sore agaynſt their willes, are forced, for ſundry cauſes, to ſatiffie the workemans requeſt, in endyng forthwith: He, ſo feareth this, ſo new an attempt, & ſo coſtly: And in matter ſo ſlenderly (hetherto) among the common Sorte of Studentes, conſidered or eſtemed.

And where I was willed, ſomewhat to alledge, why, in our vulgare Speche, this part of the Principall Science of *Geometrie*, called *Euclides Geometricall Elementes*, is publiſhed, to your handlyng: being vnlatined people, and not Vniuerſitie Scholers: Verily, I thinke it nedeleſſe.

1. For, the Honour, and Eſtimation of the Vniuerſities, and Graduates, is, hereby, nothing diminiſhed. Seing, from, and by their Nurſe Children, you receaue all this Benefite: how great ſoeuer it be.

Neither

Neither are their Studies, hereby, any whit hindred. No more, then the Italian *Vniuersities*, as *Academia Bononiensis, Ferrariensis, Florentina, Mediolanensis, Patauina, Papiensis, Perusina, Pisana, Romana, Senensis*, or any one of them, finde them selues, any deale, disgraced, or their Studies any thing hindred, by *Frater Lucas de Burgo*, or by *Nicolaus Tartalea*, who in vulgar Italian language, haue published, not onely *Euclides Geometrie*, but of *Archimedes* somewhat: and in Arithmetike and Practicall Geometrie, very large volumes, all in their vulgar speche. Nor in Germany haue the famous *Vniuersities*, any thing bene discontent with *Albertus Durerus*, his Geometricall Institutions in Dutch: or with *Gulielmus Xylander*, his learned translation of the first sixe bookes of *Euclide*, out of the Greke into the high Dutch. Nor with *Gualterus H. Riffius*, his Geometricall Volume: very diligently translated into the high Dutch tounge, and published. Nor yet the *Vniuersities* of Spaine, or Portugall, thinke their reputation to be decayed: or suppose any their Studies to be hindred by the Excellent *P. Nonnius*, his Mathematicall workes, in vulgare speche by him put forth. Haue you not, likewise, in the French tounge, the whole Mathematicall Quadriuie? and yet neither Paris, Orleance, or any of the other Vniuersities of Fraunce, at any time, with the Translaters, or Publishers offended: or any mans Studie thereby hindred?

2.

And surely, the Common and Vulgar Scholer (much more, the Gramarian) before his comming to the *Vniuersitie*, shall (or may) be, now (according to *Plato* his Counsell) sufficiently instructed in *Arithmetike* and *Geometrie*, for the better and easier learning of all maner of *Philosophie, Academicall*, or *Peripateticall*. And by that meanes, goe more cheresully, more skilfully, and spedily forwarde, in his Studies, there to be learned. And, so, in lesse time, profite more, then (otherwise) he should, or could do.

3.

Also many good and pregnant English wittes, of young Gentlemen, and of other, who neuer intend to meddle with the profound search and Studie of Philosophie (in the *Vniuersities* to be learned) may neuertheless, now, with more ease and libertie, haue good occasion, vertuously to occupie the sharpnesse of their wittes: where, els (perchance) otherwise, they would in fond exercises, spend (or rather leese) their time: neither seruing God: nor furdering the Weale, common or priuate.

4.

And great Comfort, with good hope, may the *Vniuersities* haue, by reason of this *English* Geometrie, and Mathematicall Præface, that they (hereafter) shall be the more regarded, esteemed, and resorted vnto. For, when it shall be knowen and reported, that of the *Mathematicall Sciences* onely, such great Commodities are ensuing (as I haue specified): and that in dede, some of you vnlatined Studentes, can be good witnesse, of such rare fruite by you enioyed (thereby): as either, before this, was not heard of: or els, not so fully credited: Well, may all men coniecture, that farre greater ayde, and better furniture, to winne to the Perfection of all Philosophie, may in the Vniuersities be had: being the Storehouses & Threasory of all Sciences, and all Artes, necessary for the best, and most noble State of Common Wealthes.

5.

Vniuersities.

Besides this, how many a Common Artificer, is there, in these Realmes of England and Ireland, that dealeth with Numbers, Rule, & Cumpasse: Who, with their owne Skill and experience, already had, will be hable (by these good helpes and informations) to finde out, and deuise, new workes, straunge Engines, and Instrumentes: for sundry purposes in the Common Wealth? or for priuate pleasure? and for the better maintayning of their owne estate? I will not (therefore)

6.

fight againſt myne owne ſhadowe. For, no man (I am ſure) will open his mouth againſt this Enterpriſe. No mā (I ſay) who either hath Charitie toward his brother (and would be glad of his furtherance in vertuous knowledge): or that hath any care & zeale for the bettering of the Cōmon ſtate of this Realme. Neither any, that make accompt, what the wiſer ſort of men (Sage and Stayed) do thinke of them. To none (therefore) will I make any *Apologie*, for a vertuous acte doing: and for cōmending, or ſetting forth, Profitable Artes to Engliſh men, in the Engliſh toung.

" But, vnto God our Creator, let vs all be thankefull: for that, *As he, of his Good-*
" *nes, by his Powre, and in his wiſedome, hath Created all thynges, in Number,*
☞ " *Waight, and Meaſure*: So, to vs, of hys great Mercy, he hath reuealed Meanes,
" whereby, to atteyne the ſufficient and neceſſary knowledge of the foreſayd hys
" three principall Inſtrumentes: Which Meanes, I haue abundantly proued vnto
" you, to be the *Sciences* and *Artes Mathematicall*.

And though I haue ben pinched with ſtraightnes of tyme: that, no way, I could ſo pen downe the matter (in my Mynde) as I determined: hopyng of conuenient layſure: Yet, if vertuous zeale, and honeſt Intent prouoke and bryng you to the readyng and examinyng of this Compendious treatiſe, I do not doute, but, as the veritie therof (accordyng to our purpoſe) will be euident vnto you: So the pith and force therof, will perſuade you: and the wonderfull frute therof, highly pleaſure you. And that you may the eaſier perceiue, and better remember, the principall pointes, where of my Preface treateth, I will giue you the Groundplatt of my whole diſcourſe, in a Table annexed: from the firſt to the laſt, ſomewhat Methodically contriued.

The Ground platt of this Præface in a Table.

If Haſt, hath cauſed my poore pen, any where, to ſtumble: You will, (I am ſure) in part of recompence, (for my earneſt and ſincere good will to pleaſure you), Conſider the rockiſh huge mountaines, and the perilous vnbeaten wayes, which (both night and day, for the while) it hath toyled and labored through, to bryng you this good Newes, and Comfortable profe, of Vertues frute.

So, I Commit you vnto Gods Mercyfull direction, for the reſt: hartely beſechyng hym, to proſper your Studyes, and honeſt Intentes: to his Glory, & the Commodity of our Countrey. *Amen.*

Written at my poore Houſe
At Mortlake.

Anno. 1570. Februarij. 9.

J. DEE.

Here haue you (according to my promisse) the Groundplat of my MATHEMATICALL Præface: annexed to *Euclide* (now first) published in our Englishe tounge. An. 1570. Febr. 3.

Sciences, and Artes Mathematicall, are, either

- *Principall, which are two, onely,*
  - Arithmetike.
    - *Simple,* Which dealeth with Numbers onely: and demonstrateth all their properties and appertenances: where, an Vnit, is Indiuisible.
    - *Mixt,* Which with aide of Geometrie principall, demonstrateth some Arithmeticall Conclusion, or Purpose.
  - Geometrie.
    - *Simple,* Which dealeth with Magnitudes, onely: and demonstrateth all their properties, passions, and appertenances: whose Point, is Indiuisible.
    - *Mixt,* Which with aide of Arithmetike principall, demonstrateth some Geometricall purpose: as EVCLIDES ELEMENTES.

  The vse whereof, is either,
  - *In thinges Supernaturall, æternall, & Diuine: By Application,* Ascending.
  - *In thinges Mathematicall: Without farther Application.*
  - *In thinges Naturall: both Substãtiall, & Accidentall, Visible, & Inuisible. &c. By Application:* Descending.

  The like Vses and Applications are, (though in a degree lower) in the Artes Mathematicall Deriuatiue.

- *Deriuatiue frõ the Principalls: of which, some haue*

  The names of the Principalls: as,
  - Arithmetike, *vulgar: which considereth*
    - Arithmetike of most vsuall whole Numbers: And of Fractions to them appertaining.
    - Arithmetike of Proportions.
    - Arithmetike Circular.
    - Arithmetike of Radicall Nũbers: Simple, Compound, Mixt: And of their Fractions.
    - Arithmetike of Cosike Nũbers: with their Fractions: And the great Arte of Algiebar.
  - Geometrie, *vulgar: which teacheth Measuring*
    - *At hand —*
      - All Lengthes. —— Mecometrie.
      - All Plaines: As, Land, Borde, Glasse, &c. —— Embadometrie.
      - All Solids: As, Timber, Stone, Vessels, &c. —— Stereometrie.
    - *With distãce from the thing Measured, as,*
      - *How farre, from the Measurer, any thing is: of him sene, on Land or Water: called* Apomecometrie.
      - *How high or deepe, from the leuell of the Measurers standing, any thing is: Seene of hym, on Land or Water: called* Hypsometrie.
      - *How broad, a thing is, which is in the Measurers vew: so it be situated on Land or Water: called* Platometrie.

      Of which are growen the Feates & Artes of
      - Geodesie: *more cunningly to Measure and Survey Landes, Woods, Waters. &c.*
      - Geographie.
      - Chorographie.
      - Hydrographie.
      - Stratarithmetrie.

  Propre names as,
  - Perspectiue, —— *Which demonstrateth the maners and properties of all Radiations: Directe, Broken, and Reflected.*
  - Astronomie, —— *Which demonstrateth the Distances, Magnitudes, and all Naturall motions, Apparences, and Passions, proper to the Planets and fixed Starres: for any time, past, present, and to come: in respecte of a certaine Horizon, or without respecte of any Horizon.*
  - Musike, —— *Which demonstrateth by reason, and teacheth by sense, perfectly to iudge and order the diuersitie of Soundes, hie or low.*
  - Cosmographie, — *Which, wholy and perfectly maketh description of the Heauenly, and also Elementall part of the World: and of these partes, maketh homological application, and mutuall collation necesary.*
  - Astrologie, —— *Which reasonably demonstrateth the operations and effectes of the naturall beames of light, and secrete Influence of the Planets, and fixed Starres, in euery Element and Elementall body: at all times, in any Horizon assigned.*
  - Statike, —— *Which demonstrateth the causes of heauines and lightnes of all thinges: and of the motions and properties to heauines and lightnes belonging.*
  - Anthropographie, *Which describeth the Nũber, Measure, Waight, Figure, Situation, and colour of euery diuers thing contained in the perfecte body of MAN: and geueth certaine knowledge of the Figure, Symmetrie, Waight, Characterization, & due Locall motion of any percell of the sayd body assigned: and of numbers to the said percell appertaining.*
  - Trochilike, —— *Which demonstrateth the properties of all Circular motions: Simple and Compound.*
  - Helicosophie, — *Which demonstrateth the designing of all Spirall lines: in Plaine, on Cylinder, Cone, Sphere, Conoïd, and Spharoid: and their properties.*
  - Pneumatithmie, — *Which demonstrateth by close hollow Geometricall figures (Regular and Irregular) the straunge properties (in motion or stay) of the Water, Ayre, Smoke, and Fire, in their Continuitie, and as they are ioyned to the Elementes next them.*
  - Menadrie, —— *Which demonstrateth, how, aboue Natures Vertue, and power simple: Vertue and force, may be multiplied: and so to directe, to lift, to pull, and to put or cast fro, any multiplied, or simple determined Vertue, Waight, or Force: naturally, not, so, directible, or moueable.*
  - Hypogeiodie, — *Which demonstrateth, how, vnder the Spharicall Superficies of the Earth, at any depth, to any perpendicular line assigned (whose distance fro the perpendicular of the entrance: and the Azimuth likewise, in respecte of the sayd entrance, is knowen) certaine way, may be described and gone, &c.*
  - Hydragogie, —— *Which demonstrateth the possible leading of water by Natures law, and by artificiall helpe, from any head (being Spring, standing, or running water) to any other place assigned.*
  - Horometrie, —— *Which demonstrateth, how, at all times appointed, the precise, vsuall denomination of time, may be knowen, for any place assigned.*
  - Zographie, —— *Which demonstrateth and teacheth, how, the Intersection of all visuall Pyramids, made by any plaine assigned (the Center, distance, and lightning determined) may be, by lines, and proper colours represented.*
  - Architecture, — *Which is a science garnished with many doctrines, and diuers Instructions: by whose iudgement, all workes by other workmen finished, are iuged.*
  - Nauigation, —— *Which demonstrateth, how, by the Shortest good way, by the aptest direction, and in the shortest time: a sufficient Shippe, betwene any two pact (in passage nauigable) assigned, may be conducted: and in all stormes and naturall disturbances chauncing, how to vse the best possible meanes, to recouer the place first assigned.*
  - Thaumaturgike, — *Which geueth certaine order to make straunge Workes, of the sense to be perceiued: and of men greatly to be wondred at.*
  - Archemastrie, — *Which teacheth to bring to actuall experience sensible, all worthy conclusions, by all the Artes Mathematicall purposed: and by true Naturall Philosophie, concluded: And both addeth to them a farder Scope, in the termes of the same Artes: and also, by his proper Method, and in peculiar termes, procedeth, with helpe of the forsayd Artes, to the performance of complete Experiences: which, of no particular Arte, are hable (Formally) to be challenged.*

Imprinted by *Iohn Day*.
An. 1570. Feb. 25.

¶ The first booke of Euclides Elementes.

The argument of the first Booke.

IN THIS FIRST BOOKE is intreated of the most simple, easie, and first matters and groundes of Geometry, as, namely, of Lynes, Angles, Triangles, Parallels, Squares, and Parallelogrammes. First of theyr definitions, shewyng what they are. After that it teacheth how to draw Parallel lynes, and how to forme diuersly figures of three sides, & foure sides, according to the varietie of their sides, and Angles: & cõpareth them all with Triangles, & also together the one with the other. In it also is taught how a figure of any forme may be chaunged into a Figure of an other forme. And for that it entreateth of these most common and generall thynges, thys booke is more vniuersall then is the seconde, third, or any other, and therefore iustly occupieth the first place in order: as that without which, the other bookes of *Euclide* which follow, and also the workes of others which haue written in Geometry, cannot be perceaued nor vnderstanded. And forasmuch as all the demonstrations and proofes of all the propositions in this whole booke, depende of these groundes and principles following, which by reason of their playnnes neede no greate declaration, yet to remoue all (be it neuer so litle) obscuritie, there are here set certayne shorte and manifest expositions of them.

¶ *Definitions.*

Definition of a poynt.

1. *A signe or point is that, which hath no part.*

The better to vnderstand what maner of thing a signe or point is, ye must note that the nature and propertie of quantitie (wherof Geometry entreateth) is to be deuided, so that whatsoeuer may be deuided into sundry partes, is called quantitie. But a point, although it pertayne to quantitie, and hath his beyng in quantitie, yet is it no quantitie, for that it cannot be deuided. Because (as the definition saith) it hath no partes into which it should be deuided. So that a pointe is the least thing that by minde and vnderstandirg can be imagined and conceyued: then which, there can be nothing lesse, as the poirt *A* in the margent.

A

Definition of a poynt after Pithagoras.

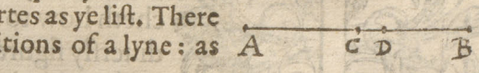
A signe or point is of *Pithagoras* Scholers after this manner defined: *A poynt is an vnitie which hath position.* Nũbers are conceaued in mynde without any forme & figure, and therfore without matter wheron to receaue figure, & consequently without place and position. Wherfore vnitie beyng a part of number, hath no position, or determinate place. Wherby it is manifest, that number is more simple and pure then is magnitude, and also immateriall: and so vnity which is the beginning of number, is lesse materiall then a signe or poynt, which is the beginnyng of magnitude. For a poynt is materiall, and requireth position and place, and therby differeth from vnitie.

Definition of a line.

2. *A line is length without breadth.*

There pertaine to quantitie three dimensions, length, bredth, & thicknes, or depth: and by these thre are all quãtities measured & made known. There are also, according

to these three dimensions, three kyndes of continuall quantities: a lyne, a superficies, or plaine, and a body. The first kynde, namely, a line is here defined in these wordes, *A lyne is length without breadth*. A point, for that it is no quantitie nor hath any partes into which it may be deuided, but remaineth indiuisible, hath not, nor can haue any of these three dimensions. It neither hath length, breadth, nor thickenes. But to a line, which is the first kynde of quantitie, is attributed the first dimension, namely, length, and onely that, for it hath neither breadth nor thicknes, but is conceaued to be drawne in length onely, and by it, it may be deuided into partes as many as ye list, equall, or vnequall. But as touching breadth it remaineth indiuisible. As the lyne A B, which is onely drawen in length, may be deuided in the pointe C equally, or in the point D vnequally, and so into as many partes as ye list. There are also of diuers other geuen other definitions of a lyne: as these which follow.



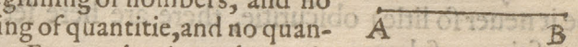
An other definition of a line. *A lyne is the mouyng of a poynte*, as the motion or draught of a pinne or a penne to your sence maketh a lyne.

An other. Agayne, *A lyne is a magnitude hauing one onely space or dimension*, namely, *length wantyng breadth and thicknes.*

The endes of a line.

3 *The endes or limites of a lyne, are pointes.*

For a line hath his beginning from a point, and likewise endeth in a point: so that by this also it is manifest, that pointes, for their simplicitie and lacke of composition, are neither quantitie, nor partes of quantitie, but only the termes and endes of quantitie. As the pointes *A*, *B*, are onely the endes of the line *A B*, and no partes thereof. And herein differeth a poynte in quantitie, from vnitie in number: for that although vnitie be the beginning of nombers, and no number (as a point is the beginning of quantitie, and no quantitie) yet is vnitie a part of number. For number is nothyng els but a collection of vnities, and therfore may be deuided into them, as into his partes. But a point is no part of quantitie, or of a lyne: neither is a lyne composed of pointes, as number is of vnities. For things indiuisible being neuer so many added together, can neuer make a thing diuisible, as an instant in time, is neither tyme, nor part of tyme, but only the beginning and end of time, and coupleth & ioyneth partes of tyme together.



Difference of a poynt frō vnity.

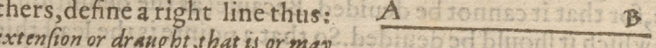
Vnitie is a part of number.

A poynt is no part of quantitie.

Definition of a right line.

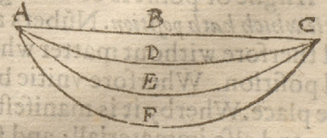
4 *A right lyne is that which lieth equally betwene his pointes.*

As the whole line *A B* lyeth straight and equally betwene the poyntes *A B* without any going vp or comming downe on eyther side.



Definition of a right line after Campanus. *Campanus* and certain others, define a right line thus: *A right line is the shortest extension or draught, that is or may be from one poynt to an other.* *Archimedes* defineth it thus.

Definitiō therof after Archimedes. *A right line is the shortest of all lines, which haue one and the self same limites or endes:* which is in maner al one with the definitiō of *Campanus*. As of all these lines *A B C*, *A D C*, *A E C*, *A F C*, which are all drawen from the point *A*, to the poynte *B*, as *Campanus* speaketh, or which haue the self same limites or endes, as *Archimedes* speaketh, the lyne *A B C*, beyng a right line, is the shortest.



Definitiō therof after Plato. *Plato* defineth a right line after this maner: *A right line is that whose middle part shadoweth the extremes.* As if you put any thyng in the middle of a right lyne, you shall not see from the one ende to the other, which thyng happeneth not in a crooked lyne. The Ecclipse of the Sunne (say Astronomers) then happeneth, when the Sunne, the Moone, & our eye are in one right line. For the Moone then being in the midst betwene vs and the Sunne, causeth it to be darkened. Diuers other define a right line diuersly, as followeth.

An other definition. *A right lyne is that which standeth firme betwene his extremes.*

An other. Agayne, *A right line is that which with an other line of lyke forme cannot make a figure.*

Agayne.

Agayne, *A right lyne is that which hath not one part in a plaine superficies, and an other erected on high.* *An other.*

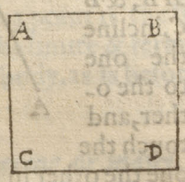
Agayne. *A right lyne is that, all whose partes agree together with all his other partes.* *An other.*

Agayne, *A right lyne is that, whose extremes abiding, cannot be altered.* *An other.*

Euclide doth not here define a crooked lyne, for it neded not. It may easely be vnderstand by the definition of a right lyne, for euery contrary is well manifested & set forth by hys contrary. One crooked lyne may be more crooked then an other, and from one poynt to an other may be drawen infinite crooked lynes: but one right lyne cannot be righter then an other, and therfore from one point to an other, there may be drawen but one right lyne. As by the figure aboue set, you may see. *VVhy Euclide here defineth not a crooked lyne.*

5 *A superficies is that, which hath onely length and breadth.* *Definition of a superficies.*

A superficies is the second kinde of quantitie, and to it are attributed two dimensions, namely length, and breadth. As in the superficies *A B C D*, whose length is taken by the lyne *A B*, or *C D*, and breadth by the lyne *A C*. or *B D*: and by reason of those two dimensions a superficies may be deuided two wayes, namely by his length, and by hys breadth, but not by thickneſſe, for it hath none. For, that is attributed onely to a body, which is the third kynde of quantitie, and hath all three dimensions, length, breadth, and thicknes, and may be deuided according to any of them. *A superficies may be deuided two wayes.*



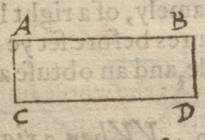
Others define a superficies thus: *A superficies is the terme or ende of a body*. As a line is the ende and terme of a superficies. *An other definition of a superficies.*

6 *Extremes of a superficies, are lynes.* *The extremes of a superficies.*

As the endes, limites, or borders of a lyne, are pointes, inclosing the line: so are lines the limites, borders, and endes inclosing a superficies. As in the figure aforesayde you maye see the superficies inclosed with foure lynes. The extremes or limites of a bodye, are superficiesles. And therfore a superficies is of some thus defined: *A superficies is that, which endeth or inclofeth a body*: as is to be sene in the sides of a die, or of any other body. *Another definition of a superficies.*

7 *A plaine superficies is that, which lieth equally betwene his lines.* *Definition of a plaine superficies.*

As the superficies *A B C D* lyeth equally and smoothe betwene the two lines *A B*, and *C D*: or betwene the two lines *A C*, and *B D*: so that no part therof eyther swelleth vpward, or is depressed downward. And this definitiō much agreeth with the definition of a right line. A right line lieth equally betwene his points, and a plaine superficies lyeth equally betwene his lynes. Others define a plaine superficies after this maner:



A plaine superficies, is the shortest extension or draught from one lyne to an other like as a right lyne is the shorteſt extension or draught from one point to an other. *Another definition of a playne superficies.*

Euclide also leaueth out here to speake of a crooked and hollow superficies, because it may easely be vnderstand by the diffinition of a plaine superficies, being hys contrary. And euen as from one point to an other may be drawen infinite crooked lines, & but one right line, which is the shorteſt: so from one lyne to an other may be drawen infinite croked superficiesles, & but one plain superficies, which is the shorteſt. Here must you consider when there is in Geometry mention made of pointes, lines, circles, triangles, or of any other figures, ye may not conceyue of them as they be in matter, as in woode, in mettall, in paper, or in any such lyke, for so is there no lyne, but hath some breadth, and may be deuided: nor points, but that shal haue some partes, and may also be deuided, and so of others. But you must conceiue them in mynde, plucking them by imagination from all matter, so shall ye vnderstande them truely and perfectly, in their owne nature as they are defined. As a lyne to be long, and not broade: and a poynte to *NOTE.*

be so little, that it shall haue no part at all.

Another definition of a playne superficies. Others otherwyse define a playne superficies: *A plaine superficies is that, which is firmly set betwene his extremes*, as before was sayd of a right lyne.

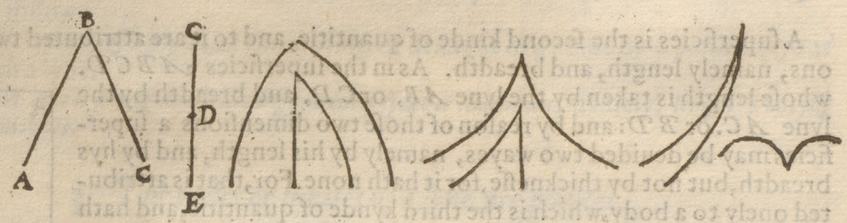
An other definition. Agayne, *A plaine superficies is that, vnto all whose partes a right line may well be applied.*

An other definition. Again, *A plaine superficies is that, which is the shortest of al superficies, which haue one & the self extremes:* As a right line was the shortest line that can be drawen betwene two pointes.

An other definition. Againe, *A playne superficies is that, whose middle darkeneth the extremes*, as was also sayd of a right lyne.

Definition of a playne angle.

8 *A plaine angle is an inclination or bowing of two lines the one to the other and the one touching the other, and not beyng directly ioyned together.*



As the two lines *A B*, & *B C*, incline the one to the other, and touch the one the other in the point *B*, in which point by reason of the inclination of the sayd lines, is made the angle *A B C*. But if the two lines which touch the one the other, be without all inclination of the one to the other, and be drawne directly the one to the other, then make they not any angle at all, as the lines *C D*, and *D E*, touch the one the other in the point *D*, and yet as ye see they make no angle.

Definition of a rectilined angle.

9 *And if the lines which containe the angle be right lynes, then is it called a rightlyned angle.*

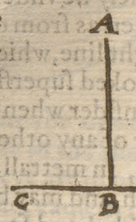
As the angle *A B C*, in the former figures, is a rightlined angle, because it is contained of right lines: where note, that an angle is for the most part described by thre letters, of which the second or middle letter representeth the very angle, and therfore is set at the angle.

By the contrary, a crooked lyned angle, is that which is contained of crooked lines, which may be diuersly figured. Also a mixt angle is that which is caused of them both, namely, of a right line and a crooked, which may also be diuersly figured, as in the figures before set ye may see. There are of angles thre kindes, a right angle, an acute angle, and an obtuse angle, the definitions of which now follow.

Three kindes of angles.

What a right angle, & what also a perpendicular lyne is.

10 *When a right line standing vpon a right line maketh the angles on either side equall: then either of those angles is a right angle. And the right lyne which standeth erected, is called a perpendiculer line to that vpon which it standeth.*



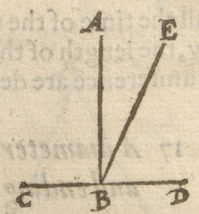
As vpon the right line *C D*, suppose there do stand an other line *A B*, in such sort, that it maketh the angles on either side therof equall: namely, the angle *A B C* on the one side equall to the angle *A B D* on the other side: then is eche of the two angles *A B C*, and *A B D* a right angle, and the line *A B*, which standeth erected vpon the line *C D*, without inclination to either part is a perpendicular line, commonly called among artificers a plumbe lyne.

What an obtuse angle is.

11 *An obtuse angle is that which is greater then a right angle.*

As

As the angle *C B E* in the example is an obtuſe angle, for it is greater then the angle *A B C*, which is a right angle, becauſe it contayneth it, and containeth moreouer the angle *A B E*.



12 *An acute angle is that, which is leſſe then a right angle.*

What an acute angle is.

As the angle *E B D* in the figure before put is an acute angle, for that it is leſſe then the angle *A B D*, which is a right angle, for the right angle contayneth it, and moreouer the angle *A B E*.

13 *A limite or terme, is the ende of euery thing.*

The limite of any thing.

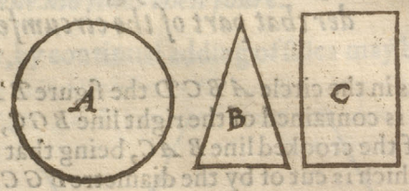
For as much as of thinges infinite (as *Plato* ſaith) there is no ſcience, therefore muſt magnitude or quantitie (wherof Geometry entreateth) be finite, and haue borders and limites to incloſe it, which are here defined to be the endes therof. As a point is the limite or terme of a line, becauſe it is thend therof: A line likewiſe is the limite & terme of a ſuperficies: and likewiſe a ſuperficies is the limite and terme of a body, as is before declared.

No ſcience of thinges infinite

14 *A figure is that, which is contayned vnder one limite or terme, or many.*

Definition of a figure.

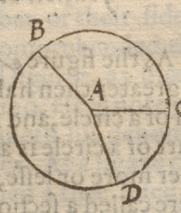
As the figure *A* is contained vnder one limit, which is the round line. Alſo the figure *B* is contayned vnder three right lines. And the figure *C* vnder foure, and ſo of others, which are their limites or termes.



15 *A circle is a plaine figure, conteyned vnder one line, which is called a circumference, vnto which all lynes drawen from one poynt within the figure and falling vpon the circumference therof are equall the one to the other.*

Definition of a circle.

As the figure here ſet is a plaine figure, that is a figure without groſſenes or thicknes, and is alſo contayned vnder one line, namely, the crooked lyne *B C D*, which is the circumference therof, it hath moreouer in the middle therof a point, namely, the point *A*, from which, all the lynes drawen to the ſuperficies, are equal: as the lines *A B, A C, A D*, and other how many ſoeuer.



Of all figures a circle is the moſt perfect, and therfore is it here firſt defined.

A circle the moſt perfect of all figures.

16 *And that point is called the centre of the circle, as is the point A, which is ſet in the middes of the former circle.*

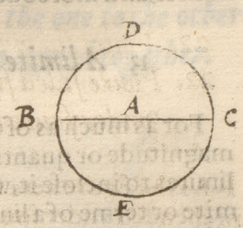
The centre of a circle.

For the more eaſy declaration, that all the lines drawen from the centre of the circle to the circumference, are equall, ye muſt note, that although a line be not made of pointes: yet a point, by his motion or draught, deſcribeth a line. Likewiſe a line drawen, or moued, deſcribeth a ſuperficies: alſo a ſuperficies being moued maketh a ſolide or bodie. Now thē imagine the line *A B*, (the point *A* being fixed) to be moued about in a plaine ſuperficies, drawing the point *B* continually about the point *A*, till it returne to the place where it began firſt to moue: ſo ſhall the point *B*, by this motion, deſcribe the circumference of the circle, and the point *A* being fixed, is the centre of the circle. Which in

all the time of the motion of the line, had like distance: from the circumference, namely, the length of the line *A B*. And for that al the lines drawn from the centre to the circumference are described of that line, they are also equal vnto it, & betwene thēselues.

Definition of a diameter.

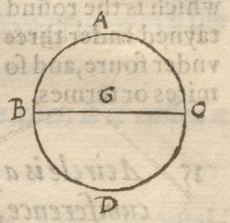
17 *A diameter of a circle, is a right line, which drawen by the centre thereof, and ending at the circumference on either side, deuideth the circle into two equall partes.*



As the line *B A C* in this circle presēt is the diameter, becausē it passeth from the point *B*, of the one side of the circumferēce, to the point *C*, on the other side of the circumference, & passeth also by the point *A* being the centre of the circle. And moreouer it deuideth the circle into two equall partes: the one, namely *B D C*, being on the one side of the line, & the other namely, *B E C*, on the other side, which thing did *Thales Miletius* (which brought Geometry out of Egipt into Grece) first obserue and proue. For if a line drawen by the centre, do not deuide a circle into two equal partes: all the lines drawen from the centre to the circumference should not be equall.

Definition of a semicircle.

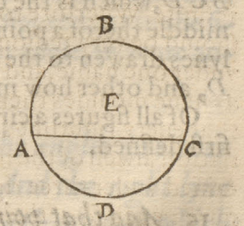
18 *A semicircle, is a figure which is contayned vnder the diameter, and vnder that part of the circumference which is cut of by the diametre.*



As in the circle *A B C D* the figure *B A C* is a semicircle, becausē it is contained of the right line *B G C*, which is the diametre, and of the crooked line *B A C*, being that part of the circumference, which is cut of by the diametre *B G C*. So likewise the other part of the circle, namely *B D C*, is a semicircle as the other was.

Definition of a section of a circle.

19 *A section or portion of a circle, is a figure whiche is contayned vnder a right lyne, and a parte of the circumference, greater or lesse then the semicircle.*



As the figure *A B C*, in the example, is a section of a circle, & is greater then halfe a circle, and the figure *A D C*, is also a section of a circle, and is lesse then a semicircle. A section, portion, or part of a circle is all one, and signifieth such a part which is either more or lesse, then a semicircle: so that a semicircle is not here called a section or portion of a circle. A right lyne drawen from one side of the circumference of a circle to the other, not passyng by the centre, deuideth the circle into two vnequall partes, which are two sections, of which that which contayneth the centre is called the greater section, and the other is called the lesse section. As in the example, the part of the circle *A B C*, which containeth in it the centre *E*, is the greater section, being greater then the halfe circle: the other part, namely *A D C*, which hath not the centre in it, is the lesse section of the circle, being lesse then a semicircle.

Definition of rectilined figures.

20 *Right lined figures are such which are contayned vnder right lynes.*

As are such as followeth, of which some are contayned vnder three right lines, some vnder foure, some vnder fiue, and some vnder mo.

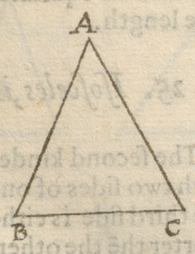
Definition of three sided figures.

21 *Thre sided figures, or figures of thre sydes, are such which are contayned vnder three right lines.*

As

As the figure in the example *A B C*, is a figure of three sides, becauſe it is cõtained vnder thre right lines, namely, vnder the lines *A B, B C, C A*.

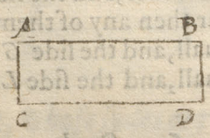
A figure of three ſides, or a triangle, is the firſt figure in order of all right lined figures, and therfore of all others it is firſt defined. For vnder leſſe then three lines, can no figure be comprehended.



22. *Foure ſided figures or figures of foure ſides are ſuch, which are contained vnder foure right lines.*

Definition of foure ſided figures.

As the figure here ſet, is a figure of foure ſides, for that it is cõprehended vnder foure right lines, namely, A B, B D, D C, C A.

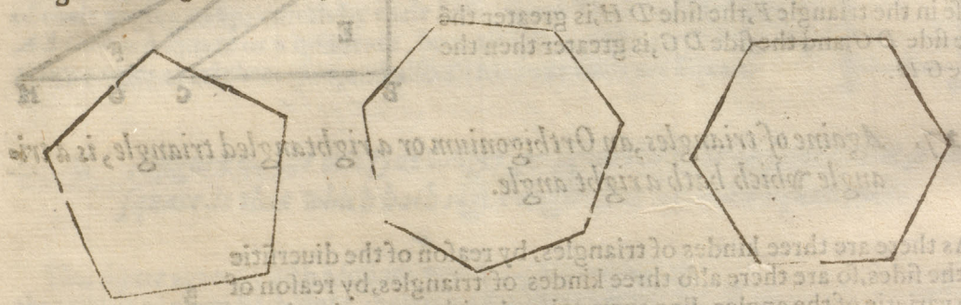


Triangles, and foure ſided figures ſerue commonly to many vſes in demonſtrations of Geometry. Wherfore the nature and properties of them, are much to be obſerued, the vſe of other figures is more obſcure.

23. *Many ſided figures are ſuch which haue mo ſides then foure.*

Definition of many ſided figures.

Right lined figures hauing mo ſides then fower, by continual adding of ſides may be

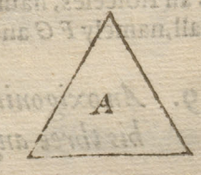


infinite. Wherfore to define them all ſeuerally, according to the number of their ſides, ſhould be very tedious, or rather impoſſible. Therfore hath *Euclide* comprehended thẽ vnder one name, and vnder one diffinition: calling them many ſided figures, as many as haue mo ſides then foure: as if they haue fiue ſides, ſixe, ſeuen, or mo. Here note ye, that euery rightlined figure hath as many angles, as it hath ſides, & taketh his denomination aſwell of the number of his angles, as of the number of his ſides. As a figure cõtained vnder three right lines, of the number of his three ſides, is called a thre ſided figure: euen ſo of the number of his three angles, it is called a triangle. Likewiſe a figure contained vnder foure right lines, by reaſon of the number of his ſides, is called a foure ſided figure: and by reaſon of the number of his angles, it is called a quadrangled figure, and ſo of others.

24. *Of three ſided figures or triangles, an equilatre triangle is that, which hath three equall ſides.*

Definition of an equilater triangle.

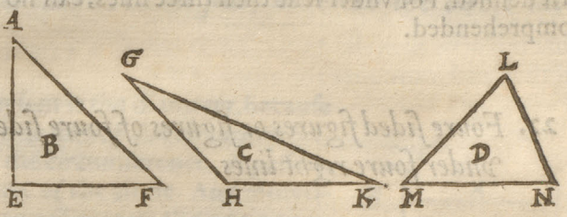
Triangles haue their differences partly of their ſides, and partly of their angles. As touching the differences of their ſides, there are three kindes. For either all thre ſides of the triangle are equall, or two onely are equall, & the third vnequal: or els all three are vnequall the one to the other. The firſt kind of triangles, namely, that which hath three equall ſides, is moſt ſimple, and eaſieſt to be knowen: and is here firſt defined, and



is called an equilater triangle, as the triangle *A* in the example, all whose sides are of one length.

Definition of an Isosceles.

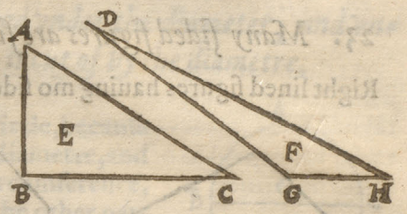
25. *Isosceles, is a triangle, which hath onely two sides equall.*



The second kinde of triangles hath two sides of one length, but the third side is either longer or shorter thē the other two, as are the triangles here figured, *B*, *C*, *D*. In the triangle *B*, the two sides *A E* and *E F* are equal the one to the other, and the side *A F*, is lōger then any of them both: Likewise in the triangle *C* the two sides *G H* and *H K*, are equall, and the side *G K* is greater. Also in the triangle *D*, the sides *L M* and *M N*, are equall, and the side *L N* is shorter.

Definition of a Scalenum.

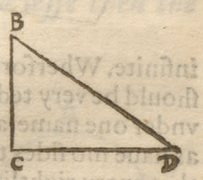
26. *Scalenum is a triangle, whose three sides are all vnequall.*



As are the triangles *E*, *F*, in which there is no one side equall to any of the other. For in the triangle *E*, the side *A C* is greater then the side *B C*, and the side *B C* is greater thē the side *A B*. Likewise in the triangle *F*, the side *D H*, is greater thē the side *D G*, and the side *D G*, is greater then the side *G H*.

An Orthigonium triangle.

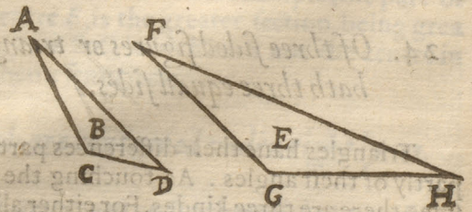
27. *Againe of triangles, an Orthigonium or a rightangled triangle, is a triangle which hath a right angle.*



As there are three kindes of triangles, by reason of the diuersitie of the sides, so are there also three kindes of triangles, by reason of the varietie of the angles. For euery triangle either containeth one right angle, & two acute angles: or one obtuse angle, & two acute: or three acute angles: for it is impossible that one triangle should containe two obtuse angles, or two right angles, or one obtuse angle, and the other a right angle. All which kindes are here defined. First a rightangled triangle whiche hath in it a right angle. As the triangle *B C D*, of which the angle *B C D*, is a right angle.

An Ambligonium triangle.

28. *An ambligonium or an obtuse angled triangle, is a triangle which hath an obtuse angle.*



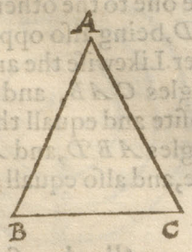
As is the triangle *B*, whose angle *A C D*, is an obtuse angle, and is also a Scalenon, hauing his three sides vnequall: the triangle *E*, is likewise an Ambligonion, whose angle *F G H*, is an obtuse angle, & is an Isosceles, hauing two of his sides equall, namely *F G* and *G H*.

An Oxigonium triangle.

29. *An oxigonium or an acuteangled triangle, is a triangle which hath all his three angles acute.*

As

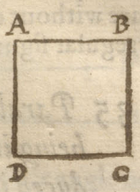
As the triangles *A,B,C*, in the example, al whose angles are acute: of which *A* is an equilater triangle, *B*, an Isosceles, and *C* a Scalenon. An equilater triangle is most simple, and hath one vniforme construction, and therfore all the angles of it are equall, and neuer hath in it either a right angle, or an obtuse: but the angles of an Isosceles or a Scalenon, may diuersly vary. It is also to be noted that in comparison of any two sides of a triangle, the third is called a base. As of the triangle *A B C* in respect of the two lines *A B* and *A C*, the line *B C*, is the base: and in respect of the two sides *A C* and *C B*, the line *A B*, is the base, and likewyse in respect of the two sides *C B* & *B A*, the line *A C*, is the base.



30 *Of foure syded figures, a quadrate or square is that, whose sydes are equall, and his angles right.*

Definition of a square.

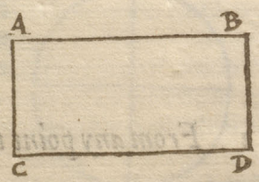
As triangles haue their difference and varietie by reason of their sides and angles: so likewise do figures of foure sides, take their varietie and difference partly by reason of their sides, & partly by reason of their angles, as appeareth by their definitions. The four sided figure *A B C D* is a square or a quadrate, because it is a right angled figure, al hys angles are right angles, and also all his four sides are equall.



31 *A figure on the one syde longer, or squarelike, or as some call it, a long square, is that which hath right angles, but hath not equall sydes.*

Definition of a long square.

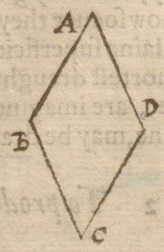
This figure agreeth with a square touching his angles, in that either of them hath right angles, and differeth from it onely by reason of his sides, in that the sides therof be not equall, as are the sides of a square. As in the example, the angles of the figure *ABCD*, are right angles, but the two sides thereof *A B*, and *C D*, are longer then the other two sides *A C*, & *B D*.



32 *Rhombus (or a diamonde) is a figure hauing foure equall sydes, but it is not right angled.*

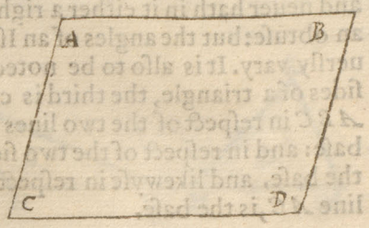
Definition of a Diamond figure

This figure agreeth with a square, as touching the equallitie of lines, but differeth from it in that it hath not right angles, as hath the square. As of this figure, the foure lines *A B, B C, C D, D A*, be equall, but the angles therof are not right angles. For the two angles *A B C* and *A D C*, are obtuse angles, greater then right angles, & the other two angles *B A D* and *B C D*, are two acute angles lesse then two right angles. And these foure angles are yet equall to foure right angles: for, as much as the acute angle wanteth of a right angle, so much the obtuse angle excedeth a right angle.



Definition of a diamondlike figure.

33 *Rhombaides (or a diamond like) is a figure, whose oppoſite ſides are equall, and whoſe oppoſite angles are alſo equall, but it hath neither equall ſides, nor right angles.*

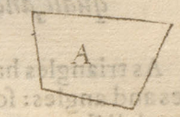


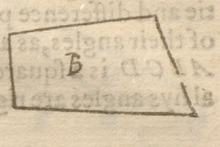
As in the figure *A B C D*, all the foure ſides are not equall, but the two ſides *A B* and *C D*, being oppoſite the one to the other, alſo the other two ſides *A C* and *B D*, being alſo oppoſite, are equall the one to the other. Likewiſe the angles are not right angles, but the angles *C A B*, and *C D B*, are obtuſe angles, and oppoſite and equall the one to the other. Likewyſe the angles *A B D*, and *A C D*, are acute angles, and oppoſite, and alſo equall the one to the other.

Trapezia or tables.

34 *All other figures of foure ſides beſides theſe, are called trapezia, or tables.*

Such are all figures, in which is obſerued no equallitie of ſides nor anges: as the figures *A* and *B*, in the margēt, which haue neither equall ſides, nor equal angles, but are deſcribed at all aduenture without obſeruation of order, and therefore they are called irregular figures.

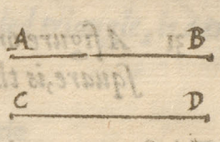




Definition of Parallel lines.

35 *Parallel or equidiſtant right lines are ſuch, which being in one and the ſelfe ſame ſuperficies, and produced infinitely on both ſydes, do neuer in any part concurre.*

As are the lines *A B*, and *C D*, in the example.

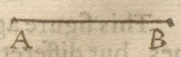


Peticions or requeſtes.

1 *From any point to any point, to draw a right line.*

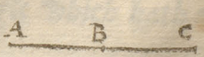
What Peticions are.

After the definitions, which are the firſt kind of principles, now follow petitions, which are the ſecond kynd of principles: which are certain general ſentences, ſo plain, & ſo perſpicuous, that they are perceiued to be true as ſoone as they are vttered, & no man that hath but common ſence, can, nor will deny them. Of which, the firſt is that, which is here ſet. As from the point *A*, to the point *B*, who wil deny, but eaſily graunt that a right line may be drawn? For two points howſoeuer they be ſet, are imagined to be in one and the ſelfe ſame plaine ſuperficies, wherfore from the one to the other there is ſome ſhorteſt draught, whiche is a right line. Likewiſe any two right lines howſoeuer they be ſet, are imagined to be in one ſuperficies, and therefore from any one line to any one line, may be drawen a ſuperficies.



2 *To produce a right line finite, ſtraight forth continually.*

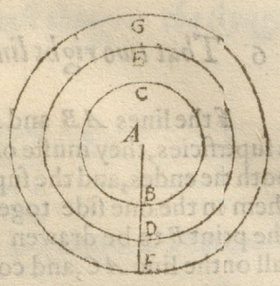
As to draw in length continually the right line *A B*, who will not graunt? For there is no magnitude ſo great, but that there may bea greter, nor any ſo litle, but that there may be a leſſe. And a line



a line is a draught from one point to an other, therfore from the point *B*, which is the ende of the line *A B*, may be drawn a line to ſome other point, as to the point *C*, and from that to an other, and ſo infinitely.

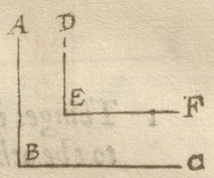
3 *Vpon any centre and at any diſtance, to deſcribe a circle.*

A playne ſuperficies may in compaſſe be extended infinitely: as from any pointe to any pointe may be drawen a right line, by reaſon wherof it commeth to paſſe that a circle may be deſcribed vpon any centre and at any ſpace or diſtance. As vpon the centre *A*, and vpon the ſpace *A B*, ye may deſcribe the circle *B C*, & vpon the ſame centre, vpon the diſtance *A D*, ye may deſcribe the circle *D E*, or vppon the ſame centre *A*, according to the diſtaunce *A F*, ye may deſcribe the circle *F G*, and ſo infinitely extendyng your ſpace.



4 *All right angles are equall the one to the other.*

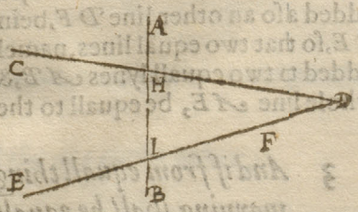
This peticion is moſt plaine, and offreth it ſelfe euen to the ſence. For as much as a right angle is cauſed of one right lyne falling perpendicularly vppon an other, and no one line can fall more perpendicularly vpō a line then an other: therfore no one right angle can be greater thē an other: neither do the length or ſhortenes of the lines alter the greatnes of the angle. For in the example, the right angle *A B C*, though it be made of much longer lines then the right angle *D E F*, whoſe lines are much ſhorter, yet is that aigle no greater then the other. For if ye ſet the point *E* iuſt vpon the point *B*, then ſhal the line *E D*, euenly and iuſtly fall vpon the line *A B*, and the line *E F*, ſhall alſo fall equaly vpon the line *B C*, and ſo ſhal the angle *D E F*, be equall to the angle *A B C*, for that the lines which cauſe them, are of like inclination.



It may euidently alſo be ſene at the centre of a circle. For if ye draw in a circle two diameters, the one cutting the other in the centre by right angles, ye ſhall deuide the circle into fowre equall partes, of which eche contayneth one right angle, ſo are all the foure right angles about the centre of the circle equall.

5 *VVhen a right line falling vpon two right lines, doth make on one & the ſelfe ſame ſyde, the two inwarde angles leſſe then two right angles, then ſhal theſe two right lines beyng produced at length concurre on that part, in which are the two angles leſſe then two right angles.*

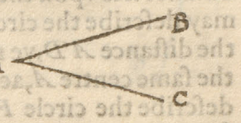
As if the right line *A B*, fall vpon two right lines, namely, *C D* and *E F*, ſo that it make the two inward angles on the one ſide, as the angles *D H I* & *F I H*, leſſe then two right angles (as in the example they do) the ſaid two lines *C D*, and *E F*, being drawen forth in lēgth on that part, wheron the two angles being leſſe thē two right angles conſiſt, ſhal at lēgth concurre and meete together: as in the point *D*, as it is eaſie to ſee. For the partes of the lines towardes *D F*, are more enclined th: one to



the other, then the partes of the lines towardes *C E* are. Wherfore the more these parts are produced, the more they shall approch neare and neare, till at length they shal mete in one point. Contrariwise the same lines drawn in lēgth on the other side, for that the angles on that side, namely, the angle *C H B*, and the angle *E I A*, are greater then two right angles, so much as the other two angles are lesse then two right angles, shall neuer mete, but the further they are drawen, the further they shalbe distant the one from the other.

6 *That two right lines include not a superficies.*

If the lines *A B* and *A C*, being right lines, should inclose a superficies, they muste of necessitie bee ioyned together at both the endes, and the superficies must be betwene thē. Ioyne them on the one side together in the pointe A, and imagine the point *B* to be drawen to the point *C*, so shall the line *A B*, fall on the line *A C*, and couer it, and so be all one with it, and neuer inclose a space or superficies.



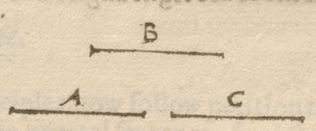
Common sentences.

1 *Thinges equall to one and the selfe same thyng: are equall also the one to the other.*

What common sentences are.

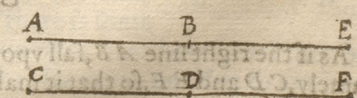
Difference betwene peticions & common sentences.

After definitions and petitions, now are set common sentences, which are the third and last kynd of principles. Which are certaine general propositiōs, commonly known of all men, of themselues most manifest & cleare, & therfore are called also dignities not able to be denied of any. Peticions also are very manifest, but not so fully as are the cōmon sentences, and therfore are required or desired to be graunted. Peticions also are more peculiar to the arte whereof they are: as those before put are proper to Geometry: but common sentences are generall to all things wherunto they can be applied. Agayne, peticions consist in actions or doing of somewhat most easy to be done: but common sentences consist in consideration of mynde, but yet of such thinges which are most easy to be vnderstanded, as is that before set. As if the line *A* be equall to the line *B*, And if the line *C* be also equall to the line *B*, then of necessitie the lines *A* and *C*, shalbe equal the one to the other. So is it in all superficiesses, angles, & numbers, & in all other things (of one kynde) that may be compared together.



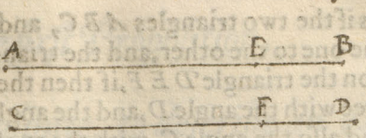
2 *And if ye adde equall thinges to equall thinges: the whole shalbe equall.*

As if the line *A B* be equal to the line *C D*, & to the line *A B*, be added the line *B E*, & to the line *C D*, be added also an other line *D F*, being equal to the line *B E*, so that two equal lines, namely, *B E*, and *D F*, be added to two equall lynes *A B*, & *C D*: then shal the whole lyne *A E*, be equall to the whole lyne *C F*, and so of all quantities generally.



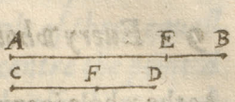
3 *And if from equall thinges, ye take away equall thinges: the thinges remayning shall be equall.*

As if from the two lines *AB* and *CD*, being equal, ye take away two equall lines, namely, *EB* and *FD*, then maye you conclude by this common ſentence, that the partes remayning, namely, *AE*, and *CF* are equall the one to the other: and ſo of all other quantities.



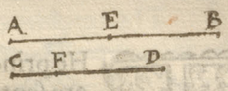
4 *And if from vnequall thinges ye take away equall thinges: the thynges which remayne ſhall be vnequall.*

As if the lines *AB*, and *CD*, be vnequall, the line *AB*, beyng longer then the line *CD*, & if ye take frō them two equall lines, as *EB*, and *FD*: the partes remayning, which are the lines *AE* and *CF*, ſhall be vnequall the one to the other, namely, the lyne *AE*, ſhall be greater then the line *CF*, which is euer true in all quantities whatſoeuer.



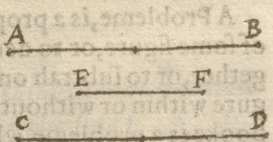
5 *And if to vnequall thinges ye adde equall thinges: the whole ſhall be vnequall.*

As if ye haue two vnequal lines, namely, *AE* the greater, and *CF* the leſſe, & if ye adde vnto thē two equall lines, *EB* & *FD*, then maye ye conclude that the whole lines compoſed are vnequall: namely, that the whole lyne *AB*, is greater then the whole line *CD*, and ſo of all other quantities.



6 *Thinges which are double to one and the ſelfe ſame thing: are equall the one to the other.*

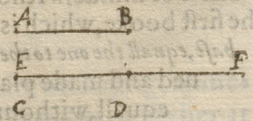
As if the line *AB* be double to the line *EF*, and if alſo the line *CD*, be double to the ſame line *EF*: thē may you by this common ſentence conclude, that the two lines *AB*, & *CD*, are equall the one to the other. And this is true in all quantities, and that not only, when they are double, but alſo if they be triple or quadruple, or in what proportion ſoeuer it be of the greater inequallitie. Which is when the greater quantitie is compared to the leſſe.



What proportion of the greater inequality is

7 *Thinges which are the halfe of one and the ſelfe ſame thing: are equal the one to the other.*

As if the line *AB*, be the halfe of the line *EF*, and if the lyne *CD*, be the halfe alſo of the ſame line *EF*: then may ye conclude by this common ſentence, that the two lines *AB* and *CD*, are equall the one to the other. This is alſo true in all kyndes of quantitie, and that not onely when it is a halfe, but alſo if it be a third, a quarter, or in what proportion ſoeuer it be of the leſſe in equallitie. Which is when the leſſe quantitie is cōpared to the greater.

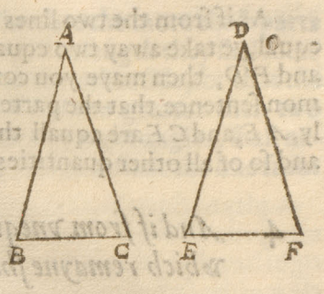


What proportion of the leſſe inequalitie is.

8 *Thinges which agree together: are equall the one to the other.*

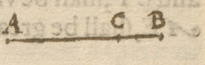
Such thinges are ſayd to agree together, whiche when they are applied the one to the other: or ſet the one vpon the other, the one excedeth not the other in any thyng.

As if the two triangles *A B C*, and *D E F*, were applied the one to the other, and the triangle *A B C*, were set vpon the triangle *D E F*, if then the angle *A*, do iustly agree with the angle *D*, and the angle *B*, with the angle *E*, and also the angle *C*, with the angle *F*: and moreouer if the line *A B*, do iustly fall vpon the line *D E*, and the line *A C*, vpon the line *D F*, and also the line *B C*, vppon the line *E F*, so that on euery part of these two triangles, there is iust agreement, then may ye conclude that the two triangles are equall.



9 *Euery whole is greater then his part.*

As the whole is equal to all his partes taken together, so is it greater then any one part therof. As if the line *C B* be a part of the line *A B*, then by this common sentence ye may conclude that the whole line *A B*, is greater then the part, namely, thē the line *C B*. And this is generall in all thinges.



What a Proposition is.

THe principles thus placed & ended, now follow the propositions, which are sentences set forth to be proued by reasoning and demonstrations, and therfore they are agayne repeated in the end of the demonstration. For the proposition is euer the conclusion, and that which ought to be proued.

Propositions of two sortes.

Propositions are of two sortes, the one is called a Probleme, the other a Theoreme.

What a Probleme is.

A Probleme, is a proposition which requireth some action, or doing: as the makyng of some figure, or to deuide a figure or line, to apply figure to figure, to adde figures together, or to subtrah one from an other, to describe, to inscribe, to circumscribe one figure within or without another, and suche like. As of the first proposition of the first booke is a probleme, which is thus: *Vpon a right line geuen not being infinite, to describe an equilater triangle, or a triangle of three equall sides*. For in it, besides the demonstration and contemplation of the mynde, is required somewhat to be done: namely, to make an equilater triangle vpon a line geuen. And in the ende of euery probleme, after the demonstration, is concluded after this manner, *Which is the thing, which was required to be done.*

What a Theoreme is.

A Theoreme, is a proposition, which requireth the searching out and demonstration of some propertie or passion of some figure: Wherin is onely speculation and contemplation of minde, without doing or working of any thing: As the fifth proposition of the first booke, which is thus, *An Isosceles or triangle of two equall sides, hath his angles at the base, equall the one to the other, &c.* is a Theoreme. For in it is required only to be proued and made plaine by reason and demonstratiō, that these two angles be equall, without further working or doing. And in the ende of euery Theoreme, after the demonstration is concluded after this maner, *Which thyng was required to be demonstrated or proued.*

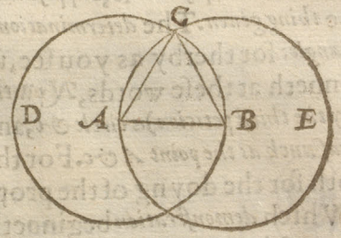
The first Probleme. The first Proposition.

Vpon a right line geuen not beyng infinite, to describe an equilater triangle, or a triangle of three equall sides.

SVppose that the right line geuen be AB. It is required vpon the line AB, to describe an equilater triangle, namely, a triangle of three equall sides. Now therfore making the centre the point A and the space AB, describe (by the third petition) a circle BCD: and agayne (by the same) makyng the centre the point B, and the space BA, describe an other circle ACE. And (by the first petition) from the point C, wherin the circles cut the one the other, draw one right line to the point A, and an other right line to the point B. And forasmuche as the point A is the centre of the circle CBD, therfore (by the 15. definition) the line AC is equall to the line AB: Agayne forasmuch as the point B is the centre of the circle CAE, therfore (by the same definition) the line BC is equall to the line BA. And it is proued, that the line AC is equall to the line AB: wherfore either of these lines CA and CB, is equall to the line AB: but thinges which are equall to one and the same thing, are also equall the one to the other (by the first common sentence) wherfore the line CA, also is equall to the line CB. VVherfore these three right lines CA, AB, and BC are equal the one to the other. VVherfore the triangle ABC is equilater. VVherfore vppon the line AB, is described an equilater triangle ABC. VVherfore vppon a line geuen not being infinite, there is described an equilater triangle, VVhich is the thing, which was required to be done.

Construction.

Demonstration



A triangle or any other rectilined figure is then said to be set or described vpon a line, when the line is one of the sides of the figure.

This first proposition is a *Probleme*, because it requireth acte or doyng, namely, to describe a triangle. And this is to be noted, that euery *Proposition*, whether it be a *Probleme*, or a *Theoreme*, commonly containeth in it *a thing geuen, and a thing required to be searched out:* although it be not alwayes so. And the *thing geuen*, is euer set before the *thing required*. In some propositions there are more things geuē then one, and mo thinges required then one. In some there is nothing geuen at all.

Thing geuen.
Thing required

Moreouer euery *Probleme & Theoreme*, beyng perfect and absolute, ought to haue all these partes, namely. First the *Proposition*, to be proued. Then the *exposition*

Proposition.
Exposition.

Determination. *Conſtruction.* *Demonſtration.* *Concluſion.*

which is the explication of the thing geuen. After that followeth the *determinatiõ*, which is the declaration of the thing required. Then is ſet the *conſtruction* of ſuche things which are neceſſary ether for the doing of the propoſitiõ, or for the demõſtration. Afterward followeth the *demonſtration*, which is the reaſon and proofe of the propoſition. And laſt of all is put the *concluſion*, which is inferred & proued by the demonſtration, and is euer the propoſition. But all thoſe partes are not of neceſſitie required in euery Probleme and Theoreme. But the *Propoſition, demonſtration, and concluſion*, are neceſſary partes, & can neuer be abſent: the other partes may ſometymes be away.

Caſe.

Further in diuers propoſitions, there happen diuers caſes: which are nothing els, but varietie of delineatiou and conſtruction, or chaunge of poſition, as when pointes, lines, ſuperficieſſes, or bodies are chaunged. VVhich thinges happen in diuers propoſitions.

The thing geuen in this Probleme. *The thing required.* *The propoſition.* *The expoſition.* *The determination.* *The conſtructiõ*

NOw then in this *Probleme*, the *thing geuen*, is the line geuẽ: the *thing required*, to be ſerched out is, how vpõ that line to deſcribe an equilater triangle. The *Propoſition* of this *Probleme* is, *Vpon a right line geuen not beyng infinite, to deſcribe an equilater triangle.*

The *expoſition* is, *Suppoſe that the right line geuen be A B*, and this declareth onely the *thing geuen*. The *determination* is, *It is required vpon the line A B, to deſcribe an equilater triangle:* for therby as you ſee, is declared onely *the thing required*. The *conſtruction* beginneth at theſe words, *Now therfore making the cẽtre the point A, & the ſpace A B, deſcribe (by the third petition) a circle &c*, and continueth vntil you come to theſe wordes, *And foraſmuch as the point A &c.* For thether to are deſcribed circles and lines, neceſſarye both for the doyng of the propoſition, and alſo for the demonſtration therof. VVhich *demonſtration* beginneth at theſe wordes: *And foraſmuche as the point A is the centre of the circle C B D &c*: And ſo procedeth till you come to theſe wordes, *VVherfore vpon the line A B is deſcribed an equilater triangle A B C.* For vntill you come thether, is, by groundes before ſet and conſtructions had, proued, and made euident, that the triangle made, is equilater. And then in theſe wordes, *wherfore vpon the line A B, is deſcribed an equilater triangle A B C*, is put the firſt *concluſion*. For there are commonly in euery propoſition two concluſions: the one perticuler, the other vniuerſal: and from the firſt you go to the laſt. And this is the firſt and perticuler concluſion, for that it concludeth, that vpon the lyne A B is deſcribed an equilater triãgle, which is according to the expoſition. After it, followeth the laſt and vniuerſal concluſion, *wherfore vpon a right line geuen not being infinite is deſcribed an equilater triangle.* For whether the line geuen be greater or leſſe then thys lyne, the ſame conſtructiõs and demonſtrations proue the ſame concluſion. Laſt of all is added this clauſe, *Which is the thing which was required to be done:* wherby as we haue before noted, is declared, that this propoſition is a *Probleme* and not a *Theoreme*. As for varietie of caſes in this propoſition there is none, for that the line geuen, can haue no diuerſitie of poſition.

The perticular concluſion. *The vniuerſall concluſion.* *The note where by it is knowne to be a Probleme.* *No caſes in thys propoſition.*

As you haue in this *Probleme* ſene plainely ſet foorthe *the thing geuen, and the thing required*, moreouer the *propoſition, expoſition, determination, conſtruction, demonſtration, and concluſion* (which are generall alſo to many other both *Problemes and Theoremes*) ſo may you by the example therof diſtinct them, and ſearche them out in other *Problemes*, and alſo *Theoremes*.

Three kindes of demonſtration.

This alſo is to be noted, that there are three kyndes of demonſtration, The one is called *Demonſtratio a priori*, or compoſition. The other is called *Demonſtratio a poſteriori.*

a posteriori, or resolution. And the third is a demonstration leadyng to an impossibilitie.

A demonstration *a priori*, or composition is, when in reasoning, from the principles and first groundes, we passe discending continually, till after many reasons made, we come at the length to conclude that, which we first chiefly entend. And this kinde of demonstration vseth Euclide in his bookes for the most part. *Demonstration a priori, or composition.*

A demonstration *a posteriori*, or resolution is, when contrariwise in reasoning, we passe from the last conclusion made by the premisses, and by the premisses of the premisses, continually ascending, til we come to the first principles and grounds, which are indemonstrable, and for their simplicity can suffer no farther resolution. *Demonstration a posteriori, or resolution.*

A demonstration leadyng to an impossibilitie is that argument, whose cõclusion is impossible: that is, when it concludeth directly against any principle, or against any proposition before proued by principles, or propositions before proued. *Demonstration leading to an impossibility.*

Premisses in an argument, are propositions goyng before the conclusion by which the conclusion is proued. *Premisses what they are.*

Composition passeth from the cause to the effect, or from thinges simple to thinges more compounded. Resolution contrariwise passeth from thinges compounded to thinges more simple, or from the effect to the cause.

Composition or the first kynde of demonstration, which passeth from the principles, may easely be sene in this first proposition of Euclide. The demonstration wherof beginneth thus. And forasmuch as the point A is the centre of the circle C B D, therfore the line A C, is equal to the line A B. This reason (you see) taketh his beginnyng of a principle, namely, of the definition of a circle. And this is the first reason. Agayne forasmuch as B is the centre of the circle C A E, therfore the line B C is equall to the lyne B A: which is the second reason. And it was before proued that the lyne A C is equall to the line A B, wherfore either of these lines C A & C B is equal to the lyne A B. And this is the third reasõ. But things which are equall to one & the selfe same thyng, are also equall the one to the other. Wherfore the line C A is equal to the line C B. And this is the fourth argument. Wherfore these three lines C A, A B, and B C are equall the one to the other which is the conclusion, and the thing to be proued.

An example of composition in the first proposition.
First reason.
Second reason.
Third reason.
Fourth reason.
Conclusion.

You may also in the same first Propositiõ, easely take an exãple of Resolutiõ: vsing a contrary order passyng backward frõ the last conclusiõ of the former demonstration, til you come to the first principle or ground wheron it began. For the last argument or reason in composition, is the first in Resolution: & the first in composition, is the last in resolution. Thus therfore must ye procede. The triangle A B C is contained of three equall right lines, namely, A B, A C, and B C, and therfore it is an equilater triangle by the definition of an equilater triangle; and this is the first reason. That the three lines be equall, is thus proued. The lines A C and C B are equall to the line A B, wherfore they are equall the one to the other: and this is the second reason. That the lines A B and B C, are equal is thus proued: The lines A B and A C, are drawen from the centre of the circle A C E, to the circumference of the same: wherfore they are equall by the definitiõ of a circle: and this is the third reason. Likewise that the lines A C and A B, are equall

Example of resolution in the first proposition.
First reason.
Second reason.
Third reason.
Fourth reason which is the end of the whole resolution.

equall, is proued by the same reason. For the lines A C and A B are drawn from the centre of the circle B C D: wherfore they are equall by the same definition of a circle: this is the fourth reason or sillogisme. And thus is ended the whole resolution: for that you are come to a principle, which is indemõstrable, & can not be resolued.

Of a demonstration leading to an impossibilitie, or to an absurditie, you may haue an example in the fourth proposition of this booke.

An addition of Campanus.

BVt nowe if vpon the same line geuen, namely, *A B*, ye wil describe the other two kinds of triangles, namely, an *Isosceles* or a triãgle of two equal sides, & a *Scalenon*, or a triangle of three vnequall sides. First for the describing of an *Isosceles* triangle produce the line *A B* on ether side, vntill it concur with the circumferences of both the circles in the pointes *D* and *F*, and making the centre the point *A*, describe a circle *H F G* according to the quãtity of the line *A F*. Likewise making the centre the poynte *B*, describe a circle *H D G*, according to the quantitie of the line *B D*. Now thẽ these circles shall cut the one the other in two poyntes, which let be *H*, and *G*: And let the endes of the line geuen be ioyned with one of the sayd sections by two right lines, which let be *A G* and *B G*. And forasmuche as these two lines *A B* and *A D* are drawen frõ the centre of the circle *C D E* vnto the circumferẽce therof, therfore ar they equal. Likewise the lines *B A* and *B F*, for that they are drawen from the centre of the circle *E A C F* to the circumference therof, are equal. And forasmuch as ether of the lines *A D* and *B F* is equall to the line *A B*, therfore they are equal the one to the other. Wherfore putting the line *A B* cõmõ to thẽ both, the whole line *B D* shal be equall to the whole line *A F*. But *B D* is equal to *B G*, for they are both drawen frõ the cẽtre of the circle *H D G* to the circumferẽce therof. And likewise by the same reason the line *A F* is equal to the line *A G*. Wherfore by the cõmõ sentẽce the lines *A G* and *B G* are equal the one to the other, and either of them is greater then the line *A B*, for that either of the two lines *B D* and *A F* is greater then the line *A B*. Wherfore vpon the line geuen is described an *Isosceles* or triangle of two equall sides.

How to describe an Isosceles triangle.

How to describe a Scalenum.

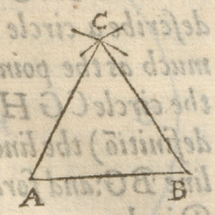
Ye may also describe vpon the selfe same line a *Scalenon*, or triangle of three vnequall sides, if by two right lines, ye ioyne both the endes of the line geuen to some one point that is in the circumference of one of the two greater circles, so that that poynt be not in one of the two sections, and that the line *D F* do not concur with it, when it is on either side produced continuallye and directlye. For let the poynte *K* be taken in the circumference of the circle *H D G*, and let it not be in any of the sections, neyther let the line *D F* concur with it, when it is produced continually and directly vnto the circumference therof. And draw these lines *A K* and *B K*, and the line *A K* shal cut the circumference of the circle *H F G*. Let it cut it in the poynte *L*: now then by the common sentence the line *B K* shalbe equal to the line *A L*, for (by the definition of a circle) the line *B K* is equall to the line *B G*, and the line *A L* is equall to the line *A G* which is equal to the line *B G*. Wherfore the line *A K* is greater then the line *B K* and by the same reason maye it be proued that the line *B K* is greater then the line *A B*.

Wherfore

Wherfore the triangle *ABK* consisteth of three vnequal sides. And so haue ye vpon the line geuen, described all the kindes of triangles.

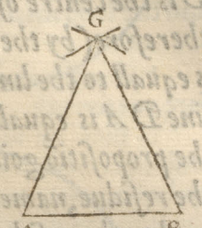
This is to be noted, that if a man will mechanically and redely, not regarding demonstration vpon a line geuen describe a triangle of three equall sides, he needeth not to describe the whole foresayd circle, but onely a little part of eche: namely, where they cut the one the other, and so from the point of the section to draw the lines to the endes of the line geuen. As in this figure here put.

How to describe an equilater triangle redily & mechanically.



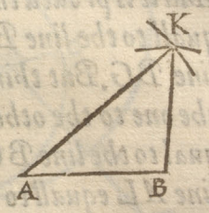
And likewise, if vpon the said line he will describe a triangle of two equall sydes, let him extende the compasse according to the quantitie that he will haue the syde to be, whether longer then the line geuen or shorter: and so draw onely a litle part of eche circle, where they cut the one the other, & frō the point of the section draw the lines to the ende of the line geuen. As in the figures here put. Note that in this the two sydes must be such, that beyng ioyned together, they be longer then the line geuen.

How to describe an Isosceles triangle redily.



And so also if vpon the sayd right line he will describe a triangle of three vnequal sydes, let him extend the compasse. First, according to the quantitie that he will haue one of the vnequall sydes to be, and so draw a little part of the circle, & then extend it according to the quantitie that he wil haue the other vnequal syde to be, and draw likewyse a little part of the circle, and that done, from the point of the section draw the lines to the endes of the line geuen, as in the figure here put. Note that in this the two sides must be such, that the circles described according to their quātitie may cut the one the other.

How to describe a Scalenum triangle redily.



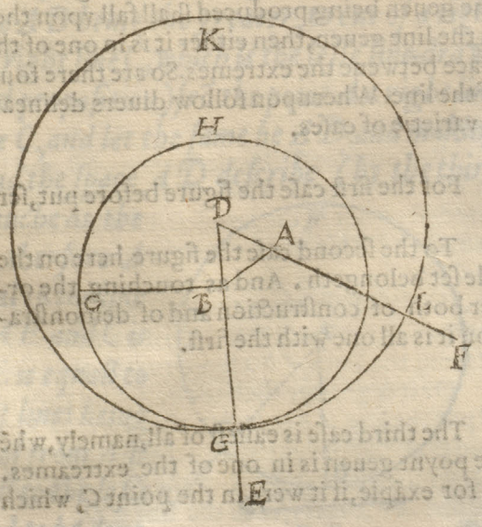
The second Probleme. The second Proposition.

Frō a point geuen, to draw a right line equal to a right line geuen.

Vppose that the point geuē be A, & let the right line geuen be B C. It is required frō the point A, to drawe a right lyne equall to the line B C.

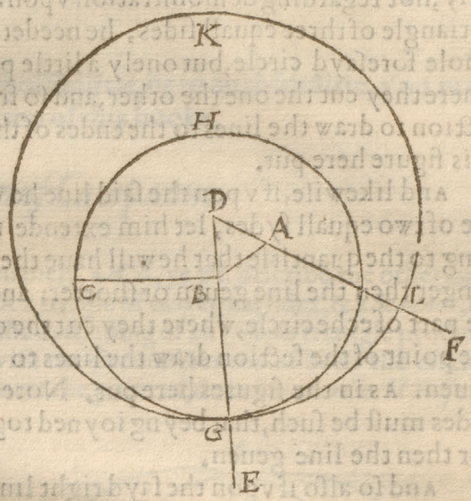
Draw (by the first peticiō) from the point A to the poynte B a right line A B: and vpon the line A B describe (by the first propositiō) an equilater triangle, and let the same be D A B, and extēd, by the second peticiō, the right lines D A & D B, to the poyntes E

Construction.



and F, & (by the third peticiō) making the centre B and the ſpace BC deſcribe a circle CG H: & againe (by the ſamē) making the centre D and the ſpace DG deſcribe a circle GKL. And foraſmuch as the pointe B is the centre of the circle CGH, therfore (by the 15. definitiō) the line BC is equal to the line BG: and foraſmuch as the poynt D is the centre of the circle GKL: therefore (by the ſame) the line DL is equall to the line DG: of which the line DA is equall to a line DB (by the propoſitiō going before) wherfore the reſidue, namely, the line AL is equal to the reſidue, namely, to the line BG (by the third common ſentence) And it is proued that the line BC is equall to the line BG VVherfore eyther of theſe lines AL & BC is equal to the line BG. But things which are equall to one and the ſame thing are alſo equall the one to the other (by the firſt commō ſentence.) VVherfore the line AL is equal to the line BC. VVherfore from the poynt geuē, namely, A, is drawn a right line AL equall to the right line geuen BC: which was required to be done.

Demonſtration.



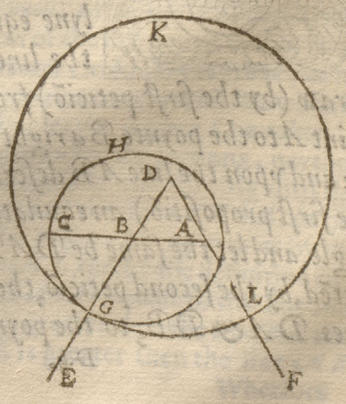
Of Problemes and Theorems, as we haue before noted, ſome haue no caſes at all, which are thoſe which haue onely one poſition and conſtruction: and other ſome haue many and diuers caſes: which are ſuch propoſitions which haue diuers deſcriptions & conſtructions, and chaunge their poſitions. Of which ſorte is this ſecond propoſition, which is alſo a Probleme. This propoſition hath two thinges geuen: Namely, a pointe, and a line: the thing required is, that from the pointe geuen whereſoeuer it be put, be drawen a line equall to the line geuen. Now this poynt geuen may haue diuers poſitiōs For it may be placed eyther without the right line geuen, or in ſome point in it. If it be without it, either it is on the ſide of it, ſo that the right line drawen from it to the ende of the right line geuen maketh an angle: or els it is put directly vnto it, ſo that the right line geuen being produced ſhall fall vpon the point geuen which is without. But if it be in the line geuen, then either it is in one of the endes or extreames thereof: or in ſome place betwene the extremes. So are there foure diuers poſitions of the poynt in reſpect of the line. Wherupon follow diuers delineations and conſtructions, and conſequently varietie of caſes.

Two thinges geuen in this propoſition.

Fower caſes in this propoſition.

The firſt caſe. For the firſt caſe the figure before put, ſerueth.

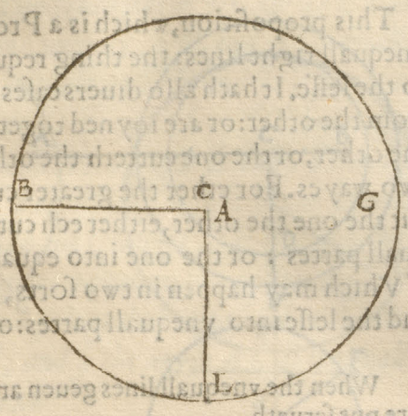
The ſecond caſe. To the ſecond caſe the figure here on the ſide ſet belongeth. And as touching the order both of conſtruction and of demonſtration it is all one with the firſt.



The third caſe. The third caſe is eaſieſt of all, namely, whē the poynt geuen is in one of the extreames. As for exāple, if it were in the point C, which

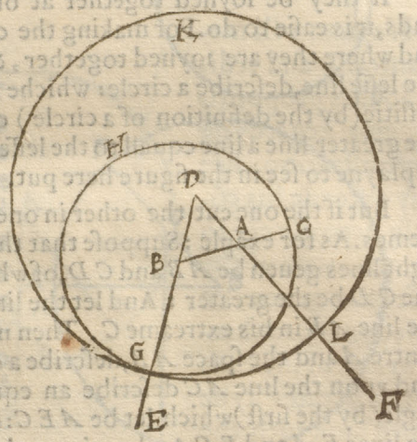
is

is one of the extreames of the line *B C*. Then making the centre the poynt *C*, and the space *C B* describe a circle *B L G*: and from the centre *C* drawe a line vnto the circumference, which I set the *C L*, which by the definition of a circle, shalbe equall to the line geuen *B C*.



The fourth case.

The fourth case as touching construction herein differeth from the two firste, for that whereas in thē you are willed to draw a right line from the poynt geuen, namely, *A*, to the poynt *B* which is one of the endes of the line geuē, here you shal not nede to draw that line, for that it is already drawen. As touching the rest, both in construction and demonstration you may proceede as in the two firste. As it is manifeste to see in thys figure here on the side put.



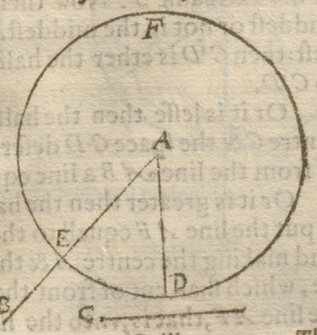
This proposition though it be very easie to be done mechanically, yet is no principle.

This proposition for the playnes & easines thereof, seemeth to be as it were a principle, and may easly mechanically be done. For opening the compasse to the quantitye of the line geuen, and setting on foote of it fixed in the poynt geuen and marking with the other another poynt whersoeuer it fall, & so by the firste peticion drawing a right line frō the one of those poyntes to the other, the sayd righte line shall beequall to the right line geuen: yet in deede is it no principle, for that it may by demonstration be proued: but principles can not be proued, as we haue before declared.

The 3. Probleme. The 3. Proposition.

Two vnequal right lines being geuen, to cut of from the greater, a right lyne equall to the lesse.

Vppose that the two vnequal right lines geuen be A B & C, of which let the lyne A B be the greater. It is required from the line A B being the greater, to cut of a right line equal to the right line C, which is the lesse line drawen (by the second proposition) frō the point A a right line equall to the line C, and let the same be A D: and making the centre A and the space A D describe (by the third peticion) a circle D E F. And forasmuche as the point A is the centre of ỹ circle D E F, therfore A E is equal to A D, but the line C is equal to the line A D. Wherfore either of these lines A E and C is equall to A D, wherfore the line A E is equall to the line C, wherfore two vnequall right lines being geuen, namely, A B and C, there is cut of from A B being ỹ greater, a right line A E equall to the lesse lyne, namely, to C: which was required to be done.



Two thinges geuen in this propoſition. Diuers caſes in it.

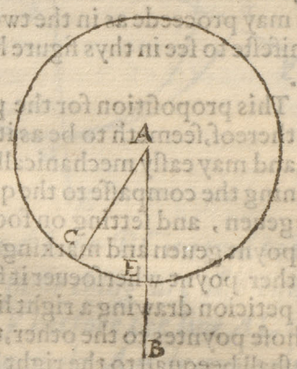
This propoſition, which is a Probleme, hath two thinges geuen, namely, two vnequall right lines: the thing required is, from the greater to cut of a line equal to the leſſe. It hath alſo diuers caſes. For the lines geuen either are diſtinct th'one from the other: or are ioyned together at one of their endes: or they cut the one the other, or the one cutteth the other in one of the extreames. VVhich may be two wayes. For ether the greater cutteth the leſſe, or the leſſe the greater. If they cut the one the other, either ech cutteth th'other into equall partes: or into vnequall partes: or the one into equall partes, and the other into vnequall partes. VVhich may happen in two ſorts, for the greater may be cut into equall partes, and the leſſe into vnequall partes: or contrariwiſe.

The firſt caſe.

When the vnequall lines geuen are diſtinct the one from the other, the figure before put ſerueth.

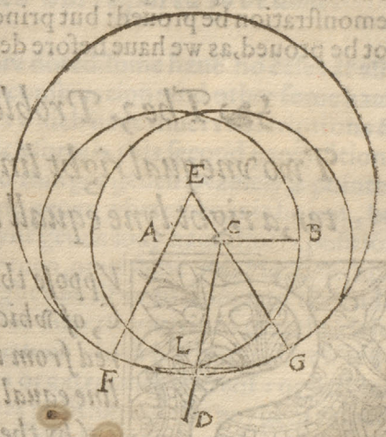
The ſecond caſe.

If they be ioyned together at one of their ends, it is eaſie to do. For making the centre that end where they are ioyned together, & the ſpace the leſſe line, deſcribe a circle: whiche ſhall of neceſſitie (by the definition of a circle) cut off from the greater line a line equall to the leſſe line, as it is playne to ſee in the figure here put.



The third caſe.

But if the one cut the other in one of the extremes. As for exãple: Suppoſe that the vnequall right lines geuen be *A B* and *C D*, of which let the line *C D* be the greater: And let the line *C D* cut the line *A B* in his extreame *C*. Then making the centre *A* and the ſpace *A B*, deſcribe a circle *B F*. And vpon the line *A C* deſcribe an equilater triangle (by the firſt) which let be *A E C*: & produce the lines *E A* and *E C*. And againe making the cẽtre *E* and the ſpace *E F* deſcribe a circle *G F*. Likewiſe making the centre *C* and the ſpace *C G*, deſcribe a circle *G L*. Now foraſmuch as the line *E F* is equall to the line *E G* (For *E* is the centre) of which the line *E A* is equall to the line *E C*: therfore the reſidue *A F* is equall to the reſidue *C G*. But the line *A F* is equall to the line *A B*, for *A* is the centre, wherefore alſo the line *C G* is equall to the line *A B*. But the line *C G* is alſo equall to the line *C L*, for the point *C* is the centre. Wherefore the line *A B* is equall to the line *C L*. Wherefore from the line *C D* is cut of the line *C L* which is equall to the line *A B*.



The fourth caſe

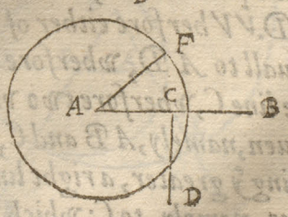
But now let *C D* be leſſe then *A B*, and let it cut *A B* by his extreame *C*. Now then eyther it cutteth it in the middeſt or not in the middeſt. Firſt let it cut it in the middeſt: then *C D* is ether the halfe of *A B*, & ſo is *A C* equal to *C D*.

The fifth caſe.

Or it is leſſe then the halfe: and then making the centre *C* & the ſpace *C D* deſcribe a circle, which ſhall cut of from the line *A B* a line equal to the line *C D*.

The ſixt caſe.

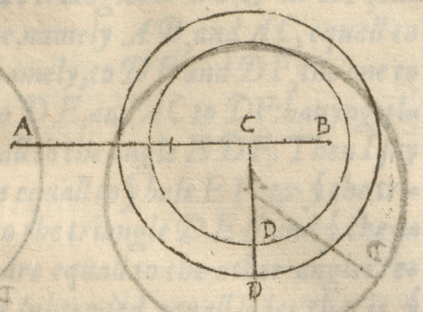
Or it is greater then the half. And thẽ vnto the point *A* put the line *A F* equall to the line *C D*, by the ſecond. And making the centre *A* & the ſpace *A F* deſcribe a circle, which ſhall cut of from the line *A B* a line equall to the line *A F*, that is, vnto the line *C D*.



But

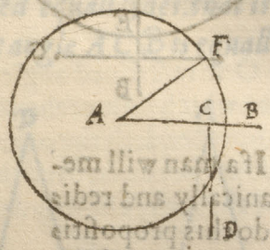
But if the line *C D* do not cut the line *A B* in the midst: *C D* shal either be the halfe of the line *A B*: or greater then the halfe, or lesse. If *C D* be the halfe of *A B*, or lesse then the half of *A B*, thé making the centre *C*, and the space *C D* describe a circle whiche shall cut of from the line *A B* a line equal to the line *C D*.

The seuenth & eight cases.



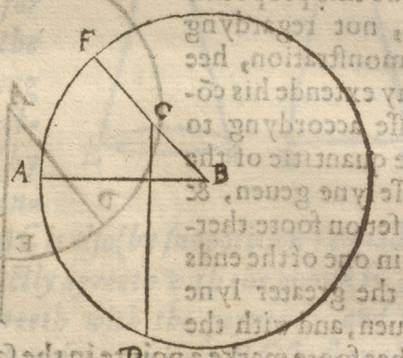
But if it be greater then the halfe, then againe vnto the point *A* put the line *A F* equal to the line *C D* (by the second propositiō:) & making the centre *A*, and the space *A F* describe a circle which shall cut of from the line *A B* a line equall to the line *A F*, that is, to the line *C D*.

The ninth case.



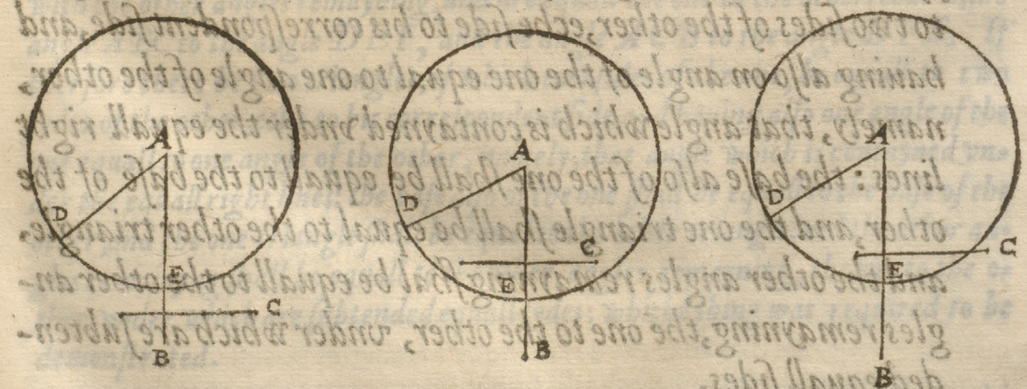
But if they cut the one the other as the lines *C D* & *A B* do. Thē making the cētre *B* & the space *B A* describe a circle *A F*, & draw a line from the point *B* to the point *C*, & produce it to the point *F*. And forasmuch as the two right lines *B F* and *C D* are vnequall, and the line *C D* cutteth the line *B F* by one of his extreames, therefore it is possible to cut of from *C D* a line equall to the line *B F*. For how to do it we haue before declared, wherefore it is possible from the line *C D* to cut of a line equall to the line *A B*: or *A B* and *B F* are equall the one to the other.

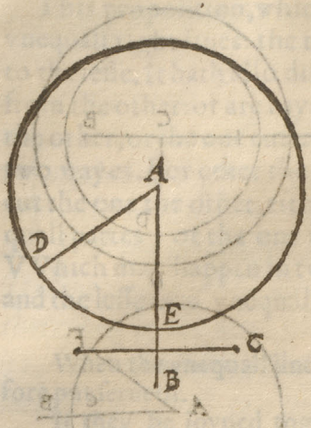
The tenth case.

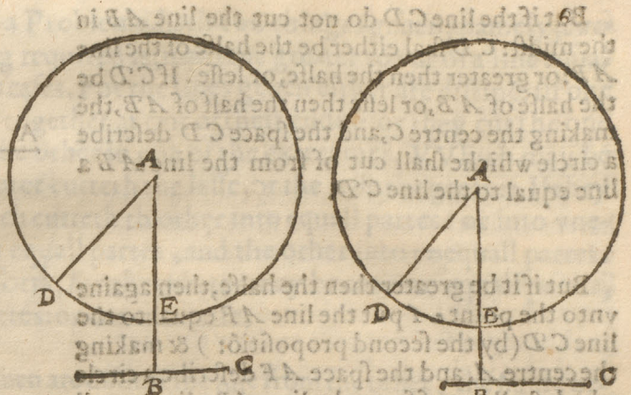


This is to be noted, that in all these cases, a man may both as touchyng construction and demonstration, procede as in the first case. For it is possible in any position to put to the ende of the greater lyne a line equall to the lesse lyne, and so makyng the centre the sayd ende, and the space the lesse line, to describe a circle, which shall cut of frō the greater lyne a lyne equall to the line put, namely, to the lesse line geuen, as it is manifest to see in the figures partly here vnder set, and partly in the beginning of the other side put.

In all these cases the construction and demōstration of the first case will serue.

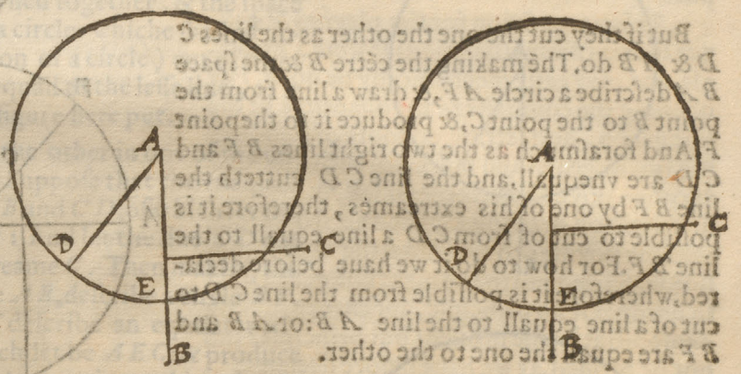






This Proposition, thoughe also it be most easie to be done mechanically. yet is no principle.

If a man will mechanically and redily do this propositi, on, not regardyng demonstration, hee may extende his cõpasse accordyng to the quantitie of the lesse lyne geuen, & so set on foote therof in one of the ends of the greater lyne geuen, and with the other foote marke a pointe in the said greater line, which shall cutte of from the greater line a line equall to the lesse. The easines of doing wherof may cause this proposition also to seeme vnto some to be rather a principle, then a proposition. But to that we haue before in the former proposition answered.



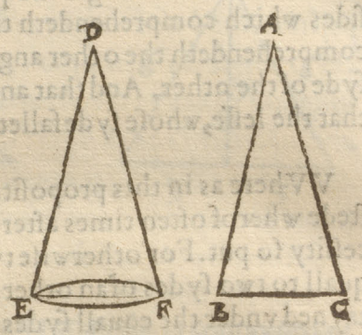
The first Theoreme. The 4. Proposition.

If there be two triangles, of which two sides of th'one be equal to two sides of the other, eche side to his correspondent side, and hauing also on angle of the one equal to one angle of the other, namely, that angle which is contayned vnder the equall right lines: the base also of the one shall be equall to the base of the other, and the one triangle shall be equal to the other triangle, and the other angles remayning shal be equall to the other angles remayning, the one to the other, vnder which are subtended equall sides.

Suppose

SVppose that there be two triangles A B C, & D E F, hauing two sides of the one, namely A B, and A C, equall to two sides of the other, namely, to D E and D F, the one to the other, that is, A B to D E, and A C to D F: hauyng also the angle B A C, equall to the angle E D F. Then I say that the base also B C is equall to ỹ base E F: & ỹ the triangle A B C, is equall to the triangle D E F: and ỹ the other angles remainyng are equall to the other angles remayning, the one to the other, vnder which are subtended equall sydes: that is, ỹ the angle A B C is equall to the angle D E F, and ỹ the angle A C B is equall to to the angle D F E. For the triangle A B C exactly agreyng with the triangle D E F, and the point A being put vpō the point D, & the right line A B vpon the right line D E, the pointe B also shall exactly agree with the pointe E: for that (by supposition) the line A B is equal to the line D E. And the line A B exactly agreeyng with the line D E, the right line also A C exactly agreeth with the right line D F, for that (by supposition) the angle B A C is equall to the angle E D F. And forasmuch as the right line A C is also (by supposition) equall to the right line D F, therfore the pointe C exactly agreeth with the pointe F. Againe forasmuch as the pointe C exactly agreeth with the poynte F, and the point B exactly agreeth with the point E: therefore the base B C shall exactly agree with the base E F. For if the point B do exactly agree with the point E, and the point C with the point F, and the base B C do not exactly agre wyth the base E F, then two right lines do include a superficies: which (by the 10. cōmon sentence) is impossible. VVherfore the base B C exactly agreeth wt the base E F, and therfore is equall vnto it. VVherfore the whole triangle A B C exactly agreeth with the whole triangle D E F, & therfore (by the 8. common sentence) is equall vnto it. And (by the same) the other angles remayning exactly agree with the other angles remayning, and are equall the one to the other: that is, the angle A B C to the angle D E F, and the angle A C B to the angle D F E. If therfore there be two triangles, of which two sides of the one, be equall to two sydes of the other, eche to his correspondent side, and hauing also one angle of the one equall to one angle of the other, namely, that angle which is contayned vnder the equall right lines: the base also of the one shall be equall to the base of the other, and the one triangle shall be equal to the other triangle, and the other angles remainyng shall be equall to the other angles remayning, the one to the other, vnder which are subtended equall sydes: whiche thing was required to be demonstrated.

Demonstration leading to an absurditie.



Two thinges geuen in this proposition.

This Proposition which is a Theoreme, hath two things geuen: namely, the

Three things required in it.

equality of two sides of the one triangle, to two sides of the other triangle, and the equalitie of two angles contayned vnder the equall sydes. In it also are thre thinges required. The equality of base to base: the equality of field to field: and the equality of the other angles of the one triangle to the other angles of the other triangle, vnder which are subtended equall sides.

How one side is equall to an other, & so generally how one right line is equall to an other.

One side of a playne figure is equall to an other, and so generally one right lyne is equall to an other, when the one being applied to the other, theyr extreames agree together. For otherwise euery righte line applied to any right lyne, agreeth therwith: but equall right lines only, agree in the extremes.

How one rectilined angle is equal to an other

One rectilined angle is equall to an other rectilined angle, when one of the sides which comprehendeth the one angle, being set vpon one of the sides which comprehendeth the other angle, the other side of the one agreeth with the other syde of the other. And that angle is the greater, whose syde falleth without: and that the lesse, whose syde falleth within.

Why this particle, ech to his correspondent side, is put.

VVhere as in this proposition is put this particle *eche to his correspondent side,* (in stede wherof often times afterward is vsed this phrase *the one to the other*) it is of necessity so put. For otherwise two sydes of one triangle added together, may be equall to two sydes of an other triangle added together, and the angles also contayned vnder the equall sydes may be equall: and yet the two triangles may notwithstanding be vnequall. VVhere note that a triangle is sayd to be equall to an other triangle, when the field or area of the one is equall to the area of the other.

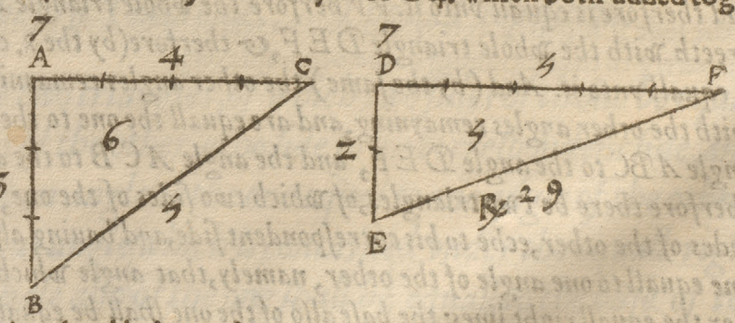
How one triangle is equall to an other.

What the fielde or area of a triangle is, and so of any rectilined figure.

And the area of a triangle, is that space, which is contayned within the sydes of a triangle.

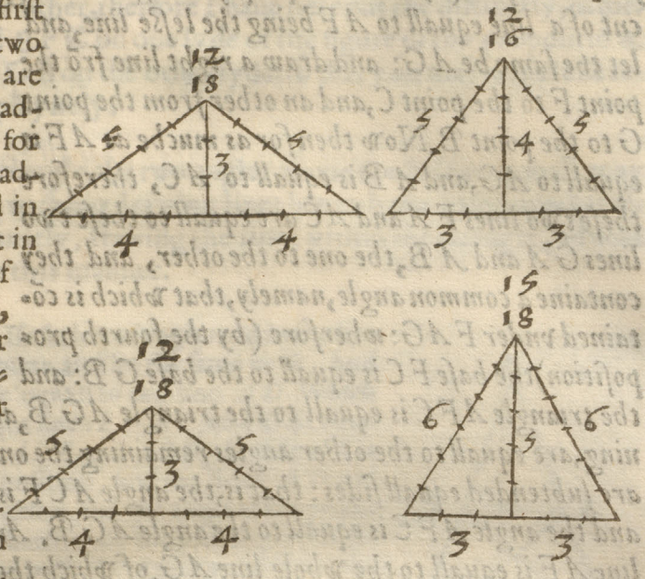
What the circuite or compasse of a triangle is, and so also of any rectilined figure.

And the circuite or compasse of a triangle is a line composed of all the sides of a triangle. And so may you think of all other rectilined figures. And now to proue that there may be two triangles, two sydes of one of which being added together, may be equall to two sydes of the other added together, and the angles contayned vnder the equall sydes may be equall, and yet notwithstanding the two triangles vnequall. Suppose that there be two rectangle triangles: namely, A B C, and D E F, and let their right angles be B A C and E D F. And in the triangle A B C let the syde A B be 3. and the syde A C 4. which both added together make 7.



And in the triãgle D E F, let the side D E be 2. and the side D F be 5. whiche added together make also 7. & so the sydes of the one triangle added together, are equall to the sides of the other triãgle added together. Yet are both the triangles vnequall, and also their bases. For the area of the triangle A B C is 6. and his base is 5. And the area of the triangle D E F is 5: and his base is 29. So that to haue the areas of two triangles to be equall, it is requisite that all the sydes of the two triangles be equall, eche to hys correspondent syde. It happeneth also sometymes in triangles, that the areas of them beyng equall, their sydes added together shall be vnequall. And contrariwise

wise, their sides beyng equall, their areas be vnequall. As in these figures here put it is plaine to see. In the first example the areas of the two triangles are equal, for they are eche 12. and the sides in ech added together are vnequall, for in the one triangle the sides added together make 18. and in the other they make 16. But in the seconde exaple the areas of the two triagles are vnequal, for the one is 12. and th'other is 15. But the sides added together in eche are equall, for in eche they make 18.



That angle is said to subtend a side of a triagle, which is placed directly opposite, & against that side. That side also is sayd to subtend an angle, which is opposite to the angle. For euery angle of a triangle is contayned of two sydes of the triangle, and is subtended to the third side.

How an angle is sayd to subtēd a side: and a side an angle.

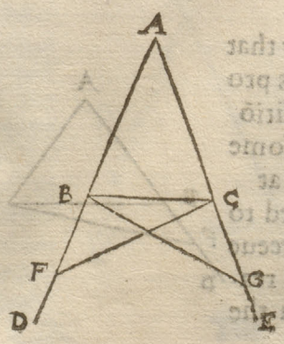
This is the first Proposition in which is vsed a demonstration leading to an absurditie, or an impossibilitie. Which is a demonstration that proueth not directly the thing entended, by principles, or by thinges before proued by these principles: but proueth the contrary therof to be impossible, & so doth indirectly proue the thing entended.

This proposition proued by a demonstration leading to an absurdity.

The 2. Theoreme. The 5. Proposition.

An Isosceles, or triangle of two equal sides, hath his angles at the base equall the one to the other. And those equal sides being produced, the angles which are vnder the base are also equall the one to the other.

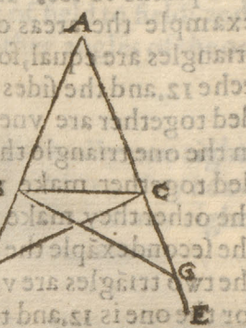
Suppose that A B C be a triangle of two equall sydes called Isosceles, hauing the syde A B equall to the side A C. And (by the second peticiō) produce the lines A B & A C directly to ỹ points D & E. Thē I say, that ỹ angle A B C is equal to the angle A C B: and ỹ ỹ angle C B D is equal to th'angle B C E. Take in the line B D a point at all aduentures, and let the



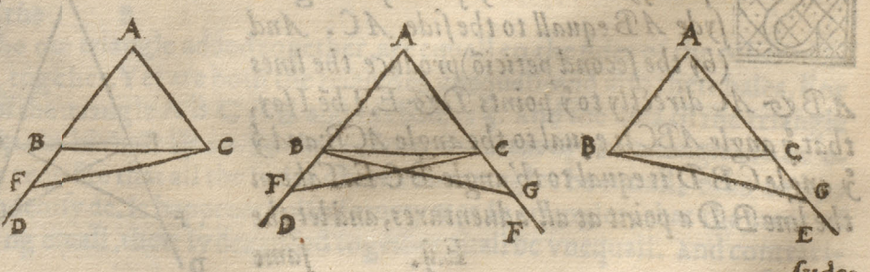
Construction.

same be F, and (by the third proposition) from the greater line, namely, AE, cut of a line equall to AF being the lesse line, and let the same be AG: and draw a right line frō the point F to the point C, and an other from the point G to the poin: B. Now then for as muche as AF is equall to AG, and AB is equall to AC, therefore these two lins FA and AC are equall to these two lines GA and AB, the one to the other, and they containe a conmon angle, namely, that which is cōtained vnder FAG: wherfore (by the fourth proposition) the base FC is equall to the base GB: and the triangle AFC is equall to the triangle AGB, and the other angles remaining, are equll to the other angles remaining the one to the other, vnder which are subtendel equall sides: that is, the angle ACF is equall to the angle ABG, and the angle AFC is equall to the angle AGB. And forasmuch as the whole line AF is equall to the whole line AG, of which the line AB is equal to ỹ lyne AC, therfore the residue of the line AF, namely, the line BF, is equal to the residue of the lne AG, namely, to the line CG (by the third common sentence) And it is proued that CF is equal to BG. Now therfore these two BF & FC are equall to these two CG and GB the one to the other, and the angle BFC is equall to the angle CGB, and they haue one base, namely, BC, common to them both: wherfore (by the 4. proposition) the triangle BFC is equalt to the triangle CGB, and the other angles remaynyng are equall to the other angles remaining eche to other, vnder which are subtended equall sides. VVherfore the angle FBC is equall to the angle GCB, and the angle BCF is equall to the angle CBG. Now forasmuch as the whole angle ABG is equall to the whole angle ACF (as it hath bene proued) of which the angle CBG is equal to the angle BCF: therfore the angle remayning: namely, ABC is equall to the angle remaining, namely, to ACB (by the third common sentence) And they ar the angles at the base of the triangle ABC. And it is proued that the angle FBC is equall to the angle GCB, and they are angles vnder the base. VVherfore a triangle of two equall sides hath his angles at the base equall the one to ỹ other. And those equall sides being produced, the angles which are vnder the base are also equall the one to the other: which was required to be proued.

Demonstration.



For that this propositiō is some what hard to perceue by reason the



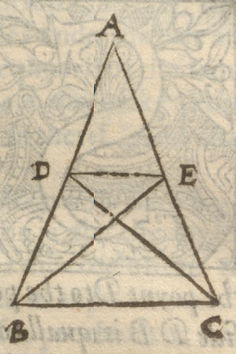
sydes

ſides of the triangles A F C and A G B & alſo the ſydes of the triãgles B F C & C G B run ſo one within an other, therfore I haue here put thẽ diſtinctly, namely, the triangles F A C and B F C on one ſyde of the figure of the propoſitiõ & the triangles A G B and C G B on the other ſyde: ſo that you may with leſſe labor ſee the demonſtration playnely.

That in an Iſoſceles triangle, the two angles aboue the baſe are equall, may otherwiſe be demonſtrated without drawing lines beneath the baſe ſomwhat altering the conſtruction. Namely, drawing the lines within the triangle, whiche before were without it after this manner.

An other demõſtration inuẽted by Proclus.

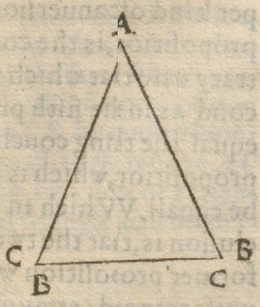
Suppoſe that *A B C* be an Iſoſceles triangle: and in the line *A B* take a point at all aduentures, and let the ſame be *D*, And from the line *A C* cut of a line eqnall to the line *A D*. Which let be *A E*. And draw theſe right lines *B E*, *D C*, and *D E*. Now foraſmuch as in the triangles, *A B E*, and *A C D*, the ſide *A B* is equall to the ſide *A C*, by ſuppoſition, and the ſides *A D* and *A E* are alſo equall by conſtruction, and the angle at the poynt *A* is common to them both: therfore, by the fourth propoſition, the baſe *B E* is equall to the baſe *D C*. And, by the ſame, the angles remayning of the one triangle are equall to the angles remayning of the other triangle. Wherefore the angle *A B E* is equal to the angle *A C* D. Againe foraſmuch as in the triangles *B D E*, and *C E D* the ſide *D B* is equall to the ſide *E C*, and the ſide *B E* to the ſide *D C*, and the angle *D B E* is equal to the angle *E C D*, and the baſe *D E* being common to both triangles is equall to it ſelfe: therefore the angles remayning of the one triangle, are equall to the angles remayning of the other triangle. Wherfore the angle *E D B* is equall to the angle D *E C*: & the angle *D E B* is equal to the angle *E D C*. And foraſmuch as the angle E D B is equal to the angle D E C, frõ which are taken away equall angles D E B, & E D C, therfore by the cõmon ſentence the angles remayning, namely, B D C and *C E B* are equall: And as it was before manifeſt the ſides *B* D and D *C* are equall to the ſides *C E* and *E B* the one to the other, that is, ech to his correſpondent ſide: and the baſe *B C* is common to both the triangles: wherfore the angles remayning are equall to the angles remayning the one to the other, vnder which are ſubtẽded equall ſides. Wherfore the angle E *B C* is equall to the angle *E C B*. For the line D *C* ſubtendeth the angle D *B C*, and the line *E B* ſubtẽdeth the angle *E C B*: which two lines are as we haue before proued equall. Wherfore in an Iſoſceles triangle, the angles at the baſe are equall, though the righ: lines be not produced.



To proue this alſo, there is an other demonſtration of Pappus much ſhorter which needeth no kind of addition of any thing at all: as followeth.

An other demõſtration inuẽted by Pappus.

Suppoſe that *A B C* be an Iſoſceles triangle, & let the ſide *A B* be equall to the ſide *A C*. Now then vnderſtand this one triangle to be as it were two triangles. And thus reaſon. Foraſmuch as in the two triangles *A B C* and *A C B*, *A B* is equal to *A C* & *A C* to *A B*, therfore two ſides of the one are equall to two ſides of the other, ech to his correſpondent ſide, & the angle *B A C* is equall to the angle *C A B*, for it is one and the ſelfe ſame angle. Wherfore by the 4. propoſition the baſe *C B* is equall to the baſe *B C*, and the triangle *A B C* is equall to the triangle *A C B*: and the angle *A B C* is equall to the angle *A C B*, and the angle *A C B* to the angle *A B C*: for vnder them are ſubtended equall ſides, namely, the lines *A B* & *A C*. Wherfore in an Iſoſceles triangle, the angels at the baſe are equall.



Thales Milesius the inuentor of this proposition.

The old Philosopher Thales Milesius was the first inuenter of this fifth proposition, as also of many other.

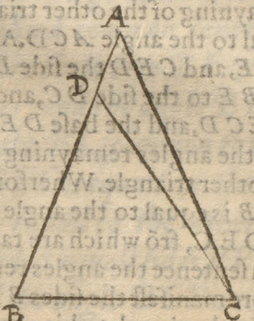
The third Theoreme. The sixt Proposition.

If a triangle haue two angles equall the one to the other: the sides also of the same, which subtend the equall angles, shalbe equall the one to the other.

Construction.

Demonstration leading to an impossibility.

Suppose that ABC be a triangle, hauing the angle ABC equall to the angle ACB. Then I say that the side AB is equall to the side AC. For if the side AB be not equal to the side AC, then one of them is greater. Let AB be the greater. And by the third proposition, from AB being the greater cut of a line equal to the lesse line, which is AC. And let the same be DB. And drawe a line from the poynt D to the poynt C. Now forasmuch as the side DB is equall to the syde AC, and the line BC is common to the both: therefore these two sydes DB and BC are equall to these two sydes AC & CB the one to the other. And the angle DBC is by suppositon equall to the angle ACB. Wherefore (by the 4 proposition) the base DC is equall to the base AB: & (by the same) the triangle DBC is equall to the triangle ACB: namely, the lesse triangle vnto the greater triagle, which is impossible. Wherefore the syde AB is not vnequal to the side AC. Wherefore it is equal. If therfore a triangle haue two angles equall the one to the other: the sydes also of the same, which subtende the equall angles, shall be equall the one to the other: which was required to be demonstrated.



The chiefest and most proper kind of conuersion.

In Geometrie is oftentimes vsed conuersion of propositions. As this proposition is the conuerse of the proposition next before. The chiefest and most proper kind of conuersion is, when that which was the thing supposed in the former proposition, is the conclusion of the conuerse and second proposition: and contrary wise that which was concluded in the first, is the thing supposed in the second as in the fifth proposition it was supposed the two sides of a triangle to be equal, the thing concluded is, that the two angles at the base are equall & in this proposition, which is the conuerse therof is supposed that the angles at the base be equall. Which in the former proposition was the conclusion. And the conclusion is, that the two sydes subtending the two angles are equall, which in the former proposition was the suppositon. This is the chiefest kind of conuersion vniforme and certayne.

There

There is an other kind of conuerſion, but not ſo full a conuerſion nor ſo perfect as the firſt is. VVhich happeneth in compoſed propoſitions, that is, in ſuch, which haue mo ſuppoſitions then one, and paſſe from theſe ſuppoſitions to one concluſion. In the cõuerſes of ſuch propoſitiõs, you paſſe from the concluſion of the firſt propoſition, with one or mo of the ſuppoſitions of the ſame: & conclude ſome other ſuppoſition of the ſelfe firſt propoſition: of this kinde are many in Euclide. Therof you may haue an example in the 8. propoſition being the conuerſe of the fourth. This conuerſion is not ſo vniforme as the other, but more diuers and vncertaine according to the multitude of the things geuen, or ſuppoſitions in the propoſition.

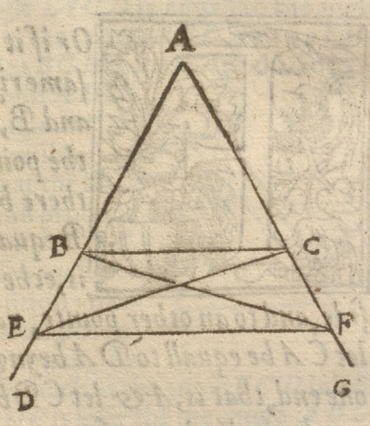
An other kind of conuerſion not ſo perfect as the firſt.

But becauſe in the fifth propoſition there are two concluſions, the firſt, that the two angles at the baſe be equall: the ſecond, that the angles vnder the baſe are equall: this is to be noted, that this ſixt propoſition is the conuerſe of the ſame fifth as touching the firſt concluſion onely. You may in like maner make a conuerſe of the ſame propoſition touching the ſecond concluſion therof. And that after this maner.

Two concluſions in the fifth propoſition.

The ſixt propoſition is the conuerſe as touching the firſt concluſion onely. The conuerſe as touching the ſecond concluſion.

THe two ſides of a triangle beyng produced, if the angles vnder the baſe be equall, the ſaid triangle ſhall be an Iſoſceles triangle. In which propoſitiõ the ſuppoſition is, that the angles vnder the baſe are equall: which in the fifth propoſition was the concluſion: & the concluſion in this propoſition is, that the two ſides of the triangle are equal, which in the fift propoſition was the ſuppoſition. But now for proofe of the ſaid propoſition:



Conſtruction.

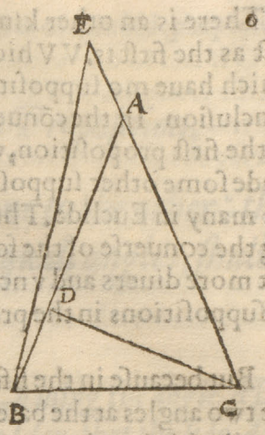
Demonſtration.

Suppoſe that there be a triangle *A B C*, & let the ſides *A B*, and *A C* be produced to the poyntes *D* and *G*, and let the angles vnder the baſe be equall, namely, the angles *D B C*, and *G C B*. Then I ſay that the triangle *A B C* is an *Iſoſceles* triangle. For take in the line *A D* a point which let be *E*. And vnto the line *B E* put the line *C F* equall (by the 3. propoſitiõ). And draw theſe lines *E C*, *B F*, and *E F*. Now foraſmuch as *B E* is equall to *C F*, and *B C* is common to thẽ both, therfore theſe two lines *B E* & *B C*, are equall to theſe two lines *C F* and *C B* the one to the other, & the angle *E B C* is equall to the angle *F C B* by ſuppoſition. Wherfore (by the 4. propoſition) the baſe of the one is equall to the baſe of the other, and the one triangle is equal to the other triangle, & the other angles remayning are equal vnto the other angles remayning, the one to the other, vnder which are ſubtended equall ſides. Wherfore the baſe *E C* is equall to the baſe *F B*, and the angle *B E C* to the angle *C F B*, and the angle *C B F* to the angle *B C E*. But the whole angle *E B C* is equall to the whole angle *F C B*, of which the angle *F B C* is equall to the angle *E C B*: wherefore the angle remayning *E B F* is equall to the angle remayning *F C E*. But the line *B E* is equall to the line *C F*, & the line *B F* to the line *C E*, and they contayne equall angles: wherfore by the ſame fourth propoſition the angle *B E F* is equall to the angle *C F E*. Wherfore by this ſixt propoſition the ſide *A E* is equall to the ſide *A F*: of whiche *B E* is equall to *C F*, by conſtruction: wherfore (by the third common ſentence) the reſidue *A B* is equall to the reſidue *A C*. Wherfore the triangle *A B C* is an *Iſoſceles* triangle. If therfore the two ſides of a triangle being produced, the angles vnder the baſe be equall, the ſayd triangle ſhall be an *Iſoſceles* triangle: which was required to be proued.

This moreouer is to be noted, that in this propoſition there may be an other caſe: for in taking an equall line to *A C* from *A B*, you may take it from the poynte *A* and

An other caſe in this ſixt propoſition.

not from the poynt *B*. And yet though this ſuppoſition alſo be put the ſelfe ſame abſurdity will follow.

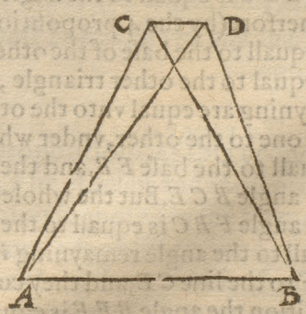


For ſuppoſe that *A C* be equall to *A D*: and produce the line *C A* to the poynt *E*: and put the line *A E* equall to the line *D B* (by the third propoſition) wherefore the whole line *C E* is equall to the whole line *A B* (by the ſecond common ſentẽce) Draw a line from the poynt *E* to the point *B*. And foraſmuch as the line *A B* is equall to the line *E C*, and the line *B C* is common to them both, and the angle *A C B* is ſuppoſed to be equall to the angle *A B C*: Wherfore (by the fourth propoſition) the triangle *E B C* is equall to the triangle *A B C*, namelye, the whole to the part: which is impoſſible.

The 4. Theoreme. The 7. Propoſition.

If from the endes of one line, be drawn two right lynes to any pointe: there can not frõ the ſelf ſame endes on the ſame ſide, be drawn two other lines equal to the two firſt lines, the one to the other, vnto any other point.

Or if it be poſſible: then from the endes of one & the ſelf ſame right line, namely, A B, from the pointes (I ſay) A and B, let there be drawn two right lines A C and C B to the point C: and from the ſame endes of the line A B, let there be drawen two other right right lines A D and D B equall to the lines A C and C B the one to the other, is, eche to his correſpondent line, and on one and the ſame ſide, and to an other pointe, namely, to D: ſo that let C A be equall to D A beyng both drawen from one end, that is, A: & let C B be equall to D B, beyng both alſo drawn from one ende, that is, B. And (by the firſt petition) draw a right line from the point C to the point D. Now foraſmuch as A C is equal to A D, the angle A C D alſo is (by the 5. propoſition) equall to the angle A D C: wherfore the angle A C D is leſſe thẽ the angle B D C. VVherfore the angle B C D is much leſſe then the angle B D C. Againe foraſmuch as B C is equall to B D, and therfore alſo the angle B C D is equall to the angle B D C. And it is proued that it is much leſſe then it: which is impoſſible. If therfore from the endes of one line, be drawen two right lines to any pointe: there can not from the ſelfe ſame endes on the ſame ſide, be drawen two other lines equall to the two firſt lines, the one to the other, vnto any other point: VVhich was required to be demonſtrated.



Demonſtration leading to an abſurditie.

In

In this propoſition the concluſion is a negation, which very rarely happeneth in the mathematicall artes. For they euer for the moſt part vſe to conclude affirmatiuely, & not negatiuely. For a propoſitiō vniuerſall affirmatiue is moſt agreable to ſciences, as ſaith *Ariſtotle*, and is of it ſelfe ſtrong, and nedeth no negatiue to his prooſe. But an vniuerſall propoſition negatiue muſt of neceſſitie haue to his prooſe an affirmatiue, For of onely negatiue propoſitions there can be no demonſtrations. And therfore ſciences vſing demonſtration, conclude affirmatiuely, and very ſeldome vſe negatiue concluſions.

Negatiue concluſions rarely vſed in the mathematicall artes.

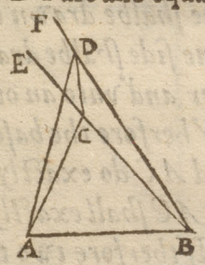
An other demonſtration after Campanus.

Suppoſe that there be a line *A B*, from whoſe ends *A* and *B*, let there be drawen two lines *A C* and *B C* on one ſide, which let concur in the poynt *C*. Then I ſay that on the ſame ſide there cannot be drawen two other lines, from the endes of the line *A B*, which ſhall concur at any other poynt, ſo that that which is drawē from the point *A* ſhall be equall to the line *A C*, and that which is drawen from the point *B* ſhalbe equall to the line *B C*. For if it be poſſible, let there be drawn two other lines on the ſelfe ſame ſide,

which let concurre in the point *D*, and let the line *A D* be equall to the line *A C*, & the line *B D* equall to the line *B C*. Wherfore the poynt *D* ſhall fall either within the triangle *A B C*, or without. For it cannot fall in one of the ſides, for then a parte ſhould be equall to his whole. If therfore it fall without; then either one of the lines *A D* and *D B* ſhall cut one of the lines *A C* and *C B*, or els neither ſhall cut neyther. Firſte let one cut the other and draw a right line from *C* to *D*. Now foraſmuch as in the triangle *A C D*, the two ſides *A C* and *A D* are equall, therfore the angle *A C D* is equall to the angle *A D C*, by the fifth propoſitiō: likewiſe foraſmuch as in the triāgle *B C D*, the two ſides *B C* and *B D* are equall, therfore by the ſame, the angles *B C D* & *B D C* are alſo equall. And foraſmuch as the angle *B D C* is greter thē the angle *A D C*, it followeth that the angle *B C D* is greater then the angle *A C D*, namely, the part greater then the whole: which is impoſſible.

Diuers caſes in this demonſtration.

Firſt caſe.



Second caſe.

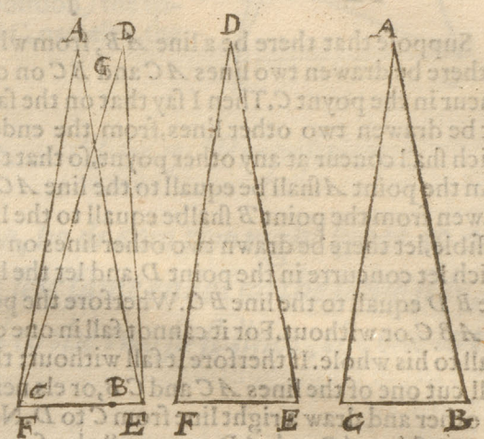
But if the point *D* fal without the triangle *ABC*, ſo that the lines cut not the one the other, draw a line from *D* to *C*. And produce the lines *B D* & *B C* beyond the baſe *C D*, vnto the points *E* & *F*. And foraſmuch as the lines *A C* and *A D* are equall, the angles *A C D* and *A D C* ſhall alſo be equall, by the fifth propoſition: likewiſe for aſmuch as the lines *B C* and *B D* are equal, the angles vnder the baſe, namely, the angles *F D C* and *E C D* are equall, by the ſeconde part of the ſame propoſition. And for as much as the angle *E C D* is leſſe then the angle *A C D*: It followeth that the angle *F D C* is leſſe thē the angle *A D C*: which is impoſſible: for that the angle *A D C* is a part of the angle *F D C*. And the ſame inconuenience will follow if the poynt D fall within the triangle *A B C*.

The fift Theoreme. The 8. Propoſition.

If two triangles haue two ſides of th'one equall to two ſides of the other, eche to his correſpondent ſide, & haue alſo the baſe of the one equall to the baſe of the other: they ſhall haue alſo the angle contained vnder the equall right lines of the one, equall to the angle contayned vnder the equall right lynes of the other.

SVppose that there be two triangles A B C and D E F: & let these two sides of the one A B and A C, be equall to these two sides of the other D E, and D F, ech to his correspondent side, that is, A B to D E, and A C, to D F, & let the base of the one, namely, B C be equal to the base of the other, namely, to E F. Then I say, that the angle B A C is equall to the angle E D F. For the triangle A B C exactly agreing with the triangle D E F, and the point B being put vpon the point E, and the right line B C vpon the right line E F: the point C shall exactly agree with the point F (for the line B C is equall to the line E F) And B C exactly agreeing with E F the lines also B A and A C shall exactly agree with the lines E D & D F. For if the base B C do exactly agree with the base F E, but the sides B A & A C doo not exactly agree w the sides E D & D F, but differ as F G & G F do: thē from ỹ endes of one lyne shalbe drawn two right lines to a poynt, & from the self same endes on the same side shalbe drawn two other lines, equal to the two first lines, ỹ one to the other, and vnto an other poynt: but that is impossible, (by the seuenth propositiō) VVherfore the base B C exactly agreeing with the base E F, the sides also B A and A C do exactly agre with the sides E D and D F. VVherfore also the angle B A C shall exactly agre w the angle E D F, and therfore shall also be equal to it. If therfore two triangles haue two sides of the one equall to two sides of the other, ech to his correspondent side, and haue also the base of ỹ one equall to the base of the other: they shall haue also the angle contayned vnder the equall right lines of the one, equall to the angle contayned vnder the equall right lines of the other: which was required to be proued.

Demonstration leading to an impossibility.



This proposition is the conuerse of the fourth, but not the chiefest kind of conuersion.

This Theoreme is the conuerse of the fourth, but it is not the chiefest and principall kind of conuersion. For it turneth not the whole supposition into the conclusion, and the whole conclusion into the supposition. For the fourth propositiō whose conuerse this is, is a cōpound theoreme, hauing two things geuē or supposed, which are these: the one, that two sides of the one triāgle be equal to two sides of the other triāgle: th'other, that the angle cōtained of the two sides of th'one is equal to the angle contained of the two sides of th'other: but hath amōgest other one thing required, whiche is, that the base of the one, is equal to the base of the other. Now in this 8. propositiō, being the conuerse therof: that the base of the one is equal to the base of th'other, is the supposition, or the thing geuē; which in the former propositiō was the concluſiō. And this, that two sides of

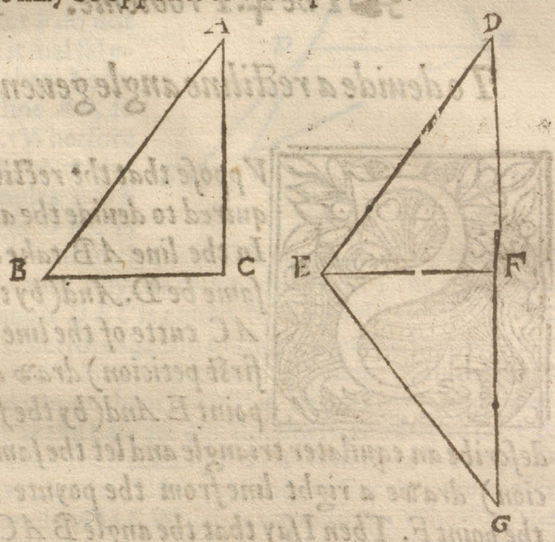
the

the one are equall to two sides of the other, is in this proposition also a supposition, like as it was in the former proposition: so that it is a thing geuen in either proposition. The conclusion of this proposition is that the angle enclosed of the two equall sides of the one triangle is equall to the angle enclosed of the two equall sides of the other triangle: which in the former proposition was one of the things geuen.

Philo and his scholas demonstrate this proposition without the helpe of the former proposition, in this maner.

An other demonstration inuented by Philo.

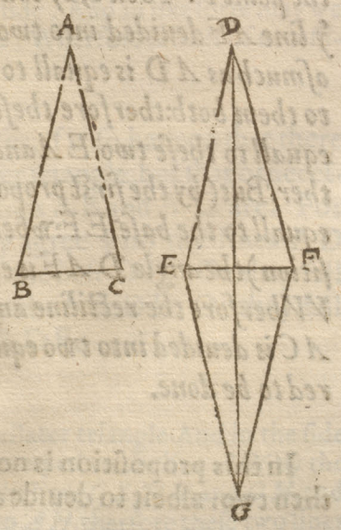
Suppose that there be two triangles *A B C* and *D E F*, hauing two sydes of the one equall to two sydes of the other, namely, *A B* and *A C* equall to D E and D *F*, the one to the other, & the base *B C* equal to the base E *F*. And for that the base *B C* is equall to the base E *F*, therfore the one being applied to the other they agree. Place the two triãgles *A B C* & D *E F* in one & the self same plaine superficies, & apply the base of the one to the base of the other: But yet so that the triãgle *A B C* be set one the other side of the right line *E F*, that the top of the one may be opposite to the top of the other. And in stead of the triangle *A B C* put the triangle *E F G* as in the figure. And let D E be equall to *E G*, and *D F* to *F G*. Nowe then by this meanes shall happen diuers cases. For the line F *G* may fall directly vpõ the line D F, or it may so fall that it may make with the line D F an angle within the figure, or without.



After this demonstratiõ three cases in this preposition.

The first case.

First let it fall directlye. And forasmuche as the line D E is equall to the line E *G*, and D FG, is one righte line: therfore D E G is an Isosceles triãgle: and so, by the fifth proposition, the angle at the point D is equal to the angle at the poynt *G*: which was required to be proued.



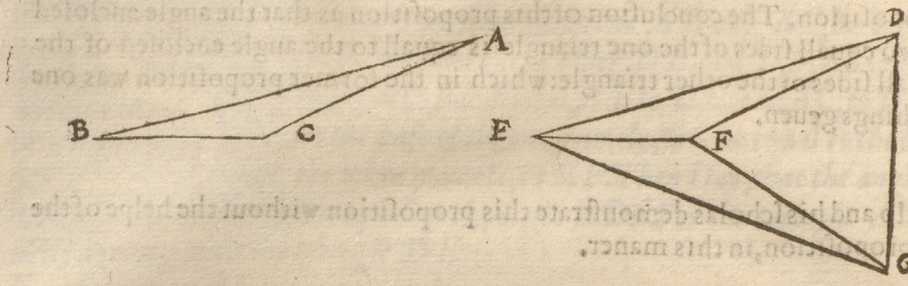
The second case.

But if it fall not directly, but make with the line D F an angle within the figure, drawe a line from D to G. Now forasmuch as E D and E G are equall, and the line D G is the base: therfore by the fifth propositiõ, the angle E D G is equall to the angle E G D. Agayne forasmuch as D F is equall to F G, and D G is the base: therfore by the same, the angle F D G is equal to the F G D: and it was proued that the angle E D G is equall to the angle E G D: wherfore the whole angle E D F is equal to the whole angle F G E: whiche was required to be proued.

The third case.

But if the line F G make with the line D F an angle without the figure: draw a right line without the figure

from the poynt D to the poynt G. And forasmuch as D E and E G are equall, and D G is the base, therfore by the fifth proposition, the angles E D G and D G E are equall. A-



gaine forasmuch as D F is equall to F G, and D G is the base, therfore, by the same, the angle F D G is equall to the angle F G D. And it was proued that the whole angles E D G & D G E are equall the one to the other: wherfore the angles remayning E D F & E G F are equall the one to the other, which was required to be proued.

The 4. Probleme. The 9. Proposition.

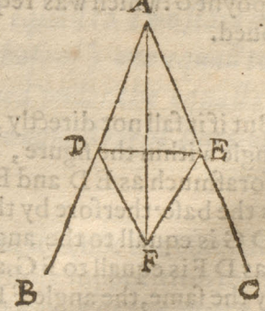
To deuide a rectiline angle geuen, into two equall partes.

Construction.

Vppose that the rectiline angle geuen be B A C. It is required to deuide the angle B A C into two equal partes. In the line A B take a point at all aduentures, & let the same be D. And (by the third proposition) from the lyne A C cutte of the line A E equall to A D. And (by the first peticion) draw a right line from the point D to the point E. And (by the first proposition) vpon the line D E describe an equilater triangle and let the same be D F E, and (by the first peticion) drawe a right line from the poynte A to the point F. Then I say that the angle B A C is by ỹ line A F deuided into two equal partes.

Demonstration.

For, forasmuch as A D is equall to A E, and A F is cõmon to them both: therfore these two D A and A F, are equall to these two E A and A F, the one to the other. But (by the first proposition) the base D F is equall to the base E F: wherfore (by the 8. proposition) the angle D A F is equal to the angle F A E. VVherfore the rectiline angle geuen, namely, B A C is deuided into two equal partes by the right line A F: VVhich was required to be done.



In this proposition is not taught to deuide a right lined angle into mo partes then two: albeit to deuide an angle, so it be a right angle, into three partes, it is

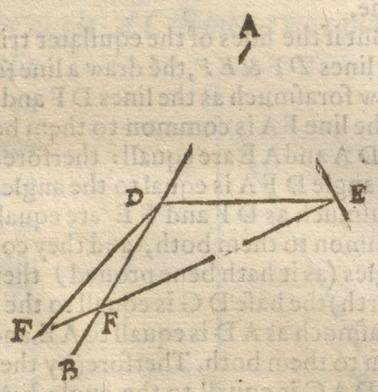
not

not hard. And it is taught of *Vitellio* in his first boke of Perspectiue, the 28. Proposition. For to deuide an acute angle into three equal partes, is (as saith *Proclus*) impossible: vnles it be by the helpe of other lines which are of a mixt nature. Which thing *Nicomedes* did by such lines which are called *Concoides linea*, who first serched out the inuention, nature, & properties of such lines. And others did it by other meanes, as by the helpe of quadrant lines inuented by *Hippias & Nicomedes*. Others by *Helices* or Spiral lines inuented of *Archimedes*. But these are things of much difficulty and hardnes, and not here to be intreated of.

It is impossible to deuide an acute rectilne angle into three equall partes without the helpe of lines which are of a mixt nature.

Here against this proposition may of the aduersary be brought an *instance. For he may cauill that the hed of the equilater triangle shall not fall betwene the two right lines, but in one of them, or without them both. As for example.

**An instance is an obiection or a doubt, wherby is letted or troubled the construction, or demonstration, & contayneth an vnruth, and an impossibility: and therfore it must of necessity be answered vnto, and the falsehode thereof made manifest.*



Suppose that the angle to be deuided into two equall partes be *B A C*, and in the line *B A* take the poynt *D*, and vnto the line *D A* put the line *A E* equal (by the third proposition.) And draw a line frō *D* to *E*. And vpon the line *D E* describe (by the first) an equilater triangle, which let be *D F E*. Now then if it be possible that the point F do not fal betwene the lines *A B* & *A C*, then it shal fal ether in the line *AB* or *AC*, or without them both. Suppose that the point F be fall vpon line *A B*, so that let D F E be an equilater triangle. Wherfore the line D F is equal to the line F E: & the angles at the base are equall, namely, the angles E D F and D E F. Wherefore the whole angle *D E C* is greater then the angle E D F. Againe forasmuch as *A D* is equall to *A E*, therefore *A D E* is an Isosceles triangle. Wherefore (by the fifth proposition) the angles vnder the base are equall. Wherfore the angle *D E C* is equall to the angle *E D B*. But it was also greater: which is impossible. Wherfore the top of the equilater triangle cānot be in the right line *A B*. And in like sort also may we proue that it cānot be in the right line *A C*. Wherfore suppose that it be without them both, if it be possible. And forasmuch as D F is equal to F E, the angles at the base are equal, namely, the angles D E F & E D F. Wherfore the angle D E F is greater then the angle *E D* F. Wherfore the angle D E C is much greater then the angle E D F. But it is also equal vnto it. For they are angles vnder the base *D E* of the Isosceles triangle *A D E*. Which is impossible. Wherfore the poynt F shall not fall without the two right lines on that side. And in like sorte may we proue that it shall not fall without them on the other side. Wherfore it shall of necessity fall betwene them: which was required to be proued.

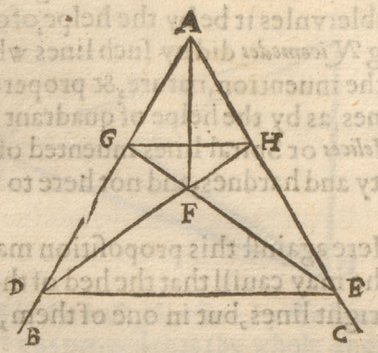
There may also in this proposition be diuers cases, if it so happen that there be no space vnder the base D E to describe an equilater triangle, but that of necessitie you must describe it on the same side that the lines A B and A C are. For then the sides of the equilater triangle either exactly agree with the lines A D and A E, if the said lines A D and A E be equall with the base D E. Or they fall without them, if the lines A D and A E be lesse then the base D E. Or they fall within them, if the said lines be greater then the base D E.

Diuers cases in this proposition.

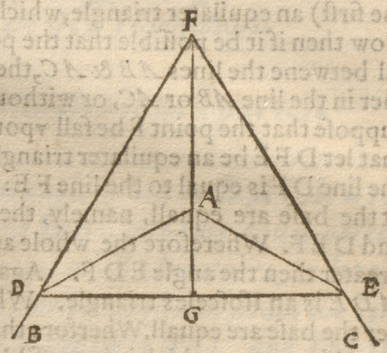
The first case.

First let them exactly agree. And let *D A E* be an equilater triangle. And in the side *A D* take the poynt *G*. And from the side *A E* cut of a line equal to the line *A G* (by the third proposition) which let be *A H*. And draw these right lines *G E*, *H D* and *G H*, and A F. Now forasmuch as *AD* is equal to *AE*, and *A G* vnto *A H*, therfore these two lines

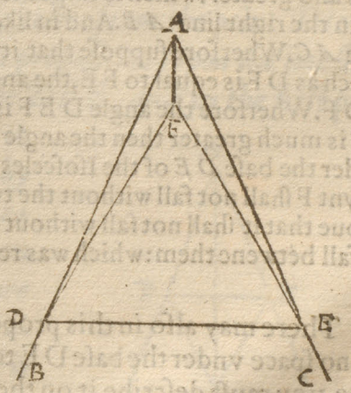
D A and A H are equall to these two lines E A and A G: and they contayne one and the selfe same angle. Wherfore by the fourth proposition, the angle G D H is equal to the angle H E G. And the base D H is equall to the base E G. But the line D G is equal to the line E H: wherfore againe by the fourth proposition, the angle E G H is equall to the angle D H G. Wherfore by the sixt proposition, the base G F is equal to the base H F. And forasmuch as A H is equall to A G, and A F is common to thē both, and the base G F is equall to the base H F, therfore the angle G A F is equall to the angle H A F. Wherefore the angle G A H is deuided into two equall partes: which was required to be done.



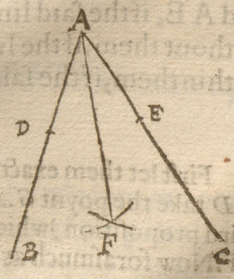
The second case. But if the fides of the equilater triangle fall without the right lines B A & A C, as do the lines D F & E F, thē draw a line from F to A & produce the line F A to the point G. Now forasmuch as the lines D F and F E are equal, & the line F A is common to them both, & the bases D A and A E are equall: therfore (by the eight) the angle D F A is equal to the angle E F A. Againe forasmuch as D F and F E are equall, and F G is common to them both, and they containe equall angles (as it hath bene proued) therefore (by the fourth) the base D G is equall to the base G E. And forasmuch as A D is equall to A E, and A G is common to them both. Therfore (by the eight) the angle D A G is equall to the angle E A G, wherefore the angle D A E is deuided into two equall partes: Which was required to be done.



The third case. But if the sides of the equilater triangle fal within the right lines B A and A C, as do the lines D F and F E, then againe draw a line from A to F. And forasmuch as D A is equall to A E, and A F is common to them both, and the base D F is equal to the base F E: therfore the angle D A F is (by the eight) equall to the angle E A F. Wherefore the angle at the point A is deuided into two equall partes, how soeuer the equilater triangle be placed: which was required to be done.



To deuide a rectilne angle into two equall partes Mechanically. This is to be noted, that if a man will mechanically or readily, not regardyng demonstration, deuide the foresaid rectiline angle B A C, and so any other rectiline angle geuen whatsoeuer, into two equall partes, he shall neede onely with one opening of the compasse taken at all aduentures to marke the two pointes D and E, which cut of equal partes of the lines A B and A C, howsoeuer they happen, and so making the centres the two points D and E, to describe two circles according to the openyng of the compasse: and from the point A to their intersection, which let be the point F, to draw a right line: which shall deuide the angle B A C into two equall partes. And here note, that you shall not nede to draw the circles all whole, but one-



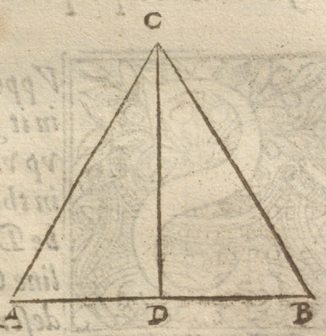
ly a portion where they cut the one the other: As in the figure here in the end of the other side put.

The 5. Probleme. The 10. Proposition.

To deuide a right line geuen being finite, into two equall partes.

Vppose that the right line geuen be AB. It is required to deuide the line AB into two equal partes. Describe (by the first proposition) vpon the line AB an equilater triangle and let the same be ABC. And (by the former proposition) deuide the angle ACB into two equall partes by the right line CD. Then I say that the right line AB is deuided into two equall partes in the poynt D. For forasmuch as (by the first proposition) AC is equall to CB, and CD is common to thē both: therfore these two lines AC & CD are equal to these two lines BC & CD, ỹ one to ỹ other, and the angle ACD is equall to the angle BCD. Wherfore (by the 4. proposition) the base AD is equall to the base BD. Wherefore the righte line geuen AB, is deuided into two equall partes in the poynt D: which was required to be done.

Construction.

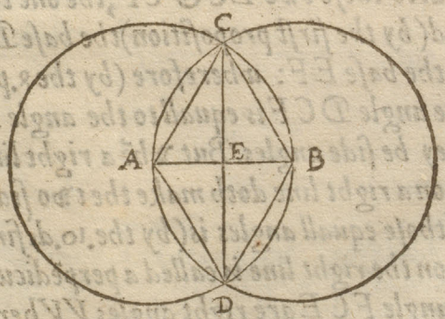


Demonstration.

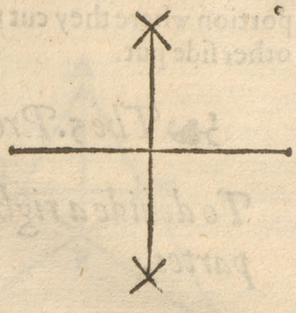
Apollonius teacheth to deuide a right line being finite into two equall partes after this manner.

An other way to deuide a right line being finite, inuented by Apollonius.

Suppose that the right line being finite be *AB*: which it is required to deuide in to two equal parts. Now thē making the centre the point *A* & the space *AB* describe a circle. Again making the centre the poynt *B* & the space BA describe an other circle: and from the cōmon sections draw the right line *CD*, which let cut the line AB in the poynt E. Then I say, that the right line *CD* deuideth the line *AB* into two equall partes in the point E. For draw these right lines *DA*, *DB*, *CA* and *CB*. Which shal be equal the one to the other, for that they are drawne frō the centres to the circumferences of equall circles. And forasmuch as the lines *CA* & *AD* are equall to the lines *CB* and *BD*, and the line *CD* is common to either of them: therefore, by the eight, the angle *ACD* is equall to the angle *BCD*. Again forasmuch as the line *CE* is common to the lines *AC* and *CB*, which are equall the one to the other, therfore these two lines *AC* and *CE* are equal to these two lines *BC* and *CE*, the one to the other, and they contayne equall angles, as it hath bene proued. Wherefore by the 4. proposition the base *AE* is equall to the base *BE*. Wherefore the line *AB* is deuided into two equall partes in the poynt *E*: which was required to be done.



By this way of deuiding a right line, into two equall parts inuented by Apollonius, it is manifest, that if a man wil mechanically, or redely, not considering the demonstratiõ, deuide the said right line, and so any right line geuen whatsoeuer, into two equall: partes he nede onely to marke the poynts of the intersections of the circles, & to draw a line frõ the sayd intersections, which shall deuide the right line geuen into two equall partes: as in the figure here put.



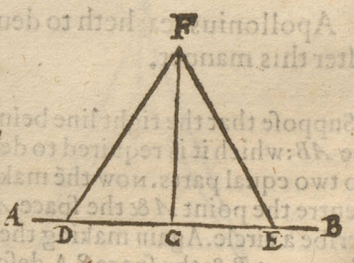
The 6. Probleme. The 11. Proposition.

Vpon a right line geuen, to rayse vp from a poynt geuen in the same line a perpendicular line.

Construction.

Vppose that the right line geuen be AB, & let the point in it geuen be C. It is required from the poynte C to rayse vp vnto the right line AB a perpendicular line. Take in the line AC a poynt at all aduentures, & let the same be D, and (by the 3. proposition) put vnto DC an equall line CE. And by the first proposition) vpon the line DE describe an equilater triangle FDE, & draw a line frõ F to C. Then I say that vnto the right line geuen AB, and from the poynte in it geuen, namely, C is raysed vp a perpendicular line FC. For forasmuch as DC is equal to CE, & the line CF is cõmon to them both: therfore these two DC and CF, are equal to these two EC & CF, the one to the other: and (by the first proposition) the base DF is equal to the base EF: wherefore (by the 8. proposition) the angle DCF is equall to the angle ECF: and they be side angles. But whẽ a right line standing vpon a right line doth make the two side angles equall the one to the other, ether of those equall angles is (by the 10. definition) a right angle: & the line standing vpon the right line is called a perpẽdicular line. Wherfore the angle DCF, & thangle FCE are right angles. Wherfore vnto the right line geuẽ AB, & frõ the poynt in it C, is raysed vp a perpendicular line CF: which was required to be done.

Demonstration.



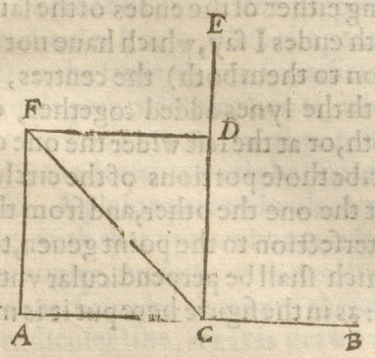
Although the poynte geuen should be set in one of the endes of the righte line geuen, it is easy so do it as it was before. For producing the line in length from the poynt by the second peticion, you may worke as you did before. But if one require to erect a right line perpendicularly from the poynt at the end of the line

lyne, without producing the right lyne, that also may well bee done after thys maner.

An other case in this proposition.

Construction.

Suppose that the right line geuen be *A B*, & let the point in it geuen be in one of the endes therof, namely, in *A*. And take in the line *A B* a point at all aduentures, and let the same be *C*. And from the said point raise vp (by the foresaid proposition) vnto *A B* a perpendiculer line, which let be *C E*. And (by the 3. proposition) from the line *C E* cut of the line *C D* equall to the line *C A*. And (by the 9. Proposition) deuide the angle *A C D* into two equall partes by the line *C F*. And from the point *D* raise vp vnto the line *C E* a perpēdiculer line, *D F*, which let concurre with the line *C F* in the point *F*. And drawe a right line from *F* to *A*. Then I say that the angle at the point *A* is a right angle. For, forasmuch as *D C* is equall to *C A*, and *C F* is common to them both, and they containe equall angles (for the angle at the point *C* is deuided into two equall partes) therefore (by the 4. Proposition) the line *D F* is equall to the line *F A*, and so the angle at the point *A* is equal to the angle at the point *D*. But the angle at the point *D* is a right angle. Wherfore also the angle at the point *A* is a right angle. Wherefore from the point *A* vnto the line *A B*, is raised vp a perpendiculer line *A F*, without producing the line *A B*. Which was required to be done.



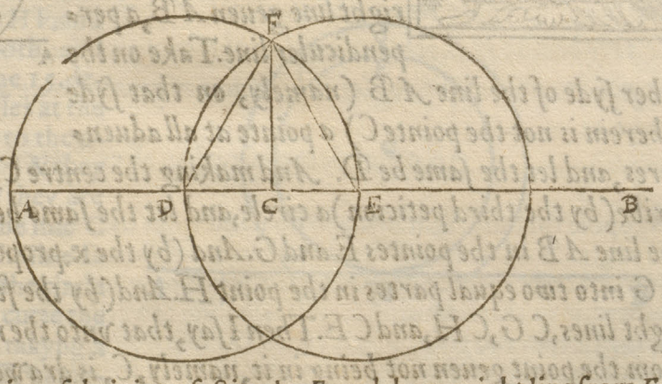
Demonstration.

Appollonius teacheth to rayse vp vnto a line geuen, from a point in it geuen, a perpendiculer line, after this maner.

An other way to erect a perpēdicular line inuented by Appolonias.

Construction.

Suppose that the right line geuen be *A B*. And let the point in it geuē, be *C*. And in the line *A C*, take a point at all aduētures, & let the same be *D*. And frō the lyne *C B*, cut of a line equall to the line *C D*, whiche let be *C E*: and makyng the centre *D*, and the space *D E*, describe a circle. And againe making the centre *C*, & the space *E D*, describe another circle, and let the point of their intersection be *F*. And draw a right line from *F* to *C*. Then I say that the line *F C* is erected perpendicularly vnto the line *A B*. For drawe these lines *F D* and *F E*: which shal by the definition of a circle be either of them equal to the line *D E*: and therfore (by the first common sentence) are equall the one to the other. But the lines *D C* and *C E* are by construction equall, and the line *F C* is common to them both. Wherfore the angles also at the point *C* are equal (by the 8. propositiō:) wherfore they are right angles. Wherfore the line *C F* is erected perpendicularly vnto the line *A B* from the point *C*: which was required to be done.

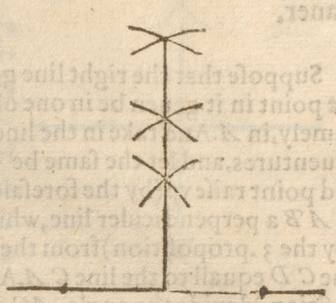


Demonstration.

How to erect a perpendicular line mechanically.

By this way of erecting a perpendiculer line inuented by Appollonius, it is also manifest, that if a man will mechanically, without demonstration, erect vnto

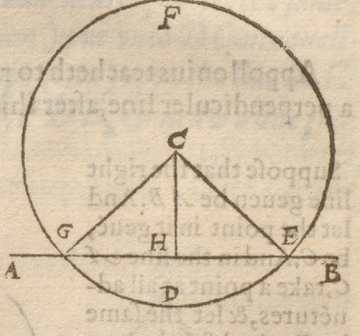
a line geuen from a point geuē in it a perpendiculer line: he neede onely on either ſide of the pointe geuen, to cut of equall lines: and ſo making either of the endes of the ſaid lines (either of th'endes I ſay, which haue not one point cōmon to them both) the centres, and the ſpace both the lynes added together, or wider then both, or at the leſt wider thē one of them, to deſcribe thoſe portions of the circles where they cut the one the other, and from the point of the interſection to the point geuen, to draw a lyne, which ſhall be perpendicular vnto the lyne geuē: as in the figure here put it is manifeſt to ſee.



The 7. Probleme. The 12. Propoſition.

Vnto a right line geuen being infinite, and from a point geuen not being in the ſame line, to draw a perpendicular line.

LEt the right line geuen being infinite be AB, & let ỹ point geuen not being in the ſaid line AB, be C. It is required from the point geuē, namely, C, to draw vnto the right line geuen AB, a perpendiculer line. Take on the other ſyde of the line AB (namely, on that ſyde wherein is not the pointe C) a pointe at all aduentures, and let the ſame be D. And making the centre C, and the ſpace CD, deſcribe (by the third peticion) a circle, and let the ſame be EFG, which let cutte the line AB in the pointes E and G. And (by the x. propoſition) deuide the lyne EG into two equal partes in the point H. And (by the firſt peticion) draw theſe right lines, CG, CH, and CE. Then I ſay, that vnto the right line geuen AB, & from the point geuen not being in it, namely, C, is drawen a perpendiculer lyne CH. For foraſmuch as GH is equall to HE, and HC is common to them both: therfore theſe two ſydes GH and HC, are equall to theſe two ſydes EH & HC, the one to the other: and (by the 15 definitiō) the baſe CG is equal to the baſe CE: wherfore (by the 8. propoſition) the angle CHG is equall to the angle CHE: and they are ſyde angles: but when a right line ſtanding vpon a right line maketh the two ſyde angles equall the one to the other, either of thoſe equall angles is (by the 10. definition) a right angle, and the line ſtanding vpon the ſayde right line is called a perpendiculer line. Wherfore vnto the right line geuē AB, and from the point geuen C, which is not in the line AB, is drawn a perpendiculer line CH: which was required to be done.



Conſtruction.

Demenſtration.

This

This Probleme did *Oenopides* first finde out, considering the necessary vse therof to the study of Astronomy.

Oenopides the first inuenter of this probleme.

There are two kindes of perpendiculer lines: wherof one is a plaine perpendiculer lyne, the other is a solide. A plaine perpendiculer line is, when the point from whence the perpendiculer line is drawen, is in the same plaine superficies with the line wherunto it is a perpendicular. A solide perpendiculer line is, whē the point, from whence the perpendiculer is drawne, is on high, and without the plaine superficies. So that a plaine perpendiculer line is drawen to a right line: & a solide perpendiculer line is drawn to a superficies. A plaine perpendiculer line causeth right angles with one onely line, namely, with that vpon whome it falleth. But a solide perpendiculer line causeth right angles, not only with one line, but with as many lynes as may be drawn in that superficies, by the touch therof.

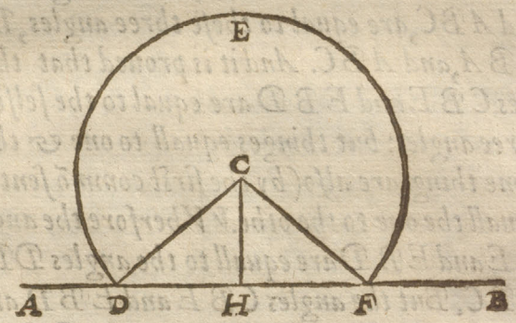
Two kindes of perpendiculer lines, namely, a plaine perpendiculer line and a solide.

This proposition teacheth to draw a plaine perpendiculer line, for it is drawn to one line, and supposed to be in the selfe same plaine superficies.

This proposition teacheth to draw a playne perpendiculer line.

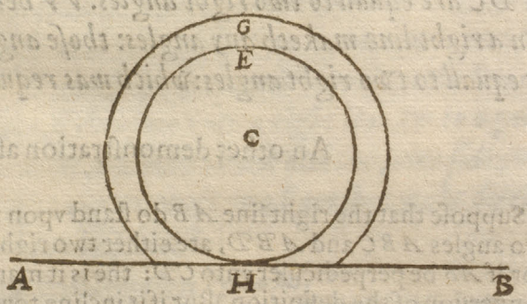
There may be in this proposition an other case. For if it be so, that on the other side of the line *A B*, there be no space to take a pointe in but onely on that side wherein is the point C. Then take some certaine point in the line *A B*, which let be D. And making the centre the point *C*, and the space C D, describe a part of the circumference of a circle, which let be D E F: which let cut the line A B in the two pointes D and F. And deuide the line D F into two equall partes in the poynt H. And draw these lines C D, C H and C F. And forasmuch as D H is equal to H F, and C H is common to them both, and C D is equall to C F (by the 15. definition:) therfore the angles at the point H are equal the one to the other (by the 8. proposition:) & they are side angles, wherefore they are right angles. Wherfore the line C H is a perpendiculer to the line D F. But if it happen so that the circle which is described do not cutte the lyne, but touche it, then takyng a point without the point E, namely, the point G, and making the centre the point C, and the space CG, describe a part of the circumference of a circle: which shall of necessitie cut the line AB: and so may you proceede as you did before. As you may see in the second figure.

An other case in this proposition.



Construction.

Demonstration.



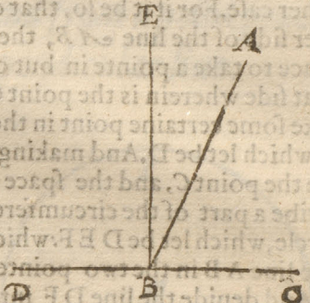
The 6. Theoreme. The 13. Proposition.

When a right line standing vpon a right line maketh any angles: those angles shall be either two right angles, or equall to two right angles.

Construction.

Demonstration

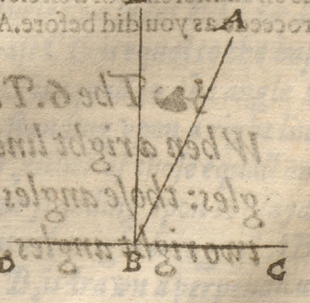
Suppose that the right line A B standing vppon the right line C D do make these angles C B A and A B D. Then I say, that the angles C B A and A B D are eyther two right angles, or els equall to two right angles. If the angle C B A be equall to the angle A B D: then are they two right angles (by the tenth difinition.) But if not, raise vp (by the 11 proposition) vnto the right line C D, and from the pointe geuen in it, namely, B, a perpendiculer line B E. VVherfore (by the x. definition) the angle C B E and E B D are right angles. Now forasmuch as the angle C B E, is equall to these two angles C B A and A B E, put the angle E B D common to them both: wherfore the angles C B E and E B D, are equal to these three angles C B A, A B E, and E B D. Agayne forasmuch as the angle D B A is equall vnto these two angles D B E and E B A, put the angle A B C common to them both: wherfore the angles D B A and A B C, are equal to these three angles, D B E, E B A, and A B C. And it is proued that the angles C B E and E B D are equal to the selfe same three angles: but thinges equall to one & the self same thing, are also (by the first commõ sentence) equall the one to the othe. VVherfore the angles C B E and E B D are equall to the angles D B A & A B C. But the angles C B E and E B D are two right angles: wherfore also the angles D B A and A B C are equall to two right angles. VVherfore when a right line standing vpon a right line maketh any angles: those angles shalbe either two right angles, or equall to two right angles: which was required to be demonstrated.



An othet demonstration after *Pelitarius*.

An other demonstration after Pelitarius.

Suppose that the right line *A B* do stand vpon the right line *C D*. Then I say, that the two angles *A B C* and *A B D*, are either two right angles, or equal to two right angles. For if *A B* be perpẽdiculer vnto *C D*: thẽ is it manifest, that they are right angles (by the conuersion of the definition) But if it incline towardes the end *C*, then (by the 11. proposition) from the point *B*, erect vnto the line *C D* a perpendiculer line *B E*. By whiche construction the propositiõ is very manifest. For forasmuch as the angle *A B D* is greater then the right angle *D B E* by the angle *A B E*, and the other angle *A B C* is lesse then the right angle *C B E* by the selfe same angle *A B E*: if from the greater bee taken away the excesse, and the same bee added to the lesse, they shall be made two right angles. That is, if from the obtuse angle *A B D* be taken away the angle *A B E*, there shal remayne the right angle *D B E*. And then if the same angle *A B E* be added to the acute angle *C B A*, there shall bee made the right angle *C B E*. Wherefore it is manifest, that the two angles, namely, the obtuse angle *A B D*, & the acute angle *A B C*, are equall to the two right angles *C B E* and *D B E*: which was required to be proued.

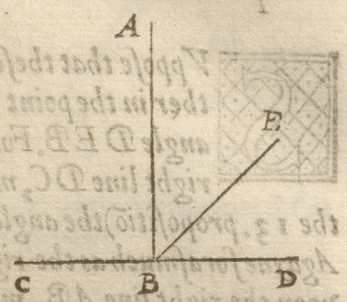


The

The 7. Theoreme. The 14. Proposition.

If vnto a right line, and to a point in the same line, be drawn two right lines, not both on one and the same side, making the side angles equall to two right angles: those two right lines shall make directly one right line.

VNto the right line AB, & to ỳ point in it B, let there be drawn two right lines BC, and BD, vnto contrary sides, making the syde angles, namely, ABC & ABD, equall to two right angles. Then I say, that ỳ right lines BD and BC make both one right line. For if CB and BD do not make both one right line, let the right line BE be so drawn to BC, that they both make one right line. Now forasmuch as the right line AB standeth vpon the right line CBE, therfore the angles ABC and ABE are equall to two right angles (by the 13. proposition) But (by supposition) the angles ABC and ABD are equall to two right angles: wherfore the angles CBA, and ABE, are equall to the angles CBA, and ABD: take away the angle ABC, which is common to them both. VVherfore the angle remayning ABE, is equall to the angle remaining ABD, namely, the lesse to the greater, which is impossible. VVherefore the line BE is not so directly drawen to BC, that they both make one right line. In like sorte may we proue, that no other line, besides BD, can so be drawne. VVherfore the lines CB and BD, make both one right line. If therfore vnto a right line, & to a point in the same line, be drawn two right lines, not both on one and the same side, making the side angles equall to two right angles: those two lines shal make directly one right line: which was required to be proued.

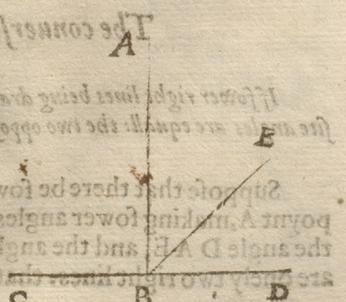


Demoustration leading to an absurditie.

An other demonstration after *Pelitarius*.

An other demonstration after Pelitarius.

Suppose that there be a right line *AB*, vnto whose pointe *B*, let there be drawen two right lines *CB* and *BD*, vnto contrary sides: and let the two angles *CBA*, and *DBA*, be either two right angles, or equall to two right angles. Then I say, that the two lines *CB* and *BD*, do make directly one right line, namely, *CD*. For if they do not, thē let *BE* be so drawn vnto *CB*, that they both make directly one right line *CBE*: which shall passe either aboue the line *BD*, or vnder it. First let it passe aboue it. And for as much as the two angles *CBA* and *ABE*, are (by the former proposition) equall to two right angles, and are a part of the two angles, *CBA* and *ABD*: but the angles *CBA* and *ABD* are by (supposition) equall also to two right angles: therefore the parte is equall to the whole which is impossible. And the like absurditie will follow if *CB*



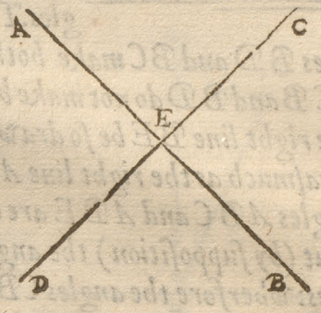
E passe vnder the line B D: namely, that the whole shalbe equall to the part: which is also impossible. Wherefore C D is one right line: which was required to be proued.

The 8. Theoreme. The 15. Proposition.

If two right lines cut the one the other: the hed angles shal be equal the one to the other.

Demonstration.

SVppose that these two right lines A B and C D, do cut the one the other in the point E. Then I say, that the angle A E C, is equall to the angle D E B. For forasmuch as the right line A E, standeth vpon the right line D C, making these angles C E A, and A E D: therefore (by the 13. propositiō) the angles C E A, and A E D, are equall to two right angles. Agayne forasmuch as the right line D E, standeth vpon the right line A B, making these angles A E D, and D E B: therfore (by the same proposition) the angles A E D, and D E B, are equall to two right angles: and it is proued, that the angles C E A, and A E D, are also equall to two right angles. VVherfore the angles C E A, and A E D, are equall to the angles A E D, and D E B. Take away the angle A E D, which is common to them both. VVherefore the angle remayning C E A, is equall to the angle remayning D E B. And in like sort may it be proued, that the angles C E B, and D E A, are equall the one to the other. If therefore two right lines cut the one the other, the hed angles shalbe equall the one to the other: which was required to be demonstrated.



Thales Milesius the first innenter of this proposition.

Thales Milesius the Philosopher was the first inuenter of this Proposition, as witnesseth *Eudemius*, but yet it was first demonstrated by *Euclide*. And in it there is no construction at all. For the exposition of the thing geuē, is sufficient inough for the demonstration.

No construction in this proposition.

What hed angles are.

Hed Angles, are apposite angles, caused of the intersection of two right lines: and are so called, because the heddes of the two angles are ioyned together in one pointe.

The conuerse of this proposition after Pelitarius.

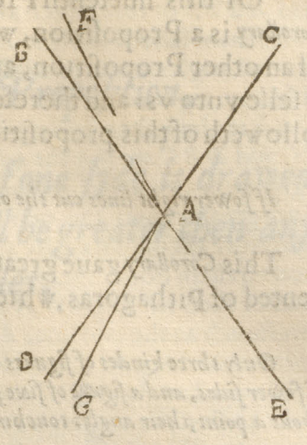
The conuerse of this propositiō after Pelitarius.

If fower right lines being drawen from one point, do make fower angles, of which the two opposite angles are equall: the two opposite lines shalbe drawen directly, and make one right line.

Suppose that there be fower right lines A B, A C, A D, and A E, drawen from the poynt A, making fower angles at the point A: of which let the angle B A C be equall to the angle D A E, and the angle B A D to the angle C A E. Then I say, that B E and C D are onely two right lines: that is, the two right lines B A and A E are drawen directly,

and

and doo make one right line, and likewiſe the two right lines *C A* and *A D* are drawen directly, and do make one right line. For otherwiſe if it be poſſible, let *E F* be one right line, and likewiſe let *C G* be one right line. And foraſmuch as the right line *E A* ſtandeth vpon the right line *C G*, therefore the two angles *E A C* and *E A G*, are (by the 13 propoſition) equall to two right angles. And foraſmuch as the right line *G A* ſtandeth vpon the right line *E F*: therefore (by the ſelfe ſame) the two angles *E A G* and *F A G*, are alſo equall to two right angles. Wherefore taking away the angle *E A G*, which is common to them both, the angle *E A C*, ſhall (by the thirde common ſentence) be equall to the angle *F A G*: but the angle *E A C* is ſuppoſed to be equall to the angle *B A D*. Wherefore the angle *B A D* is equall to the angle *F A G*, namely a part to the whole: which is impoſſible. And the ſelfe ſame abſurditie will follow, on what ſide ſoeuer the lines be drawen. Wherefore *B E* is one line, and *C D* alſo is one line: which was required to be proued.



Demonſtration leading to an abſurditie.

The ſame conuerſe after Proclus.

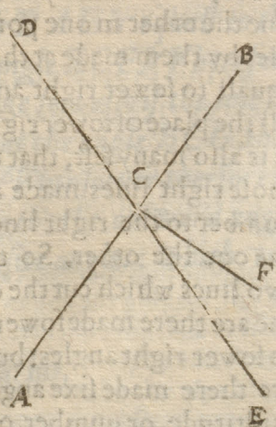
If vnto a right line, and to a point thereof be drawen two right lines, not on one and the ſame ſide, in ſuch ſort that they make the angles at the toppe equall: thoſe right lines ſhalbe drawen directly one to the other, and ſhal make one right line.

The ſame conuerſe after Peletarius, which is demonſtrated directly.

Suppoſe that there be a right line A B, and take a point in in *C*. And vnto the point in it *C*, draw theſe two right lines *C D* and *C E* vnto contrary ſides, making the angles at the hed equal, namely, the angles *A C D* and *B C E*. Then I ſay, that the lines *C D* and *C E* are drawen directly, and do make one right line. For foraſmuch as the right line *C D* ſtanding vpō the right line *A B*, doth make the angles *D C A* and *D C B* equall to two right angles (by the 13 propoſition:) and the angle *D C A* is equall to the angle *B C E*: therefore the angles *D C B* and *B C E* are equal to two right angles. And foraſmuch as vnto a certayne right line *B C*, and to a point thereof *C*, are drawen two right lines not both on one and the ſame ſide, making the ſide angles equall to two right angles, therefore (by the 14 propoſition) the lines *C D* and *C E* are drawen directly, & do make one right line, which was required to be proued.

The ſame conuerſe after Proclus demonſtrated indirectly.

The ſame may alſo be demonſtrated by an argument leading to an abſurditie. For if *C E* be not drawen directly to *C D*, ſo that they both make one right line, then (if it bee poſſible) let *C F* bee drawne directly vnto it. So that let *D C F* be one right line. And foraſmuch as the two right lines *A B* and *D F* do cutte the one the other, they make the hed angles equall (by the 15. propoſition) Wherefore the angles *A C D* and *B C F* are equall: but (by ſuppoſition) the angles *A C D* and *B C E* are alſo equall. Wherefore (by the firſt common ſentence) the angle *B C E* is equall to the angle *B C F*: namely, the greater to the leſſe: which is impoſſible. Wherefore no other right line beſides *C E* is drawen directly to *C D*. Wherefore the lines *C D* and *C E* are drawen directly, and make one right line: which was required to be proued.



What a Corollary is.

Of this fiuetenth Proposition followeth a *Corollary*. VVhere note that a *Corollary* is a Proposition, whose demonstration dependeth of the demonstration of an other Proposition, and it appeareth sodenly, as it were by chance offering it selfe vnto vs: and therefore is reckoned as lucre or gayne. The *Corollary* which followeth of this proposition, is thus.

A Corollary following of this proposition.

If fower right lines cut the one the other: they make fower angles equall to fower right angles.

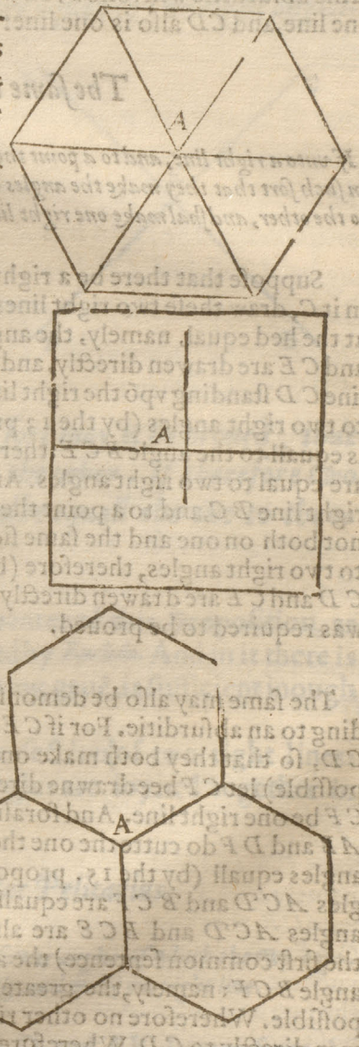
This *Corollary* gaue great occasion to finde out that wonderfull proposition inuented of Pithagoras, which is thus.

A wonderfull proposition inuented by Pithagoras.

Only three kindes of figures of many angles, namely, an equilater triangle, a right angled figure of fower sides, and a figure of sixe sides, hauing equall sides and equal angles, can fill the whole space about a point, their angles touching the same point.

Euery angle of an equilater triangle is equal to two third partes of a right angle

Euery angle of an equilater triangle contayneth two third partes of a right angle: sixe tymes two thirdes of a right angle make fower right angles. VVherefore sixe equilater triangles fill the whole space about a point which is equal to fower right angles, as in the 1. figure. Also euery angle of a rectangle quadrilater figure is a right angle: wherfore fower of them fill the whole space as in the 2. figure. Euery angle of a sixe angled figure is equal to a right angle, and moreouer to a third part of a right angle. But a right angle, and a third part of a right angle, take thre times, make 4. right angles: wherefore three equilater sixe angled figures fill the whole space about a point: which space (by this *Corollary*) is equall to fower right angles: as in the third figure. Any other figure of many sids, howsoeuer you ioyne the together at the angles, shal either want of fower angles, or exceede them. By this *Corollary* also it is manifest that if mo then two lines, that is, three, or fower, or how many soeuer do cut the one the other in one point, all the angles by them made at the point shalbe equall to fower right angles. For they fill the place of fower right angles. And it is also many fest, that the angles by those right lines made are double in number to the right lines which cutte the one the other. So that if there be two lines which cut the one the other, the are there made fower angles equall to fower right angles: but if thre, then are there made sixe angles: if fower, eight angles, and so infinitly. For euer the multitude, or number of of the angles is dubled to the multitude of the right lines which cut the one the other. And as the angles increase in multitude, so



Euery angle of a sixe angled figure is equall to a right angle, and to a third part of a right angle.

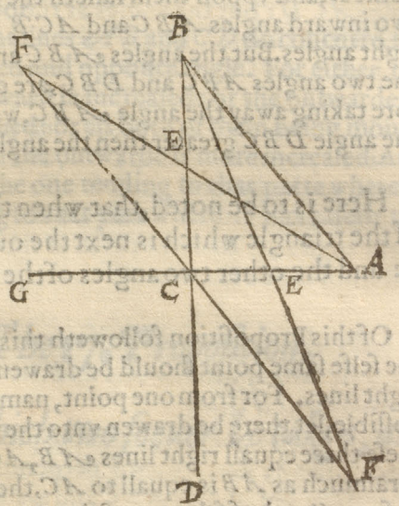
diminish

diminish they in magnitude. For that that which is deuided is alwayes one and the selfe same thing, namely, fower right angles.

The 9. Theoreme. The 16. Proposition.

Whensoeuer in any triangle, the lyne of one syde is drawen forth in length: the outwarde angle shall be greater then any one of the two inwarde and opposite angles.

Vppose that A B C be a triangle: & let one of ỹ sides therof, namely, B C be produced vntō the point D. Then I say, that the outwarde angle A C D, is greater then any one of ỹ two inward and opposite angles, that is, then the angle C B A, or then the angle B A C. Deuide the line A C (by the 10. proposition) into two equall partes, in the point E. And draw a line from the point B to the point E. And (by the 2. peticion) extend B E to the point F. And (by the 2. proposition) vnto the line B E put an equall line E F. And (by the first peticion) draw a line from F to C: and (by the 2. peticion) extend the line A C to the point G. Now forasmuch as the line A E, is equall to the line E C, and B E is equall to E F: therfore these two sides A E and E B, are equall to these two sides C E and E F, the one to the other: and the angle A E B, is (by the 15. proposition) equall to the angle F E C, for they are hed angles: wherefore (by the 4. proposition) the base A B is equall to the base F C: And the triangle A B E is equall to the triangle F E C: and the other angles remayning are equall to the other angles remayning, the one to the other, vnder which are subtended equall sides. Wherefore the angle B A E is equall to the angle E C F. But the angle E C D, is greater then the angle E C F. Wherefore the angle A C D, is greater then the angle B A C. In like sort also if the line B C be deuided into two equall partes, may it be proued, that the angle B C G, that is, the angle A C D, is greater then the angle A B C. Whensoeuer therfore in any triangle, the line of one side is drawen forth in length: the outward angle shalbe greater then any one of the two inward and opposite angles: which was required to be demonstrated.



Construction.

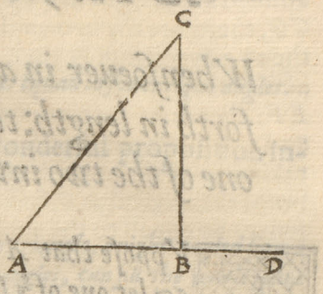
Demonstration.

Another demonstration after Pelitarius.

An other Demonstration after Pelitarius.

Suppose that the triangle geuen be A B C. Whose side A B let be produced vnto the

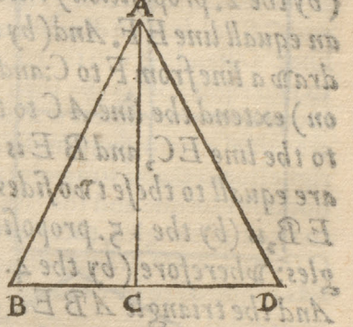
the point *D*. Then I say, that the angle *D B C* is greater then either of the angles *B A C* and *A C B*. For forasmuch as the two lines *A C* and *B C* do concurre in the point *C*, and vpon them falleth the line *A B*: therefore (by the conuerse of the first petition) the two inward angles on one and the selfe same side, are lesse then two right angles. Wherefore the angles *A B C* and *C A B* are lesse then two right angles: but the angles *A B C* and *D B C* are (by the 13 proposition) equal to two right angles. Wherefore the two angles *A B C* and *D B C* are greater then the two angles *A B C* and *B A C*. Wherfore taking away the angle *A B C*, which is common to them both, there shall be left the angle *D B C* greater then the angle *B A C*. And by the same reason, forasmuch as the two lines *B A* and *C A* concurre in the point *A*, and vppon them falleth the right line *C B*, the two inward angles *A B C* and *A C B* are lesse then two right angles. But the angles *A B C* and *D B C* are equall to two right angles. Wherfore the two angles *A B C* and *D B C*, are greater then the two angles *A B C* & *A C B*. Wherfore taking away the angle *A B C*, which is common to them both, there shal remaine the angle *D B C* greater then the angle *A C B*: which was required to be proued.



Here is to be noted, that when the side of a triangle is drawen forth, the angle of the triangle which is next the outwatd angle, is called an angle in order vnto it: and the other two angles of the triangle are called opposite angles vnto it.

A Corrollary following of this proposition.

Of this Proposition followeth this *Corrollary*, that it is not possible that from one & the selfe same point should be drawen to one and the selfe same right line, three equall right lines. For from one point, namely, *A*, if it be possible, let there be drawen vnto the right line *B D*, these three equall right lines *A B*, *A C*, & *A D*. And forasmuch as *A B* is equall to *A C*, the angles at the base are (by the fifth proposition) equall. Wherfore the angle *A B C* is equal to the angle *A C B*. Agayne forasmuch as *A B* is equall to *A D*, the angle *A B D* is (by the same) equall to the angle *A D B*: but the angle *A B C* was equall to the angle *A C B*. Wherefore the angle *A C B* is equall to the angle *A D B*; namely, the outward angle to the inwarde & opposite angle: which is impossible. Wherfore from one and the selfe same point, can not be drawn to one & the selfe same right line three equall right lynes: which was required to be proued.



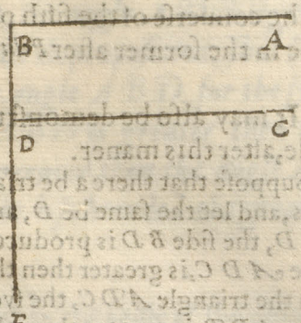
An other Corrollary following also of the same.

By this Proposition also may this be demonstrated, that if a right line falling vpon two right lines, do make the outward angle equall to the inward and opposite angle, those right lines shall not make a triangle, neither shal they concurre. For otherwise one & the selfe same angle should be both greater, and also equal: which is impossible. As for example.

Suppose that there be two right lines *A B* and *C D*, and vpon them let the right line *B E* fall, making the angles *A B D* and *C D E* equall. Then I say, that the right lines *A B* and *C D* shall not concurre. For if they concurre, the foresaide angles abidyng equall, namely, the angles *C D E* and *A B D*: Then forasmuch as the angle *C D E* is the outward angle it is of necessitie greater then the inward and opposite angle, & it is also equal vnto it: which is impossible. Wherfore if the said lines cõcurre, thẽ shal not the angles remayne equall, but the angle at the point *D* shall be encreased. For whether *A B*

abiding

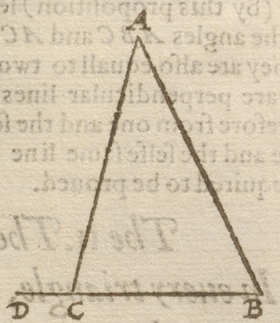
abiding fixed you suppose the line *C D* to be moued vnto it, so that they concurre, the space and distance in the angle will be greater: for how much more *C D* approcheth to *A B*, so much farther of goeth it from *D E*. Or whether *C D* abiding fixed, you imagine the line *A B* to be moued vnto it, so that they concurre, the angle *A B D* will be lesse, for therewith all it commeth neere vnto the lines *C D* & *B D*. Or whether you imagine either of them to be moued the one to the other, you shall finde that the line *A B* comming neere to *C D*, maketh the angle *A B E* lesse, and *C D* going farther from *D E* by reason of his motion to the line *B D*, maketh the angle *C D E* to increase. Wherefore it followeth of necessitie, that if it be a triangle, and that the right lines *A B* and *C D* do concurre, the outward angle also shall be greater then the inward and opposite angle. For either the inward and opposite angle abiding fixed, the outward is increased: or the outwarde abiding fixed, the inward and opposite is diminished: or els both of them being moued till they concurre, the inwarde is diminished, and the outwarde is more increased. And the cause hereof is the motion of the right lines the one tending to that parte where it diminisheth the inwarde angle, the other tending to that part where it increaseth the outward angle.



The 10. Theoreme. The 17. Proposition.

In euery triangle, two angles, which two soeuer be taken, are lesse then two right angles.

Vppose that A B C be a triangle. Then I saye that two angles of the sayd triangle A B C, which two angles soeuer be taken, are lesse then two right angles. Extend (by the 2. peticiō) the line B C, to the point D. And forasmuch as (by the proposition going before) the outward angle of the triangle A B C, namely, the angle A C D is greater then the inward and opposite angle A B C: put the angle A C B commō to them both: wherefore the angles A C D and A C B are greater then the angles A B C and B C A. But (by the 13 proposition) the angles A C D and A C B are equall to two right angles. Wherefore the angles A B C and B C A are lesse then two right angles. In like sort also may we proue, that the angles B A C and A C B are lesse then two right angles: and also that the angles C A B & A B C are lesse then two right angles. Wherefore in euery triangle, two angles, which two soeuer be taken, are lesse then two right angles: which was required to be proued.



Construction.

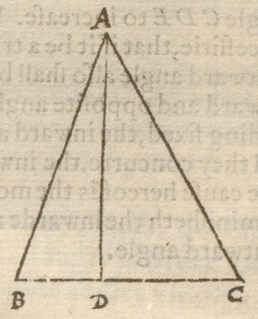
Demonstration.

This may also be demonstrated without the helpe of the former proposition, by the conuerse of the fifth petition, and by the 13. proposition as you saw was done in the former after *Pelitarius*.

It may also be demonstrated without producing any of the sides of the triangle, after this maner.

An other demõstration inuented by Proclus.

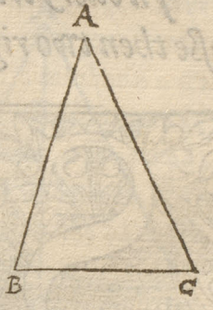
Suppose that there a be triangle *ABC*. And in the side *BC* take a point at all aduentures, and let the same be *D*, and draw the line *AD*. And forasmuch as in the triangle *ABD*, the side *BD* is produced, therefore (by the former proposition) the outward angle *ADC*, is greater then the inward and opposite angle *ABD*. Agayne forasmuch as in the triangle *ADC*, the syde *CD* is produced, therefore (by the same) the outward angle *ADB*, is greater then the inwarde and opposite angle *ACD*: but the angles at the point *D* are equall to two right angles (by the 13. proposition:) wherfore the angles *ABC* and *ACB* are lesse then two right angles. And by the same reason may we proue that the angles *BAC* and *BCA* are lesse then two right angles, if we take a poynt in the line *AC*, and draw a right line from it to the point *B*: and so also may it be proued that the angles *CAB* and *ABC* are lesse the two ryght angles, if there be taken in the lyne *AB* a point, and from it be a line drawen to the point *C*.



A Corrollary following this Proposition.

By this proposition also may be proued this Corrollary, that from one and the selfe same point to one and the selfe same right line, can not be drawen two perpendicular lines.

For if it be possible, from the point *A*, let there be drawen vnto the right line *BC*, two perpendicular lines *AB*, and *AC*: wherefore the angles *ABC* and *ACB* are right angles. But forasmuch as *ABC* is a triangle. therefore any two angles therof are (by this proposition) lesse then two right angles. Wherefore the angles *ABC* and *ACB* are lesse then two right angles: but they are also equall to two right angles, by reason *AB* and *AC* are perpendicular lines vpon *BC*: which is impossible. Wherefore from one and the selfe same point cannot be drawē to one and the selfe same line two perpendicular lines: which was required to be proued.

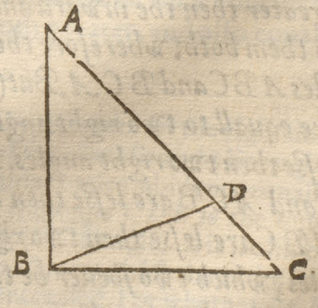


The 11. *Theoreme.* *The* 18. *Proposition.*

In euery triangle, to the greater side is subtended the greater angle.

Construction.

Vppose that ABC be a triangle, hauing the side AC greater then the side AB. Then I say that the angle ABC is greater then the angle BCA. For forasmuch as AC is greater thē AB, put (by the 3. proposition) vnto AB an equall line AD. And (by



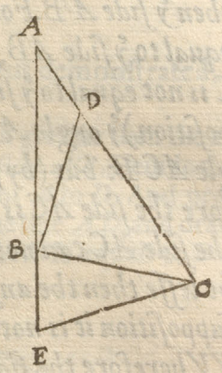
the

the first petition) draw a line from the point B to the point D. And forasmuch as the outward angle of the triangle D B C, namely, the angle A D B is greater then the inward and opposite angle D C B (by the 16. proposition,) but (by the 5. proposition) the angle A D B is equall to the angle A B D, for the syde A B is equall to the syde A D: therefore the angle A B D is greater then the angle A C B. Wherefore the angle A B C is much greater then the angle A C B. Wherefore in euery triangle, to the greater syde is subtended the greater angle: which was required to be proued.

Demonstration.

Another demonstration after Prophyrius.

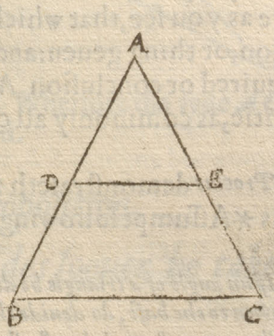
You may also proue the angle at the point *B* greater then the angle at the point *C* (the side *A C* being greater then the side *A B*) if from the line *A C* you cut of a line equall to the line *A B*, beginning at the point *C*, as before you beganne at the point *A*: and that after this manner. Let the line D C be equall to the line *A B* and draw the line *B D*: and produce *A B* to the point *E*: and put the line *B E* equal to the line *A D*. Wherefore the whole line *A E* is equall to the whole line *A C*: draw a line from *E* to *C*. And forasmuch as *A E* is equal to *A C*, therfore the angle *A E C* is also equall to the angle *A C E* (by the 5. proposition:) but the angle *A B C* is greater then the angle *A E C*. For one of the sides of the triangle *C B E*, namely, the side *B E* is produced, and so the outward angle *A B C* is greater then the inward and opposite *B E C* (by the 16 proposition:) wherefore the angle *A B C* is much greater then the angle *A C B*: which was required to be proued.



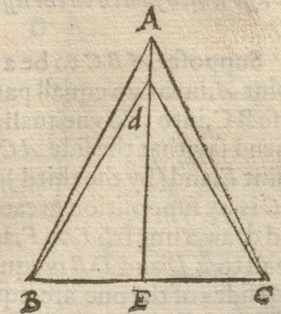
That which is spoken in this Propesition is to be vnderstanded in one and the selfe same triangle.

Note that that which is here spoken in this proposition, is to be vnderstanded in one and the selfe same triangle. For it is possible that one and the selfe same angle may be subtended of a greater line, and of a lesse line: and one and the selfe same right line may subtend a greater angle, and a lesse angle. As for example.

Suppose that there be an Isosceles triangle *A B C*, & in the side *AB* take the point *D* at all aduentures: & frō the line *A C* cut of (by the 3. proposition) the lyne *A E* equall to the line *A D*. And draw a right line from *D* to *E*. Wherfore the right lines *DE* and *BC* do subtend the angle at the point *A*, & of them the one is greater, and the other lesse. And after the selfe same manner a man may put infinite right lines greater & lesse, subtending the angle at the point *A*.



Agayne suppose that *A B C* be an Isosceles triangle. And let *B C* be lesse then either of the lines *B A* and *AC*. And vpon *B C* describe (by the first) an equilater triangle *B C D*. And draw a line from *A* to *D*, and produce it to the point *E*. And forasmuch as in the triangle *A B D*, the outward angle *B D E*, is greater rhen the inward & opposite angle *B A D* (by the 16. proposition) And by the same in the triāgle *A C D*, the outward angle *C D E*, is greater then the inward & opposite angle *C A D*: therfore the whole angle *B D C* is greater thē the whole angle *B A C*. And one and the selfe same right line subtendeth both these angles, namely, the greater angle & the lesse. And it is also proued, that greater right lines & lesse subtende one and the selfe same angle. But in



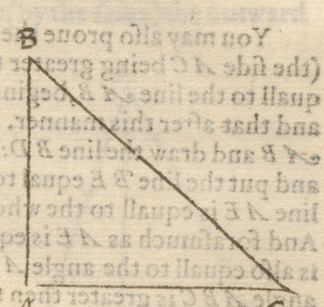
one and the selfe same triangle one right line subtendeth one angle, and the great right line euer subtendeth the great angle, and the lesse the lesse, as it was proued in the proposition.

The 12. Theoreme. The 19. Proposition.

In euery triangle, vnder the greater angle is subtended the greater side.

Demonstration leading to an impossibilitie.

SVppose that *A B C* be a triangle, hauyng the angle *A B C* greater then the angle *B C A*. Then I say that the side *A C* is greater then ỹ side *A B*. For if not, thē the side *A C* is ether equal to ỹ side *A B*, or els it is lesse thē it. The side *A C* is not equal to ỹ side *A B*, for then (by the 5. proposition) ỹ angle *A B C* should be equall to the angle *A C B*: but (by suppositiō) it is not. VVherefore the side *A C* is not equall to the side *A B*. And the side *A C* can not be lesse then the side *A B*, for then the angle *A B C* shoulde be lesse then the angle *A C B* (by the proposition next goyng before). But (by supposition it is not) VVherefore the side *A C* is not lesse then the side *A B*. VVherefore the side *A C* is greater then the side *A B*. VVherefore in euery triangle, vnder the greater angle is subtended the greater side: which was required to be demonstrated.



This proposition is the conuerse the former.

This proposition is the conuerse of the proposition next going before. VVherefore as you see, that which was the conclusion in the former, is in this the supposition, or thing geuen: and that which was there the thing geuen, is here the thing required or conclusion. And it is proued by an argument leading to an impossibilitie, as commonly all conuerses are.

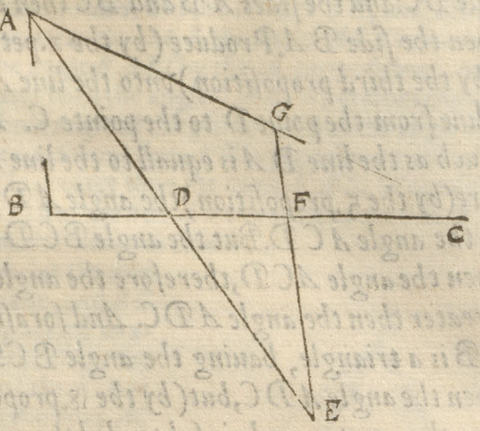
Proclus demonstrateth this proposition after an other way: but first he putteth this *Assumpt following.

**An Assumpt is a Proposition taken of necessitie to the helpe of a demonstration, the certainty whereof is not so plaine, and therfore nedeth it selfe first to be demonstrated. An assumpt put by Proclus for the demonstration of this Proposition.*

If an angle of a triangle be deuided into two equall partes, and if the line which deuideth it being drawen to the base, do deuide the same into two vnequall partes: the sides which contayne that angle shalbe vnequall, and that shalbe the greater side, which falleth on the grater side of the base, and that the lesse which falleth on the lesse side of the base.

Suppose *A B C* to be a triangle, and (by the 9. proposition) deuide the angle at the point *A*, into two equall partes, by the right line *A D*. And let the line *A D* deuide the base B C, into two vnequall partes, and let the part *C D* be greater then the parte *B D*. Then I say, that the side *A C* is greater then the side *A B*. Produce the line *A D* to the point *E*, and (by the third) put the line *D E* equall to the line *D A*. And forasmuch as *D C* is by supposition greater then *D B*, put (by the 3. proposition) *D F* equal to *B D*, and draw a line frō *E* to *F*, and produce it to the point *G*. Now forasmuch as *A D* is equall to *E D* and *D B* is equall to *D F*, therfore in the two triangles *A B D*, and *E F D*, two sides of the one are equall to two sides of the other, eche to his correspondent side: and (by the 15. proposition) they contayne equall angles, namely, the hed angles

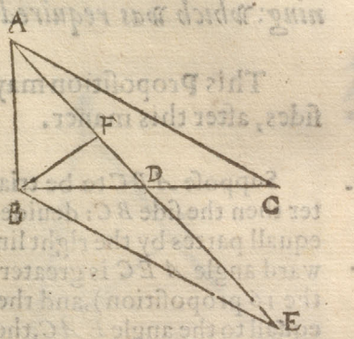
gles: wherfore (by the fourth propoſition) the baſe *B A* is equall to the baſe *E F*: and the angle *D E F* is equall to the angle *D A B*. But the angle *D A G* is by conſtruction equall to the ſame angle *D A B*: wherefore (by the firſt common ſentence) the angles *E A G* and *A E G* are equall. Wherefore (by the 6. propoſion) the ſide *A G* is equall to the ſide *E G*. Wherfore the ſide *A C* is greater then the ſide *E G*. Wherefore it is much greater then the ſide *E F*. But the ſide *E F* is equall to the ſide *A B*, as it hath bene proued. Wherefore the ſide *A C* is greater then the ſide *A B*: which was required to be proued.



This aſſumpt being put, this Propoſition is of Proclus thus demonſtrated.

An other demonſtration after Proclus.

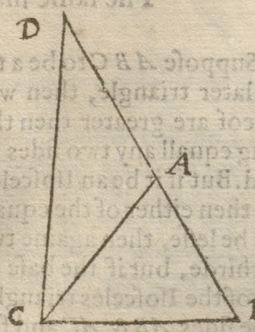
Suppoſe *A B C* to be a triangle, hauing his angle at the point *B* greater then the angle at the point *C*. Then I ſay that the ſide *A C* is greater then the ſide *A B*. Deuide the line *B C* into two equall partes in the point *D*, and draw a line from *A* to *D*. And produce the line *A D* to the point *E*: and put the line *D E* equall to the line *A D*, and draw a line from *B* to *E*. Now foraſmuch as *B D* is equall to *D C*, and *A D* is equall to *D E* therefore in the two triangles *A D C* and *B D E*, two ſides of the one are equall to two ſides of the other, ech to his correſpondent ſide, and they containe equall angles (by the 15. propoſition): wherefore (by the fourth propoſition) the baſe *B E* is equall to the baſe *A C*, and the angle *D B E* is equal to the angle at the point *C*. Deuide alſo th'angle *A B E* into two equal parts by the line *B F*: wherfore the line *E F* is greater then the line *F A*. And foraſmuch as in the triangle *A B E*, the angle at the point *B* is deuided into two equall partes by the right line *B F*, and the line *E F* is greater then the line *A F*: therefore by the former *Aſſumpt* the ſide *B E* is greater then the ſide *B A*: but the line *B E* is equall to the line *A C*. Wherfore the ſyde *A C* is greater then the ſide *A B*: which was required to be proued.



The 13. Theoreme. The 20. Propoſition.

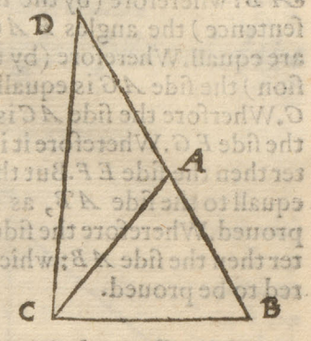
In euery triangle two ſides, which two ſides ſoeuer be taken, are greater then the ſide remayning.

Suppoſe that A B C be a triangle. Then I ſay that two ſides of the triangle A B C, which two ſides ſoeuer be taken, are greater then the ſide remayning that is, the ſides B A and A C are greater then the



Construction. *Demonstration.*

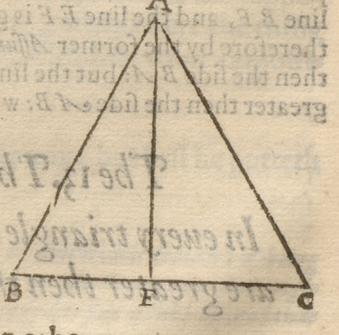
side BC: and the sides AB and BC then the side AC: and the sides AC and BC then the side BA. Produce (by the 2. petition) the line BA to the point D. And (by the third proposition) vnto the line AC put an equall line AD: and drawe a line from the point D to the pointe C. And forasmuch as the line DA is equall to the line AC, therefore (by the 5. proposition) the angle ADC, is equall to the angle ACD. But the angle BCD is greater then the angle ACD, therefore the angle BCD is greater then the angle ADC. And forasmuch as DCB is a triangle, hauing the angle BCD greater then the angle ADC, but (by the 18. proposition) vnder the greater angle is subtended the greater side: Wherfore DB is greater then BC. But the line DB is equall to the lines AB and AC (for the line AD is equall to the line AC) Wherfore the sides BA and AC, are greater then the side BC. And in like sorte may we proue, that the sides AB and BC are greater then the side AC: & that the sides BC and CA are greater then the side AB. Wherfore in euery triangle two sides, which two sides soeuer be taken, are greater then the side remayning: which was required to be demonstrated.



This Proposition may also be demonstrated without producing any of the sides, after this maner.

Another demonstration without producing one of the sides.

Suppose *ABC* to be triangle. Then I say, that the two sides *AB* and *AC* are greater then the side *BC*: deuide the angle at the point *A* (by the 9. proposition) into two equall partes by the right line *AE*. And forasmuch as in the triangle *ABE*, the outward angle *AEC* is greater then the angle *BAE* (by the 16 proposition), and the angle *BAE* is put to be equall to the angle *EAC*, therefore the side *AC* is greater then the side *CE*. And by the same reason, the side *AB* is greater thē the side *BE*. For in the triangle *AEC* the outward angle *AEB*, is greater then the angle *CAE*, that is then the angle *EAB*. Wherefore also the side *AB* is greater then the side *BE*. Wherfore the sides *AB* and *AC* are greater then the whole side *BC*. And after the same maner may you proue touching the other sides also.



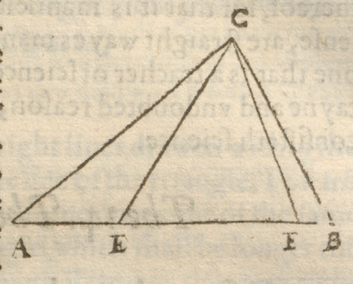
The same may yet also be demonstrated an other way.

Another Demonstration.

Suppose *ABC* to be a triangle. Now if *ABC* be an equilater triangle, then without doubt any two sides thereof are greater then the third. For the three sides being equall any two sides of them are double to the third. But if it be an Isosceles triangle, either the base is lesse then either of the equall sides or it is greater. If the base be lesse, then againe two of them are greater then the thirde, but if the base be greater: let *BC* being the base of the Isosceles triangle *ABC* be greater thē either of the sides *AB* & *AC* and from it cut of (by the 3. proposition

position) a line equall to any one of the other sides, whiche let bee BE, and drawe a line from A to E. And forasmuch as in the triangle AEB, the angle AEC is an outward angle: therefore it is greater then the angle BAE (by the 16. proposition). And by the same reason, the angle AEB is greater then the angle CAE. Wherefore the angles at the point E are greater then the whole angle at the pointe A. But the angle BEA is equal to the angle BAE (by the 5. proposition) for AB is put to be equall to BE. Wherefore the angle remayning AEC is greater then the angle CAE. Wherefore also the side AC is greater then the side EC. But the side AB is equall to the side BE. Wherefore the sides AB and AC are greater then the side BC.

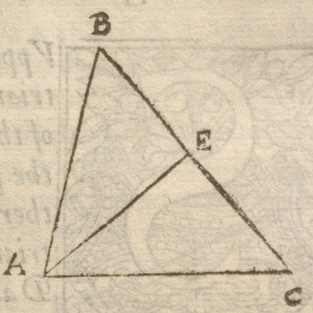
But if the triangle ABC be a Scalenum, let the side AB be the greatest, and let AC be the meane, and BC the least. Wherefore the greatest side being added to any one of the two sides must nedes be greater then the third. For of it selfe it is greater then any of them. But if AB being the greatest, you would proue the sides AC and CB to be greater then it. Then as you did in the Isoceles triangle, cut of from the greatest a line equall to one of them, and from the point C to the point of the intersection draw a right line, and reason as you did before by the outward angles of the triangle, and you shal haue your purpose.



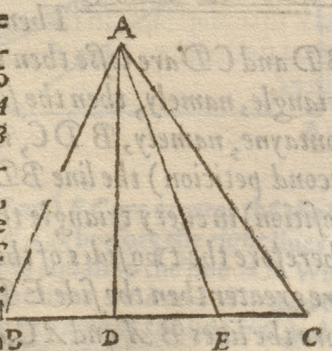
This proposition may yet moreouer be demonstrated by an argument leading to an absurditie, and that after this manner.

An other demonstration leading to an absurditie

Suppose ABC to be a triangle. Then I say that the sides AB and AC, are greater then the side BC. For if they be not greater, they are either equall or lesse. First let them be equall, and from the line BC cut of the line BE equall to the line AB (by the 3. proposition) wherfore the residue EC is equall to AC. Now forasmuch as AB is equall to BE they subtend equall angles. Likewise forasmuch as AC is equall to CE they subtend equal angles. Wherfore the angles which are at the point E are equall to the angles whiche are at the pointe A, which is impossible (by the 16. proposition).



But now let the sides AB and AC be lesse then the side BC, and from the line BC cut of (by the 3. proposition) the line BD equall to the line AB, and likewise frō the same line BC cut of the line CE equall to the line AC. And forasmuch as AB is equall to BD, the angle BDA also is equall to the angle BAD (by the fifth proposition). Againe forasmuch as AC is equall to CE, therefore (by the same) the angle CEA is equall to the angle EAC. Wherefore these two angles BDA and CEA are eqnall to these two angles BAD and EAC. Agayne forasmuch as the angle BDA is the outward angle of the triangle ADC, therefore it is greater then the angle EAC. For it is greater then the angle DAC (by the 16. proposition). And by the same reason, forasmuch as CEA is the the outward angle of the triangle ABE, therefore it is greater then the angle BAD (for it is greater then the angle BAE). Wherfore the angles BDA and CEA are greater then the two angles BAD and EAC. But they were also proued equall vnto them: which is impossible. Wherefore the sides AB and AC are neither equall to the side BC, nor lesse then it, but greater. And so also may it be proued of the rest.



A briefe demonstration by the definition of a right line.

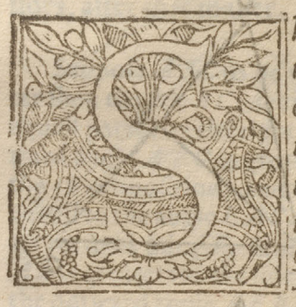
A man may also more briefely demonstrate this proposition by Campanus definition of a right line, which as we haue before declared is thus: *A right line is the shortest extension or drawght that is or may be from one point to another.* Wherfore any one side of a triangle, for that it is a right line drawen from some one point to some other one point, is of necessitie shorter then the other two sides drawen from and to the same pointes.

Not all thinges manifest to the sense, are straight way manifest to reason and vnderstanding.

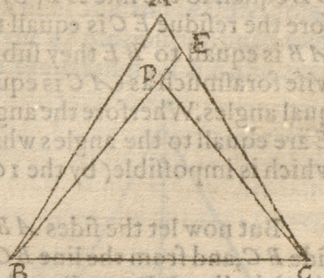
Epicurus and such as followed him derided this proposition, not counting it worthy to be added in the number of propositions of Geometry for the easines thereof, for that it is manifest euen to the sense. But not all thinges manifest to sense, are straight wayes manifest to reason and vnderstanding. It pertayneth to one that is a teacher of sciences, by prose and demonstration to render a certayne and vndoubted reason, why it so appeareth to the sense: and in that onely consisteth science.

The 14. Theoreme. The 21. Proposition.

If from the endes of one of the sides of a triangle, be drawen to any point within the sayde triangle two right lines: those right lines so drawen, shalbe lesse then the two other sides of the triangle, but shall containe the greater angle.



Vppose that A B C be a triangle: and frō the endes of the side B C, namely, frō the pointes B and C, let there be drawen within ỹ triangle two right lines B D and C D to ỹ point D. Then I say, that the lines B D and C D are lesse then the other sides of the triangle, namely, then the sides B A and A C: and that the angle which they contayne, namely, B D C, is greater then the angle B A C. Extend (by the second peticion) the line B D to the point E. And forasmuch as (by the 20. proposition) in euery triangle the two sides are greater then the side remayning, therefore the two sides of the triangle A B E, namely, the sides A B and A E, are greater then the side E B. Put the line E C common to them both. Wherefore the lines B A and A C, are greater then the lines B E and E C. Againe forasmuch as (by the same) in the triangle C E D, the two sides C E and E D, are greater then ỹ side D C, put ỹ line D B common to them both: wherfore ỹ lines C E and E B, are greater then the lines C D and D B. But it is proued that the lines B A and A C, are greater then the lines B E and E C. Wherefore the lines B A and A C, are much greater then the lines B D and D C. Agayne forasmuch

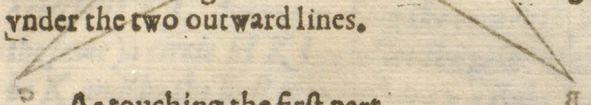


Demonstration.

as

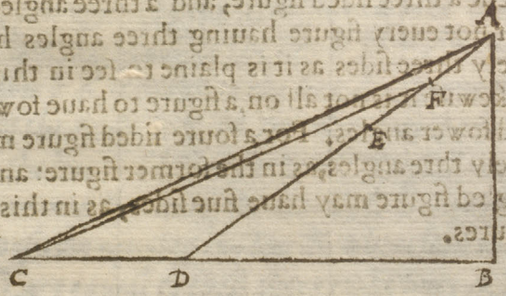
as (by the 16. propoſition) in euery triangle, the outward angle is greater then the inward and oppoſite angle, therefore the outward angle of the triangle C D E, namely, B D C, is greater then the angle C E D. VVherefore alſo (by the ſame) the outward angle of the triangle A B E, namely, the angle C E B is greater then the angle B A C. But it is proued, that the angle B D C is greater then the angle C E B. VVherfore the angle B D C is much greater then the angle B A C. VVherefore if from the endes of one of the ſides of a triangle, be drawen to any point within the ſayde triangle two right lines: thoſe right lines ſo drawn ſhalbe leſſe then the two other ſides of the triangle, but ſhall containe the greater angle: which was required to be demonſtrated.

In this propoſition is expreſſed, that the two right lines drawen within the triangle, haue their beginning at the extremes of the ſide of the triangle. For frō the one extreme of the ſide of the triangle, and from ſome one point of the ſame ſide, may be drawen two right lines within the triangle, which ſhall be longer thē the two outward lines: which is wonderfull and ſeemeth ſtraunge, that two right lines drawen vpon a parte of a line, ſhould be greater then two right lines drawen vpon the whole line. And agayne it is poſſible from the one extreme of the ſide of a triangle, and from ſome one point of the ſame ſide to drawe two right lynes within the triangle which ſhall containe an angle leſſe then the angle contayned vnder the two outward lines.



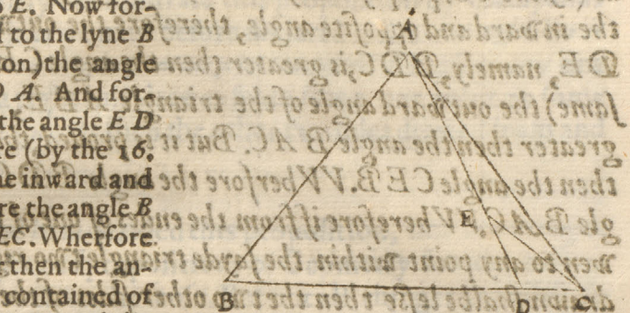
As touching the firſt part.

Suppoſe *A B C* to be a rectangle triangle, whoſe right angle let be at the point *B*. And in the ſide *B C* take a point at al aduentures, which let be *D*: and draw a right line frō *A* to *D*. Wherfore the line *A D* is greater then the line *A B* (by the 19. propoſitiō) From the line *A D* cut of (by the thirde) a line equall to the line *A B*, which let be *D E*. And deuide the line *E A* into two equall partes in the point *F* (by the 10. propoſition) And draw a line frō *F* to *C*. Now foraſmuch as *A F C* is a triangle, therfore the lines *A F* and *F C* are greater thē the line *A C* (by the former propoſition): but *A F* is equal to *F E*: wherfore the right lines *F E* and *F C* are greater then the line *A C*. And the line *D E* is equall to the line *A B*. Wherfore the right lines *F C* and *F D* are greater then the right lines *A B* and *A C*: and they are drawen within the triangle *A B C*, the one from one extreme of the ſide *B C*, and the other from a point in the ſame ſide *B C*: whiche was required to be proued.

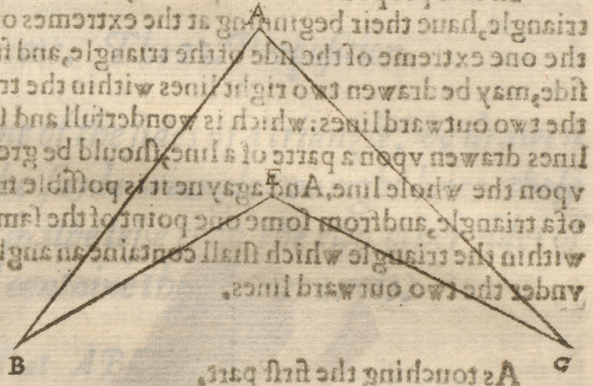


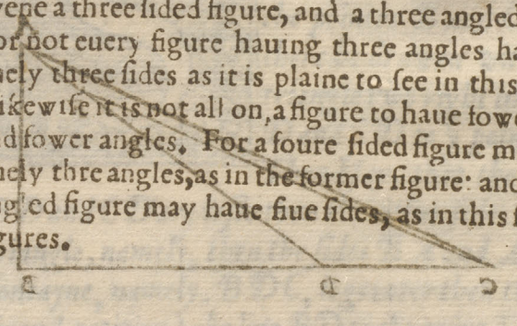
As touchyng the ſeconde parte. Suppoſe *A B C* to bee an Iſoſceles triangle, and let the baſe thereof namelye, *B C* bee greater then either of the equall ſydes, and from the lyne *B C* cutte of a line equall to the lyne *A B* (by the thirde propoſition) whiche let bee *B D*: and drawe a line from *A* to *D*: and in the

line *A D* take a point at aduentures, which let be *E*, & draw a line from *C* to *E*. Now forasmuch as the line *A B*, is equal to the lyne *B D*, therfore (by the fift proposition) the angle *B A D* is equall to the angle *B D A*. And forasmuch as in the triangle *E D C* the angle *E D B* is an outwarde angle, therefore (by the 16. proposition) it is greater then the inward and opposite angle *D E C*. Wherefore the angle *B A D* is greater then the angle *D E C*. Wherfore the angle *B A C* is much greater then the angle *D E C*, and the angle B A C is contained of the outward right lines *B A* and A C, and the angle *D E C* is contayned of the inward right lines *D E* and *E C*: which was required to be proued.



By meanes of this proposition also is described that kynd of triangles, which contayneth foure sides. As for example this figure A B C. For it is cõtained of fower sides B A, A C, C E, & E B. But it hath onely three angles, one at the point B, an other at the point A, and the third at the point C. VVherefore this present figure A B C is a quadrilater triangle: which of olde philosophers hath euer bene counted wonderfull. And here is to be noted, that there is difference betwene a three sided figure, and a three angled figure. For not euery figure hauing three angles hath also onely three sides as it is plaine to see in this figure. Likewise it is not all on, a figure to haue fower sides, and fower angles. For a foure sided figure may haue onely thre angles, as in the former figure: and a foure angled figure may haue fiue sides, as in this figure folowing. And so of all other figures.



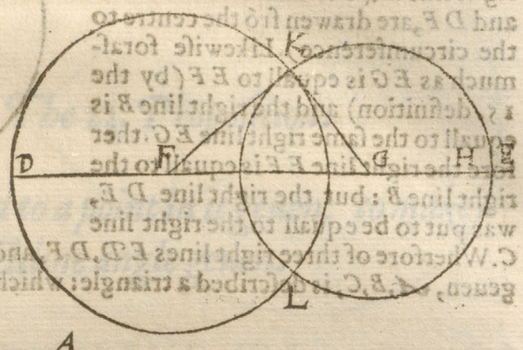


The 8. Probleme. The 22. Proposition.

Of thre right lines, which are equall to thre right lines geue, to make a triangle. But it behoueth two of those lines, which two soeuer be taken, to be greater then the third. For that in euery triangle two sides, which two sides soeuer be taken, are grea-

greater then the ſide remayning.

SVppoſe that the three right lines geuē be *A*,*B*,*C*: of which let two of them, which two ſoeuer be taken, be greater then the third, that is, let the lines *A*,*B*, be greater then the line *C*, and the lines *A*,*C*, then the line *B*, and the lines *B*, *C*, then the line *A*. It is required of three right lines equall to the right lines *A*,*B*,*C*, to make a triangle. Take a right line hauing an appointed ende on the ſide *D*, and being infinite on the ſide *E*. And (by the 3. propoſition) put vnto the line *A*, an equall line *D F*, and put vnto the line *B*, an equall line *F G*, and vnto ẏ line *C*, an equall line *G H*. And making the centre *F*, and the ſpace *D F*, deſcribe (by the 3. petitiō) a circle *D K L*. Agayne making the centre *G*, and the ſpace *G H*, deſcribe (by the ſame) a circle *H K L*: and let the point of the interſection of the ſayd circles be *K*, and (by the firſt petition) draw a right line from the point *K* to ẏ point *F*, & an other from the point *K* to the point *G*. Then I ſay, that of thre right lines equall to the lines *A*,*B*,*C*, is made a triangle *K F G*. For foraſmuch as the point *F* is the centre of the circle *D K L*, therefore (by the 15. definition) the line *F D* is equall to the line *F K*. But the line *A* is equall to the line *F D*. Wherefore (by the firſt common ſentence) the line *F K* is equall to the line *A*. Agayne foraſmuch as the point *G*, is the centre of the circle *L K H*, therefore (by the ſame definition) the line *G K* is equall to the line *G H*. But the line *C* is equall to the line *G H*: wherefore (by the firſt common ſentence) the line *K G* is equall to the line *C*. But the line *F G* is by ſuppoſition equal to the line *B*: wherefore theſe three right lines *G F*, *F K*, and *K G*, are equall to theſe three right lines *A*,*B*, *C*. Wherefore of three right lines, that is, *K F*, *F G*, and *G K*, which are equall to the thre right lines geuen, that is to *A*, *B*, *C*, is made a triangle *K F G*: which was required to be done.



Conſtruction.

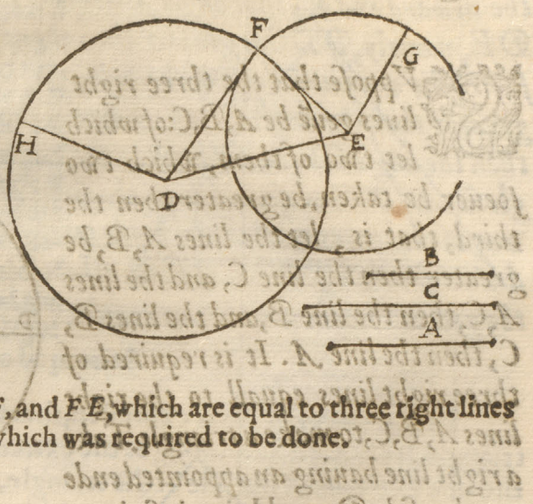
Demonſtration.

An other conſtruction, and demonſtration after Fluſſates.

Suppoſe that the three right lines be *A*,*B*,*C*. And vnto ſome one of them, namely, to *C*, put an equall line *D E*, and (by the ſecond propoſition) from the point *E*, draw the line *E G*, equall to the line *B*: and (by the ſame) vnto the point *D* put the line *D H* equal to the line *A*. And making the centre the point *E*, & the ſpace *E G*, deſcribe a circle *F G*: likewiſe making the centre the point *D*, and the ſpace *D H*, deſcribe an other circle *H F*: which circles let cutte the one the other in the point *F*. And draw

An other conſtruction and demonſtration after Fluſſates.

theſe lines *D F* and *E F*. Then I ſaye that *D F E* is a triangle deſcribed of 3. right lines equall to the right lines *A,B,C*. For foraſmuch as the line *D H* is equall to the right line *A*, the line *D F*, ſhall alſo be equall to the ſame right line *A*, (For that the lines *D H* and *D F*, are drawen frō the centre to the circumference). Likewiſe foraſmuch as *E G* is equall to *E F* (by the 15. definition) and the right line *B* is equall to the ſame right line *E G*. therfore the right line *E F* is equall to the right line *B*: but the right line *D E*, was put to be equall to the right line *C*. Wherfore of three right lines *E D, D F*, and *F E*, which are equal to three right lines geuen, *A,B,C*, is deſcribed a triangle: which was required to be done.

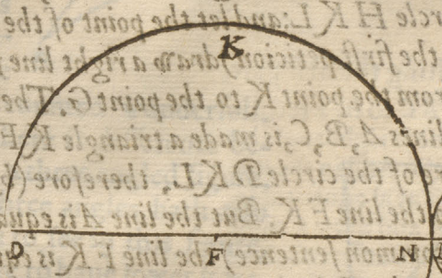


Inſtances in this Probleme.

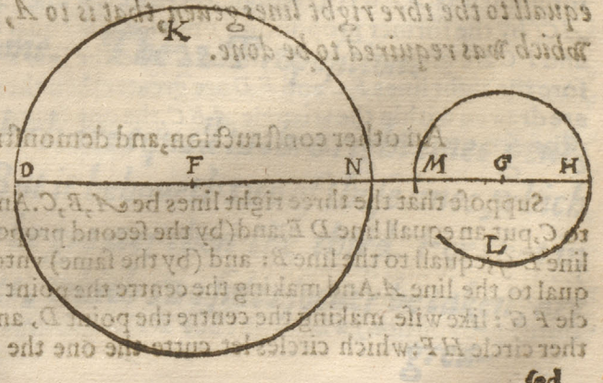
In this propoſition the aduerſary paraduenture will cauill, that the circles ſhall not cut the one the other (which thing *Euclide* putteth them to do) But now if they cutte not the one the other, either they touch the one the other, or they are diſtaunte the one from the other. Firſt if it be poſſible let them tooche the one the other: as in the figure here put (the conſtruction whereof anſwereth to the conſtruction of *Euclide*).

Firſt inſtance.

And foraſmuch as *F* is the centre of the circle *D K*, therfore the line *D F* is equal to the line *F N*. And foraſmuch as the point *G*, is the centre of the circle *H L*, therfore the line *H G*, is equall to the line *G M*. Wherefore theſe two lines *D F*, and *G H*, are equall to one line, namely, to *F G*. But they were put to be greater then it: for the lines *D F, F G*, and *G H*, were put to be equall to the lines *A,B,C*, euery two of which are ſuppoſed to be greater then the thirde: wherefore they are both greater, and alſo equall, which is impoſſible. Agayne if it be poſſible, let the circles be diſtant the one from the other, as are the circles *D K* and *H L*. And foraſmuch as *F* is the centre of the circle *D K*, therfore the line *D F* is equal to the line *F N*. And foraſmuch as *G* is the centre of the circle *L H*, therefore the line *H G* is equall to the line *G M*: wherefore the whole line *F G* is greater then the two lines *D F*, and *G H*, (for the line *F G*, exceedeth the lines *D F*, and *G H*, by the line *N M*) but it was ſuppo-



Second inſtance

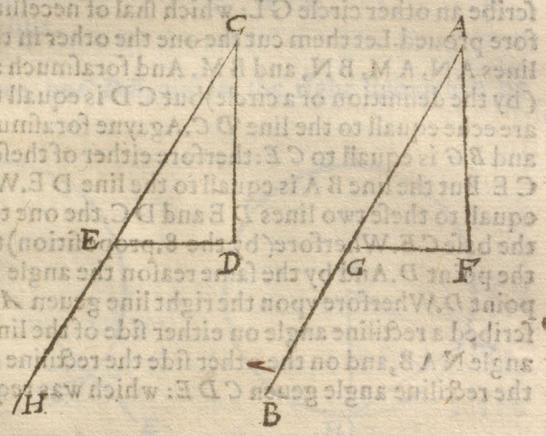


ſed

ſed that the lines *D F* and *H G* are greater then the line *F G*: as alſo it was ſuppoſed that the lines *A* and *C*, were greater then the line *B* (for the line *D F* is put to be equall to the line *A*, and the line *F G* to the line *B*, and the line *H G* to the line *C*.) Wherefore they are both greater and alſo equall: which is impoſſible. Wherefore the circles neyther tooch the one the other, nor are diſtant the one from the other. Wherefore of neceſſitie they cut the one the other: which was required to be proued.

The 9. Probleme. The 23. Propoſition.

Vpon a right line geuen, and to a point in it geuen: to make a rectiline angle equall to a rectiline angle geuen.



Conſtruction.

Suppoſe ỹ the right line geuẽ be AB, & let ỹ point in it geuen be A. And let alſo the rectiline angle geuẽ be DC H. It is required vpon the right line geuen AB, and to the point in it geuen A, to make a rectiline angle equall to the rectiline angle geuen DC H. Take in either of the lines CD and CH a point at all aduentures, & let the ſame be D and E. And (by the firſt peticion) draw a right line frõ D to E. And of thre right lines, AF, FG and GA, which let be equall to the three right lines geuen, that is, to CD, DE, and EC, make (by ỹ propoſition goyng before) a triangle, and let the ſame be AFG: ſo that let the line CD be equall to the line AF, and the line CE to the line AG, and moreouer the lyne DE to the line FG. And foraſmuch as theſe two lines DC and CE are equall to theſe two lines FA and AG, the one to the other, and the baſe DE is equall to the baſe FG: therfore (by the 8. propoſition) the angle DCE is equall to the angle FAG. Wherfore vpon the right line geuen AB, and to the point in it geuen namely A, is made a rectiline angle FAG, equal to the rectiline angle geuen DCH: which was required to be done.

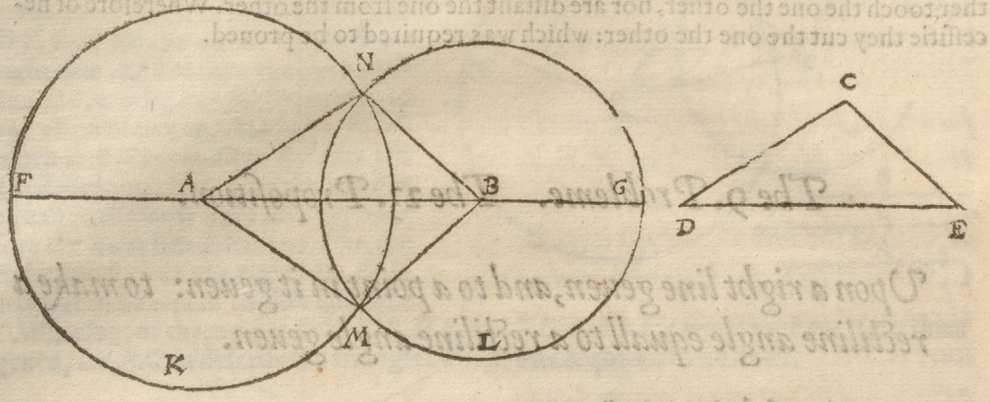
Demonſtration.

An other conſtruction and demonſtration after Proclus.

An other conſtruction and demonſtration after Proclus.

Suppoſe that the right line geuen be *AB*: & let the point in it geuen be *A*, & let the rectiline angle geuẽ be *CDE*. It is required vpõ the right line geuẽ *AB*, & to the point in it geuẽ *A*, to make a rectiline angle equal to the rectiline angle geuẽ *CDE*. Drawe a line frõ *C* to *E*. And produce the line *AB* on either ſide to the points *F* and *G*. And vn-

to the line *CD*, put the line *FA* equal, & vnto the line *DE* let the line *AB* be equal, & vnto the line *EC* put the line *BG* equal. And making the cẽtre the point *A*, & the ſpace *AF*, deſcribe a circle K F. And agayne making the centre the point *B* and the ſpace *BG* de-

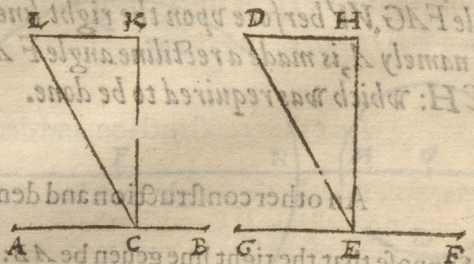


ſcribe an other circle G L: which ſhal of neceſſitie cut the one the other, as we haue before proued. Let them cut the one the other in the pointes M & N. And draw theſe right lines A N, A M, B N, and B M. And foraſmuch as F A is equall to A M: and alſo to A N (by the definition of a circle) but C D is equall to F A, wherfore the lines A M and A N are eche equall to the line *D C*. Agayne foraſmuch as *B G*, is equall to B M, and to B N, and *B G* is equall to *C E*: therfore either of theſe lines B M and B N is equall to the line C E. But the line B A is equall to the line D E. Wherfore theſe two lines B A & A M, are equall to theſe two lines *D* E and D C, the one to the other, and the baſe B M is equal to the baſe *C E*. Wherfore (by the 8. propoſition) the angle M A B, is equall to the angle at the point *D*. And by the ſame reaſon the angle N A B, is equall to the ſame angle at the point *D*. Wherfore vpon the right line geuen *A B*, and to the point in it geuen *A*, is deſcribed a rectiline angle on either ſide of the line *A B*: namely, on one ſide the rectiline angle N A B, and on the other ſide the rectiline angle M A B, either of which is equall to the rectiline angle geuen *C D E*: which was required to be done.

An other conſtruction alſo, and demonſtration after Pelitarius.

Another Demonſtration after Pelitarius.

Suppoſe that the right line geuen be *A B*: and let the point in it geuen be *C*, and let the rectiline angle geuen be *D E F*. It is required vpon the line geuen *AB*, and to the point in it geuen *C*, to deſcribe a rectiline angle equall to the rectiline angle geuen *D E F*. Produce the line *F E* to the point *G*: and from the point *E* erect (by the 11. propoſition) vnto the line *G F* a perpendiculer line *E H*, which if it exactly agree with the lyne *E D*, then was the angle geuẽ a right angle. Wherfore if from the pointe *C* you erecte a perpendiculer line vnto the line *A B*, that ſhall be done which was required to be done. But if it do not, then from the point *H*, erect vnto the line *H E*, a perpendiculer lyne *H D*, whiche being produced ſhall (by the fifth peticion) concurre with the line *E D* being alſo produced: for the angle *D E H* is leſſe then a right angle (when as *GEH* is a right angle). Wherefore let them concurre in the point *D*, and ſo is made the triangle *D E H*. After the ſame maner frõ the point geuen *C*, erect vnto the line *A B* a perpendiculer line *C K*: which let be equall to



the

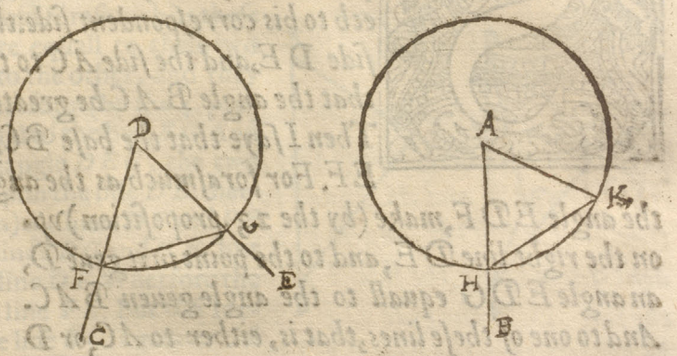
the perpendicular line *EH* (by the 3. propoſition): and from the point *K* erect vnto the line *KC*, a perpendiculer line *KL*, whiche let be equall to the perpendiculer lyne *HD*. And draw a line from *C* to *L*. Then I ſay that the angle *LCB*, is equall to the angle geuen *DEF*. For the two triangles *HED* and *KCL*, are (by the fourth propoſition) equal, and equilater the one to the other: and the two angles *LCK* and *DEH* are equal. And the two angles *BCK* and *FEH* are equal, for either of them is a right angle. Wherfore (by the 2. common ſentence) the whole angle *LCB*, is equall to the whole angle *DEF*. Which was required to be done.

And if the perpendiculer line chaunce to fall without the angle geuen, namely, if the angle geuen be an acute angle, the ſelfe ſame manner of demonſtration will ſerue: but onely that in ſtede of the ſecond common ſentence, muſt be vſed the 3. common ſentence.

Appollonius putteth another conſtruction & demonſtration of this propoſitiō: which (though the demonſtration thereof depende of propoſitions put in the third booke, yet for that the conſtruction is very good for him that wil redely, and mechanically, without demonſtration, deſcribe vpon a line geuen, and to a point in it geuen, a rectiline angle equall to a rectiline angle geuen) I thought not amiſſe here to place it. And it is thus.

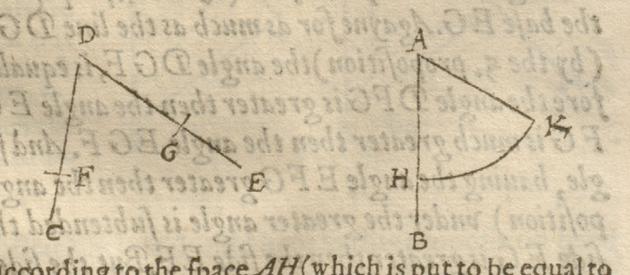
An other conſtruction & demonſtration after Appollonius.

Suppoſe that the rectiline angle geuen be *CDE*, and let the right line geuen be *AB*, and let the point in it geuen be *A*. Take in the line *CD*, a point at all aduentures, which let be *F*. And making the cētre the point *D*, and the ſpace *DF*, deſcribe a circle *FG*, cutting the line *DE*, in the point *G*, and draw a ryght line from *F* to *G*. Likewiſe from the line *AB*, cut of a line equall to the line *DF*, which let be *AH*. And making the cētre the point *A*, and the ſpace *AH* deſcribe a circle *HK*, and from the point *H*, ſubtend vnto the circumference of the circle a right lyne equall to the right line *FG*, whiche let bee *HK*: and drawe a right lyne from *A* to *K*. Then I ſay that the angle *HAK*, is equall to the angle *CDE*. The proofe whereof I now omitte for that it dependeth of the 28 and 27 propoſitions of the third booke.



How to do this propoſition redily and mechanically.

But now, as I ſayd, by this you may very redily, and mechanically, without demonſtration, vpon a line geuen, and to a point in it geuen deſcribe a rectiline angle equall to a rectiline angle geuen. For in the rectiline angle geuē, you neede onely to marke the two pointes, where the circumference of the circle cutteth the lines contayning the angle geuen: as the points *F* and *G*: and likewiſe to marke in the line geuen as in *AB*, the point *H*, & ſo making the centre the point *A*, according to the ſpace *AH* (which is put to be equal to *FD*) deſcribe a peece of a circumference on that ſide that you wil haue the angle to be, as for example the circumference *HK*, and opening your compaſſe to the wideth from



the point *F*, to the point *G*, set one foote thereof fixed in the point *H*, and marke the point where the other foote cutteth the sayde circumference, which point let be *K*. And from that point to the point *A*, draw a right line: and so shall you haue described at the point *A*, an angle equal to the angle at the point *D*. As in the figures in the end of the other side put.

Oenopides the first inuenter of this proposition.

Oenopides was the first inuenter of this proposition as witnesseth Eudemius.

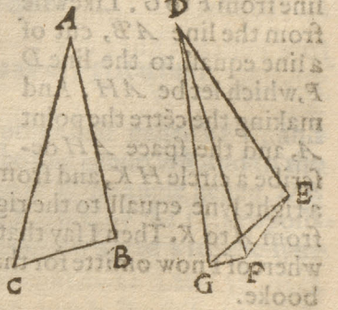
The 15. Theoreme The 24. Proposition.

If two triangles haue two sides of the one equall to two sides of the other, ech to his correspondent side, and if the angle cõtained vnder the equall sides of the one, be greater then the angle contayned vnder the equall sides of the other: the base also of the same, shalbe greater then the base of the other.

Suppose that there be two triangles A B C, and D E F, hauing two sides of the one, that is, A B, and A C, equall to two sides of the other, that is, to D E, and D F, ech to his correspondent side: that is, the side A B, to the side D E, and the side A C to the side D F: and suppose that the angle B A C be greater then the angle E D F. Then I saye that the base B C, is greater then the base E F. For forasmuch as the angle B A C is greater then

Construction.

thẽ angle E D F, make (by the 23. proposition) vpon the right line D E, and to the point in it geuẽ D, an angle E D G equall to the angle geuen B A C. And to one of these lines, that is, either to A C, or D F, put an equall line D G. And (by the first peticiõ) draw a right line from the point G, to the point E, and an other from the point F, to the point G. And



Demonstration.

forasmuch as the line A B is equall to the line D E, and the line A C to the line D G, the one to ỹ other, and the angle B A C is (by construction) equall to the angle E D G, therefore (by the 4. proposition) the base B C, is equall to the base E G. Agayne for as much as the line D G is equall to the line D F, ther- (by the 5. proposition) the angle D G F, is equall to the angle D F G. Wherefore the angle D F G is greater then the angle E G F. Wherefore the angle E F G is much greater then the angle E G F. And forasmuch as E F G is a triangle, hauing the angle E F G greater then the angle E G F, and (by the 18. proposition) vnder the greater angle is subtended the greater side, therefore the side E G is greater then the side E F. But the side E G is equall to the side B C: wherefore the side B C is greater then the side E F. If therefore two triangles

haue

haue two sides of the one equall to two sides of the other, eche to his correspondent side, and if the angle contayned vnder the equall sides of the one, be greater then the angle contayned vnder the equall sides of the other: the base also of the same shalbe greater then the base of the other: which was required to be proued.

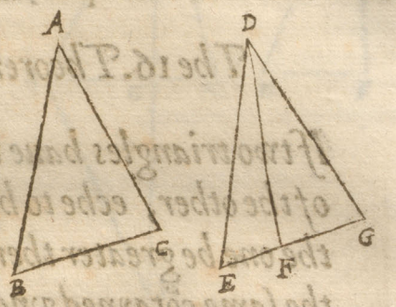
Three cases in this proposition.

In this Theoreme may be three cases. For the angle $E D G$, being put equall to the angle $B A C$, and the line $D G$, being put equall to the line $A C$, and a line being drawen from E to G, the line $E G$ shall either fall aboue the line $G F$, or vpon it, or vnder it. *Euclides* demonstration serueth, when the line $G E$ falleth aboue the line $G F$, as we haue before manifestly seene.

The first case.

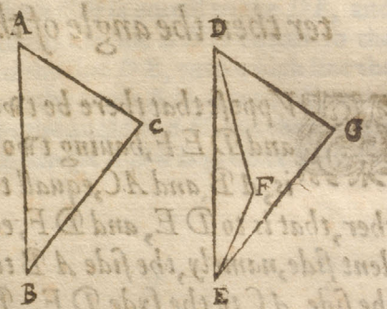
Second case.

But if it fall vpon it, as in this figure here put. Then forasmuch as the two lines AB and AC, are equal to the two lines $D E$ and $D G$, the one to the other, and they contayne equall angles by construction: therefore (by the 4. proposition) the base $B C$, is equall to the base $E G$: but the base $E G$, is greater then the base $E F$: wherfore also the base $B C$, is greater then the base $E F$: which was required to be proued.



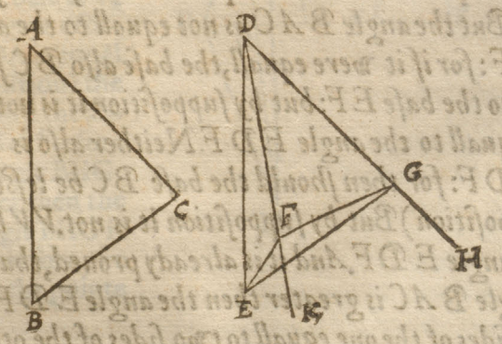
Third case.

But now let the line $E G$, fall vnder the line $E F$, as in the figure here put. And forasmuch as these two lines AB, and AC, are equall to these two lines $D E$ and $D G$, the one to the other, and they contayne equall angles, therefore (by the 4. proposition) the base $B C$, is equal to the base $E G$. And forasmuch as within the triangle $D E G$, the two linnes $D F$ and $F E$, are set vpon the side $D E$: therfore (by the 21. proposition) the lines $D F$ and $F E$ are lesse then the outward lines $D G$ and $G E$: but the line $D G$ is equal to the line $D F$. Wherfore the line $G E$ is greater then the line $F E$. But $G E$ is equall to $B C$. Wherefore the line $B C$ is greater the the line $E F$. Which was required to be proued.



Another demonstration of the third case.

This third case may also another way be demonstrated. Produce the lines $D F$ and $D G$, which are equal, vnto the points K and H: and draw a line from F to G: wherefore (by the second part of the fifth proposition) the angles $K F G$ and $F G H$, which are vnder the base $F G$, are equall: therefore the angle $E F G$ is greater then the angle $F G E$. Wherfore (by the 18 proposition) the side $E G$ is greater then the side $E F$: but the base $B C$ is equal vnto the base $E G$: Wherefore the base $B C$, is greater then the base $E F$: Which was required to be proued.



It may peraduenture seme, that Euclide should here in this propositon haue proued that not onely the bases of the triangles are vnequall, but also that the areas of the same are vnequall: for so in the fourth proposition, after he had pro-

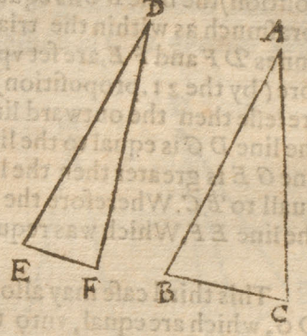
Why Euclide &c.

ued the base to be equall, he proued also the areas to be equall. But hereto may be answered, that in equall angles and bases, and vnequall angles and bases, the consideration is not like. For the angles and bases being equall, the triangles also shall of necessitie be equall, but the angles and bases being vnequall, the areas shall not of necessitie be equall. For the triangles may both be equall and vnequall: and that may be the greater, whiche hathe the greater angle, and the greater base, and it may also be the lesse. And for that cause Euclide made no mencion of the comparison of the triangles. VVhereof this also mought be a cause, for that to the demonstration thereof are required certayne Propositions concerning parallel lines, which we are not as yet come vnto. Howbeit after the 37. proposition of this booke you shal find the comparison of the areas of triangles, which haue their sides equall, and their bases and angles at the toppe vnequall.

after the 37. &c.

The 16. Theoreme. The 25. Proposition.

If two triangles haue two sides of the one equall to two sydes of the other, eche to his correspondent syde, and if the base of the one be greater then the base of the other: the angle also of the same cõtayned vnder the equall right lines, shall be greater then the angle of the other.



Demonstration leading to an absurdity.

SVppose that there be two triangles, A, B, C, and D E F, hauing two sides of th' one, that is, A B, and A C, equall to two sides of the other, that is to D E, and D F, eche to his correspondent side, namely, the side A B to the side D F, and the side A C to the syde D F. But let the base B C be greater then the base E F. Thẽ I say, that the angle B A C is greater then the angle E D F. For if not, then is it either equall vnto it, or lesse then it. But the angle B A C is not equall to the angle E D F: for if it were equall, the base also B C should (by the 4. proposition) be equal to the base E F: but by supposition it is not. VVherfore the angle B A C is not equall to the angle E D F. Neither also is the angle B A C lesse then the angle E D F: for then should the base B C be lesse thẽ the base E F (by the former proposition) But by supposition it is not. VVherfore ỹ angle B A C is not lesse thẽ ỹ angle E D F. And it is already proued, that it is not equall vnto it: wherfore ỹ angle B A C is greater then the angle E D F. If therfore two triangles haue two sides of the one equall to two sides of the other, eche to his correspondent side, & if the base of the one be greater then the base of the other, the angle also of the same contayned vnder ỹ equal right lines shall be greater thẽ the angle of the other: which was required to be proued.

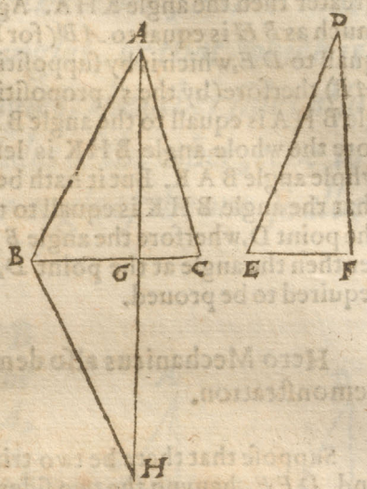
This

This propoſition is plaine oppoſite to the eight,& is the cõuerſe of the foure and twenty which went before,and it is proued(as commonly all conuerſes are) by a reaſon leading to an abſurditie.But it may after Menelaus Alexandrinus be demonſtrated directly,after this maner.

All conuerſes are commonly indirectly demonſtrated.

An other demõſtration after Menelaus Alexandrinus.

Suppoſe that there be two triangles *A B C* & *D E F*: haning the two ſides *A B* and *A C* equal to the two ſides *D E* and *D F*,the one to the other: and let the baſe *B C* be greater then the baſe *E F*.Then I ſay that the angle at the point *A*,is greater thẽ the angle at the point *D*.For from the baſe *B C* cut of (by the thirde) a line eqnall to the baſe *E F*, and let the ſame be *B G*.And vpon the line *G B* and to the point *B* put (by the 23,propoſition) an angle equal to the angle *D E F*: which let be *G B H*: and let the line *B H* be equall to the line *D E*.And drawe a lyne from *H* to *G*,and produce it beyond the point *G*: whiche being produced ſhall fal either vppon the angle *A*, or vpon the line *A B*,or vpon the line *A C*. Firſt let it fall vpon the angle *A*.And foraſmuch as theſe two lines *B G* and *B H* are equall to theſe two lines *E F* and *E D*,the one to the other,and they contayne equall angles(by conſtruction)namely,the angles *G B H* and *D E F*: therfore(by the 4.propoſition)the baſe *G H* is equall to the *D F*, and the angle *B H G* to the angle *E D* F.Agayne foraſmuch as the line *B H* is equall to the line *B A* (for the line *A B* is ſuppoſed to be equal to the line *D* E, vnto which line the line *B H* is put equal) therfore(by the 5.propoſition)the angle *B H A* is equall to the angle *B A H*: wherfore alſo the angle E D F is equal to the angle *B A H*.But the angle *B A C* is greater then the angle *B A H*: wherfore alſo the angle *B A C* is greater then the angle E D F.

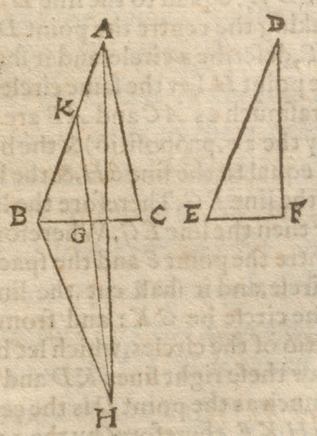


Diuers caſes in this demonſtration.

Firſt caſe.

Second caſe.

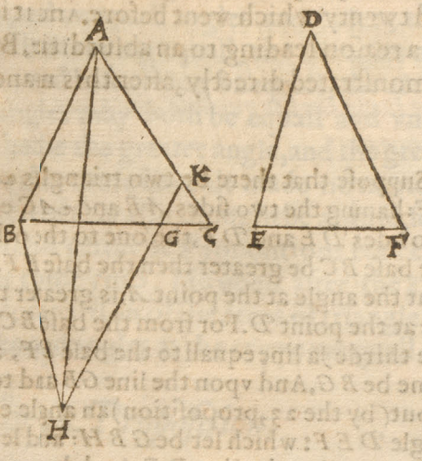
But now let it fall vpon the line *A B* in the point K, and drawe a line from *A* to *H*. And foraſmuch as theſe two lines *B G* and B H are equall to theſe two lines E F and E D,the one to the other,& they containe equal angles(by conſtruction)namely,the angles G B H and D E F: therfore (by the 4. propoſition)the baſe G H is equall to the baſe D F, and the angle B H G to the angle EDF.Agayne foraſmuch as in the triangle B A H,the ſide BA is equal to the ſide B H,therfore (by the 5.propoſition) the angle B A H is equal to the angle B H A.But the angle B H A is greater then the angle B H G: wherfore alſo the angle BAH is greater then the angle BHG. Wherfore the angle B A C is much greater then the angle B H G.But it is proued that the angle BHG is equall to the angle at the point D. Wherefore the angle B A C is greater then the angle at the pointe D: Which was required to be proued.



Third caſe.

But now ſuppoſe that the line H G beyng produced doo fall vppon the line *A C*, namely,in the point K.And agayne draw alſo a line from *A* to *H*.And foraſmuch as *B G* is equall to *E F*,and *B H*,to *E D*,therefore theſe two lines *B G* and *B H* are equall to theſe two lines *E F* and *E D*,the one to the other,and (by conſtruction)they contayne equall angles,namely,the angles GBH and *FED*.Wherfore (by the fourth propoſitiõ)

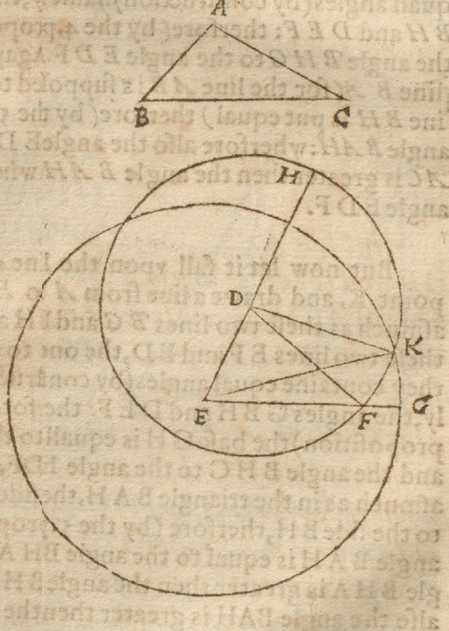
the base G H is equal to the base D F: & th'angle B H G is equall to th'angle E D F. And forasmuch as G H is equall to D F, and D F is equall to A C: therfore G H also is equall to A C. Wherfore H K is greater then A C, wherefore H K is much greater then A K. Wherfore (by the 18. proposition) the angle K A H is greater then the angle K H A. Agayne forasmuch as B H is equal to A B (for B H is put equall to D E, which is by supposition equal to A B) therfore (by the 5. proposition) the angle B H A is equall to the angle B A H. Wherfore the whole angle B H K is lesse then the whole angle B A K. But it hath bene proued, that the angle B H K is equall to the angle at the point D, wherfore the angle B A C is greater then the angle at the point D, which was required to be proued.



Hero Mechanicus also demonstrateth it an other way, and that by a direct demonstration.

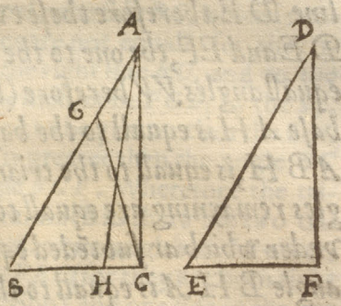
An other demonstration after Hero Mechanicus.

Suppose that there be two triangles A B C and D E F, hauyng the two sides A B, and A C, equall to the two sides D E, & D F, the one to the other, and let the base B C, be greater then the base E F. Then I say, that the angle at the point A, is greater then the angle at the point D. For forasmuch as B C, is greater thē E F, produce E F to the point G, and put the line E G, equall to the line B C. Likewise produce the line D E to the point H, and put the line D H, equall to the line D F. Wherefore making the centre the point D, and the space D F, describe a circle, and it shall passe also by the point H. Let the same circle be F K H. And forasmuch as A C and A B are greater thē B C (by the 20. propositiō) & the lines A B & A C, ar equal to the line E H, & the line B C is equal to the line E G. Therefore the line E H is greater then the line E G. Wherefore making the centre the point E and the space E G describe a circle, and it shall cut the line E H. Let the same circle be G K: and from the common sectiō of the circles, which let be the point K, draw these right lines K D and K E. And forasmuch as the point D is the centre of the circle H K F, therefore (by the 15. definition) the line D K, is equall to the line D H, that is vnto the line A C. Agayne forasmuch as E is the centre of the circle G K, therefore the line E K is equal to the line E G, that is, to the line B C. And forasmuch as these two lines A B and A C, are equall to these two lines D E and D K, and the base B C is equal to the base E K (for E K is equall to E G (by the 15. definition) & E G is put to be equal to B C). Wherefore (by the 4. proposition) the angle B A C is equal to the angle E D K. But the angle E D K is greater then the angle E D F: wherefore also the angle B A C, is greater then the angle E D F: which was required to be proued.



The 17. Theoreme. The 26. Proposition.

If two triangles haue two angles of the one equall to two angles of the other, ech to his correspondent angle, and haue also one side of the one equal to one side of the other, either that side which lieth betwen the equall angles, or that which is subtended vnder one of the equall angles: the other sides also of the one, shalbe equall to the other sides of the other, eche to his correspondent side, and the other angle of the one shalbe equall to the other angle of the other.

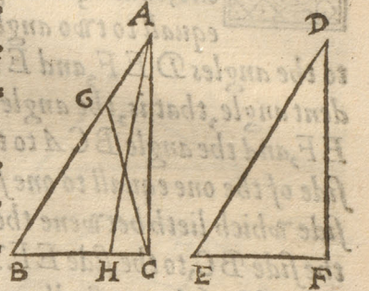


Suppose that there be two triangles A B C, and D E F, hauing two angles of the one, that is, the angles A B C, and B C A, equall to two angles of the other, that is, to the angles D E F, and E F D, ech to his corresp̄ dent angle, that is, the angle A B C, to the angle D E F, and the angle B C A to the angle E F D, and one side of the one equall to one side of y other, first that side which lieth betwene the equall angles, that is, the side B C, to the side E F. The I say that the other sides also of the one shalbe equall to the other sides of the other, ech to his correspondent side, that is, the side A B, to the side D E, and the side A C, to the side D F, and the other angle of the one, to the other angle of the other, that is, the angle B A C to the angle E D F. For if the side A B be not equall to the side D E, the one of them is greater. Let the syde A B be greater: and (by the 3. proposition) vnto the line D E, put an equall line G B, and draw a right line from the point G, to the point C. Now forasmuch as the line G B, is equall to the line D E, and the line B C to the line E F, therefore these two lines G B and B C, are equall to these two lines D E and E F, the one to the other, and the angle G B C is (by supposition) equall to the angle D E F. Wherefore (by the 4. proposytion) the base G C is equall to the base D F, and the triangle G C B is equall to the triangle D E F, and the angles remayning are equall to the angles remayning vnder which are subtended equall sydes. Wherefore the angle G C B is equall to the angle D F E. But the angle D F E is supposed to be equall to the angle B C A. Wherefore (by the first common sentence) the angle B C G is equal to the angle B C A, the lesse angle to the greater: which is impossible. Wherefore the line A B is not vnequall to the line D E. Wherefore it is equall. And the the line B C is equall to the line E F: now therefore there are two sydes A B and

Demonstration leading to an absurditie.

BC equall to two sydes DE and EF, the one to the other, and the angle ABC, is equall to the angle DEF. VVherefore (by the 4. proposition) the base AC is equall to the base DF, and the angle remayning BAC is equall to the angle remayning EDF.

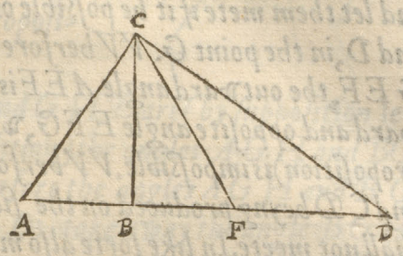
Agayne suppose that the sydes subtending the equall angles be equall the one to the other, let the syde I say AB be equall to the syde DE. Then agayne I say, that the other sydes of the one are equall to the other sydes of the other, ech to his correspondent syde, that is the syde AC to the syde DF, and the syde BC to the syde EF: and moreouer the angle remayning, namely, BAC, is equall to the angle remayning, that is, to the angle EDF. For if the syde BC be not equall to the syde EF, the one of them is greater: let the syde BC, if it be possible, be greater. And (by the third proposytion) vnto the line EF, put an equall line BH, and drawe a right line from the point A to the point H. And forasmuch as the line BH is equall to the line EF, and the line AB to the line DE, therefore these two sydes AB and BH, are equall to these two sydes DE and EF, the one to the other, and they containe equall angles. VVherefore (by the 4. proposition) the base AH is equall to the base DF, and the triangle ABH, is equall to the triangle DEF, and the angles remayning are equall to the angles remayning, vnder which ar subtēded equal sydes. VVherfore the angle BHA is equall to the angle EFD. But the angle EFD is equall to the angle BCA. VVherefore the angle BHA is equal to the angle BCA. VVherefore the outward angle of ỹ triangle AHC, namely, the angle BHA, is equall to the inward and opposite angle, namely, to the angle HCA, which (by the 16 proposition) is impossible. VVherfore the syde EF is not vnequall to the syde BC, wherefore it is equall. And the syde AB is equall to ỹ syde DE: wherefore these two sydes AB and BC, are equall to these two sydes DE and EF, the one to the other, and they contayne equall angles: VVherfore (by the 4. proposition) the base AC is equall to the base DF: and the triangle ABC, is equall to the triangle DEF, and the angle remayning, namely, the angle BAC is equall to the angle remayning, that is, to the angle EDF. If therefore two triangles haue two angles of the one equall to two angles of the other, ech to his correspondent angle, and haue also one syde of the one equall to one syde of the other, either that syde which lieth betwene the equall angles, or that which is subtended vnder one of the equall angles: the other sydes also of the one shalbe equall to the other sydes of the other, eche to his correspondent syde, and the other angle of the one shalbe equall to the other angle of the other: which was required to be proued.



VVhereas in this proposition it is sayde, that triangles are equall, which hauing two angles of the one equall to two angles of the other, the one to the other,

ther, haue alſo one ſide of the one equall to one ſide of the other, either that ſide which lieth betwene the equall angles, or that ſide which ſubtendeth one of the equall angles: this is to be noted that without that caution touching the equal ſide, the propoſition ſhall not alwayes be true. As for example.

Suppoſe that there be a rectangle triangle *A B C*, whoſe right angle let be at the point *B*, & let the ſide *B C* be greater thē the ſide *B A*: and produce the line *A B*, frō the point *B* to the point *D*. And vpō the right line *B C* & to the point in it *C*, make vnto the angle *B A C* an equal angle (by the 23. propoſition), which let be *B C D*, & let the lines *B D* & *C D*, being produced cōcurre in the point *D*. Now thē there are two triangles *A B C*, and *B C D*, which haue two angles of the one equall to two angles of the other, the one to the other, namely, the angle *A B C* to the angle *D B C* (for they are both right angles), & the angle *B A C*, to the angle *B C D* (by conſtruction) and haue alſo one ſide of the one equall to one ſide of the other, namely, the ſide *B C*, which is cōmon to them both. And yet notwithſtanding the triangles are not equall: for the triangle *B D C*, is greater then the triangle *A B C*. For vpon the right line *B C*, and to the point in it *C*, deſcribe an angle equall to the angle *A C B*: which let be *F C B* (by the 23. propoſition). And foraſmuch as the ſide *B C* was ſuppoſed to be greater then the ſide *A B*, therefore (by the 18. propoſition) the angle *B A C* is greater then the angle *B C A*, wherefore alſo the angle *B C D* is greater then the angle *B C F*. Wherefore the triangle *B C D* is greater then the triangle *B C F*. Agayne foraſmuch as there are two triangles *A B C* and *B C F*, hauing two angles of the one equal to two angles of the other, the one to the other, namely, the angle *A B C* to the angle *F B C* (for they are both right angles) and the angle *A C B* to the angle *F C B* (by conſtruction), and one ſide of the one is equall to one ſide of the other, namely, that ſide which lieth betwene the equall angles, that is, the ſide *B C* which is common to both triangles. Wherefore (by this propoſition) the triangles *A B C* and *F B C* are equal. But the triangle *D B C* is greater thē the triangle *F B C*. Wherefore alſo the triangle *D B C* is greater then the triangle *A B C*. Wherefore the triangles *A B C* and *D B C*, are not equall: notwithſtanding they haue two angles of the one equall to two angles of the other, the one to the other, and one ſide of the one equall to one ſide of the other.



The reaſon wherof is, for that the equal ſide in one triangle, ſubtēdeth one of the equall angles, and in the other lieth betwene the equal angles. So that you ſee that it is of neceſſitie that the equall ſide do in both triangles, either ſubtend one of the equall angles, or lie betwene the equall angles.

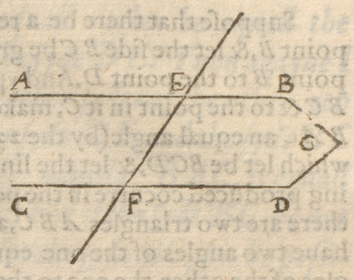
Of this propoſition was Thales Mileſius the inuentor, as witneſſeth Eudemus in his booke of Geometricall enarrations.

Thales Mileſius the inuentor of this propoſition.

The 18. Theoreme. The 27. Propoſition.

If a right line falling vpon two right lines, do make the alternate angles equall the one to the other: thoſe two right lines are parallels the one to the other.

Suppose that the right line E F falling vppon these two right lines A B and C D, do make the alternate angles, namely, the angles A E F & E F D equall the one to the other. Then I say that A B is a parallel line to C D. For if not, then these lines produced shall mete together, either on the syde of B and D, or on the syde of A & C. Let them be produced therfore, and let them mete if it be possible on the syde of B and D, in the point G. Wherfore in the triangle G E F, the outward angle A E F is equal to the inward and opposite angle E F G, which (by the 16. proposition) is impossible. Wherfore the lines A B and C D beyng produced on the side of B and D, shall not meete. In like sorte also may it be proued that they shall not mete on the syde of A and C. But lines whiche being produced on no syde meete together, are parrallell lines (by the last definition:) wherfore A B is a parrallel line to C D. If therfore a right line falling vpon two right lines, do make the alternate angles equall the one to the other: those two right lines are parrallels the one to the other: which was required to be demonstrated.



Demonstration leading to an absurditie.

This worde alternate is of Euclide in diuers places diuersly taken: somtimes for a kind of situation in place, and somtime for an order in proportion, in which signification he vseth it in the v. booke, and in his bokes of numbers. And in the first signification he vseth it here in this place, and generally in all hys other bokes, hauing to do with lines & figures. And those two angles he calleth alternate, which beyng both contayned within two parallel or equidistant lynes are neither angles in order, nor are on the one and selfe same side, but are seperated the one from the other by the line which falleth on the two lines: the one angle beyng aboue, and the other beneath.

This worde alternate vsed in diuers senses.

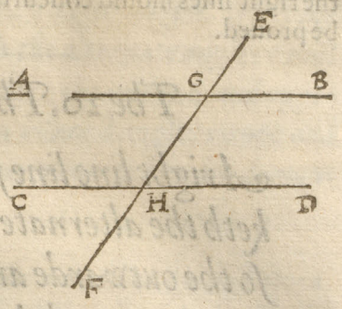
How it is taken in this place. Which angles are called alternate.

The 19. Theoreme. The 28. Proposition.

If a right line falling vpon two right lines, make the outward angle equall to the inward and opposite angle on one and the same syde, or the inwarde angles on one and the same syde, equall to two right angles: those two right lines shall be parallels the one to the other.

Suppose that the right line E F, fallyng vppon these two right lines A B and C D, do make the outward angle E G B equall to the inward and opposite angle G H D, or do make the inward angles on one and the

the ſame ſide, that is, the angles B G H and G H D equall to two right angles. Then I ſay that the lyne A B is a parallel line to the lyne C D For foraſmuch as the angle E G B is (by ſuppoſition) equall to the angle G H D, and the angle E G B is (by the 15. propoſition) equall to the angle A G H: therfore the angle A G H is equall to the angle G H D: and they are alternate angles. VVherfore (by the 27. propoſition) A B is a parallel line to C D.



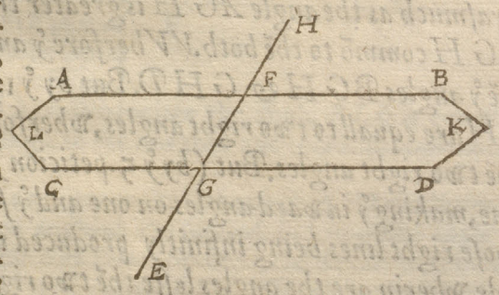
Demonſtration

Agayne foraſmuch as the angles B G H and G H D are (by ſuppoſition) equall to two right angles, & (by the 13. propoſition) the angles A G H and B G H, are alſo equall to two right angles, wherefore the angles A G H and B G H, are equall to the angles B G H and G H D: take away the angle B G H which is common to them both VVherfore the angle remainyng, namely, A G H is equall to the angle remayning, namely, to G H D. And they are alternate angles. VVherfore (by the former propoſition) A B is a parallell line to C D. If therfore a right line fallyng vpon two right lines, do make the outward angle equall to the inward and oppoſite angle on one and the ſame ſide, or the inwarde angles on one and the ſame ſide, equall to two right angles, thoſe two right lines ſhall be parallels the one to the other: which was required to be proued.

Ptolomeus demonſtrateth the ſecond part of this propoſition, namely, that the two inward angles on one and the ſame ſide being equall, the right lines are parellels, after this manner.

An other demõſtration of the ſecond part of this propoſition after Ptolomeus

Suppoſe that there be two right lines *A B* and *C D*, and let a certayne right line *E F G H* cutte them in ſuche ſorte, that it make the angles *B F G* and *F G D* equall to two right angles. Then I ſay, that theſe right lines *A B* and *C D* are parallel lines, that is, they ſhall not concurre. For if it be poſſible, let the lines *B F* and *G D* being produced concurre in the pointe K. Nowe foraſmuche as the right line *E F* ſtandeth vppon the right line *A B*, therfore (by the 13. propoſition) it maketh the angles *A F E*, and *B F E* equall to two right angles: likewiſe foraſmuch as the line *F G* ſtandeth vpõ the line *C D*, therfore (by the ſame propoſition) it maketh the angles *C G F* and *D G F* equall to two right angles. Wherfore the foure angles *B F E*, *A F E*, *C G F*, and *D G F* are equal to foure right angles: of which the two angles *B F G* and *F G D* are (by ſuppoſition) equall to two right angles, wherfore the angles remaining, namely, *A F G* and *C G F* are alſo equall to two right angles. If therfore the right lines F B and *G D* being produced (the inward angles being equall to two right angles) do concurre, then ſhall the lynes *F A* and *G C* being produced concurre. For the angles *A F G* and *C G F* are equall to two right angles. For either the right lines ſhall concurre on either ſide, or els on neither ſide. For that on either ſide the angles are equall to two right angles. Wherefore let the right lines F *A* and *G C* concurre in the point L. Wherefore the two right lines L A F K and L C G K do comprehend a ſpace, which (by the 6. petition) is impoſſi-

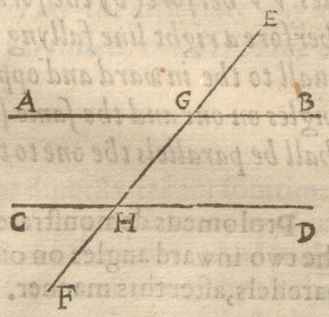


ble. Wherfore it is not possible that the inward angles being equal to two right angles, the right lines should concurre. Wherefore they are parallels: which was required to be proued.

The 20. Theoreme. The 29. Proposition.

A right line line falling vppon two parallel right lines: maketh the alternate angles equall the one to the other: and also the outwarde angle equall to the inwarde and opposite angle on one and the same side: and moreouer the inwarde angles on one and the same side equall to two right angles.

Vppose that vpon these parallel lines A B and C D do fal the right line E F. Then I say that the alternate angles which it maketh, namely, the angles A G H and G H D, are equall the one to the other: and ỹ the outward angle E G B is equal to the inwarde and opposite angle on the same side, namely, to ỹ angle G H D: and ỹ the inward angles on one and the selfe same side, that is, the angles B G H and G H D, are equall to two right angles. For if the angle A G H be not equal to the angle G H D, the one of them is greater. Let the angle A G H be greater. And forasmuch as the angle A G H is greater then the angle G H D, put the angle B G H commõ to thẽ both. Wherfore ỹ angles A G H and B G H, are greater thẽ ỹ angles B G H & G H D. But by ỹ 13. propopositiõ ỹ angles A G H & B G H are equall to two right angles, wherfore ỹ angles B G H & G H D are lesse thẽ two right angles. But (by ỹ 5. petition) if vpõ two right lines do fall a right line, making ỹ inward angles on one and ỹ same side, lesse thẽ two right angles, those right lines being infinitly produced must needes at ỹ length meete on the side wherin are the angles lesse thẽ two right angles. Wherfore the right lines A B and C D being infinitely produced will at ỹ length meete. But they cannot meete, becaus they are parallels (by supposition): wherfore the angle A G H is not vnequall to the angle G H D: wherfore it is equall.



Demonstration leading to an absurdity. Ffrst part.

And the angle A G H is (by the 15. proposition) equall to the angle E G B. Wherfore (by the first common sentence) the angle E G B is equall to the angle G H D.

Second part.

Put the angle B G H common to them both: wherfore the angles E G B and B G H, are equall to the angles B G H and G H D. But the angles E G B

Third part.

and

and B G H are (by the 13. propoſition) equall to two right angles. Wherefore the angles B G H and G H D are alſo equall to two right angles. If a lyne therfore do fall vpō two parallel right lines: it maketh the alternate angles equal the one to the other: and alſo the outward angle equall to the inward and oppoſite angle on one and the ſame ſide: and moreouer the inward angles on one and the ſame ſyde equall to two right angles: whiche was required to be demonſtrated.

This propoſition is the conuerſe of the two propoſitions next going before. For, that which in either of them is the thing ſought, or cōcluſion, is in this the thing geuen, or ſuppoſition. And contrariwiſe the thinges which in them were geuen or ſuppoſitions, are in this proued, and are concluſions.

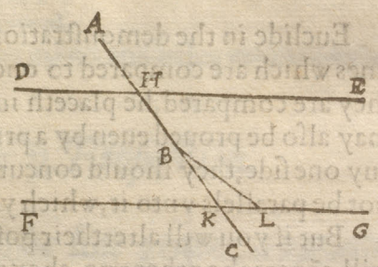
This propoſition is the conuerſe of the two former propoſitiōs.

Pelitarius after this propoſition addeth this witty concluſion.

If two right lines which cut two parallel lines, do betwene the ſayde parallel lines concurre in a point, and make the alternate angles equall, or the outward angle equall to the inward and oppoſite angle on the ſame ſide, or finally the two inward angles on one and the ſelfe ſame ſide equall to two right angles: thoſe two right lines are drawen directly and make one right line.

An addition of Pelitarius.

Suppoſe that there be two right lines *A B* and *C B*, which let cut two parallel lines *D E* and F G: and let *A B* cut the line *D E* in the point *H*: and let *C B* cut the line F G in the point *K*: & let the lines *A B* & *C B*, concurre betwene the two parallel lines *D E* & F G in the point *B*: and let the angle *D H B* be equal to the angle B K G: or let the angle *A H D* be equall to the angle B K F: or finally let the angles *B H D* and B K F be equal to two right angles. Thē I ſay that the two lines *A B* and *B C* are drawen directly, and do make one right line. For if they be not, then produce *A B* vntil it cut F G in the point L, and let A L be one right line, and ſo ſhal be made the triangle *B L K*. Now then (by the firſt part of this 29. propoſition) the angle *D H B* ſhalbe equal to the alternate angle *G L B*: but (by ſuppoſition) the angle *D H B* is equall to the angle *B K G*. Wherefore the angle *B L G* is equall to the angle *B K L*, namely, the outward angle to the inwarde and oppoſite angle: which (by the 16. propoſition) is impoſſible.



Demonſtration leading to an abſurditie. Firſt part.

Moreouer (by the ſecōd part of this 29. propoſitiō) the angle A H D ſhalbe equal to the angle *B L K*, namely, the outward angle to the inward and oppoſite angle on one and the ſame ſide. But the ſame angle *A H D* is ſuppoſed to be equall to the angle B K F: wherefore the angle B K F is equall to the angle *B L K*. Which (by the ſelfe ſame 16. propoſition) is impoſſible.

Second part.

Laſtly foraſmuch as the angles *B H D* and B K F are ſuppoſed to be equall to two right angles, & the angles *B H D* & B L K are alſo by the laſt part of this 29 propoſition equal to two right angles, therefore the angle B K F ſhalbe equal to the angle B L K: which agayne by the ſelfe ſame 16. propoſition is impoſſible.

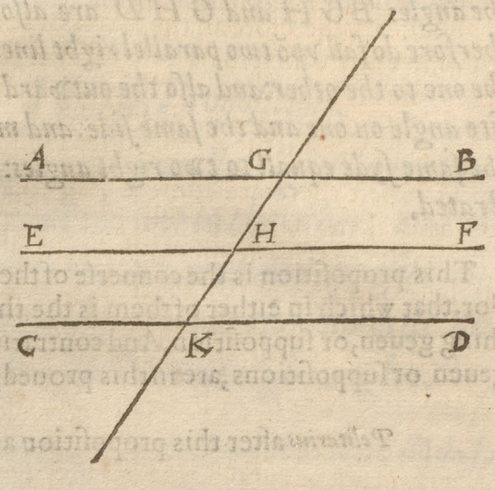
Third part.

The 21. Theoreme The 30. Propoſition.

Right lines which are parallels to one and the ſelfe ſame right line: are alſo parrallel lines the one to the other.

Demonstration.

Vppose that these right lines AB and CD, be parallel lines to the right line EF. Then I say, that the line AB is a parallel line to CD. Let there fall vpon these thre lines a right line GHK. And forasmuch as the right line GHK falleth vppon these parallell right lines AB and EF, therfore (by the proposition going before) the angle AGH is equall to the angle GHF. Agayne forasmuch as the right line GK falleth vppon these parallell right lines EF and CD, therefore (by the same) the angle GHF is equall to the angle GKD. Now then it proued that the angle AGH is equall to ỹ angle GHF, and ỹ the angle GKD is equall to the angle GHF. Wherfore the angle AGK is equall to the angle GKD. And they are alternate angles: wherfore AB is a parallel line to CD. Right lines therfore which are parallels to one and the selfe same right line, are also parallel lines the one to the other: which was required to be proued.

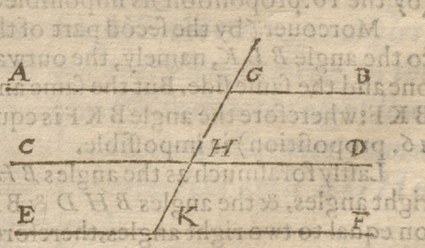


Euclide in the demonstration of this proposition, setteth the two parallel lines which are compared to one, in the extremes, and the parrallel to whome they are compared, he placeth in the middle, for the easier demonstration. It may also be proued euen by a principle onely. For if they shoulde concurre on any one side, they should concurre also with the middle line, and so should they not be parallels vnto it, which yet they are supposed to be.

An other case in this Probleme.

But if you will alter their position and placing and set that line to which you will cõpare the other two, aboue, or beneath: you may vse the same demonstration which you had before. As for example.

Suppose that the lines *AB* and *CD* be parallels to the line *EF*: and let both the lines *AB* and *CD*, be aboue, and let the line *EF* be beneath, and not in the middest. Vpon which let the right line *GHK* fall. And forasmuch as either of the angles *KHD* and *HGB* is equall to the angle *HKE*, (for they are alternate angles) therfore they are (by the first common sentence) equall the one to the other. Wherefore (by the 28 proposition) the right lines *AB* and *EF*, are parallels.



An obiection.

Answer.

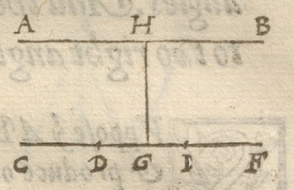
But here if a man will obiect that the lines EK and KF, are parallels vnto the line CD, and therefore are parallels the one to the other. VVe will answere that the lines EK and KF are partes of one parallel line, and are not two parallel lines.

lines. For parallel lines are vnderstanded to be produced infinitly But EK being produced, falleth vpon K F. Wherefore it is one and the selfe same with it, and not an other. Wherefore all the partes of a parrallel line are parallels, both to the right line vnto which the whole parallel line is a parallel, and also to all the parts of the same right line. As the line E K is a parallel vnto H D, and the line K E to the line C H. For if they be produced infinitly, they will neuer concurre.

Parallel lines are vnderstanded to be produced infinitely.

Howbeit there are some which like not, that two distinct parellel lines, should be taken and counted for one parallel line: for that the continuall quantity, namely, the line is cut asonder, and cesseth to be one. Wherefore they say, that there ought to be two distinct parallel lines compared to one. And therfore they adde to the proposition a correction, in this maner. *Two lines being parallels to one line: are either parallels the one the other, or els the one is set directly against the other, so that if they be produced they should make one right line. As for example.*

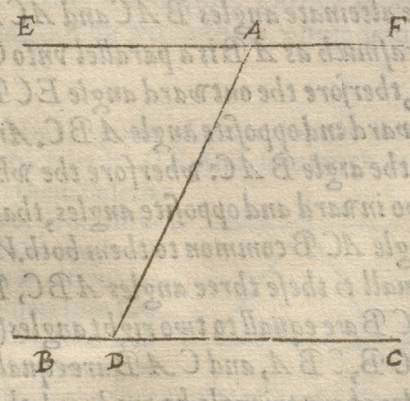
Suppose that the lines *C D* and *E F* be parallels to one and the selfe same line *A B*, and let them not be parallels the one to the other. Then I say, that the two lines *C D* & *E F*, are directly set the one to the other. For for as much as they are not parallel lines, let them concurre in the point *G*, And from the point *G* draw a line cutting the line *AB* in the point *H*. Now by the former proposition the angles *A H G* & *H G C* are equall to two right angles, but by the same propositiō, the angle *A H G*, is equall to the alternate angle *H G F*. Wherefore the angles *H G C* and *H G F* are equal to two right angles. Wherefore (by the 14 proposition) the lines *C G* and *F G* are drawen directly and make one right line. Wherfore also the lines *C D* and *E F* are set directly the one to the other: and being produced they will make one right line.



The 10. Probleme. The 31. Proposition.

By a point geuen, to draw vnto a right line geuen, a parallel line.

Vppose that the point geuen be A, and let the right line geuen be B C. It is required by the point geuen, namely A, to draw vnto the right line B C, a parallel line. Take in the line B C a point at all aduentures, and let the same be D. and (by the first petitiō) draw a right line from the point A, to the point D. And (by the 23. proposition) vpon the right line geuen A D, and to the point in it geuen A, make an angle D A E, equall to the angle geuen A D C. And (by the 14. proposition) put vnto the line A E the line A F directly, in such sorte that they



Construction.

both make one right line. And forasmuch as the right line AD falling vpon the right lines BC and EF, doth make the alternate angles, namely, EAD, and ADC equall ỹ one to the other, therfore (by the 27. proposition) EF is a parallel line to BC. VVherfore by the point geuen, namely A, is drawne to the right line geuen BC a parallel line EAF: which was required to be done.

This proposition is to be vnderstanded of a point geuen without the line geuen, and in such sorte also, that the same line geuen being produced, doo not fall vppon the pointe geuen.

The 22. Theoreme. The 32. Proposition.

If one of the sydes of any triangle be produced: the outwarde angle that it maketh, is equal to the two inward and opposite angles. And the three inwarde angles of a triangle are equall to two right angles.

Vppose ỹ ABC be a triangle, & produce one of ỹ sides therof namely, CB to the pointe D. Then I say, that the outwarde angle ACD is equall to the two inwarde and opposite angles CAB & ABC: and ỹ the three inwarde angles of the triangle, that is, the angles ABC, BCA, and CAB are equall to two right angles. For (by the proposition going before) rayse vp frō the point C, a parallel to the right line AB, and let the same be CE. And forasmuch as AB is a parallel to CE, and vpon them falleth the right line AC: therefore the alternate angles BAC and ACE are equall the one to the other. Agayne forasmuch as AB is a parallel vnto CE, and vpon them falleth the right line BD, therfore the outward angle ECD is (by the 29. proposition) equall to the inward and opposite angle ABC. And it is proued that the angle ACE is equal to the angle BAC: wherfore the whole outwarde angle ACD is equall to the two inward and opposite angles, that is, to the angles BAC and ABC. Put the angle ACB common to them both. VVherfore the angles ACD and ACB, are equall to these three angles ABC, BCA, and BAC. But the angles ACD & ACB are equall to two right angles (by the 13. proposition): wherfore the angles ACB, CBA, and CAB are equall to two right angles. If therfore one of the sides of any triangle be produced, the outward angle that it maketh, is equall to the two inward and opposite angles. And the three inward angles of a triangle

Construction.

Demonstration.

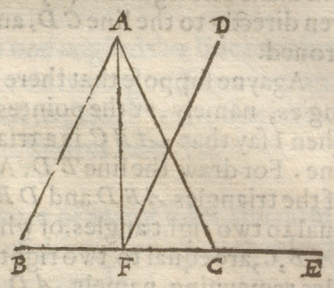
are

are equall to two right angles: which was required to be demonſtrated.

Euclide demonſtrateth either part of this compoſed Theoreme, by drawyng frō one angle of the triangle a parrallel line to one of the ſides of the ſame triangle, without the triangle. Either part therof may alſo be proued without drawyng of a parallel line without the triangle, only chaunging the order of the thinges required or concluſions. For Euclide firſt proueth that the outwarde angle of a triangle (one of his ſides beyng produced) is equall to the two inwarde and oppoſite angles: and by that he proueth the ſecond part: namely, that the 3. inward angles of a triangle are equall to two right angles. But here it is contrariwiſe. For firſt is proued that the three inward angles of a triangle are equall to two right angles, and by that is proued the other part of the Theoreme, namely, that one ſide of a triāgle beyng produced, the outward angle is equal to the two inward and oppoſite angles. And that after this maner.

An other demonſtration.

Suppoſe that there be a triangle *ABC*, and produce the ſide *BC* to the point *E*. And take in the line *BC* a point at al auentures which let be F: & draw a line from *A* to F. And by the point F drawe vnto the line *AB* a parallel line (by the former propoſition) which let be F D. Now foraſmuch as F D is a parallell vnto *AB*, and vpon them falleth the right line A F, and alſo the right line *BC*, therfore the alternate angles are equall, and alſo the outward angle is equall to the inward angle. Wherefore the whole angle *AFC* is equall to the angles F A B and ABF. And by the ſame reaſon (if by the point F we draw a parallel line to the line *AC*) may we proue that the angle A F B is equall to the angles F A C, and *ACF*. Wherfore the two angles *AFB* & *AFC* are equall to the three angles of the triangle *ABC*. But the two angles *AFB* & *AFC* are (by the 13. propoſition) equall to two right angles. Wherfore alſo the three angles of the triangle *ABC* are equall to two right angles.

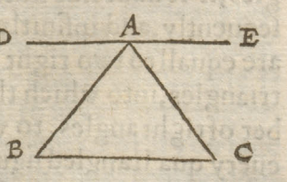


But the angles *ACF* and *ACE* are alſo (by the 13. propoſition) equall to two right angles. Take away the angle *ACF* which is common, wherfore the angle remaining, namely, the outward angle *ACE* is equall to the two angles remaining, namely, to the two inwarde and oppoſite angles *ABC* and *CAB*: which was required to be proued.

The latter part of this Theoreme firſt found out by Pithagoras. The demonſtration thereof after him.

Eudemus affirmeth that the latter part of this Theoreme, *The three angles of a triangle are equall to two right angles*, was firſt found out by Pithagoras, whoſe demonſtration thereof is thus.

Suppoſe that there be a triangle *ABC*: and by the point *A*, draw (by the former propoſition) vnto the line *BC*, a parallel line, which let be *DE*. And foraſmuch as the right lines *BC* and *DE* are parallels, and vpon them falleth the right lines *AB* and *AC*, therefore (by the 29. propoſition) the alternate angles are equall. Wherefore the angle *DAB* is equall to the angle *ABC*, and the angle *EAC* to the angle *ACB*. Adde the angle *BAC* common. Wherefore the angles *DAB*, *BAC*, *CAE*, that is, the angles *DAB* and *BAE*, namely, two angles equal to two right angles, are equal to the thre angles of the triangle *ABC*. Wherfore the thre angles of a triangle are equall to two right angles: which was required to be proued.



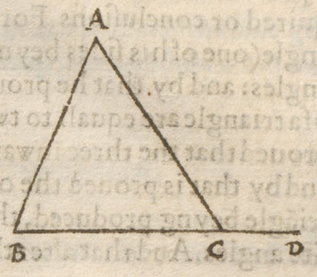
The conuerſe of this propoſition is thus,

The conuerse of this proposition.

If the outward angle of a triangle be equall to the two inward angles opposite against it: one of the sides of the triangle is produced, and the line without the triangle, is drawen directly to the side of the triangle, & maketh one right line with it. And if the thre inward angles of a rectiline figure be equal to two right angles, the same rectiline figure is a triangle.

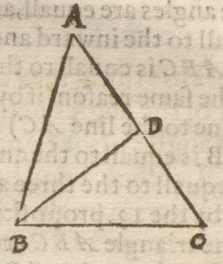
Demonstration of the first part of the conuerse.

Suppose that there be a triangle *ABC*: and let the outward angle *ACD* be equal to the two inward & opposite angles *ABC* and *CAB*. Then I say that the side *BC* is produced to the poynt *D*. And that *BCD* is one right line. For forasmuch as the angle *ACD* is equal to the two inward & opposite angles, adde the angle *ACB* common. Wherefore the angles *ACD* and *ACB* are equal to the three angles of the triangle *ABC*. But the three angles of the triangle *ABC* are equall to two right angles. Wherefore also the two angles *ACD* and *ACB* are equall to two right angles. But if vnto a right line, and to a point in the same line be drawen two right lines, no: both on one and the same side, making the side angles equal to two right angles: those two right lines shal be drawē directly, and make one right line (by the 14. proposition.) Wherefore the right line *BC* is drawen directly to the line *CD*, and so is *BCD* one right line: which was required to be proued.



Demonstration of the second part of the conuerse.

Agayne suppose that there be a certayne rectiline figure *ABC*, hauing onely three angles, namely, at the pointes *A*, *B*, *C*: which angles let be equal to two right angles Then I say that *ABC* is a triangle. First *AC* is one right line. For draw the line *BD*. And forasmuch as in either of the triangles *ABD* and *DBC*, the three angles are equal to two right angles, of which the angles at the points *A*, *B*, *C*, are equal to two right angles. Wherefore the angles remayning, namely, *ADB* and *CDB* are equall to two right angles. Wherefore (by the 14. proposition) the line *DC* is set directly to the line *DA*. Wherefore the side *AC* is one right line. And in like sort may we proue that the side *AB* is one right line, and also that the side *BC* is one right line. Wherefore the figure *ABC* is a triangle: which was required to be proued.



A Corrollary.

By the secōd part of this 29. propositiō, namely, *three angles of a triangle are equall to two right angles*, may easely be knowen, to how many right angles, the angles within any figure hauing right lines and many angles are equall. As are figures of fower angles, of fiue angles, of fixe angles, and so consequently: and infinitly. And this is to be noted, that euery right lined figure is resolued into triangle. For that a triangle is the first of all figures. For two lines accomplish no figures VVherfore how many sides the figure hath, into so many triangles may it be resolued, sauing two. As if the figure haue fower sides, it is resolued into two triangles, if it haue fiue sides, into 3. triangles: if 6 sides into 4. triangles, and so consequently, and infinitly. And it is proued that the three angles of euery triangle are equall to two right angles. VVherefore if you multiply the number of the triangles, into which the figure is resolued, by two, you shall haue the number of right angles, to which the angles of the figure are equall. So the angles of euery quadrangled figure are equall to 4. right angles. For it is composed of two triangles. And the angles of a fiue angled figure are equal to 6. right angles, for it is composed of three triangles, and so forth in like order.

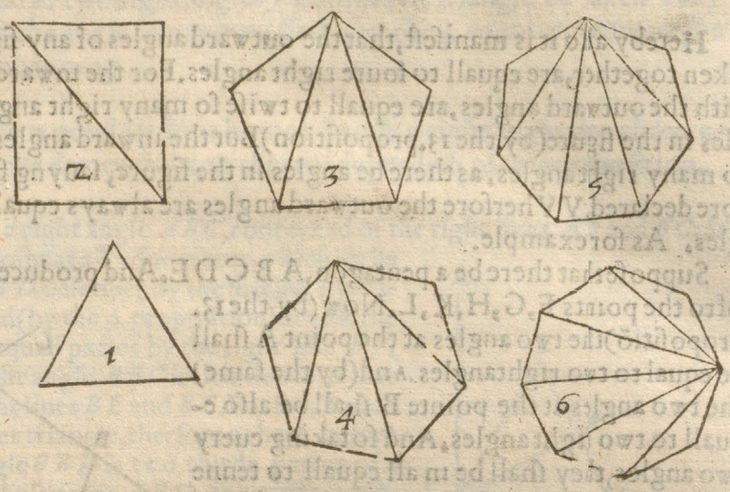
Euery right lined figure is resolued in triangles.

A triangle is the first of all figures.

Into how many triangles a figure may be resolued.

The redi'est and apte'st maner to reduce any rectiline figure into triangles, is thus

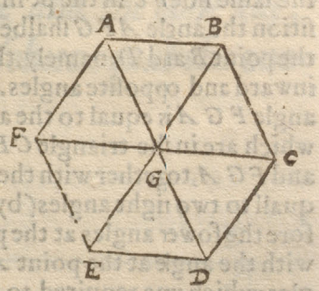
thus. From any one angle of the figure to euery other angle (of the same) beyng oppoſite vnto it, drawe a right line, ſo ſhall you haue all the triangles of that figure deſcribed. In a quadrãgle, from one angle you can drawe but one lyne to the oppoſite angle, by which it is deuided into two triangles only. In a pentagon figure, from one angle you may draw lines to two oppoſite angles, and ſo you ſhal haue three triangles. In an Hexagon, you may from one angle draw lines to thre oppoſite angles, and ſo ſhall you haue 4. triangles. In an heptagon, from one angle may be drawne lines to foure oppoſite angles, and ſo ſhal there be fiue triangle. And ſo conſequently of the reſt. As you ſee in the figures here ſet.



An other way to know the nũber of right angles vnto which the angles of any figure are equall.

This thing may alſo be thus expreſſed. In any figure of many ſides, the number of the angles of the figure doubled, is the nũber of the right angles to which the angles of the figure are equall, ſauing foure. As for example.

Let there be an hexãgon figure A B C D E F, and within it take a point at all auentures, namely, G. And draw from the ſame point to euery one of the angles a right line, & ſo ſhal there be comprehended in the figure ſo many triangles, as there are angles in the ſame. VVherefore by this 32. propoſition all the angles of theſe triangles taken together, are equall to double ſo many right angles, as there be angles in the figure. VVherefore for aſmuch as there are ſixe triangles, there are twelue right angles. But all the angles at the point G are equall to 4. right angles by the 13. propoſition. VVherefore take away foure out of twelue, and there reſt eight. VVherefore the ſixe angles in the Hexagon figure are equall to eight right angles.



Another expreſſion of the former Corollary.

By that which hath now bene declared, it foloweth that all the angles of any figure hauing many ſides, takẽ together, are equal to twiſe ſo many right angles, as the figure is in the reaw or order of figures. A triãgle is the firſt figure in order, & his angles are equal to two right angles, which are twiſe one. A quadrangle is the ſecond figure in order. VVherfore his angles are equal to fower right angles, which are twiſe two. The order of figures is gathered of the ſides. For if you take two from the number of the ſides of a figure, the number of the ſides remayning, is the number of the order of the figure. As if you will know, how muny in order is a figure of ſixe ſides: from ſix (which is the number of his ſides) take away two, and there will remaine foure. VVherfore a figure of ſixe ſides is the burth figure

A triangle the firſt figure in order.

A quadrangle the ſecond, and ſo conſequently.

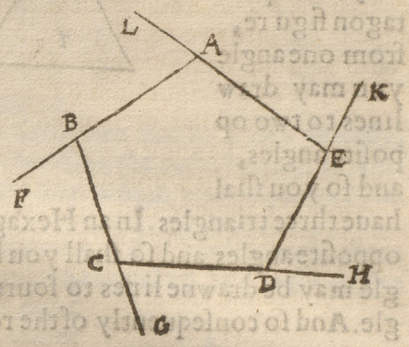
How the order of figures is gathered.

in the order of figures. Then double foure, so shall you haue eight. VVherefore the angles therof are equall to eight right angles. And so of all other figures.

An other Corollary.

Hereby also it is manifest, that the outward angles of any figure of many sides taken together, are equall to foure right angles. For the inwarde angles together with the outward angles, are equall to twise so many right angles, as there be angles in the figure (by the 13. proposition) But the inward angles are equal to twise so many right angles, as there be angles in the figure, sauyng foure: as it was before declared. VVherfore the outward angles are always equal to foure right angles. As for example.

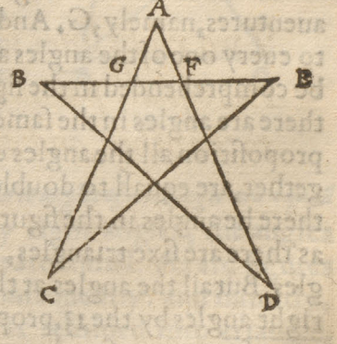
Suppose that there be a pentagon, A B C D E. And produce the fiue sides therof to the points F, G, H, K, L. Now (by the 13. propositiō) the two angles at the point A shall be equal to two right angles. And (by the same) the two angles at the pointe B shall be also equall to two right angles. And so taking euery two angles, they shall be in all equall to tenne right angles. VVherfore takyng away the inward angles, whiche (as hath before bene proued) are equall to sixe righte angles, the outward angles shall be equall to fower right angles. And so of all other figures.



An other Corollary.

It is also manifest, that euery pentagō, which is so described, that ech side therof deuideth two of the other sides, hath his fiue angles equall to two right angles.

For suppose that *ABCDE*: be such a pentagon as is there required so that let the side *AC* cut the side *B E* in the point *G*: & let the side *AD*, cut the same side *B E* in the point *F*. Now thē by this proposition the angle *A F G* shalbe equall to the two angles at the point *B* and *D*: namely, the outward angle to the two inward and opposite angles. And by the same reason the angle *F G A* is equal to the angles at the points *C* and *E* which are in the triangle *C E G*. But the two angles *AFG* and *F G A*, together with the angle at the point *A*, are equall to two right angles (by this proposition). Wherefore the fower angles at the pointes, *B, C, D, E*, together with the angle at the point *A*, are equal to two right angles: which was required to be proued.



An other Corollary.

By this proposition also it is manifest, that euery angle of an equilate triangle is two third partes of a right angle. And that in a triangle of two equall sides hauing a right angle at the toppe, either of the two angles at the base is the halfe of a right angle. And in a triangle called Scalenum, such a Scalenū (I say) which is made by the drawght of a perpendicular line from any one of the angles of an equilater triāgle to the opposite side therof, one angle is a right angle, an other is two third parts of a right angle, namely, that angle which was also an angle of the equilater triangle, wherfore of necessity the angle remaining is one third part of a right angle. For the three angles of a triāgle must be equall to two right angles.

An other Corollary.

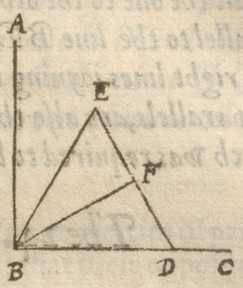
Moreouer by this proposition it is manifest, that if there be two triangles, and if two angles of the one be equal to two angles of the other: the angle remaining

ning ſhall alſo be equall to the angle remayning. For foraſmuch as three angles of any triangle are equal to three angles of any other triangle (for that in ech the three angles are equal to two right angles): If from ech triangle be taken away the two equall angles, the angle remayning ſhall (by the 3. common ſentence) be equall to the angle remayning.

And here I thinke it good to ſhew how to deuide a right angle into three equall partes, for that the demonſtration thereof dependeth of this propoſition.

How to deuide a right angle into three equall partes.

Suppoſe that there be a right angle *ABC*, contayned of the right lines *AB* and *BC*: & in the line *BC*, take a point at all aduentures, which let be *D*. And vpon the line *BD* deſcribe (by the firſt) an equilater triangle *BDE*. And (by the 9. propoſition) deuide the angle *DBE* into two equall partes by the right line *BF*. Then *I* ſay that the right angle *ABC* is deuided into thre equal parts by the right lines *BE* and *BF*. For foraſmuch as *EBD* is an equilater triangle, therfore as hath before bene declared, the angle *EBD* is two thirde partes of a right angle. But the whole angle *ABC*, is a right angle. Wherfote the angle remaining, namely, *ABE* is one third part of a right angle. Again foraſmuch as the angle *EBD*, is two third partes of a right angle, and it is deuided into two equall parts by the right line *BF*, therefore either of theſe two angles *EBF* & *FBD* is one third part of a right angle. Wherefore the three angles *ABE*, *EBF* and *FBD* are equall the one to the other. Wherefore the right angle *ABC* is deuided into three equall partes by the right lines *BE* and *BF*: which was required to be done.

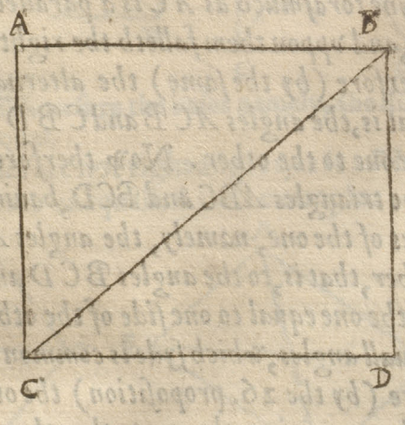


The 23. Theoreme. The 33. Propoſition.

Two right lines ioyning together on one and the ſame ſide, two equall parallel lines: are alſo them ſelues equall the one to the other, and alſo parallels.

Demonſtration.

SVppoſe that AB and CD be right lines equal, and parallels: and let theſe two right lines AC and BD ioyne the̅ together, the one on the one ſide, and the other on y other ſide. Then I ſay that the lines AC & BD are both equall, & alſo parallels. Draw (by the firſt petition) a right line from the point B to the point C. And foraſmuch as AB is a parallel to CD, and vpō them falleth the right line BC, therfore the alternate angles ABC and BCD, are equall the one to the other (by the 29. propoſition). And foraſmuch as the line AB is equall to the line CD, and the line BC is common to them both,



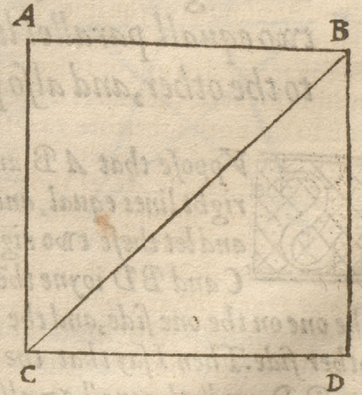
therefore these two lines A B and B C, are equall to these two lines B C and C D, and the angle A B C is equall to the angle B C D. Wherfore (by the 4. propoposition) the base B D is equall to the base A C, and the triangle A B C, is equall to the triangle B C D, and the angles remayning are equall to the angles remayning the one to the other, vnder which are subtended equall sides, wherefore the angle A C B is equall to the angle C B D, and the angle B A C to the angle B D C. And forasmuch as vpon these right lines A C and B D falleth the right line B C making the alternate angles, that is the angles A C B and C B D, equall the one to the other, therefore (by the 27. proposition) the line A C is a parallel to the line B D. And it is proued that it is equall vnto it. Wherefore two right lines ioyning together on one and the same side two equall lines which are parallels, are also themselues equall the one to the other, and also parallels: which was required to be proued.

The 24. Theoreme. The 34 Proposition.

In parallelogrammes, the sides and angles which are opposite the one to the other, are equall the one to the other, and their diameter deuideth them into two equall partes.

SVppose that A B C D be a parallelogramme and let the diameter thereof be B C. Then I say that the opposite sides and angles of the parallelogramme A C D B are equall the one to the other, and y the diameter B C deuideth it into two equall partes. For forasmuch as A B is a parallel line vnto C D, and vpon them falleth a right line B C: therfore (by the 29. proposition) the alternate angles A B C and B C D are equall the one to the other. Agayne forasmuch as A C is a parallel line to B D, and vppon them falleth the right lyne B C: therfore (by the same) the alternate angles, that is, the angles A C B and C B D are equall the one to the other. Now therfore there are two triangles A B C and B C D, hauing two angles of the one, namely, the angles A B C and A C B equall to two angles of the other, that is, to the angles B C D and C B D, the one to the other, and one side of the one equal to one side of the other, namely, that syde that lieth betwene the equall angles, which syde is common to them both, namely, the side B C. Wherfore (by the 26. proposition) the other sides remaining are equall to the other sides remaining, the one to the other, and the angle remaining is equal to the angle remayning. Wherfore the side A B is equall to the side C D, and the side A C

Demonstration



to

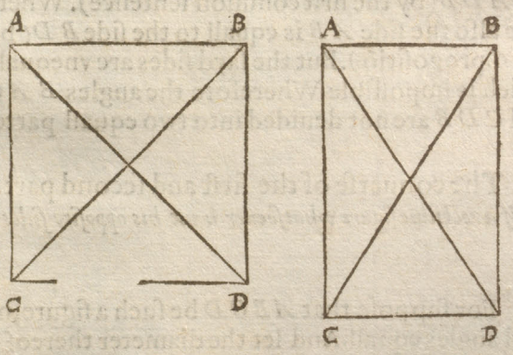
to the ſide BD, & the angle BAC is equal to the angle BDC. And foraſmuch as the angle ABC is equal to the angle BCD, and the angle CBD to the angle ACB: therfore (by the ſecond common ſentence) the whole angle ABD is equall to the whole angle ACD. And it is proued that the angle BAC is equall to the angle CDB. Wherfore in parallelogrammes, the ſides and angles which are oppoſite are equall the one to the other. I ſay alſo that the diameter therof deuideth it into two equall partes. For foraſmuch as AB is equall to CD, and BC is common to them both, therfore theſe two AB and BC are equall to theſe two BC and CD, and the angle ABC is equal to the angle BCD. Wherfore (by the 4. propoſition) the baſe AC is equall to the baſe BD, and the triangle ABC is equall to the triangle BCD. Wherfore the diameter BC deuideth the parallelogramme ABCD into two equall partes: which is all that was required to be proued.

Thre paſsions of parallelogrãmes demõſtrated in this Thoreme.

In this Theoreme, are demonſtrated three paſſions or properties of parallelogrammes. Namely, that their oppoſite ſides are equall; that their oppoſite angles are equall: and that the diameter deuideth the parallelogramme into two equall partes. Which is true in all kindes of parallelogrammes. There are fower kindes of parallelogrammes, a ſquare, a figure of one ſide longer then the other, a Rhombus, or diamond figure, and a Rhomboides or diamondlike figure. And here is to be noted, that in thoſe parallelogrammes, all whoſe angles ar right angles (as is a ſquare, and a figure on the one ſide longer) the diameters do not only deuide the figure into two equall partes, but alſo they are equal the one to the other. As for example.

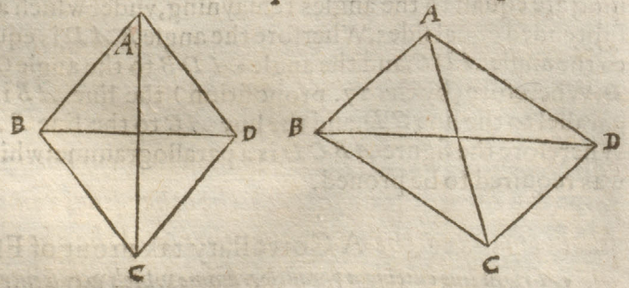
Fower kindes of parallelogrammes.

Suppoſe that *ABCD* be a ſquare, or a figure on the one ſide longer, and draw in it theſe diametres *AD* and *BC*. And foraſmuch as the line *AB* is equall to the line *CD* (by the definitiõ of a ſquare, and of a figure on the alſo one ſide lõger) & the line *AC* is common to thẽ both: therfore two ſides of the triangle *ABC* are equal to two ſides of the triangle *ACD*, the one to the other, and the angles which they contayne are equall, namely, the angles *BAC* & *ACD*, for they are right angles. Wherefore the baſes namely, the diameters *AD* and *BC*, are (by the 4. propoſition) equal.



But in thoſe parallelogrames whoſe angles are not right angles, as is a Rhombus, and a Rhomboides, the diameters be euer vnequall. As for example.

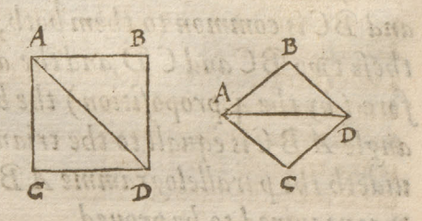
Suppoſe that *ABCD* be a Rhombus, or a Rhombaides and drawe in it theſe diameters *AC* and *BD*. And foraſmuch as *AB* is equall to *CD*, and *BC* is common to them both, & the angle *ABC* is not equall to the angle *BCD* (by the definition of a Rhombus and alſo of a Rhombaides)



therefore (by the 24.propoſition) the baſes alſo are vnequall, namely, the diameters *A C* and *B D*.

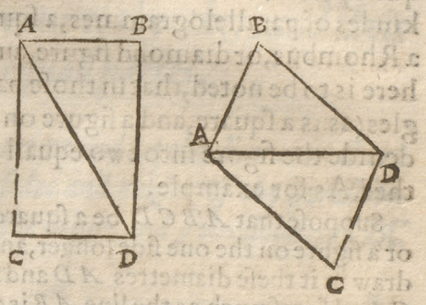
Agayne. In parallelogrammes of equall ſides, as are a ſquare, and a Rhombus, the diameters do not onely deuide the figures into two equall partes, but alſo they deuide the angles into two equall partes.

For ſuppoſe that there be a ſquare or Rhombus *A B C D*, and draw the diameter *A D*. And foraſmuch as the ſides *A B* and *B D* are equall to the ſides *A C* and *C D* (for the figures are equilater) and the angles *A B D* and *A C D* are equall (for they are oppoſite angles) and the baſe A D is common to both triangles. Therefore (by the fourth propoſition) the angles *B A D* & *C A D* are equall, and ſo alſo are the angles *B D A* and *C D A* equall. Wherfore the angles *B A C* and *C D B* are deuided into two equall partes.



But in parallogrammes whoſe ſides are not equall, ſuch as are a figure on the one ſide longer, and a Rhomboides it is not ſo.

For ſuppoſe *A B C D* to be a figure on the one ſide longer or a Romboides. And draw the diameter *A D*. And now if the angles *B A C* and *C D B*, be deuided into two equall partes by the diameter *A D*, then foraſmuch as the angle *C A D* is (by the 29.propoſition) equall to the angle *A D B*, the angle alſo *B A D* ſhal be equal to the angle *A D B* (by the firſt common ſentence). Wherfore alſo the ſide *A B* is equall to the ſide *B D* (by the 6.propoſitiõ). But the ſayd ſides are vnequal: which is impoſſible. Wherefore the angles *B A C* and *C D B* are not deuided into two equall partes.

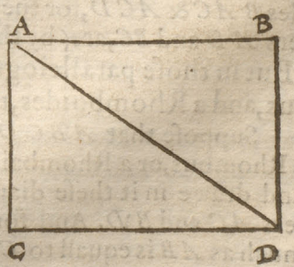


The conuerſe of this propoſition after Proclus.

The conuerſe of the firſt and ſecond part of this propoſition after Proclus.

If a rectiline figure whatſoeuer haue his oppoſite ſides and angles equall: then is a parallelograme.

For ſuppoſe that *A B C D* be ſuch a figure, namely, which hath his oppoſite ſides and angles equall. And let the diameter thereof be *A D*. Now foraſmuch as the ſides *A B* and *B D* are equall to to the ſides *D C* and *A C*, and the angles which they cõtayne are equall, and the baſe *A D* is common to ech triangle, therefore (by the 4.propoſition) the angles remayning are equall to the angles remayning, vnder which are ſubtended equal ſides. Wherfore the angle *B A D* is equal to the angle *A D C*, and the angle *A D B* to the angle *C A D*. Wherefore (by the 27. propoſition) the line *A B* is a parallel to the line *C D*, and the line *A C* to the line *B D*. Wherefore the figure *A B C D* is a parallogramme: which was required to be proued.



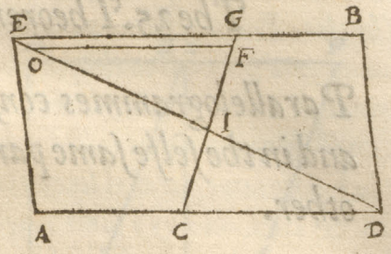
A Corollary taken out of Fluſſates.

A Corrollary taken out of Fluſſates.

A right line cutting a parallelogramme which way ſoeuer into two equall partes, ſhall alſo deuide the diameter thereof into two equall partes.

For

For if it be possible let the right line *G C* deuide the parallelogramme *A E B D* into two equal partes, but let it deuide the diameter *D E* into two vnequall partes in the point *I*. And let the part *I E* be greater then the part *I D*, And vnto the line *I D* put the line *I O* equall (by the 3. propositiō). And by the point *O*, draw vnto the lines *A D* and *B E* a parallel line *O F* (by the 31. proposition.) Wherefore in the triangles *F O I* and *C D I*, two angles of the one are equal to two angles of the other, namely, the angles *I O F* and *I D C* (by the 29. proposition), & the angles *F I O* & *C I D* (by the 15. propositiō), & the side *I D* is equal to the side *I O*. Wherefore (by the 26. proposition) the triangles are equall. Wherefore the whole triangle *E I G* is greater then the triangle *D I C*. And forasmuch as the trapesium *G B D C* is supposed to be the halfe of the parrallelograme, and the halfe of the same parallelograme is the triangle *E B D* (by this proposition). From the trapesium *G B D C* and the triangle *E B D* which are equall, take away the trapesium *G B D I* which is common to them both, and the residue namely, the triangle *D I C* shalbe equall to the residue, namely, to the triāgle *E I G*: but it is also lesse (as hath before ben proued): which is impossible. Wherefore a right line deuiding a parallelogramme into two equall partes, shall not deuide the diameter thereof vnequally. Wherefore it shall deuide it equally: which was required to be proued.



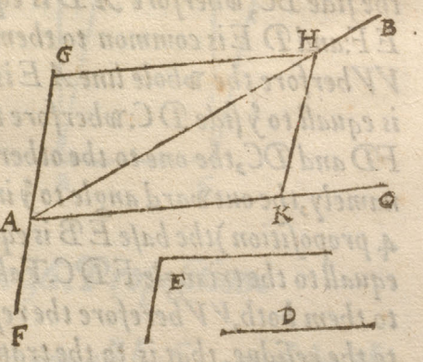
Demonstration leading to an absurditie.

An addition of Pelitarius.

Betwene two right lines being infinite and making an angle geuen: to place a line equall to a line geuen, in such sorte, that it shall make with one of those lines an angle equall to an other angle geuen. Now it behoueth that the two angles geuen be lesse then two right angles.

An addition of Pelitarius.

Suppose that there be two lines *A B* and *A C*, making an angle geuen *B A C*: and let them be infinite on that side where they open one from the other. And let the line geuen be D, and let the other angle geuen be *E*. And let the two angles *A* and *E* be lesse then two right angles (otherwise there coulde not be made a triangle, as it is manifest by the 17 proposition). It is required betwene the lines *A B* and *A C* to place a line equall to the line geuen *D*, which with one of them as for exāple with the line *A C*, may make an angle equal to the angle geuen *E*. Now then vpon the line *A C* and to the point in it *A*, make an angle equall to the angle geuē *E* (by the 23. proposition), which let be *C A F*. And produce the line *F A* on the other side of the point *A* to the point *G*: and let *A G* be equall to the line geuen *D* (by the 3. proposition). And by the point *G*, draw (by the 31. proposition) a parallel line to the line *A C*, which let be *G H*, and produce it vntil it concurre with the line *A B*: which concurse let be in the point *H*. And agayne by the point *H* draw the line *H K* parallel vnto the line *G F*: which let cut the line *A C* in the point *K*. Then I say that the line *H K* is placed betwene the lines *A B* & *A C* & is equall to the line *D*. And that the angle at the point *K* is equall to the angle geuen *E*. For forasmuch as (by construction) *A G H K* is a parallelogramme the line *K H* is equall to the line *A G* (by this proposition). Wherefore also it is equall to the line *D*. And forasmuch as the line *A K* falleth vppon the two parallel lines, *F G* and *K H*, therfore the angle *A K H* is equal to the angle *F A K* (by the 29. propositiō) for that they are alternate angles. Wherfore also the same angle at the point *K* is equal to the angle geuen *E*. Wherefore the line *H K* being placed betwene the two lines *A B* and *A C*, and being equall to the line *D*, maketh the angle at the point K equall to the angle geuen *E*: which was required to be done.



Construction.

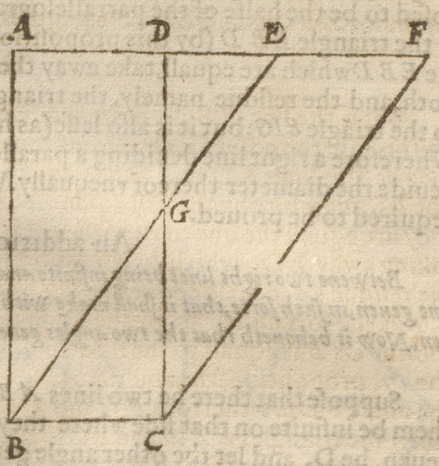
Demonstration

Though this addition of Pelitarius be not so muche pertayning to the

propoſition: yet becauſe it is witty and ſemeth ſomewhat difficult, I thought it good here to anexe it.

The 25. Theoreme. The 35. Propoſition.

Parallelogrammes conſiſting vppon one and the ſame baſe, and in the ſelfe ſame parallel lines, are equall the one to the other.



Demonſtration. *VPpoſe that theſe parallelogrammes ABCD and EBCF do conſiſt vpon one and the ſame baſe, that is, vppon BC, and in the ſelfe ſame parallel lines, that is AF, and BC. Then I ſay, that the parallelograme ABCD is equal to the parallelograme EBCF. For foraſmuch as ABCD is a parallelogramme, therfore (by the 34. propoſition) the ſide AD, is equall to the ſide BC, and by the ſame reaſon alſo the ſide EF is equall to the ſide BC, wherfore AD is equall to EF: and DE is common to them both. VVherfore the whole line AE is equall to the whole line DF. And the ſide AB is equall to ỹ ſide DC: wherfore theſe two EA and AB are equall to theſe two FD and DC, the one to the other: and ỹ angle FDC is equall to the angle EAB, namely, the outward angle to ỹ inward angle (by ỹ 29. propoſitiõ): wherfore (by ỹ 4 propoſition) the baſe EB is equall to the baſe FC, and the triangle EAB is equall to the triangle FDC. Take away the triangle DGE, which is common to them both. VVherefore the reſidue, namely, the trapeſium ABGD is equall to the reſidue, that is, to the trapeſium EGCF. Put the triangle GBC commõ to them both. VVherefore the whole parallelogramme ABCD is equall to the whole parallelogramme EBCF. VVherefore parallelogrammes conſiſting vpon one and the ſame baſe, and in the ſelfe ſame parallel lines, are equal the one to the other: which was required to be demonſtrated.*

Parallelogrammes are ſayde to be in the ſelfe ſame parallel lines, when their baſes, and the oppoſite ſides vnto them, are one and the ſelfe ſame lynes wyth the parallels.

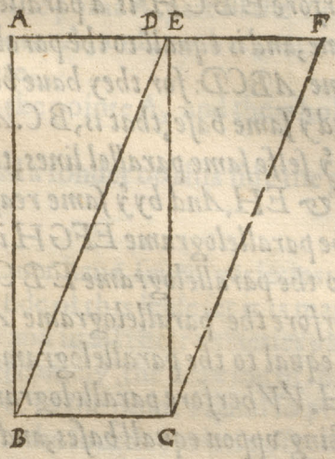
Three caſes in this propoſition. The firſt caſe. In this propoſition are three caſes. For the line BE may cutte the line AF, either beyond the point D, or in the point D, or on this ſide the point D. When

it

it cutteth the line A F beyond the point D the demonstration before put ſeth.

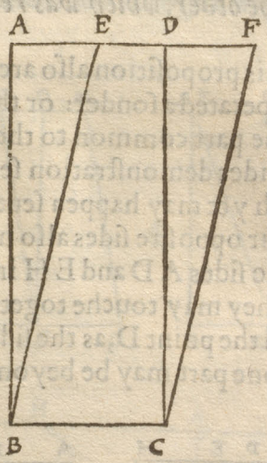
The ſecond caſe.

But if the line *B E* do cutte the line *A F* in the point D, then foraſmuch as (by the former propoſition) the triangle *B C D* or *B C E* is the halfe of either of theſe parallelogrammes *A B C D* and *E B C F* (for in the parallelogramme *A B C D* the diameter *B D* maketh the triangle *B D C* the halfe of the ſame parallelogramme, and in the parallelogramme *E B C F* the diameter *E C* or D C maketh the ſelfe ſame triangle *B D C* the halfe of the parallelograme *E B C F*) therfore (by the 7. common ſentence) the parallelogrammes *A B C* D and *E B C F* are equall.



The third caſe.

But if the liue *B E* do cutte the lyne *A F* on this ſide the point D, then foraſmuch as eyther of the lines A D and *E F* is equall to the line *B C*, therefore by the firſt common ſentence they are equall the one to the other. Wherefore taking away E D, which is common to both, the reſidue *A E* ſhalbe equall to the reſidue D F. Agayne foraſmuch as (by the 34. propoſitiō) the line *A B* is equall to the line C D, and (by the 27. propoſition) the angle *E A B* is equal to the angle F D C: therfore (by the 4. propoſition) the triangles *E B A* and F C D are equal. Adde the trapeſium C D E B common to them both: and ſo (by the ſeconde common ſentence) the two parallelogrammes *A B C* D and *E B C F* ſhalbe equall: which was required to be proued.



The 26. Theoreme. The 36. Propoſition.

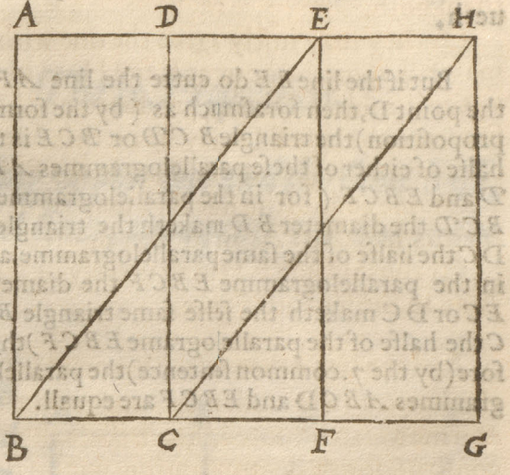
Parallelogrammes conſiſting vpon equall baſes, and in the ſelfe ſame parallel lines, are equall the one to the other.

Vppoſe that theſe parallelogrammes A B C D and E F G H do conſiſt vpon equall baſes, that is, vpon B C and F G, and in the ſelfe ſame parallel lines, that is, A H and B G. Then I ſay, that the parallelogramme A B C D is equall to the parallelogramme E F G H. Draw a right line from the point B to the point E, and an other from the point C to the point H. And foraſmuch as B C is equall to F G, but F G is equall to E H, therfore B C alſo is equall to E H, and they are parallel lines, and the lines B E and C H ioyne them together: but two right lynes ioynyng together two equall right

Conſtruction.

Demonſtration

lines being parallels, are themselues also (by the 33 proposition) equall the one to the other, and parallels. Wherfore EBCH is a parallelogramme, and is equall to the parallelograme ABCD: for they haue both one and ỹ same base, that is, BC. And are in ỹ selfe same parallel lines, that is, BC & EH. And by ỹ same reason also the parallelograme EFGH is equal to the parallelograme EBCH. Wherfore the parallelograme ABCD is equal to the parallelograme EFGH. Wherfore parallelogrames consisting vppon equall bases, and in the selfe same parrallel lines, are equall the one to the other: which was required to be proued.



Three cases in this proposition.

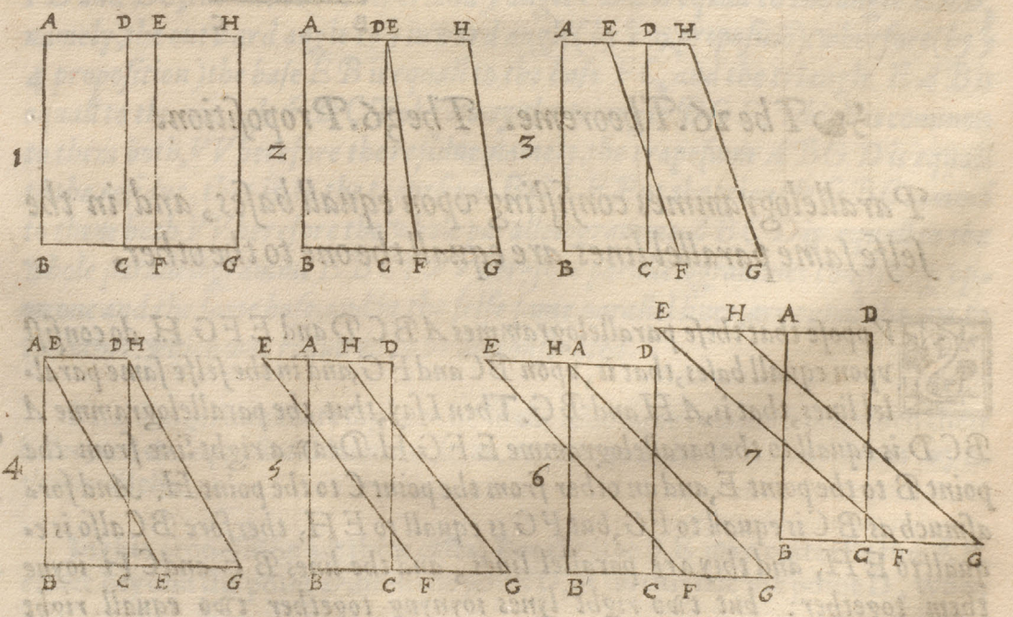
In this proposition also are three cases. For the equall bases may either be vtterly seperated a sonder: or they may touche at one of the endes: or they may haue one part common to them both.

The first case. Euery case may happen seuen diuers wayes.

Euclides demonstration serueth when the bases be vtterly seperated a sonder. Which yet may happen seuen diuers wayes. For the bases being seperated asõder, their opposite sides also may be vtterly seperated a sonder beyond the point D, as the sides AD and EH in the first figure.

2. Or they may touche together in one of the endes, and the whole side may be beyond the point D, as the sides AD and EH do in the second figure.

3. Or one part may be beyond the point D, and an other part common to them



both, as in the third figure, the sides A D and E H haue the part E D common to them both.

Or they may iustly agree the one with the other, that is, the pointes A and D may fall vpon the pointes E and H: as in the fourth figure. 4.

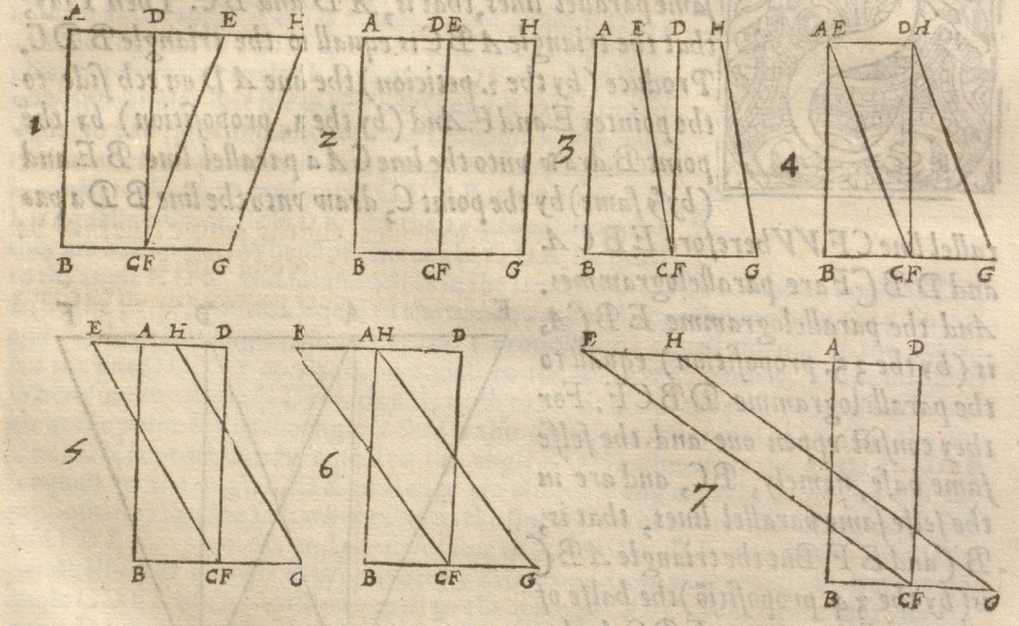
Or the side A D being produced on this side the point A, part of the opposite side vnto the base F G may be on this side the point A, and an other part may be common with the line A D, as in the fifth figure. 5.

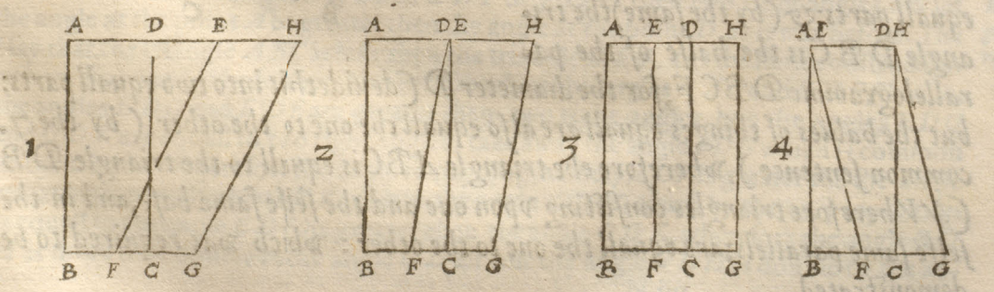
Or one ende of the side E H may light vpon the pointe A, and the whole side on this side of it: As in the sixt figure. 6.

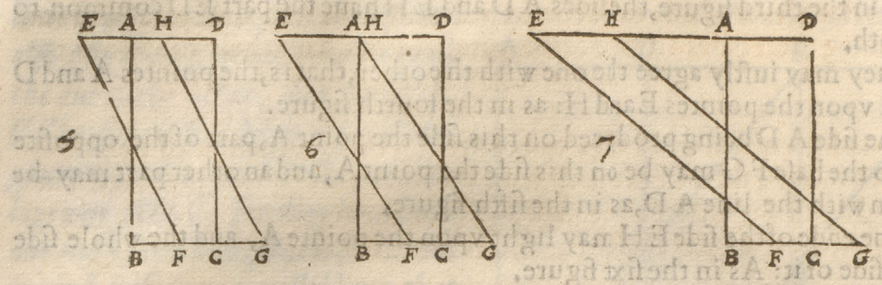
Or the said side E H may vtterly be seperated a sonder on this side the pointe A, as in the seuenth figure. 7.

The like Varietie in ech of the other two cases. Euclides construction and demonstration serueth in all these cases, and in their Varities also.

And the two other cases also may in like maner haue seuen varieties: as in the figures here vnderneth and on the other side of this leafe set it is manifest. And here is to be noted, that in these three cases and in all their varieties also, the construction & demonstration put by Euclide (namely, the drawing of lines frō the point B to the point E & from the pointe C to the point H, and so prouing it by the former proposition) will serue onely in the fourth varietie of ech case, there nedeth no farther construction: for that the conclusion straight way followeth by the former proposition.







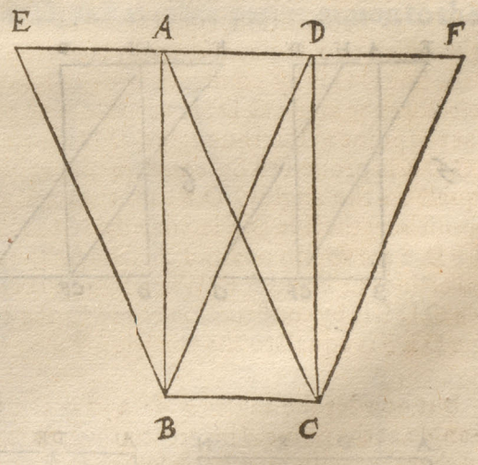
The 27. Theoreme. The 37. Proposition.

Triangles consisting vpon one and the selfe same base, and in the selfe same paralles: are equall the one to the other.

Construction.

Demonstration.

SVppose that these triangles A B C and D B C do consist vpon one and the same base, namely, B C, and in the selfe same parallel lines, that is, A D and B C. Then I say, that the triangle A B C is equall to the triangle B D C. Produce (by the 2. peticion) the line A D on ech side to the pointes E and F. And (by the 31. proposition) by the point B, draw vnto the line C A a parallel line B E and (by ỹ same) by the point C, draw vnto the line B D a parallel line C F. VVherefore E B C A. and D B C F are parallelogrammes. And the parallelogramme E B C A, is (by the 35. proposition) equall to the parallelogramme D B C F. For they consist vppon one and the selfe same base, namely, B C, and are in the selfe same parallel lines, that is, B C and E F. But the triangle A B C is (by the 34. propositiõ) the halfe of the parallelogramme E B C A, for the diameter A B deuideth it into two equall parts: & (by the same) the triangle D B C is the halfe of the parallelogramme D B C F, for the diameter D C deuideth it into two equall parts: but the halues of thinges equall are also equall the one to the other (by the 7. common sentence), wherefore ehe triangle A B C is equall to the triangle D B C. VVherefore triangles consisting vpon one and the selfe same base, and in the selfe same parallels: are equall the one to the other: which was required to be demonstrated.



Those

Thoſe triangles are ſaide to be contayned within the ſelfe ſame parallel lines, which hauing their baſes in one of the parallel lines, haue their toppes in the other.

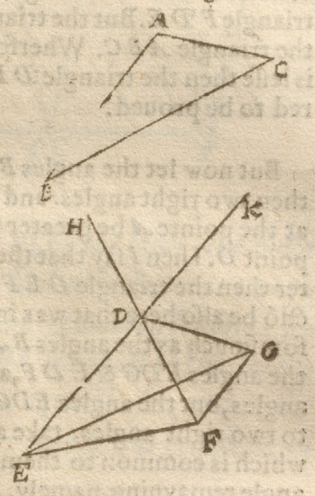
How triangles are ſayde to be in the ſelfe ſame parallel lines.

Here as I promiſed will I ſhew out of Proclus the compariſon of two triangles, which hauing their ſides equall, haue the baſes and angles at the toppe vnequall. And firſt I ſay that the vnequall angles at the toppe being equall to two right angles, the triangles ſhalbe equall. As for example.

Compariſon of two triangles whoſe ſides being equal, their baſes and angles at the toppe are vnequall.

Suppoſe that theſe two triangles *A B C* and *D E F* haue two ſides of the one, namely, *A B* and *A C*, equall to two ſides of the other, namely, to *D E* and *D F*, eche to his correſpondent ſide, that is, *A B* to *D E*, and *A C* to *D F*, and let the baſe *B C* be greater then the baſe *E F*: and let the angle at the point *A* be greater then the angle at the point D. But let the ſayde angles at the pointes *A* and D, be equall to two right angles. Then I ſay that the triangles *A B C* and D *E F* are equall. For foraſmuch as the angle *B A C* is greater then the angle *E D F*, vpon the line *E* D, and to the point D deſcribe (by the 23. propoſition) an angle equall to the angle *B A C*, which let be E D G: and put the line D G equall to the line *A C*: and draw a line from E to G, and an other from *F* to G: and produce the lines E D & F D beyond the poynt D to the pointes *H* and *K*. Now for aſmuch as the angle *B A C* is equall to the angle E D G, and the angles *B A C* and E D F are equall to two right angles, therefore the angles E D G and E D F are equall to two right angles. But the angles E D G and K D G, are alſo equal to two right angles: take away the angle F D G common to them both: wherefore the angle remayning E D F is equall to the angle remayning G D K. But the angle E D F is equall to the angle H D K (by the 15. propoſition) for they are hed angles. Wherefore the angle G D K is equall to the angle H D K. And foraſmuch as in the triangle G D F the outward angle G D H is (by the 32. propoſition) equal to the two inward and oppoſite angles at the points *G* and *F*: which two angles alſo are (by the 5. propoſition) equall the one to the other: for the line D G is by conſtruction equall to the line *A C*, namely, to the line D F. Wherefore the angle G D H is double both to the angle at the point *G*, and to the angle at the point *F*. But the angle *G D H* is alſo double to the angle G D K (for the angle G D K is proued to be equall to the angle K D H) wherefore the angle at D G F is equall to the angle G D K: and they are alternate angles. Wherefore (by the 27. propoſition) the line D E is a parallel to the line *F G*. Wherefore the triangles G D E and F D E are vppon one and the ſelfe ſame baſe, namely, *D E*, and in the ſelfe ſame parallel lines D E and *G F*. Wherefore by this propoſition they are equall. But the triangle G D E is by conſtruction equall to the triangle *A B C*. Wherefore alſo the triangle D E F is equall to the triangle *A B C*: which was required to be proued.

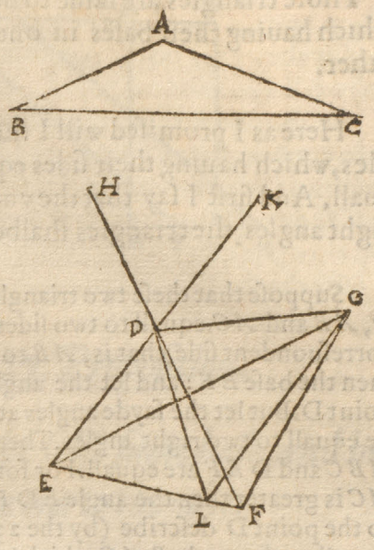
When the two angles at the toppes are equall to two right angles.



But now let the angles *B A C* and *E D F* be greater then two right angles: & let the angle at the point *A* be greater then the angle at the point *D*, as it was before. Thē I ſay that the triangle *A B C* is leſſe then the triangle *D E F*. Let the ſame conſtruction be here that was in the former. And foraſmuch as the angles *B A C* and *E D F*, that is, the angles *E D G* and *E D F* are greater then two right angles, but the angles *E D G* and *G D K* are equall to two right angles: take away the angle *F D G* which is common to them both. Wherefore the angle remayning, namely, *E D F* is greater thē the angle remayning, namely, then *G D K*: that is, the angle *K D H* (which by the 15. propoſition is equall to the angle *E D F*) is greater then the angle *G D K*, wherefore the angle *G D H* is more then double to the angle *G D K*: but the angle *G D H* is double to the an-

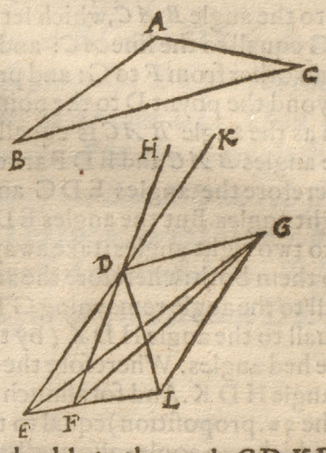
When they are greater thē two right angles.

gle *D G F*, as was before proued. Wherefore the angle *G D K* is lesse then the angle *D G F*. Vnto the angle *G D K* put (by the 23. proposition) the angle *D G L* equall: and produce the line *G L* till it concurre with the line *E F* in the pointe *L*. And draw a line from *D* to *L*. Wherefore (by the 27. proposition) *G L* is a parallel line to *D E*, for that the alternate angles *D G L* and *G D K* are equal. Wherfore the triangles *G D E* and *L D E* are (by this proposition) equal (for they consist vpon one and the self same base, namely, *D E*, and are in the selfe same parallel lines, namely, *E D* and *G L*) But the triangle *L D E* is lesse then the triangle *F D E*. Wherfore also the triangle *G D E* is lesse then the triangle *F D E*. But the triangle *G D E* is equal to the triangle *A B C*. Wherfore the triangle *A B C* is lesse then the triangle *D E F*: which was required to be proued.



When they are lesse then two right angles.

But now let the angles *B A C* and *E D F* be lesse then two right angles: and agayne let the angle at the pointe *A* be greater then the angle at the point *D*. Then *I* say that the triãgle *A B C* is greater then the triangle *D E F*. Let the same constructiõ be also here that was in the two former. And forasmuch as the angles *B A C* and *E D F*, that is, the angles *E D G* & *E D F*, are lesse then two right angles, but the angles *E D G* and *G D K* are equal to two right angles, take away the angle *F D G* which is common to them both, wherefore the angle remayning, namely, *E D F* is lesse then the angle remayning, namely, then *G D K*: that is, the angle *H D K* (which by the 15. proposition is equall to the angle *E D F*) is lesse then the angle *G D K*. Wherfore the whole angle *G D H* is lesse then double to the angle *G D K*. But it is double to the angle D G F (as before it was proued): wherfore the angle *G D K* is greater then the angle D G F. Put the angle D G L equall to the angle *G D K* (by the 23. proposition) and produce the line *G L* till it concurre with the line *E F* also produced, & let the concurse be in the point *L*. And draw a line from D to *L*. And for as much as the angle D *G L* is equall to the angle *G D K*, and they are alternate angles, therefore the line *G L* is a parallel to D *E* (by the 27. proposition). Wherefore (by this proposition) the triangles *G D E* and *L D E* are equal: but the triangle L D E is greater then the triangle *F D E*, and the triangle *G D E* is equall to the triangle *A B C*. Wherefore the triangle *A B C* is greater then the triangle D *E F*: which was required to be proued.



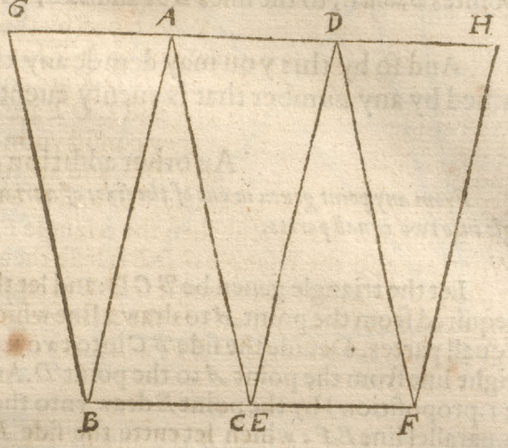
The 28. Theoreme. The 38. Proposition.

Triangles which consist vppon equall bases, and in the selfe same parallel lines, are equall the one to the other.

Svppose that these triangles A B C and D E F do consist vpon equal bases, that is, vpon B C and E F, and in the selfe same parallel lines, that is B F and A D. Then I say that the triangle A B C is equall to the triangle

gle ABC is equall to the triangle DEF. Produce (by the second peticion) the line AD on eche side to the pointes G and H. And (by the 31. proposition) by the point B drawe vnto CA a parallel line BG, and (by the same) by the pointe F drawe vnto DE a parallel line FH. VVherfore GBCA and DEFH are parallelogrammes. But the parallelograme GBCA is (by the 36 proposition) equal to the parallelogramme DEFH, for they consist vpon equall bases, that is, BC and EF, and are in the selfe same parallel lines, that is, BF and GH. But (by the 34. proposition) the triangle ABC is the halfe of the parallelogramme GBCA, for the diameter AB deuideth it into two equall partes: and the triangle DEF is (by the same) the halfe of the parallelogramme DEFH, for the diameter FD deuideth it into two equall partes. But the halues of thinges equall are (by the 7. common sentence) equall the one to the other. VVherfore the triangle ABC is equall to the triangle DEF. VVherefore triangles which consist vppon equall bases, and in the selfe same parallel lines, are equall the one to the other: which was required to be proued.

Construction.



Demonstration.

In this proposition are three cases. For the bases of the triangles either haue one part common to them both or the base of the one toucheth the base of the other onely in a point: or their bases are vtterly seuered a sunder. And ech of these cases may also be diuersly, as we before haue sene in parallelogrammes consisting on equall bases, and being in the selfe same parallel lines. So that he which diligently noteth the variety that was there put touching them, may also easely frame the same varietie to ech case in this proposition. VVherefore I thinke it nedeles here to repeate the same agayne: for how soeuer the bases be put, or the toppes, the manner of construction and demonstration here put by Euclide will serue: namely, to draw parallel lines to the sides.

Thre cases in this proposition.

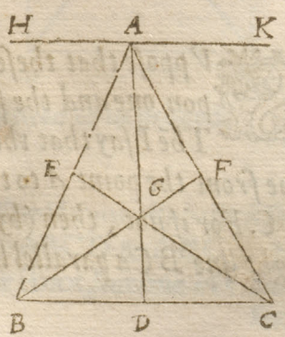
Ech of these cases also may be diuersly.

An addition of Pelitarius,

To deuide a triangle geuen into two equall partes.

An addition of Pelitarius, to deuide a triangle into two equall partes.

Suppose that the triangle geuen to be deuided in to two equall partes be *ABC*. Deuide one of the sides therof, namely, *BC* into two equall partes (by the 10. proposition) in the point D. And draw a line from the point D to the point *A*. Thē I say that the two triangles *ABD* & *ACD*, are equal, which is easy to proue (by the 38. proposition) if by the point *A* we drawe vnto the line *BC* a parallel line (by the 31. proposition), which let by *HK*: for so the triangles *ABD* and *ADC*, consisting vppon equal bases *BD* & *DC*, and being in the selfe same parallel lines *HK* and *BC* are of necessitie equall. The selfe



same thing also wil happen if the side *B A* be deuided into two equall parts in the point *E*, and so be drawen a right line from the point *E*, to the point *C*. Or if the side *A C* be deuided into two equall partes in the point *F*, and so be drawen a right line from the point *F* to the point *B*: which is in like manner proued by drawing parallel lines by the pointes *B*, *and C*, to the lines *B A* and *A C*,

Note. And so by this you may deuide any triangle into so many partes as are signified by any number that is euenly euen: as into 14. 16. 32. 64. &c.

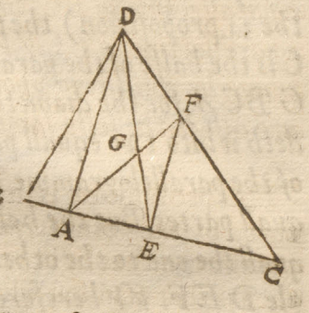
An other addition of Pelitarius.

An other addition of Pelitarius. *From any point geuen in one of the sides of a triangle, to draw a line which shal deuide the triangle into two equall partes.*

Let the triangle geuen be *B C D*: and let the point geuen in the side *B C* be *A*. It is required from the point *A* to draw a line which shal deuide the triangle *B C D* into two equall partes. Deuide the side *B C* into two equall partes in the point *E*. And drawe a right line from the point *A* to the point *D*. And (by the 31. proposition) by the point *E* draw vnto the line *A D* a parallel line *E F*: which let cutte the side *D C* in the point *F*. And draw a line from the point *A* to the point *F*. Then I say that the line *A F* deuideth the triangle *B C D* into two equall partes: namely, the trapesium *A B D F* is equall to the triangle *A C F*. For draw a line from *E* to *D*, cutting the line *A F* in the point *G*. Now then it is manifest (by the 38. proposition) that the two triangles *B E D* and *C E D* are equall (if we vnderstand a line to be drawen by the point D parallel to the line *B C*, for the bases *B E* and *E C* are equal). The two triangles also D *E F* and *A E F* are (by the 37. proposition) equall: for they consist vpon one and the selfe same base *E F*, and are in the selfe same parallel lines *A* D and *E F*. Wherefore taking away the triangle *E F G* which is cõmõ to thẽ both, the triangle *A E G* shalbe equall to the rriangle D*F*G: wherfore vnto either of thẽ adde the trapesiũ *C F G E*, and the triangle *ACF* shalbe equal to the triangle *DEC*. But the triangle *D E C* is the halfe part of the whole triangle *B C D* wherefore the triangle *A C F* is the halfe part of the same triangle *B C D*. Wherfore the residue, namely, the trapesium *A B F D* is the other halfe of the same triangle. Wherefore the line *A F* deuideth the whole triangle *B C D* into two equall partes: which was required to be done.

Construction.

Demonstration



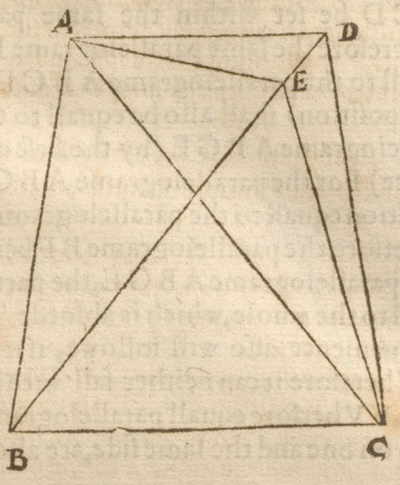
The 29. Theoreme. The 39. Proposition.

Equall triangles consisting vpon one and the same base, and on one and the same side: are also in the selfe same parallel lines.

Suppose that these two equall triangles A B C and D B C do consist vpon one and the same base, namely, B C and on one and the same side. Thẽ I say that they are in the selfe same parallel lines. Drawe a right line from the point A to the point D. Now I say that A D is a parallel line to B C. For if not, then (by the 31. proposition) by the point A drawe vnto the right line B C a parallel line A E, and draw a right line from the point E to the

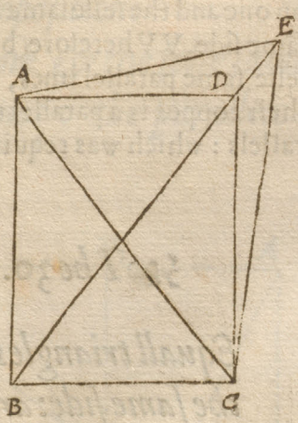
point

point C. VVherfore ỹ triangle E B C is (by ỹ 37. propoſitiō) equal to the triangle A B C, for they conſiſt vpon one and the ſelfe ſame baſe, namely, BC, and are in ỹ ſelfe ſame parallels, that is, A E and B C. But the triangle D B C is (by ſuppoſition) equall to the triangle A B C. VVherfore the triangle D B C is equal to the triangle E B C, the greater vnto the leſſe: which is impoſſible. VVherefore the line A E, is not a parallel to the line B C. And in like ſorte may it be proued that no other line beſides A D is a parallel line to B C, wherefore A D is a parallel line to B C. VVherfore equall triangles conſiſting vpon one and the ſame baſe, and on one and the ſame ſide, are alſo in the ſelfe ſame parallel lines: which was required to be proued.



This propoſition is the conuerſe of the 37. propoſition. And here is to be noted that if by the point A, you draw vnto the line B C a parallel line, the ſame ſhal of neceſſitie either light vpō the point D, or vnder it, or aboue it. If it light vpō it, then is that manifeſt which is required: but if it light vnder it, then foloweth that abſurditie which Euclide putteth, namely, that the greater triangle is equall to the leſſe: which ſelfe ſame abſurditie alſo will follow, if it fall aboue the point D. As for example. *This Theoreme the conuerſe of the 37. propoſition.*

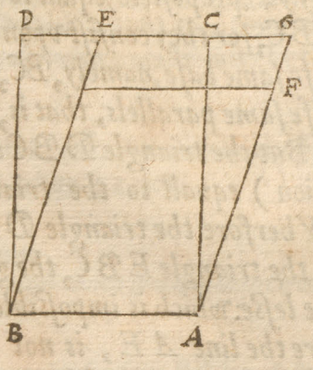
Suppoſe that theſe equall triangles *A B C* and D *B C* do conſiſt vppon one and the ſelfe ſame baſe *B C*, and on one and the ſame ſide. Then *I* ſay, that they are in the ſelfe ſame parallel lines, and that a right line ioyning together their toppes is a parallel to the baſe *B C*. Draw a right line frō *A* to *D*. Now if this be not a parallel to the baſe *B C*, let *A E* be a parallel vnto it, and let *A E* fall without the line *A D*. And produce the line *B D* till it concurre with the line *A E* in the pointe *E* and draw a line from *E* to *C*. Wherfore the triangle *A B C* is equal to the triangle *E B C*: but the triangle *A B C* is equall to the triangle *D B C*: Wherfore the triangle *E B C* is equall to the triangle D *B C*. Namely, the whole to the part: which is impoſſible. Wherfore the parallel line falleth not without the line *A D*. And Euclide hath proued that it falleth not within. Wherfore the line *A D* is a parallel vnto the line *B C*. Wherfore equall triangles which are on the ſelfe ſame ſide, and on one and the ſelfe ſame baſe, are alſo in the ſelfe ſame parallel lines: which was required to be proued.



An addition of Fluſſates.

The ſelfe ſame alſo followeth in parallelogrames. For if vpon the baſe AB be ſet on one & the ſame ſide theſe equal parallelogrames ABCD & A B G E, they ſhall of neceſſitie be in the ſelfe ſame parallel lines. For if not, but one of them is *An addition of Fluſſates.*

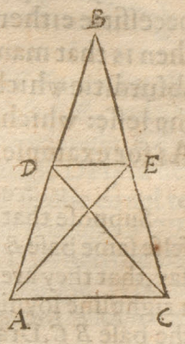
set eyther within or without, let the parallelograme B F being equall to the parallelograme A B C D be set within the same parallel lines: wherefore the same parallelograme B F beyng equall to the parallelograme A B C D (by the 35. proposition) shall also be equall to the other parallelograme A B G E (by the first common sentence) For the parallelograme A B G E is by supposition equall to the parallelogramme A B C D: whetfore the parallelograme B F being equall to the parallelograme A B G E, the parte shall bee equal to the whole, which is absurde. The same inconuenience also will followe, if it fall without, VVherefore it can neither fall within nor without. VVherfore equall parallelogrames beyng vpon one and the selfe same base and on one and the same side, are also in the selfe same parallel lines.



An addition of Campanus.

An addition of Campanus. *If a right line deuide two sides of a triangle into two equall partes: it shall be equidistant vnto the third side.*

Suppose that there be a triangle A B C: and let there bee a right lyne D E, which let deuide the two sides A B and B C into two equall partes in the pointes D and E. Then I say, that the line D E is a parallel to the line A C. Drawe these two lines A E and D C. Now then imagining a line to be drawne by the point E parallel to the line A B, the triangle B D E shall (by the 38. proposition) bee equall to the triangle D A E (for their two bases A D and D B are put to be equall) And by the same reason the triangle B D E is equall to the triangle C E D. VVherefore (by the first common sentēce) the triangles E A D and E C D are equall, and they are erected on one and the selfe same base, namely, DE, and on one and the same side. VVherefore (by the 39. proposition) they are in the selfe same parallel lines, and the line which ioyneth together their toppes is a parallel to their base. VVherfore the lynes DE and A C are parallels: which was required to be proued.

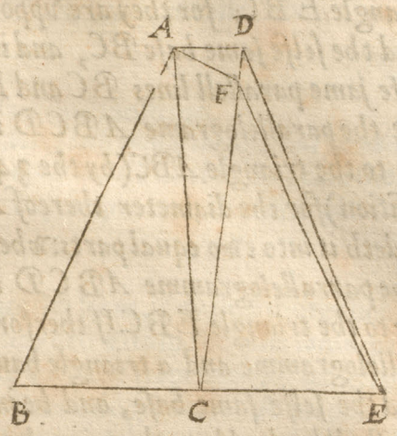


The 30. Theoreme. The 40. Proposition.

Equall triangles consisting vpon equall bases, and in one and the same side: are also in the selfe same parallel lines.

Suppose that these equall triangles A B C and C D E do consist vppon equall bases, that is, vppon B C and C E, and on one and the same syde, namely, on the side of A. Then I say that they are in the selfe same parallel

lell lines. Drawe by the first peticion a right line from the point A to the pointe D. Now I say that AD is a parallel line to BE. For if not, then (by the 31. proposition) by the point A drawe vnto the line BE a parallel line AF. And drawe a right line from the point F to the pointe E. VVherfore (by the 38. propositiõ) the triangle BAC is equall to the triangle CFE: for they consist vpon equal bases, that is BC and CE, and are in the selfe same parallel lines, namely, BE and AF. But by supposition the triangle ABC is equal to the triangle CDE. VVherfore the triangle DCE is equall to the triangle FCE, namely, the greater vnto the lesse, which is impossible. VVherfore AF is not a parallel line to BE. And in like sorte may we proue that no other line besides AD is a parallell line to BE. VVherfore AD is a parallel lyne to BE Equall triangles therfore consisting vppon equall bases, and in one and the same side: are also in the selfe same parallel lines: which was required to be proued.



Construction.

Demonstration leading to an absurditie.

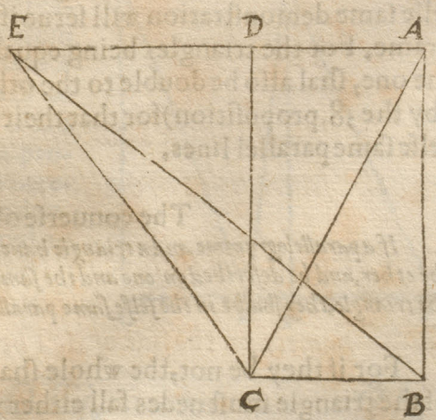
This proposition is the conuerse of the 38. propositiõ.

This proposition is the conuerse of the 38. proposition. And in this as in the former proposition, if the parallel line drawen by the point A, should not passe by the point D, it must passe eyther beneath it, or aboue it. Euclide setteth forth onely the absurdity which should follow if it passe beneath it: but the selfe same absurditie also wil follow if it should passe aboue it: as it is not hard to see by the gathering thereof in the former proposition. And therefore here I omitte it.

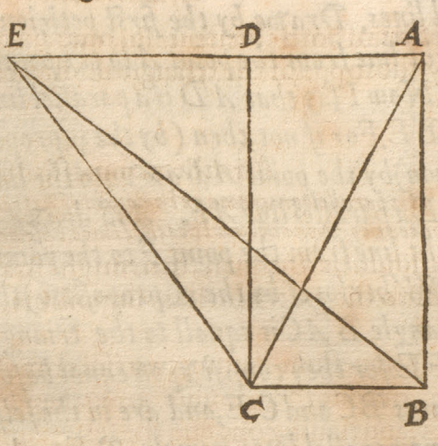
The 31. Theoreme. The 41. Proposition.

If a parallelograme & a triangle haue one & the selfe same base, and be in the selfe same parallel lines: the parallelograme shalbe double to the triangle.

Vppose that the parallelograme ABCD and the triangle EBC haue one & the same base, namely, BC, and let thẽ be in the selfe same parallel lines, namely, BC & AE. Thẽ I say, that the parallelograme ABCD is double to the triangle BEC. Draw (by the first peticion) a ryght line from the pointe A to ỹ pointe C. VVherfore (by the 37. propo-



Demonstration.



sition) the triangle ABC is equall to the triangle EBC: for they are vppon one and the selfe same base BC, and in the selfe same parallell lines BC and EA: but the parallelograme ABCD is double to the triangle ABC (by the 34. proposition) for the diameter thereof AC deuideth it into two equal parts: wherfore the parrallelogramme ABCD is double to the triangle EBC. If therfore a parallelogramme and a triangle haue one and the selfe same base, and be in the selfe same parallels, the parallelograme shall be double to the triangle: which was required to be proued.

Two cases in this proposition.

This proposition hath two cases. For the base beyng one, the triangle may haue his toppe without the parallelograme, or within. The first case is demonstrated of the author. The second case is thus.

Suppose that there be a parallelograme *ABCD*, and let the triangle be *ECD*, either of which let haue one and the selfe same base, namely, *CD*, and let them be in the selfe same parallel lines *CD* and *AB*, and let the toppe of the triangle *ECD*, namely, the point *E*, be within the parallelograme *ABCD*. Then *I* say that the parallelograme *ABCD* is double to the triangle *ECD*. Draw a right line frō the point *A* to the point *D*. Now forasmuch as the parallelograme *ABCD* is double to the triangle *ACD* (by the 34. proposition): and the triangle *ADC* is equall to the triangle *EDC* (by the 37. proposition). Therfore the parallelogramme *ABCD* is double to the triangle *ECD*: which was required to be proued.

A corollary.

By this proposition it is manifest that if the base be doubled, the triangle erected vppon it shalbe equall to the parallelogramme.

The selfe same demonstration will serue if the triangle & the parallelograme be vpon equall bases.

And if the bases of the triangle and of the parallelogramme be equall, the selfe same demonstration will serue if you drawe the diameter of the parallelograme. For the triangles being equal, the parallelogramme which is double to the one, shal also be double to the other. And the triangles must nedes be equall (by the 38. proposition) for that their bases are equal, and for that they are in the selfe same parallel lines.

The conuerse of this proposition is thus.

If a parallelogramme and a triangle haue one and the selfe same base, or equall bases the one to the other, and be described on one and the same side of the base: if the parallelogramme be double to the triangle, they shalbe in the selfe same parallel lines.

The conuerse of this proposition.

For if they be not, the whole shalbe equall to his parte. For then the toppe of the triangle must nedes fall either within the parallel lines or without. And whether

whether of both soeuer it do, one and the selfe same impossibilitie will follow, if by the toppe of the triangle be drawen vnto the base a parallel line.

An other conuerse of the same proposition.

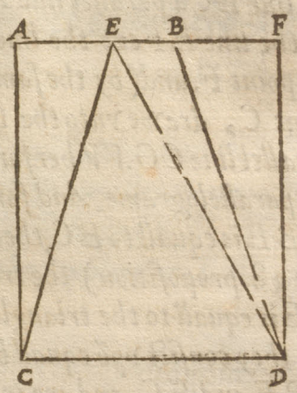
If a parallelogramme be the double of a triangle, being both within the selfe same parallel lines: then are they vpon one and the selfe same base, or vpon equall bases. For if in that case their bases should be vnequal, then might straight way be proued, that the whole is equall to his part: which is impossible.

An other conuerse of the same propositiõ.

A trapesium hauing two sides onely parallel lines, is eyther more then double, or lesse then double to a triangle contayned within the selfe same parallel lines, and hauing one and the selfe same base with the trapesium, or table. Iust the double it cannot be, for then it should be a parallelogramme. A trapesium hauing two sides parallels hath of necessitie the one of them longer then the other: for if they were equall then should the other two sides enclosing them be also equall (by the 33. proposition.) If the greater side of the trapesium be the base of the triangle, then shal the trapesium be lesse then the double of the triangle And if the lesse side of the trapesium be the base of the triangle then shall the trapesium be greater then the triangle.

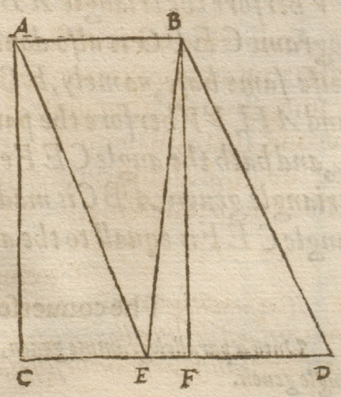
Comparison of a triangle and a trapesium being vpon one & the selfe same base, and in the selfe same parallel lines.

For suppose that *A B C D* be a trapesium, and let two sides thereof, namely, *A B* and *C D* be parallel lines, and let the side *A B* be lesse then the side *C D*, & produce the side *A B* infinitlye on the side *B* to the point *F*. And let the triangle *E C D* haue one and the selfe same base with the trapesium, namely, the line *C D*. Then I say that the trapesium *A B C D* is lesse thē the double of the triangle *E C D*. For put the line *A F* equall to the line C D (by the 3. propositió) and draw a line from D to *F*. Wherefore *A C D F* is a parallelogramme (by the 33. proposition). Wherefore (by the 34 proposition) it is double to the triangle *E C D*. But the trapesium *A B C D* is a part of the parallelogramme *A C D F*. Wherefore the trapesium *A B C D* is lesse then the double of the triangle *E C D*: which was required to be proued.



When the greater side of the trapesium is the base of the triangle.

Agayne let the triangle haue to his base the side *A B*. Then I say that the trapesium *A B C D* is greater then the double of the triangle *A E B*. For from the side *C D* cut of the line *C F* equall to the line *A B* (by the 3. proposition). And draw a line from *B* to *F*. Wherfore (by the 33. proposition) *A B C F* is a parallelogramme: and therefore is (by the 34. proposition) double to the triangle *A E B*. Wherefore the trapesium *A B C D* is more then the double of the triangle *A E B*: which was required to be proued.



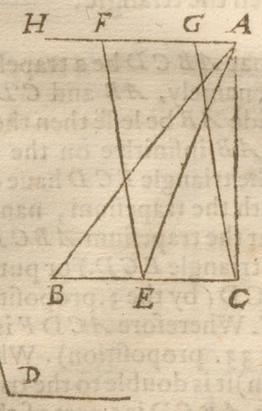
When the lesse side is the base.

The 11. Probleme. The 42. proposition.

Vnto a triangle geuen, to make a parallelograme equal, whose angle shall be equall to a rectiline angle geuen.

Construction.

Suppose that the triangle geuen be A B C, and let the rectiline angle geuen be D. It is required that vnto the triangle A B C there be made a parallelograme equall, whose angle shal be equall to the rectiline angle geuen, namely, to the angle D. Deuide (by the 10. propositiō) the line B C into two equall partes in the pointe E. And (by the first peticion) draw a right line from the point A to the point E. And (by the 23. proposition) vpon the right line geuen E C, and to the point in it geuen E, make the angle C E F equal to the angle D. And (by the 31. proposition) by the point A draw vnto the line E C a parallel line A H: and let the line E F cut the line A H in the point F. and (by the same) by the point C, drawe vnto the line E F a parallel line C G. Wherfore F E C G is a parallelograme. And forasmuche as B E is equall to E C, therfore (by the 38. proposition) the triangle A B E is equall to the triangle A E C, for they consist vpō equall bases that is B E and E C, and are in the selfe same parallel lines, namely, B C and A H. Wherfore the triangle A B C is double to the triangle A E C. And the parallelograme C E F G is also double to the triangle A E C: for they haue one & the selfe same base, namely, E C: and are in the selfe same parallel lines, that is, E C and A H. Wherfore the parallelograme F E C G is equall to the triangle A B C, and hath the angle C E F equall to the angle geuen D. Wherefore vnto the triangle geuen A B C is made an equall parallelograme, namely, F E C G, whose angle C E F is equall to the angle geuen D: which was required to be done.



Demonstration

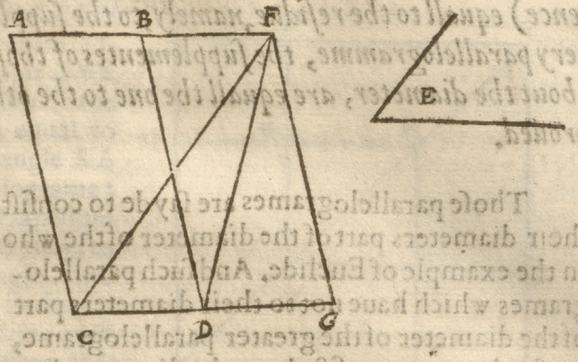
The conuerse of this proposition after Pelitarius.

The conuerse of this former proposition.

Vnto a parallelogramme geuen, to make a triangle equall, hauyng an angle equall to a rectiline angle geuen.

Suppose that the parallelograme geuen be *A B C D*, and let the angle geuen be *E*. It is required vnto the parallelograme *A B C D* to make a triangle equall hauyng an angle

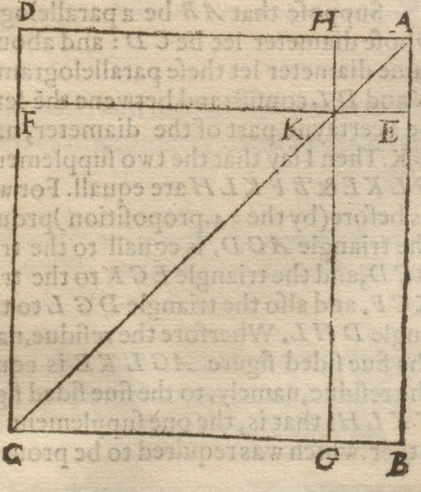
angle equal to the angle *E*. Vpon the line *C D* and to the pointe in it *C*, deſcribe (by the 23. propoſition) an angle equall to the angle *E*: which let be *D C F*: aud let the line *C F* cut the line *A B* being produced, in the point *F*: and produce the line *C D* (which is parallel to the line *A F*) to the point *G*. And put the line *D G* equall to the line *C D* and draw a line from *F* to *G*. Then *I* ſay that the triangle *C F G* is equal to the parallelograme *ABCD*. For foraſmuch as (by the 38. propoſition) the whole triangle *C F G* is double to the triangle *C D F*. Aud (by the 41. propoſition) the parallelograme *A B C D* is double to the ſame triangle *C D F*: therfore the parallelograme *A B C D* and the triangle *C F G* are equall the one to the other: which was required to be done.



The 32. Theoreme. The 43. Propoſition.

In euery parallelograme, the ſupplementes of thoſe parallelogrammes which are about the diameter, are equall the one to the other.

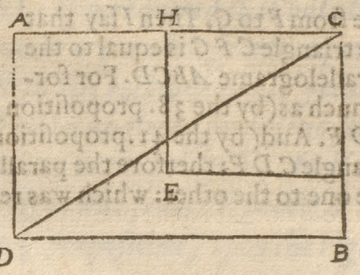
SVppoſe that A B C D be a parallelograme, and let the diameter therof be A C: and about the diameter A C let theſe parallelogrames E H and G F conſiſt: and let the ſupplementes be B K and K D. Then I ſay that the ſupplement B K is equall to the ſupplement K D. For foraſmuch as A B C D is a parallelograme and the diameter therof is A C, therfore (by the 34. propoſition) the triangle A B C is equall to the triangle A D C. Agayne foraſmuch as A E K H is a parallelograme, and the diameter therof is A K, therfore (by the ſame) the triangle A E K is equall to the triangle A H K. And by the ſame reaſon alſo the triangle K F C is equall to the triangle K G C. And foraſmuch as the triangle A E K is equall to the triangle A H K, and the triangle K F C to the triangle N G C, therfore the triangles A E K and K G C are equall to the triangles A H K and K F C: and the whole triangle A B C is equall to the whole triangle A D



C: Wherfore the residue, namely, the supplement B K is (by the 3. common sentence) equall to the residue, namely, to the supplement K D. VVherefore in euery parallelogramme, the supplementes of those parallelogrammes whiche are about the diameter, are equall the one to the other: whiche was required to be proued.

How parallelogrammes are sayde to consiste about a diameter.

Those parallelograms are sayde to consist about a diameter which haue to their diameters part of the diameter of the whole and great parallelograme, as in the example of Euclide. And such parallelograms which haue not to their diameters part of the diameter of the greater parallelograme, are sayde not to consist about the diameter. For thē the diameter of the greater parallelograme cutteth the side of the lesse cōtayned wythin it. As in the parallelogramme *A B*, the diameter *C D*, cutteth the side *E H* of the parallelogramme *C E*. Wherefore the parallelogramme *C E* is not about one and the selfe same diameter with the parallelogramme *C D*.



Supplements & Complementes.

Supplementes or Complementes are those figures which with the two parallelogrammes accomplish the whole parallelogramme. Although Pelitarius for distinction sake putteth a difference betwene Supplementes and Complementes. The parallelogrammes about the diameter he calleth Complementes, the other two figures he calleth Supplementes.

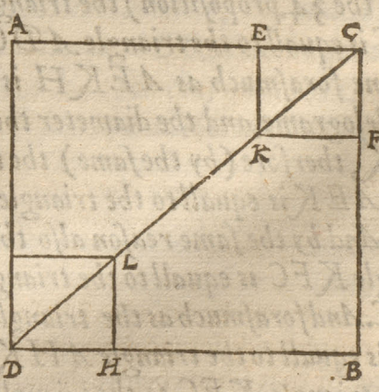
Three cases in this Theoreme.

This theoreme hath three cases onely. For the parallelogrammes which consist about the diameter, eyther touch the one the other in a point: or by a certayne parte of the diameter are seuered the one from the other: or els they cutte the one the other. For the first case is the example of Euclide before set. The second case is thus.

The first case.

The second case.

Suppose that *A B* be a parallelograme: whose diameter let be *C D*: and aboute the same diameter let these parallelogrammes *C K* and *D L* consist: and betwene thē let there be a certayne part of the diameter, namely, *L K*. Then I say that the two supplementes *A G L K E* & *B F K L H* are equall. For we may as before (by the 34. proposition) proue that the triangle *A C D*, is equall to the triangle *B C D*, and the triangle *E C K* to the triangle *K C F*, and also the triangle *D G L* to the triangle *D H L*. Wherfore the residue, namely, the fiue sided figure *A G L K E* is equall to the residue, namely, to the fiue sided figure *B F K L H*: that is, the one supplement to the other. which was required to be proued.

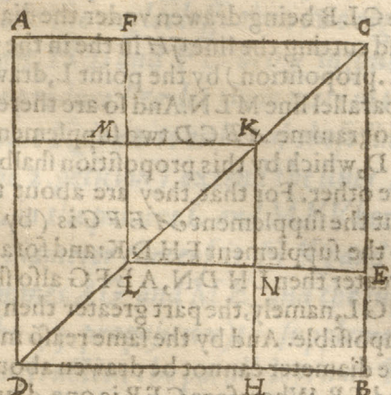


The third case.

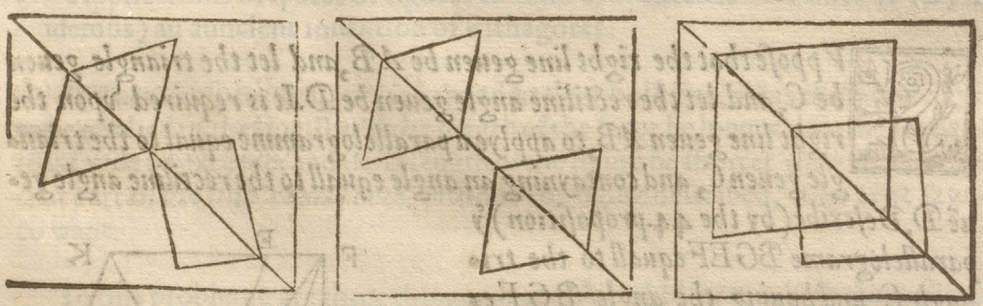
But now suppose *A B* to be a parallelogramme, and let the diameter thereof be *C D*: and let the one of the parallelogrammes about it be *E C F L*, and let the other be

DGK*H*,

DGKH, of which let the one cut the other. Then I say that the supplementes *FG* and *EH* are equall. For forasmuch as the whole triangle *DGK* is equal to the whole triangle *DHK* (by the 34 proposition), and part also of the one, namely, the triangle *KLM* is equal to part of the other, namely, to the triangle *KLN* (by the same), for LK is a parallelograme: therefore the residue, namely, the Trapesium *DLNH* is equall to the residue, namely, to the trapesiũ *DLMG*: but the triangle *ADC* is equal to the triangle *BCD*, and in the parallelograme *EF*, the triangle *FCL* is equall to the triangle *ECL*, and the trapesium *DGML* is (as it hath bene proued) equall to the trapesium *DHNL*. Wherefore the residue, namely, the quadrilater figure *GF* is equall to the residue, namely, to the quadrilater figure *EH*, that is, the one supplement to the other: which was required to be proued.



This is to be noted that in ech of those three cases it may so happen, that the parallelogrammes aboute the diameter shall not haue one angle common wyth the whole parallelogramme, as they haue in the former figures. But yet though they haue not, the selfe same demonstration wil serue, as it is playne to see in the figures here vnderneath put. For alwayes, if from thinges equall be taken away thinges equall, the residue shalbe equall.



This proposition Pelitarius calleth Gnomicall, and misticall, for that of it (sayth he) spring infinite demonstrations, and vses in geometry. And he putteth the conuerse thereof after this manner.

This proposition called Gnomical and misticall.

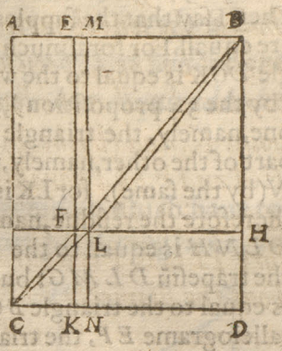
If a parallelogramme be deuided into two equall supplementes, and into two complements whatsoeuer: the diameter of the two complementes shall be set directly, and make one diameter of the whole parallelogramme.

The conuerse of this proposition.

Here is to be noted as I before admonished that Pelitarius for distinction sake putteth a difference betwene supplementes and complementes, which difference, for that I haue before declared, I shall not neede here to repete agayne.

Suppose that there be a parallelogramme *ABCD*, whose two equall supplements let be *AEFG* and *FHDK*, and let the two complementes thereof be *GFCK* and *EBFH*: whose diameters let be *CF* and *FB*. Then I say that *CFB* is one right line, and is the diameter of the whole parallelogramme *ABCD*: for if it be not, then is there an

other diameter of the whole parrallelogramme, which let be C L B being drawen vnder the diameters C F and F B, and cutting the line *G H* in the in the point L. And (by the 31. propoſition) by the point L, draw vnto the line *A C* a parallel line M L N. And ſo are there in the whole parallelogramme *A B C D* two ſupplements A M G L and L H N D, which by this propoſition ſhalbe equall the one to the other. For that they are about the diameter C L B. But the ſupplement *A E F G* is (by ſuppoſition) equall to the ſupplement F H D K: and foraſmuch as F H D K is greater then L H *D* N, A E F G alſo ſhalbe greater then A M G L, namely, the part greater then the whole: which is impoſſible. And by the ſame reaſō may it be proued, that the diameter cannot be drawen aboue the diameters C F and F B. Wherefore C F B is one diameter of the whole parallelogramme *A B C D*: which was required to be proued.



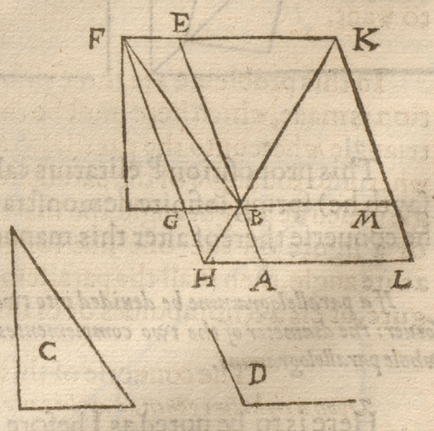
The 12. Probleme. The 44. Propoſition.

Vppon a right line geuen, to applye a parallelograme equall to a triangle geuen, and contayning an angle equall to a rectiline angle geuen.

Vppoſe that the right line geuen be A B, and let the triangle geuen be C, and let the rectiline angle geuen be D. It is required vpon the right line geuen A B, to applye a parallelogramme equal to the triangle geuen C, and contayning an angle equall to the rectiline angle geuē D.

Conſtruction.

Deſcribe (by the 44. propoſition) ỹ parallelograme B G E F equall to the triangle C, and hauing the angle B G F equall to the angle D. And vnto the line E B ioyne the line A B in ſuch ſort that they make both one right line. And extend the line F G beyond the point G to the poynte H. And (by the 31. propoſition) by the point A drawe to either of theſe lines B G and E F a parallel line A H. And (by the firſt peticion) drawe a right line from the point H to the point B. And foraſmuch as vpon the parallel lines A H and E F falleth a certayne right line H F, therefore (by the 29 propoſition) the angles A H F and H F E are equall to two right angles: wherefore the angles B H G and G F E are leſſe then two right angles: but if vpon two right lines fall a right line making the inward angles on one and the



ſame

ſame ſide leſſe then two right angles, thoſe right lines being infinitly produced ſhall at the length mete on that ſide in which are the angles leſſe then two right angles (by the 5. petition). Wherfore the lines HB and FE being infinitly produced will at the length mete. Let them be produced, & let them mete in the point K. And (by the 31 propoſition) by the point K draw to either of theſe lines EA and FH a parallel line KL. And (by the 2. petition) extend the lines HA and GB till they cõcurre with the line KL in the pointes L and M. Wherefore HLKF is a parallelogramme, and the diameter thereof is HK: and about the diameter HK are the parallelogrammes AG and ME, and the ſupplementes are LB and BF: wherefore (by the 43. propoſition) the ſupplement LB is equall to the ſupplement BF: but by conſtruction the parallelograme BF is equall to the triangle C: wherefore alſo the parallelogramme LB is equall to the triangle C. And foraſmuch as the line FH is a parallel to the line KL, and vpon them lighteth the line GM, therefore (by the 27. propoſition) the angle FGB is equall to the angle BML. But the angle FGB is equall to the angle D, therfore the angle BML is equal to the angle D. Wherfore vpõ the right line geuen AB is applied the parrallelograme LB, equal to the triangle geuen C, and contayning the angle BML equal to the rectilme angle geuen D: which was required to be done.

Demonſtration

Applications of ſpaces with exceſſes or wants an auncient inuention of Pithagoras.

Applications of ſpaces or figures to lines with exceſſes or wantes is (ſayth Eudemus) an auncient inuention of Pithagoras.

How a figure is ſayde to be applied to a line.

When the ſpace or figure is ioyned to the whole line, thẽ is the figure ſayd to be applied to the line. But if the length of the ſpace be longer then the line, thẽ it is ſayde to exceede: and if the length of the figure be ſhorter then the line, ſo that part of the line remayneth without the figure deſcribed, then is it ſayde to want.

Three thinges geuen in this propoſition.

In this probleme are three thinges geuen. A right line to which the application is made, which here muſt be the one ſide of the parallelogramme applied. A triangle whereunto the parallelogramme applied muſt bee equall: and an angle wherunto the angle of the parallelograme applied muſt be equall. And if the angle geuen be a right angle, thẽ ſhal the parallelograme applied be either a ſquare, or a figure on the one ſide longer. But if the angle geuen be an obtuſe or an acute angle, then ſhall the parallelograme applied be a Rhombus or diamond figure, or els a Rhomboides or diamondlike figure.

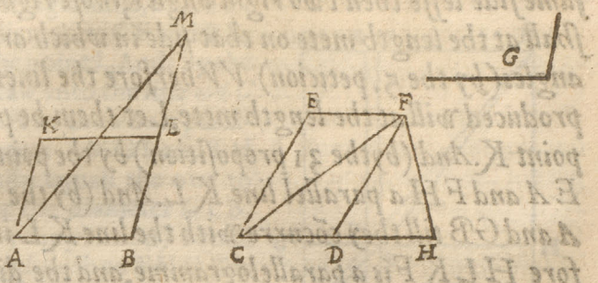
The conuerſe of this propoſition after Pelitarius.

The conuerſe of this propoſition.

Vpon a right line geuen, to applie vnto a parallelograme geuen an equall triangle hauyng an angle equall to an angle geuen.

Suppoſe that the right line geuen be *AB*, and let the parallelograme geuen be *CDEF*, and let the angle geuen be *G*. It is required vpon the line *AB* to deſcribe a triãgle equall to the parallelograme *CDEF*, hauing an angle equall to the angle *G*. Drawe the diameter *CF* & produce *CD* beyond the point D to the point *H*. And put the line

D H equall to the line *C D*. And draw a line from *F* to *H*. Now thē (by the 41. propoſition) the triāgle *C H F* is equall to the parallelograme *C D E F*. And (by this propoſition) vppon the line *A B* deſcribe a parallelograme *A B K L* equall to the triangle *C H F*, hauing the angle *A B L* equal to the angle geuen *G*: and produce the line *B L* beyonde the pointe L to the point M. And put the line L M equall to the line B L, and draw a line from *A* to M. Then I ſay that vpon the line *A B* is deſcribed the triangle *A B* M, which is ſuch a triangle as is required. For (by the 41. propoſition) the triangle *A B* M is equal to the parallelogramme *A B K L* (for that they are betwene two parallel lines *B* M and *A K*, & the baſe of the triangle is double to the baſe of the parallelogramme): but *A B K L* is by conſtruction equall to the triangle *C H F*: and the triangle *C H F* is equall to the parallelograme *C D E F*. Wherfore (by the firſt common ſentence) the triangle *A B* M is equall to the parallelograme geuen *C D E F*, and hath his angle *A* B M equal to the angle geuen *G*: which was required to be done.



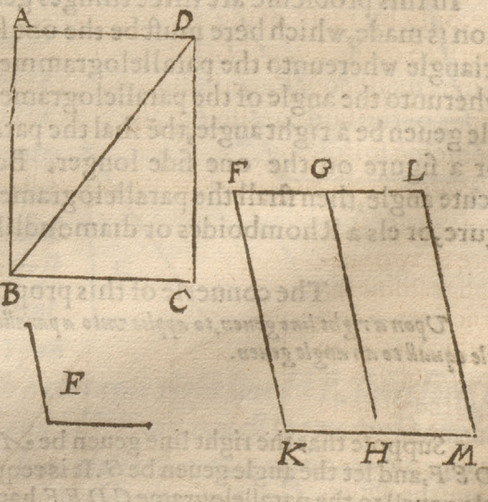
The 13. Probleme. The 45 Propoſition.

To deſcribe a parallelograme equal to any rectiline figure geuen, and contayning an angle equall to a rectiline angle geuē.

VPpoſe that the rectiline figure geuen be A B C D, and let the rectiline angle geuē be E. It is required to deſcribe a parallelograme equall to the rectiline figure geuen A B C D, and contayning an angle equal to the rectiline angle geuen E. Draw (by the firſt peticion) a right line frō the point D to the point B. And (by the 42. propoſition) vnto the triangle A B D deſcribe an equall parallelograme F H, hauing his angle F K H equall to the angle E. And (by the 44. of the firſt) vpō the right line G H apply the parallelogramme G M equal to the triangle D B C, hauing his angle G H M equall to the angle E. And foraſmuch as eyther of thoſe angles H K F and G H M is equall to the angle E: therefore the angle H K F is equall to the angle G H M: put the angle K H G common to them both, wherfore the angles F K H and K H G are equall to the angles K H G and G H M. but the angles F K H and K H G are (by the 29. propoſition) equall to two right angles. Wherfore the angles K H G and G H M are equall to two right

Conſtruction.

Demonſtration.



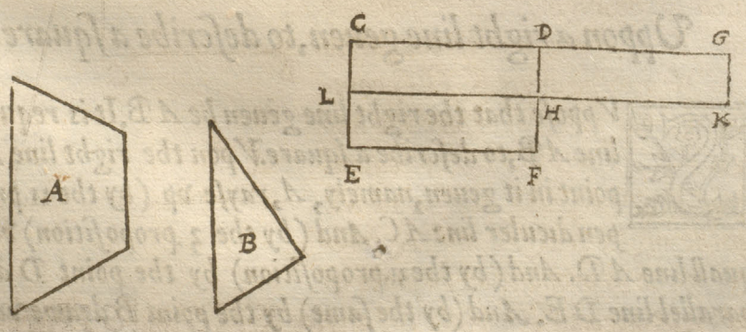
right angles. Now then vnto a right line G H, and to a point in the same H, are drawen two right lines K H and H M not both on one and the same side, making the side angles equall to two right angles. Wherfore (by the 14. proposition) the lines K H and H M make directly one right line. And forasmuch as vpon the parallel lines K M and F G falleth the right line H G, therefore the alternate aagles M H G and H G F are (by the 29 proposition) equall the one to the other: put the angle H G L common to them both. Wherfore the angles M H G and H G L are equall to the angles H G F and H G L. But the angles M H G & H G L are equall to two right angles (by the 29. proposition). Wherfore also the angles H G F and H G L are equall to two right angles. Wherfore (by the 14. proposition) the lines F G and G L make directly one right line. And forasmuch as the line K F is (by the 34. proposition) equal to the lyne H G, and it is also parallel vnto it: and the line H G is (by the same) equall to the line M L, therfore (by the first common sentence) the line F K is equall to the lyne M L, and also a parallel vnto it (by the 30. proposition). But the right lynes K M and F L ioyne them together. Wherfore (by the 33. proposition) the lines K M and F L are equall the on to the other and parallel lines. Wherfore K F L M is a parallelograme. And forasmuch as the triangle A B D is equal to the parallelograme F H, and the triangle D B C to the parallelogramme G M: therfore the whole rectiline figure A B C D is equall to the whole parallelograme K F L M. Wherfore to the rectiline figure geuen A B C D is made an equall parallegrame K F L M, whose angle F K M is equal to the angle geuen, namely, to E: which was required to be done.

The rectiline figure geuē is in the example of Euclide is a parallelograme. But if the rectiline figure be of many sides, as of 5. 6. or mo, thē must you resolue the figure into his triangles, as hath bene before taught in the 32. proposition. And thē apply a parallelograme equal to euery triangle vpon a line geuē, as before in the example of the author. And the same kind of reasoning wil serue that was before, only by reasō of the multitude of triangles, you shall haue neede of oftener repeticiō of the 29. and 14. propositiōs to proue that the bases of al the parallelogrames made equall to all the triangles make one right line, and so also of the toppes of the said parallelogrames. Pelitarius addeth vnto this proposition this Probleme following.

Two vnequall rectiline superficieces beyng geuen, to find out the excesse of the greater aboue the lesse.

An addition of Pelitarius.

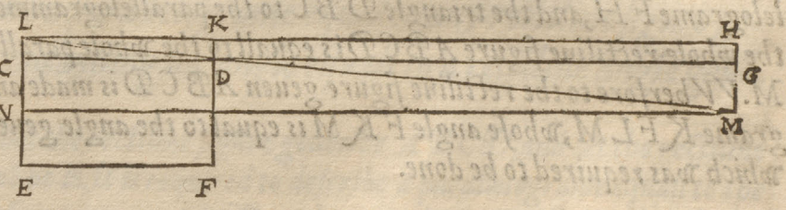
Suppose that there be two vnequall rectiline superficieces *A* & *B* of which let *A* be the greater. It is required to finde out the excesse of the superficies *A* aboue the superficieces *B*. Describe (by the 44.



propoſition) the parallelograme *C D E F* equall to the rectiline figure *A*, contayning a right angle. And produce the line *C D* beyond the point *D* to the point *G*: & put the line *D G* equall to the line *C D*. And againe (by the 44. propoſition) vpon the line *D G* deſcribe the parallelograme *D G H K* equall to the rectiline figure *B*, and hauyng the angle *D G K* a right angle. And produce the line K H beyond the point H, vntill it cutte the line *C E* in the point L. Then I ſay that H L E F, is the exceſſe of the rectiline figure *A* aboue the rectiline figure *B*. For firſt that C G K L is a parallelogramme it is manifeſt, neither nedeth it to be demonſtrated. And foraſmuch as the lines *C D* and *D G* are by ſuppoſition equal and either of them is a parallel to K L, therfore (by the 36. propoſition) the two parallelogrames *C H* and *D K* are equall. And foraſmuch as D K is ſuppoſed to be equall to the rectiline figure *B*, *C H* alſo ſhall be equall to the ſame rectiline figure *B*. Wherfore foraſmuch as the whole parallelograme *C F* is equall to the rectiline figure *A*, and L F is the exceſſe of *C F* aboue D L or D K, it followeth that L F is the exceſſe of the rectiline figure *A* aboue the rectiline figure *B*: whiche was required to be done.

An other more redy way.

Let the parallelograme C *D E F* remayne equall to the rectiline figure *A*, & produce the line *C D* beyond the point *D* to the pointe G. And vpon the line *D G* deſcribe the parallelograme D G H K equall to the rectiline figure B. And produce the lines E C & H K beyond the points C and K till they concurre in the poiut L. And by the pointe D draw the diameter L D M, which let cutte the line H G beyng produced beyonde the pointe G in the point M, & by the pointe M drawe vnto the line H L a parallel M N cuttyng the line E L in the pointe N: and by that meanes is H L M N a parallelograme. Then *I* ſay that N F is the evceſſe of the rectiline figure *A* aboue the rectiline figure *B*. For foraſmuch as the parallelograme H D is equall to the rectiline figure *B*, & the ſupplementes H D and D N ar (by the 43. propoſition) equall: therfore D N alſo is equall to the rectiline figure B, which rectiline figure D N being taken away frō the parallelograme C F (which is ſuppoſed to be equall to the rectiline figure *A*) the reſidue N F ſhall be the ezceſſe of the rectiline figure *A* aboue the rectiline figure *B*: which was required to be done.

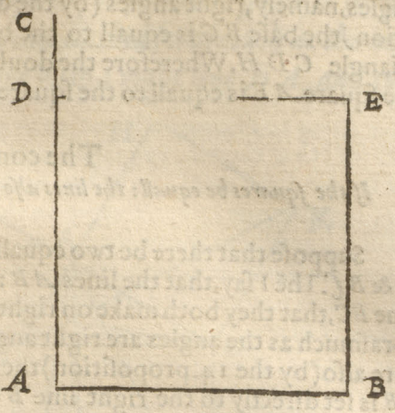


The 14. Probleme. The 46. Propoſition.

Vppon a right line geuen, to deſcribe a ſquare.

SVppoſe that the right line geuen be A B. It is required vpō the right line A B, to deſcribe a ſquare. Vpon the right line A B, and from a point in it geuen, namely, A, rayſe vp (by the 11. propoſition) a perpendiculer line A C. And (by the 3. propoſition) vnto A B put an equall line A D. And (by the 31. propoſition) by the point D drawe vnto A B a parallel line D E. And (by the ſame) by the point B drawe vnto A D a parallel

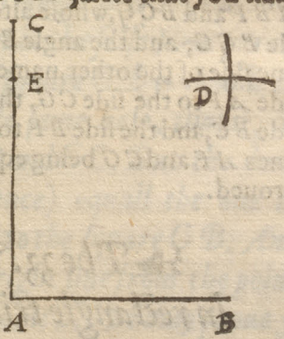
line



line BE. VVherefore ADEB is a parallelogramme. VVherefore the line AB is equall to the line DE, and the line AD to the line BE: but the line AB is equall to the line AD wherefore these fower lines BA, AD, DE, EB, are equall the one to the other. VVherefore the parallelogramme ADEB consisteth of equall sides. I say also that it is rectangle. For forasmuch as vpon the parallel lines AB and DE falleth a right line AD: therefore (by the 29. proposition) the angle: BAD and ADE are equal to two right angles: but the angle BAD is a right angle. VVherfore the angle ADE also is a right angle. But in parallelogrames the sides and angles which are opposite are equall the one to the other (by the 34 proposition). VVherefore the two opposite angles ABE and BED are ech of them a right angle. VVherefore the parallelograme ABED is rectangle: & it is also proued that it is equilater. VVherfore it is a square, & it is described vpon the right line geuen AB: which was required to be done.

To describe a square mechanically.

This is to be noted that if you will mechanically and redily, not regarding demõstration describe a square vpon a line geuen, as vpon the line *AB*, after that you haue erected the perpendiculer line *CA* vpon the line *AB*, and put the line *AE* equall to the line *AB*: then open your compasse to the wydth of the line *AB* or *AE*, & set one foote thereof in the point *E*, and describe a peece of the circumference of a circle: and againe make the centre the point *B*, and describe also a piece of the circumference of a circle, namely, in such sort that the peece of the circumferẽce of the one may cut the peece of the circumference of the other, as in the point *D*: and from the point of the intersection, draw vnto the points *E* & *B* right lines: & so shalbe described a square. As in this figure here put, wherin *I* haue not drawen the lines *ED* and *DB*, that the peeces of the circumference cutting the one the other might the plainlier be sene.

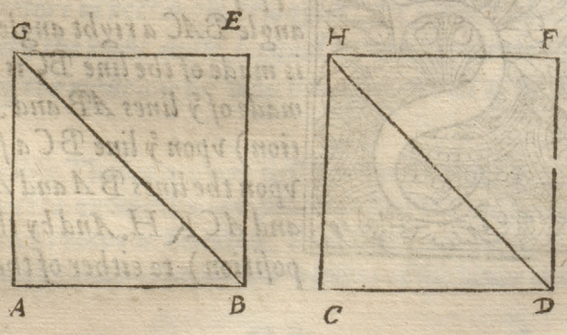


An addition of Proclus.

If the lines vpon which the squares be described be equall, the squares also are equall.

An addition of Proclus.

Suppose that these right lines *AB* and *CD* be equall, & vpon the line *AB* describe a square *ABEG*: and vpon the line *CD* describe a square *CDHF*. Then *I* say that the two squares *ABEG* & *CDHF* are equal. For draw these right lines *GB* and *HD*. And for as much as the right lines *AB* and *CD* are equall, & the lines *AG* and *HC* are also equall, and they contayne eqaul

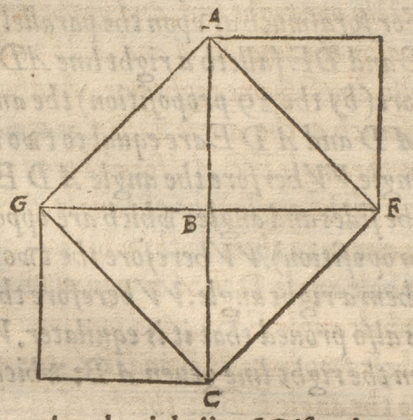


angles, namely, right angles (by the definition of a square) therefore (by the 4. proposition) the base *B G* is equall to the base *H D*. And the triangle *A B G* is equall to the triangle *C D H*. Wherefore the doubles of the saide triangles are equall. Wherefore the square *A E* is equall to the square *C F*: which was required to be proued.

The conuerse thereof is thus.

The conuerse thereof. *If the squares be equall: the lines also vppon which they are described are equall.*

Suppose that there be two equall squares *A F* and *C G* described vpon the lines *A B* & *B C*. Thē I say, that the lines *A B* and *B C* are equall. Put the line *A B* directly to the line *BC*, that they both make on right line. And forasmuch as the angles are right angles, therfore also (by the 14. proposition) the right line *FB* is set directly to the right line *B G*. Drawe these right lines *FC*, *AG*, *AF*, and *C G*. Now for asmuch as the square *AF* is equal to the square *CG*, the triangle also *A F B* shalbe equall to the triangle *C B G*: put the triangle *BC F* cōmon to them both. Wherfore the whole triangle *AC F* is equall to the whole triangle *C F G*. Wherefore the line *A G* is a parallel vnto the line *C F* (by the 38. propositiō): for the triangles consist vpon one and the selfe same base, namely *C F*. Againe forasmuch as either of these angles *AF G* & *CBG* is the halfe of a right angle, therfore (by the 17. proposition) the line *AF* is a parallel to the line *C G*. Wherfore the right line *AF* is equal to the right line *CG* (for the opposite sides of a parallelograme are equall). And forasmuch as there are two triangles *A B F* and *B C G*, whose alternate angles are equall, namely, the angle *A F B* to the angle *B G C*, and the angle *B A F* to the angle *B C G*, and one side of the one is equall to one side of the other, namely, the side which lieth betwene the equal angles, that is, the side *A F* to the side *C G*, therefore (by the 26. proposition) the side *A B* is equal to the side *B C*, and the side *B F* to the side *B G*. Wherefore it is proued that the squares of the lines *A F* and *C G* being equall, their sides also shalbe equall: which was required to be proued.



The 33. Theoreme. The 47. Proposition.

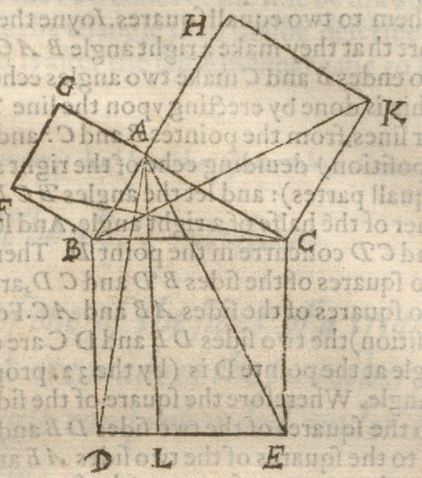
In rectangle triangles, the square whiche is made of the side that subtendeth the right angle, is equal to the squares which are made of the sides contayning the right angle.

Construction. *Vppose that A B C be a rectangle triangle, hauyng the angle BAC a right angle. Thē I say ẏ the square which is made of the line BC is equall to the squares which are made of ẏ lines AB and AC. Describe (by ẏ 46. proposition) vpon ẏ line B C a square BDCE, and (by ẏ same) vpon the lines B A and A C describe the squares ABFG and A C K H. And by the point A draw (by the 31. proposition) to either of these lynes B D and C E a parallel*

line

line AL. And (by the first petition) draw a right lyne from the point A to the point D, and an other from the point C to the point E. And forasmuch as the angles BAC and BAG are right angles, therfore vnto a right line BA, and to a point in it geuen A, are drawen two right lines AC and AG, not both on one and the same side, makyng the two side angles equall to two right angles: wherfore (by the 14. proposition) the lines AC and AG make directly one right line. And by the same reason the lines BA and AH make also directly one right line. And forasmuch as the angle DBC is equall to the angle FBA (for either of the is a right angle) put the angle ABC common to them both: wherfore the whole angle DBA is equall to the whole angle FBC. And forasmuch as these two lines AB and BD are equal to these two lines BF and BC, the one to the other, and the angle DBA is equal to the angle FBC: therfore (by the 4. proposition) the base AD is equall to the base FC, and the triangle ABD is equall to the triangle FBC. But (by the 31. proposition) the parallelogramme BL is double to the triangle ABD, for they haue both one and the same base, namely, BD, and are in the selfe same parallel lynes, that is, BD and AL and (by the same) the square GB is double to the triangle FBC, for they haue both one and the selfe same base, that is, BF, and are in the selfe same parallel lynes, that is, FB and GC. But the doubles of thinges equall, are (by the sixte common sentence) equall the one to the other. Wherfore the parallelograme BL is equall to the square GB. And in like sorte if (by the first petition) there be drawen a right line from the point A to the point E, and an other from the point B to the point K, we may proue ỹ the parallelograme CL is equal to the square HC. Wherfore the whole square BDEC is equall to the two squares GB and HC. But the square BDEC is described vpon the line BC, and the squares GB and HC are described vppon the lines BA & AC: wherfore the square of the side BC, is equal to the squares of the sides BA and AC. Wherefore in rectangle triangles, the square whiche is made of the side that subtendeth the right angle, is equal to the squares which are made of the sides contayning the right angle: which was required to be demonstrated.

Demonstration.



This most excellent and notable Theoreme was first inuented of the greate philosopher Pithagoras, who for the exceeding ioy conceiued of the inuention therof, offered in sacrifice an Oxe, as recorde Hierone, Proclus, Lycius, & Vitruuius. And it hath bene commōly called of barbarous writers of the latter time Dulcarnon.

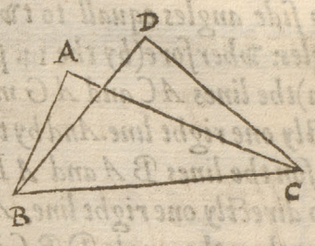
Pithagoras the first inuenter of this proposition.

An addition of Pelitarius.

An addition of Pelitarius.

To reduce two vnequall squares to two equall squares.

Suppose that the squares of the lines *AB* and *AC* be vnequall. It is required to reduce them to two equall squares. Ioyne the two lines *AB* and *AC* at their endes in such sort that they make a right angle *BAC*. And draw a line from *B* to *C*. Then vppon the two endes *B* and *C* make two angles eche of which may be equal to halfe a right angle (This is done by erecting vpon the line *BC* perpendiculer lines, from the pointes *B* and *C*: and so (by the 9. proposition) deuiding eche of the right angles into two equall partes): and let the angles *BCD* and *CBD* be either of thē halfe of a right angle. And let the lines *BD* and *CD* concurre in the point *D*. Then *I* say that the two squares of the sides *BD* and *CD*, are equall to the two squares of the sides *AB* and *AC*. For (by the 6 proposition) the two sides *DB* and DC are equall, and the angle at the pointe D is (by the 32. proposition) a right angle. Wherefore the square of the side *BC* is equal to the squares of the two sides *DB* and *DC* (by the 47. proposition): but it is also equall to the squares of the two sides *AB* and *AC* (by the self same proposition) wherfore (by the common sentence) the squares of the two sides *BD* and *DC* are equall to the squares of the two sides *AB* and *AC*: which was required to be done.



An other aditiō of Pelitarius.

An other addition of Pelitarius.

If two right angled triangles haue equall bases. the squares of the two sides of the one are equall to the squares of the two sides of the other.

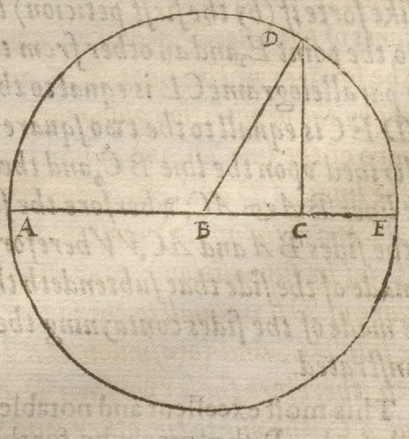
This is manifest by the former construction and demonstration.

An other addition of Pelitarius.

An other addition of Pelitarius.

Two vnequall lines being geuen, to know how much the square of the one is greater then the square of the other.

Suppose that there be two vnequal lines *AB* and *BC*: of which let *AB* be the greater. It is required to search out how much the square of *AB* excedeth the square of *BC*. That is *I* wil finde out the square, which with the square of the line *BC* shalbe equal to the square of the line *AB*. Put the lines *AB* and *BC* directly, that they make both one right line: and making the centre the point *B*, and the space *BA* describe a circle *ADE*. And produce the line *AC* to the circumference, and let it concurre with it in the point *E*. And vpon the lyne *AE* and frō the point *C* erect (by the 11. proposition) a perpendiculer line *CD*, which produce till it concurre with the circumference in the point *D*: & draw a line from *B* to *D*. Then *I* say, that the square of the line *CD*, is the excesse of the square of the line *AB* aboue the square of the line *BC*. For forasmuch as in the triangle *BCD*, the angle at the point *C* is a right angle, the square of the base *BD* is equall to the squares of the two sides *BC* and *CD* (by this 47. proposition). Wherefore also the square of the line *AB* is equall to the selfe same squares of the lines *BC* and *CD*. Wherefore the square of the line *BC* is so much lesse then the square of the line *AB*, as is the square of the line *CD*: which was required to search out.



An

An other addition of Pelitarius.

The diameter of a ſquare being geuen, to geue the ſquare thereof.

An other aditiō of Pelitarius.

This is eaſie to be done. For if vpon the two endes of the line be drawen two halfe right angles, and ſo be made perfect the triangle then ſhalbe deſcribed half of the ſquare: the other halfe whereof alſo is after the ſame manner eaſie to be deſcribed.

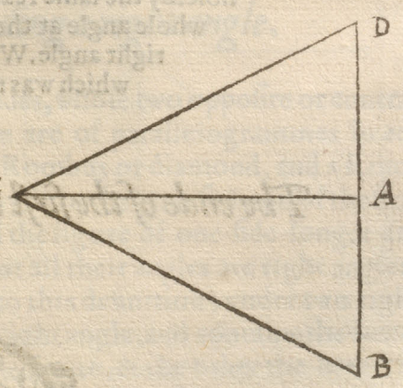
Hereby it is manifeſt, that the ſquare of the diameter is double to that ſquare whoſe diameter it is.

A Corrollary.

The 34. Theoreme. The 48. Propoſition.

If the ſquare which is made of one of the ſides of a triangle, be equall to the ſquares which are made of the two other ſides of the ſame triangle: the angle comprehended vnder thoſe two other ſides is a right angle.

SVppoſe that A B C be a triangle, and let the ſquare which is made of one of the ſides there, namely, of the ſide B C, be equall to the ſquares which are made of the ſides B A and A C. Then I ſay that the angle B A C is a right angle. Rayſe vp (by the 11. propoſitiō) from the point A vnto the right line A C a perpendicular line A D. And (by the thirde propoſition) vnto the line A B put an equall line A D. And by the firſt petition draw a right line from the point D to the point C. And foraſmuch as the line D A is equall to the line A B, the ſquare which is made of the line D A is equall to the ſquare whiche is made of the line A B. Put the ſquare of the line A C, common to them both. Wherefore the ſquares of the lines D A and A C are equal to the ſquares of the lines B A and A C. But (by the propoſition going before) the ſquare of the line D C is equal to the ſquares of the lines A D and A C. (For the angle D A C is a right angle) and the ſquare of B C is (by ſuppoſition) equall to the ſquares of A B and A C. Wherefore the ſquare of D C is equall to the ſquare of B C: wherefore the ſide D C is equall to the ſide B C. And foraſmuch as A B is equall to A D and A C is common to them both, therefore theſe two ſides D A and A C are equall to theſe two ſides B A and A C, the one to the other, and the baſe D C is equall to the baſe B C: wherefore (by the 8. propoſition) the angle D A C is equall to the angle B A C. But the angle D A C is a right angle, wherefore alſo the angle B A C is a right angle. If



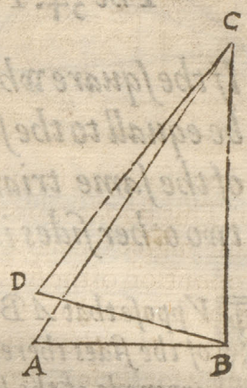
therefore the ſquare which is made of one of the ſides of a triangle, be equall to the ſquares which are made of the two other ſides of the ſame triangle, the angle comprehended vnder thoſe two other ſides is a right angle: which was required to be proued.

This propoſition is the conuerſe of the former.

This propoſition is the conuerſe of the former, and is of Pelitarius demonſtrated by an argument leading to an impoſſibilitie after this maner.

An other Demonſtration after Pelitarius.

Suppoſe that *ABC* be a triangle: & let the ſquare of the ſide *AC*, be equal to the ſquares of the two ſides *AB* and *BC*. Then *I* ſay that the angle at the point *B*, which is oppoſite to the ſide *AC*, is a right angle. For if the angle at the point *B* be not a right angle, then ſhal it be eyther greater or leſſe thē a right angle. Firſt let it be is greater. And let the angle *DBC* be a right angle, by erecting from the point *B* a perpendicular line vnto the line *BC* (by the 11. propoſition) which let be *BD*: and put the line *BD* equall to the lyne *AB* (by the thirde propoſition). And drawe a line from *C* to D. Now (by the former propoſition) the ſquare of the ſide *CD* ſhalbe equall to the ſquares of the two ſides *BD* and *BC*: wherefore alſo to the ſquares of the two ſides *BA* and *BC*. Wherefore the baſe *CD* ſhalbe equall to the baſe *CA*, when as their ſquares are equall: which is contrary to the 24. propoſition. For foraſmuch as the angle *ABC* is greater then the angle *DBC*, and the two ſides *AB* and *BC* are equall to the two ſides *DB* and *BC*, the one to the other, the baſe *CA* ſhall be greater then the baſe *CD*. It is alſo contrary to the 7. propoſition, for from the two endes of one & the ſame line, namely, frō the points *B* & *C* ſhould be drawn on one and the ſame ſide two lines *BD* and *DC* ending at the pointe *D*, equall to two other lines *BA* and *AC* drawen from the ſame endes and ending at an other point, namely, at *A*, which is impoſſible. By the ſame reaſon alſo may we proue that the whole angle at the pointe *B* is not leſſe then a right angle. Wherfore it is a right angle: which was required to be proued.



(∴)

The ende of the firſt booke of Euclides Elementes.

¶ The second booke of Euclides Elementes.

IN this second booke Euclide sheweth, what is a Gnomõ, and a right angled parallelogramme. Also in this booke are set forth the powers of lines, deuided euenly and vneuenly, and of lines added one to an other. The power of a line, is the square of the same line: that is, a square, euery side of which is equall to the line. So that here are set forth the qualities and proprieties of the squares and right lined figures, which are made of lines & of their parts. The Arithmetician also out of this booke gathereth many compendious rules of reckoning, and many rules also of Algebra, with the equatiõs therein vsed. The groundes also of those rules are for the most part by this second booke demonstrated. This booke moreouer contayneth two wonderfull propositions, one of an obtuse angled triangle, and the other of an acute: which with the ayde of the 47. proposition of the first booke of Euclide, which is of a rectangle triangle, of how great force and profite they are in matters of astronomy, they knowe which haue trauayled in that arte. VVherefore if this booke had none other profite beside, onely for these 2. propositions sake it were diligently to be embraced and studied.

The argument of the second booke.

What is the power of a line.

Many compẽdious rules of reckoning gathered out of this booke, and also many rules of Algebra.

Two wonderfull propositions in this booke.

The definitions.

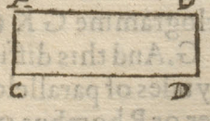
1. *Euery rectangled parallelogramme, is sayde to be contayned vnder two right lines comprehending a right angle.*

First definition.

A parallelogramme is a figure of fower sides, whose two opposite or contrary sides are equall the one to the other. There are of parallelogrammes fower kyndes, a square, a figure of one side longer, a Rombus or diamond, and a Romboides or diamond like figure, as before was sayde in the 33. definition of the first booke. Of these fower sortes, the square and the figure of one side longer are onely right angled Parallelogrammes: for that all their angles are right angles. And either of them is contayned (according to this definition) vnder two right lynes which concurre together, and cause the right angle, and containe the same. Of which two lines the one is the length of the figure, & the other the breadth. The parallelogramme is imagined to be made by the draught or motion of one of the lines into the length of the other. As if two numbers shoulde be multiplied the one into the other. As the figure A B C D is a parallelograme, and is sayde to be contayned vnder the two right lines A B and A C, which contayne the right angle B A C, or vnder the two right lines A C and C D, for they likewise contayne the right angle A C D: of which 2. lines the one, namely, A B is the length, and the other, namely, A C is the breadth. And if we imagine the line A C to be drawen or moued directly according to the lẽgth

What a parallelogramme is.

Fower kindes of parallelogrammes.



of the line A B, or contrary wiſe the line A B to be moued directly according to the length of the line A C, you ſhall produce the whole rectangle parallelogramme A B C D which is ſayde to be contayned of them: euen as one number multiplied by an other produceth a plaine and righte angled ſuperficiall number, as ye ſee in the figure here ſet, where the number of ſixe or ſixe vnities, is multiplied by the number of fiue or by fiue vnities: of which multiplication are produced 30, which number being ſet downe and deſcribed by his vnities repreſenteth a playne and a right angled number. VVherefore euen as equall numbers multipled by equal numbers produce numbers equall the one to the other: ſo rectangle parallelogrames which are comprehended vnder equal lines are equal the one to the other.

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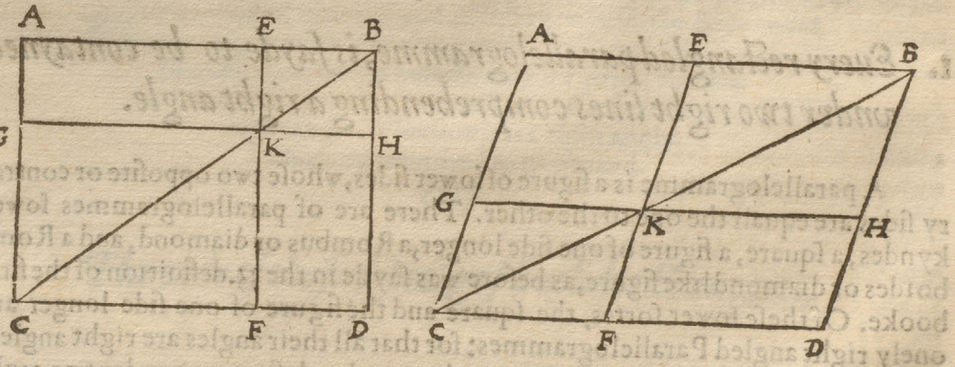
Second definition.

2. *In euery parallelogramme, one of thoſe parallelogrammes, which ſoeuer it be, which are about the diameter, together with the two ſupplementes, is called a Gnomon.*

Thoſe perticuler parallelogrames are ſayde to be about the diameter of the parallelograme, which haue the ſame diameter which the whole parallelograme hath. And ſupplementes are ſuch, which are without the diameter of the' whole parallelograme. As of the parallelograme ABCD the partial or perticuler parallelogrames G K C F and E B K H are parallelogrames about the diameter, for that ech of them hath for his diameter a part of the diameter of the whole parallelogramme. As C K and K B the perticuler diameters, are partes of the line C B, which is the diameter of the whole parallelogramme. And the two parallelogrammes A E G K and K H F D, are ſupplementes, becauſe they are wythout the diameter of the whole parallelogramme, namely, C B. Now any one of thoſe partiall parallelogrammes about the diameter together with the two ſupplementes make a gnomon. As the parallelograme E B K H, with the two ſupplementes A E G K and K H F D make the gnomon F G E H. Likewiſe the parallelogramme G K C F with the ſame two ſupplementes make the gnomon E H F G. And this diffinition of a gnomon extendeth it ſelfe, and is generall to all kyndes of parallelogrammes, whether they be ſquares or figures of one ſide longer or Rhombus or Romboides. To be ſhorte, if you take away from the whole parallelogramme one of the partiall parallelogrammes which are about the di-



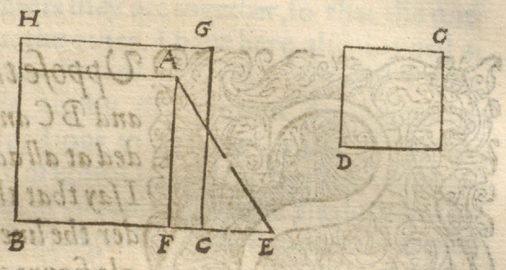
ameter

ameter whether ye will, the reſt of the figure is a gnomon.

A propoſition added by Campane after the laſt propoſition of the firſt booke.

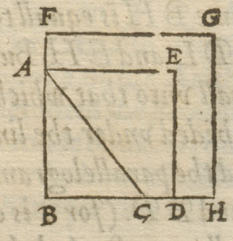
Campane after the laſt propoſition of the firſt booke addeth this propoſitiõ. *Two ſquares being geuen, to adioyne to one of them a Gnomon equall to the other ſquare:* which, for that as then it was not taught what a Gnomon is, I there omitted, thinking that it might more aptly be placed here. The doing and demonſtration whereof, is thus.

Suppoſe that there be two ſquares *A B* and *C D*: vnto one of which, namely, vnto *A B*, it is required to adde a Gnomon equall to the other ſquare, namely, to *C D*. Produce the ſide *B F* of the ſquare *A B* directly to the point *E*: and put the line *F E* equall to the ſide of the ſquare *C D*. And draw a line from *E* to *A*. Now then foraſmuch as *E F A* is a rectangle triangle, therefore (by the 47. of the firſt) the ſquare of the line *E A* is equall to the ſquares of the lines *E F* & *F A*. But the ſquare of the line *E F* is equall to the ſquare *C D*, & the ſquare of the ſide *F A* is the ſquare *A B*. Wherefore the ſquare of the line *A E* is equall to the two ſquares *C D* and *A B*. But the ſides *E F* and *F A* are (by the 21. of the firſt) longer then the ſide *A E*, and the ſide *F A* is equall to the ſide *F B*. Wherfore the ſides *E F* and *F B* are longer thē the ſide *A E*. Wherefore the whole line *B E* is longer then the line *A E*. From the line *B E* cut of a line equall to the line *A E*, which let be *B C*. And (by the 46. propoſition) vpon the line *B C* deſcribe a ſquare, which let be *B C G H*: which ſhalbe equal to the ſquare of the line *A E*, but the ſquare of the line *A E* is equal to the two ſquares *A B* and *D C*. Wherefore the ſquare *B C G H* is equal to the ſame ſquares. Wherfore foraſmuch as the ſquare *B C G H* is compoſed of the ſquare *A B* and of the gnomon *F G A H*, the ſayde gnomon ſhalbe equall vnto the ſquare *C D*: which was required to be done.



Another more redy way after Pelitarius.

Suppoſe that there be two ſquares, whoſe ſides let be *A B* and *B C*. It is required vnto the ſquare of the line *A B*, to adde a gnomon equall to the ſquare of the line *B C*. Set the lines *A B* and *B C* in ſuch ſort that they make a right angle *A B C*. And draw a line frō *A* to *C*. And vpō the line *A B* deſcribe a ſquare which let be *A B D E*. And produce the line *B A* to the point *F*, and put the line *B F* equall to the line *A C*. And vpon the line B F deſcribe a ſquare which let be B F G H: which ſhalbe equal to the ſquare of the line *A C*, whē as the lines *B F* and *A C* are equal: and therefore it is equal to the ſquares of the two lines *A B* and *B C*. Now foraſmuch as the ſquare *B F G H* is made complete by the ſquare *A B D E* and by the gnomon *F E G D*, the gnomon *F E G D* ſhalbe equal to the ſquare of the line *B C*: which was required to be done.

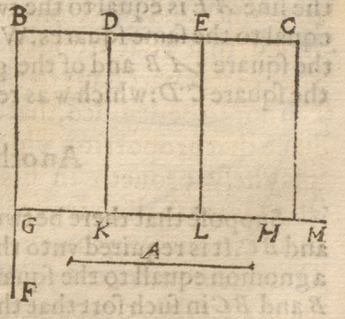


The 1. Theoreme. The 1. Proposition.

If there be two right lines, and if the one of them be deuided into partes howe many soeuer: the rectangle figure comprehended vnder the two right lines, is equall to the rectangle figures whiche are comprehended vnder the line vndeuided, and vnder euery one of the partes of the other line.

Construction.

Demonstratiõ



Suppose that there be two right lynes A and BC and let one of them, namely, BC be deuided at all aduentures in the pointes D and E. Then I say that the rectangle figure comprehended vnder the lines A, and BC, is equall vnto the rectangle figure comprehended vnder the lines A, and BD, & vnto the rectangle figure which is cõprehended vnder the lines A and DE, and also vnto the rectangle figure which is comprehended vnder the lines A and EC. For from the pointe B rayse vp (by the 11. of the first) vnto the right line BC a perpendiculer line BF, & vnto the line A (by the third of the first) put the line BG equall, and by the point G (by the 31. of the first) draw a parallel line vnto the right line BC and let the same be GM, and (by the selfe same) by ẏ points D, E, and C, draw vnto the line BG these parallel lines DK, EL and CH. Now then the parallelograme BH is equall to these parallelogrammes BK, DL, and EH. But the parallelograme BH is equall vnto that which is contayned vnder the lines A and BC. (For it is comprehẽded vnder the lines GB & BC, and the line GB is equall vnto the line A) And the parallelograme BK is equall to that which is contayned vnder the lines A and BD: (for it is comprehended vnder the line GB and BD, and BG is equall vnto A) And the parallelograme DL is equall to that which is contayned vnder the lines A and DE (for the line DK, that is, BG is equal vnto A) And moreouer likewise the parallelograme EH is equall to that which is contained vnder the lines A & EC. Wherfore that which is comprehẽded vnder ẏ lines A & BC is equall to that which is comprehended vnder the lines A & BD, & vnto ẏ which is comprehẽded vnder the lines A and DE, and moreouer vnto that which is comprehended vnder the lines A and EC. If therfore there be two right lines, and if the one of them be deuided into partes how many soeuer, the rectangle

gle figure comprehended vnder the two right lines, is equall to the rectangle figures which are comprehended vnder the line vndeuided and vnder euery one of the partes of the other line: which was required to be demonstrated.

Because that all the Propositions of this second booke for the most part are true both in lines and in numbers, and may be declared by both: therefore haue I haue added to euery Proposition conuenient numbers for the manifestation of the same. And to the end the studious and diligent reader may the more fully perceaue and vnderstand the agrement of this art of Geometry with the science of Arithmetique, and how nere & deare sisters they are together, so that the one cannot without great blemish be without the other, I haue here also ioyned a little booke of Arithmetique written by one *Barlaam*, a Greeke authour a man of greate knowledge. In whiche booke are by the authour demonstrated many of the selfe same proprieties and passions in number, which *Euclide* in this his second boke hath demonstrated in magnitude, namely the first ten propositions as they follow in order. VVhich is vndoubtedly great pleasure to cõsider, also great increase & furniture of knowledge. VVhose Propositiõs are set orderly after the propositiõs of *Euclide*, euery one of *Barlaam* correspõdent to the same of *Euclide*. And doubtles it is wonderful to see how these two cõtrary kynds of quantity, quantity discrete or number, and quantity continual or magnitude (which are the subiectes or matters of Arithmitique and Geometry) shoulde haue in them one and the same proprieties common to them both in very many points, and affections, although not in all. For a line may in such sort be deuided, that what proportion the whole hath to the greater parte the same shall the greater part haue to the lesse. But that can not be in number. For a number can not so be deuided, that the whole number to the greater part thereof, shall haue that proportion which the greater part hath to the lesse, as Iordane very playnely proueth in his booke of Arithmetike, which thynge Campane also (as we shall afterwardin the 9. booke after the 15. proposition see) proueth. And as touching these tenne firste propositions of the seconde booke of Euclide, demonstrated by Barlaam in numbers, they are also demõstrated of Campane after the 15. proposition of the 9. booke, whose demonstrations I mynde by Gods helpe to set forth when I shal come to the place. They are also demõstrated of Iordane that excellet learned authour in the first booke of his Arithmetike. In the meane tyme I thought it not amisse here to set forth the demonstrations of Barlaam, for that they geue great light to the seconde booke of Euclide, besides the inestimable pleasure, which they bring to the studious considerer. And now to declare the first Proposition by numbers. I haue put this example following.

Take two numbers the one vndeuided as 74, the other deuided into what partes and how many you list, as 37. deuided into 20. 10. 5. and 2: which altogether make the whole 37. Then if you multiply the number vndeuided, namely, 74, into all the partes of the number deuided as into 20. 10. 5. and 2. you shall produce 1480. 740. 370. 148. which added together make 2738: which self number is also produced if you multiplye the two numbers first geuen the one into the other. As you see in the example on the other side set.

Multiplication of the whole nũber vndeuided into the partes of the whole number deuided.	74	37
	1480	20
	740	10
	370	5
	148	2
	2738	the number produced of the one whole number into the partes of the other whole number
Multiplication of the one whole number into the other.	74	equall to
	37	
	518	
	222	
	2738	the number produced of the same whole into the other whole

So by the aide of this Proposition is gotten a compendious way of multiplication by breaking of one of the numbers into his partes: which oftentimes serueth to great vse in working chiefly in the rule of proportions. The demonstration of which proposition followeth in Barlaam. But first are put of the author these principles following.

Barlaam.

¶ Principles.

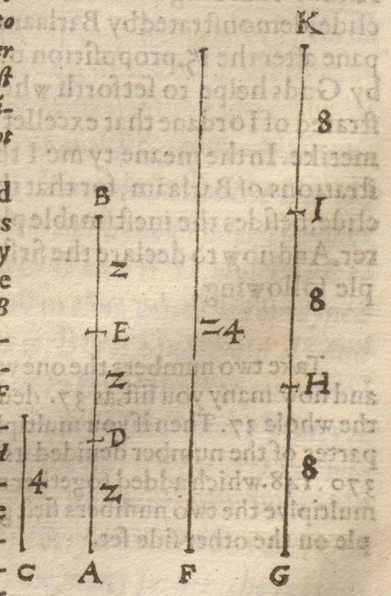
1. *A number is sayd to multiply an other number: when the number multiplied is so oftentymes added to it selfe, as there be vnities in the number, which multiplieth: wherby is produced a certaine number which the number muleiplied measureth by the vnities which are in the number which multiplieth.*
2. *And the number produced of that a multiplication is called a plaine or superficiall number.*
3. *A square number is that which is produced of the multiplicatian of any number into it selfe.*
4. *Euery lesse number compared to a greater is sayd to be a part of the greater, whether the lesse measure the greater, or measure it not.*
5. *Numbers, whome one and the selfe same number measureth equally, that is, by one and the selfe same number are equall the one to the other,*
6. *Numbers that are equemultiples to one and the selfe same number, that is, which contayne one and the same number equally and alike, are equall the one to the other.*

The first Proposition.

Barlaam.

Two numbers beyng geuen, if the one of them be deuided into any numbers how many soeuer: the playne or superficiall number which is produced of the multiplication of the two numbers first geuen the one into the other, shall be equall to the superficiall nũbers which are produced of the multiplication of the number not deuided into euery part of the number deuided.

Suppose that there be two numbers *AB* and *C*. And deuide the number *AB* into certayne other numbers how many soeuer, as into *AD*, *DE*, and *EB*. Then I say that the superficiall number which is produced of the multiplication of the number *C* into the number *AB* is equall to the superficiall numbers which arẽ produced of the multiplication of the number *C* into the nũber *AD*, and of *C* into *DE*, and of *C* into *EB*. For let *F* be the superficiall number produced of the multiplication of the number *C* into the number *AB*, and let *GH* be the superficiall number produced of the multiplication of *C* into *AD*: And let *HI* be produced of the multiplication of *C* into *DE*: and finally of the multiplication of *C* into *FB* let there be produced the number

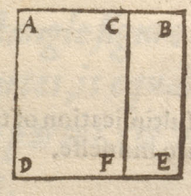


ber *I K*. Now forasmuch as *A B* multiplying the number *C* produced the number *F*: therefore the number *C* measureth the number *F* by the vnities which are in the nnmber *A B*. And by the same reason may be proued that the number *C* doth also measure the number *G H*, by the vnities which are in the number *A D*, and that it doth measure the number *H I* by the vnities which are in the number *D F* and finally that measureth the number *I K* by the vnities which are in the number *E B*. Wherefore the nũber *C* measureth the whole number *G K* by the vnities which are in the number *A B*. But it before measured the number *F* by tne vnities which are in the number *AB*, wher fore either of these numbers *F* and *G K* is equemultiplex to the number *C*. But numbers which are equemultiplices to one of the selfe same numbers are equall the one to the other (by the *6*. definition. Wherefore the number *F* is equa'l to the number *G K*. But the number *F* is the superficiall number produced of the multiplication of the nũber *C* idto the number *A B*: and the number *G K* is composed of the superficiall numbers produced of the multiplication of the nũber *C* not deuided into euery one of the numbers *A D*, *D E*, and *E B*. If therefore there be two numbers geuen and the one of them be deuided &c. Which was required to be proued.

The 2. Theoreme. The 2. Proposition.

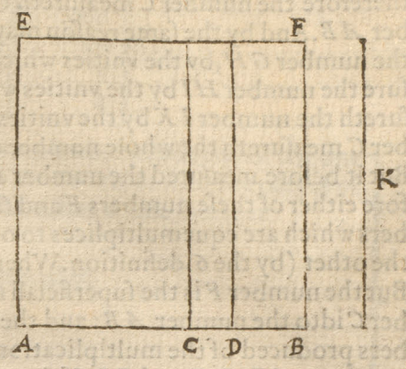
If a right line be deuided by chaunce, the rectangles figures which are comprehended vnder the whole and euery one of the partes, are equall to the square whiche is made of the whole.

Svppose that the right line AB be by chaunse denided into the point C. Then I say that the rectãgle figure comprehended vnder AB and BC together with the rectangle comprehendep vnder AB and AC is equall vnto the square made of AB. Describe (by the 46. of the first) vpon AB a square ADEB: and (by the 31 of the first) by the point C draw a line parallel vnto either of these lines AD aud BE, and let the same be CF. Now is the parallelogramme AE equall to the parallelogrammes AF and CE, by the first of this booke. But AE is the square made of AB. And AF is the rectangle parallelogramme comprehended vnder the lines BA and AC: for it is comprehended vnder the lines DA and AC: but the line AD is equall vnto the line AB. And likewise the parrallelogramme CE is equall to that which is contayned vnder the lynes AB and BC: for the line BE is equal vnto the line AB. Wherfore that which is contayned vnder BA and AC together with that which is contayned vnder the lines AB and BC, is equall to the square made of the line AB. If therefore a right line be deuided by chaunce the rectãgle figures which are comprehended vnder the whole, and euery one of the partes, are equall to the square which is made of the whole: which was required to be demonstrated.



An other demonstration of Campane.

Suppoſe that the line *AB* be deuided into the lines *AC*, *CD*, and *DB*. Then I ſay that the ſquare of the whole line *AB*, which let be *AEBF*, is equal to the rectangle figures which are contayned vnder the whole and euery one of the partes: for take the line *K*, which let be equal to the line *AB*. Now then by the firſt propoſition the rectangle figure contained vnder the lines *AB* and *K*, is equall to the rectangle figures contayned vnder the line *K* and al the partes of the line *AB*. But that which is contayned vnder the lines *K* and *AB* is equall to the ſquare of the line *AB*, and the rectangle figures contayned vnder the line *K* and al the partes of *AB*, are equall to the rectangle figures contayned vnder the line *AB* and all the partes of the line *AB*: for the lines *AB* and *K* are equall: wherefore that is manifeſt which was required to be proued.



An example of this Propoſition in numbers.

Take a number, as 11. and deuide it into two partes, namely, 7. and 4: and multiply 11. into 7, and then into 4. and there ſhalbe produced 77. and 44: both which numbers added together make 121. which is equall to the ſquare number produced of the multiplication of the number 11. into himſelfe, as you ſee in the example.

Multiplication of the whole into his partes.

11 × 7 = 77
11 × 4 = 44
77 + 44 = 121 — the number produced of the whole into his partes.

Multiplication of the whole into himſelfe.

11 × 11 = 121 — the number produced of the whole into himſelfe.

121 equal to 121

The demonſtration whereof followeth in Barlaam.

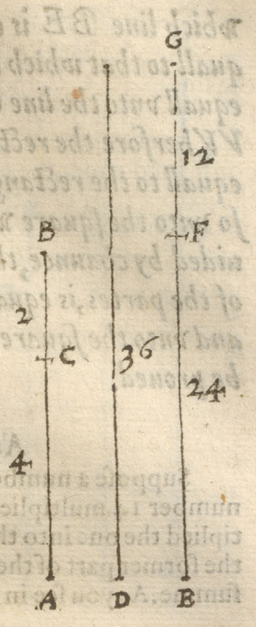
The ſecond Propoſition.

Barlaam.

If a number geuen be deuided into two other numbers: the ſuperficiall numbers, which are produced of the multiplication of the whole into either part, added together, are equall to the ſquare number of the whole number geuen.

Suppoſe that the number geuen be *AB*: and let it be deuided into two other numbers *AC* and *CB*. Then I ſay that the two ſuperficiall numbers, which are produced of the multiplication of *AB* into *AC*, and of *AB* into *BC*, thoſe two ſuperficiall numbers (I ſay) beyng added together, ſhalbe equall to the ſquare number produced of the multiplicatiō of the number *AB* into it ſelfe. For let the number *AB* multiplying it ſelfe produce the number *D*. Let the number *AC* alſo multiplying the number *AB* produce

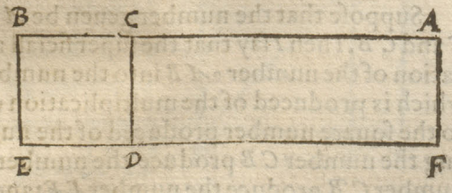
produce the number *E F*: agayne let the number *C B* multiplying the selfe same number *A B* produce the number F G. Now forasmuch as the number *A C* multiplying the number *A B* produced the number E F: therefore the number *A B* measureth the number E F by the vnities which are in *A C*. Againe forasmuch as the number *C B* multiplied the number *A B*, and produced the number F G: therfore the number *A B* measureth the number F G by the vnities which are in the number *C B*. But the same number *A B* before measured the number E F by the vnities which are in the number *A C*. Wherefore the number *A B* measureth the whole number *E G* by the vnities whcih are in *A B*. Farther forasmuch as the number *A B* multiplying it selfe produced the number *D*: therefore the number *A B* measureth the number *D* by the vnities which are in himselfe. Wherfore it measureth either of these numbers, namely, the number *D*, and the number *E G*, by the vnities which are in himselfe. Wherfore how multiplex the number *D* is to the number *A B*, so multiplex is the number *E G* to the same number *A B*. But numbers which are equemultiplices to one and the selfe same number, are equal the one to the other. Wherefore the number *D* is equall to the number *E G*. And the number *D* is the square number made of the number *A B*, and the number *E G* is composed of the two superficiall numbers produced of *A B* into *B C*, and of *B A* into *A C*. Wherefore the square number produced of the number *A B* is equall to the superficial numbers, produced of the number A B into the number *B C*, and of A B into AC, added together. If therefore a number be deuided into two other numbers &c. which was required to be proued.



The 3. Theoreme. The 3. Proposition.

If a right line be deuided by chaunce: the rectangle figure cõprehended vnder the whole and one of the partes, is equall to the rectangle figure comprehended vnder the partes, & vnto the square which is made of the foresaid part.

Vppose that the right line geuen A B be deuided by chaunce in the point C. Then I say that the rectangle figure comprehẽded vnder the lines A B and B C is equall vnto the rectangle figure comprehended vnder the lines A C and C B, and also vnto the square which is made of the line B C. Describe (by the 46. of the first) vpon the line B C a square C D E B: and (by the second peticion) extend E D vnto F. And by the point A draw (by the 31. of the first) a line parallel vnto either of these lines C D and B E, and let the same be A F. Now the parallelograme A E is equall vnto the parallelogrammes A D and C E. And A E is the rectangle figure comprehended vnder the lines A B and B C. For it is comprehended vnder the lines A B and B E, Construction.



Demonstratiõ

which line BE is equall vnto the line BC. And the parallelograme AD is equall to that which is contayned vnder the lines AC and CB: for the line DC is equall vnto the line CB. And DB is the ſquare which is made of the lyne CB. Wherfore the rectangle figure comprehended vnder the lynes AB and BC is equall to the rectangle figure comprehended vnder the lines AC and CB & alſo vnto the ſquare which is made of the line BC. If therfore a right line be deuided by chaunce, the rectangle figure comprehended vnder the whole and one of the partes, is equall to the rectangle figure comprehended vnder the partes, and vnto the ſquare which is made of the foreſayd part: which was required to be proued.

An example of this Propoſition in numbers.

Suppoſe a number, namely, 14. to be deuided into two partes 8. and 6. The whole number 14. multiplied into 8. one of his partes, produceth 112: the partes 8. & 6. multiplied the one into the other produce 48, which added to 64 (which is the ſquare of 8. the former part of the number) amounteth alſo to 112: whiche is equall to the former ſumme. As you ſee in the example.

Multiplication of the whole into one of his partes.

14 × 8 } the partes.
8 × 6 }

112 — the number produced of the whole into one of his partes.

Multiplication of the one part into the other.

8 × 6 = 48

Multiplication of the former part into it ſelfe.

8 × 8 = 64

48 + 64 = 112 — the number compoſed of the one part into the other, and of the former part into himſelfe.

112 equal to 48 + 64

The demonſtration hereof followeth in Barlaam.

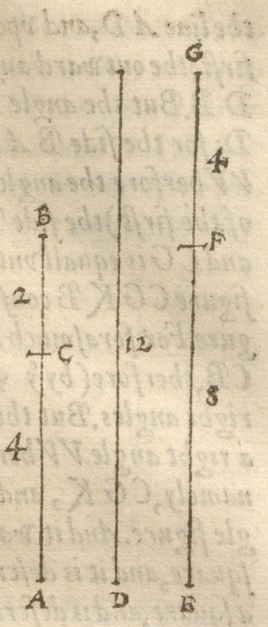
The third propoſition.

Barlaam. *If a number geuen be deuided into two numbers: the ſuperficiall number which is produced of the multiplication of the whole into one of the partes, is equall to the ſuperficiall number which is produced of the partes the one into the other, and to the ſquare number produced of the foreſayd part.*

Suppoſe that the number geuen be *AB*, which let be deuided into two numbers *AC* and *CB*. Then I ſay that the ſuperficiall number whiche is produced of the multiplication of the number *AB* into the number *BC*, is equall to the ſuperficiall number which is produced of the multiplication of the number *AC* into the number *CB*, and to the ſquare number produced of the number *CB*. For let the number *AB* multiplieng the number *CB* produce the number *D*. And let the number *AC* multiplieng the number *CB* produce the number *EF*: and finally let the number *CB* multiplieng himſelfe produce the number *FG*. Now foraſmuch as the number *AB* multiplieng the

number

number *C B* produced the number *D*. Therfore the number *C B* measureth the number *D* by the vnities whiche are in the number *A B*. Agayne forasmuch as the number *A C* multiplied the number *C B*, and produced the number *E F*, therefore the number *C B* measureth the nũber *E F* by the vnities which are in *A C*. Agayne forasmuch as the number *C B* multiplied it selfe and produced the number *F G*: therfore the number *C B* measureth the number *F G* by the vnities which are in it selfe. But as we haue before proued the selfe same nũber *C B* measureth also the number *E F* by the vnities which are in the nũber *A C*, wherfore the number *C B* measureth the whole number *E G* by the vnities which are in the number *A B*. And it also measureth the number *D* by the vnities whiche are in the number *A B*. Wherfore the number *C B* equally measureth either number, namely, the number *D*, and the number *E G*. But those numbers whome one and the selfe same number measureth equally, are equall the one to the other. Wherfore the number *D* is equall to the number *E G*. But the number *D* is a superficiall number produced of the multiplication of the number *A B* into the number *B C*, and the number *E G* is the superficial number produced of the multiplication of the nũber *A C* into the number *C B*, and of the square of the number *C B*. Wherfore the superficial number produced of the multiplication of the number *A B* into the number *C B* is equal to the superficiall number produced of the number *A C* into the number *C B*, and to the square of the number *C B*. If therfore a number be deuided into two numbers, the superficiall nũber &c: which was required to be proued.

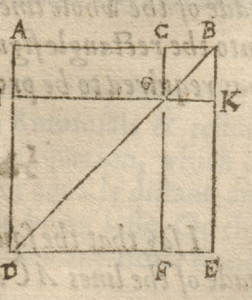


The 4. Theoreme. The 4. Proposition.

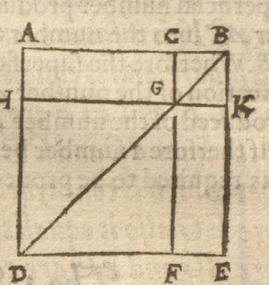
If a right line be deuided by chaunce, the square whiche is made of the whole line is equal to the squares which are made of the partes, & vnto that rectangle figure which is comprehended vnder the partes twise.

Suppose that the right lyne A B be by chaunce deuided in the pointe C. Then I say that the square made of the line A B is equall vnto ỹ squares which are made of the lines A C and C B, and vnto the rectangle figure contained vnder the lines A C and C B twise. Construction.

Describe (by ỹ 46. of the first) vpon the line A B a square A D E B: and draw a line from B to D, and (by the 31. of the first) by the point C draw a line parallel vnto either of these lines A D and B E cutting the diameter B D in the point G, and let the same be C F. And (by the point G (by the selfe same) draw a line parallel vnto eyther of these lines A B and D E, and let the same be H K. And forasmuch as the line C F is a parallel vnto Demonstratiõ



the line AD, and vpon them falleth a right line BD: therfore (by the 29. of the first) the outward angle CG B is equall vnto the inward and opposite angle A D B. But the angle A D B is (by the 5. of the first) equall vnto the angle A B D: for the side B A is equall vnto the side A D (by the definition of a square). VVherfore the angle CG B is equall vnto the angle G BC: wherfore (by the 6. of the first) the side BC is equall vnto the side CG. But C B is equall vnto G K, and C G is equall vnto K B: wherfore G K is equall vnto K B. VVherfore the figure C G K B consisteth of foure equall sides. I say also that it is a rectangle figure. For forasmuch as CG is a parallel vnto BK & vpon thē falleth a right line CB, therfore (by ỹ 9. of the 1.) the angles K B C, and G C B are equal vnto two right angles. But the angle KBC is a right angle, wherfore ỹ angle BCG is also a right angle. VVherfore (by the 34. of the first) the angles opposite vnto them, namely, C G K, and G K B are right angles. VVherfore C G K B is a rectangle figure. And it was before proued that the sides are equall. VVherfore it is a square, and it is described vpon the line BC. And by the same reason also H F is a square, and is described vpon the line H G, that is, vpon the line AC. VVherfore the squares HF and CK are made of the lines AC and C B. And forasmuch as the parallelograme AG is (by the 43. of the first) equall vnto the parallelogramme G E. And AG is that which is contayned vnder AC and CB, for CG is equal vnto C B, wherfore G E is equall to that which is contained vnder AC and CB. VVherefore AG and GE are equall vnto that which is comprehended vnder AC and CB twise. And the squares HF and CK are made of the lines AC and CB. VVherfore these foure rectangle figures HF, CK, AG, and GE are equall vnto the squares whiche are made of the lines AC and CB, and to the rectangle figure which is comprehended vnder the lines AC and CB twise. But the rectangle figures HF, CK, AG, and GE are the whole rectangle figure ADEB which is the square made of the line AB. VVherfore the square which is made of the line AB is equall to the squares which are made of the lines AC and CB, and vnto the rectangle figure which is comprehended vnder the lines AC and CB twise. If therfore a right line be deuided by chaunce, the square whiche is made of the whole line, is equall to the squares which are made of the partes, & vnto the rectangle figure which is comprehended vnder the partes twise: which was required to be proued.

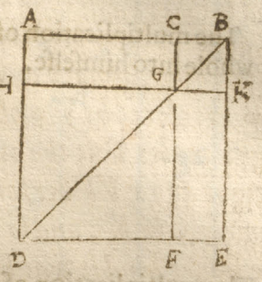


An other demonstration.

I say that the square of the line AB is equall vnto the squares whiche are made of the lines AC and CB, & vnto the rectangle figure which is comprehēded vnder the lines AC and CB twise. For the selfe same discription abiding, for-

asmuch

aſmuch as the line AB is equall vnto ỹ line AD, ỹ angle ABD is (by the 5. of the firſt) equall vnto the angle ADB. And foraſmuch as the three angles of euery triangle are equal to two right angles (by the 32. of the firſt). therefore ỹ three angles of the triangle ABD, namely, the angles ADB, DBA, and BAD, are equall to two right angles. But the angle BAD is a right angle, wherefore the angles remayning ABD, and ADB, are equall vnto one right angle: and they are equall ỹ one to the other, wherfore either of theſe angles ABD, & ADB, is the halfe of a right angle. And the angle BCG is a right angle, for it is equall vnto the oppoſite angle at the point A (by the 29. of the firſt). VVherefore the angle remayning CGB is the halfe of a right angle. VVherefore the angle CGB is equal vnto the angle CBG: wherefore alſo the ſide BC is equall vnto the ſide CG. But BC is equall vnto GK, and CG is equal vnto BK. VVherefore the figure CK conſiſteth of equall ſides: and in it is a right angle CBK. VVherefore CK is a ſquare, and is made of the line BC. And by the ſame reaſon HF is a ſquare, and is equall vnto that ſquare which is made of the line AC. VVherefore CK and HF are ſquares, and are equall to thoſe ſquares which are made of the lines AC and CB. And foraſmuch as AG is equall vnto EG: and AG is that which is contayned vnder AC and CB, for GC is equal vnto CB: wherefore EG alſo is equall to that which is cõprehended vnder AC and CB: wherefore AG and EG are equall vnto that rectangle figure which is comprehended vnder AC, and CB twiſe. And CK, and HF are equal vnto the ſquares which are made of AC, and CB: wherefore CK, HF, AG, and GE are equal vnto thoſe ſquares which are made of AC, and CB, and vnto that rectangle figure which is comprehended vnder AC and CB twiſe. But CK, HF, AG, and GE are the whole ſquare AE which is made of AB. VVherefore the ſquare which is made of AB, is equall to the ſquares which are made of AC and CB, and vnto the rectangle figure which is comprehended vnder AC and CB twiſe: which was required to be demonſtrated.

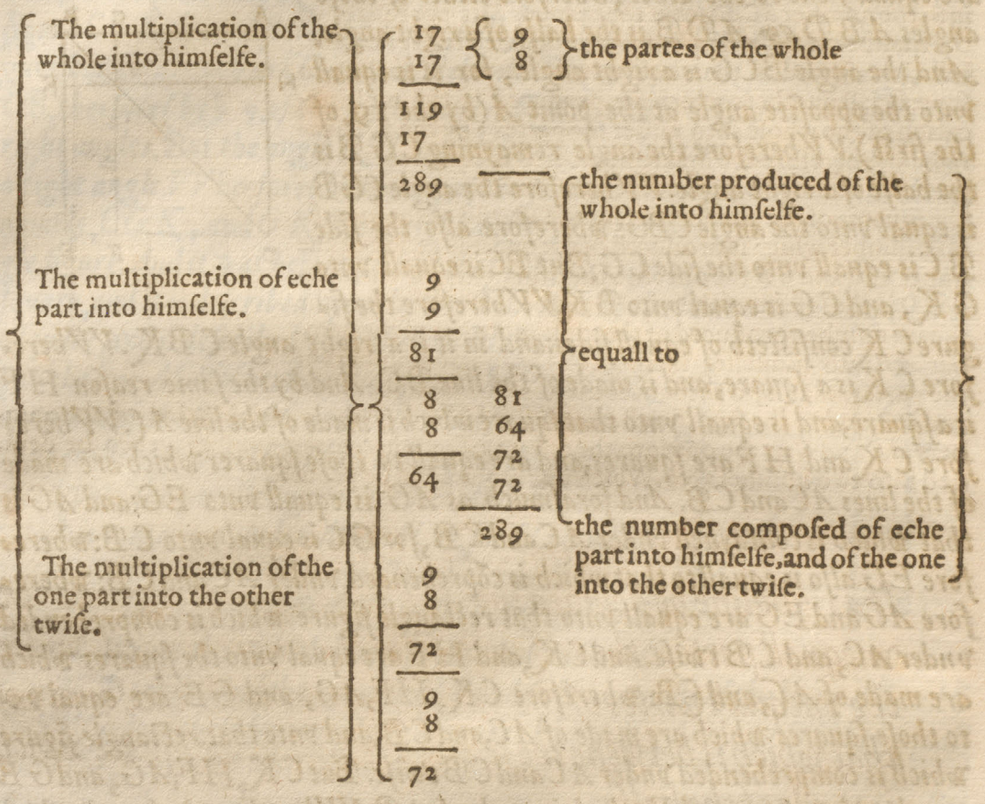


Hereby it is manifeſt that the parallelogrames which conſiſt about the diameter of a ſquare muſt needes be ſquares. A Corollary.

This propoſition is of infinite vſe chiefely in ſurde numbers. By helpe of it is made in thẽ additiõ & ſubſtraction, alſo multiplicatiõ in Binomials & reſiduals. And by helpe hereof alſo is demonſtrated that kinde of equation, which is, when there are three denominations in naturall order, or equally diſtant, and two of the greater denominations are equall to the thirde being leſſe. On this propoſition is grounded the extraction of ſquare roots. And many other things are alſo by it demonſtrated.

An example of this Propoſition in numbers.

Suppoſe a number namely, 17. to be deuided into two partes 9. and 8. The whole number 17. multiplied into himſelfe, produceth 289. The ſquare numbers of 9. and 8. are 81. and 64: the numbers produced of the multiplication of the partes the one into the other twiſe are 72. and 72: which two numbers added to the ſquare numbers of 9. and 8. namely, to 81. and 64. make alſo 289. which is equall to the ſquare number of the whole number 17. As you ſee in the example.



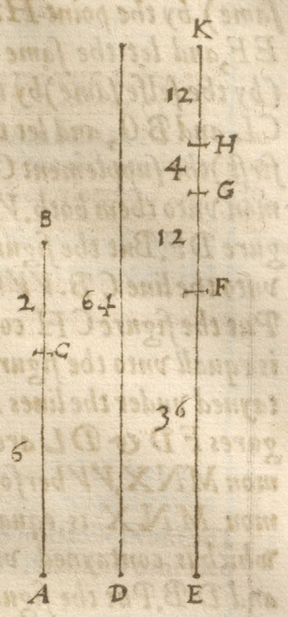
The demonſtration wherof followeth in Barlaam.

The fourth Propoſition.

Barlaam. *If a number geuen be deuided into two numbers: the ſquare number of the whole, is equall to the ſquare numbers of the partes, and to the ſuperficiall number which is produced of the multiplication of the partes the one into the other twiſe.*

Suppoſe that the number geuen be *AB*: which let be deuided into two numbers *AC* and *CB*. Then I ſay that the ſquare number of the whole number *AB*, is equall to the ſquares of the partes, that is, to the ſquares of the numbers *AC* and *CB*, and to the ſuperficiall number produced of the multiplication of the numbers *AC* and *CB* the one into the other twiſe. Let the ſquare number produced of the multiplication of the whole number *AB* into himſelfe be *D*. And let *CA* multiplied into himſelfe produce the number *EF*: And *CB* multiplyed into it ſelfe let it produce *GH*: and finally of the multiplicatiō of the numbers *AC* and *CB* the one into the other twiſe let there be produced either of theſe ſuperficiall numbers *FG* and *HK*. Now foraſmuche as the number *AC* multiplying it ſelf produced the number *EF*: therefore the number *AC* meaſureth the number *EF* by the vnities which are in it ſelfe. And foraſmuch as the number *CB* multiplyed the number *CA* and produced the number *FG*: therefore

fore the number *AC* meaſureth the nũber *F G* by the vnities whiche are in the number *C B*. But it before alſo meaſured the number *E F* by the vnities which are in it ſelfe. Wherefore the number *A B* multiplying the number *A C* produceth the number *E G*. And therefore the number *E G* is the ſuperficiall number produced of the multiplication of the number *B A* into the number *A C*. And by the ſame reaſon may we proue that the number *G K* is the ſuperficiall number produced of the multiplication of the number *A B* into the number *BC*. Farther the number *D* is the ſquare of the number *A B*. But if a number be deuided into two numbers, the ſquare of the whole number is equall to the two ſuperficiall numbers which are produced of the multiplication of the whole into either the partes (by the 2. Theoreme.) Wherefore the ſquare number *D* is equall to the ſuperficiall number *E K*. But the number *EK* is compoſed of the ſquares of the numbers *A C* and *C B*, and of the ſuperficial number which is produced of the multiplication of the nũber *A C* and *C B* the one into the other twiſe: & the number *D* is the ſquare of the whole number *AB*. Wherfore the ſquare number produced of the multiplication of the number *AB* into himſelfe, is equall to the ſquare numbers of the partes, that is, to the ſquare numbers of the nũbers *A C* and *C B*, and to the ſuperficiall number produced of the multiplication of the numbers *A C* and *C B*, the one into the other twiſe. If therefore a number geuen be deuided into two numbers &c. Which was required to be proued.

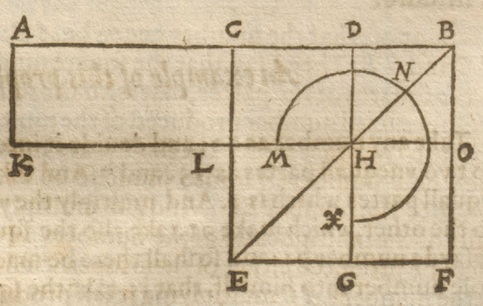


The 5. Theoreme. The 5. Propoſition.

If a right line be deuided into two equall partes, & into two vnequall partes: the rectangle figures comprehended vnder the vnequall partes of the whole, together with the ſquare of that which is betwene the ſectiõs, is equal to the ſquare which is made of the halfe.

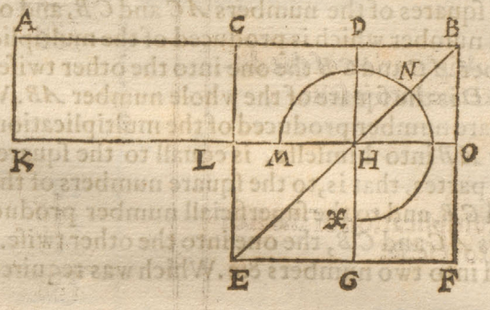
Vppoſe that the right line A B be deuided into two equall partes in the point C, and into two vnequall partes in the point D. Then I ſay that the rectangle figure comprehended vnder A D and D B together with the ſquare which is made of C D, is equall to the ſquare which is made of C B. Conſtruction. *De-*

ſcribe (by the 46. of the firſt) vppon C B a ſquare, and let the ſame be C E F B. And (by the firſt peticion) drawe a line from E to B. And by ỹ point D drawe (by the 31. of the firſt) a line parallel vnto each of theſe lines C E and B F cutting the diameter B E in the point H, and let ỹ ſame be D G. And agayne (by the ſelfe



Demonstratiõ

same) by the point H drawe a line parallel vnto eche of these lines A B and E F, and let the same be K O: and let K O be equall vnto A B. And againe (by the selfe same) by the point A draw a line parallel vnto either of these lines C L and B O, and let the same be A K. And forasmuch as (by the 43. of the first) the supplement C H is equall to the supplemẽt H F. put the figure D O cõmon vnto them both. VVherefore the whole figure C O is equall to the whole figure D F. But the figure C O is equall vnto the figure A L, for ỹ line A C is equall vnto the line C B. VVherefore the figure A L also is equal vnto the figure D F. Put the figure C H common vnto them both. VVherfore the whole figure A H is equall vnto the figures D L and D F. But A H is equall to that which is cõtayned vnder the lines A D and D B, for D H is equall vnto D B. And the figures F D & D L are the Gnomon M N X. VVherfore ỹ Gnomon M N X is equall to that which is contayned vnder A D and D B. Put the figure L G cõmon vnto them both, which is equal to the square which is made of C D. VVherefore the Gnomõ M N X and the figure L G are equall to the rectangle figure cõprehended vnder A D and D B and vnto the square which is made of C D. But the Gnomon M N X, and the figure L G are the whole square C E F B, which is made of B C. VVherefore the rectangle figure comprehended vnder A D and D B, together with the square which is made of C D, is equall to the square which is made of C B. If therefore a right line be deuided into two equall parts, and into two vnequall partes, the rectangle figure comprehended vnder the vnequall partes of the whole, together with the square of that which is betwene the sections, is equall to the square which is made of the halfe: which was required to be proued.



This Proposition also is of greate vse in Algebra. By it is demonstrated that equation wherein the greatest and least karectes or numbers are equall to the middle.

An example of this proposition in numbers.

Take any number as 20: and deuide it into two equall partes 10. and 10. and then into two vnequall partes as 13. and 7. And take the differẽce of the halfe to one of the vnequall partes which is 3. And multiply the vnequall partes, that is, 13 and 7. the one into the other, which make 91. take also the square of 3. which is 9. and adde it to the foresayde number 91: and so shall there be made 100. Then multiply the halfe of the whole number into himself, that is, take the square of 10. which is 100. which is equal to the number before produced of the multiplication of the vnequal parts the one into the other, & of the difference into it selfe which is also 100. As you se in the example.

The

The whole euen number. 20

10 } the equall partes
10 }

13 } the vnequall partes
7 }

3 } The difference of one of the vnequal partes to the halfe.

Multiplication of the vnequall partes the one into the other.

13
7
91 91

Multiplication of the difference into it ſelfe.

9
3 100
3
9

the nũber compoſed of the multiplication of the vnequal partes the one into the other, & of the difference into it ſelfe equall to

Multiplication of the halfe into it ſelfe.

10
10
100

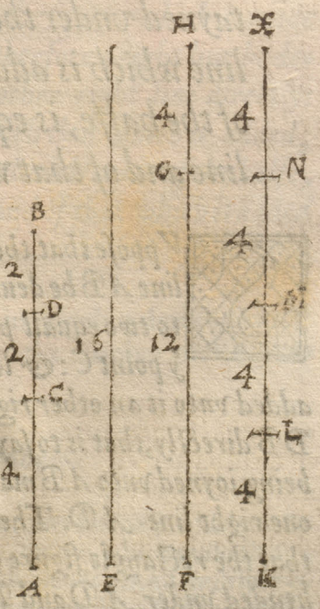
the number produced of the halfe into it ſelfe.

The demonſtration wheroffolloweth in Barlaam.

The fifth propoſition.

If an euen number be deuided into two equall partes, and againe alſo into two vnequall partes: the ſuperficiall number which is produced of the multiplication of the vnequall partes the one into the other, together with the ſquare of the number ſet betwene the parts, is equal to the ſquare of halfe the number.

Suppoſe that *AB* be an euen number: which let be deuided into two equall numbers *AC* and *CB*, and into two vnequall numbers *AD* and *DB*. Then *I* ſay, that the ſquare number which is produced of the multiplication of the halfe number *CB* into it ſelfe, is equall to the ſuperficiall number produced of the multiplication of the vnequall numbers *AD* and *DB* the one into the other, and to the ſquare number produced of the number *CD* which is ſet betwene the ſayde vnequall partes. Let the ſquare number produced of the multiplication of the halfe number *CB* into it ſelfe be *E*. And let the ſuperficiall number produced of the multiplication of the vnequal nũbers *AD* and *DB* the one into the other, be the number *FG*: and let the ſquare of the number *DC* which is ſet betwene the partes be *GH*. Now foraſmuch as the number *BC* is deuided into the numbers *BD* and *DC*, therfore the ſquare of the number *BC*, that is, the number *E*, is equall to the ſquares of the numbers *BD* and *DC*, and to the ſuperficiall number which is compoſed of the multiplication of the numbers *BD* and *DC* the one into the other twiſe, (by the 4. propoſition of this boke) Let the ſquare of the number *BD* be the number *KL*: & let NX be the ſquare of the number *DC*: and finally of

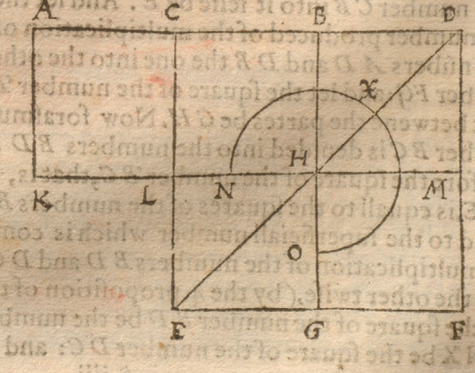


the multiplication of the numbers *BD* and *DC* the one into the other twise, let be produced either of these numbers *LM* and *MN*. Wherefore the whole number *KX* is equall to the number *E*. And forasmuch as the number *BD* multipliyng it selfe produced the number *KL*, therefor it measureth it by the vnities which are in it selfe. Moreouer forasmuch as the number *CD* multiplying the number *BD* produced the number *LM*, therefore also *DB* measureth *LM* by the vnities which are in the number *CD*: but it before measured the number *KL* by the vnities which are in it selfe. Wherefore the number *DB* measureth the whole number *KM* by the vnities which are in *CB*. But the number *CB* is equall to the number *CA*. Wherefore the number *DB* measureth the number *KM* by the vnities which are in *CA*. Agayne forasmuch as the nũber *CD* multiplyng the number *DB* produced the number *MN*: therefore the number *DB* measureth the number *MN* by the vnities which are in the number *CD*: but it before measured the number *KM* by the vnities which are in the number *AC*. Wherfore the number *BD* measureth the whole number *KN* by the vnities which are in the number *AD*. Wherefore the number *FG* is equall to the number *KN*. For numbers which are equemultiplices to one and the selfe same number, are equall the one to the other. But the number *GH* is equall to the number NX: for either of them is supposed to be the square of the number *CD*. Wherefore the whole number KX is equall to the whole number *FH*. But the number KX is equall to the number *E*. Wherefore also the number *FH* is equall to the number *E*. And the number *FH* is the superficial number produced of the multiplication of the numbers *AD* and *DB* the one into the other together with the square of the number *DC*. And the number *E* is the square of the number *CB*. Wherfore the superficiall number produced of the multiplication of the vnequal partes *AD* and *DB* the one into the other, together with the square of the nũber *DC* which is set betwene those vnequall partes, is equall to the square of the number *CB*, which is the halfe of the whole number *AB*. If therfore an euen number be deuided into two equall partes, &c. which was required to be proued.

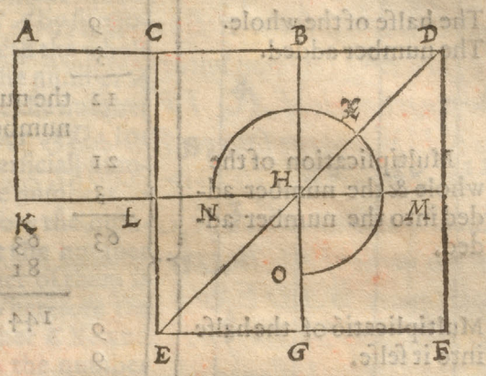
The 6. Theoreme. The 6. Proposition.

If a right line be deuided into two equal partes, and if vnto it be added an other right line directly, the rectangle figure contayned vnder the whole line with that which is added, & the line which is addedt, ogether with the square which is made of the halfe, is equall to the square which is made of the halfe line and of that which is added as of one line.

Vppose that the righte line AB be deuided into two equall partes in ỹ point C: & let there added vnto it an other right line DB directly, that is to say, which being ioyned vnto AB make both one right line AD. Then I say, that the rectangle figure comprehended vnder AD and DB, together



gether with the ſquare whiche is made of B C is equall to the ſquare whiche is made of D C. Deſcribe (by the 46. of the 1.) vpon C D a ſquare C E F D, and (by the firſt peticion) draw a line from D to E: and (by the 31. of the firſt) by ỹ point B draw a line parallel vnto either of theſe lines E C & D F, cutting the diameter D E in the point H, and let the ſame be B G, & (by the ſelf ſame) by ỹ point H draw to either of theſe lines A D and E F a parallel line K M: and moreouer by the pointe A drawe a line parallel to either of theſe lines C L and D M: and let the ſame be A K. And foraſmuch as A C is equall vnto C B, therfore (by the 36. of the firſt) the figure A L is equal vnto the figure C H. But (by the 43. of the firſt) C H is equal vnto the figure H F, wherfore A L is equall vnto H F. Put the figure C M common to them both, wherfore the whole line A M is equal vnto the gnomon N X O. But A M is that which is contayned vnder A D and D B: for D M is equal vnto D B: wherfore the gnomon N X O, is equall vnto the rectangle figure contained vnder A D and D B. Put the figure L G common to them both, which is equall to the ſquare which is made of C B. VVherefore the rectangle figure which is contayned vnder A D and D B together with the ſquare which is made of C B is equall to the gnomon N X O, and vnto L G. But the gnomon N X O and L G are the whole ſquare C E F D which is made of C D. VVherfore the rectangle figure contayned vnder A D and D B together with the ſquare which is made of C B is equall to the ſquare which is made of C D. If therfore a right line be deuided into two equall partes, and vnto it be added an other right line directly: the rectangle figure contayned vnder the whole line with that which is added, and the line which is added, together with the ſquare which is made of the halfe, is equall to the ſquare which is made of the halfe line and of that which is added as of one line: which was required to be demonſtrated.



By this Propoſition (beſides many other vſes) is in Algebra demonſtrated that equation wherin the two leſſe numbers be equall to the number of the greateſt denomination.

An example of this propoſition in numbers.

Take any euen number as 18. and adde vnto it any other number as 3. which make in all 21. And multiply 21. into the number added, namely, into 3. which maketh 63. Take alſo the halfe of the whole euen number, that is, of 18. which is 9. And multiply 9. into it ſelf which maketh 81. which adde vnto 63. (the number produced of the whole euen number, and the number added into the number added) and you ſhal make 144.

Then adde 9. the halfe of the whole euen number vnto 3. the number added which maketh 12. And multiply 12. into it ſelfe, that is, take the ſquare of 12. which is 144. which is equall to the number compoſed of the multiplication of the whole number and the number added into the number added, and of the ſquare of the number added, which is alſo 144. As you ſee in the example.

The whole euen number. 18
The number added. 3

21 the number compoſed of the whole number, & the number added.

The halfe of the whole. 9
The number added. 3

12 the number compoſed of the halfe and of the number added.

Multiplication of the whole & the number added into the number added. 21 × 3 = 63

Multiplicatiõ of the halfe into it ſelfe. 9 × 9 = 81

Multiplicatiõ of the halfe and the number added in to it ſelfe. 12 × 12 = 144

63 + 81 = 144

the nũber compoſed of the whole and the number added into the number added and of the ſquare of the halfe equall to the ſquare number made of the number compoſed of the halfe and the number added.

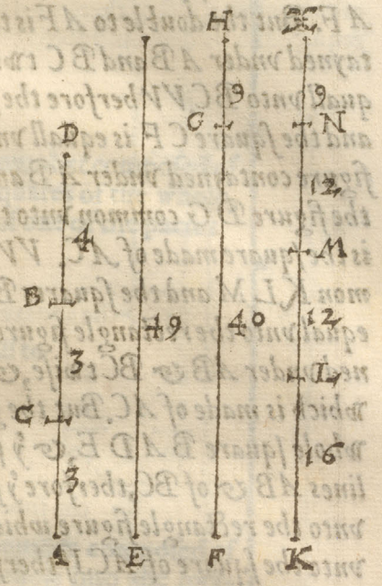
The demonſtration wherof followeth in Barlaam.

The ſixt Propoſition.

If an euen number be deuided into two equall numbers, and vnto it be added ſome other number: the ſuperficiall number which is made of the multiplication of the number compoſed of the whole nũber and the number added, into the number added, together with the ſquare of the halfe number, is equall to the ſquare of the number compoſed of the halfe and the number added.

Suppoſe that *A B* be an euen number, and let it be deuided into two equall numbers *A C* and *C B*: and vnto it let there be added an other number *B D*. Then *I* ſay that the ſuperficiall number produced of the multiplication of the number *A D* into the number *D B* is equall to the ſquare of the number *C D*. For let the ſquare number of the number *C D* be the number *E*, and let the ſuperficial number produced of the multiplication of the number *A D* into the number *D B* be the number *F G*: and finally let the ſquare number of *C B* be the number *G H*. And foraſmuch as the ſquare of the nũber *C D* is (by the 4. propoſition) equall to the ſquares of the numbers *D B* and *BC* together with the ſuperficiall number which is produced of the multiplication of the numbers *D B* and *B C* the one into the other twiſe. Let the ſquare of the number *B D* be the number K L: and let the ſuperficiall numbers produced of the multiplication of the numbers *D B* and *B C* the one into the other twiſe be either of theſe numbers

LM and M N: and finally let the ſquare of the number *B C* be the number N X. Wherfore the whole number K *X* ſhall be equall to the ſquare of the number *C D*. But the ſquare of the number *C D* is the number *E*. Wherfore the number K X is equall to the number *E*. And foraſmuch as the number *B D* multiplieng it ſelfe produced the number K L: therfore the number *B D* meaſureth the number K L, by the vnities which are in it ſelfe, but it alſo meaſureth the number L M by the vnities which are in the number *C B*. Wherfore the number *D B* meaſureth the whole number K M by the vnities which are in the number *C D*. The nũber *D B* alſo meaſureth the number M N by the vnities which are in the number *C B*: & the number *C B* is equall to the number *C A* by ſuppoſition. Wherfore the number *D B* meaſureth the whole number K N by the vnities which are in the number *A D*. But the number *D B* doth alſo meaſure the number *F G* by the vnities which are in the number *A D*: for by ſuppoſition the number *F G* is the ſuperficiall number produced of the multiplication of the numbers *A D* and *D B* the one into the other. Wherfore the number *F G* is equall to the number K N. But the number *H G* is equall to the number N X: for either of them is the ſquare number of the number *C B*. Wherefore the whole number *F H* is equall to the number K X: and the number K X is proued to be equall to the number *E*. Wherfore the number *F H* ſhall alſo be equall to the number *E*. And the number *F H* is the ſuperficiall number produced of the multiplication of the numbers *A D* and *D B* the one into the other, together wyth the ſquare of the number *C B*: and the number *E* is the ſquare of the number *C D*. Wherfore the ſuperficiall number produced of the multiplication of the numbers *A D* and *D B* the one into the other, together with the ſquare of the number *C B*, is equall to the ſquare of the number *C D*. *If* therfore an euen number &c.

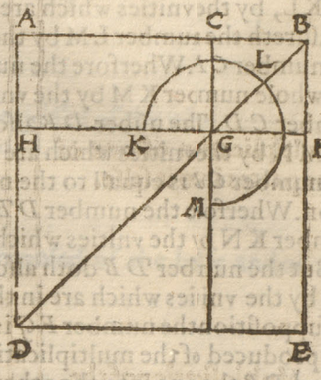


The 7. Theoreme. The 7. Propoſition.

If a right lyne be deuided by chaunce, the ſquare whiche is made of the whole together with the ſquare which is made of one of the partes, is equall to the rectangle figure which is cõtayned vnder the whole and the ſaid parte twiſe, and to the ſquare which is made of the other part.

SVppoſe that the right line A B be deuided by chaunce in the point C. Then I ſay that the ſquare which is made of A B, together with the ſquare which is made of B C, is equal vnto the rectangle figure which is contayned vnder the lines A B and B C twiſe, and vnto the ſquare which is made of A C. Deſcribe (by the 46. of the firſt) vppon A B a ſquare A D E B, and make complete the figure. And foraſmuch as (by the 43. of the firſt) the figure A G is equall vnto the figure G E. Put the figure C F common to the

both: wherfore the whole figure AF is equall to the whole figure CE. VVherfore the figures AF and CE are double to the figure AF. But the figures AF and CE are the gnomon KLM, and the square CF: wherfore the gnomon KLM, and the square CF is double to the figure AF. But the double to AF is that which is contayned vnder AB and BC twise, for BF is equall vnto BC. VVherfore the gnomon KLM and the square CF is equall vnto the rectangle figure contayned vnder AB and BC twise. Put the figure DG common vnto them both, which is the square made of AC. VVherfore the gnomon KLM and the squares BG and GD are equal vnto the rectangle figure which is contained vnder AB & BC twise, & vnto the square which is made of AC. But the gnomon KLM, & the squares BG, & DG are ỳ whole square BADE, & ỳ part or square CF, which squares are made of the lines AB & of BC, therfore ỳ squares which are made of AB & BC are equall vnto the rectangle figure which is contayned vnder AB and BC twise, and also vnto the square of AC. If therfore a right line be deuided by chaunce: the square which is made of the whole together with the square which is made of one of the partes, is equall to the rectangle figure which is contayned vnder the whole and the sayd part twise, and to the square which is made of the other parte: Whiche was required to be demonstrated.



Flussates addeth vnto this Proposition this Corollary.

The squares of two vnequall lines do exceede the rectangle figures contayned vnder the said lines by the square of the excesse wherby the greater lyne excedeth the lesse.

For if the line *AB* be the greater, and the line *BC* the lesse, it is manifest that the squares of *AB* and *BC* are equall to the rectangle figure contayned vnder the lynes *AB* and *BC* twise, and moreouer to the square of the line *AC*, wherby the line *AB* excedeth the line *BC*.

By this proposition most wonderfully was found out the extraction of roote squares in irrationall numbers, beside many other straunge thinges.

An example of this propósition in numbers.

Take any number as 13. and deuide it into two partes as into 4. & 9. Take the square of 13. which is 169. take also the square of 4. which is 16. and adde these two squares together which make 185. Then multiply the whole number 13. into 4. the foresayde part twise, and you shall produce 52. and 52: take also the square of the other part, that is, of 9. which is 81. And adde it to the productes of 13. into 4. twise, that is, vnto 52. and 52. and those three numbers added together shall make 185. whiche is equall to the number composed of the squares of the whole and of one of the partes, which is also 185. As you see in the example.

The whole

13 { 4 / 9 } partes of the whole.

Multiplication of the whole into it ſelfe.

13
13
39
13
169 169

Multiplication of one of the partes into it ſelfe.

16 185

4
4
16

The number compoſed of the ſquares of the whole, and of one of the partes equall to the number compoſed of the whole into the foreſaid part twiſe, and of the ſquare of the other part.

Multiplicationof the whole into the foreſayde part twiſe.

13
4
52 52
52 81

Multiplication of the other part into it ſelfe.

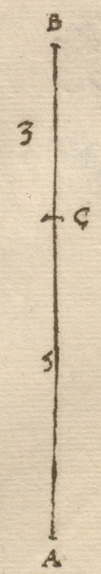
185

9
9
81

The demonſtration wheroffolloweth in Barlaam.

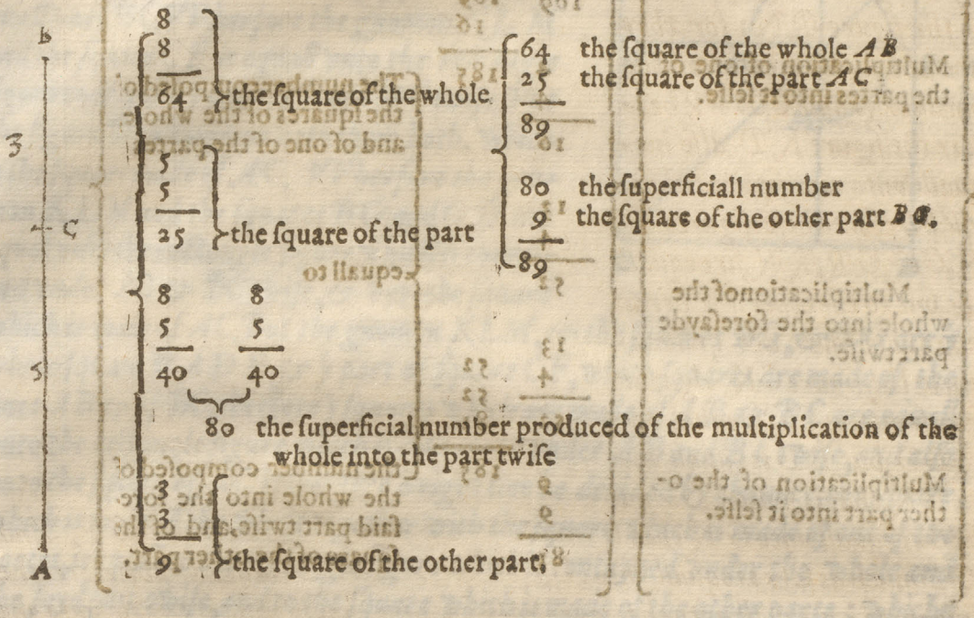
The ſeuenth propoſition.

If a number be deuided into two numbers: the ſquare of the whole number together with the ſquare of one of the partes, is equall to the ſuperficiall number produced of the multiplication of the whole number into the foreſaid part twiſe, together with the ſquare of the other part.



Suppoſe that the number *AB* be deuided into the numbers *AC* and *CB*. Then I ſay that the ſquare numbers of the numbers *BA* and *AC* are equall to the ſuperficiall number produced of the multiplicatiõ of the number *BA* into the number *AC* twiſe, together with the ſquare of the number *BC*. For foraſmuch as (by the 4. of this booke) the ſquare of the number *AB* is equall to the ſquares of the numbers *BC* and *CA*, and to the ſuperficiall number produced of the multiplication of the numbers *BC* and *CA* the one into the other twiſe: adde the ſquare of the number *AC* common to them both. Wherfore the ſquare of the number *AB* together with the ſquare of the number *AC* is equall to two ſquares of the number *AC* and to one ſquare of the number *CB*, and alſo to the ſuperficiall number produced of the multiplication of the numbers *BC* and *CA* the one into the other twiſe. And foraſmuch as the ſuperficial number produced of the multiplication of the numbers *BA* and *CA* the one into the other once, is equall to the ſuperficiall nũber produced of the multiplication of *BC* into *CA* once, and to the ſquare of the number *CA* (by the third of this booke): therfore the number produced of the multiplication of *BA* into *AC* twiſe is equall to the number produced of the multiplication of *BC* into *CA* twiſe, and alſo to two ſquares of the number *CA*. Adde the ſquare number of *BC* common to them both. Wherfore two ſquares of the number *AC* and one ſquare of the number *CB* together with the ſuperficiall number

duced of the multiplication of *B C* into *C A* twise are equall to the superficiall number produced of the multiplication of the number *B A* into the number *A C* twise together with the square of the number *C B*. Wherfore the square of the number *A B* together with the square of the nũber *A C* is equal to the superficial nũber produced of the multiplication of the number *B A* into the number *A C* twise, together with the square of the number *C B*. If therfore a number be deuided into two numbers &c. which was required to be demonstrated.



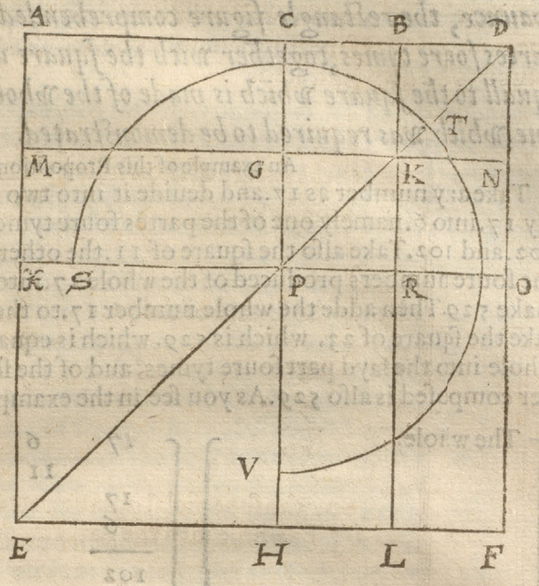
The 8. Theoreme. The 8. Proposition.

If a right line be deuided by chaũce, the rectangle figure comprehended vnder the whole and one of the partes foure times, together with the square which is made of the other parte, is equall to the square which is made of the whole and the foresaid part as of one line.

Vppose that there be a certayne right line A B, and let it be deuided by chaunce in the point C. Then I say that the rectangle figure comprehended vnder A B and B C foure tymes, together with the square which is made of A C is equall to the square made of A B and B C as of one line. Extend the line A B (by the second petition). And (by the third of the first) vnto C B put an equall lyne B D. And (by the 46. of the first) describe vppon A D a square A E F D. And describe a double figure. And forasmuch as C B is equall vnto B D, but C B is equall vnto G K (by the 34. of the first) and likewise B D

is

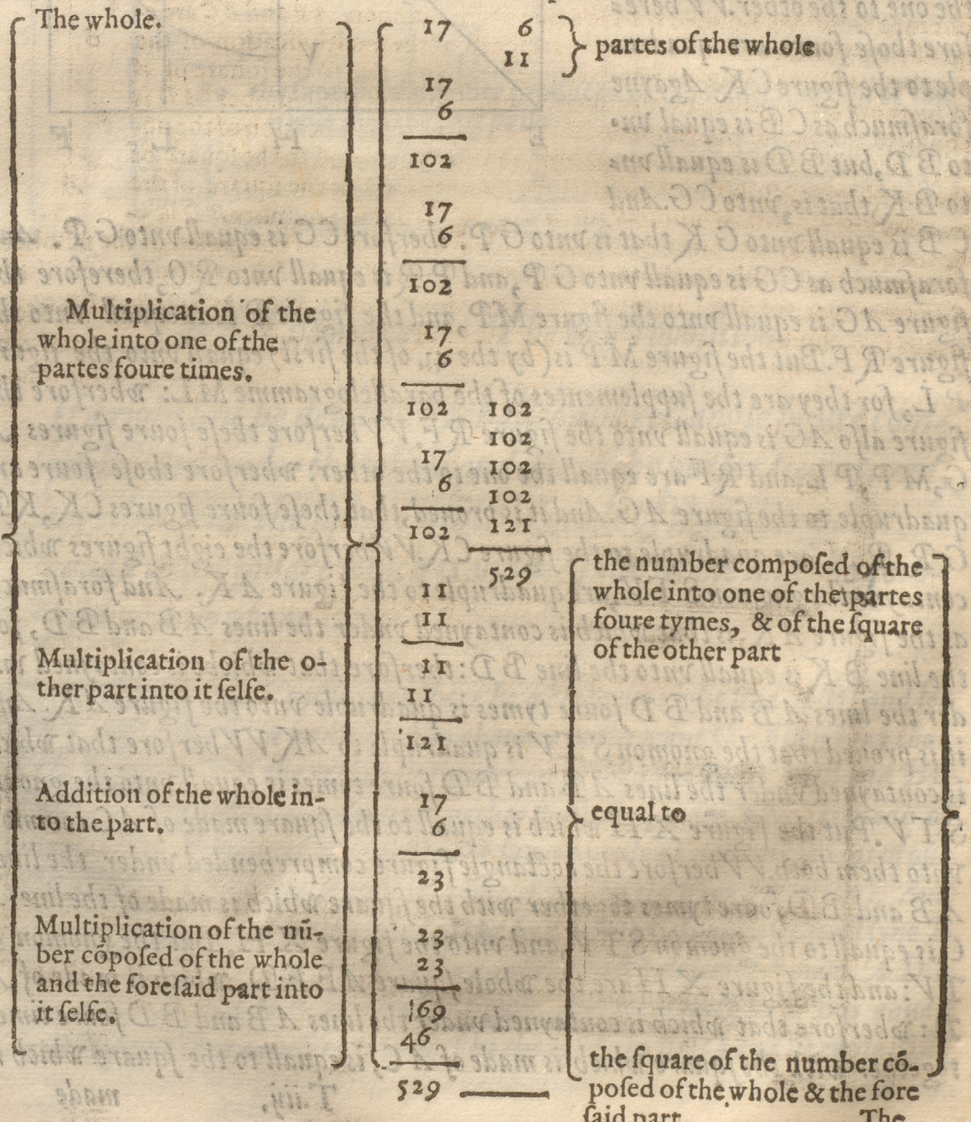
is equall vnto KN, wherefore GK also is equall vnto KN: and by the same reason also PR, is equall vnto RO. And forasmuch as BC is equall vnto BD, and GK vnto KN, therfore (by the 36. of the first) the figure CK is equall vnto the figure KD, and the figure GR is equall vnto the figure RN. But (by ỹ 43. of ỹ 1.) the figure CK is equall vnto the figure RN: for they are the supplementes of the parallelograme CO. Wherefore the figure KD also is equall vnto the figure NR. Wherefore these figures DK, CK, GR, RN, are equall the one to the other. Wherefore those foure are quadruple to the figure CK. Agayne forasmuch as CB is equal vnto BD, but BD is equall vnto BK, that is, vnto CG. And CB is equall vnto GK that is vnto GP: therfore CG is equall vnto GP. And forasmuch as CG is equall vnto GP, and PR is equall vnto RO, therefore the figure AG is equall vnto the figure MP, and the figure PL is equall vnto the figure RF. But the figure MP is (by the 43. of the first) equall vnto the figure PL, for they are the supplementes of the parallelogramme ML: wherfore the figure also AG is equall vnto the figure RF. Wherfore these foure figures AG, MP, PL, and RF are equall the one to the other: wherfore those foure are quadruple to the figure AG. And it is proued, that these foure figures CK, KD GR, RN, are quadruple to the figure CK. Wherfore the eight figures which contayne the gnomon STV, are quadruple to the figure AK. And forasmuch as the figure AK, is that which is contayned vnder the lines AB and BD, for the line BK is equall vnto the line BD: therfore that whiche is contayned vnder the lines AB and BD foure tymes is quadruple vnto the figure AK. And it is proued that the gnomon STV is quadruple to AK. Wherfore that which is contayned vnder the lines AB and BD foure tymes is equall vnto the gnomō STV. Put the figure XH which is equall to the square made of AC common vnto them both. Wherfore the rectangle figure comprehended vnder the lines AB and BD foure tymes together with the square which is made of the line AC, is equall to the gnomon STV, and vnto the figure XH. But the gnomon STV: and the figure XH are the whole square AEFD, which is made of AD: wherfore that which is contayned vnder the lines AB and BD foure times together with the square which is made of AC, is equall to the square which is



made of A D. But B D is equall vnto B C. Wherfore the rectangle figure contayned foure tymes vnder A B and B C together with the ſquare which is made of A C, is equall vnto the ſquare which is made of A D, that is, vnto that whiche is made of A B and B C as of one line. If therefore a right lyne be deuided by chaunce, the rectangle figure comprehended vnder the whole and one of the partes foure tymes, together with the ſquare which is made on the other part, is equall to the ſquare which is made of the whole and the foreſaid part, as of one line, which was required to be demonſtrated.

An example of this Propoſition in numbers.

Take any number as 17. and deuide it into two partes, as into 6. and 11. And multiply 17. into 6. namely one of the partes foure tymes, and you ſhall produce 102. 102. 102. and 102. Take alſo the ſquare of 11. the other part, which is 121: and adde it vnto the foure numbers produced of the whole 17. into the part 6. foure tymes, & you ſhall make 529. Then adde the whole number 17. to the foreſaid part 6. which make 23. & take the ſquare of 23. which is 529. which is equall to the number compoſed of the whole into the ſayd part foure tymes, and of the ſquare of the other part, which number compoſed is alſo 529. As you ſee in the example.



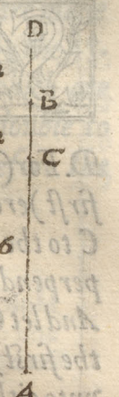
The

The demonſtration wheroffolloweth in Barlaam.

The eight propoſition.

If a number be deuided into two numbers, the ſuperficiall number produced of the multiplication of the whole into one of the partes foure tymes, together with the ſquare of the other parte, is equall to the ſquare of the number compoſed of the whole number and the foreſayd part.

Suppoſe that the number *AB* be deuided into two numbers *AC* and *CB*. Then *I* ſay that the ſuperficiall number produced of the multiplication of the number *AB* in to the number *CB* foure tymes together with the ſquare of the number *AC*, is equall to the ſquare of the number compoſed of the numbers *AB* & *CB*. For vnto the number *BC* let the number *BD* be equall. Now foraſmuch as the ſquare of the number *AD* is equal to the ſquares of the numbers *AB* and *BD*, & to the ſuperficiall number produced of the multiplication of the numbers *AB* & *BD* the one into the other twiſe (by the 4. of this booke) And the numb er *BD* is equall to the number *BC*: therefore the ſquare of the number *AD* is equall to the ſquares of the numbers *AB* and *BC*, and to the ſuperficiall number produced of the multiplication of the numbers *AB* and *BC* the one into the other twiſe. But the ſquares of the numbers *AB* and *BC* are equall vnto the ſuperficiall number produced of the multiplication of the numbers *AB* and *BC* the one into the other twiſe, and to the ſquare of *AC* (by the former propoſition) Wherfore the ſquare of the number *AD* is equall to the ſuperficial number produced of the multiplication of the nũbers *AB* and *BC* the one into the other foure tymes, and to the ſquare of the number *AC*. But the ſquare of the number *AD* is the ſquare of the number compoſed of the numbers *AB* and *BC*: for the number *BD* is equal to the number *BC*. Wherfore the ſquare of the number compoſed of the numbers *AB* and *BC* is equall to the ſuperficiall number produced of the multiplication of the numbers *AB* and *BC* the one into the other foure tymes, & to the ſquare of the number *AC*. If therfore a number be deuided into two numbers, &c.



8 8 8 8
2 2 2 2
16 16 16 16

64. the ſuperficiall number produced of the multiplication of the numbers *AB* and *BC* the one into the other foure tymes.

6
6
36 the ſquare of *AC*.

10
10
100 the ſquare of the number compoſed of *AB* and *BC*.

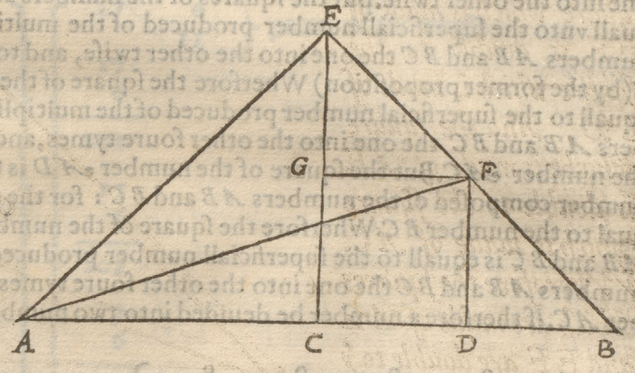
64 the ſuperficial nũber produced of the multiplicatiõ made 4. times
36 the ſquare number of *AC*.
100

The 9. Theoreme. The 9. Proposition.

If a right line be deuided into two equall partes, and into two vnequall partes, the squares which are made of the vnequall partes of the whole, are double to the squares, which are made of the halfe lyne, and of that lyne which is betwene the sections.

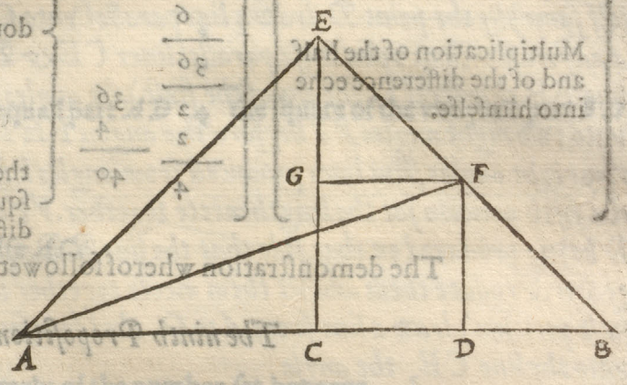
Suppose that a certayne right line A B be deuided into two equall partes in the pointe C, and into two vnequall partes in the pointe D. Then I say that the squares which are made of the lines A D and D B, are double to the squares whiche are made of the lynes A C and C D. For (by the 11. of the first) erect from ỹ point C to the right line A B a perpendiculer line C E. And let C E (by the 3. of the first) be put equall vnto either of these lines A C & C B: and (by the first peticiō) draw lines from A to E, and from E to B. And (by the 31. of the first) by the point D draw vntō the line E C a parallel lyne, and let the same be D F: and (by the selfe same) by the point F draw vnto A B a line parallel, and let the same be F G. And (by the first petition) draw a line from A to F. And forasmuch as A C is equall vnto C E, therfore (by the 5. of the first) the angle E A C is equal vnto the angle C E A. And forasmuch as the angle at the point C is a right angle: therfore the angles remayning E A C, and A E C, are equall vnto one right angle, wherefore eche of these angles E A C and A E C is the halfe of a right angle. And by the same reason also eche of these angles E B C and C E B is the halfe of a right angle. Wherfore the whole angle A E B is a right angle. And forasmuch as the angle G E F is the halfe of a right angle, but E G F is a right angle. For (by the 29. of the first) it is equall vnto the inward and oppositе angle, that is, vnto E C B: wherfore the angle remayning E F G is the halfe of a right angle. Wherefore (by the 6. common sentence) the angle G E F is equall vnto the angle E F G. Wherfore also (by the 6. of the first) the side E G is equall vnto the side F G. Againe forasmuch as the angle at the point B is the halfe of a right angle, but the angle F D B is a right angle, for it also (by the 29. of the first) is equall vnto the inward

Construction.



Demonstration.

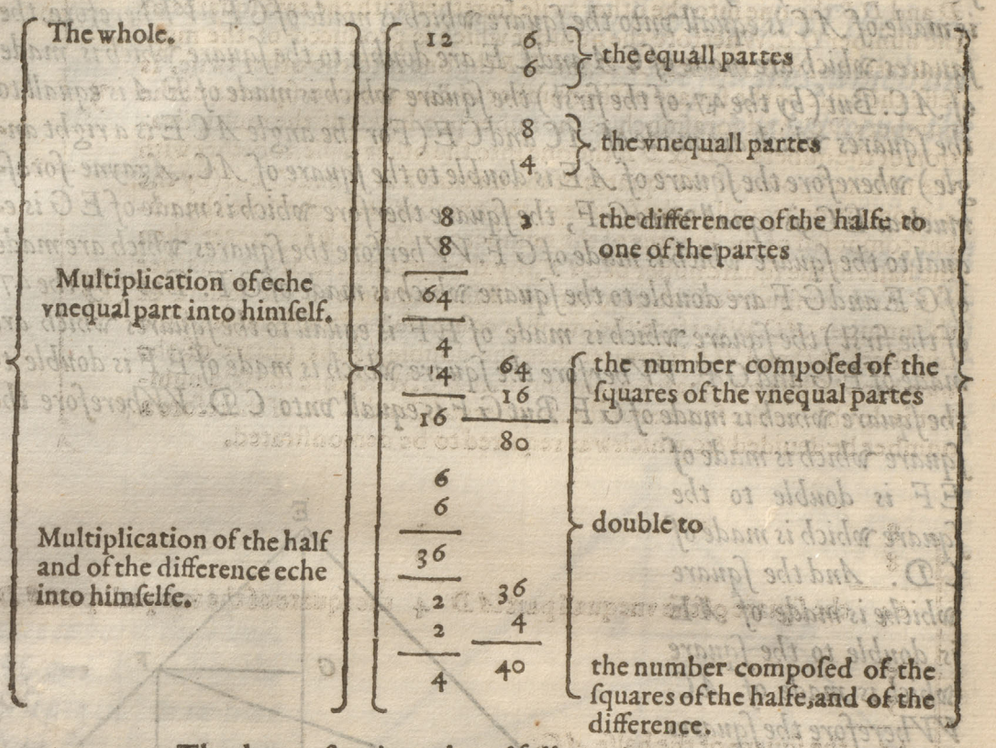
inwarde and opposite angle E C B. VVherefore the angle remayning B F D is the halfe of a right angle. VVherfore the angle at the point B is equall vnto the angle D F B. VVherfore (by the 6. of the first) the side D F is equall vnto the side D B. And forasmuch as A C is equall vnto C E, therfore the square which is made of A C is equall vnto the square which is made of C E. VVherefore the squares which are made of C A and C E are double to the square which is made of A C. But (by the 47. of the first) the square which is made of E A is equall to the squares which are made of A C and C E (For the angle A C E is a right angle) wherefore the square of A E is double to the square of A C. Agayne forasmuch as E G, is equall vnto G F, the square therfore which is made of E G is equal to the square which is made of G F. VVherfore the squares which are made of G E and G F are double to the square which is made of G F. But (by the 47. of the first) the square which is made of E F is equall to the squares which are made of E G and G F. VVherfore the square which is made of E F is double to the square which is made of G F. But G F is equall vnto C D. VVherefore the square which is made of E F is double to the square which is made of C D. And the square whiche is made of A E is double to the square which is made of A C. VVherefore the squares which are made of A E and E F are double to ỹ squares which are made of A C and C D. But (by the 47. of the first) the square which is made of A F is equal to the squares which are made of A E and E F (For ỹ angle A E F is a right angle). VVherfore the square which is made of A F is double to the squares which are made of A C & C D. But (by the 47. of the first) ỹ squares which are made of A D and D F are equall to ỹ square which is made of A F. For ỹ angle of ỹ point D is a right angle. VVherfore the squares which are made of A D and D F are double to ỹ squares which are made of A C and C D. But D F is equall vnto D B. VVherfore the squares which are made of A D and D B, are double to the squares which are made of A C and C D. If therfore a right line be deuided into two equall partes and into two vnequall partes, the squares which are made of the vnequall partes of the whole, are double to the squares which are made of the halfe lyne, and of that lyne which is betwene the sections: which was required to be proued.



¶ *An example of this proposition in numbers.*

Take any euen number as 12. And deuide it first equally as into 6. and 6. & then vnequally as into 8. & 4. And take the difference of the halfe to one of the vnequal partes

which is 2. And take the ſquare numbers of the vnequall partes 8, and 4, which are 64, and 16: and adde them together, which make 80. Then take the ſquares of the halfe 6, and of the differēce 2: which are 36, and 4: which added together make 40. Vnto which number, the number compoſed of the ſquares of the vnequall partes, whiche is 80, is double. As you ſee in the example.



The demonſtration wherof followeth in Barlaam.

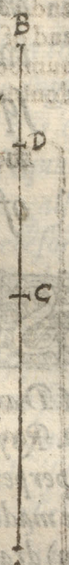
The ninth Propoſition.

If a number be deuided into two equall numbers, and againe be deuided into two inequall partes: the ſquare numbers of the vnequall numbers, are double to the ſquare which is made of the multiplication of the halfe number into it ſelfe, together with the ſquare whiche is made of the number ſet betwene them.

For let the number *AB* being an euen number be deuided into two equall numbers *AC* & *CB*: & into two vnequall nūbers *AD* and *DB*. Then *I* ſay that the ſquare numbers of *AD* and *DB*, are double to the ſquares which are made of the multiplication of the numbers *AC* and *CD* into themſelues. For foraſmuch as the number *AB* is an euen number, and is deuided alſo into two equal numbers *AC* and *CB*, and afterward into two vnequal nūbers *AD* and *DB*: therefore the ſuperficial nūber produced of the multiplicatiō of the nūbers *AD* & *DB*, th'one into the other, together with the ſquare of the number *DC*, is equal to the ſquare of the number *AC* (by the fift propoſition) Wherfore the ſuperficiall number produced of the multiplication of the numbers *AD* and *DB* the one into the other twiſe, together with two ſquares of the number *CD*, is double to the ſquare of the number *AC*. Foraſmuch as alſo the number *AB* is deuided into two equal numbers *AC* and *CB*, therfore the ſquare number of *AB* is quadruple to the ſquare number produced of the multiplication of the number *AC* into it ſelfe (by the 4. propoſition). Moreouer foraſmuch as the ſuperficiall number produced of the multiplication of the numbers *AD* & *DB* the one into the other twiſe together, with two ſquares of the number *DC*, is double to the ſquare number of *CA*: &

foraſmuch

foraſmuch as there are two numbers, of whiche the one is quadruple to one and the ſelfe ſame number, and the other is double to the ſame number: therefore that number whiche is quadruple ſhall be double to that number whiche is double. Wherefore the ſquare of the number *A B* is double to the number produced of the multiplicatiō of the numbers *A D* and *D B* the one into the other twiſe together with the two ſquares of the number *D C*. Wherfore the number which is produced of the multiplication of the numbers *A D* and *D B* the one into the other twiſe, is leſſe thē halfe of the ſquare of the number *A B* by the two ſquares of the numbers *D C*. And foraſmuch as the nūber produced of the multiplication of the nūbers *A D* & *D B* the one into the other twiſe, together with the nūber cōpoſed of the ſquares of the numbers *A D* and *D B* is (by the 4. propoſition) equall to the ſquare of the number *A B*: therfore the nūber compoſed of the ſquares of the numbers *A D* & *D B* is greater then the halfe of the ſquare nūber of *A B*, by the two ſquares of the number *D C*. And the ſquare of the number *A B* is quadruple to the ſquare of the number *A C*. Wherfore the number compoſed of the ſquares of the numbers *A D* and *D B* is greater then the double of the ſquare of the number *A C* by two ſquares of the number *D C*. Wherfore the ſaid number is double to the ſquares of the numbers *A C* and *C D*. If therefore a number be deuided &c. which was required to be demonſtrated.



8 × 8 = 64 the ſquare of the vnequall part *A D*

2 × 2 = 4 the ſquare of the vnequall part *B D*

5 × 5 = 25 the ſquare of the halfe *A C*.

3 × 3 = 9 the ſquare of *C D*, namely, of the number ſet betwene.

25 + 9 = 34 the ſquares of the halfe, and of the number ſet betwene.

64 + 4 = 68 the ſquares of the vnequall partes.

68 | 2
34

The 10. Theoreme. The 10. Propoſition.

If a right line be deuided into two equal partes, & vnto it be added an other right line directly: the ſquare which is made of the whole & that which is added as of one line, together with

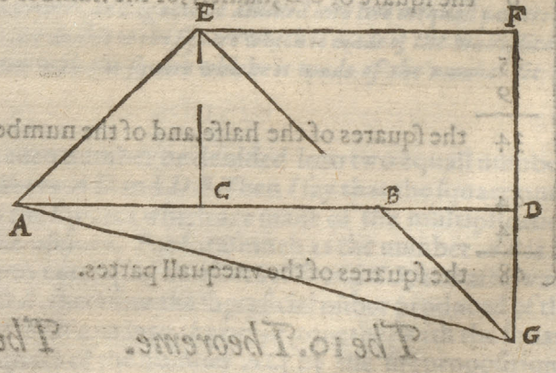
the ſquare whiche is made of the lyne whiche is added, theſe two ſquares (I ſay) are double to theſe ſquares, namely, to the ſquare which is made of the halfe line, & to the ſquare which is made of the other halfe lyne and that whiche is added, as of one lyne.

Conſtruction.

Suppoſe that a certayne right line A B be deuided into two equall partes in the point C. And vnto it let there be added an other right line directly, namely, B D. Thē I ſay, that the ſquares which are made of the lines A D and D B are double to the ſquares which are made of the lines A C and C D. Rayſe vp (by the 11. of the firſt) from the point C vnto the right line A C D a perpendiculer lyne, and let the ſame be C E. And let C E (by the 3. of the firſt) be made equall vnto either of theſe lines A C and C B. And (by the firſt petition) draw right lines from E to A, and from E to B. And (by the 31. of the firſt) by the point E, draw a line parallel vnto C D, and let the ſame be E F. And (by ỹ ſelf ſame) by the point D draw a line parallel vnto C E and let the ſame be D F. And foraſmuch as vpon theſe parallel lines C E & D F lighteth a certain right line E F, therfore (by the 29. of the firſt) the angles C E F and E F D are equal vnto two right angles. VVherfore the angles F E B, and E F D are leſſe then two right angles. But lines produced from angles leſſe then two right angles (by the fifth petition) at the length meete together. VVherfore the lines E B and F D beyng produced on that ſide that the line B D is, will at the length meete together. Produce them and let them meete together in the point G. And (by the

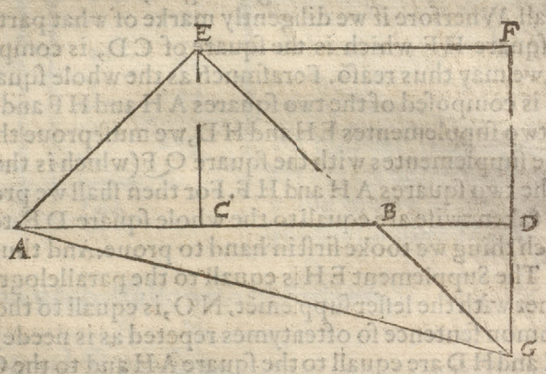
Demonſtration.

firſt petition) draw a lne from A to G. And foraſmuch as the line A C is equall vnto the line C E, the angle alſo A E C is (by the 5. of the firſt) equall vnto the angle E A C. And the angle at ỹ point C is a right angle. VVherfore eche of theſe angles E A C & and A E C is the halfe of a right angle. And by thi ſame reaſon eche of theſe angles C E B, and E B C is the halfe of a right angle. VVherefore the angle A E B is a right angle. And foraſmuch as the angle E B C is the halfe of a right angle, therfore (by the 15. of the firſt) the angle D B G is the half of a right angle. But ỹ angle B D G is a right angle (for it is equal vnto the angle D C E, for they are alternate angles) VVherfore the angle remaining D G B is the halfe of a right angle. VVherfore (by the 6 common ſentence of the firſt) the angle D G B is equall to the angle D B G. VVherfore (by the 6. of the firſt) the



ſide

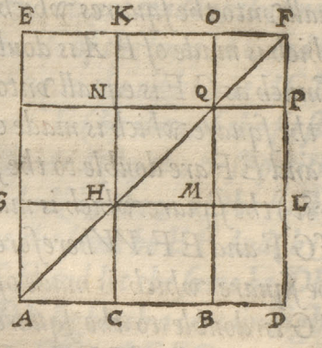
B D is equall vnto the side G D. Agayne forasmuch as the angle E G F is the halfe of a right angle: and the angle at the pointe F is a right angle: for (by the 34. of the first) it is equall vnto the opposite angle E C D. VVherefore the angle remayning F E G is the halfe of a right angle. VVherfore the angle E G F is equall to the angle F E G. VVherfore (by the 6. of the first) the side F E is equall vnto the side F G. And forasmuch as E C is equall vnto C A, the square also which is made of E C is equall to the square which is made of C A. VVherefore the squares which are made of C E and C A are double to the square which is made of A C. But the square which is made of E A is (by the 47. of the first) equall vnto the squares which are made of E C and C A. VVherefore the square which is made of E A is double to the square which is made of A C. Againe forasmuch as G F is equall vnto E F, the square also which is made of G F is equall to the square which is made of F E. VVherfore the squares which are made of G F and E F are double to the square which is made of E F. But (by the 47. of the first) the square which is made of E G is equall to the squares which are made of G F and E F. VVherefore the square which is made of E G is double to the square which is made of E F. But E F is equall vnto C D, wherfore the square which is made of E G is double to the square which is made of C D. And it is proued the square which is made of E A is double to the square which is made of A C. VVherfore the squares which are made of A E and E G are double to the squares which are made of A C and C D. But (by the 47. of ths first) the square which is made of A G is equall to the squares which are made of A E and E G. VVherefore the square which is made of A G is double to the squares which are made of A C and C D. But vnto the square whiche is made of A G are equall the squares which are made of A D and D G. VVherfore the squares which are made of A D and D G are double to the squares which are made of A C and D C. But D G is equall vnto D B. VVherfore the squares which are made of A D and D B are double to the squares which are made of A C and D C. If therfore a right line be deuided into two equall partes, and vnto it be added an other lyne directly, the square which is made of the whole and that which is added, as of one line, together with the square which is made of the line which is added, these two squares (I say) are double to these squares, namely, to the square which is made of the halfe lyne, and to the square which is made of the other halfe lyne and that which is added, as of one lyne: which was required to be proued.



¶ An other demonstration after Pelitarius.

Suppoſe that the lyne A B be deuided into two equall partes in the poynte C. And vnto it let there be added an other right lyne directly, namely, B D. Then I ſay that the ſquare of A D together with the ſquare of B D is double to the ſquares of A C and C D.

Vpon the whole line A D deſcribe a ſquare A D E F. And vpon the halfe lyne A C deſcribe the ſquare A C G H. And produce the ſides G H and C H till they cut the ſides E F & D F, wherby ſhalbe deſcribed the figure H L K F, which ſhalbe the ſquare of the line C D: as (by the Corollary of the 4. of this boke, & by the 34. Propoſition of the 1.) it is manifeſt if we draw the diameter C D. For the lyne K F is equall to the line C D. And making alſo the lines H M and H N equall to either of theſe lynes A C and C B, drawe the lynes M O and N P cutting the one the other right angled wiſe in the point Q. Either of which lynes let cut the ſides of the ſquare A D: E F in the pointes O and P. Now it nedeth not to proue that the figure H Q is the ſquare of the lyne A C, ſeyng that it is the ſquare of the line C B: as the figure Q F is the ſquare of the line B D: neither alſo needeth it to proue that the parallelograme H P is equall to either of the ſupplementes E H and H D: nor that the ſupplementes N O and Q L are equall. For all this is manifeſt euē by the forme of the figure, for that all the angles about the diameter are half right angles, & the ſides are equall. Wherfore if we diligently marke of what partes the ſquare H F which is the ſquare of C D, is compoſed, we may thus reaſō. Foraſmuch as the whole ſquare E D is compoſed of the two ſquares A H and H F and of the two ſupplementes E H and H D, we muſt proue that theſe ſupplementes with the ſquare Q F (which is the ſquare of the line B D) are equall to the two ſquares A H and H F. For then ſhall we proue that theſe two ſquares A H & H F taken twiſe are equall to the whole ſquare D E together with the ſquare of Q F, which thing we tooke firſt in hand to proue. And thus do I proue it.



The Supplement E H is equall to the parallelograme H P. And the ſquare A H together with the leſſer ſupplemēt, N O, is equall to the other ſupplemēt H D (by the firſt common ſentence ſo oftentymes rēpeted as is neede) wherfore the two ſupplementes E H and H D are equall to the ſquare A H and to the Gnomon K H L P Q O. If therfore vnto either of them be added the ſquare Q F: the two ſupplementes E H and H D together with the ſquare of Q F ſhal be equal to the ſquare A H, & to the Gnomon K H L P Q O and to the ſquare Q F. But theſe three figures do make the two ſquares A H and H F. Wherfore the two ſupplementes E H and H D together with the ſquare Q F are equall to the two ſquares A H and H F, which was the ſecond thing to be proued. Wherfore the two ſquares A H and H F beyng taken twiſe are equall to the whole ſquare D E together with the ſquare of Q F. Wherfore the ſquare D E together with the ſquare Q F is double to the ſquares A H and H F: which was required to be proued.

¶ *An example of this Propoſition in numbers.*

Take any euen number as 18: and take the halfe of it which is 9. and vnto 18. the whole, adde any other number as 3. which maketh 21. Take the ſquare number of 21. (the whole number and the number added) which maketh 441. Take alſo the ſquare of 3 (the number added) which is 9. which two ſquares added together make 450. Then adde the halfe number 9. to the number added 3. which maketh 12. And take the ſquare of 9. the halfe number and of 12. the halfe number and the number added which ſquares are 81. and 144. and which two ſquares alſo added together make 225: vnto which ſumme the foreſayd number 450. is double. As you ſee in the example.

The

The whole. 18
The number added. 3 9 } the halfe
21

21
21
21
Multiplication of the whole and the number added into himselfe. 42
441

441
Multiplication of the number added, into himselfe. 3 9
3 450 The number composed of the square of the whole & the number added and of the square of the number added,
9
Multiplication of the halfe into himselfe. 9
9
81 double to
Multiplication of the halfe, & the number added into it selfe. 12 81
12 144
24 225 the number composed of the square of the halfe and of the square of the halfe and the number added.
12
144

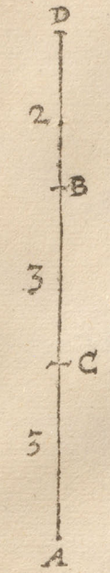
The demonstration wherof followeth in Barlaam.

The tenth Proposition.

If an euen nomber be deuided into two equall nombers, and vnto it be added any other nomber: the square nomber of the whole nomber composed of the nõber and of that which is added, and the square nomber of the nõber added: these two square nõbers (I say) added together, are double to these square nombers, namely, to the square of the halfe nomber, and to the square of the nomber composed of the halfe nomber and of the nomber added.

Suppose that the nomber *AB* being an euen nomber be deuided into two equall nombers *AC* and *CB*: and vnto it let be added an other nomber *BD*. Then I say, that the square nombers of the nombers *AD* and *DB* are double to the square nombers of *AC* and *CD*. For forasmuch as the nomber *AD* is deuided into the nombers *AB* and *BD*: therefore the square nombers of the nombers *AD* and *DB* are equall to the superficiall nomber produced of the multiplication of the nombers *AD* and *DB* the on into the other twise, together with the square of the nomber *AB* (by the 7 propositiõ) But the square of the nomber *AB* is equal to fower squares of either of the nombers *AC* or *CB* (for *AC* is equall to the nomber *CB*): wherfore also the squares of the nombers *AD* and *DB* are equall to the superficiall nomber produced of the multiplication of the nombers *AD* and *DB* the one into the other twise, and to fower squares of the nomber *BC* or *CA*. And forasmuch as the superficiall nomber produced of the multiplication of the nombers *AD* and *DB* the one into the other, together with the square of the nomber *CB*, is equal to square of the nomber *CD* (by the 6 propositiõ): therfore the nomber produced of the multiplication of the nombers *AD* and *DB* the one into the other twise together with two squares of the nomber *CB*, is equall to two squares of the nomber *CD*. Wherefore the squares of the nombers *AD* and *DB* are equall to

two ſquares of the nomber CD, and to two ſquares of the nomber AC. Wherefore they are double to the ſquares of the numbers AC and CD. And the ſquare of the nomber AD is the ſquare of the whole and of the nomber added: And the ſquare of DB is the ſquare of the nomber added: the ſquare alſo of the nomber CD is the ſquare of the nomber compoſed of the halfe and of the nomber added: If therefore an euen nomber be deuided. &c. Which was required to be proued.



8 × 8 = 64 the ſquare of AD

2 × 2 = 4 the ſquare of DB

5 × 5 = 25 the ſquare of CD, namely, of the number compoſed of the halfe and of the number added.

3 × 3 = 9 the ſquare of the halfe AC.

25 − 9 = 34; 64 − 4 = 68; 68 | 34 | 2.

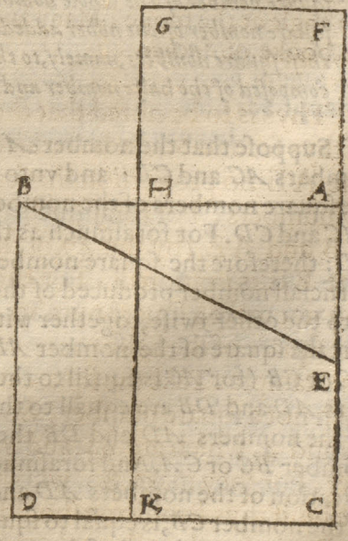
The I. Probleme. The II. Propoſition.

To deuide a right line geuen in ſuch ſort, that the rectangle figure comprehended vnder the whole, and one of the partes, ſhall be equall vnto the ſquare made of the other part.

Suppoſe that the right line geuen be AB. Now it is required to deuide the line AB in ſuch ſort, that the rectangle figure contayned vnder the whole and one of the partes, ſhall be equall vnto the ſquare which is made of the other part. Deſcribe (by the 46. of the firſt) vpon AB

Conſtruction. a ſquare $ABCD$. And (by the 10. of the firſt) deuide the line AC into two equall partes in the point E, and draw a line from B to E. And (by the ſecond petition) extend CA vnto the point F. And (by the 3. of the firſt) put the line EF equall vnto ỹ line BE. And (by the 46. of the firſt) vpon the line AF deſcribe a ſquare $FGAH$. And (by the 2. petition) extend GH vnto the point K. Then I ſay that the line AB is deuided in the point H in ſuch ſort, that the rectangle figure which is comprehẽded vnder AB and BH is equall to the ſquare which is made of

Demonſtratiõ AH. For foraſmuch as the right line AC is deuided into two equall partes in the poynt E, and vnto it is added an other right line



AF.

AF. Therefore (by the 6. of the ſecond) the rectangle figure contayned vnder CF and FA together with the ſquare which is made of AE is equall to ỹ ſquare which is made of EF. But EF is equall vnto EB. VVherefore the rectangle figure contayned vnder CF, and FA together with the ſquare which is made of EA is equall to the ſquare which is made of EB. But (by 47. of the firſt) vnto the ſquare which is made of EB are equall the ſquares which are made of BA and AE. For the angle at the poynt A is a right angle. VVherefore that which is contayned vnder CF and FA, together with the ſquare which is made of AE, is equall to the ſquares which are made of BA and AE. Take away the ſquare which is made of AE which is common, to them both: VVherfore the rectangle figure remayning contayned vnder CF and FA is equall vnto the ſquare which is made of AB. And that which is contained vnder the lines CF and FA is the figure FK. For the line FA is equall vnto the line FG. And the ſquare which is made of AB is the figure AD. VVherefore the figure FK is equall vnto the figure AD. Take away the figure AK which is common, to them both. VVherefore the reſidue, namely, the figure FH is equall vnto the reſidue, namely, vnto the figure HD. But the figure HD is that, which is contayned vnder the lines AB and BH, for AB is equall vnto BD. And the figure FH is the ſquare which is made of AH. VVherfore the rectangle figure comprehended vnder the lines AB and BH is equall to the ſquare which is made of the line HA. VVherefore the right line geuen AB is deuided in the point H, in ſuch ſort that the rectangle figure contayned vnder AB and BH is equall to the ſquare which is made of AH: which was required to be done.

Thys propoſition hath many ſingular vſes. Vpon it dependeth the demonſtration of that worthy Probleme the 10. Propoſition of the 4. booke: which teacheth to deſcribe an Iſoſceles triangle, in which eyther of the angles at the baſe ſhall be double to the angle at the toppe. Many and diuers vſes of a line ſo deuided ſhall you finde in the 13. booke of *Euclide*.

Many and ſinguler vſes of this propoſition.

Thys is to be noted that thys Propoſition can not as the former Propoſitions of thys ſecond booke be reduced vnto numbers. For the line EB hath vnto the line AE no proportion that can be named, and therefore it can not be expreſſed by numbers. For foraſmuch as the ſquare of EB is equall to the two ſquares of AB and AE (by the 47. of the firſt) and AE is the halfe of AB, therefore the line BE is irrationall. For euen as two equall ſquare numbers ioyned together can not make a ſquare number: ſo alſo two ſquare numbers, of which the one is the ſquare of the halfe roote of the other, can not make a ſquare number. As by an example. Take the ſquare of 8. which is 64. which doubled, that is, 128. maketh not a ſquare number. So take the halfe of 8. which is 4. And the ſquares of 8. and 4. which are 64. and 16. added together likewyſe make not a ſquare number. For they make 80. who hath no roote ſquare. Which thyng muſt of neceſſitie be if thys Probleme ſhould haue place in numbers.

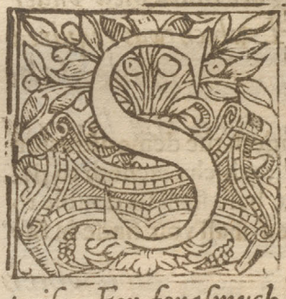
This propoſition can not be reduced vnto numbers.

But in Irrationall numbers it is true, and may by thys example be declared.

Let 8. be ſo deuided, that that which is produced of the whole into one of his partes ſhall be equall to the ſquare number produced of the other part. Multiply 8. into him ſelfe and there ſhall be produced 64. that is, the ſquare *A B C D*. Deuide 8. into two equall partes, that is, into 4, and 4. as the line *A E* or *E C*. And multiply 4. into hym ſelfe, and there is produced 16, which adde vnto 64, and there ſhall be produced 80: whoſe roote is $\sqrt{80}$: which is the line *E B* or the line *E F* by the 47. of the firſt. And foraſmuch as the line *E F* is $\sqrt{80}$. & the lyne *E A* is 4. therfore the lyne *A F* is $\sqrt{80}-4$. And ſo much ſhall the line A H be. And the line *B H* ſhall be $8-\sqrt{80}-4$, that is, $12-\sqrt{80}$. Now thē $12-\sqrt{80}$ multiplied into 8 ſhal be as much as $\sqrt{80}-4$. multiplied into it ſelfe. For of either of them is produced $96-\sqrt{5120}$.

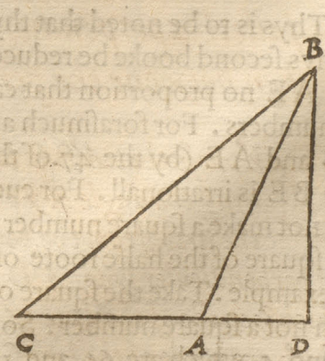
The 11. Theoreme. The 12. Propoſition.

In obtuſeangle triangles, the ſquare which is made of the ſide ſubtending the obtuſe angle, is greater then the ſquares which are made of the ſides which comprehend the obtuſe angle, by the rectangle figure, which is comprehended twiſe vnder one of thoſe ſides which are about the obtuſe angle, vpon which being produced falleth a perpendicular line, and that which is outwardly taken betwene the perpendicular line and the obtuſe angle.



Vppoſe that A B C be an obtuſeangle triangle hauing the angle B A C obtuſe, and from the point B (by the 12. of the firſt) draw a perpendicular line vnto C A produced and let the ſame be B D. Then I ſay that the ſquare which is made of the ſide B C, is greater then the ſquares which are made of the ſides B A and A C, by the rectangle figure comprehended vnder the lines C A and A D twiſe. For foraſmuch as the right line C D is by chaunce deuided in the poynt A, therefore (by the 4. of the ſecond) the ſquare which is made of C D is equall to the ſquares which are made of C A and A D, and vnto the rectangle figure contayned vnder C A and A D twiſe. Put the ſquare which is made of D B common vnto them both. VVherefore the ſquares which are made of C D and D B are equall to the ſquares which are made of the lines C A, A D, and D B, and vnto the rectangle figure contayned vnder the lines C A and A D twiſe. But (by the 47. of the firſt) the ſquare which is made of C B is equall to the ſquares which are made of the lines

Demonſtration.



C D

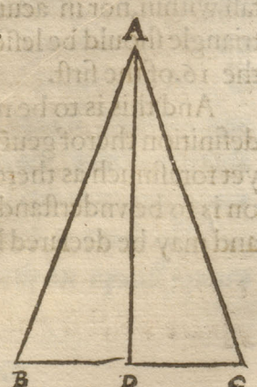
CD and DB. For the angle at the point D is a right angle. And vnto the ſquares which are made of AD and DB (by the ſelfe ſame) is equall the ſquare which is made of AB. Wherfore the ſquare which is made of CB, is equall to the ſquares which are made of CA and AB and vnto the rectangle figure contayned vnder the lines CA and AD twiſe. Wherfore ỹ ſquare which is made of CB, is greater then the ſquares which are made of CA and AB by the rectangle figure contayned vnder the lines CA and AD twiſe. In obtuſe-angle triangles therefore, the ſquare which is made of the ſide ſubtending the obtuſe angle, is greater then the ſquares which are made of the ſides which comprehend the obtuſe angle, by the rectangle figure which is comprehended twiſe vnder one of thoſe ſides which are about the obtuſe angle, vpon which being produced falleth a perpendiculer lyne, and that which is outwardly taken betwene the perpendiculer lyne and the obtuſe angle: which was required to be demonſtrated.

Of what force thys Propoſition, and the Propoſition following, touching the meaſuring of the obtuſeangle triangle and the acuteangle triangle, with the ayde of the 47. Propoſition of the firſt booke touching the rightangle triangle, he ſhall well perceaue, which ſhall at any time neede the arte of triangles in which by thre thinges knowen is euer ſearched out three other thinges vnknowen, by helpe of the table of arkes and cordes.

The 12. Theoreme. The 13. Propoſition.

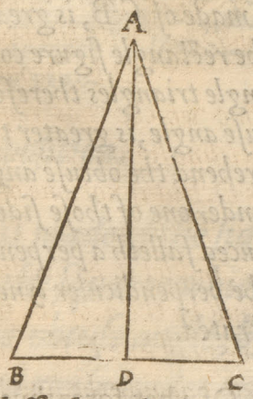
In acuteangle triangles, the ſquare which is made of the ſide that ſubtendeth the acute angle, is leſſe then the ſquares which are made of the ſides which comprehend the acute angle, by the rectangle figure which is cõprehended twiſe vnder one of thoſe ſides which are about the acuteangle, vpõ which falleth a perpendiculer lyne, and that which is inwardly taken betwene the perpendiculer lyne and the acute angle.

Vppoſe that ABC be an acuteangle triangle hauyng the angle at ỹ point B acute, & (by the 12. of the firſt) from the point A draw vnto the lyne BC a perpendiculer lyne AD. Then I ſay that the ſquare which is made of the lyne AC is leſſe then the ſquares which are made of the lyne CB and BA, by the rectangle figure conteyned vnder the lines CB and BD twiſe. For foraſmuch as the right lyne BC is by chaunce deuided in the point D, therfore (by the 7. of the ſecond) the ſquares



Demonſtratiõ

which are made of the lines C B and B D are equall to the rectangle figure contained vnder the lines C B and D B twiſe and vnto the ſquare whiche is made of line C D. Put the ſquare which is made of the line D A common vnto them both. VVherfore the ſquares which are made of the lines C B, B D, and D A, are equall vnto the rectangle figure contayned vnder the lines C B and B D twiſe, and vnto the ſquares which are made of A D and D C. But to the ſquares whiche are made of the lines B D and D A is equall ỹ ſquare which is made of the line A B: for th'angle at ỹ point D is a right angle. And vnto the ſquares whiche are made of the lines A D and D C is equall the ſquare whiche is made of the line A C (by the 47. of ỹ firſt): wherfore the ſquares which are made of the lines C B and B A are equal to the ſquare which is made of the line A C, and to that which is contained vnder the lines C B and B D twiſe. VVherfore the ſquare which is made of the line A C beyng taken alone, is leſſe then the ſquares which are made of the lines C B and B A by the rectangle figure, which is contained vnder the lines C B and B D twiſe. In rectangle triangles therfore the ſquare which is made of the ſide that ſubtendeth the acute angle, is leſſe then the ſquares which are made of the ſides which comprehend the acute angle, by the rectangle figure which is comprehended twiſe vnder one of thoſe ſides which are about the acute angle, vpon which falleth a perpendicular line, and that which is inwardly taken betwene the perpendicular line and the acute angle: which was required to be proued.



¶ *A Corollary added by Orontius.*

A Corollary. Hereby is eaſily gathered, that ſuch a perpendicular line in rectangle triangles falleth of neceſſitie vpon the ſide of the triangle, that is, neyther within the triangle, nor without. But in obtuſeangle triangles it falleth without, and in acuteangle triangles within. For the perpendicular line in obtuſeangle triangles, and acuteangle triangles can not exactly agree with the ſide of the triangle: for then an obtuſe & an acuteangle ſhould be equal to a right angle, contrary to the eleuenth and twelfth definitions of the firſt booke. Likewiſe in obtuſeangle triangles it can not fall within, nor in acuteangle triangles without: for then the outward angle of a triangle ſhould be leſſe then the inward and oppoſite angle, which is contrary to the 16. of the firſt.

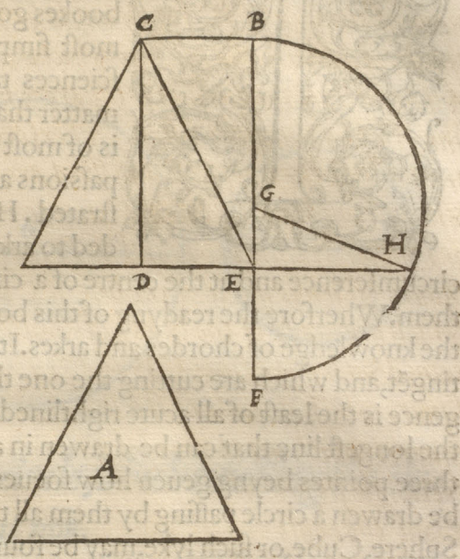
This Propoſition true in all kindes of triangles. And this is to be noted, that although properly an acuteangle triangle, by the definition therof geuē in the firſt booke, be that triangle, whoſe angles be all acute: yet foraſmuch as there is no triangle, but that it hath an acute angle, this propoſition is to be vnderſtanded, & is true generally in all kindes of triangles whatſoeuer, and may be declared by them, as you may eaſily proue.

The 2. Probleme. The 14. Proposition.

Vnto a rectiline figure geuen, to make a square equall.

SVppose that the rectiline figure geuen be A. It is required to make a square equall vnto the rectiline figure A. Make (by the 45. of ỹ first) vnto the rectiline figure A an equall rectangle parallelogramme BC DE. Now if ỹ line BE be equall vnto the line ED, then is ỹ thyng done whiche was required: for vnto the rectiline figure A is made an equal square BD. But if not, one of these lines BE & is ED the greater. Let BE be the greater, and let it be produced vnto ỹ point F. And (by the 3. of the first) put vnto ED an equall line EF. And (by the 10. of the first) deuide the line BF into two equall partes in the point G. And making the centre the point G, and the space GB or GF describe a semicircle BHF. And (by the 2. petition) extend the line DE vnto ỹ point H. And (by the 1. petition) draw a line from G to H. And forasmuch as the right line FB is deuided into two equall partes in the point G, and into two vnequal partes in the point E, therfore (by the 5. of the second) the rectangle figure comprehended vnder the lines BE and EF together with the square which is made of the line EG, is equall to the square which is made of the line GF. But the line GF is equall vnto the line GH. VVherfore the rectangle figure comprehended vnder the lines BE and EF together with the square which is made of the line GE is equall to square which is made of the line GH. But vnto the square which is made of the line GH are equall the squares whiche are made of the lines HE and GE (by the 47. of the first.) VVherfore ỹ which is contained vnder ỹ lines BE and EF together with ỹ square which is made of GE is equall to ỹ squares which are made of HE and GE. Take away the square of the line EG common to them both. VVherfore the rectangle figure contained vnder the lines BE & EF is equall to the square which is made of the line EH. But that whiche is contained vnder the lines BE and EF is the parallelogramme BD, for the line EF is equall vnto the line ED. VVherfore the parallelogramme BD is equall to ỹ square whiche is made of the line HE. But the parallelograme BD is equall vnto the rectiline figure A. VVherfore ỹ rectiline figure A is equall to the square which is made of ỹ line HE. VVherfore vnto the rectiline figure geuen A, is made an equall square described of the line EH: which was required to be done.

Construction.



Demonstratiõ

¶ The ende of the second Booke of Euclides Elementes.

¶ The third booke of Euclides Elementes.

The argument of this booke.

THis third booke of Euclide entreateth of the moſt perfect figure, which is a circle. Wherefore it is much more to be eſtemed then the two bookes goyng before, in which he did ſet forth the moſt ſimple proprieties of rightlined figures. For ſciences take their dignities of the worthynes of the matter that they entreat of. But of al figures the circle is of moſt abſolute perfection, whoſe proprieties and paſsions are here ſet forth, and moſt certainely demõſtrated. Here alſo is entreated of right lines ſubtended to arkes in circles: alſo of angles ſet both at the circumference and at the centre of a circle, and of the varietie and differences of them. Wherfore the readyng of this booke, is very profitable to the attayning to the knowledge of chordes and arkes. It teacheth moreouer which are circles contingẽt, and which are cutting the one the other: and alſo that the angle of contingence is the leaſt of all acute rightlined angles: and that the diameter in a circle is the longeſt line that can be drawen in a circle. Farther in it may we learne how, three pointes beyng geuen how ſoeuer (ſo that they be not ſet in a right line), may be drawen a circle paſſing by them all three. Agayne, how in a ſolide body, as in a Sphere, Cube, or ſuch lyke, may be found the two oppoſite pointes. Whiche is a thyng very neceſſary and commodious: chiefly for thoſe that ſhall make inſtrumentes ſeruyng to Aſtronomy, and other artes.

Definitions.

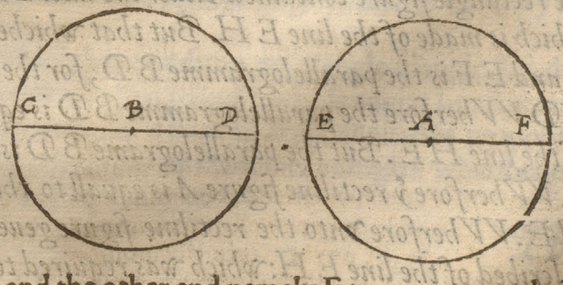
The firſt definition.

Equall circles are ſuch, whoſe diameters are equall, or whoſe lynes drawen from the centres are equall.

The circles A and B are equal, if theyr diameters, namely, E F and C D be equall: or if their ſemidiameters, whiche are lynes drawen from the center to the circumference, namely A F and B D be equall.

Why circles take their equality of their diameters or ſemidiameters.

The reaſon why circles take theyr equalitie, of the equalitie of their diameters or ſemidiameters is, for that a circle is deſcribed by one reuolution or turnyng about of the ſemidiameter, hauing one of his endes fixed. As if you imagine the lyne A E to haue his one point namely A faſtened, and the other end namely E to moue round till



it

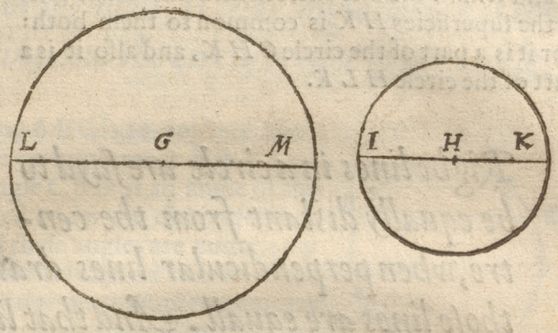
it come to the place where it begā to moue, it shal fully describe the whole circle. Wherefore if the semidiameters bee equall, the circles of necessitye must also be equall: and also the diameters.

By thys also is knowen the definition of vnequall circles.

Definition of vnequall circles.

Circles whose diameters or semidiameters are vnequall, are also vnequall. And that circle which hath the greater diameter or semidiameter, is the greater circle: and that circle which hath the lesse diameter or semidiameter, is the lesse circle.

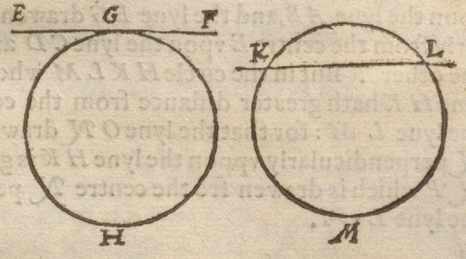
As the circle L M is greater then the circle I K, for that the diameter L M is greater then the diameter I K: or for that the semidiameter G L is greater then the semidiameter H I.



A right line is sayd to touch a circle, which touching the circle and being produced cutteth it not.

Second definition.

As the right lyne E F drawen from the point E, and passyng by a point of the circle, namely, by the point G to the point F only toucheth the circle G H, and cutteth it not, nor entreth within it. For a right line entryng within a circle, cutteth and deuideth the circle. As the right lyne K L deuideth and cutteth the circle K L M, and entreth within it: and therfore toucheth it in two places. But a right lyne touchyng a circle, which is commonly called a cōtingent lyne, toucheth the circle onely in one point.

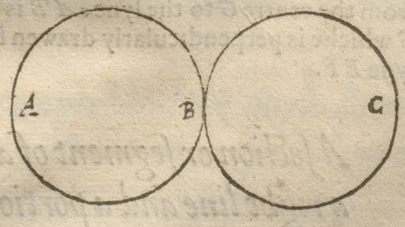


A contigent line.

Circles are sayd to touch the one the other, which touching the one the other, cut not the one the other.

Third definition.

As the two circles A B and B C touch the one the other. For theyr circumferences touch together in the poynt B. But neither of them cutteth or deuideth, the other. Neither doth any part of the one enter within the other. And such a touch of circles is euer in one poynt onely: which poynt onely is common to them both. As the poynt B is in the conference of the circle A B, and also is in the circūference of the circle B C.

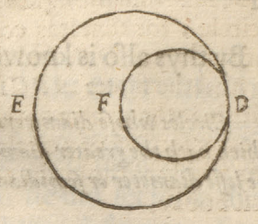


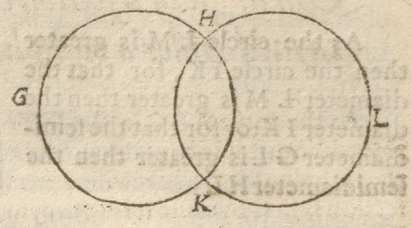
The touch of circles is euer in one poinc onely.

Circles may touch togeather two maner of wayes.

Circles may touch together two maner of wayes, either outwardly the one wholy without the other: or els the one being contayned within the other.

As the circles *D E* and *D F*: of which the one *D E* contayneth the other, namely *D F*: and touch the one the other in the poynt *D*: and that onely poynt is common to them both: neither doth the one enter into the other. If any part of the one enter into any part of the other, then the one cutteth and deuideth the other, and toucheth the one the other not in one poynt onely as in the other before, but in two pointes, and haue also a superficies common to them both. As the circles *G H K* and *H L K* cut the one the other in two poyntes *H* and *K*: and the one entreth into the other: Also the superficies *H K* is common to them both: For it is a part of the circle *G H K*, and also it is a part of the circle *H L K*.

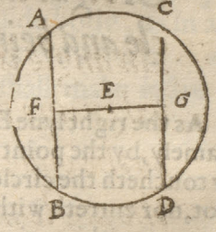


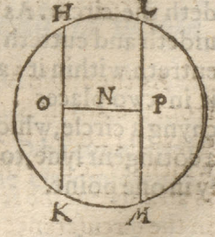


Fourth definition.

Right lines in a circle are sayd to be equally distant from the centre, when perpendicular lines drawen from the centre vnto those lines are equall. And that line is sayd to be more distant, vpon whom falleth the greater perpendicular line.

As in the circle *A B C D* whose centre is *E*, the two lynes *A B* and *C D* haue equall distance from the centre *E*: bycause that the lyne *E F* drawen from the centre *E* perpendicularly vpon the lyne *A B*, and the lyne *E G* drawen likewise perpendicularly from the centre *E* vpon the lyne *C D* are equall the one to the other. But in the circle *H K L M* whose centre is *N* the lyne *H K* hath greater distance from the centre *N* then hath the lyne *L M*: for that the lyne *O N* drawen from the centre *N* perpendicularly vppon the lyne *H K* is greater then the lyne *N P* which is drawen frō the centre *N* perpendicularly vpon the lyne *L M*.





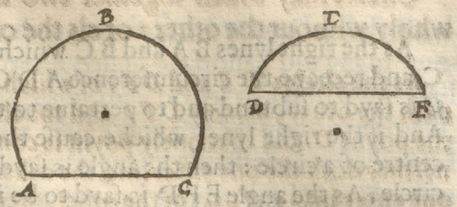
So likewise in the other figure the lynes *A B* and *D C* in the circle *A B C D* are equidistant from the centre *G*, bycause the lynes *O G* and *G P* perpendicularly drawen from the centre *G* vppon the sayd lynes *A B* and *D C* are equall. And the lyne *A B* hath greater distance from the centre *G* then hath the the lyne *E F*, bycause the lyne *O G* perpendicularly drawen from the centre *G* to the lyne *A B* is greater then the lyne *H G* whiche is perpendicularly drawen from the centre *G* to the lyne *E F*.

Fift definition.

A section or segment of a circle, is a figure cōprehended vnder a right line and a portion of the circumference of a circle.

As the

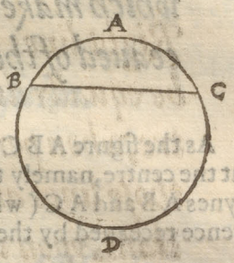
As the figure *A B C* is a section of a circle bycause it is comprehended vnder the right lyne *A C* and the circumference of a circle *A B C*. Likewise the figure *D E F* is a section of a circle, for that it is comprehended vnder the right lyne *D F*, and the circūference *D E F*. And the figure *A B C* for that it cōtaineth within it the centre of the circle is called the greater section of a circle: and the figure *D E F* is the lesse section of a circle, bycause it is wholy without the centre of the circle as it was noted in the 16. Definition of the first booke.



An angle of a section or segment, is that angle which is contayned vnder a right line and the circūference of the circle.

Sixt definition.

As the angle A B C in the section A B C is an angle of a section, bycause it is contained of the circumference B A C and the right lyne B C. Likewise the angle C B D is an angle of the section B D C bycause it is contayned vnder the circumference B D C, and the right lyne B C. And these angles are commonly called mixte angles, bycause they are contayned vnder a right lyne and a crooked: And these portions of circumferences are commonly called arkes, and the right lynes are called chordes, or right lynes subtended. And the greater section hath euer the greater angle, and the lesse section the lesse angle.



Mixt angles.

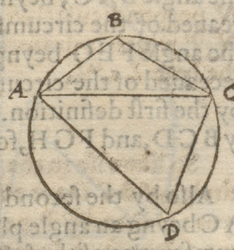
Arkes.

Chordes.

An angle is sayd to be in a section, whē in the circumference is taken any poynt, and from that poynt are drawen right lines to the endes of the right line which is the base of the segment, the angle which is contayned vnder the right lines drawen from the poynt, is (I say) sayd to be an angle in a section.

Seuenth definition.

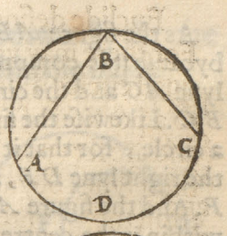
As the angle A B C is an angle in the section A B C, bycause from the poynt B beyng a poynt in the circumference A B C are drawen two right lynes B C and B A to the endes of the lyne A C which is the base of the section A B C. Likewise the angle A D C is an angle in the section A D C, bycause from the poynt D beyng in the circūference A D C are drawen two right lynes, namely, D C & D A to the endes of the right line A C which is also the base to the sayd section A D C. So you see, it is not all one to say, an angle of a section, and an angle in a section. An angle of a section cōsisteth of the touch of a right lyne and a crooked. And an angle in a section is placed on the circumference, and is contayned of two right lynes. Also the greater section hath in it the lesse angle, and the lesse section hath in it the greater angle.



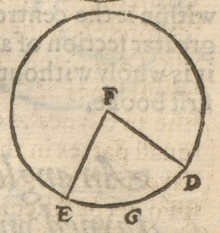
Difference of an angle of a Section, and of an angle in a Section.

But when the right lines which comprehend the angle do receaue any circumference of a circle, then that angle is sayd to be correspondent, and to pertaine to that circumference.

Eight definition.



As the right lynes B A and B C which containe the angle A B C, and receaue the circumference A D C therfore the angle A B C is ſayd to ſubtend and to pertaine to the circũference A D C. And if the right lynes whiche cauſe the angle, concurre in the centre of a circle: then the angle is ſayd to be in the centre of a circle. As the angle E F D is ſayd to be in the centre of a circle, for that it is comprehended of two right lynes F E and F D: whiche concurre and touch in the centre F. And this angle likewiſe ſubtendeth the circumference E G D: whiche circumference alſo, is the meaſure of the greatnes of the angle E F D.



Ninth deſinition.

A Sector of a circle is (an angle being ſet at the centre of a circle) a figure contayned vnder the right lines which make that angle, and the part of the circumference receaued of them.

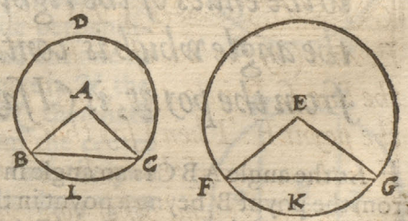
As the figure A B C is a ſector of a circle, for that it hath an angle at the centre, namely the angle B A C, & is cõtained of the two right lynes A B and A C (whiche contayne that angle and the circumference receaued by them.

Tenth definition.

Like ſegmentes or ſections of a circle are thoſe, which haue equall angles, or in whom are equall angles.

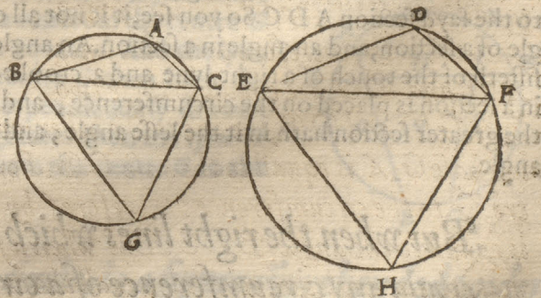
Two deſinitions. First.

Here are ſet two definitions of like ſections of a circle. The one pertaineth to the angles whiche are ſet in the centre of the circle and receaue the circumferẽce of the ſayd ſections: the other pertaineth to the angle in the ſection, whiche as before was ſayd is euer in the circumference. As if the angle B A C, beyng in the centre A and receaued of the circumference B L C be equall to the angle F E G beyng alſo in the centre E and receaued of the circumference F K G, then are the two ſections B C L and F G K lyke by the firſt definition. By the ſame definition alſo are the other two ſections like, namely B C D, and F G H, for that the angle B A C is equall to the angle F E G.



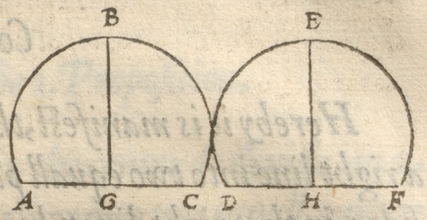
Second.

Alſo by the ſecond definition if B A C beyng an angle placed in the circumference of the ſection B C A be e- angle E D F beyng an angle in the ſection E F D placed in the circumference, there are the two ſections B C A, and E F D lyke the one to the other. Likewiſe alſo if the angle B G C beyng in the ſection B C G be equall to the angle E H F beyng in the ſectiõ E H F the two ſections B C G and E F H are lyke. And ſo is it of angles beyng equall in any poynt of the circumference.



Euclide defineth not equall Sections: for they may infinite wayes be deſcribed. For there may vppon vnequall right lynes be ſet equall Sections (but yet in vnequall circles) For from any circle beyng the greater, may be cut of a portion equall to a portion of an other circle beyng the leſſe. But when the Sections are equall, and are ſet vpon equall right lynes, theyr circumferences alſo ſhalbe equall. And right lynes beyng deuided into two equall partes, perpendicular lynes drawen from the poyntes of the diuiſion to the circumferẽces ſhalbe equall. As if the two ſections *A B C* and *D E F*, beyng ſet vppon equall ryght lynes *A C* & *D F*, be equall: then if ech of the two lynes *A C* & *D F* be deuided into two equall partes in the poyntes *G* and *H*, & from the ſayd poyntes be drawen to the circumferences two perpendicular lynes *B G* and *E H*, the ſayd perpendicular lynes ſhalbe equall.

Why Euclide defineth not equall Sections.



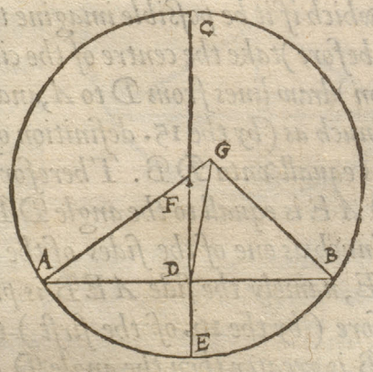
The 1. Probleme. The 1. Propoſition.

To finde out the centre of a circle geuen.

Vppoſe that there be a circle geuen A B C. It is required to finde out the centre of the circle A B C. Draw in it a right line at all aduentures, and let the ſame be A B. And (by the 10. of the firſt) deuide the line A B into two equall partes in the poynt D. And (by the 11. of the ſame) frõ the poynt D raiſe vp vnto A B a perpendicular line D C, & (by the ſecond petition) extend D C vnto ỹ point E. And (by the 10. of the firſt) deuide the line C E into two equall partes in the poynt F. Then I ſay that the point F is the centre of the circle A B C. For if it be not, let ſome other point, namely G, be the centre. And (by the firſt petition) draw theſe right lines G A, G D, and G B. And for aſmuch as A D is equall vnto D B, and D G is common vnto thẽ both, therefore theſe two lines A D and D G are equall to theſe two lines G D and D B, the one to the other, and (by the 15. definition of the firſt) the baſe G A is equall to the baſe G B. For they are both drawen from the centre G to the circumference: therefore (by the 8. of the firſt) the angle A D G is equall to the angle B D G. But when a right line ſtanding vpon a right line maketh the angles on eche ſide equall the one to the other, eyther of thoſe angles (by the 10. definition of the firſt) is a right angle. Wherefore the angle B D G

Conſtruction.

Demonſtration leading to an impoſſibilitie.



is a right angle: but ỹ angle F D B is also a right angle by construction. Wherfore (by the 4. petition) the angle F D B is equall to the angle B D G, the greater to the lesse, which is impossible. Wherefore the poynt G is not the centre of the circle A B C. In like wise may we proue that no other poynt besides F is the centre of the circle A B C. Wherefore the poynt F is the centre of the circle A B C: which was required to be done.

Correlary.

Correlary.

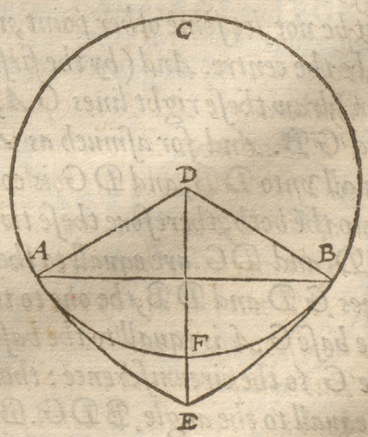
Hereby it is manifest, that if in a circle a right line do deuide a right line into two equall partes, and make right angles on eche side: in that right line which deuideth the other line into two equall partes is the centre of the circle.

The 1. Theoreme. The 2. Proposition.

If in the circūference of a circle be takē two poyntes at all aduentures: a right line drawen from the one poynt to the other shall fall within the circle.

Demonstratiō leading to an impossibilitie.

Suppose that there be a circle A B C. And in the circumference therof, let there be takē at all aduentures these two poyntes A & B. Then I say that a right line drawen from A to B shall fall within the circle A B C. For if it do not, let it fall without the circle, as the line A E B doth, which if it be possible imagine to be a right line. And (by the Proposition going before) take the centre of the circle, and let the same be D. And (by the first petition) draw lines from D to A, and from D to B. And extend D F to E. And for asmuch as (by the 15. definition of ỹ first) D A is equall vnto D B. Therefore the angle D A E is equall to the angle D B E. And for asmuch as one of the sides of the triangle D A E, namely the side A E B is produced, therefore (by the 16. of the first) the angle D E B, is greater then the angle D A E. But the angle D A E is equall vnto the angle D B E. Wherfore the angle D E B is greater then the angle D B E. But (by the 18. of the first) vnto the greater angle is subtended the greater side. Wherefore the side D B is greater then the side D E. But (by the 15. definition of the first) the line D B is



equall

equall vnto the line D F. VVherfore the line D F is greater then the line D E, namely, the leſſe greater then the greater: which is impoſsible. VVherfore a right line drawen from A to B falleth not without the circle. In like ſort alſo may we proue that it falleth not in the circumference: VVherefore it falleth within the circle. If therefore in the circumference of a circle be taken two poyntes at all aduentures: a right line drawen from the one poynt to the other ſhall fall within the circle: which was required to be proued.

The 2. Theoreme. The 3. Propoſition.

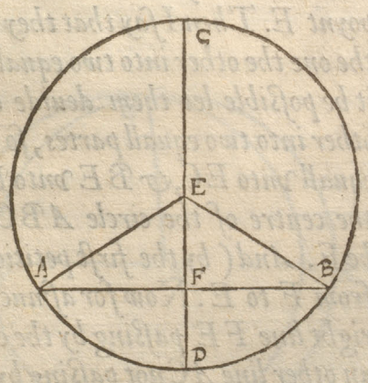
If in a circle a right line paſsing by the centre do deuide an other right line not paſsing by the cẽtre into two equall partes: it ſhall deuide it by right angles. And if it deuide the line by right angles, it ſhall alſo deuide the ſame line into two equall partes.

The first part of this Propoſition.

SVppoſe that there be a circle A B C, and let there be in it drawen a right line paſsing by the centre, and let the ſame be C D, deuiding an other right line A B not paſsing by ẏ centre into two equall partes in the poynt F. Then I ſay that the angles at the poynt of the deuiſion are right angles.

Conſtruction.

Take (by ẏ firſt of the third) the centre of the circle A B C, and let the ſame be E. And (by the firſt petition) drawe lines from E to A & from E to B.



Demonſtration.

And for aſmuch as the line A F is equall vnto the line F B, and the line F E is common to them both, therfore theſe two lines E F and F A are equall vnto theſe two lines E F & F B. And the baſe E A is equall vnto the baſe E B (by the 15. definition of the firſt). VVherefore (by the 8. of the firſt) the angle A F E is equall to the angle B F E. But when a right line ſtanding vpon a right line doth make the angles on eche ſide equall the one to the other, eyther of thoſe angles is (by the 10. definition of the firſt) a right angle. VVherfore either of theſe angles A F E, & B F E is a right angle. VVherefore the line C D paſsing by the centre, and deuiding the line A B not paſsing by the centre into two equall partes, maketh at the point of the deuiſion right angles.

The ſecond part conuerſe of the firſt.

But now ſuppoſe that the line C D do deuide the line A B in ſuch ſort that it maketh right angles. Then I ſay that it deuideth it into two equall partes, that is, ẏ the line A F is equall vnto the line F B. For the ſame order of conſtruction remayning, for aſmuch as the line E A is equall vnto the line E B (by the 15. de-

Demonſtration.

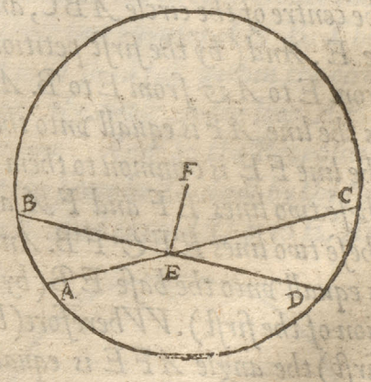
finition of the first). Therefore the angle EAF is equall vnto the angle EBF (by the 5. of the first). And the right angle AFE is (by the 4. petition) equall to the right angle BFE. Wherefore there are two triangles EAF, & EBF hauing two angles equall to two angles, & one side equall to one side, namely the side EF which is common to them both, and subtendeth one of the equall angles, wherefore (by the 26. of the first) the sides remayning of the one, are equall vnto the sides remayning of the other. Wherefore the line AF is equall vnto the line FB. If therefore in a circle a right line passing by the centre do deuide an other right line not passing by the centre into two equall partes, it shall deuide it by right angles. And if it deuide the line by right angles it shall also deuide the same line into two equall partes: which was required to be demonstrated.

The 3. Theoreme. The 4. Proposition.

If in a circle two right lines not passing by the centre, deuide the one the other: they shall not deuide eche one the other into two equall partes.

Demonstration leading to an impossibilitie.

Svppose that there be a circle ABCD, and let there be in it drawen two right lines not passing by the centre and deuiding the one the other, and let the same be AC and BD, which let deuide the one the other in the poynt E. Then I say that they deuide not eche the one the other into two equall partes. For if it be possible let them deuide eche the one the other into two equall partes, so that let AE be equall vnto EC, & BE vnto ED. And take the centre of the circle ABCD, which let be F. And (by the first petition) draw a line from F to E. Now for asmuch as a certaine right line FE passing by the centre deuideth an other line AC not passing by the centre into two equall partes, it maketh where the deuision is right angles (by the 3. of the third). Wherfore the angle FEA is a right angle. Againe for asmuch as the right line FE, passing by the centre, deuideth the right line BD not passing by the centre into two equall partes, therefore (by the same) it maketh where ỹ deuision is right angles. Wherfore the angle FEB is a right angle. And it is proued that the angle FEA is a right angle. Wherfore (by the 4. petition) the angle FEA is equall vnto the angle FEB, namely the lesse angle vnto the greater: which is impossible. Wherefore the right lines AC and BD deuide not eche one the other into two equall partes. If therfore in a circle two right lines not passing by the centre, deuide the one the other, they



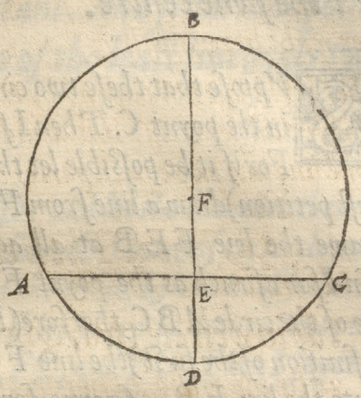
shall

ſhall not deuide eche one the other into two equall partes: which was required to be demonſtrated.

In this Propoſition are two caſes. For the lines cutting the one the other, do eyther, neyther of them paſſe by the centre, or the one of them doth paſſe by the centre, & the other not. The firſt is declared by the author. The ſecond is thus proued.

Two caſes in this Propoſition.

Suppoſe that in the circle *A B C D* the line *B D* paſſing by the centre doe cut the line *A C* not paſſing by the centre. Then I ſay that the lines *A C* and *B D* do not deuide the one the other into two equall partes. For by the former Propoſition the line *B D* paſſing by the centre and deuiding the line *A C* into two equall partes, it ſhall alſo deuide it perpendicularly. And for aſmuch as the line *A C* deuideth the line *B D* into two equall partes & right angled wiſe: therfore by the Correllary of the firſt of thys booke, the line *A C* paſſeth by the centre of the circle: which is cõtrary to the ſuppoſition. Wherfore the lines *A C* and *B D* do not deuide the one the other into two equall partes: which was required to be proued.



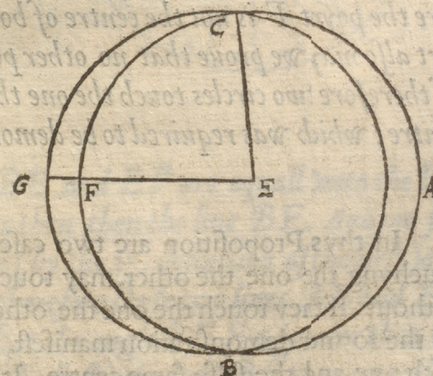
Conſtruction for the ſecond caſe.

Demonſtration.

The 4. Theoreme. The 5. Propoſition.

If two circles cut the one the other, they haue not one and the ſame centre.

Vppoſe that theſe two circles A B C, and C B G do cut the one the other in the poyntes C and B. Then I ſay that they haue not one & the ſame centre. For if it be poſſible let E be centre to them both. And (by the firſt petition) draw a line from E to C. And draw an other right line E F G at all aduentures. And for aſmuch as the poynt E is the centre of the circle A B C, therefore (by the 15. definition of the firſt) the line E C is equall vnto the line E F. Agayne for aſmuch as the poynt E is the centre of the circle C B G, therefore (by the ſame definition) the line E C is equall vnto the line E G. And it is proued that the line E C is equall vnto the line E F: wherefore the line E F alſo is equall vnto the line E G, namely the leſſe vnto the greater: which is impoſſible. Wherfore the poynt E is not the centre of both the circles A B C, & C B G. In like ſort alſo may we proue that no other poynt is the centre of both the ſayd circles.



Conſtruction.

Demonſtratiõ leading to an impoſſibilitie.

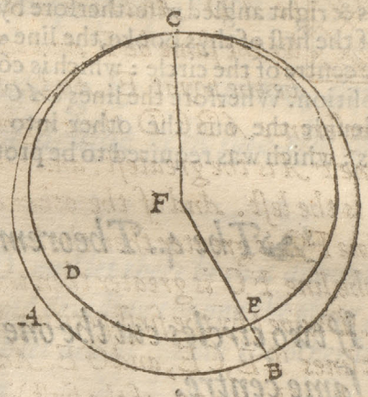
circles. If therefore two circles cut the one the other, they haue not one and the same centre: which was required to be proued.

The 5. Theoreme. The 6. Proposition.

If two circles touch the one the other, they haue not one and the same centre.

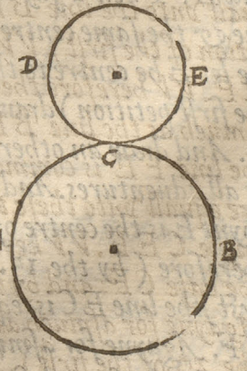
Demonstration leading to an impossibilitie.

SVppose that these two circles ABC, & CDE do touch the one the other in the poynt C. Then I say that they haue not one and the same centre. For if it be possible let the point F be centre vnto them both. And (by the first petition) draw a line from F to C: and drawe the line FEB at all aduentures. And for asmuch as the poynt F is the centre of the circle ABC, therfore (by the 15. definition of the first) the line FC is equall vnto the line FB. Agayne forasmuch as the poynt F is the centre of ỹ circle CDE, therefore (by the same definition) the line FC is equall vnto the line FE. And it is proued, that the line FC is equall vnto the line FB, wherefore the line FE also is equall vnto the line FB, namely the lesse vnto ỹ greater: which is impossible. Wherefore the poynt F is not the centre of both the circles ABC and CDE. In like sort also may we proue that no other poynt is the centre of both the sayd circles. If therefore two circles touch the one the other: they haue not one and the same centre: which was required to be demonstrated.



Two cases in thys Proposition.

In thys Proposition are two cases: for the circles touchyng the one the other, may touch eyther within or without. If they touch the one the other within, then is it by the former demonstration manifest, that they haue not both one and the selfe same centre. It is also manifest if they touch the one the other without: for that euery centre is in the middest of hys circle.

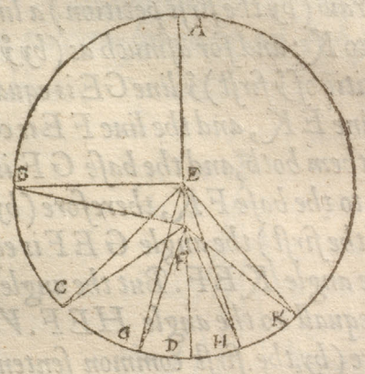


The 6. Theoreme. The 7. Proposition.

If in the diameter of a circle be taken any poynt, which is not the

the centre of the circle, and from that poynt be drawen vnto the circumference certaine right lines: the greatest of those lines shall be that line wherein is the centre, and the lest shall be the residue of the same line. And of all the other lines, that which is nigher to the line which passeth by the centre is greater then that which is more distant. And from that point can fall within the circle on ech side of the least line onely two equall right lines.

SVppose that there be a circle *ABCD*: and let the diameter thereof be *AD*. And take in it any poynt besides the centre of the circle, and let the same be *F*. And let the centre of the circle (by the 1. of ỹ third) be the poynt *E*. And from the poynt *F* let there be drawen vnto the circumference *ABCD* these right lines *FD*, *FC*, and *FG*. Then I say that the line *FA* is the greatest: and the line *FD* is the lest. And of the other lines, the line *FB* is greater then the line *FC*, and the line *FC* is greater then the line *FG*. Drawe (by the first petition) these right lines *BE*, *CE*, and *GE*. And for asmuch as (by the 20. of the first) in euery triangle two sides are greater then the third, therefore ỹ lines *EB* and *EF* are greater then the residue, namely then the line *FB*. But the line *AE* is equall vnto the line *BE* (by the 15. definition of the first). VVherefore the lines *BE* and *EF* are equall vnto the line *AF*. VVherefore the line *AF* is greater then then the line *BF*. Agayne for asmuch as the line *BE* is equall vnto *CE* (by the 15. definition of the first) and the line *FE* is common vnto them both, therefore these two lines *BE* and *EF* are equall vnto these two *CE* and *EF*. But the angle *BEF* is greater then the angle *CEF*. VVherefore (by the 24. of the first) the base *BF* is greater then the base *CF*: and by the same reason the line *CF* is greater then the line *FG*. Agayne for asmuch as the lines *GF* and *FE* are greater then the line *EG* (by the 20. of the first). But (by the 15. definition of the first) the line *EG* is equall vnto the line *ED*: VVherefore the lines *GF* and *FE* are greater then the line *ED*, take away *EF*, which is cõmon to thẽ both, wherfore ỹ residue *GF* is greater then the residue *FD*: VVherefore the line *FA* is the greatest, and the line *FD* is the lest, and the line *FB* is greater then the line *FC*, and the line *FC* greater



Construction.

The first part of this Propositon.

Demonstration.

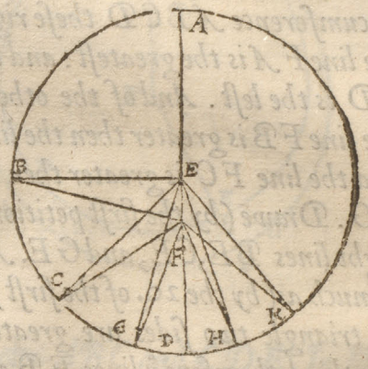
Second part.

Third part.

is greater then the line FG. Now also I say that from the poynt F there can be drawen onely two equall right lines into the circle ABCD on eche side of the least line, namely FD. For (by the 23. of the first) vpon the right line geuen EF and to the poynt in it, namely E, make vnto the angle GEF an equall angle FEH: and (by the first petition) draw a line from F to H. Now for asmuch as (by the 15. definition of the first) the line EG is equall vnto the line EH, and the line EF is common vnto them both, therefore these two lines GE and EF are equall vnto these two lines HE and EF, and (by construction) the angle GEF is equall vnto the angle HEF. VVherefore (by the 4. of ỹ first) the base FG is equall vnto the base FH. I say moreouer that from the poynt F can be drawen into the circle no other right line equall vnto the line FG. For if it possible let the line FK be equall vnto the line FG. And for asmuch as FK is equall vnto FG. But the line FH is equall vnto the line FG, therefore the line FK is equall vnto the line FH. VVherfore the line which is nigher to the line which passeth by the centre is equall to that which is farther of, which we haue before proued to be impossible.

This demonstrated by an argument leading to an impossibilie.

An other demonstration of the latter part of the Proposition leading also to an impossibilitie.



Or els it may thus be demonstrated. Draw (by the first petition) a line from E to K: and for asmuch as (by ỹ 15. definitiō of ỹ first) ỹ line GE is equall vnto ỹ line EK, and the line FE is common to them both, and the base GF is equall vnto the base FK, therefore (by the 8. of the first) the angle GEF is equall to the angle KEF. But the angle GEF is equall to the angle HEF. VVherefore (by the first common sentence) the angle HEF is equall to the angle KEF the lesse vnto the greater: which is impossible. VVherefore from the poynt F there can be drawen into the circle no other right line equall vnto the line GF. VVherefore but one onely. If therefore in the diameter of a circle be taken any poynt, which is not the centre of the circle, and from that poynt be drawen vnto the circumference certaine right lines: the greatest of those right lines shall be that wherein is the centre: and the least shall be the residue. And of all the other lines, that which is nigher to the line which passeth by the centre is greater then that which is more distant. And from that poynt can fall within the circle on ech side of the least line onely two equall right lines: which was required to be proued.

¶ A Corollary.

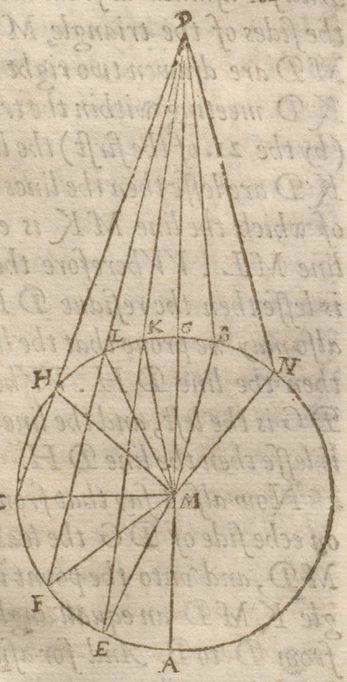
A Corollary.

Hereby it is manifest, that two right lines being drawen frō any one poynt of the diameter, the one of one side, and the other of the other side, if with the diameter they make equall angles, the sayd two right lines are equall. As in thys place are the two lines FG and FH.

¶ The

The 7. Theoreme. The 8. Proposition.

If without a circle be taken any poynt, and from that poynt be drawen into the circle vnto the circumference certayne right lines, of which let one be drawen by the centre and let the reſt be drawen at all aduentures: the greateſt of thoſe lines which fall in the concauitie or hollownes of the circumference of the circle, is that which paſſeth by the centre: and of all the other lines that line which is nigher to the line which paſſeth by the centre is greater then that which is more diſtant. But of thoſe right lines which end in the conuexe part of the circumference, that is the leaſt which is drawen from the poynt to the diameter: and of the other lines that which is nigher to the leaſt is alwaies leſſe then that which is more diſtant. And from that poynt can be drawen vnto the circumference on ech ſide of the leaſt onely two equall right lines.

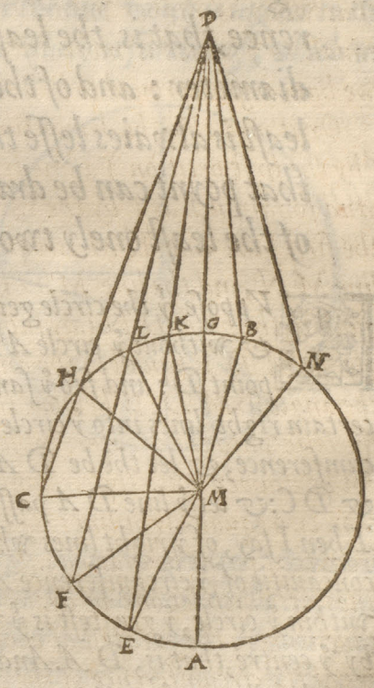


SVppoſe ẏ the circle geuen be *ABC*, & without ẏ circle *ABC*, take the point *D*: and frō ẏ ſame point draw certain right lines into ẏ circle vnto the circumference, & let thē be *DA*, *DE*, *DF*, & *DC*: & let ẏ line *DA* paſſe by ẏ centre. Then I ſay, of ẏ right lines which fall in the concauitie of ẏ circumference *AEFC*, ẏ is, within ẏ circle, ẏ greateſt is ẏ which paſſeth by ẏ centre, that is, *DA*. And of thoſe lines which fall vpon ẏ conuex part of ẏ circumference, ẏ leſt is ẏ which is drawen frō ẏ point *D* vnto ẏ end of ẏ diameter *AG*. And of the right lines falling win the circumferēce, the line *DE* is greater then ẏ line *DF*, & the line *DF* is greater then ẏ line *DC*. And of the right lines which end in ẏ conuex part of the circumference, ẏ is, without ẏ circle, that which is nigher vnto *DG* ẏ leſt, is alwayes leſſe then ẏ which is more diſtāt, that is, the line *DK* is leſſe then the line *DL*, and the line *DL* is leſſe then the line *DH*. Take (by the firſt of the third) the centre of the circle *ABC*, and let the *Conſtruction.*

The first part of this Propositon.

same be M: and (by the first petition) drawe these right lines ME, MF, MC, MH, ML, and MK. And for asmuch as (by the 15. definition of the first) the line AM is equall vnto the line EM, put the line MD common to them both. VVherefore the line AD is equall vnto the lines EM and MD. But the lines EM and MD are (by the 20. of the first) greater then the line ED. VVherefore the line AD also is greater then the line ED. Agayne for asmuch as (by the 15. definition of the first) the line ME is equall vnto the line MF, put the line MD common to them both: VVherefore the lines EM and MD are equall to the lines FM and MD, and the angle EMD is greater then the angle FMD: VVherefore (by the 19. of the first) the base ED is greater then the base FD. In like sort also may we proue that the line FD is greater then the line CD. VVherefore the line DA is the greateſt, and the line DE is greater then the line DF, and the line DF is greater then the line DC.

Second part.



And for asmuch as (by the 20. of the first) the lines MK and KD are greater then the line MD. But (by the 15. definition of the first) the line MG is equall vnto the line MK. VVherefore the residue KD is greater then ỹ residue GD. VVherefore the line GD is lesse then the line KD. And for asmuch as from the endes of one of the sides of the triangle MLD, namely, MD are drawen two right lines MK and KD meeting within the triangle, therfore (by the 21. of the first) the lines MK and KD are lesse then the lines ML & LD, of which the line MK is equall vnto the line ML. VVherefore the residue DK is lesse then the residue DL. In like sort also may we proue that the line DL is lesse then the line DH. VVherefore the line DG is the leſt, and the line DK is lesse then the line DL, and the line DL is lesse then the line DH.

Third part.

Now also I say that from the poynt D can be drawen vnto the circumference on eche side of DG the leaſt onely two equall right lines. Vpon the right line MD, and vnto the poynt in it M make (by the 23. of the first) vnto the angle KMD an equall angle DMB. And (by the first petition) drawe a line from D to B. And for asmuch as (by the 15. definition of the first) the line MB is equall vnto the line MK put the line MD common to thē both, wherefore these two lines MK and MD are equall to these two lines BM and MD the one to the other, and the angle KMD is (by the 23. of the first) equall to the angle BMD: VVherefore (by the 4. of the first) the base DK is equall to the

to the base DB.

Now I say that from the poynt D on that side that the line DL is, can not be drawen vnto the circumference any other line besides DB equall vnto the right line DK. For if it be possible let there be drawen an other line besides DB, and let the same be DN. And for asmuch as the line DK is equall vnto the line DN. But vnto the line DK is equall the line DB. Therfore (by the first common sentence) the line DB is equall vnto the line DN. Wherefore that which is nigher vnto DG the least, is equall to ỹ which is more distant: Which we haue before proued to be impossible.

This is demõstrated by an argument leading to an absurdity.

Or it may thus be demonstrated. Draw (by the first petition) a line from M to N. And for asmuch as (by the 15. definition of the first) the line KM is equall vnto the line MN, and the line MD is common to them both. And the base KD is equall to the base DN (by supposition) therefore (by the 8. of the first) the angle KMD is equall to the angle DMN. But the angle KMD is equall to the angle BMD. Wherfore the angle BMD is equall to the angle NMD, the lesse vnto the greater: which is impossible. Wherefore from the poynt D can not be drawen vnto the circumference ABC on eche side of DG the lest, more then two equall right lines. If therefore without a circle be taken any poynt and from that poynt be drawen into the circle vnto the circumference certaine right lines, of which let one be drawen by the centre, and let the rest be drawen at all aduentures: the greatest of those right lines which fall in ỹ concauitie or hollownes of the circumference of the circle is that which passeth by the centre. And of all the other lines, that line which is nigher to the line which passeth by the centre, is greater then that which is more distant. But of those right lines which end in the conuexe part of the circumference, that line is the lest which is drawen from the poynt to the dimetient: and of the other lines that which is nigher to the least is alwayes lesse then that which is more distant. And from that poynt can be drawen vnto the circumference on ech side of the lest only two equall right lines: which was required to be proued.

An other demonstration of the latter part, leading also to an impossibility.

Thys Proposition is called commonly in old bookes amongest the barbarous, *Cauda Pauonis*, that is, the Peacockes taile.

This Proposition is commõly called Cauda Pauonis.

¶ A Corollary.

Hereby it is manifest, that the right lines, which being drawen from the poynt

A Corollary.

geuen without the circle, and fall within the circle, are equally distant from the least, or from the greatest (which is drawen by the centre) are equall the one to the other: but contrarywyse if they be vnequally distant, whether they light vpon the concaue or conuexe circumference of the circle, they are vnequall.

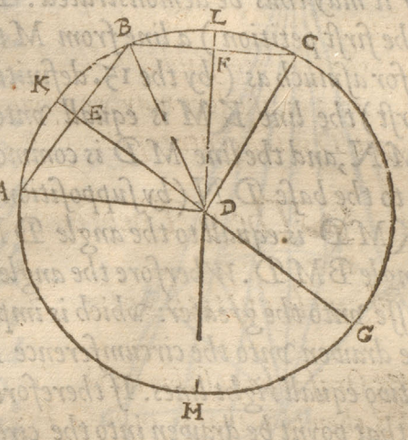
The 8. Theoreme. The 9. Proposition.

If within a circle be taken a poynt, and from that poynt be drawen vnto the circumference moe then two equall right lines, the poynt taken is the centre of the circle.

Construction.

Demonstration.

Suppose that the circle be A B C, and within it let there be taken the poynt D. And from D let there be drawen vnto the circumference A B C moe then two equall right lines, that is, D A, D B, and D C. Then I say that the poynt D is the centre of the circle A B C. Draw (by the first petition) these right lines A B and B C: and (by the 10. of the first) deuide thē into two equall partes in the poyntes E and F: namely, the line A B in the poynt E, and the line B C in the poynt F. And draw ẏ lines E D and F D, and (by the second petition) extend the lines E D and F D on eche side to the poyntes K, G, and H, L. And for asmuch as the line A E is equall vnto the line E B, and the line E D is common to them both, therefore these two sides A E and E D are equall vnto these two sides B E, and E D: and (by supposition) the base D A is equall to the base D B. Wherfore (by the 8. of the first) the angle A E D is equall to the angle B E D. Wherfore eyther of these angles A E D and B E D is a right angle. Wherefore the line G K deuideth ẏ line A B into two equall partes and maketh right angles. And for asmuch as, if in a circle a right line deuide an other right line into two equall partes in such sort that it maketh also right angles, in ẏ line that deuideth is the centre of the circle (by the Correllary of the first of the third). Therfore (by the same Correllary) in the line G K is the centre of the circle A B C. And (by the same reason) may we proue that in ẏ line H L is the centre of the circle A B C, and the right lines G K, and H L haue no other poynt common to them both besides the poynt D. Wherefore the poynt D is the centre of the circle A B C. If therefore within a circle be taken a poynt, and from that point be drawen vnto the circumference more then two equall right lines, the poynt taken is the centre of the circle: which was required to be proued.

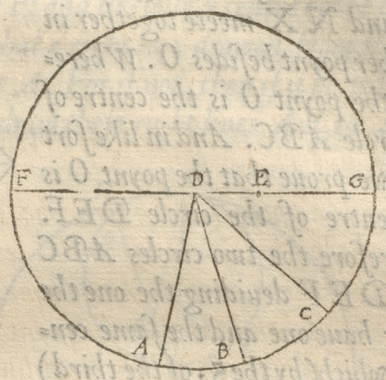


¶ An

¶ *An other demonstration.*

An other demonstration of the same leading also to an impossibilitie.

Let there be taken within the circle A B C the poynt D. And from the poynt D let there be drawen vnto the circumference more then two equall right lines, namely, D A, D B, and D C. Then I say that the poynt D is the centre of the circle. For if not, then if it be possible let the point E be the centre: and draw a line from D to E, and extend D E to the poyntes F and G. Wherefore the line F G is the diameter of the circle A B C. And for asmuch as in F G the diameter of the circle A B C is taken a poynt, namely D, which is not the centre of that circle, therefore (by the 7. of the third) the line D G is ỹ greateſt, and the line D C is greater then the line D B, and the line D B is greater then the line D A. But the lines D C, D B, D A, are also equall (by supposition): which is impoſsible. Wherefore the poynt E is not the centre of the circle A B C. And in like sort may we proue that no other poynt besides D. Wherefore the poynt D is the centre of the circle A B C: which was required to be proued.

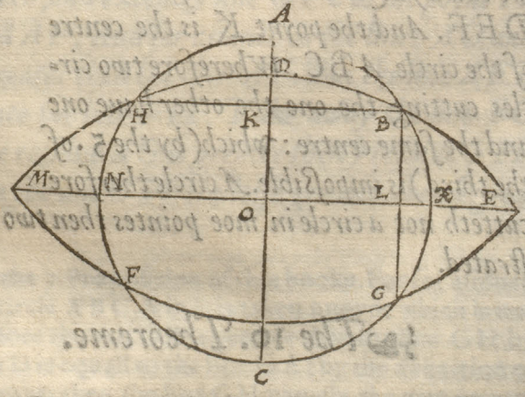


The 9. Theoreme. The 10. Proposition.

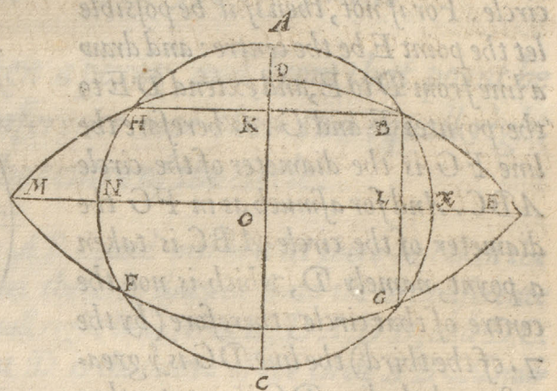
A circle cutteth not a circle in moe pointes then two.

Demonstration leading to an impoſsibilitie.

For if it be possible let the circle A B C cut the circle D E F in mo pointes then two, that is, in B, G, H, & E. And drawe lines frō B to G, and from B to H. And (by ỹ 10. of the first) deuide either of the lines B G & B H into two equall partes, in ỹ pointes K and L. And (by the 11. of the first) from the poynt K raiſe vp vnto ỹ line B H a perpendicular line K C, and likewiſe from the poynt L raiſe vp vnto ỹ line B G a perpendicular line L M, and extend the line C K to the poynt A, and L N M to the poyntes X and E. And for asmuch as in the circle A B C, the right line A C deuideth the right line B H into two equall partes and maketh right angles, therfore (by the 3. of the third)



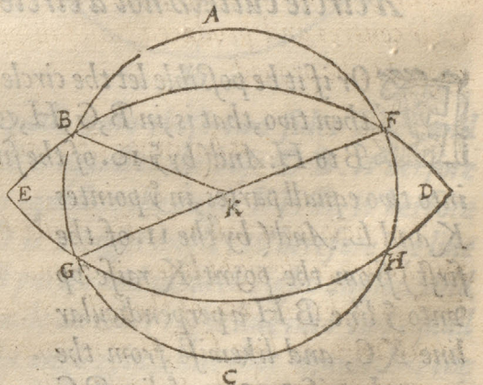
in the line AC is the centre of the circle ABC. Agayne, for asmuch as in the selfe same circle ABC the right line NX, that is, the line ME deuideth the right line BG into two equall partes and maketh right angles, therefore (by the third of the third) in the line NX is the centre of the circle ABC. And it is proued that it is also in the line AC. And these two right lines AC and NX meete together in no other poynt besides O. Wherefore the poynt O is the centre of the circle ABC. And in like sort may we proue that the poynt O is the centre of the circle DEF. Wherefore the two circles ABC and DEF deuiding the one the other haue one and the same centre: which (by the 5. of the third) is impossible. A circle therfore cutteth not a circle in moe poyntes then two: which was required to be proued.



❧ An other demonstration to proue the same.

An other demonstration of the same leading also to an impossibilitie.

Suppose that the circle ABC do cut the circle DGF in mo poyntes then two, that is, in B, G, F, and H. And (by the first of the third) take the centre of the circle ABC and let the same be the poynt K. And draw these right lines KB, KG, and KF. Now for asmuch as within the circle DEF is taken a certaine poynt K, and from that poynt are drawen vnto the circumference moe then two equall right lines, namely, KB, KG, and KF: therefore (by the 9. of the third) K is the centre of the circle DEF. And the poynt K is the centre of the circle ABC. Wherefore two circles cutting the one the other haue one and the same centre: which (by the 5. of the third) is impossible. A circle therfore cutteth not a circle in moe pointes then two: which was required to be demonstrated.



The 10. Theoreme. The 11. Proposition.

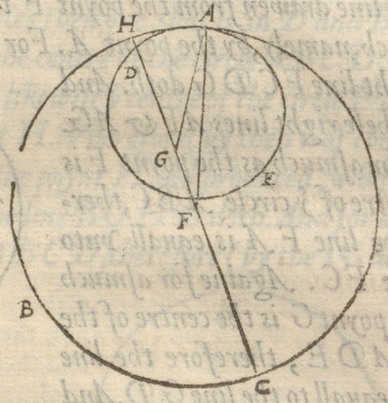
If two circles touch the one the other inwardly, their centres being

being geuen: a right line ioyning together their centres and produced, will fall vpon the touch of the circles.

Vppoſe that theſe two circles ABC, and ADE do touch the one the other in the poynt A. And (by the firſt of the third) take the centre of the circle ABC, and let the ſame be F: and likewiſe ỹ centre of the circle ADE, and let the ſame be G. Then I ſay that a right line drawen from F to G and being produced, will fall vpon the poynt A. For if not, then if it be poſsible let it fall as the line FGDH doth. And draw theſe right lines AF, & AG. Now for aſmuch as the lines AG and GF are (by the 20. of the firſt) greater then the line FA, that is, then the line FH, take away the line GF which is common to them both. Wherefore the reſidue AG is greater then the reſidue GH. But the line DG is equall vnto the line GA (by the 15. definition of the firſt). Wherefore the line GD is greater then ỹ line GH: the leſſe then the greater: which is impoſsible. Wherfore a right line drawen from the poynt F to the poynt G and produced, falleth not beſides the poynt A, which is ỹ point of the touch. Wherefore it falleth vpon the touch. If therefore two circles touch the one ỹ other inwardly, their centres being geuen, a right line ioyning together their centres and produced, will fall vpon the touch of the circles: which was required to be proued.

Conſtruction.

Demonſtration leading to an impoſsibilitie.



An other demonſtration to proue the ſame.

But now let it fall as GFC falleth, and extend ỹ line GFC to the poynt H: and drawe theſe right lines AG and AF. And for aſmuch as the lines AG and GF are (by the 20. of the firſt) greater then the line AF. But the line AF is equall vnto the line CF, that is, vnto the line FH. Take away the line FG common to them both. Wherfore the reſidue AG is greater then ỹ reſidue GH, that is, the line GD is greater then the line GH: the leſſe greater then ỹ greater: which is impoſsible.

An other demonſtration of the ſame leading alſo to an impoſsibilitie.

Which thing may alſo be proued by the 7. Propoſition of this booke. For for aſmuch as the line HC is the diameter of the circle ABC, & in it is taken a poynt which is not the centre, namely, the poynt G, therefore the line GA is greater then the line GH by the ſayd 7. Propoſition. But the line GD is equall to the line GA (by the definition of a circle). Wherefore the line GD is greater then the line GH, namely, the part greater then the whole: which is impoſsible.

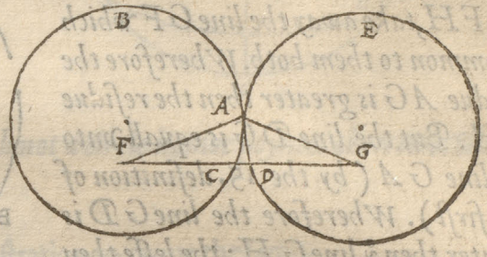
The ſame againe demonſtrated by an argument leading to an abſurditie.

The 11. Theoreme. The 12. Propoſition.

If two circles touch the one the other outwardly, a right line drawen by their centres ſhall paſſe by the touch.

Demonſtratiō leading to an impoſſibilitie.

SVppoſe that theſe two circles *ABC* and *ADE* do touch the one the other outwardly in the poynt *A*. And (by the third of the third) take the centre of the circle *ABC*, and let the ſame be the poynt *F*: and likewiſe the centre of the circle *ADE*, and let the ſame be the poynt *G*. Then I ſay that a right line drawen from the poynt *F* to the poynt *G* ſhall paſſe by the poynt of the touch, namely, by the poynt *A*. For if not, then if it be poſſible, let it paſſe as the right line *FCDG* doth. And draw theſe right lines *AF* & *AG*. And for aſmuch as the poynt *F* is the centre of ỹ circle *ABC*, therefore the line *FA* is equall vnto the line *FC*. Againe for aſmuch as the poynt *G* is the centre of the circle *ADE*, therefore the line *GA* is equall to the line *GD*. And

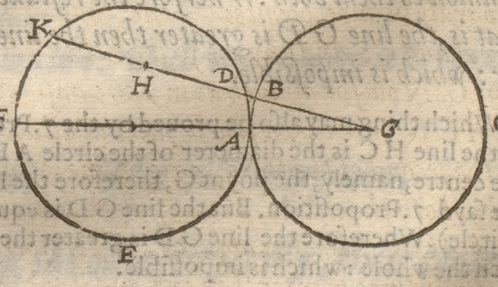


And it is proued that the line *FA* is equall to the line *FC*. Wherefore the lines *FA* and *AG* are equall vnto the lines *FC* and *GD*: Wherefore the whole line *FG* is greater then the lines *FA* and *AG*. But it is alſo leſſe (by the 20. of the firſt): which is impoſſible. Wherfore a right line drawen from the poynt *F* to the poynt *G* ſhall paſſe by the poynt of the touch, namely, by the poynt *A*. If therefore two circles touch the one the other outwardly, a right line drawen by their centres, ſhall paſſe by the touch: which was required to be demonſtrated.

¶ An other demonſtration after Pelitarius.

An other demonſtration after Pelitarius leading also to an abſurditie.

Suppoſe that the two circles *ABC* and *DEF* do touch the one the other outwardly in the poynt *A*: And let *G* be the centre of the circle *ABC*: From which poynt produce by the touch of the circles the line *GA* to the poynt *F* of the circumference *DEF*. Which for aſmuch as it paſſeth not by the centre of the circle *DEF* (as the aduerſary affirmeth) draw from the ſame centre *G* an other right line *GK*, which if it be poſſible let paſſe by the centre of the circle *DEF*, namely, by the poynt *H*: cutting the circumference *ABC* in the poynt *B*, & the circumference *DEF* in the poynt *D*; & let the oppoſite poynt therof be in the poinct *K*. And for aſmuch as frō the poynt *G* taken without the circle *DEF* is drawen the line *GK* paſſing by the centre *H*, and frō the ſame poynt is drawen alſo an



other

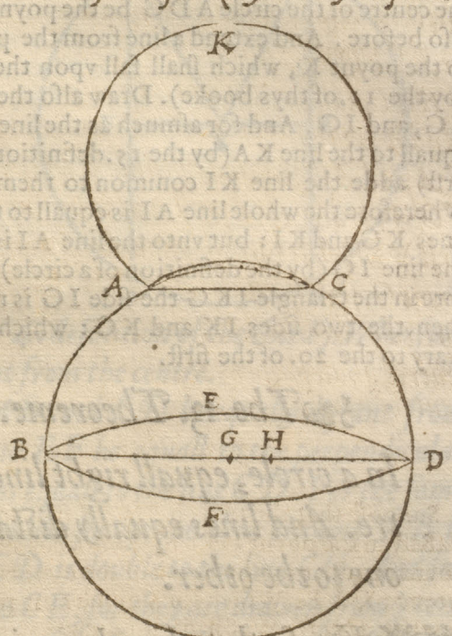
other line not passing by the centre, namely, the line $G F$. Therefore (by the 8. of thys booke) the outward part $G D$ of the line $G K$ shall be lesse then the outward part $G A$ of the line $G F$. But the line $G A$ is equall to the line $G B$. Wherfore the line $G D$ is lesse then the line $G B$, namely, the whole lesse then the part: which is absurde.

The 12. Theoreme. The 13. Proposition.

A circle can not touch an other circle in moe poyntes then one, whether they touch within or without.

Of circles which touch the one the other inwardly.

IFor if it be possible, let the circle $A B C D$ touch ỹ circle $E B F D$ first inwardly in moe poyntes then one, that is, in D and B. Take (by the first of the third) the centre of the circle $A B C D$, and let the same be ỹ point G: and likewise ỹ centre of the circle $E B F D$, and let the same be ỹ poynt H. Wherefore (by the 11. of the same) a right line drawen from the poynt G to the poynt H and produced, will fall vpon the poyntes B and D: let it so fall as the line $B G H D$ doth. And for asmuch as the poynt G is the centre of the circle $A B C D$, therefore (by the 15. definition of the first) the line $B G$ is equall to the line $D G$. Wherfore the line $B G$ is greater then then the line $H D$: Wherfore the line $B H$ is much greater then the line $H D$. Agayne for asmuch as the poynt H is the centre of the circle $E B F D$, therefore (by the same definition) the line $B H$ is equall to the line $H D$: and it is proued that it is much greater then it: which is impossible. A circle therefore can not touch a circle inwardly in moe poyntes then one.



Of circles which touch the one the other outwardly.

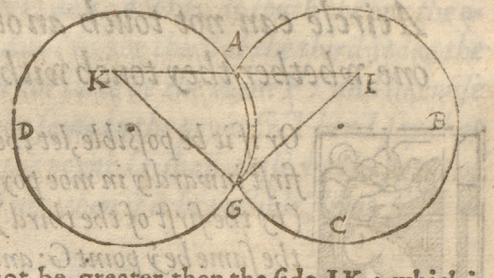
Now I say that neither outwardly also a circle toucheth a circle in moe poyntes then one. For if it be possible, let the circle $A C K$ touch ỹ circle $A B C D$ outwardly in moe poyntes thē one, that is, in A and C: And (by the first petition) draw a line from the poynt A to the poynt C. Now for asmuch as in ỹ circumference of either of the circles $A B C D$, and $A C K$, are taken two poyntes at all aduentures, namely, A and C, therefore (by the second of the third) a right line ioyning together those poyntes shall fall within both the circles. But it falleth within the circle $A B C D$, & without the circle $A C K$: which is absurde. Wherefore a circle shall not touch a circle outwardly in moe pointes then one, and it is proued ỹ neither also inwardly. Wherefore a circle can not touch an other circle in moe poyntes then one, whether they touch

touch within or without: which was required to be demonstrated.

¶ *An other demonstration after Pelitarius and Flussates.*

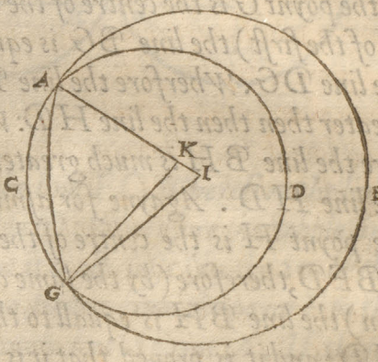
An other demonstration after Pelitarius & Flussates, of circles which tooch the one the other outwardly.

Suppose that there be two circles A B G and A D G, which if it be possible, let touch the one the other outwardly in moe poyntes then one, namely, in A and G. Let the centre of the circle A B G be the poynt I, and let the centre of the circle A D G be the poynt K. And draw a right line from the poynt I to the poynt K, which (by the 12. of thys booke) shall passe both by the poynt A and by the poynt G: which is not possible: for then two right lines should include a superficies, contrary to the last common sentence. It may also be thus demonstrated. Draw a line from the centre I to the centre K, which shall passe by one of the touches, as for example by the poynt A. And draw these right lines G K and G I, and so shall be made a triangle, whose two sides G K and G I shall not be greater then the side I K: which is contrary to the 20. of the first.



Of circles which tooch the one the other inwardly.

But now if it be possible, let the foresayd circle A D G touch the circle A B C inwardly in moe poyntes then one, namely, in the pointes A and G: and let the centre of the circle A B C be the poynt I, as before: and let the centre of the circle A D G be the poynt K, as also before. And extend a line from the poynt I to the poynt K, which shall fall vpon the touch (by the 11. of thys booke). Draw also these lines K G, and I G. And for asmuch as the line K G is equall to the line K A (by the 15. definition of the first) adde the line K I common to them both. Wherefore the whole line A I is equall to the two lines K G and K I: but vnto the line A I is equall the line I G (by the definition of a circle). Wherfore in the triangle I K G the side I G is not lesse then the two sides I K and K G: which is contrary to the 20. of the first.



The 13. Theoreme. The 14. Proposition.

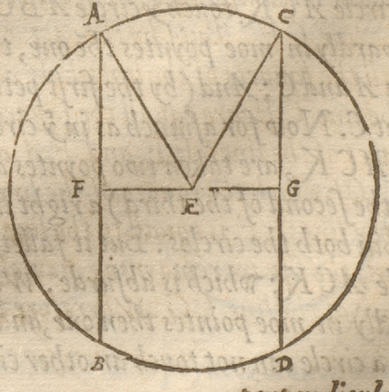
In a circle, equall right lines, are equally distant from the cẽtre. And lines equally distant from the centre, are equall the one to the other.

The first part of this Theoreme.

Suppose that there be a circle A B C D, and let there be in it drawen these equall right lines A B and C D. Then I say that they are equally distant from the centre.

Construction.

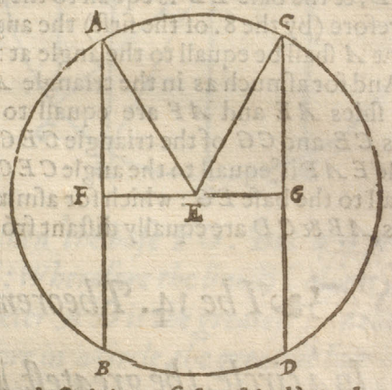
Take (by the first of the third) the centre of the circle A B C D, and let the same be ỹ poynt E. And (by the 12. of the first) from the point E draw vnto the lines A B & C D



perpendicular

perpendicular lines E F and E G. And (by the first petition) draw these right lines A E and C E. Now for asmuch as a certaine right line E F drawen by the centre cutteth a certaine other right line A B not drawen by the centre, in such sort that it maketh right angles, therefore (by the third of the third) it deuideth it into two equall partes. Wherefore the line A F is equall to the line F B. Wherfore the line A B is double to the line A F: and by the same reason also the line C D is double to the line C G. But the line A B is equall to the line C D. Wherfore the line A F is also equall to the line C G. And for asmuch as (by the 15. definition of the first) the line A E is equall to the line E C, therefore the square of the line E C is equall to the square of the line A E. But vnto the square of the line A E, are equall (by the 47. of the first) the squares of the lines A F & F E: for the angle at the poynt F is a right angle. And (by ỹ selfe same) to the square of the line E C are equall the squares of the lines E G and G C: for the angle at the poynt G is a right angle. Wherefore the squares of the lines A F and F E are equall to the squares of the lines C G and G E: of which the square of the line A F is equall to the square of the line C G: for the line A F is equall to the line C G. Wherefore (by the third common sentence) the square remayning, namely, the square of the line F E, is equall to the square remayning, namely, to the square of the line E G. Wherefore the line E F is equall to the line E G. But right lines are sayd to be equally distant from ỹ centre, when perpendicular lines drawen frō the centre to those lines, are equall (by the 4. definition of the third). Wherfore the lines A B and C D are equally distant from the centre.

Demonstration.



Demonstration.

But now suppose that the right lines A B and C D be equally distant from the centre, that is, let the perpendicular line E F be equall to the perpendicular line E G. Then I say that the line A B is equall to the line C D. For the same order of construction remayning, we may in like sort proue that the line A B is double to the line A F, and that the line C D is double to the line C G. And for asmuch as the line A E is equall to the line C E, for they are drawen from ỹ centre to the circumference, therfore the square of the line A E is equall to ỹ square of the line C E. But (by the 47. of the first) to the square of the line A E are equall the squares of the lines E F and F A. And (by the selfe same) to ỹ square of the line C E are equall the squares of the lines E G and G C. Wherfore the squares of the lines E F and F A are equall to the squares of the lines E G and G C. Of which the square of the line E G is equall to the square of the line E F, for the line E F is equall to the line E G. Wherefore (by the third common sentence) the square remayning, namely, the square of the line A F, is equall to the square of the line C G. Wherefore the line A C is equall vnto the line C G. But

The second part which is the conuerse of the first.

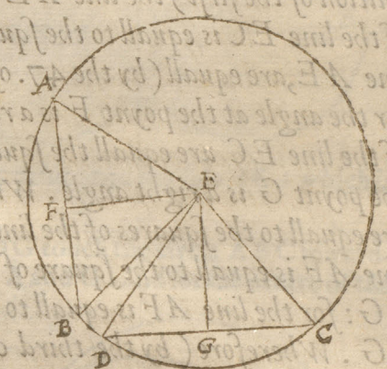
the

the line AB is double to the line AF, and the line CD is double to the line CG. Wherefore the line AB is equall to the line CD. Wherefore in a circle equall right lines are equally distant from the centre. And lines equally distant from the centre, are equall the one to ỹ other: which was required to be proued.

¶ An other demonstration for the first part after Campane.

An other demonstration of the first part after Campane.

Suppose that there be a circle *ABDC*, whose centre let be the poynt *E*. And draw in it two equall lines *AB* and *CD*. Then I say that they are equally distant from the centre. Draw from the centre vnto the lines *AB* and *CD*, these perpendicular lines *EF* and *EG*. And (by the 2. part of the 3. of this booke) the line *AB* shall be equally deuided in the poynt *F*, and the line *CD* shall be equally deuided in the poynt *G*. And draw these right lines *EA*, *EB*, *EC*, and *ED*. And for asmuch as in the triangle *AEB* the two sides *AB* and *AE* are equall to the two sides *CD* and *CE* of the triangle *CED*, & the base *EB* is equall to the base *ED*, therefore (by the 8. of the first) the angle at the point *A* shall be equall to the angle at the point *C*. And for asmuch as in the triangle *AEF* the two sides *AE* and *AF* are equall to the two sides *CE* and *CG* of the triangle *CEG*, and the angle *EAF* is equall to the angle *CEG*, therefore (by the 4. of the first) the base *EF* is equall to the base *EG*: which for asmuch as they are perpendicular lines, therefore the lines *AB* & *CD* are equally distant frō the centre, by the 4. definition of this booke.

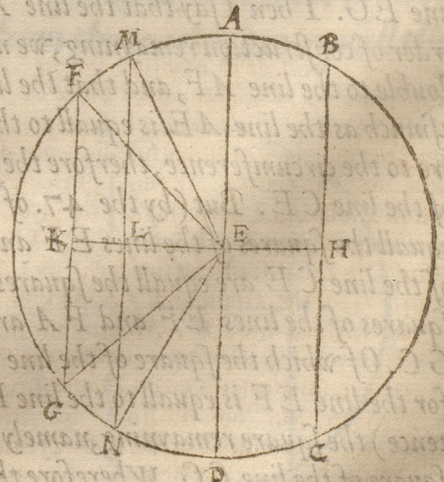


The 14. Theoreme. The 15. Proposition.

In a circle, the greateſt line is the diameter, and of all other lines that line which is nigher to the centre is alwayes greater then that line which is more distant.

Suppose that there be a circle ABCD, and let the diameter thereof be the line AD, and let the centre thereof be the poynt E. And vnto the diameter AD let the line BC be nigher then the line FG. Then I say that the line AD is the greateſt, and the line BC is greater then ỹ line FG.

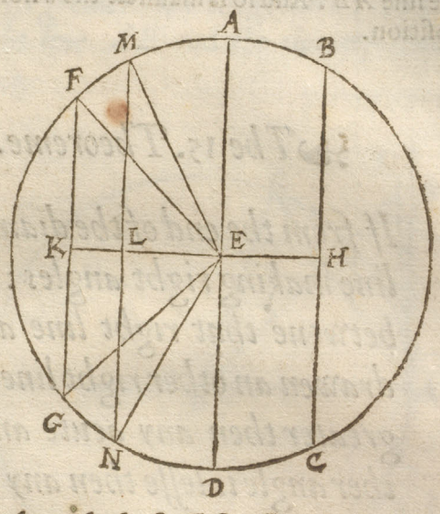
Construction. *Draw (by the 12. of the first) from the centre E to the lines BC and FG perpendicular lines EH and EK. And for asmuch as the line BC is nigher vnto the centre then the line FG, therfore*



(by the

(by the 4. definition of the third) the line EK is greater then the line EH. And (by the third of the first) put vnto the line EH an equall line EL. And (by the 11. of the first) from the point L raise vp vnto the line EK a perpendicular line LM: and extend the line LM to the poynt N. And (by the first petition) draw these right lines, EM, EN, EF, and EG. And for asmuch as the line EH is equall to the line EL, therefore (by the 14. of the third, and by the 4. definition of the same) the line BC is equall to the line MN. Againe for asmuch as the line AE is equall to the line EM, and the line ED to the line EN, therefore the line AD is equall to the lines ME and EN. But the lines ME and EN are (by the 20. of the first) greater then the line MN. Wherefore the line AD is greater then the line MN. And for asmuch as these two lines ME and EN are equall to these two lines FE and EG (by the 15. definition of the first) for they are drawen from the centre to the circumference, and the angle MEN is greater then the angle FEG, therefore (by the 24. of the first) the base MN is greater then the base FG. But it is proued that the line MN is equall to the line BC: Wherefore the line BC also is greater then the line FG. Wherefore the diameter AD is the greatest, and the line BC is greater then the line FG. Wherefore in a circle, the greatest line is the diameter, and of all the other lines, that line which is nigher to ỹ centre is alwaies greater then that line which is more distant: which was required to be proued.

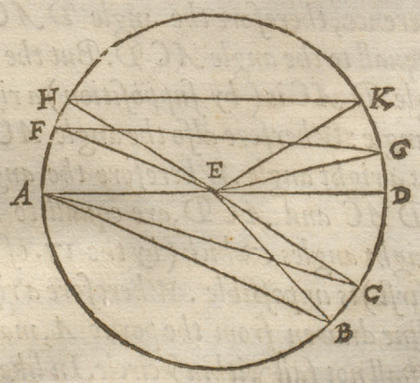
Demonstration.



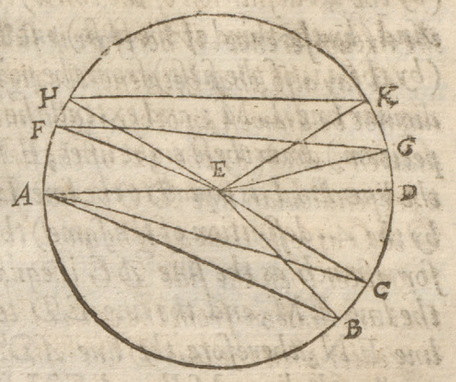
¶ *An other demonstration after Campane.*

In the circle ABCD, whose centre let be the poynt E, draw these lines, AB, AC, AD, FG, and HK, of which let the line AD be the diameter of the circle. Then I say that the line AD is the greatest of all the lines. And the other lines eche of the one is so much greater then ech of the other, how much nigher it is vnto the centre. Ioyne together the endes of all these lines with the centre, by drawing these right lines EB, EC, EG, EK, EH, and EF. And (by the 20. of the first) the two sides EF and EG of the triangle EFG, shall be greater then the third side FG. And for asmuch as the sayd sides EF & EG are equall to the line AD (by the definition of a circle) therefore the line AD is greater then the line FG. And by the same reason it is greater then euery one of the rest of the lines, if they be put to be bases of triangles: for that euery two sides drawen frō the

An other demonstration after Campane.



centre are equall to the line *A D*. Which is the first part of the Proposition. Agayne, for asmuch as the two sides *E F* and *E G* of the triangle *E F G*, are equall to the two sides *E H* and *E K* of the triangle *E H K*, and the angle *F E G* is greater then the angle *H E K*, therfore (by the 24. of the first) the base *F G* is greater then the base *H K*. And by the same reason may it be proued, that the line *A C* is greater then the line *A B*. And so is manifest the whole Proposition.



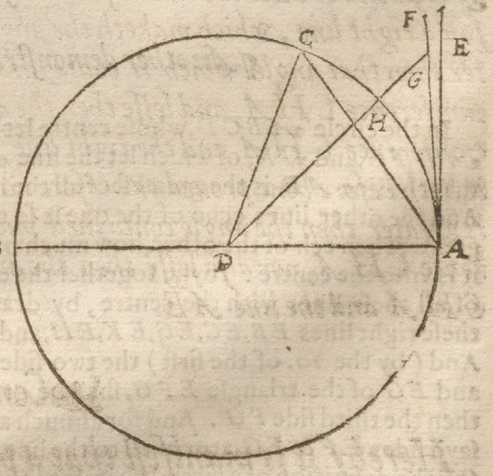
The 15. Theoreme. The 16. Proposition.

If from the end of the diameter of a circle be drawen a right line making right angles: it shall fall without the circle: and betwene that right line and the circumference can not be drawen an other right line: and the angle of the semicircle is greater then any acute angle made of right lines, but the other angle is lesse then any acute angle made of right lines.

Vppose that there be a circle A B C: whose centre let be the point D, and let the diameter therof be A B. Then I say ỹ a right line drawen from the poynt A, making with the diameter A B right angles, shall fall without the circle. For if it do not, then if it be possible, let it fall within the circle as the line A C doth, and draw a line from the point D to the point C. And for asmuch as (by the 15. definition of the first) the line D A is equall to the line D C, for they are drawen from the centre to the circumference, therefore the angle D A C is equall to the angle A C D. But the angle D A C is (by supposition) a right angle: Wherfore also the angle A C D is a right angle. Wherefore the angles D A C and A C D, are equall to two right angles: which (by the 17. of the first) is impossible. Wherefore a right line drawen from the poynt A, making with the diameter A B right angles, shall not fall within ỹ circle. In like sort also may we proue, that it falleth not in the

The first part of this Theoreme.

Demonstration leading to an absurditie.



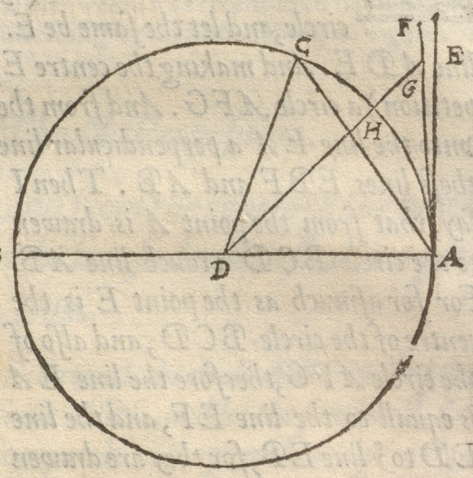
the circumference. Wherefore it falleth without, as the line AE doth.

Second part.

I say also, that betwene the right line AE, and the circumference ACB, can not be drawen an other right line. For if it be possible, let the line AF so be drawen. And (by the 12. of the first) from the poynt D draw vnto the line FA a perpendicular line DG. And for asmuch as AGD is a right angle, but DAG is lesse then a right angle, therefore (by the 19. of the first) the side AD is greater then the side DG. But the line DA is equall to the line DH, for they are drawen from the centre to the circumference. Wherefore the line DH is greater then the line DG: namely, the lesse greater then the greater: which is impossible. Wherefore betwene the right line AE and the circumference ACB, can not be drawen an other right line.

Third part.

I say moreouer, that the angle of the semicircle contayned vnder ỹ right line AB and the circūference CHA, is greater then any acute angle made of right lines. And the angle remayning cōtayned vnder ỹ circumference CHA and the right line AE, is lesse then any acute angle made of right lines. For if there be any angle made of right lines greater then that angle which is contayned vnder the right line BA and the circumference CHA, or lesse then that which is contayned vnder the circumference CHA and the right line AE, then betwene the circumference CHA and the right line AE, there shall fall a right line, which maketh the angle contayned vnder the right lines, greater then that angle which is contayned vnder the right line BA and the circumference CHA, and lesse then the angle which is contayned vnder the circumference CHA and the right line AE. But there can fall no such line, as it hath before bene proued. Wherfore no acute angle contained vnder right lines, is greater then the angle contayned vnder the right line BA and the circumference CHA, nor also lesse then the angle contayned vnder the circumference CHA and the line AE.



Correlary.

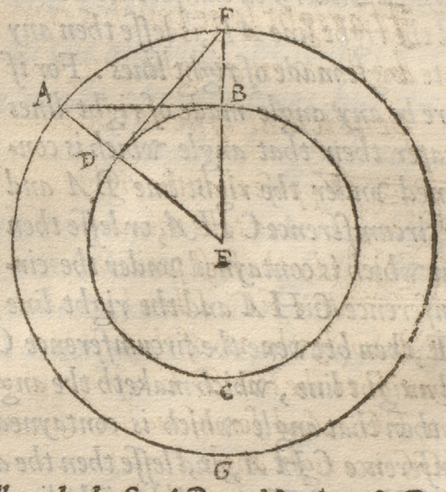
Hereby it is manifest that a perpendicular line drawen frō the end of the diameter of a circle toucheth the circle: and that a right line toucheth a circle in one poynt onely. For it was proued (by the 2. of the third) that a right line drawen from two pointes taken in

the circumference of a circle, ſhall fall within the circle. Which was required to be demonſtrated.

The 2. Probleme. The 17. Propoſition.

From a poynt geuen, to draw a right line which ſhall touch a circle geuen.

Conſtruction. *Vppoſe that the poynt geuen be A, and let the circle geuen be BCD. It is required from the poynt A to draw a right line which ſhall touch the circle BCD. Take (by the firſt of the third) the centre of the circle, and let the ſame be E. And (by the firſt petition) draw the right line ADE. And making the centre E, and the ſpace AE, deſcribe (by ỹ third petition) a circle AFG. And from the poynt D raiſe vp (by the 11. of the firſt) vnto the line EA a perpendicular line DF. And (by the firſt petition) drawe theſe lines EBF and AB. Then I ſay, that from the point A is drawen to the circle BCD a touch line AB.*



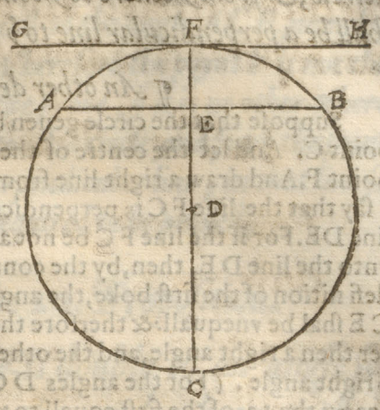
Demonſtration. *For for aſmuch as the point E is the centre of the circle BCD, and alſo of the circle AFG, therfore the line EA is equall to the line EF, and the line ED to ỹ line EB, for they are drawen from the centre to the circumference. Wherefore to theſe two lines AE and EB, are equall theſe two lines EF & ED, and the angle at the poynt E is common to them both: Wherefore (by the 4. of the firſt) the baſe DF is equall to the baſe AB, and ỹ triangle DEF is equall to the triangle EBA, and the reſt of the angles remayning to the reſt of the angles remayning. Wherefore the angle EDF is equall to ỹ angle EBA. But the angle EDF is a right angle: Wherfore alſo the angle EBA is a right angle, and the line EB is drawen from the centre. But a perpendicular line drawen from the end of the diameter of a circle, toucheth the circle (by ỹ Corellary of the 16. of the third). Wherefore the line AB toucheth the circle BCD. Wherfore from the point geuen, namely, A, is drawen vnto ỹ circle geuē BCD, a touch line AB: which was required to be done.*

¶ *An addition of Pelitarius.*

An addition of Pelitarius. Vnto a right lyne which cutteth a circle, to drawe a parallel line which ſhall touch the circle.

Suppoſe

Suppose that the right lyne A B do cut the circle A B C in the poyntes A and B. It is required to drawe vnto the line A B a parallel lyne which shall touche the circle. Let the centre of the circle be the point D. And deuide the lyne A B into two equall partes in the poinct E. And by the poinct E and by the centre D, draw the diameter C D E F. And from the poinct F (which is the ende of the diameter) rayse vp (by the 11. of the first) vnto the diameter C F a perpendicular line G F H. Then I say that the lyne G F H (which by the correllary of the 16. of this booke toucheth the circle) is a parallel vnto the line A B. For forasmuch as the right line C F falyng vpon either of these lines A B & G H maketh all the angles at the point E right angles (by the 3. of this boke) and the two angles at the point F are supposed to be right angles: therfore (by the 29. of the first) the lines A B and G H are parallels: which was required to be done. And this Probleme is very commodious for the inscribing or circumscribing of figures in or about circles.



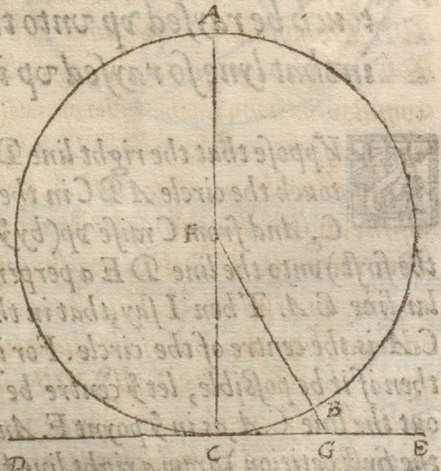
This Probleme commodious for the inscribing and circumscribing of figures in or about circles.

The 16. Theoreme. The 18. Proposition.

If a right lyne touch a circle, and from the centre to the touch be drawen a right line, that right line so drawen shalbe a perpendicular lyne to the touche lyne.

Vppose that the right line D E do touch the circle A B C in the point C. And take the centre of the circle A B C, and let the same be F. And (by the first petition) from the poynt F to the poynt C drawe a right line F C. Then I say, that C F is a perpendicular line to D E. For if not, draw (by the 12. of the first) from the poynt F to the line D E a perpendicular line F G. And for asmuch as the angle F G C is a right angle, therefore the angle G C F is an acute angle: Wherefore the angle F G C is greater then ỳ angle F C G, but vnto the greater angle is subtended the greater side (by the 19. of the first). Wherefore the line F C is greater then the line F G. But the line F C is equall to the line F B, for they are drawen from the centre to the circumference: Wherfore the line F B also is greater then the line F G, namely, the lesse then the greater: which is impossible. Wherefore the line F G is not a perpendicular line vnto the line D E. And in like sort may we proue, that no other line is a perpendicular line vnto ỳ line D E besides the line F C: Wherfore the line F C is a perpendicular line to D E. If therefore a right line touch

Demonstration leading to an impossibilitie.

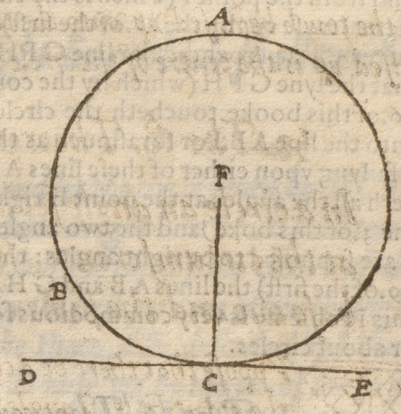


a circle, & from ỹ centre to ỹ touch be drawen a right line, ỹ right line so drawen shall be a perpendicular line to ỹ touch line: which was required to be proued.

¶ *An other demonstration after Orontius.*

An other demonstration after Orontius.

Suppose that the circle geuen be A B C, which let the right lyne D E touch in the point C. And let the centre of the circle be the point F. And draw a right line from F to C. Then I say that the line F C is perpendicular vnto the line DE. For if the line F C be not a perpẽdiculer vnto the line D E, then, by the conuerse of the x. defi nition of the first boke, the angles D C F & F C E shal be vnequall: & therfore the one is greater then a right angle, and the other is lesse then a right angle. (For the angles D C F and F C E are by the 13. of the first equall to two right angles) Let the angle FCE, if it be possible, be greater then a right angle, that is, let it be an obtuse angle. Wherfore the angle D C F shal be an acute angle. And forasmuch as by suppositiõ the right line D E toucheth the circle A B C, therefore it cutteth not the circle. Wherefore the circumference B C falleth betwene the right lines D C & C F: & therfore the acute and rectiline angle D C F shall be greater then the angle of the semicircle B C F which is contayned vnder the circumferẽce B C & the right line C F. And so shall there be geuẽ a rectiline & acute angle greater then the angle of a semicircle: which is contrary to the 16. proposition of this booke. Wherfore the angle D C F is not lesse then a right angle. In like sort also may we proue that it is not greater then a right angle. Wherfore it is a right angle, and therfore also the angle F C E is a right angle. Wherefore the right line F C is a perpendicular vnto the right line D E by the 10. definition of the first: which was required to be proued.

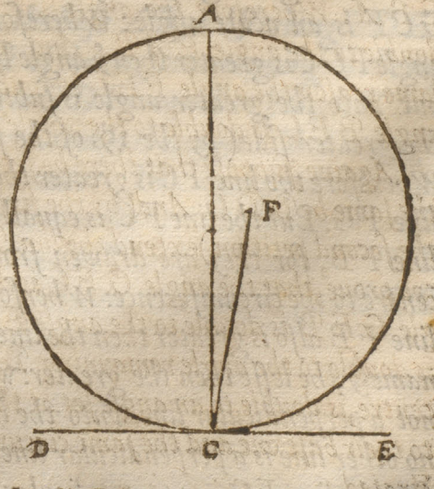


The 17. Theoreme. The 19. Proposition.

If a right lyne doo touche a circle, and from the point of the touch be rayſed vp vnto the touch lyne a perpendicular lyne, in that lyne ſo rayſed vp is the centre of the circle.

Demonstration leading to an impossibilitie.

Suppose that the right line D E do touch the circle A B C in the point C. And from C raise vp (by ỹ 11. of the first) vnto the line D E a perpendicular line C A. Then I say, that in the line C A is the centre of the circle. For if not, then if it be possible, let ỹ centre be without the line C A, as in ỹ poynt F. And (by the first petition) draw a right line from C to F. And for asmuch as a certaine right line D E toucheth the circle A B C, and from the centre to the touch is drawen a right line C F, therefore (by the 18. of the third) F C is a perpendicular line to D E.



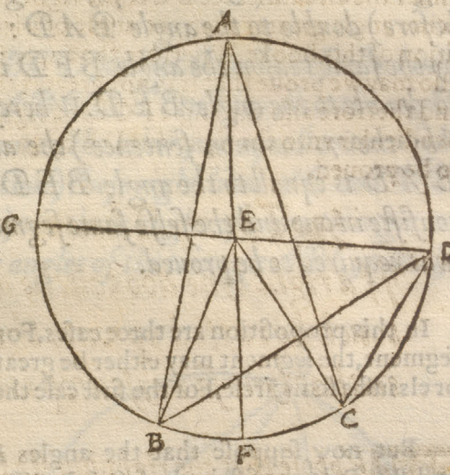
D E. Wherefore the angle F C E is a right angle. But the angle A C E is also a right angle: Wherefore the angle F C E is equall to the angle A C E, namely, the lesse vnto the greater: which is impossible Wherefore the poynt F is not the centre of the circle A B C. And in like sort may we proue, ỹ it is no other where but in the line A C. If therefore a right line do touch a circle, and from the point of the touch be raised vp vnto the touch line a perpendicular line, in that line so raised vp is the centre of the circle: which was required to be proued.

The 18. Theoreme. The 20. Proposition.

In a circle an angle set at the centre, is double to an angle set at the circumference, so that both the angles haue to their base one and the same circumference.

SVppose that there be a circle A B C, and at the centre thereof, namely, the poynt E, let ỹ angle B E C be set, & at the circumference let there be set the angle B A C, and let them both haue one and the same base, namely, the circumference B C. Then I say, that the angle B E C is double to the angle B A C. Draw ỹ right line A E, and (by the second petition) extend it to the poynt F. Now for asmuch as the line A E is equall to the line E B, for they are drawen from the centre vnto the circumference, the angle E A B is equall to the angle E B A (by the 5. of the first). Wherefore the angles E A B and E B A are double to the angle E A B. But (by the 32. of the same) the angle B E F is equall to the angles E A B and E B A: Wherefore the angle B E F is double to the angle E A B. And by the same reason the angle F E C is double to the angle E A C. Wherefore the whole angle B E C is double to the whole angle B A C.

Two cases in thys Proposition the one when the angle set at the circumference includeth the center.



Demonstration.

Againe, suppose that there be set an other angle at the circumference, and let the same be B D C. And (by the first petition) draw a line from D to E. And (by the second petition) extend the line D E vnto the poynt G. And in like sort may we proue, that the angle G E C is double to the angle E D C. Of which the angle G E B is double to the angle E D B. Wherfore the angle remayning B E C is double to the angle remayning B D C. Wherfore in a circle an angle set at the centre, is double to an angle set at the circumference, so that both the angles haue to their base one and the same circumference: which was required to be demonstrated.

The other whē the same angle set at the circumference includeth not the center.

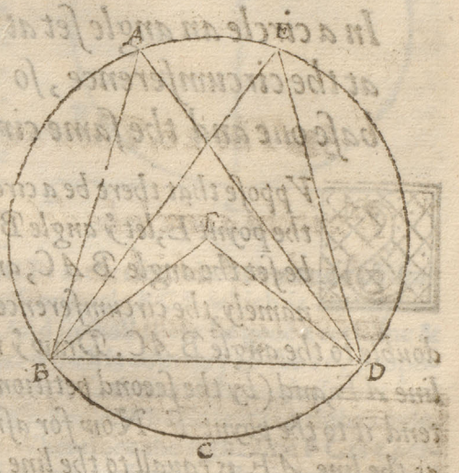
The 19. Theoreme. The 21. Proposition.

In a circle the angles which consist in one and the selfe same section or segment, are equall the one to the other.

SVppose ỹ there be a circle *ABCD*, & in the segment therof *BAED*, let there consist these angles *BAD* and *BED*. Then I say, that the angles *BAD* and *BED* are equall the one to the other. Take (by the first of the third) the centre of the circle *ABCD*, and let the same be the point *F*. And (by the first petition) draw these lines *BF* and *FD*. Now for asmuch as the angle *BFD* is set at the centre, and the angle *BAD* at the circumference, and they haue both one and ỹ same base, namely, the circumference *BCD*, therefore the angle *BFD* is (by the Proposition going before) double to the angle *BAD*: and by the same reason the angle *BFD* is also double to the angle *BED*. Wherefore (by the 7. common sentence) the angle *BAD* is equall to the angle *BED*. Wherefore in a circle the angles which consiste in one and the selfe same segment, are equall the one to the other: which was required to be proued.

Construction.

Demonstration.



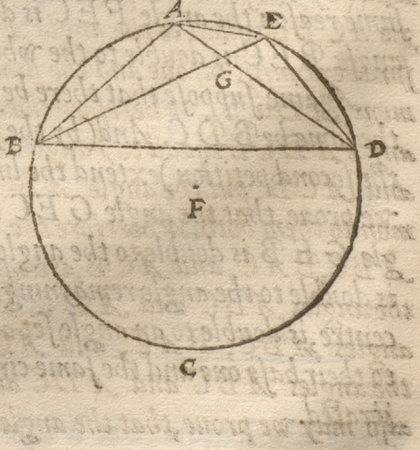
Three cases in this Proposition.

In this proposition are three cases. For the angles consisting in one and the self same segment, the segment may either be greater thé a semicircle, or lesse then a semicircle, or els iust a semicircle. For the first case the demonstration before put serueth.

The first case.

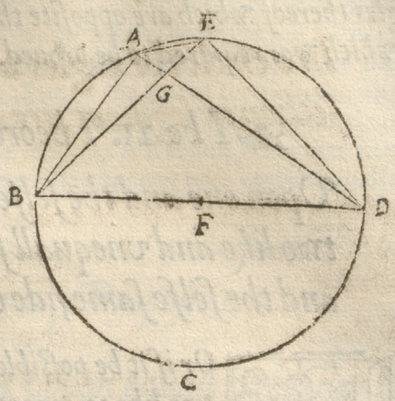
The second case.

But now suppose that the angles *BAD* and *BED* do consist in the sectiō *BAD*, which let be lesse then a semicircle. Euen in this case also I say that the angles *BAD* and *BED* are equall. For draw a right line from *A* to *E*. And let the lines *AD* and *BE* cutte the one the other in the poynt *G*, wherefore the segment *ACE* is greater then a semicircle. And therfore by the first part of this proposition the angles whiche are in it, namely, the angles *ABE* and *EDA* are equall the one to the other. And forasmuch as in the triangle *ABG* the inward and opposite angles *ABG* and *GAB* are equall to the outwarde angle *BGD*, and by the same reason the two angles *EDG* and *GED* of the triangle *DEG* are equall to the selfe same outward angle *BGD*. Wherfore the two angles *ABG* and *GAB* are equall to the two angles *EDG* and *GED*



by the

by the firſt commō ſentence. From which if there be taken equall angles, namely, *A B G*, and *E D G*, the angle remainyng *B A G* ſhall be equall to the angle remayning *D E G*, that is, the angle *B A D* to the angle *D E B* (by the third common ſentence) which was required to be proued.



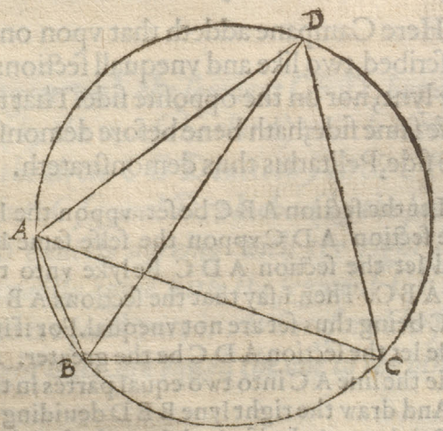
The third caſe.

The ſelfe ſame conſtruction and demonſtration will alſo ſerue, if the angles were ſet in a ſemicircle as it is playne to ſee, in the figure here ſet.

The 20. Theoreme. The 22. Propoſition.

If within a circle be deſcribed a figure of fower ſides, the angles therof which are oppoſite the one to the other, are equall to two right angles.

Vppoſe that there be a circle A B C D, and let there be deſcribed in it a figure of fower ſides, namely, A B C D. Then I ſay, that the angles thereof which are oppoſite the one to the other, are equall to two right angles. Draw (by the firſt petition) theſe right lines A C and B D. *Conſtruction.* *Now for aſmuch as (by the 32. of the firſt) the three angles of euery triangle are equall to two right angles: therfore ẏ three angles of the triangle A B C, namely,* *Demonſtration.* *C A B, A B C, and B C A, are equall to two right angles. But (by the 21. of the third) the angle C A B is equall to the angle B D C, for they conſiſt in one and the ſelf ſame ſegmēt, namely, B A D C. And (by the ſame Propoſition) the angle A C B is equall to the angle A D B, for they conſiſt in one and the ſame ſegment A D C B. Wherefore the whole angle A D C is equall to ẏ angles B A C and A C B: put the angle A B C common to them both. Wherefore the angles A B C, B A C, and A C B, are equall to the angles A B C and A D C. But the angles A B C, B A C, and A C B, are equall to two right angles. Wherefore the angles A B C and A D C are equall to two right angles. And in like ſort alſo may we proue, that the angles B A D and D C B are equall to two right angles.*



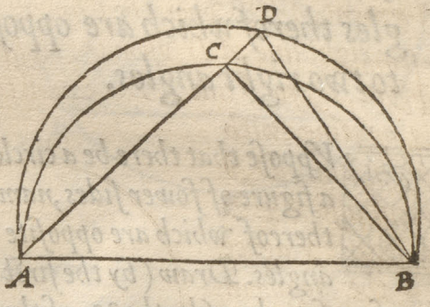
angles. If therefore within a circle be described a figure of fower sides, the angles thereof which are opposite the one to ỹ other, are equall to two right angles: which was required to be proued.

The 21. Theoreme. The 23. Proposition.

Vpon one and the selfe same right line can not be described two like and vnequall segmentes of circles, falling both on one and the selfe same side of the line.

Demonstration leading to an impossibilitie.

Or if it be possible, let there be described vpon the right line AB two like & vnequall sections of circles, namely, ACB & ADB, falling both on one and the selfe same side of the line AB. And (by the first petition) drawe the right line ACD, and (by the third petition) drawe right lines from C to B, and from D to B. And for asmuch as the segment ACB is like to the segment ADB: and like segmẽtes of circles are they which haue equall angles (by the 10. definition of the third). Wherefore the angle ACB is equall to the angle ADB, namely, the outward angle of ỹ triangle CDB to the inward angle: which (by the 16. of the first) is impossible. Wherfore vpon one and the self same right line can not be described two like & vnequall segmentes of circles, falling both on one & the selfe same side of the line: which was required to be demonstrated.

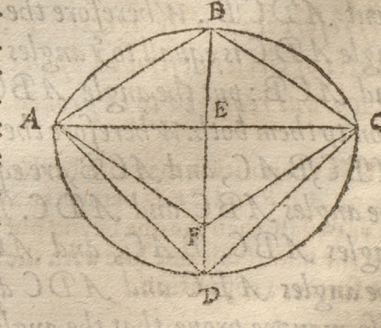


An addition of Campane demonstrated by Pelitarius.

Here Campane addeth that vpon one and the selfe same right lyne cannot be described two like and vnequall sections neither on one and the selfe same side of the lyne, nor on the opposite side. That they can not be described on one and the selfe same side, hath bene before demonstrated, and that neither also on the opposite side, Pelitarius thus demonstrateth.

Demonstration leading to an impossibilitie.

Let the section A B C be set vppon the lyne A C, and vpon the other side let be set the section A D C vppon the selfe same lyne A C, and let the section A D C be lyke vnto the section A B C. Then I say that the sections A B C and A D C being thus set are not vnequal. For if it be possible let the section A D C be the greater. And deuide the line A C into two equal partes in the point E. And draw the right lyne B E D deuiding the lyne A C right angled wise. And draw these right lynes A B, C B, A D and C D. And forasmuch as the section A D C is greater then the section A B C, the perpendicular lyne also E D shall be greater then the perpendicular lyne E B: as is before declared in the ende of the definitions of this third booke. Wher-



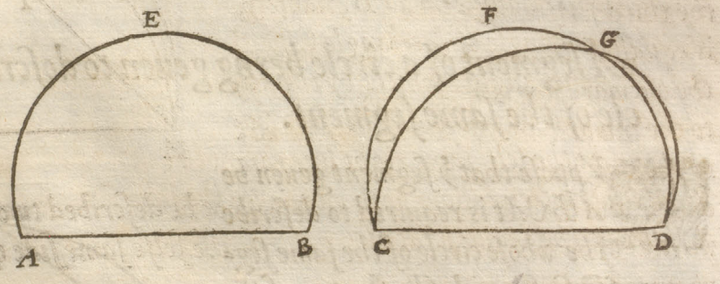
fore

fore from the lyne E D cut of a lyne equall to the lyne E B: which let be E F. And draw these right lynes A F and C F. Now then (by the 4. of the first) the triangle A E B shall be equall to the triangle A E F, and the angle E B A shall be equall to the angle E F A. And by the same reason the angle E B C shall be equall to the angle E F C. Wherefore the whole angle A B C is equall to the whole angle A F C. But by the 21. of the first, the angle A F C is greater then the angle A D C. Wherfore also the angle A B C is greater then the angle A D C. Wherefore by the definition the sections A B C and A D C are not lyke, which is contrary to the supposition. Wherefore they are not lyke and vnequall: which was required to be proued.

The 22. Theorme. The 24. Proposition.

Like segmentes of circles described vppon equall right lines, are equall the one to the other.

SVppose that vpon these equall right lines A B and C D be described these like segmentes of circles, namely, A E B and C F D. Then I say, that the segment A E B is equall to the segment C F D. For putting the segment A E B vpon the segment C F D, and the poynt A vpon ỳ poynt C, and the right line A B vpon the right line C D, the poynt B also shall fall vpon the poynt D, for ỳ line A B is equall to the line C D. And the right line A B exactly agreing with the right line C D, ỳ segment also A E B shall exactly agree with the segment C F D. For if the right line A B do exactly agree with the right line C D, and the segment A E B do not exactly agree with the segment C F D, but differeth as the segment C G D doth: Now (by the 13. of the third) a circle cutteth not a circle in more pointes then two, but the circle C G D cutteth ỳ circle C F D in more pointes thē two, that is, in the points C, G, and D: which is (by the same) impossible. Wherefore the right line A B exactly agreing with the right line C D, the segment A E B shall not but exactly agree with the segment C F D: Wherefore it exactly agreeth with it, and is equall vnto it. Wherefore like segmentes of circles described vpon equall right lines, are equall the one to the other: which was required to be proued.



Demonstration leading to an impossibilitie.

This Proposition may also be demonstrated by the former proposition. For if the sections *A E B* and *C F D* being like and set vpon equall right lines *A B* and *C D*, should be vnequall, then the one beyng put vpon the other, the greater shall exceede the lesse: but the line *A B* is one line with the line *C D*: so that therby shal follow the contrary of the former Proposition.

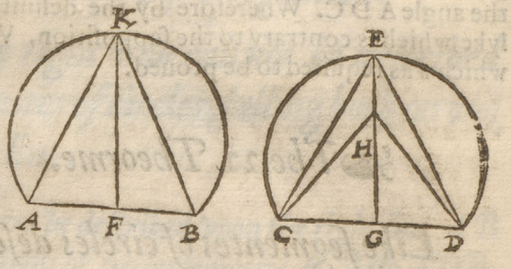
An other demonstration.

Pelitarius

Pelitarius demonstrateth this Proposition an other way.

An other demonstratiō after Pelitarius.

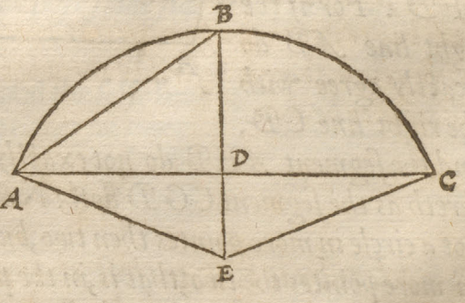
Suppose that there be two right lines *A B* & *C D* which let be equall: and vpon thē let there be set these like sections *A B K*, and *C D E*. Then I say that the said sections are equall. For if not, then let *C E D* be the greater section. And deuide the two lines *A B* and *C D* into two equall partes, the line *A B* in the pointe *F*, and the line *C D* in the point *G*. And erect two perpendicular lines *F K* and *G E*. And draw these right lines *A K* & *K B*: *E C*, *E D*. And forasmuch as the section *C E D* is the greater, therefore the perpendicular line *G E* is greater then the perpendicular *F K*: From the lyne *G E* cut of a line equall to the line *F K*, which let be *G H*: and draw these right lines *C H* and *H D*. And forasmuch as in the triangle *A K F* the two sides *A F* and *F K* are equall to the two sides *C G* & *G H* of the triangle *C H G*, and the angles at the pointes *F* and *G* are equal (for that they are right angles) therfore (by the 4. of the first) the base *A K* is equall to the base *C H*, and the angle *A K F* to the angle *C H G*. And by the same reason the angle *B K F* is equall to the angle *D H G*. Wherfore the whole angle *A K B* is equall to the whole angle *C H D*. But the angle *C H D* is greater then the angle *C E D* by the 21. of the first. Wherfore also the angle *A K D* is greater then the angle *C E D*. Wherfore the sections are not lyke, which is contrary to the supposition.



The 3. Probleme. The 25. Proposition.

A segment of a circle beyng geuen to describe the whole circle of the same segment.

Suppose that ỹ segment geuen be A B C. It is required to describe the whole circle of the same segment A B C.



Construction. *Deuide (by the 10. of the first) the line A C into two equall partes in the poynt D. And (by the 11. of the same) from the poynt D raise vp vnto the line A C a perpendicular line B D. And (by the first petition) draw a right line from A to B. Now then the angle A B D being compared to ỹ angle B A D, is either greater then it, or equall vnto it, or lesse then it.*

Three cases in this Proposition.

The first case. *First let it be greater. And (by the 23. of the same) vpon the right line B A, and vnto the poynt in it A, make vnto the angle A B D an equall angle B A E. And (by the second petition) extend the line B D vnto the poynt E. And (by the first petition) draw a line from E to C. Now for asmuch as the angle A B E is equall to the angle B A E, therefore (by the 6. of the first) the right line E B is equall to the right line A E. And for asmuch as the line A D is equall to ỹ line D C, and the line D E is common to them both: therefore these two lines A D*

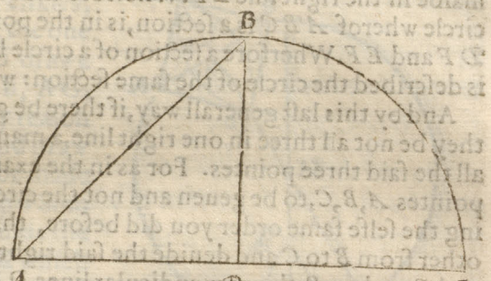
Demonstration.

and

and D E, are equall to these two lines C D and D E the one to the other. And the angle A D E is (by the 4. petition) equall to the angle C D E, for either of them is a right angle. Wherefore (by the 4. of the first) the base A E is equall to the base C E. But it is proued, that the line A E is equall to ỹ line B E. Wherfore the line B E also is equall to the line C E. Wherefore these three lines A E, E B, and E C, are equall the one to the other. Wherefore making the centre E, and the space either A E, or E B, or E C, describe (by the third petition) a circle, and it shall passe by the poyntes A, B, C. Wherefore there is described the whole circle of the segment geuen. And it is manifest, that the segment A B C is lesse then a semicircle, for the centre E falleth without it.

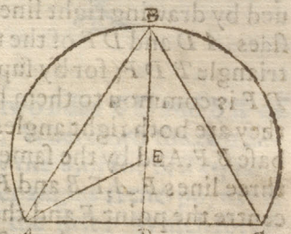
The second case.

The like demonstration also will serue if the angle A B D be equall to the angle B A D. For the line A D being equall to either of these lines, B D, and D C, there are three lines, D A, D B, and D C, equall the one to the other. So that the point D shall be the centre of the circle being complete, and A B C shall be a semicircle.



The third case.

But if the angle A B D be lesse then the angle B A D, then (by the 23. of the first) vpon the right line B A, and vnto the point in it A, make vnto the angle A B D an equall angle within ỹ segment A B C. And so the centre of the circle shall fall in ỹ line D B, and it shall be the point E: and the segment A B C shall be greater then a semicircle. Wherefore a segment being geuen, there is described the whole circle of the same segment: which was required to be done.

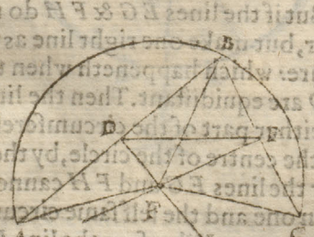


¶ A Corollary.

Hereby it is manifest, that in a semicircle the angle B A D is equall to the angle D B A: but in a section lesse then a semicircle, it is lesse: in a section greater then a semicircle, it is greater.

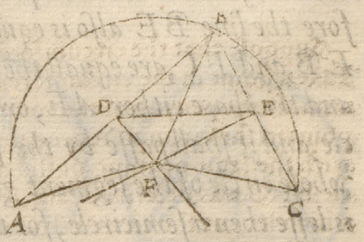
An other more ready way.

There is also an other generall way to finde out the foresaid centre, which will serue indifferently for any section whatsoeuer: And that is thus. Take in the circumference geuen or section A B C three pointes at all auentures which let be A, B, C. And draw these lines A B and B C (by the first peticion) And (by the 10. of the first) deuide into two equall partes either of the sayde lines, the line A B in the point D, & the line B C in the point E. And (by the 11. of the first) from the pointes



Demonstration.

D and *E* rayse vp vnto the lines *A B* and *B C* perpendicular lynes *D F* and *E F*. Now forasmuch as either of these angles *B D F*, and *B E F* is a right angle, a right line produced from the point *D* to the point *E*, shall deuide either of the said angles: and forasmuch as it falleth vppon the right lines *D F* and *E F*, it shall make the inward angles on one and the selfe same side, namely, the angles *D E F* and *E D F* lesse then two right angles. Wherefore (by the fift petition) the lines *D F* and *E F* being produced shall concurre. Let them concurre in the point *F*. And forasmuch as a certaine right line *D F* deuideth a certaine right lyne *A B* into two equall partes and perpendicularly, therfore (by the corollary of the first of this booke) in the line *D F* is the centre of the circle, & by the same reason the centre of the selfe same circle shalbe in the right line *E F*. Wherfore the centre of the circle wherof *A B C* is a section, is in the point *F*, which is commō to either of the lines *D F* and *E F*. Wherfore a section of a circle being geuē, namely, the section *A B C*, there is described the circle of the same section: which was required to be done.

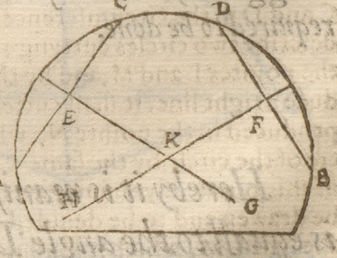


An addition.

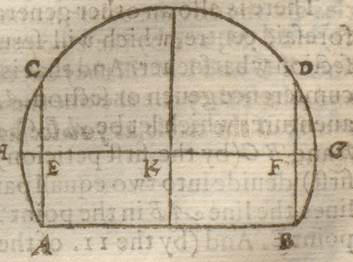
And by this last generall way, if there be geuen three pointes, set howsoeuer, so that they be not all three in one right line, a man may describe a circle which shall passe by all the said three pointes. For as in the example before put, if you suppose onely the 3. pointes *A*, *B*, *C*, to be geuen and not the circumference *A B C* to be drawen, yet following the selfe same order you did before, that is, draw a right line from *A* to *B* and an other from *B* to *C* and deuide the said right lines into two equall parts, in the points *D* and *E*, and erect the perpendicular lines *D F* and *E F*, cutting the one the other in the point *F*, and draw a right line from *F* to *B*: and making the centre the point *F*, and the space *F B* describe a circle, and it shall passe by the pointes *A* & *C*: which may be proued by drawing right lines from *A* to *F*, and from *F* to *C*. For forasmuch as the two sides *A D* and *D F* of the triangle *A D F* are equall to the two sides *B D* and *D F* of the triangle *B D F* (for by supposition the line *A D* is equall to the line *D B*, and the lyne *D F* is common to them both) and the angle *A D F* is equall to the angle *B D F* (for they are both right angles) therfore (by the 4. of the first) the base *A F* is equall to the base *B F*. And by the same reason the line *F C* is equall to the line *F B*. Wherefore these three lines *F A*, *F B* and *F C* are equall the one to the other. Wherefore makyng the centre the point *F* and the space *F B*, it shall also passe by the pointes *A* and *C*. Which was required to be done. This proposition is very necessary for many things as you shal afterward see.

An other construction and demonstration of this Proposition after Campane

Campane putteth an other way, how to describe the whole circle of a sectiō geuen. Suppose that the section be *A B*. It is required to describe the whole circle of the same section. Draw in the section two lines at all aduentures *A C* and *B D*: which deuide into two equall parts *A C* in the point *E*, and *B D* in the pointe *F*. Then from the two pointes of the deuisions draw within the section two perpendicular lines *E G* and *F H* which let cutte the one the other in the point *K*. And the centre of the circle shall be in either of the said perpendicular lines by the corollary of the first of this booke. Wherfore the point *K* is the centre of the circle: which was required to be done.



But if the lines *E G* & *F H* do not cut the one the other, but make one right line as doth *G H* in the secōd figure: which happeneth when the two lines *A C* and *B D* are equidistant. Then the line *G H*, being applyed to either part of the circumference geuen, shall passe by the centre of the circle, by the selfe same Corollary. For the lines *E G* and *F H* cannot be equidistant. For then one and the self same circumference should haue two centres. Wherfore the line *H G* being deuided into

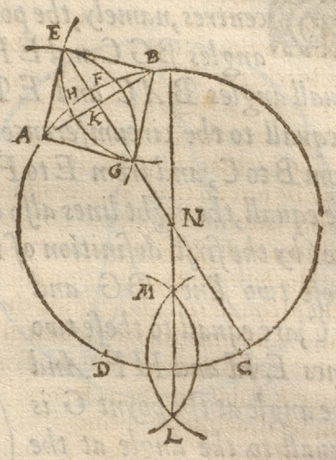


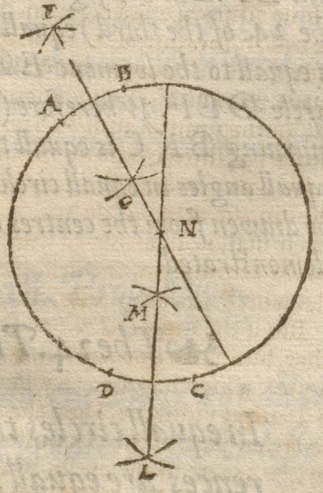
to two equall partes in the point *K*, the ſaid point *K* ſhall be the centre of the ſection.

Pelitarius here addeth a briefe way how to finde out the centre of a circle, which is commonly vſed of Artificers.

A ready way to finde out the center of a circle commõly vſed amongſt artificers.

Suppoſe that the circumference be A B C D, whoſe centre it is required to finde out. Take a point in the circumference geuen which let be A, vppon which deſcribe a circle with what openyng of the compaſſe you will, which let be E F G. Then take an other point in the circumference geuen which let be B, vpon which deſcribe an other circle with the ſame opening of the compaſſe that the circle E F G was deſcribed, and let the ſame be E H G, which let cut the circle E F G in the two pointes E and G. (I haue not here drawen the whole circles, but onely thoſe partes of them which cut the one the other for auoyding of confuſion) And drawe from thoſe centres theſe right lines A E, B E, A G, and B G, which foure lines ſhall be equall, by reaſon they are ſemidiameters of equall circles. And draw a right line from A to B, and ſo ſhall there be made two Iſoſceles triangles A E B, and A G B vnto whom the line A B is a common baſe. Now then deuide the line A B into two equal partes in the point K which muſt nedes fall betwene the two circumference E F G and E H G, otherwiſe the part ſhould be greater then his whole. Drawe a line from E to K and produce it to the point G. Now you ſee that there are two Iſoſceles triangles deuided into foure equall triangles E A K, E B K, G A K and G B K. For the two ſides A E and A K of the triangle A E K are equall to the two ſides B E and B K of the triangle B E K, and the baſe E K is common to them both. Wherefore the two angles at the point K of the two triangles A E K and B E K are by the 8. of the firſt equall: and therfore are right angles. And by the ſame reaſon the other angles at the poynte K are right angles. Wherfore E G is one right lyne by the 14. of the firſt. Which foraſmuch as it deuideth the line A B perpendicularly, therefore it paſſeth by the center by the corollary of the firſt of this booke. And ſo if you take two other poyntes, name- C and D in the circumference geuen, and vpon thē deſcribe two circles cuttyng the one the other in the pointes L and M, and by the ſaid poyntes produce a right line, it ſhall cutte the lyne E G beyng produced in the pointe N, which ſhall be the centre of the circle by the ſame Corollary of the firſt of this booke, if you imagine the right line C D to be drawen and to be deuided perpendicularly by the lyne L M, which it muſt needes be as we haue before proued. And here note that to do this mechanically not regardyng demonſtration, you neede onely to marke the poyntes where the circles cut the one the other, namely, the poyntes E, G, and L, M, and by theſe poyntes to produce the lines E G and L M till they cut the one the other, and where they cut the one the other, there is the centre of the circle, as you ſee here in the ſeconde figure.





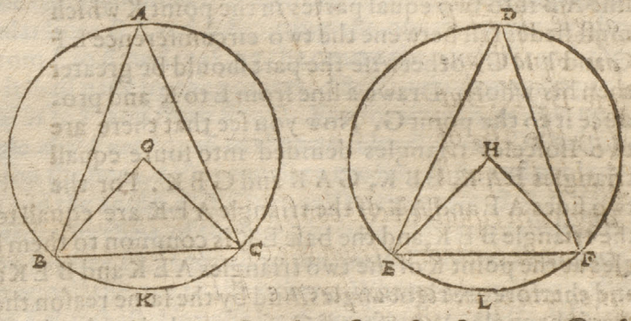
The 23. Theoreme. The 26. Proposition.

Equall angles in equall circles consist in equall circũferences, whether the angles be drawen from the centres, or from the circumferences.

Construction.

Demonstration.

SVppose that these circles *ABC* and *DEF*, be equall. And from their centres, namely, the pointes *G* and *H*, let there be drawen these equall angles *BGC* and *EHF*: and likewise from their circumferences these equall angles *BAC* and *EDF*. Then I say, that the circumference *BKC* is equall to the circumference *ELF*. Draw (by the first petition) right lines from *B* to *C*, and from *E* to *F*. And for asmuch as the circles *ABC* and *DEF* are equall, the right lines also drawen from their centres to their circumferences, are (by the first definition of the third) equall the one to the other. Wherefore these two lines *BG* and *GC*, are equall to these two lines *EH* and *HF*. And the angle at the poynt *G* is equall to the angle at the point *H*: Wherfore (by the 4. of the first) the base *BC* is equall to ỹ base *EF*. And for asmuch as the angle at the poynt *A* is equall to the angle at the point *D*, therefore the segment *BAC* is like to the segment *EDF*. And they are described vpon equall right lines *BC* and *EF*. But like segmentes of circles described vpon equall right lines, are (by the 24. of the third) equall the one to the other. Wherefore the segment *BAC* is equall to the segment *EDF*. And the whole circle *ABC* is equall to ỹ whole circle *DEF*. Wherefore (by the third common sentence) the circumference remayning *BKC* is equall to the circumference remayning *ELF*. Wherefore equall angles in equall circles consist in equall circumferences, whether the angles be drawen from the centres or from the circumferences: which was required to be demonstrated.



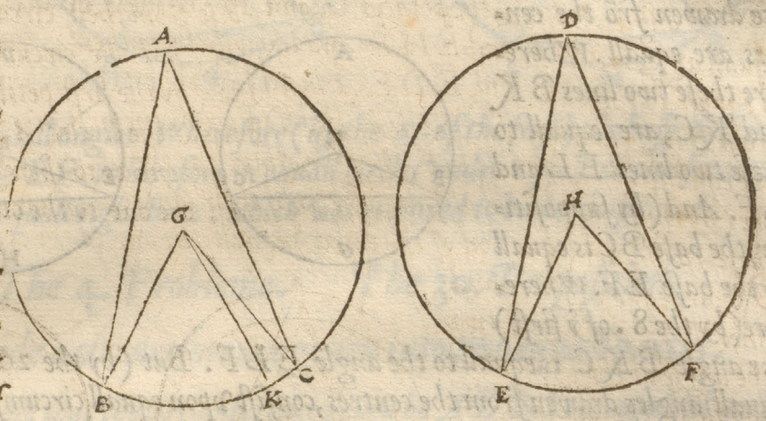
The 24. Theoreme. The 27. Proposition.

In equall circles the angles which consist in equall circumferences, are equall the one to the other, whether the angles be drawen from the centres, or from the circumferences.

Suppose

SVppose ỳ these circles ABC, and DEF, be equall. And vpon these equall circumferences of the same circles, namely, vpon BC and EF, let there consist these angles BGC and EHF drawen from the centres, and also these angles BAC and EDF drawen from the circumferences. Then I say, that the angle BGC is equall to the angle EHF, and the angle BAC to the angle EDF. If the angle BGC be equall to the angle EHF, then it is manifest, that the angle BAC is equall to ỳ angle EDF (by the 20. of the third). But if the angle BGC be not equall to ỳ angle EHF, then is the one of them greater then the other. Let the angle BGC be greater

Demonstration leading to an impossibilitie.



And (by the 23. of the first) vpon the right line BG, and vnto the point geuen in it G, make vnto the angle EHF an equall angle BGK. But (by the 26. of ỳ third) equall angles in equall circles consist vpō equall circumferences, whether they be drawen from the centres or from the circumferences. Wherefore the circumference BK is equall to the circumference EF. But the circumference EF is equall to the circumference BC: Wherefore the circumference BK also is equall to the circumference BC, the lesse to the greater: which is impossible. Wherfore the angle BGC is not vnequall to the angle EHF: Wherefore it is equall. And (by the 20. of the third) the angle at the point A is the halfe of the angle BGC: and (by the same) the angle at the point D is the halfe of the angle EHF. Wherefore the angle at the point A is equall to the angle at the point D. Wherefore in equall circles, the angles which consist in equall circumferences, are equall the one to the other, whether the angles be drawen from the centres or from the circumferences: which was required to be proued.

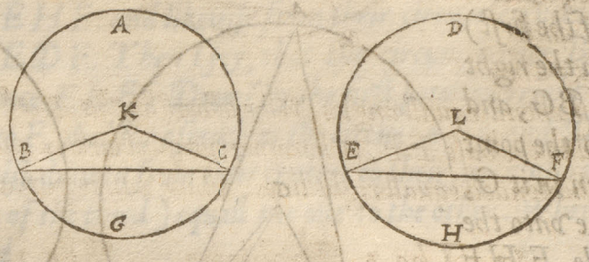
The 25. Theoreme. The 28. Proposition.

In equall circles, equall right lines do cut away equall circumferences, the greater equall to the greater, and the lesse equall to the lesse.

SVppose that these circles ABC, and DEF, be equall. And in them let there be drawen these equall right lines BC and EF, which let cut away these circumferences BAC and DEF being the greater, & also these circumferences BGC and EHF being the lesse. Then I say, that the greater circumference BAC is equall to the greater circumference EDF: and the lesse circumference BGC is equall to the lesse circumference EHF. Take (by the first of the third) the centres of the circles, and let the same be the pointes K and L. And draw these right lines, KB, KC, LE, and LF. And for asmuch as the circles are equall, therfore (by the first definition of the third) the lines which are drawen frō the centres are equall. Wherefore these two lines BK and KC, are equall to these two lines EL and LF. And (by supposition) the base BC is equall to the base EF. Wherefore (by the 8. of ỹ first) the angle BKC is equall to the angle ELF. But (by the 26. of the third) equall angles drawen from the centres, consist vpon equall circumferences. Wherfore the circumference BGC is equall to the circumference EHF: and ỹ whole circle ABC is equall to the whole circle DEF. Wherefore the circumference remayning BAC, is (by the third common sentence) equall to the circumference remayning EDF. Wherefore in circles, equall right lines do cut away equall circumferences, the greater equall to the greater, and the lesse equall to the lesse: which was required to be proued.

Construction.

Demonstration.



The 26. Theoreme. The 29. Proposition.

In equall circles, vnder equall circumferences are subtended equall right lines.

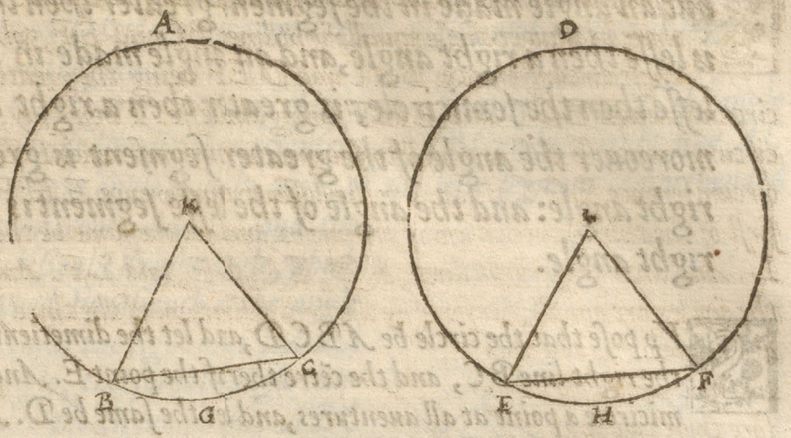
SVppose that these circles ABC and DEF, be equall. And in them let there be taken these equall circumferences, BGC and EHF: and drawe these right lines BC and EF. Then I say, that the right line BC is equall to the right line EF. Take (by the first of ỹ third) the centres of the circles, and let them be the pointes K and L, and draw these right lines KB, KC, LE, LF. And for asmuch as the circumference BGC is equall to the circumference EHF, the angle BKC is equall to ỹ angle ELF (by the 27. of the third). And for asmuch as the circles ABC and DEF are equall the one to the other, therefore (by the first definition of the third) the lines which

The conuerse of the former Proposition.

Construction.

Demonstration.

whiche are drawē from the centres are equall. Wherefore these .y. lines BK and KC, are equall to these .y. lines LE and LF, & they



comprehend equall angles. Wherfore (by the 4. of the first) the base BC is equall to the base EF. Wherefore in equall circles vnder equall circumferences, are subtended equall right lines: which was required to be demonstrated.

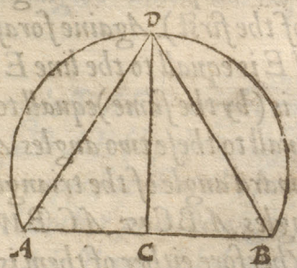
The 4. Probleme. The 30. Proposition.

To deuide a circumference geuen into two equall partes.

Construction.

Demonstration.

Vppose that the circumference geuen be ADB. It is required to deuide the circumference ADB into two equall partes. Draw a right line from A to B. And (by the 10. of the first) deuide the line AB into two equall partes in the point C. And (by the 11. of the first) from the point C rayse vp vnto AB a perpendicular line CD. And draw these right lines AD and DB. And forasmuch as the line AC is equall to the line CB, & the line CD is common to them both, therefore these two lines AC and CD are equall to these two lines BC and CD. And (by the 4. peticion) the angle ACD is equall to the angle BCD, for either of them is a right right angle. Wherfore (by the 4. of the first) the base AD is equall to the base DB. But equall right lines do cut away equall circumferences, the greater equall to the greater, & the lesse equall to the lesse (by the 28. of the third) And either of these circumferences AD and DB is lesse then a semicircle. Wherfore the circumference AD is equall to the circumference DB. Wherfore the circumference geuen is deuided into two equall partes: Which was required to be done.



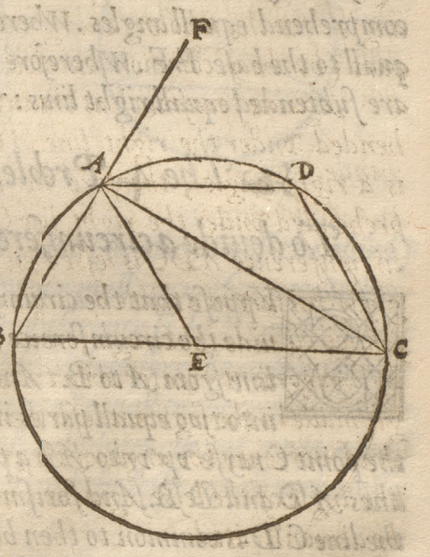
The 27. Theoreme. The 31. Proposition.

In a circle an angle made in the semicircle is a right angle:

but an angle made in the ſegment greater then the ſemicircle is leſſe then a right angle, and an angle made in the ſegment leſſe then the ſemicircle, is greater then a right angle. And moreouer the angle of the greater ſegment is greater then a right angle: and the angle of the leſſe ſegment is leſſe then a right angle.

Suppoſe that the circle be A B C D, and let the dimetient of the circle be the right line B C, and the cẽtre therof the point E. And take in the ſemicircle a point at all auentures, and let the ſame be D. And draw theſe right lines B A, A C, A D, and D C. Then I ſay that the angle in the ſemicircle B A C, namely, the angle B A C is a right angle. And the angle A B C which is in the ſegment A B C being greater then the ſemicircle, is leſſe then a right angle. And the angle A D C which is in the ſegment A D C being leſſe then the ſemicircle is greater thẽ a right angle. Draw a line from the point A to the point E, and extend the line B A vnto the point F. And foraſmuch as the line B E is equall to the line E A, (for they are drawen from the centre to the circumference) therfore the angle E A B is equall to the angle E B A (by the 5. of the firſt). Againe foraſmuch as the line A E is equall to the line E C, the angle A C E is (by the ſame) equall to the angle C A E. Wherfore the whole angle B A C is equall to theſe two angles A B C and A C B. But the angle F A C which is an outward angle of the triangle A B C is (by the 32. of the firſt) equall to the two angles A B C & A C B. Wherfore the angle B A C is equall to the angle F A C. Wherfore either of them is a right angle. Wherfore the angle B A C which is in the ſemicircle B A C is a right angle.

The firſt part of this Theoreme.



And foraſmuch as (by the 17. of the firſt) the two angles of the triangle A B C, namely, A B C and B A C are leſſe then two right angles, and the angle B A C is a right angle. Therfore the angle A B C is leſſe then a right angle, and it is in the ſegment A B C which is greater then the ſemicircle.

Seconde part.

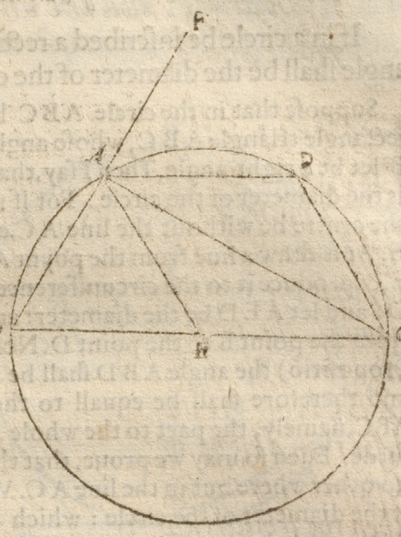
And foraſmuch as in the circle there is a figure of foure ſides, namely, A B C D. But if within a circle be deſcribed a figure of foure ſides, the angles therof which are oppoſite the one to the other are equall to two right angles (by the 22. of the third) Wherfore (by the ſame) the angles A B C and A D C are equall

Third part.

to

to two right angles. But the angle ABC is leſſe then a right angle. Wherfore the angle remayning ADC is greater then a right angle, and it is in a ſegment which is leſſe then the ſemicircle.

Now alſo I ſay that the angle of the greater ſegment, namely, the angle which is comprehended vnder the circumference ABC and the right line AC is greater then a right angle, and the angle of the leſſe ſegment comprehended vnder the circumference ADC, and the right line AC is leſſe thē a right angle, which may thus be proued. Foraſmuch as the angle comprehended vnder the right lines BA and AC is a right angle, therfore the angle comprehended vnder the circumference ABC and the right line AC is greater then a right angle: for the whole is euer greater then his part (by the 9. common ſentence. *The fourth part.*



The fift and laſt part.

Againe foraſmuch as the angle comprehended vnder the right lines AC and AF is a right angle, therfore the angle comprehended vnder the right line CA and the circumference ADC is leſſe then a right angle. Wherfore in a circle an angle made in the ſemicircle is a right angle, but an angle made in the ſegment greater then the ſemicircle is leſſe then a right angle, and an angle made in the ſegment leſſe then the ſemicircle, is greater then a right angle. And moreouer the angle of the greater ſegment is greater then a right angle: & the angle of the leſſe ſegment is leſſe then a right angle: which was required to be demonſtrated.

An other demonſtration to proue that the angle BAC is a right angle. Foraſmuch as the angle AEC is double to the angle BAE (by the 32. of the firſt) for it is equall to the two inward angles which are oppoſite. But the inwarde angles are (by the 5. of the firſt) equall the one to the other, and the angle AEB is double to the angle EAC. Wherfore the angles AEB and AEC are double to the angle BAC. But the angles AEB and AEC are equall to two right angles: Wherfore the angle BAC is a right angle. Which was required to be demonſtrated. *An other Demonſtration to proue that the angle in a ſemicircle is a right angle.*

Correlary.

Hereby it is manifeſt, that if in a triangle one angle be equall to the two other angles remayning the ſame angle is a right angle: *A Corollary.*

angle: for that the ſide angle to that one angle (namely, the angle which is made of the ſide produced without the triangle) is equall to the ſame angles, but when the ſide angles are equall the one to the other, they are alſo right angles.

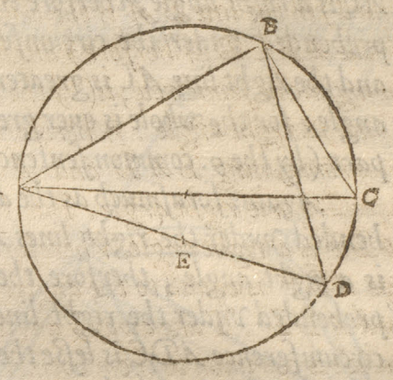
¶ *An addition of Pelitarius.*

If in a circle be inſcribed a rectangle triangle, the ſide oppoſite vnto the right angle ſhall be the diameter of the circle.

An addition of Pelitarius.

Demonſtration leading to an abſurditie.

Suppoſe that in the circle A B C be inſcribed a rectangle triangle A B C, whoſe angle at the point B let be a right angle. Then I ſay, that the ſide A C is the diameter of the circle. For if not, then ſhall the centre be without the line A C, as in the point E. And draw a line from the poynt A to the point E, & produce it to the circumference to the point D: and let A E D be the diameter: and draw a line from the point B to the point D. Now (by this 31. Propoſitiõ) the angle A B D ſhall be a right angle, and therefore ſhall be equall to the right angle ABC, namely, the part to the whole: which is abſurde. Euen ſo may we proue, that the centre is in no other where but in the line A C. Wherfore A C is the diameter of the circle: which was required to be proued.



¶ *An addition of Campane.*

An addition of Campane.

By thys 31. Propoſition, and by the 16. Propoſition of thys booke, it is manifeſt, that although in mixt angles, which are contayned vnder a right line and the circumference of a circle, there may be geuen an angle leſſe & greater then a right angle, yet can there neuer be geuẽ an angle equall to a right angle. For euery ſection of a circle is eyther a ſemicircle, or greater then a ſemicircle, or leſſe, but the angle of a ſemicircle is by the 16. of thys booke, leſſe then a right angle, and ſo alſo is the angle of a leſſe ſection by thys 31. Propoſition: Likewiſe the angle of a greater ſection, is greater then a right angle, as it hath in thys Propoſition bene proued.

❧ *The 28. Theoreme.* *The 32. Propoſition.*

If a right line touch a circle, and from the touch be drawen a right line cutting the circle: the angles which that line and the touch line make, are equall to the angles which conſiſt in the alternate ſegmentes of the circle.

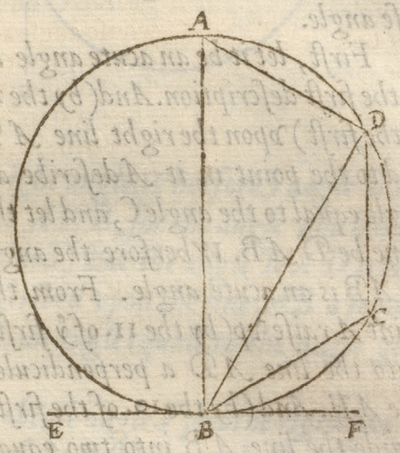
VPpoſe that the right line E F do touch the circle A B C D in the point B: and from the point B let there be drawen into the circle A B C D a right line cutting the circle, and let the ſame be B D. Then I ſay, that the angles which the line B D together with the

touch

touch line EF do make, are equall to the angles which are in the alternate segmentes of the circle, that is, the angle FBD is equall to the angle which consisteth in the segment BAD, and the angle EBD is equall to the angle which consisteth in the segment BCD. Raise vp (by the 11. of the first) from ỳ point B vnto the right line EF a perpendicular line BA. And in the circumference BD take a point at all aduentures, and let the same be C. And draw these right lines AD, DC, and CB. And for asmuch as a certaine right line EF toucheth the circle ADC in the point B, and from the point B where the touch is, is rayſed vp vnto the touch line a perpendicular BA. Therfore (by the 19. of the third) in the line BA is the centre of the circle ABCD. Wherfore ỳ angle ADB being in the semicircle, is (by the 31. of the third) a right angle. Wherefore the angles remayning BAD and ABD, are equall to one right angle. But the angle ABF is a right angle. Wherefore the angle ABF is equall to the angles BAD and ABD. Take away ỳ angle ABD which is common to them both. Wherefore the angle remayning DBF, is equall to the angle remayning BAD, which is in the alternate segment of the circle. And for asmuch as in the circle is a figure of fower sides, namely, ABCD, therfore (by the 22. of the third) the angles which are opposite the one to the other, are equall to two right angles. Wherfore the angles BAD and BCD, are equall to two right angles. But the angles DBF and DBE, are also equall to two right angles. Wherefore the angles DBF and DBE, are equall to the angles BAD and BCD. Of which we haue proued that the angle BAD is equall to the angle DBF. Wherefore the angle remayning DBE, is equall to the angle remayning DCB, which is in the alternate segment of the circle, namely, in the segment DCB. If therfore a right line touch a circle, and from the touch be drawen a right line cutting the circle: the angles which that line and the touch line make, are equall to ỳ angles which consist in ỳ alternate segmentes of the circle: which was required to be proued.

Construction.

Demonstration.



In thys Proposition may be two cases. For the line drawen from the touch and cutting the circle, may eyther passe by the centre or not. If it passe by the centre, then is it manifest (by the 18. of thys booke) that it falleth perpendicularly vpon the touch line, and deuideth the circle into two equall partes, so that all the angles in eche semicircle, are by the former Proposition, right angles, and therfore equall to the alternate angles made by the sayd perpendicular line and the touch line. If it passe not by the centre, then followe the construction and demonstration before put.

Two cases in this Proposition.

The

The 5. Probleme. The 33. Proposition.

Vppon a right lyne geuen to describe a segment of a circle, which shall contayne an angle equall to a rectiline angle geuẽ.

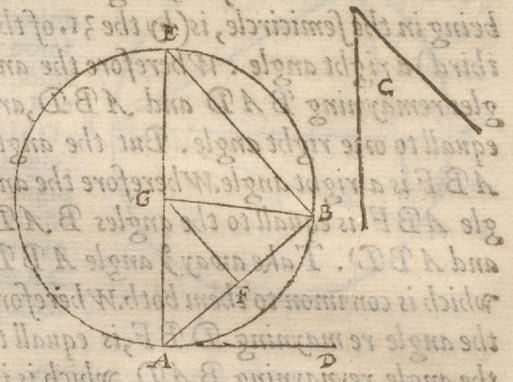
Suppose that the right line geuen be AB, and let the rectiline angle geuen be C. It is required vpon the right line geuẽ AB to describe a segment of a circle which shall contayne an angle equall to the angle C. Now the angle C is either an acute angle, or a right angle, or an obtuse angle.

Three cases in this Proposition.

The first case.

Construction.

First, let it be an acute angle as in the first description. And (by the 23 of the first) vpon the right line AB and to the point in it A describe an angle equal to the angle C, and let the same be DAB. Wherfore the angle DAB is an acute angle. From the point A raise vp (by the 11. of ỹ first) vnto the line AD a perpendiculer line AE. And (by the 10. of the first) deuide the line AB into two equall partes in the point F. And (by the 11. of the same) from the point F raise vp vnto the line AB a perpendicular lyne FG, and draw a line from G to B. And forasmuch as the line AF is equall to the line FB and the line FG is common to them both, therfore these two lines AF and FG are equall to these two lines FB and FG: and the angle AFG is (by the 4. petition) equall to the angle GFB. Wherfore (by the 4. of the same) the base AG is equall to the base GB. Wherfore making the centre G and the space GA describe (by the 3. petition) a circle and it shall passe by the point B: describe such a circle & let the same be ABE. And draw a line from E to B. Now forasmuch as from the ende of the diameter AE, namely, from the point A is drawen a right line AD making together with the right line AE a right angle, therfore (by the correllary of the 16. of the third) the line AD toucheth the circle ABE. And forasmuch as a certaine right line AD toucheth the circle ABE, & from the point A where the touch is, is drawen into ỹ circle a certaine right line AB: therfore (by the 32. of the third) the angle DAB is equall to the angle AEB which is in the alternate segment of the circle. But the angle DAB is equall to the angle C, wherfore the angle C is equall to the angle AEB. Wherfore vpon the right line geuen AB is described a segment of a circle which contayneth the angle AEB, which is equall to the angle geuen, namely, to C.



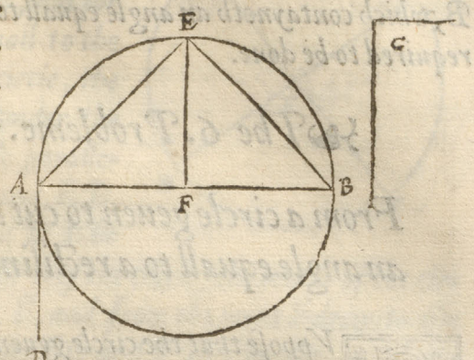
Demonstration.

The second case.

But now suppose that the angle C be a right angle. It is againe required vpon

pon the right line AB to deſcribe a ſegment of a circle, which ſhall contayne an angle equal to the right angle C. Deſcribe againe vpon the right line AB and to the point in it A an angle BAD equal to the rectiline angle geuen C (by the 23. of the firſt) as it is ſet forth in the ſecond deſcription. And (by the 10. of the firſt) deuide the line AB into two equall partes in the point F. And making the centre the point F and the ſpace FA or FB deſcribe (by the 3. peticion) ỹ circle AEB. Wherfore the right line AD toucheth the circle AEB: for that the angle BAD is a right angle. Wherfore ỹ angle BAD is equall to the angle which is in the ſegment AEB, for the angle which is in a ſemicircle is a right angle (by the 31. of the third) But the angle BAD is equal to the angle C. Wherfore there is againe deſcribed vpon the line AB a ſegment of a circle, namely, AEB, which containeth an angle equall to the angle geuen namely, to C.

Conſtruction.

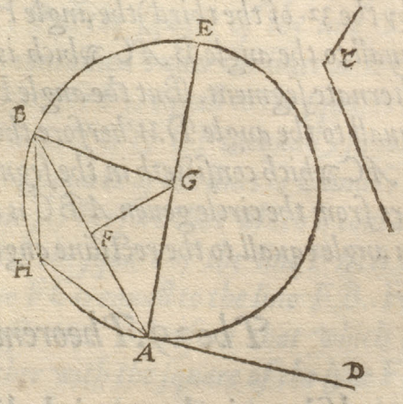


Demonſtration.

But now ſuppoſe that the angle C be an obtuſe angle. Vpon the right line AB and to the point in it A deſcribe (by the 23. of the firſt) an angle BAD equall to the angle C: as it is in the third deſcription. And from the point A rayſe vp vnto the line AD a perpendiculer line AE (by the 11. of the firſt) And agayne by the 10. of the firſt) deuide the line AB into two equall partes in the point F. And from the point F rayſe vp vnto the line AB a perpẽdicular line FG (by the 11. of the ſame) & drawe a line from G to B. And now foraſmuch as the line AF is equal to the line FB, and the line FG is common to them both, therfore theſe two lines AF and FG are equall to theſe two lines BF and FG: and the angle AFG is (by the 4. peticion) equall to the angle BFG: wherfore (by the 4. of the ſame) the baſe AG is equall to the baſe GB. Wherfore making the centre G, and the ſpace GA deſcribe (by the 3. peticion) a circle and it ſhall paſſe by the point D: let it be deſcribed as the circle AEB is. And foraſmuch as from the ende of the diameter AE is drawen a perpendiculer line AD, therefore (by the correllary of the 16. of the third) the line AD toucheth the circle AEB & from the point of the touche, namely, A, is extended the line AB. Wherfore (by the 32. of the third) the angle BAD is equall to the angle AHB which is in the alternate ſegment of the circle. But the angle BAD is equall to the angle C.

The third caſe.

Conſtruction.



Demonſtration.

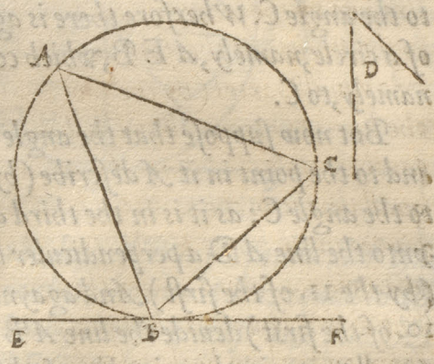
Wherefore the angle which is in the segment AHB is equall to the angle C. Wherfore vpon the right line geuen AB, is described a segment of a circle AHB, which containeth an angle equall to the angle geuen, namely, C: which was required to be done.

The 6. Probleme. The 34. Proposition.

From a circle geuen to cut away a section which shal containe an angle equall to a rectiline angle geuen.

Construction. *Suppose that the circle geuen be AC and let the rectiline angle geuen be D. It is required frō the circle ABC to cut away a segment which shall containe an angle equall to the angle D. Draw (by the 17. of the third) a line touching the circle, and let the same be EF: and let it touche in the point B. And (by the 23. of the first) vpon the right line EF and to the point in it B describe the angle FBC equall to the angle D. Now forasmuch as a certayne right line EF toucheth the circle ABC in the point B: and frora ỹ point of the touche, namely, B, is drawn into the circle a certaine right line BC, therefore (by the 32. of the third) the angle FBC is equall to the angle BAC which is in the alternate segment. But the angle FBC is equall to the angle D. Wherfore the angle BAC which consisteth in the segment BAC is equall to the angle D. Wherefore from the circle geuen ABC is cut away a segment BAC, which containeth an angle equall to the rectiline angle geuen: which was required to be done.*

Demonstration.



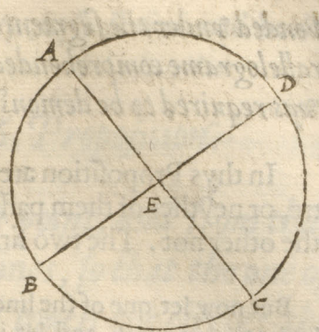
The 29. Theoreme. The 35. Proposition.

If in a circle two right lines do cut the one the other, the rectangle parallelograme comprehended vnder the segmentes or parts of the one line is equall to the rectangle parallelograme comprehended vnder the segment or partes of the other line.

Let the circle be ABCD, and in it let these two right lines AC and BD cut the one the other in the point E. Then I say that the rectangle parallelogramme contayned vnder the partes AE and EC is equall to

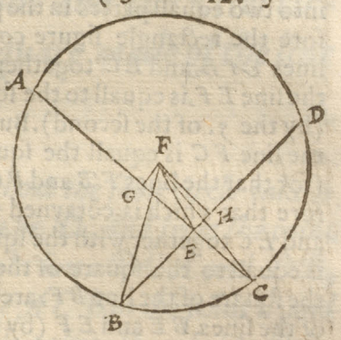
the

the rectangle parallelogramme contained vnder the partes D E and E B. For if the line A C and B D be drawen by the centre, then is it manifest, that for as much as the lines A E and E C are equall to the lines D E and E B by the definition of a circle, the rectangle parallelograme also contayned vnder the lines A E and E C is equall to ỹ rectangle parallelograme contained vnder the lines D E and E B.



Two cases in this Proposition. First case. Demonstration.

But now suppose that the lines A C and D B be not extended by the centre, and take (by the 1. of the third) the centre of the circle A B C D, and let the same be the point F, and from the point F draw to the right lines A C and D B perpendicular lines F G and F H (by the 12. of the first) and draw these right lines F B, F C, and F E. And forasmuch as a certaine right line F G drawen by the centre, cutteth a certaine right line A C not drawen by the centre in such sorte that it maketh right angles, it therfore deuideth the line A C into two equall partes (by the 3. of the third). Wherfore the line A G is equall to the line G C. And forasmuch as the right line A C is deuided into two equall partes in the point G, and into two vnequall partes in the point E: therfore (by the 5. of the second) the rectangle parallelogramme contained vnder the lines A E and E C together with the square of the line E G is equall to the square of the line G C. Put the square of the line G F common to them both, wherfore that which is contained vnder the lines A E & E C together with the squares of the lines E G and G F is equall to the squares of the lines G F & G C. But vnto ỹ squares of ỹ lines E G & G F is equall ỹ square of ỹ line F E (by the 47. of the first): and to the squares of the lines G C and G F is equall the square of the line F C (by the same) Wherfore that which is contained vnder the lines A E and E C, together with the square of the line F E is equall to the square of the line F C. But the line F C is equall to the line F B. For they are drawen from the centre to the circumference. Wherfore that which is contained vnder the lines A E and E C together with the square of the lyne F E is equal to the square of the line F B. And by the same demonstration that which is contained vnder the lines D E and E B together with the square of the line F E is equall to the square of the line F B. Wherfore that which is contained vnder the lines A E and E C together with the square of the line E F is equall to that which is contayned vnder the lines D E and E B together with the square of the line E F. Take away the square of the line F E which is common to them both. Wherfore the rectangle parallelogramme remayning which is contayned vnder the lines A E and E C is equall to the rectangle parallelogramme remayning, which is contayned vnder the lines D E and E B. If therefore in a circle two right lines do cut the one the other: the rectangle parallelogramme compre-



The second case. Construction.

Demonstration.

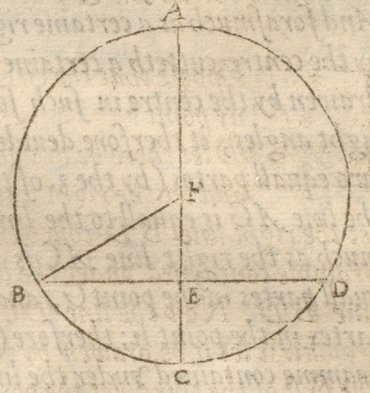
bended vnder the segmentes or parts of the one line is equall to the rectangle parallelograme comprehended vnder the segmentes or parts of the other line: which was required to be demonstrated.

Three cases in this Proposition.

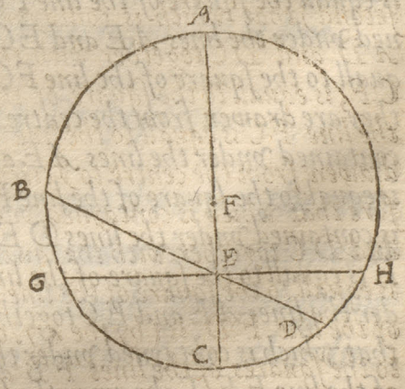
In thys Proposition are three cases: For eyther both the lines passe by the centre, or neyther of them passeth by the centre: or the one passeth by the centre and the other not. The two first cases are before demonstrated.

The third case.

But now let one of the lines onely, namely, the line AC passe by the centre, which let be the poynt F, and let it cut the other line, namely, BD, in the poynt E. Now then the line AC deuideth the line BD eyther into two equall partes, or into two vnequall partes. Fyrst let it deuide it into two equall partes: Wherefore also it deuideth it right angled wyse by the 3. of thys booke. Drawe a right line from B to F. Wherefore BEF is a right angled triangle. And for asmuch as the right line AC is deuided into two equall partes in the poynt F, & into two vnequall partes in the poynt E. Therfore the rectangle figure contayned vnder the lines AE and EC together with the square of the line EF, is equall to the square of the line FC (by the 5. of the second). But vnto the square of the line FC is equall the square of the line BF (for that the lines FB and FC are equall). Therfore that which is cõtayned vnder the lines AE and EC together with the square of the line EF, is equall to the square of the line BF. But vnto the square of the line BF, are equall the squares of the lines BE and EF (by the 47. of the first). Wherefore that which is contayned vnder the lines AE and EC together with the square of the line EF, is equall to the squares of the lines BE and EF. Take away the square of the line EF which is common to them both: Wherefore that which remayneth, namely, that which is contayned vnder the lines AE and EC, is equall to the residue, namely, to the square of the line BE. But the square of the line BE is that which is contained vnder the lines BE and ED: for (by supposition) the line BE is equall to the line ED. Wherefore that which is contayned vnder the lines AE & EC, is equall to that which is contayned vnder the lines BE and ED: which was required to be proued.



But now let the line AC passing by the centre, deuide the line BD not passing by the centre, vnequally in the poynt E. And frõ the poynt E raise vp vnto the line AC a perpendicular line EH, which produce on the other side to the poynt G. Wherefore (by the 3. of this booke) the line EH is equall to the line EG. Wherfore as we haue before proued, that which is contayned vnder the lines AE and EC, is equall to that which is contayned vnder the lines GE & EH: but that which is contayned vnder the lines BE and ED, is also equall to that which is contayned vnder the lines GE and EH, by the second case of thys Proposition: Wherfore that which is contayned vnder the lines AE and EC, is equall to that which is contayned vnder the lines BE and ED: which was agayne required to be proued.



Amongest all the Propositions in this third booke, doubtles thys is one of the chiefest. For it setteth forth vnto vs the wonderfull nature of a circle. So that by

it

it may be done many goodly conclusions in Geometry, as shall afterward be declared when occasion shall serue.

The 30. Theoreme. The 36. Proposition.

If without a circle be taken a certaine point, and from that point be drawen to the circle two right lines, so that the one of them do cut the circle, and the other do touch the circle: the rectangle parallelogramme which is comprehended vnder the whole right line which cutteth the circle, and that portion of the same line that lieth betwene the point and the vtter circũference of the circle, is equall to the square made of the line that toucheth the circle.

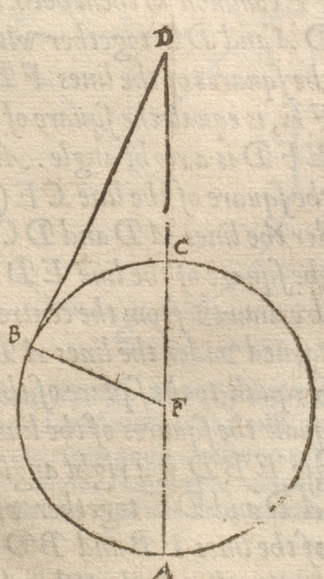
Construction.

Vppose that the circle be ABC: and without the same circle take any point at all aduentures, and let the same be D. And from the point D let there be drawen to the circle two right lines DCA and DB, and let the right line DCA cut the circle ACB in the point C, and let the right line BD touch the same. Then I say, that the rectangle parallelogramme contayned vnder the lines AD and DC, is equall to the square of the line BD. Now the line DCA is either drawen by the centre, or not.

Two cases in this Proposition.

The first case.

First let it be drawen by the centre. And (by the first of the third) let the poynt F be ỹ centre of the circle ABC, and drawe a line from F to B. Wherefore the angle FBD is a right angle. And for asmuch as ỹ right line AC is deuided into two equall partes in the poynt F, and vnto it is added directly a right line CD, therfore (by the 6. of the second) that which is contayned vnder the lines AD and DC together with the square of ỹ line CF, is equall to the square of the line FD. But the line FC is equall to the line FB, for they are drawen from the centre to ỹ circumference: Wherfore that which is contayned vnder the lines AD and DC together with the square of the line FB, is equall to the square of the line FD. But ỹ square of the line FD, is (by the 47. of the first) equall to the squares of the lines FB and BD (for the angle FBD is a right angle). Wherefore that which is contayned vnder the lines AD and DC together with the square of the line FB, is equall to the



Demonstration.

squares of the lines FB and BD. Take away the square of the line FB which is common to them both. Wherefore that which remayneth, namely, that which is contayned vnder the lines AD and DC, is equall to the square made of the line DB which toucheth the circle.

The second case.

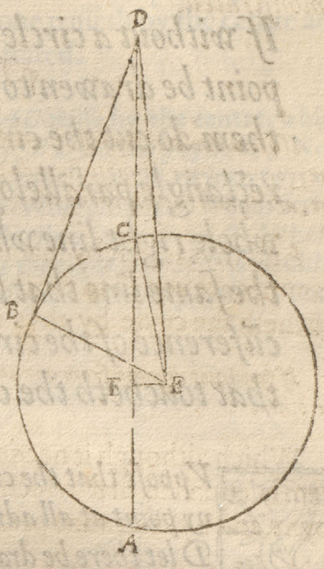
But now suppose that the right line DCA be not drawen by the centre of the circle ABC. And

Construction.

(by the first of the third) let the point E be ỹ centre of the circle ABC. And from ỹ poynt E, draw (by the 12. of the first) vnto the line AC a perpendicular line EF, and draw these right lines

Demonstration.

EB, EC, and ED. Now the angle EFD is a right angle. And for asmuch as a certaine right line EF draven by the centre, cutteth a certayne other right line AC not drawen by the centre, in such sort that it maketh right angles, it deuideth it (by ỹ third of the third) into two equall partes. Wherefore the line AF is equall to the line FC. And for asmuch as the right line AC is deuided into two equal partes in the poynt F, & vnto it is added directly an other right line making both one right line, therefore (by the 6. of the second) that which is contayned vnder the lines DA and DC together with the square of the line FC, is equall to the square of the line FD: put the square of the line FE common to them both. Wherefore that which is contayned vnder the lines DA and DC together with the squares of the lines CF and FE, is equall to the squares of the lines FD and FE. But to the squares of the lines FD and FE, is equall the square of the line DE (by the 47. of the first) for the angle EFD is a right angle. And to the squares of the lines CF and FE, is equall the square of the line CE (by the same). Wherfore that which is contayned vnder the lines AD and DC together with the square of the line EC, is equall to the square of the line ED. But the line EC is equall to the line EB: for they are drawen from the centre to the circumference. Wherefore that which is contayned vnder the lines AD and DC together with the square of the line EB, is equall to the square of the line ED. But to the square of the line ED, are equall the squares of the lines EB and BD (by the 47. of the first) for the angle EBD is a right angle: Wherefore that which is contayned vnder the lines AD and DC together with the square of the line EB, is equall to the squares of the lines EB and BD. Take away the square of the line EB which is common to them both: Wherefore the residue, namely, that which is contayned vnder the lines AD and DC, is equall to the square of the line DB. If therfore without a circle be taken a certaine point, and from that poynt be drawen to the circle two right lines, so that the one of them do cut the circle, and the other do



touch

touch the circle: the rectangle parallelogramme which is comprehended vnder the whole right line which cutteth the circle and that portion of the same line that lieth betwene the poynt and the vtter circumference of the circle, is equall to the square made of the line that toucheth the circle: which was required to be demonstrated.

¶ Two Corollaries out of Campane.

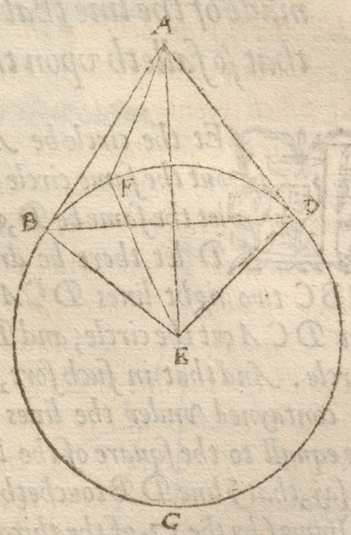
If from one and the selfe same poynt taken without a circle be drawen into the circle lines how many soeuer: the rectangle Parallelogrammes contayned vnder euery one of them and hys outward part, are equall the one to the other. *First Corollary.*

And thys is hereby manifest, for that euery one of those rectangle Parallelogrammes are equall to the square of the line which is drawen from that poynt and toucheth the circle by thys 36. Proposition. Hereunto he addeth.

If two lines drawen from one and the selfe same point do touch a circle, they are equall the one to the other. *Second Corollary.*

Which although it neede no demonstration, for that the square of eyther of them is equall to that which is contayned vnder the line drawen from the same poynt and hys outward part: yet he thus proueth it.

Suppose that there be a circle *BCD*, whose centre let be *E*, and without it take the point *A*. And from the poynt *A* drawe two lines *AB* and *AD*, which let touch the circle in the poyntes *B* and *D*. Then I say, that they are equall. Draw these right lines *EB*, *ED*, and *AE*. And by the 18. of thys booke, eyther of the angles at the poyntes *B* and *D* is a right angle. Wherefore (by the 47. of the first) the square of the line *AE*, is equall to the two squares of the lines *AB* and *EB*: and by the same reason, to the two squares of the lines *AD* and *ED*. Wherefore the two squares of the lines *AB* and *EB*, are equall to the two squares of the lines *AD* and *ED*. And for asmuch as the squares of the lines *EB* and *ED* are equall, therefore the two other squares of the lines *AB* and *AD* are also equall. Wherfore the line *AB* is equall to the line *AD*: which was required to be proued.



The same may be proued an other way: Draw a line from *B* to *D*. And (by the 5. of the first) the angle *EBD* is equall to the angle *EDB*. And for asmuch as the two angles *ABE* and *ADE* are equall, namely, for that they are right angles: if you take from them the equall angles *EBD* & *EDB*, the two other angles remayning, namely, the angles *ABD* and *ADB* shall be equall. Wherefore (by the 6. of the first) the line *AB* is equall to the line *AD*.

¶ Hereunto also Pelitarius addeth this Corollary.

From a poynt geuen without a circle, can be drawen vnto a circle onely two touch lines. *Third Corollary.*

The former description remayning, I say that from the poynt A can be drawen vnto

the circle *BCD* no more touch lines, but the two lines *AB* and *AD*. For if it be possible, let *AF* also be in the former figure a touch line, touching the circle in the poynt *F*. And prawe a line from *E* to *F*. And the angle at the point *F* shall be a right angle, by the 18. of this booke: Wherefore it is equall to the angle *EBA*, which is contrary to the 20. of the first.

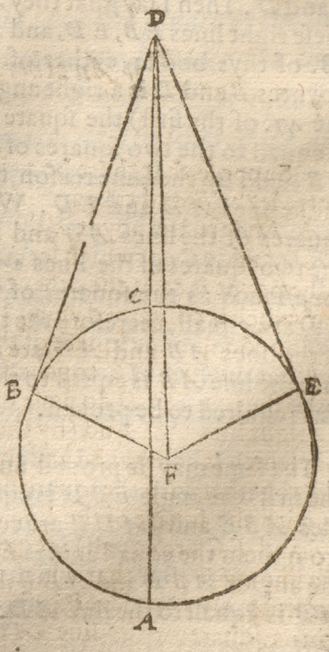
This may also be thus proued. For asmuch as all the lines drawen from one and the selfe same poynt & touching a circle are equall, as we haue before proued, but the lines *AB* and *AF* can not be equall, by the 8. Proposition of this booke, therefore the line *AF* can not touch the circle *BCD*.

The 31. Theoreme. The 37. Proposition.

If without a circle be taken a certaine point, and from that point be drawen to the circle two right lines, of which, the one doth cut the circle and the other falleth vpon the circle, and that in such sort, that the rectangle parallelogramme which is cõtayned vnder the whole right line which cutteth the circle, and that portion of the same line that lieth betwene the point and the vtter circumferẽce of the circle, is equall to the square made of the line that falleth vpon the circle: then that line that so falleth vpon the circle shall touch the circle.

This proposition is the cõuerse of the former.

LEt the circle be *ABC*: and without the same circle take a point, and let the same be *D*, & from the point *D* let there be drawen to the circle *ABC* two right lines *DCA* and *DB*: and let *DCA* cut the circle, and *DB* fall vpon the circle. And that in such sort, that that which is contayned vnder the lines *AD* and *DC*, be equall to the square of the line *DB*. Then I say, that ỹ line *DB* toucheth the circle *ABC*.



Construction. Drawe (by the 17. of the third) from the poynt *D* a right line touching the circle *ABC*, and let the same be *DE*. And (by the first of the same) let the point *F* be the centre of the circle *ABC*: and draw these right lines *FE*, *FB*, and *FD*. Wherfore the angle *FED* is a right angle. And for asmuch as the right line *DE* toucheth the circle *ABC*, and the right line *DCA* cutteth the same, therfore (by the Proposition going before) that which is contayned vnder the lines *AD* and *DC*, is equall to the square of the line *DE*. But that which is contayned

Demonstration.

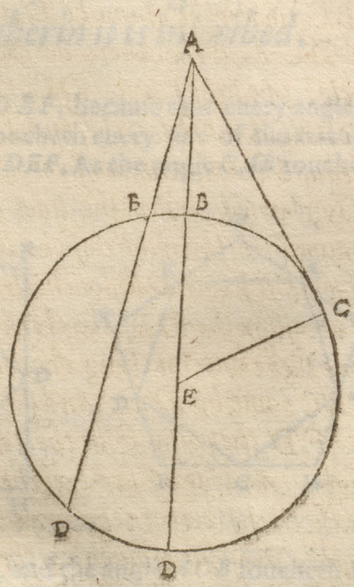
vnder

vnder the lines AD and DC, is supposed to be equall to the square of the line DB. Wherefore the square of the line DE is equall to the square of the line DB. Wherefore also the line DE is equall to the line DB. And the line FE is equall to the line FB, for they are drawen from the centre to ỹ circumference. Now therefore these two lines DE and EF are equall to these two lines DB and BF, and FD is a common base to them both. Wherefore (by the 8. of the first) the angle DEF is equall to the angle DBF. But the angle DEF is a right angle. Wherefore also the angle DBF is a right angle. And ỹ line FB being produced, shall be the diameter of the circle. But if from the end of the diameter of a circle be drawen a right line making right angles, the right line so drawen toucheth the circle (by the Correllary of the 16. of the third). Wherfore the right line DB toucheth the circle ABC. And the like demonstration will serue if the centre be in the line AC. If therefore without a circle be taken a certaine point, and from that poynt be drawen to the circle two right lines, of which the one doth cut the circle, and the other falleth vpon the circle, and that in such sort, that the rectangle parallelogramme which is contayned vnder the whole right line which cutteth the circle, and that portion of the same line that lieth betwene the poynt and the vtter circumference of the circle, is equall to the square made of the line that falleth vpon the circle: then the line that so falleth vpon the circle shall touch the circle: which was required to be proued.

¶ *An other demonstration after Pelitarius.*

An other demonstration after Pelitarius.

Suppose that there be a circle *B C D*, whose centre let be *E*: and take a point without it, namely, *A*: And frō the poynt *A* drawe two right lines *A B D*, and *A C*: of which let *A B D* cut the circle in the poynt *B*, & let the other fall vpon it. And let that which is contained vnder the lines *A D* and *A B*, be equal to the square of the line *A C*. Then I say, that the line *A C* toucheth the circle. For first if the line *A B D* do passe by the centre, draw the right line *C E*. And (by the 6. of the second) that which is contayned vnder the lines *A D* and *A B* together with the square of the line *E B*, that is, with the square of the line *E C* (for the lines *E B* and *E C* are equall) is equall to the square of the line *A E*. But that which is contained vnder the lines *A D* and *A B*, is supposed to be equall to the square of the line *A C*: Wherefore the square of the line *A C* together with the square of the line *C E*, is equall to the square of the line *A E*. Wherefore (by the last of the first) the angle at the point *C* is a right angle. Wherfore (by the 18. of this boke) the line *A C* toucheth the circle.

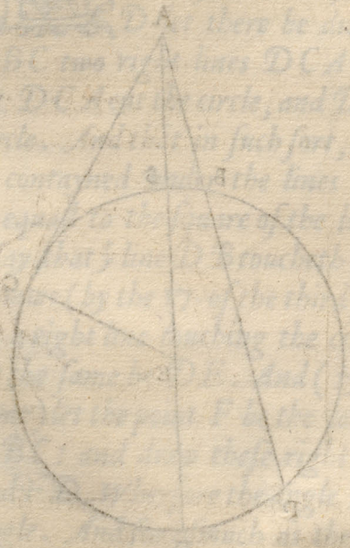


But

But if the line *A B D* doo not passe by the centre, drawe from the point *A* the line *A D*, in which let be the centre *E*. And forasmuch as that which is contained vnder this whole line and his outward part, is equall to that which is contained vnder the lines *A D* and *A B* by the first Corollary before put, therefore the same is equall to the square of the line *A C*, wherefore the angle *E C A* is a right angle as hath before bene proued in the first part of this Proposition. And therfore the line *A C* toucheth the circle: Which was required to be proued.

(.*.)

The ende of the third booke of Euclides Elementes.



¶ The fourth booke of Euclides Elementes.

THIS FOVRTH BOOKE intreateth of the inſcription & circumſcription of rectiline figures: how one right lined figure may be inſcribed within an other right lined figure, and how a right lined figure may be circumſcribed about an other right lined figure, in ſuch as may be inſcribed and circumſcribed within or about the other. For all right lined figures cannot ſo be inſcribed or circumſcribed within or about the other. Alſo it teacheth how a triangle, a ſquare, and certayne other rectiline figures being regular may be inſcribed within a circle. Alſo how they may be circumſcribed about a circle. Likewiſe how a circle may be inſcribed within them. And how it may be circumſcribed about them. And becauſe the maner of entreatie in this booke is diuers from the entreaty of the former bookes, he vſeth in this other wordes and termes then he vſed in them. The definitions of which in order here after follow.

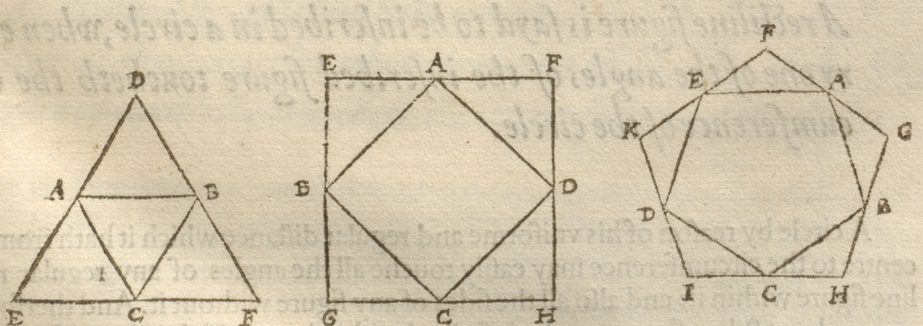
The argument of this booke.

Definitions.

A rectiline figure is ſayd to be inſcribed in a rectiline figure, when euery one of the angles of the inſcribed figure toucheth euery one of the ſides of the figure wherin it is inſcribed.

First definition.

As the triangle *A B C* is inſcribed in the triangle *D E F*, becauſe that euery angle of the triangle inſcribed, namely, the triangle *A B C* toucheth euery ſide of the triangle within which it is deſcribed, namely, of the triangle *D E F*. As the angle *C A B* toucheth



the ſide *E D*: the angle *A B C* toucheth the ſide *D F*, and the angle *A C B* toucheth the ſide *E F*. So likewiſe the ſquare *A B C D* is ſaid to be inſcribed within the ſquare *E F G H*: for euery angle of it toucheth ſome one ſide of the other. So alſo the Pentagon or fiue angled figure *A B C D E* is inſcribed within the Pentagon or fiue angled figure *F G H I K*, As you ſee in the figures.

Likewiſe

Second definition.

Likewise a rectiline figure is said to be circumscribed about a rectiline figure, when euery one of the sides of the figure circumscribed, toucheth euery one of the angles of the figure about which it is circumscribed.

As in the former descriptions the triangle *D E F* is said to be circumscribed about the triangle *A B C*, for that euery side of the figure circumscribed, namely, of the triangle *D E F* toucheth euery angle of the figure wherabout it is circumscribed. As the side *D F* of the triangle *D E F* circumscribed, toucheth the angle *A B C* of the triangle *ABC* about which it is circumscribed: and the side *E F* toucheth the angle *B C A*, and the side *C D* toucheth the angle *C A B*. Likewise vnderstand you of the square *E F G H* which is circumscribed about the square *A B C D*: for euery side of the one toucheth some one side of the other. Euē so by the same reason the Pentagon *F G H I K* is circumscribed about the Pentagon *A B C D E*, as you see in the figure on the other side. And thus may you of other rectiline figures consider.

The inscription and circumscription of rectiline figures pertaineth only to regular figures.

By these two definitions it is manifest, that the inscription and circumscription of rectiline figures here spoken of, pertayne to such rectiline figures onely, which haue equall sides and equall angles, which are commonly called regular. It is also to be noted that rectiline figures only of one kinde or forme can be inscribed or circumscribed the one within or about the other. As a triangle within or about a triangle: A square within or about a square: and so a Pentagon within or about a Pentagō, & likewise of others of one forme. But a triangle can not be inscribed or circumscribed within or aboute a square: nor a square within or about a Pentagon. And so of others of diuers kyndes. For euery playne rectiline figure hath so many angles as it hath sides. Wherfore the figure inscrided must haue so many angles as the figure in which it is inscribed hath sides: and the angles of the one (as is sayd) must touche the sides of the other. And contrariwise in circumscription of figures, the sides of the figure circumscribed must touch the angles of the figure about which it is circumscribed.

The third definition.

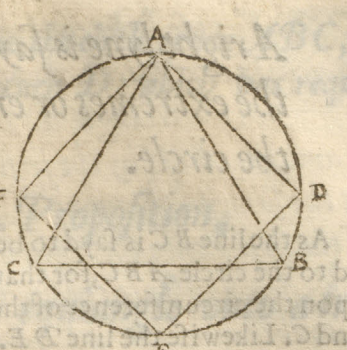
A rectiline figure is sayd to be inscribed in a circle, when euery one of the angles of the inscribed figure toucheth the circumference of the circle.

A circle by reason of his vniforme and regular distance which it hath from the centre to the circumference may easily touche all the angles of any regular rectiline figure within it: and also all the sides of any figure without it. And therfore any regular rectiline figure may be inscribed within it, and also be circumscribed about it. And agayne a circle may be both inscribed within any regular rectiline figure, and also be circumscribed about it.

As the triangle *A B C* is inscribed in the circle *A B C*: for that euery angle toucheth some one pointe of the circumference of the circle. As the angle *C A B* of the triangle *A B C* toucheth the point *A* of the circumference of the circle. And the angle *A B C*

of

of the triangle toucheth the pointe *B* of the circumference of the circle. And alſo the angle *A C B* of the triangle toucheth the pointe *C* of the circumference of the circle. In like manner the ſquare *A D E F* is inſcribed in the ſame circle *A B C*: for that euery angle of the ſquare inſcribed, toucheth ſome one poynte of the circle in which it is inſcribed. And ſo imagine you of rectilined figures.



A circle is ſayd to be circumſcribed about a rectiline figure, whē the circumference of the circle toucheth euery one of the angles of the figure about which it is circumſcribed.

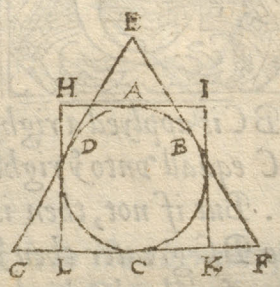
The fourth deſinition.

As in the former example of the third definition. The circle *A D E F* is circumſcribed about the triangle *A B C*, becauſe the circumference of the circle which is circumſcribed toucheth euery angle of the triangle about which it is circumſcribed: namely, the angles *C A B*, *A B C*, and *B C A*. Likewiſe the ſame circle *A D E F* is circumſcribed about the ſquare *A D E F* by the ſame definition, as you may ſee.

A circle is ſayd to be inſcribed in a rectiline figure, when the circumference of the circle toucheth euery one of the ſides of the figure within which it is inſcribed.

The fift deſinition.

As the circle *A B C D* is inſcribed within the triangle *E F G*, becauſe the circumference of the circle toucheth euery ſide of the triangle in which it is inſcribed: namely the ſide *E F* in the point *B*, and the ſide *G F* in the poinꝛe *C*, and the ſide *G E* in the point *D*. Likewiſe by the ſame reaſon the ſame circle is inſcribed within the ſquare *H I K L*. And ſo may you iudge of other rectiline figures.



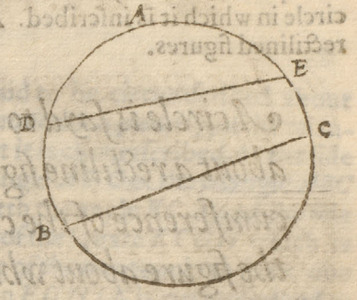
A rectilined figure is ſaid to be circumſcribed about a circle, when euery one of the ſides of the figure circumſcribed toucheth the circumference of the circle.

The ſixt deſinition.

As in the former figure of the fift definition, the triangle *E F G* is circumſcribed about the circle *A B C D*, for that euery ſide of the ſame triangle beyng circumſcribed toucheth the circumference of the circle, about which it is circumſcribed. As the ſide *E G* of the triangle *E F G* toucheth the circumference of the circle in the point *D*: and the ſide *E F* toucheth it in the point *B*: and the ſide *G F* in the point *C*. Likewiſe alſo the ſquare *H I K L* is circumſcribed about the circle *A B C D*, for euery one of his ſides toucheth the circumference of the circle, namely, in the pointes *A*, *B*, *C*, *D*. And thus conſider of all other regular right lined figures (for of them onely are vnderſtanded theſe definitions) to be circumſcribed about a circle, or to be inſcribed within a circle: or of a circle to be circumſcribed or inſcribed about or within any of them.

Seuenth definition.

A right lyne is sayd to be coapted or applied in a circle, when the extremes or endes therof, fall vppon the circumference of the circle.



As the line *B C* is sayd to be coapted or to be applied to the circle *A B C*, for that both his extremes fall vpon the circumference of the circle in the pointes *B* and *C*. Likewise the line *D E*. This definition is very necessary, and is properly to be taken of any lyne geuen to be coapted and applied into a circle, so that it exceede not the diameter of the circle geuen.

The 1. Probleme. The 1. Proposition.

In a circle geuen, to apply a right line equall vnto a right line geuen, which excedeth not the diameter of a circle.

Construction.

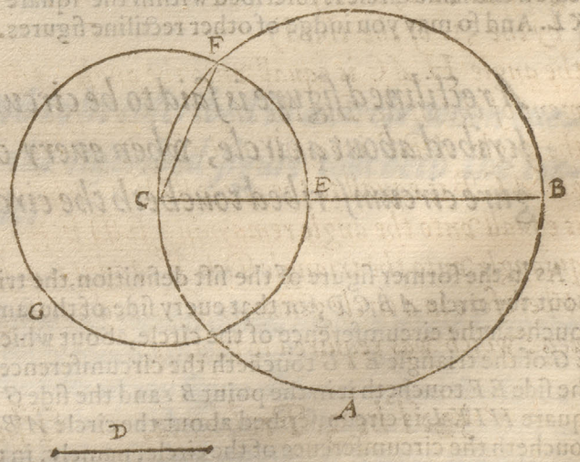
Vppose that the circle geuen be A B C, and let the right line geuen, exceding not the diameter of the same circle, be D. Now it is required in the circle geuen A B C, to apply a right line equall vnto the right line D. Draw the diameter of the circle A B C, and let the same be B C. Now if the line B C be equall vnto the line D, then is that done which was required. For in the circle geuen

Two cases in this Proposition.

First case.

Second case.

A B C is applyed a right line B C equall vnto ỹ right line D. But if not, then is the line B C greater then ỹ line D. And (by the third of the first) put vnto the line B C an equall line C E. And making the centre C, and the space C E, describe (by the third petition) a circle E G F, cutting the circle A B C in the point F, & draw a line from C to F. And for asmuch as the point C is ỹ centre of the circle E G F, therefore (by



Demonstration.

the 13. definition of the first) the line C F is equall vnto the line C E. But the line C E is equall vnto the line D. Wherefore (by the first common sentence) the

line

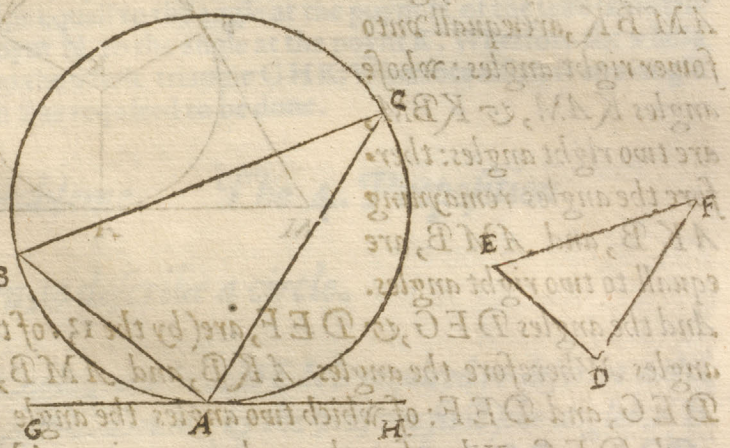
line C F also is equall vnto the line D. Wherefore in the circle geuen A B C, is applyed a right line C A equall vnto the right line geuen D: which was required to be done.

The 2. Probleme. The 2. Proposition.

In a circle geuen, to describe a triangle equiangle vnto a triangle geuen.

SVppose that the circle geuen be A B C: and let the triangle geuen be D E F. Now it is required in the circle geuen A B C to describe a triangle equiangle vnto the triangle geuen D E F. Draw (by the 17. of the third) a right line touching the circle A B C, and let the same be G A H, and let it touch in the point A. And (by the 23. of the first) vnto ŷ right line A H, and vnto the point in it A, describe an angle H A C equall vnto the angle D E F. And (by the selfe same) vnto the right line A G, and vnto the point in it A, make an angle G A B equall vnto the angle D F E. And draw a right line from B to C. And for asmuch as a certaine right line G A H toucheth ŷ circle A B C, and from the point where it toucheth, namely, A, is drawen into the circle a right line A C, therefore (by the 31. of the third) the angle H A C is equall vnto the angle A B C which is in the alternate segment of the circle. But the angle H A C is equall to the angle D E F. Wherfore the angle A B C is equall to the angle D E F. And by the same reason, the angle A C B is equall to the angle D F E. Wherefore the angle remayning, B A C, is equall vnto the angle remayning, E D F. Wherefore the triangle A B C is equiangle vnto the triangle D E F. And it is described in the circle geuen A B C. Wherefore in a circle geuen, is described a triangle equiangle vnto a triangle geuen: which was required to be done.

Construction.



Demonstration.

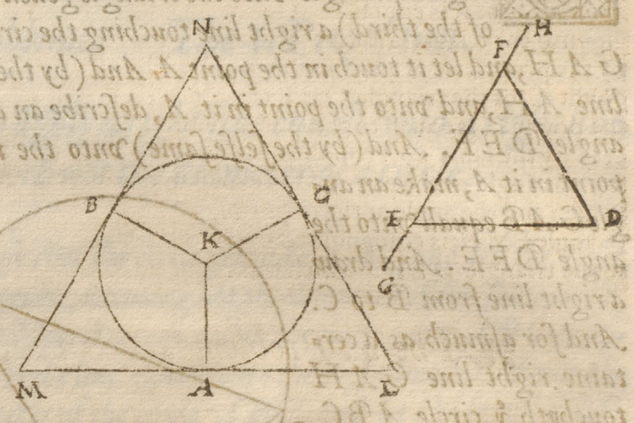
The 3. Probleme. The 3. Proposition.

About a circle geuen, to describe a triangle equiangle vnto a triangle geuen.

Construction.

Demonstration.

Vppose ẏ the circle geuen be ABC, and let the triangle geuen be DEF. It is required about the circle ABC to describe a triangle equiangle vnto the triangle DEF. Extend the line EF on ech side to the poyntes G and H. And (by the first of the third) take the centre of the circle ABC, and let the same be the point K. And then draw a right line KB. And (by the 23. of the first) vnto the right line KB, and vnto the point in it K, make an angle BKA equall vnto the angle DEG, and likewise make the angle BKC equall vnto the angle DFH. And (by the 17. of the third) draw right lines touching the circle A, B, C, in the pointes A, B, C. And let ẏ same be LAM, MBN, and NCL. And for asmuch as the right lines LM, MN, & NL, do touch the circle ABC in the pointes A, B, C, and from the centre K vnto the pointes A, B, C, are drawen right lines KA, KB, and KC, therefore the angles which are at the pointes A, B, C, are right angles (by ẏ 18. of the third). And for asmuch as the fower angles of the fower sided figure AMBK, are equall vnto fower right angles: whose angles KAM, & KBM, are two right angles: therfore the angles remayning AKB, and AMB, are equall to two right angles.



And the angles DEG, & DEF, are (by the 13. of the first) equall to two right angles. Wherefore the angles AKB, and AMB, are equall vnto the angles DEG, and DEF: of which two angles the angle AKB is equall vnto the angle DEG: Wherefore the angle remayning, AMB, is equall vnto the angle remayning, DEF. In like sort may it be proued, that the angle LNM, is equall to the angle DFE. Wherfore the angle remayning MLN, is equall vnto the angle remayning EDF. Wherefore the triangle LMN, is equiangle vnto the triangle DEF: and it is described about the circle ABC. Wherefore about a circle geuen is described a triangle equiangle vnto a triangle geuen: which was required to be done.

¶ *An other way after Pelitarius.*

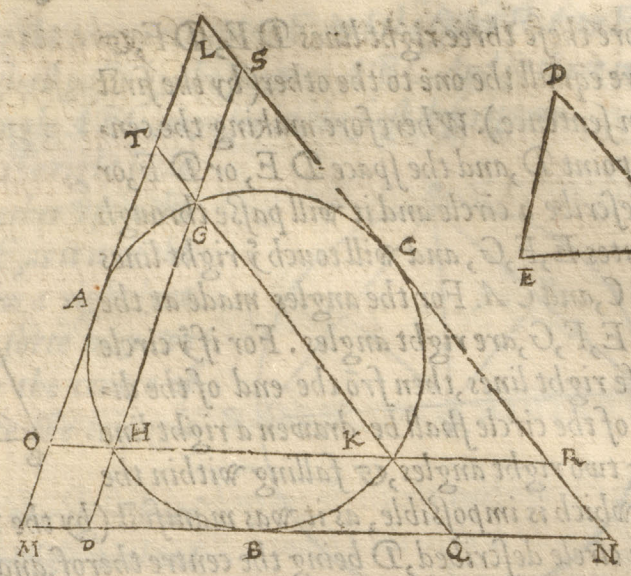
An other way after Pelitarius.

Construction.

In the circle ABC inscribe a triangle GHK equiangle to the triangle EDF (by the former Proposition): so that let the angle at the poynt G be equall to the angle D, and let the angle at the point H be equall to the angle E: and let also the angle at the poynt K be equall to the angle F. Then drawe the line LM parallel to the line GH, which let touch the circle in the poynt A (which may be done by the Proposition added of the sayd Pelitarius after the 17. Proposition). Draw likewyse the line MN parallel vnto the line HK and touching the circle in the poynt B: And also draw the line

LN

LN parallel vnto the line GK and touching the circle in the poynt C. And theſe three lines ſhall vndoubtedly concurre, as in the poyntes L, M, and N, which may eaſily be proued, if you produce on either ſide the lines GH, GK, and HK, vntill they cut the lines LM, LN, and MN, in the poyntes O, P, Q, R, S, T. Now I ſay, that the triangle LMN circũſcribed about the circle ABC, is equiangle to the triangle DEF. For it is manifeſt, that it is equiangle vnto the triãgle GHK,



Demonſtration.

by the proprietie of parallel lines. For the angle MTQ is equall to the angle at the poynt G of the triangle GHK (by the 29. of the firſt) and therefore alſo the angle at the poynt L, is equall to the ſelfe ſame angle at the poynt G (for the angle at the point L, is by the ſame 29. Propoſition, equall to the angle MTQ). And by the ſame reaſon the angle at the poynt M, is equall to the angle at the poynt H of the ſelfe ſame triangle: and the angle at the poynt N, to the angle at the poynt K. Wherfore the whole triangle LMN, is equiangle to the whole triangle GHK: Wherfore alſo it is equiangle to the triangle DEF: which was required to be done.

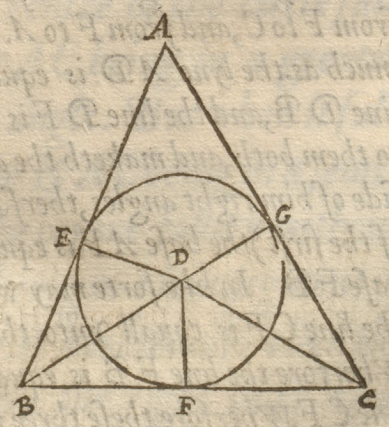
The 4. Probleme. The 4. Propoſition.

In a triangle geuen, to deſcribe a circle.

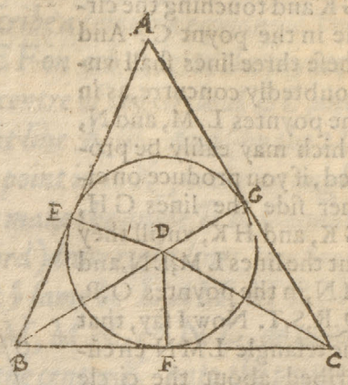
SVppoſe that the triangle geuen be ABC. It is required to deſcribe a circle in the triangle ABC. Deuide (by the 9. of the firſt) the angles ABC, and ACB into two equall partes by two right lines BD and CD. And let theſe right lines meete together in the point D. And (by the 12. of the firſt) from the point D draw vnto the right lines AB, BC and CA perpendicular lines, namely, DE, DF, and DG. And foraſmuch as the angle ABD is equall to tho angle CBD, and the right angle BED is equall vnto the right angle BFD. Now then there are two triangles EBD and FBD hauing two angles equall to two angles, and one ſide equall to one ſide, namely, BD which is common to them both, and ſubtendeth one of the equall angles. Wherfore (by the 26. of the firſt) the reſt of the ſides are equall vnto the reſt of the ſides. Wherfore the line DE is equall vnto the line DF: and by the ſame reaſon alſo the line DG is equall vnto ỹ line DF.

Conſtruction.

Demonſtration.



Wherfore these three right lines D E, D F, & D G, are equall the one to the other (by the first common sentence). Wherefore making the centre the point D, and the space D E, or D F, or D G, describe a circle and it will passe through the pointes E, F, G, and will touch ỹ right lines A B, B C, and C A. For the angles made at the pointes E, F, G, are right angles. For if ỹ circle cut those right lines, then frō the end of the diameter of the circle shall be drawen a right line making two right angles, & falling within the circle: which is impossible, as it was manifest (by the 16. of the third). Wherefore the circle described, D being the centre therof, and the space therof being either D E, or D F, or D G, cutteth not these right lines A B, B C, & C A. Wherfore (by the Corollary of the same) it toucheth them, and the circle is described in the triangle A B C. Wherfore in the triangle geuen A B C, is described a circle E F G: which was required to be done.



Demonstration leading to an impossibilitie.

The 5. Probleme. The 5. Proposition.

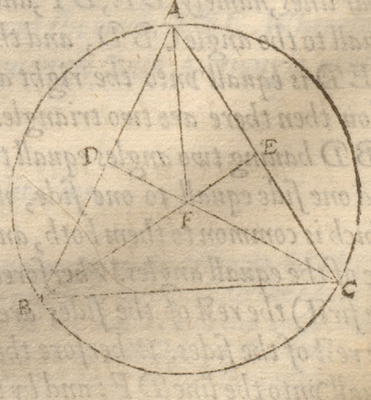
About a triangle geuen, to describe a circle.

Suppose that the triangle geuen be A B C. It is required about the triangle A B C to describe a circle. Deuide (by the 10. of the first) the right lines A B and A C into two equall partes in the pointes D and E. And from the pointes D and E (by the 11. of the first) rayse vp vnto the lines A B & A C two perpendicular lines D F and E F. Now these perpendicular lines meete together either within the triangle A B C, or in the right line B C, or els without the right line B C.

Three cases in this Proposition.

First let them meete together within the triangle in the point F. And (by ỹ first peticion) drawe right lines from F to B, from F to C, and from F to A. And forasmuch as the lyne A D is equall vnto the line D B, and the line D F is common vnto them both, and maketh the angles on ech side of him right angles, therfore (by the 4. of the first) the base A F is equall vnto the base F B. In like sorte may we proue that the line C F is equall vnto the lyne A F. Wherfore the line F B is equall vnto the line C F. Wherfore these three right lines F A, F B, and F C are equall the one to the

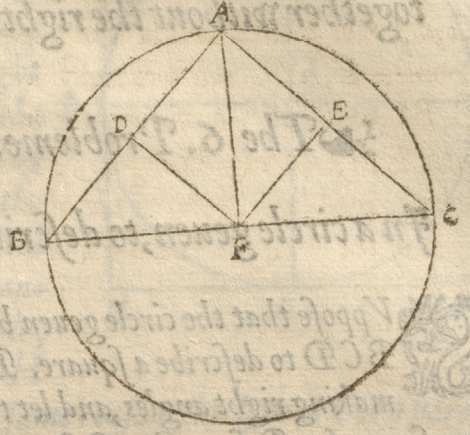
The first case.



the other. Wherfore making the centre the point F, and the ſpace F A, or F B, or F C deſcribe a circle and it ſhall paſſe by the poyntes A B C. And ſo there is a circle deſcribed about the triangle A B C, as ye ſee in the firſt deſcription.

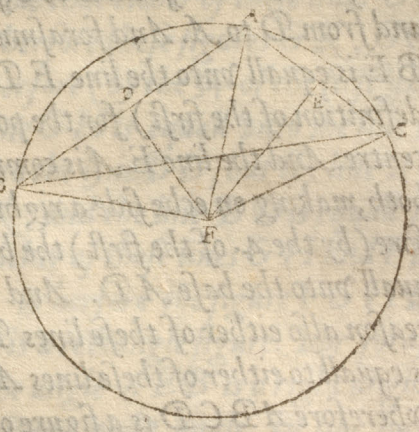
The ſecond caſe.

But now ſuppoſe that the right lines D F and E F do meete together vpon the right line B C in the point F, as it is in the ſecond deſcription, and draw a right lyne from A to F, and in like ſorte may we proue that the poynte F is the centre of the circle deſcribed aboute the triangle A B C.



The third caſe.

But now ſuppoſe that the right lines D F and E F do meete together without the triangle A B C in the poynt F. Againe as it is in the third deſcription draw right lines from F to A, from F to B, and from F to C. And foraſmuch as the line A D is equall vnto the line D B, and the line D F is common vnto them both, and maketh a right angle on eche ſide of him, wherfore (by the 4. of the firſt) the baſe A F is equall vnto the baſe B F. And in like ſort may we proue that the line C F is equal vnto the line A F. Wherfore agayne making F the centre, and the ſpace F A, or F B, or F C, deſcribe a circle and it ſhal paſſe by the pointes A, B, C, and ſo is there a circle deſcribed about the triangle A B C, as ye ſee it is in the third deſcription. Wherfore about a triangle geuen is deſcribed a circle: which was required to be done.



Correlary.

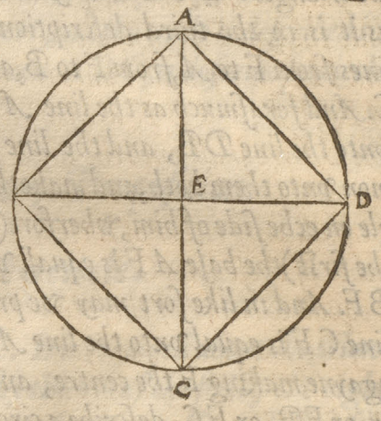
Hereby it is manifeſt, that when the centre of the circle falleth within the triangle, the angle B A C being in a greater ſegment of a circle is leſſe thē a right angle. But when it falleth vpon the right line B C the angle B A C being in a ſemicircle is a right angle. But when the centre falleth without the right line B C, the angle B A C being in a leſſe ſegment of a circle, is greater then a right angle. Wherfore alſo when the angle geuen is leſſe then a right angle, the right lines D F

and E F will meete together within the sayd triangle. But when it is a right angle they will meete together vpon the line B C. But whē it is greater then a right angle, they will meete together without the right line B C.

The 6. Probleme. The 6. Proposition.

In a circle geuen, to describe a square.

Construction. Suppose that the circle geuen be A B C D. It is required in the circle A B C D to describe a square. Draw in the circle A B C D two diameters making right angles, and let the same be A C and B D, and drawe right lines from A to B, from B to C, from C to D, and from D to A. And forasmuch as the line



Demonstration. B E is equall vnto the line E D (by the 15. definition of the first) for the point E is the centre. And the line E A is common to them both, making on eche side a right angle: therfore (by the 4. of the first) the base A B is equall vnto the base A D. And by the same reason also either of these lines B C and C D is equall to either of these lines A B and A D: wherefore A B C D is a figure of foure equal sides. I say also that it is a rectangle figure. For forasmuch as the right line B D is the diameter of the circle A B C D, therfore the angle B A D beyng in the semicircle is a right angle (by the 31. of the third) And by the same reason euery one of these angles A B C, B C D and C D A is a right angle. Wherfore the foure sided figure A B C D is a rectangle figure, and it is proued that it consisteth of equall sides. Wherfore (by the 30. definition of the first) it is a square, and it is described in the circle A B C D: which was required to be done.

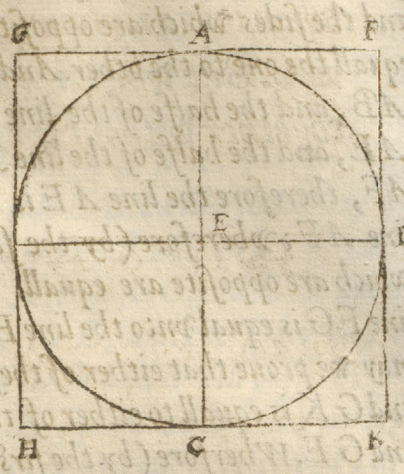
The 7. Probleme. The 7. Proposition.

About a circle geuen, to describe a square.

Construction. Suppose that the circle geuen be A B C D. It is required about the circle A B C D to describe a square. Draw in the circle A B C D two diameters making right angles, where they cut the one the other, and let the same be A C and B D. And by the pointes A, B, C, D, draw

(by

(by the 17. of the third) right lines touching the circle A B C D, and let the same be F G, G H, H K, and K F. Now forasmuch as the right line F G toucheth the circle A B C D in the point A, and from the centre E to the point A where the touch is, is drawen a right line E A, therfore (by the 18. of the third) the angles at the point A are right angles, and by the same reason the angles which are at the pointes B, C, D, are also right angles. And forasmuch as the angle A E B is a right angle, & the angle E B G is also a right angle, therfore (by the 28. of the first) the line G H is a parallel vnto the line A C: and by the same reason the lyne A C is a parallel vnto the lyne F K. In like sorte also may we proue that either of these lines G F and H K is a parallel vnto the lyne B E D. Wherfore these figures G K, G C, A K, F B, and B K are parallelogrames. Wherfore (by the 34. of the first) the line G F is equall vnto the line H K, and the line G H is equall vnto the line F K. And forasmuch as the line A C is equall vnto the lyne B D, but the line A C is equall vnto either of these lines G H and F K: and the line B D is equall to either of these lines G F and H K. Wherfore either of these lines G H and F K is equall to either of these lines G F and H K. Wherfore the figure F G H K consisteth of foure equall sides. I say also that it is a rectangle figure. For forasmuch as G B E A is a parallelogramme, and the angle A E B is a right angle: therfore (by the 34. of the first) the angle A G B is a right angle. In like sorte may we proue that the angles at the poyntes H, K, and F are right angles. Wherfore F G H K is a rectangle foure sided figure, and it is proued that it consisteth of equall sides: wherfore it is a square, and it is described about the circle A B C D. Wherfore about a circle geuen is described a square: which was required to be done.



Demonstration.

The 8. Probleme. The 8. Proposition.

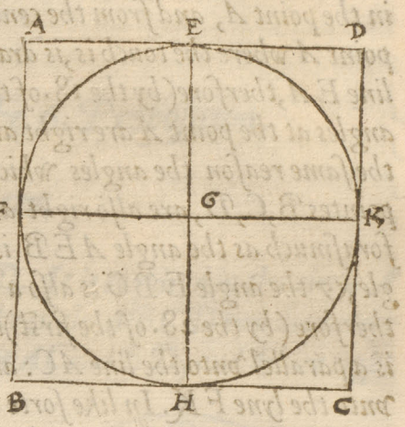
In a square geuen, to describe a circle.

Suppose that the square geuen be A B C D. It is required in the square A B C D to describe a circle. Deuide (by the 10. of the first) either of these lines A B and A D into two equall partes in the pointes E and F. And by the point E (by the 31. of the first) draw a line E H parallel vnto either of these lines A B and D C: and (by the same) by the point F draw a line F K parallel vnto either of these lines A D and B C. Wherfore euery one of these figures

Construction.

Demonstration.

gures AK, KB, AH, HD, AG, GC, BG, and GD is a parallelograme, and the sides which are opposite the one to the other, are (by the 34. of the first) equall the one to the other. And forasmuch as the line AD is equall vnto the line AB, and the halfe of the line AD is the line AE, and the halfe of the line AB, is the line AF, therefore the line AE is equall vnto the line AF: wherefore (by the same) the sides which are opposite are equall. Wherefore the line FG is equal vnto the line EG. In like sort may we proue that either of these lines GH, and GK is equall to either of these lines FG and GE. Wherfore (by the first common sentence) these foure lines GE, GF, GH, and GK are equall the one to the other. Wherfore making the centre G, and the space either GE, or GF, GH, or GK, describe a circle and it will passe by the pointes E, F, H, K, and will touche the right lines AB, BC, CD, and DA. For the angles at the pointes E, F, H, K, are right angles. For if the circle do cut the right lines AB, BC, CD, and DA, then the line which is drawen by the ende of the diameter of the circle making right angles should fall within the circle, which is impossible (by the 16. of the third) Wherfore the centre being the poynt G and the space beyng GE, or GF, or GH, or GK if a circle be described, it shall not cut the right lines AB, BC, CD, and DA. Wherefore it shall touch them. And it is described in the square ABCD: wherefore in a square geuen is described a circle: which was required to be done.



The 9. Probleme. The 9. Proposition.

About a square geuen, to describe a circle.

Suppose that the square geuen be ABCD. It is required about the square ABCD to describe a circle. Drawe

Construction. right lines from A to C, and from D to B, & let them cut the one the other in the poynt E.

Demonstration. And forasmuch as the lyne DA is equall vnto the lyne AB, and the line AC is common vnto them both, therfore these two lines DA and AC are equall vnto these two lynes BA and AC, the one to the other. And the base DC is equall vnto the base BC. Wherefore (by the 8. of the first) the angle DAC is equall vnto the angle BAC. Wherefore the angle DAB is deuided into two equall partes by the line AC. And in like

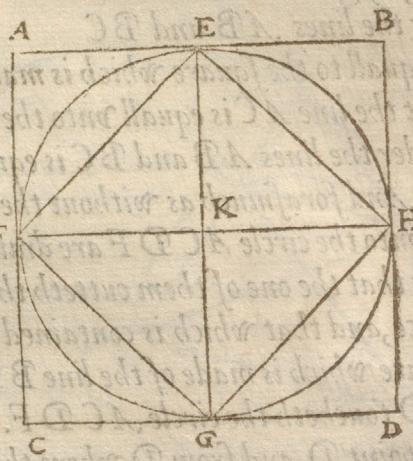
like sort may we proue that euery one of these angles ABC, BCD, and CDA is deuided into two equall partes by the right lines AC and DB. And forasmuch as the angle DAB is equall vnto the angle ABC, and of the angle DAB the angle EAB is the halfe, and of the angle ABC the angle EBA is the halfe: Therfore the angle EAB is equall vnto the angle EBA: wherfore (by the 6. of the first) the side EA is equall vnto the side EB. In like sorte may we proue that either of these right lines EA and EB is equall vnto either of these lines EC and ED. Wherfore these foure lines EA, EB, EC, and ED are equall the one to the other. Wherfore making the centre E, and the space any of these lines EA, EB, EC, or ED. Describe a circle and it will passe by the pointes A, B, C, D, and shall be described about the square $ABCD$, as it is euident in the figure $ABCD$. Wherfore about a square geuē is described a circle: which was required to be done.

¶ A Proposition added by Pelitarius.

A square circumscribed about a circle, is double to the square inscribed in the same circle.

A Proposition added by Pelitarius.

Suppose that the square $ABCD$ be circumscribed about the circle $EFGH$, whose centre let be K. And let the poyntes of the touches be E, F, G, H. And drawing these two diameters EG, and FH, and these right lines EF, FG, GH, and HB, there shall be inscribed in the circle a square $EFGH$ (by the sixt of this booke). Then I say, that the square $ABCD$, is double to the square $EFGH$. For forasmuch as the side AB of the greater square, is (by the 34. of the first) equall to FH, which is the diameter of the lesse square: but the square of FH is double to the square whose diameter it is, namely, to the square $EFGH$ (by the 47. of the first). Wherefore also the square of AB which is $ABCD$, is double to the square $EFGH$: which was required to be proued.



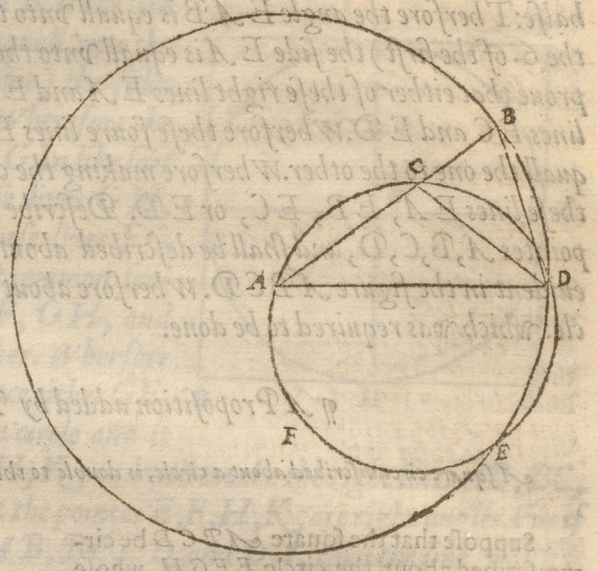
Thys may also be demonstrated by the equalite of the triangles and squares contayned in the great squares.

The 10. Probleme. The 10. Proposition.

To make a triangle of two equall sides called Isosceles, which shall haue eyther of the angles at the base double to the other angle.

Take

Construction. TAke a right line at all aduentures which let be AB, & (by ỹ 11. of the second) let it be so deuided in ỹ pointe C, ỹ the rectangle figure comprehended vnder the lines AB and BC be equall vnto ỹ square which is made of the line AC. And making the centre the point A, & the space AB, describe (by the 3. peticion) a circle BDE; and (by the 1. of the fourth) into the circle BDE apply a right line BD equall to the right lyne AC which is not greater then the diameter of the circle BDE. And draw lines from A to D and from D to C. And (by the 5. of ỹ fourth) about the triangle ACD describe a circle ACDF.



Demonstration. And forasmuch as the rectangle figure contained vnder the lines AB and BC is equall to the square which is made of the line AC: (For that is by supposition) But the line AC is equall vnto the line BD. Wherfore that which is contayned vnder the lines AB and BC is equall to the square which is made of the line BD. And forasmuch as without the circle ACDF is taken a poynt B, and from B vnto the circle ACDF are drawen two right lines BCA, and BD, in such sort that the one of them cutteth the circle, and the other endeth at the circumference, and that which is contained vnder the lines AB and BC is equall to the square which is made of the line BD, therfore (by the 17. of the third) the line BD toucheth the circle ACDF. And forasmuch as the line BD toucheth in the point D, and from D where the touche is, is drawen a right line DC, therefore (by the 32. of the same) the angle BDC is equall vnto the angle DAC, which is in ỹ alternate segment of ỹ circle. And forasmuch as ỹ angle BDC is equal vnto ỹ angle DAC, put the angle CDA common vnto thẽ both. Wherfore ỹ whole angle BDA is equal to these two angles CDA, & DAC. But vnto ỹ angles CDA, & DAC is equall the outward angle BCD (by the 32. of the 1.) Wherfore ỹ angle BDA is equal vnto ỹ angle BCD. But ỹ angle BDA is (by ỹ 5. of the first) equall vnto the angle CBD, for (by the 15. definition of ỹ first) the side AD is equall vnto the side AB: wherfore (by the 1. common sentence) the angle DBA is equall vnto the angle BCD. Wherefore these three angles BDA, DBA, and BCD are equall the one to the other. And forasmuch as the angle DBC is equall vnto the angle BCD, the side therfore BD is equall vnto the side DC. But the line BD is by supposition equall vnto the lyne CA.

Wher-

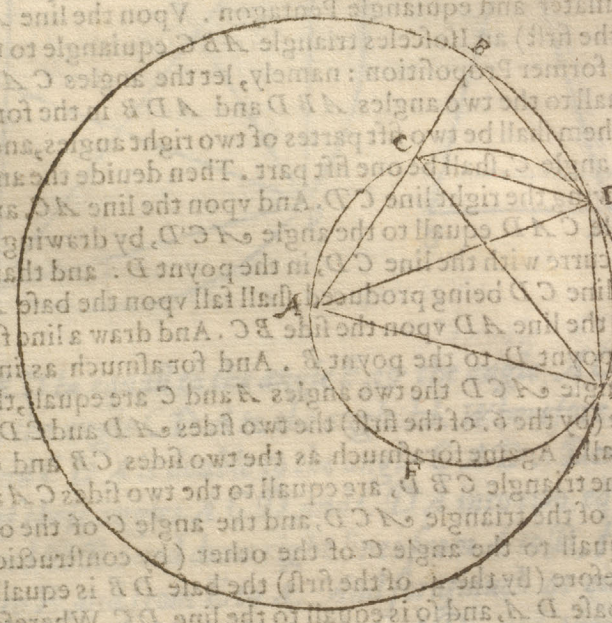
Wherfore the line AC alſo is equall vnto the lyne CD. Wherfore alſo (by the 5. of the firſt) the angle CDA is equall vnto the angle DAC. Wherfore the angles CDA and DAC are double to the angle CAD. But the angle BCD is equall vnto the angles CDA and DAC. Wherfore the angle BCD is double to the angle CAD. But the angle BCD is equall to either of theſe angles BDA & DBA. Wherfore either of theſe angles BDA & DBA is double to the angle DAB. Wherfore there is made a triangle of two equall ſids ABD hauing either of ỹ angles at the baſe DB, double to the angle remayning: which was required to be done.

Certaine additions of Campane.

Here *Campane* addeth, that the two circles *ACD* and *BDE*, do cut the one the other: and that the circle *ACD* cutteth of from the circle *BDE* an arke equall to the arke *BD*, and that the circle *BDE* cutteth of from the circle *ACD* an arke equall to the arke *DC*.

The firſt part demonſtrated.

The firſt part is manifeſt. For if the leſſe do not cut the greater but touch it as in the poynt *D*. Then (by the 11. of the third) the centre of either of them ſhall be in one line, namely, in the line *AD*: for that in it is the centre of the greater circle, and in the ſelfe ſame is the poynt of the touch. Wherefore (by the 31. of the third) the angle *ACD* is a right angle. And therfore (by the 13. of the firſt) the angle *DCB* is a right angle: and ſo ſhall the angle *ABD* be a right angle (for it is equall to the angle *DCB*, as it hath bene proued) which (by the 32. of the firſt) is impoſſible. Wherefore they ſhall cut the one the other as in the poyntes *D* & *E*. Now I ſay, that the arke *ED* of the greater circle, is equall to the arke *DB*: and that the arke *ED* of the leſſe circle, is equall to the arke *DC*.



The ſecond part demonſtrated.

Draw theſe right lines *EA*, *EC*, & *ED*. Now the (by the 27. of the third) the foure angles *DEC*, *CEA*, *DAC*, & *ADC*, are equall: for that the arkes *CA* and *CD* are equall (by the 28. of the ſame). Wherfore the whole angle *AED* is double to the angle *BAD*, & therfore is equall to either of the angles *ABD* and *ADB*. And for aſmuch as the angle *AED* is equall to the angle *ADE* (by the 5. of the firſt) for that the lines *AD* and *AE* are drawen from the centre, therefore the two angles at the pointes *E* and *D* of the triangle *AED*, ſhall be equall to the two angles at the poyntes *D* and *B* of the triangle *ADB*: and therefore the angle remayning of the one at the poynt *A*, ſhall be equall to the angle remayning of the other at the ſame point *A* (by the 32. of the firſt). Wherfore (by the 26. of the third) the arke *ED* of the greater circle, is equal to the arke *DB*: And by the ſame the arke *ED* of the leſſe circle, is equall to the arke *DC*: which was required to be proued.

Here *Pelitarius* noteth, that in euery ſuch Iſoſceles triangle as in thys place is the triangle ABD (namely, eyther of whoſe angle at the baſe, is double to the angle at the toppe) the angle at the toppe, as in thys example, the angle at the poynt A is one third part of a right angle, and moreouer one fift part of a third of a right angle: that is, two fift partes of one right angle: and to be briefe, one fift part of

two right angles. And either of the angles at the baſe, is two fift partes of two right angles, or foure fift partes of one right angle. Which ſhall manifeſtly appeare, if we deuide two right angles into fiue partes. For then in thys kinde of triangle, the angle at the toppe ſhall be one fift part, and eyther of the two angles at the baſe ſhall be two fift partes.

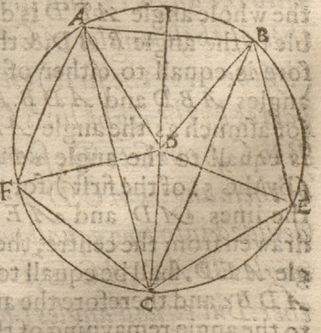
Thys alſo is to be noted, that the line *AC* is the ſide of an equilater Pentagon to be inſcribed in the circle *ACD*. For by the latter conſtruction it is manifeſt, that the three arkes *AC*, *CD*, and *DE*, of the leſſe circle, are equall. And foraſmuch as by the ſame it is manifeſt that the two lines *AD* and *AE* are equall, the arke alſo *AE* ſhall be equall to the arke *AD* (by the 20. of the third). Wherefore their halfes alſo are equall. If therefore the arke *AE* be (by the 30. of the third) deuided into two equall partes, the whole circũference *ACDEA* ſhall be deuided into fiue equall arkes. And foraſmuch as the lines ſubtending the ſayd equall arkes are (by the 29. of the ſame) equall, therefore euery one of the ſayd ſides ſhall be the ſide of an equilater Pentagon: which was required to be proued. And the ſame line *AC* ſhall be the ſide of an equilater ten angled figure to be inſcribed in the circle *BDE*: the demonſtration wherof I omitte, for that it is demonſtrated by Propoſitions following.

A Propoſition added by Petarilius.

¶ *A Propoſition added by Pelitarius.*

Vpon a right line geuen being finite, to deſcribe an equilater and equiangle Pentagon figure.

Suppoſe that the right line geuen be *AB*, vpon which it is required to deſcribe an equilater and equiangle Pentagon. Vpon the line *AB* deſcribe (by the 23. and 32. of the firſt) an Iſoſceles triangle *ABC* equiangle to the Iſoſceles triangle deſcribed by the former Propoſition: namely, let the angles *CAB* and *CBA*, at the baſe *AB*, be equall to the two angles *ABD* and *ADB* in the former conſtruction: ſo that eyther of them ſhall be two fift partes of two right angles, and the angle at the toppe, namely, the angle *C*, ſhall be one fift part. Then deuide the angle *C* into two equall partes by drawing the right line *CD*. And vpon the line *AC*, and vnto the poynt *A*, deſcribe the angle *CAD* equall to the angle *ACD*, by drawing the line *AD*, which line *AD* let concurre with the line *CD*, in the poynt *D*: and that within the triangle *ABC*, for the line *CD* being produced, ſhall fall vpon the baſe *AB*, and the line *AD* vpon the ſide *BC*. And draw a line from the poynt *D* to the poynt *B*. And for aſmuch as in the triangle *ACD* the two angles *A* and *C* are equall, therfore (by the 6. of the firſt) the two ſides *AD* and *CD* are equall. Againe foraſmuch as the two ſides *CB* and *CD* of the triangle *CBD*, are equall to the two ſides *CA* and *CD* of the triangle *ACD*, and the angle *C* of the one, is equall to the angle *C* of the other (by conſtruction) therefore (by the 4. of the firſt) the baſe *DB* is equall to the baſe *DA*, and ſo is equall to the line *DC*. Wherefore (by the 9. of the third) the poynt *D* ſhall be the centre of the circle deſcribed about the triangle *ABC*. Deſcribe the ſayd circle and let it be *ABECF*. Now then the angle *ADB* is double to the angle *ACD* (by the 20. of the third). Wherefore the angle *ADB* maketh two fift partes of two right angles, that is, one fift part of foure right angles. And for aſmuch as the ſpace about the centre *D* is equall to foure right angles, then if the ſayd ſpace be deuided into fiue angles equall to the angle *ADB*, namely, into fiue fift partes, by drawing the right lines *DE* & *DF*, which with the lines *DA*, *DB*, and *DC*, will cauſe the ſayd ſpace to be deuided into fiue equall partes, and if alſo there be drawen theſe right lines *AF*, *FC*, *CE*, and *EB*: there ſhall be deſcribed a rectiline Pentagon figure *ABECF*, which ſhall be equilater, by therule of a circle and of a circumference, and helpe of the 4. Propoſition of the firſt:



first: and shall also be equiangle (by the 4. and 5. of the same). For the fiue angles A,B,E,C,F, are deuided ech into ten equall partes: which was required to be done.

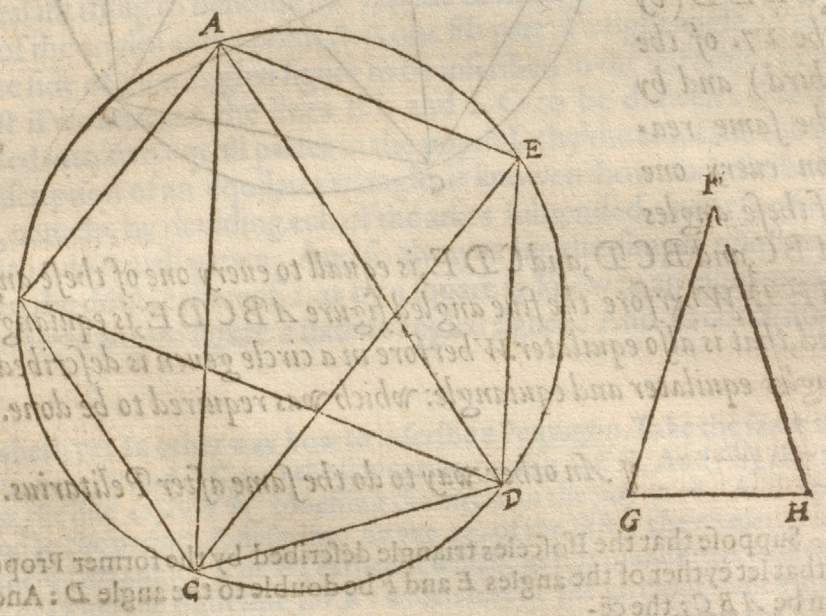
If we consider well thys demonstration of Pelitarius, it will not be hard for vs, vpon a right line geuen to describe the rest of the figures whose inscriptions hereafter followe. *Note.*

The 11. Probleme. The 11. Proposition.

In a circle geuen to describe a pentagon figure æquilater and equiangle.

Construction.

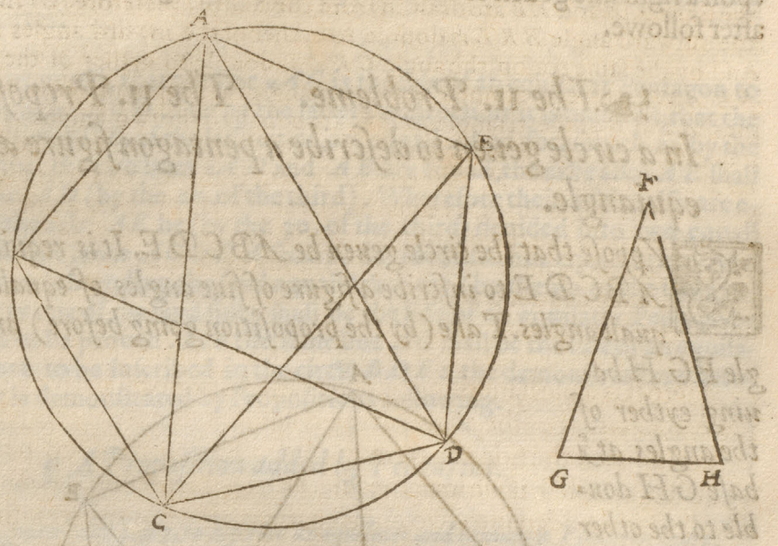
Vppose that the circle geuen be A B C D E. It is required in the circle A B C D E to inscribe a figure of fiue angles of equall sides and of equall angles. Take (by the proposition going before) an Isosceles triangle F G H hauing eyther of the angles at ẏ base G H double to the other angle, namely, vnto the angle F. And (by the 2. of ẏ fourth) in the circle A B C D E inscribe a triangle A C D equiangle vnto the triangle F G H. So that let the angle C A D be equall to the angle F, & the angle A C D vnto the angle G, and likewise the angle C D A to the angle H. Wherefore eyther of these angles A C D, and C D A is double to the angle C A D. Deuide (by the 9. of the first) either of these angles A C D, and C D A into two equall partes by the right lines C E and D B: and draw right lines from A to B, from B to C, from C to D, from D to E, and from E to A. And forasmuch as either of these angles A C D, and C D A is double to the angle C A D: and they are deuided into two equall partes by the right lines C E and D B, therfore the fiue angles D A C, A C E, E C D, C D B, and B D A are equall the one to the other. But equall angles (by the 26. of the third) subtend equall circumferences. Wherfore the fiue circumferences A B, B C, C D, D E, and E A are equall the one to ẏ other. And (by ẏ 29. of ẏ same) vnto equall circumferences are subtẽded equal right lines: wherfore ẏ fiue right lines A B, B C, C D, D E, & E A are equal ẏ one to the other. Wherfore ẏ figure A B C D E hauing fiue angles is equilater.



Demonstration.

Now also I say that it is equiangle. For forasmuch as the circumference A B

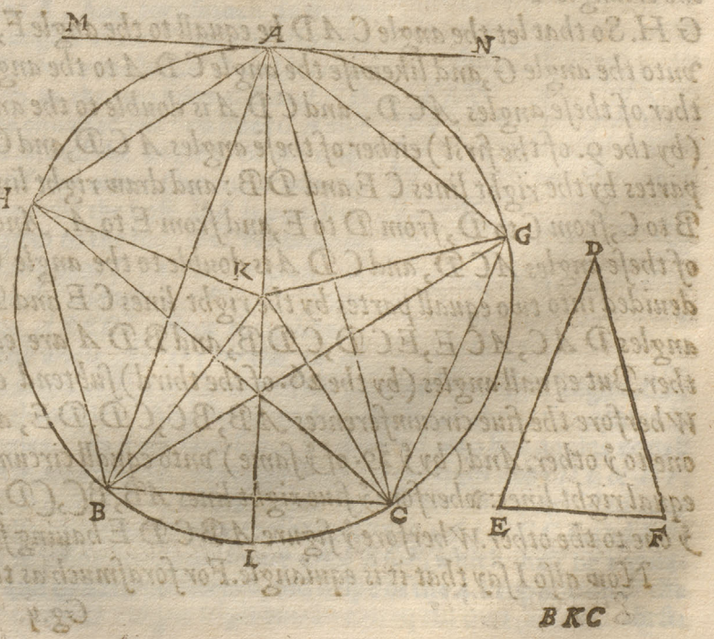
is equall to the circumference DE, put the circumference BCD common vnto them both. Wherfore the whole circumference $ABCD$ is equall to the whole circumference $EDCB$: and vpon the circumference $ABCD$ consisteth the angle AED, and vppon the circumference $EDCB$, consisteth the angle BAE. Wherefore the angle BAE is equall to ỹ angle AED (by the 27. of the third) and by the same reason euery one of these angles ABC, and BCD, and CDE, is equall to euery one of these angles BAE and AED. Wherfore the fiue angled figure $ABCDE$, is equiangle, and it is proued, that is also equilater. Wherfore in a circle geuen is described a figure of fiue angles equilater and equiangle: which was required to be done.



¶ An other way to do the same after Pelitarius.

An other way to do the same after Pelitarius.

Suppose that the Isosceles triangle described by the former Proposition, be DEF, so that let eyther of the angles E and F be double to the angle D: And let the circle geuen be ABC: the cẽtre wherof let be K. And vpon the centre K describe the angle BKC equall to one of the angles E or F of the triangle DEF. And draw a right line from B to C. Then I say, that the line BC is the side of the Pentagon figure to be inscribed in the circle ABC. Deuide the angle BKC into two equall partes by drawing the diameter AKL. And draw these right lines BA & AC. Now thẽ it is manifest (by the 20. of the third) that the angle



BKC

BKC is double to the whole angle *BAC*. Wherefore the whole angle *BAC* is equall to the angle *D*, vnto which angle, the angle *BKC* also is double. And forasmuch as in the triangle *ABK* the two angles *A* and *B* are (by the 5. of the first) equall (for the lines *KA* and *KB* are drawen from the centre) therefore (by the 32. of the same) the outward angle *BKL* is double to eyther of the inward angles *KAB* and *KBA*. And by the same reason, the angle *CKL* is double to eyther of the angles *KAC* and *KCA*. Wherfore forasmuch as the two angles at the poynt *K* are equall, the two angles *A* and *B* of the triangle *ABK*, are equall to the two angles *A* and *C* of the triangle *ACK*, the one to the other: and therefore (by the 26. of the first) the two bases *AB* and *AC* are equall. Wherefore *ABC* is an Isosceles triangle. And forasmuch as the whole angle *BAC* is equall to the angle *D*, the two angles remayning *ABC* and *ACB*, shall (by the 32. of the first) be equall to the two angles remayning *E* & *F*. Wherefore the triangle *ABC* is equiangle to the triangle *DEF*. And now you may procede in the demonstration as you did in the former, imagining first the lines *BG* and *CH* to be drawen.

Here it is a pleasant thing to beholde the varietie of triangles: for in the triangle A B C either of the angles at the point A is one fift part of a right angle. Wherby is produced the side of a ten angled figure to be inscribed in the selfsame circle: Which is manifest if we imagine the lines B L and L C to be drawen. For the arke B C is deuided into two equall partes in the poynt L (by the 26. of the third). So then by the inscription of an equilater triangle, is knowen how to inscribe an Hexagon figure, namely, by deuiding ech of the arkes subtended vnder the sides of the triangle into two equall partes. And so alwayes by the simple number of the side, is knowen the double thereof: as by a square is knowen an eight angled figure: and by an eight angled figure a sixtene angled figure. And so continually in the rest.

An other way also after Pelitarius.

Pelitarius teacheth yet an other way how to inscribe a Pentagon. Take the same circle that was before, namely, *ABC*, and the same triangle also *DEF*. And (by the 17. of the third) draw the line *MAN* touching the circle in the poynt *A*. And vpon the line *AM* and to the poynt *A*, describe (by the 23. of the first) the angle *MAB* equall to one of these two angles *E* or *F* (eyther of which, as it is manifest, is lesse then a right angle) by drawing the right line *AB*: which let cut the circumference in the poynt *B*. Agayne, vpon the line *AN* and to the poynt in it *A*, describe the angle *NAC* equall to the angle *MAB*, by drawing the right line *AC*: which let cut the circumference in the poynt *C*. And draw a right line from *B* to *C*. Then I say, that *BC* is the side of a Pentagon figure to be inscribed in the circle *ABC*. Which is manifest, if we deuide the arke *AB* into two equall partes in the poynt *H*, and draw these right lines *AH* and *BH*, and if also we deuide the arke *AC* into two equall partes in the poynt *G*, and draw these right lines *AG* and *CG*. For taking the quadrangle figure *ABCG*, it is manifest (by the 32. of the third) that the angle *ABC* is equall to the alternate angle *NAC*, and therfore is equall to the angle *E*. Likewise taking the quadrangle figure *ACBH*, the angle *ACB* shall be equall to the alternate angle *MAB*, and therefore is equall to the angle *F*. Wherefore (by the 32. of the first) as before, the triangle *ABC* is equiangle to the triangle *DEF*: And now may you procede in the demonstration as you did in the former.

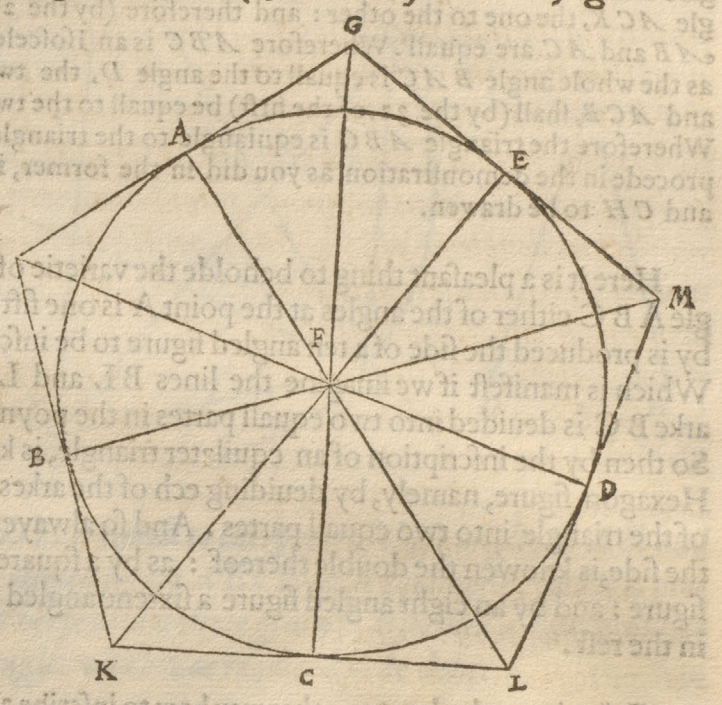
The 12. Probleme. The 12. Proposition.

About a circle geuen, to describe an equilater and æquiangle pentagon.

Construction.

SVppose that the circle geuen be A B C D E. It is required about the circle A B C D E to describe a figure of fiue angles consisting of equal sides and of equall angles. Take the pointes of the angles of a fiue angled figure described (by the 11. of the fourth) so that by the proposition goyng before, let ỹ circumferẽces A B, B C, C D, D E, and E A be equall the one to the other. And by the pointes A, B, C, D, E, draw (by the 17. of the third) right lines touching the circle, and let the same be G H, H K, K L, L M, and M G. And (by the 1. of the third) take the centre of the circle & let the same be F. And drawe right lines from F to B, from F to K, from F to C, from F to L, and from F to D. And forasmuch as the right lyne K L, toucheth the circle A B C D E in the pointe C, and from the centre F vnto the point C where the touche is, is drawen a right line F C: therefore (by the 18. of the third) F C is a perpendicular line vnto K L. Wherfore either of the angles which are at the point C is a right angle, and by the same reason the angles which are at the pointes D and B are right angles. And forasmuch as the angle F C K is a right angle, therfore the square which is made of F K is (by the 47. of the first) equall to the squares which are made of F C and C K. And by the same reason also the square which is made of F K is equall to the squares which are made of F B and B K. Wherefore the squares which are made of F C and C K are equall to the squares which are made of F B and B K, of which the square which is made of F C is equall to the square which is made of F B. Wherfore the square which is made of C K is equall to the square which is made of B K. Wherfore the line B K is equall vnto the line C K. And forasmuch as F B is equall vnto F C, and F K is common to them both, therfore these two B F and F K are equall to these two C F & F K. And the base B K is equall vnto the base C K. Wherfore (by the 8. of the first) the angle B F K is equall vnto the angle K F C: and the angle B K F to the angle F K C. Wherefore the angle B F C is double to the angle K F C. And the angle B K C is double to the angle F K C. And by the same reason the angle C F D is double to the angle C F L, and the angle D L C is double to the angle F L C. And forasmuch as the circumference B C is equall vnto the circumference C D,

Demonstration.

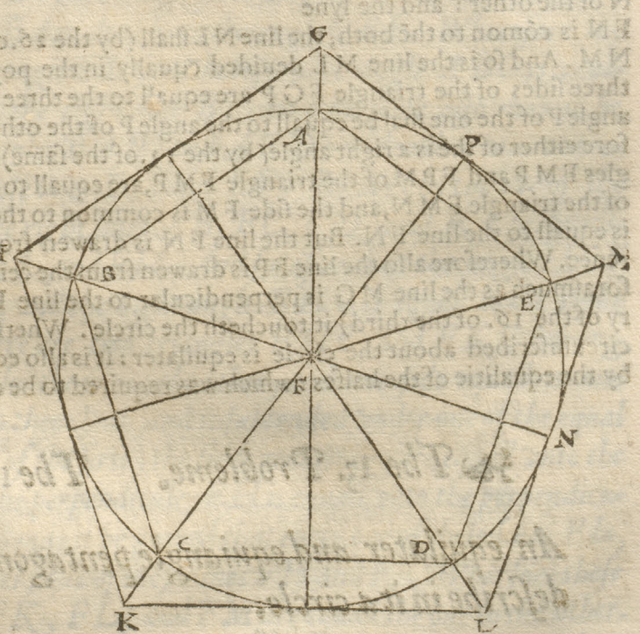


CD, therfore (by the 27. of the third) the angle BFC is equall to the angle CFD. And the angle BFC is double to the angle KFC, and the angle DFC is double to the angle LFC, wherfore the angle KFC is equall vnto the angle LFC. Now then there are two triangles FKC, and FLC, hauing two angles equall to two angles, and one side equall to one side, namely, FC, which is common to them both. Wherfore (by the 26. of the third, the other sides remayning are equall vnto the sides remayning, and the angle remayning vnto the angle remayning. Wherfore the right line KC is equall to the right line CL, and the angle FKC to the angle FLC. And forasmuch as KC is equall to CL: therefore KL is double to KC, and by the same reason also may it be proued that HK is double to BK. And forasmuch as it is proued that BK is equall vnto KC, and KL is double to KC, and HK double to BK, therfore HK is equall vnto KL. In like sort may we proue ý euery one of these lines HG, GM, & ML is equall vnto either of these lines HK and KL. Wherefore the fiue angled figure GHKLM is of equall sides. I say also that it is of equall angles. For forasmuch as the angle FKC is equall vnto the angle FLC, and it is proued that the angle HKL is double to the angle FKC, and the angle KLM is double to the angle FLC, therefore the angle HKL is equall to the angle KLM. In like sort may it be proued that euery one of these angles KHG, HGM, and GML, is equall to either of these angles HKL, and KLM. Wherefore the fiue angles GHK, HKL, KLM, LMG, and MGH are equall the one to the other. Wherfore the fiue angled figure GHKLM is equiangle, and it is also proued that it is equilater, and it is described about the circle ABCDE: which was required to be done.

¶ *An other way to do the same after Pelitarius, by parallel lines.*

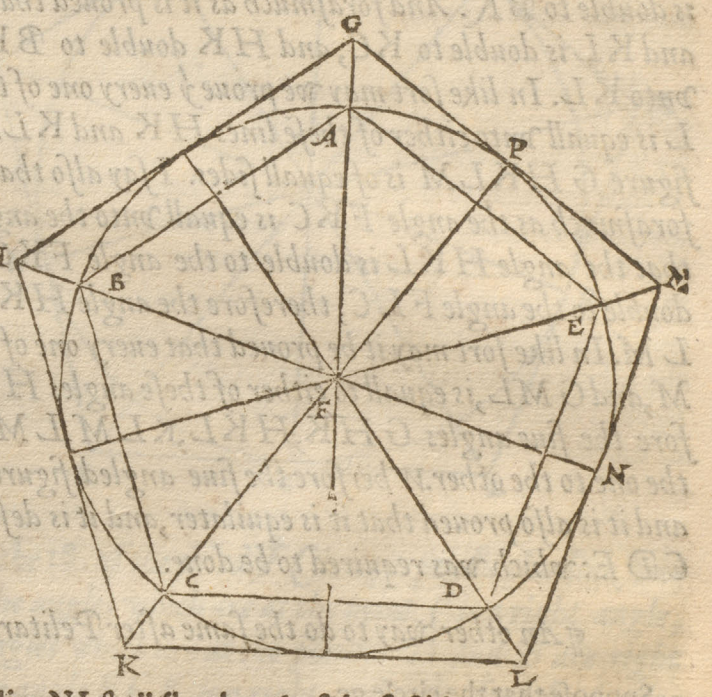
An other way to do the same after Pelitarius.

Suppose that the circle geuen be ABC, whose cẽtre let be the poynt F: and in it (by the former Proposition) inscribe an equilater and equiangle Pentagon ABCDE: by whose fiue angles drawe from the centre beyond the circũference, fiue lynes, FG, FH, FK, FL, and FM. And it is manifest, that the fiue angles at the cẽtre F, are equall, when as the fiue sides of the triangles within are equall, and also their bases. It is manifest also, that the fiue angles of the Pentagon which are at the circumference, are deuided into ten equall angles (by the 4. of the first): Now then betwene the two lines FG and FH, draw the



line G H parallel to the ſide A B, and touching the circle A B C (which is done by a Propoſition added by *Pelitarius* after the 17. of the third). And ſo likewyſe draw theſe lines H K, K L, and L M, parallel to ech of theſe ſides B C, C D, and D E, and touching the circle. And for aſmuch as the lines F G and F H fall vpon the two parallel lines A B and G H, the two angles F G H, & F H G, are equall to the two angles F A B and F B A, the one to the other (by the 29. of the firſt). Wherefore (by the ſixt of the ſame) the two lines F G and F H are equall. And by the ſame reaſon, the two angles F H K & F K H, are equall to the two angles F G H and F H G the one to the other: and the line F K is equall to the line F H, and therefore is equall to the line F G. And foraſmuch as the angles at the poynt F are equall, therefore (by the 4. of the firſt) the baſe H K is equall to the baſe G H. In like ſort may we proue, that the three lines F K, F L, and F M, are equall to the two lines F G and F H. And alſo that the two baſes K L and L M, are equall to the two baſes G H and H K: and that the angles which they make with the lines F K, F L, and F M, are equall the one to the other. Now then draw the fift line M G: which ſhall be equall to the foure former lines (by the 4. of the firſt) for that as we haue proued, the two lines F G & F M, are equall, & the angle G F M is equall to euery one of the angles at the poynt F. Thys line alſo M G toucheth the circle. For vnto the point where the line L M toucheth the circle whiche let be N, drawe the lyne F N. And it is manifeſt (by the 18. of the third) that either of the angles at the poynt N, is a right angle. Wherefore for aſmuch as the angle L of the triangle F L N, is equall to the angle M of the triangle F M N, & the angle N of the one, is equall to the angle N of the other: and the lyne F N is cõmon to thẽ both, the line N L ſhall (by the 26. of the firſt) be equall to the line N M. And ſo is the line M L deuided equally in the poynt N. And foraſmuch as the three ſides of the triangle F G P are equall to the three ſides of the triangle F M P, the angle P of the one ſhal be equall to the angle P of the other (by the 8. of the firſt). Wherfore either of thẽ is a right angle (by the 13. of the ſame). And foraſmuch as the two angles F M P and F P M of the triangle F M P, are equall to the two angles F M N & F N M of the triangle F M N, and the ſide F M is common to them both, therefore the line F P is equall to the line F N. But the line F N is drawen from the centre to the circumference. Wherefore alſo the line F P is drawen from the centre to the circumference. And foraſmuch as the line M G is perpendicular to the line F P, therefore (by the Corollary of the 16. of the third) it toucheth the circle. Wherfore the Pentagon G H K L M circumſcribed about the circle is equilater: it is alſo equiangle, as it is eaſie to proue by the equalitie of the halfes: which was required to be done.

Demonſtration.



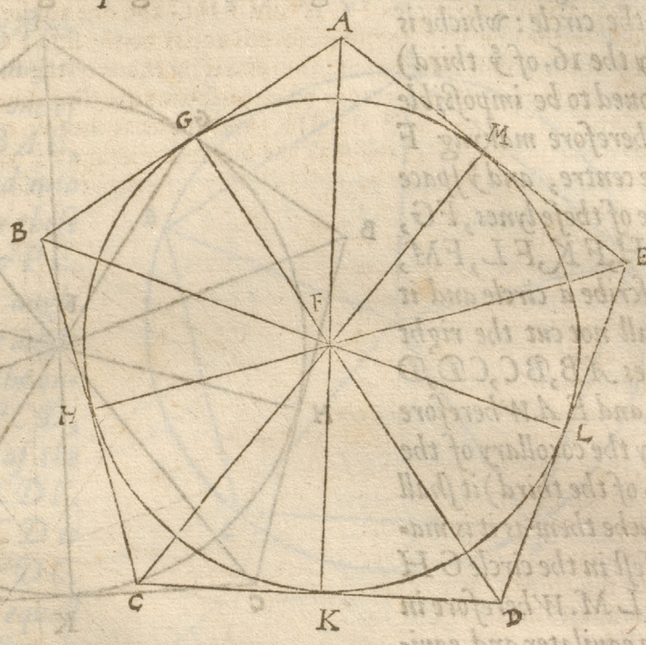
The 13. Probleme. The 13. Propoſition.

An equilater and equiangle pentagon figure beyng geuen, to deſcribe in it a circle.

Suppoſe

SVppose ẏ ẏ equilater & equiangle Pẽtagõ figure geuẽ be A B C D E. It is required in the said fiue angled figure A B C D E to describe a circle. Deuide (by the the 9. of the first) either of these angles B C D, and C D E into two equall partes, by these right lines C F and F D, and from the point F where these right lines C F and D F meete. Draw these right lines F B, F A, & F E. And forasmuch as B C is equal vnto C D and C F is common to them both. Wherefore these two lines B C and C F are equal to these two lines D C and C F: and the angle B C F is equall to the angle D C F. Wherfore (by the 4. of the first) the base B F is equall to the base D F, and the triangle B C F is equall to the triangle D C F, and the angles remayning are equall vnto the angles remaynyng the one to the other, vnder which are subtended equall sides. Wherefore the angle C B F is equall to the angle C D F. And forasmuch as the angle C D E is double to the angle C D F. But the angle C D E is equall to the angle A B C, and the angle C D F is equall to the angle C B F. Wherefore the angle C B A is double to the angle C B F. Wherefore the angle A B F is equall to the angle F B C. Wherefore the angle A B C is deuided into two equall partes by the right line B F. In lyke sorte also may it be proued, that either of these angles B A E, and A E D are deuided into two equall partes by either of these lines F A and E F. Drawe (by the 12. of the first) from the pointe F to the right lines A B, B C, C D, and E A perpendicular lines F G, F H, F K, F L, and F M. And forasmuch as the angle H C F is equall to the angle K C F, and the right angle F H C is equall to the right angle F K C: now then there are two triangles F H C, and F K C hauyng two angles equall to two angles the one to the other, and one side equall to one side: for F C is common vnto them both, and is subtended vnder one of the equal angles. Wherfore (by the 26. of the first) the sides remayning are equall vnto the sides remayning. Wherfore the perpendiculer F H is equall vnto the perpendiculer F K. And in like sort also may it be proued that euery one of these lines F L, F M, and F G is equall to euery one of these lines F H and F K. Wherfore these fiue right lines F G, F H, F K, F L, and F M are equall the one to the other.

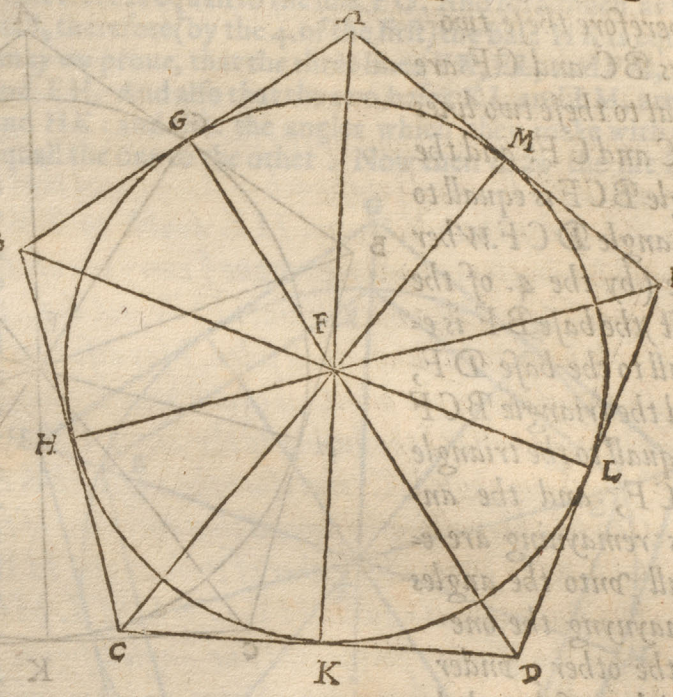
Demonstration.



Wher-

Wherfore making the centre F and the ſpace F G, or F H, or F K, or F L, or F M. Deſcribe a circle, and it will paſſe by the poyntes G, H, K, L, M. And ſhall touche the right lines A B, B C, C D, D E, and E A (by the correllary of the 16. of the third) for the angles which are at the pointes G, H, K, L, M, are right angles. For if it do not touche them but cut, then from the ende of the diameter of the circle, ſhall be drawen a right line making two right angles, and falling with in the circle: whiche is (by the 16. of ỹ third) proued to be impoſsible Wherefore makyng F the centre, and ỹ ſpace one of theſe lynes, F G, F H, F K, F L, F M, deſcribe a circle and it ſhall not cut the right lines A B, B C, C D, D E, and E A. Wherefore (by the corollary of the 16. of the third) it ſhall touche them as it is manifeſt in the circle G H K L M. Wherefore in the equilater and equiangle pentagon figure geuen, is deſcribed a circle: which was required to be done.

Demonſtration leading to an abſurditie.



A Corollary. By this propoſition and the former, it is manifeſt that perpendicular lines drawn from the middle poyntes of the ſides of an equilater and equiangle Pentagon figure, and produced, ſhall paſſe by the centre of the circle, in which the ſayd Pentagon figure is inſcribed, and ſhall alſo deuide the oppoſite angles equally, as here, A K is one right line, and deuideth the ſide C D and the angle A equally. And ſo of the reſt which is thus proued. The ſpace about the centre F is equall to foure right angles, which are deuided into ten equall angles by ten right lines metyng together in the point F. Wherfore the fiue angles A F M, M F E, E F L, L F D, and D F K are equall to two right angles. Wherfore (by the 14. of the firſt) the lynes A F and F K make one right lyne. The lyke proofe alſo will ſerue touching the reſt of the lynes. And this is alwayes true in all equilater figures which conſiſt of vneuen ſides.

The 14. Probleme. The 14. Propoſition.

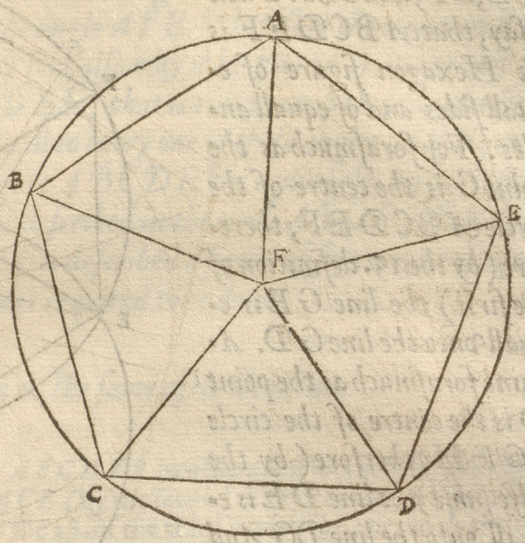
About a pentagon or figure of fiue angles geuen beyng equilater and equiangle, to deſcribe a circle.

Sup-

SVppose that ỹ Pentagon or figure of fiue angles geuen, being of equall sides and of equall angles, be ABCDE. It is required about the sayd Pentagon ABCDE, to describe a circle. Deuide (by the 9. of the first) eyther of these angles BCD, & CDE into two equall partes by either of these right lines CF, and DF. And from the poynt F where those right lines meete, draw vnto the pointes B, A, E, these right lines FB, FA, FE. And in like sort (by the Proposition going before) may it be proued, that euery one of these angles CBA, BAE, and AED is deuided into two equall partes, by these right lines FB, FA, & FE. And for asmuch as the angle BCD is equall to the angle CDE, & the halfe of the angle BCD is the angle FCD, and likewise the halfe of the angle CDE is ỹ angle CDF. Wherfore the angle FCD is equall to the angle FDC. Wherefore ỹ side FC is equall to the side FD. In like sort also may it be proued, that euery one of these lines FB, FA, and FE is equall to euery one of these lines FC and FD. Wherefore these fiue right lines FA, FB, FC, FD, and FE are equall the one to the other. Wherefore making the centre F, and the space FA, or FB, or FC, or FD, or FE. Describe a circle and it will passe by the pointes A, B, C, D, E, and shall be described about the fiue angled figure ABCDE which is equiangle and equilater. Let this circle be described and let the same be ABCDE. Wherefore about the Pentagon geuen being both equiangle and equilater, is described a circle: which was required to be done.

Construction.

Demonstration.



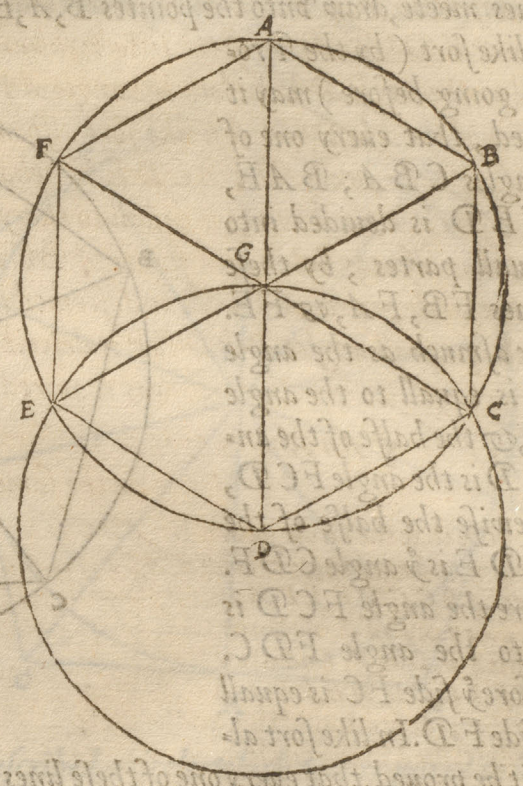
The 15. Probleme. The 15. Proposition.

In a circle geuen to describe an hexagon or figure of sixe angles equilater and equiangle.

SVppose that the circle geuen be ABCDEF. It is required in the circle geuen ABCDE to describe a figure of sixe angles of equall sides and of equall angles. Draw the diameter of the circle ABCDEF, and

let

Conſtruction. let the ſame be *AD*. And (by the firſt of the third) take the centre of the circle and let the ſame be *G*. And making the centre *D*, and the ſpace *DG*, deſcribe (by the third petition) a circle *CGEH*: and drawing right lines from *E* to *G*, and from *G* to *C*, extend them to the pointes *B* and *F* of the circumference of the circle geuen. And draw theſe right lines *AB*, *BC*, *CD*, *DE*, *EF*, and *FA*. Then I ſay, that *ABCDEF* is an Hexagon figure of equall ſides and of equall angles.



Demonſtration. For foraſmuch as the point *G* is the centre of the circle *ABCDEF*, therefore (by the 15. definition of the firſt) the line *GE* is equall vnto the line *GD*. Againe foraſmuch as the point *D* is the centre of the circle *CGEH* (therfore (by the ſelfe ſame) the line *DE* is equall vnto the line *DG*. And it is proued that the line *GE* is equall vnto the lyne *GD*. Wherfore the line *GE* is equall vnto the line *ED* (by the firſt common ſentence) wherfore the triangle *EGD* is equilater, and his three angles, namely, *EGD*, *GDE*, *DEG*, are equall the one to the other. And foraſmuch as (by the 5. of the firſt) in triangles of two equal ſides commonly called Iſoſceles, the angles at the baſe are equall the one to the other, and the three angles of a triangle are (by the 32. of the firſt) equall vnto two right angles, therfore the angle *EGD* is the third part of two right angles. And in lyke ſorte may it be proued, that the angle *DGC* is the third parte of two right angles. And foraſmuch as the right line *CG* ſtanding vpon the right line *EB* doth (by the 13. of the firſt) make the two ſide angles *EGC*, and *CGB* equall to two right angles, therfore the angle remayning *CGB* is the third part of two right angles. Wherfore the angles *EGD*, *DGC*, and *CGB* are equall the one to the other. Wherfore their hed angles, that is, *BGA*, *AGF*, and *FGE* are (by the 15. of the firſt) equall to theſe angles *EGD*, *DGC*, and *CGB*. Wherfore theſe ſixe angles *EGD*, *DGC*, *CGB*, *BGA*, *AGF*, and *FGE* are equall the one to the other. But equall angles conſiſt vpon equall circumferences (by the 26. of the third. Therfore theſe ſixe circumferences *AB*, *BC*, *CD*, *DE*, *EF*, and *FA* are equall the one to the other. But vnder equall circumferences are ſubten-

ded equall right lines (by the 29. of the same.) Wherefore these sixe right lynes AB,BC,CD,DE,EF,and FA are equall the one to the other. Wherfore the Hexagon ABCDEF is equilater. I say also that it is equiangle. For forasmuch as the circumference AF is equall vnto the circumference ED adde the circumference ABCD common to them both. Wherefore the whole circumference FABCD is equall to the whole circumference EDCBA. And vppon the circumference FABCD consisteth the angle FED: and vppon the circumference EDCBA consisteth the angle AFE. Wherefore the angle AFE is equall to the angle DEF. In like sort also may it be proued that the rest of the angles of the Hexagon ABCDEF, that is euery one of these angles FAB,ABC,BCD,and CDE is equall to euery one of these angles AFE, and FED. Wherfore the Hexagon figure ABCDEF is equiangle, and it is proued that it is also equilater, and it is described in the circle ABCDEF. Wherfore in the circle geuen ABCDEF is described a figure of sixe angles of equal sides and of equall angles: Which was required to be done.

¶ *An other way to do the same after Orontius.*

An other way to do the same after Orõtius.

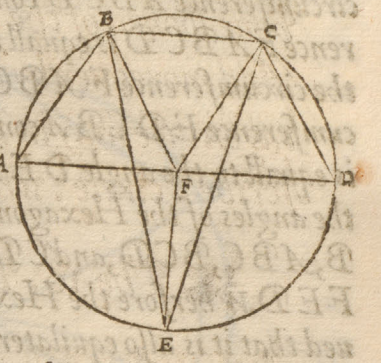
Suppose that the circle geuen be *ABCDEF* in which first let there be described an equilater and equiangle triangle *ACE* (by the second of thys booke). Wherefore the arkes *ABC,CDE,EFA* are (by the 28. of the third) equall the one to the other. Deuide euery one of those three arkes into two equall partes (by the 30. of the same) in the pointes *B,D,& F*. And draw these right lines *AB,BC,CD,DE,EF*, and *FA*. Now then by the 3. definition of this booke there shall be described in the circle geuẽ an Hexagon figure *ABCDEF*, which must nedes be equilater: for that euery one of the arkes which subtend the sides thereof are equall the one to the other. I say also that it is equiangle. For euery angle of the Hexagon figure is set vpon equall arkes, namely, vpon foure such partes of the circũference wherof the whole circumference cõtayneth sixe. Wherfore the angles of the Hexagon figure are equall the one to the other (by the 27. of the third). Wherefore in the circle geuen *ABCDEF* is inscribed an equilater and equiangle Hexagon figure: which was required to be done.

¶ *An other way to do the same after Pelitarius.*

An other way after Pelitarius.

Suppose that the circle in which is to be inscribed and equilater & equiangle Hexagon figure be *ABCDE*, whose centre let be *F*. And from the centre draw the semidiameter *FA*. And from the poynt *A* apply (by the first of thys booke) the line *AB* equall to the semidiameter. Which I say is the side of an equilater and equiangle Hexagon figure to be inscribed in the circle *ABCDE*. Draw a right line from *F* to *B*. And for asmuch as the line *AB* is equall to the line *FA*, & it is also equall to the line *FB*, ther-

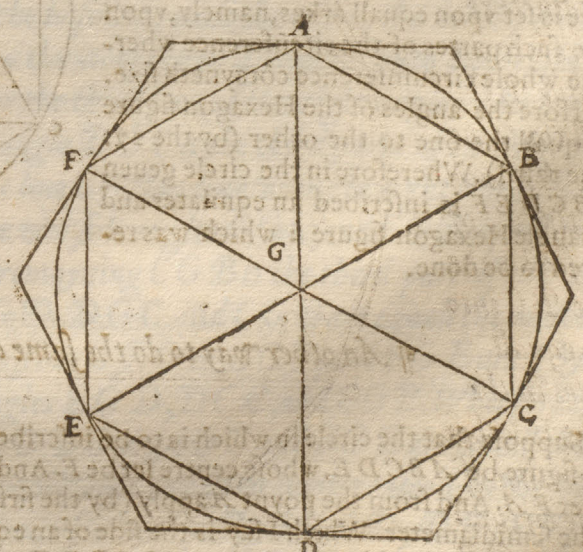
fore the triangle *AFB* is equilater, and by the 5. of the firſt equiangle. Now then vpon the centre *F* deſcribe the angle *BFC* equall to the angle *AFB*, or to the angle *FBA* (which is all one) by draving the right line *FC*. And draw a line from *B* to *C*. And for aſmuch as the angle *AFI* is the third part of two right angles by the 5. and 32. of the firſt, the angle *BFC* alſo ſhall be the third part of two right angles. Wherefore either of the two angles renaining *FBC* and *FCB*, for aſmuch as they are equall by the 5. of the firſt, ſhall be two third partes of two right angles (by the 32. of the ſame). Or (by the 4. of the frſt) foraſmuch as the angle *BFC* is equall to the angle *IBA*, and the two ſides *FB* and *FC* are equall to the two ſides *AB* and *BF*, the baſe *BC* ſhall be equall to the baſe *BF*, and therefore is equall to the line *FC*. Wherefore the triangle *FBC* is equilater and equiangle. Laſtly make the angle *CFD* equall to eyther of the angles atthe poynt *F*, by drawing the line *FD*. And draw a line from *C* to *D*. Now then by the former reaſon the triangle *FCD* ſhall be equilater and equiangle. And for aſmuch as the three angles at the poynt *F* are equall to two right angles (for ech of them is the third part of two right angles) therefore (by the 14. of the firſt) *AD* is one right line: and for that cauſe is the dameter of the circle. Wherefore if the other ſemicircle *AFD* be deuided into ſo many equall partes as the ſemicircle *ABCD* is deuided into, it ſhall comprehend ſo many equall lines ſubtended vnto it. Wherefore the line *AB* is the ſide of an equilater Hexagon figure to be inſcribed in the circle: which Hexagon figure alſo ſhall be equiangle: For the halfe of the whole angle *B* is equall to the halfe of the whole angle *C*: which was required to be done.



Now then if we draw fromthe centre *F* a perpendicular line vnto *AD*, which let be *FE*, and draw alſo theſe right lines *BE* and *CE*: there ſhall be deſcribed a triangle *BEC* whoſe angle *E* which is at the toppe ſhall be the 6. part of two right angles by the 20. of the third. For the angle *BFC* is double vnto it. And either of the two angles at the baſe, namely, the angles *IBC*, and *ECB* is dupla ſeſquialter to the angle *E*: that is, eyther of them contayneth the angle *E* twiſe and halfe the angle *E*. And by this reaſon was found out the ſide of an Hexagon figure.

Correlary.

Hereby it is manifeſt, that the ſide of an Hexagon figure deſcribed in a circle is equall to a right line drawen frō the cētre of the ſaid circle vnto the circumference. And if by the pointes A, B, C, D, E, F, *be*



drawen

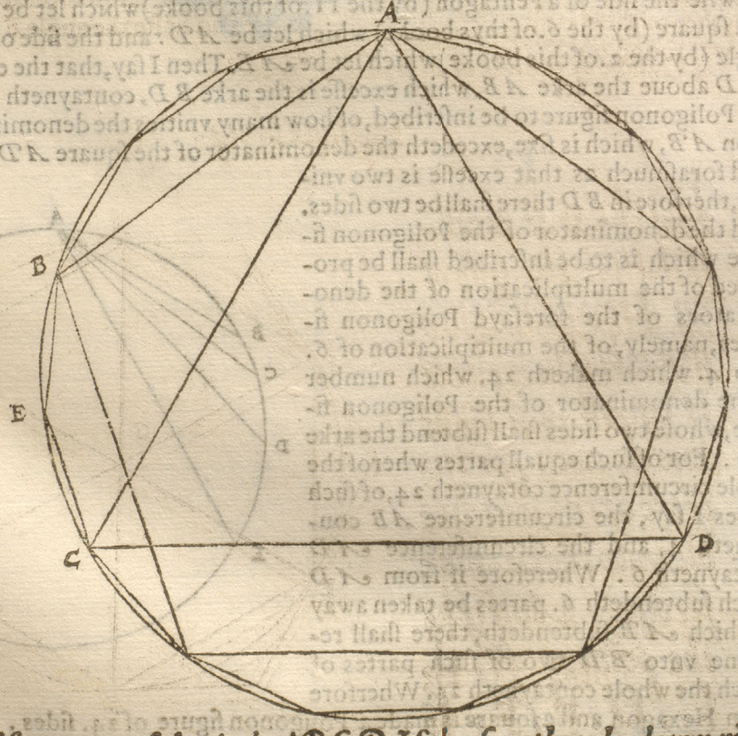
drawen right lines touching the circle, then ſhall there be deſcribed about the circle an Hexagon figure equilater and equiangle, which may be demonſtrated by that which hath bene ſpoken of the deſcribing of a Pentagon about a circle. And moreouer by thoſe thinges which haue ben ſpokē of Pentagons we may in a Hexagon geuen either deſcribe or circumſcribe a circle: which was required to be done.

The 16. Probleme. The 16. Propoſition.

In a circle geuen to deſcribe a quindecagon or figure of fiftene angles, equilater and equiangle.

SVppoſe that the circle geuen be A B C D. It is required in the circle A B C D to deſcribe a figure of fiftene angles conſiſting of equall ſides and of equall angles. Deſcribe in the circle A B C D the ſides of an equilater triangle, and let the ſame be A C, and in ỹ arke A C deſcribe the ſide of an equilater pentagon and let the ſame be A B. Now then of ſuch equall partes wherof the whole circle A B C D containeth fiftene, of ſuch partes I ſay, the circumference A B C being the third parte of the circle ſhall containe fiue. And the circumferenee A B being the fift part of a circle ſhall contain thre, wherefore the reſidue B C ſhall containe two. Deuide (by the 30. of the firſt) the arke B C into two equall partes in the point E. Wherfore either of theſe circumferences B E & E C is the fiftene part of the circle A B C D. If therfore there be drawn right

Conſtruction.



Demonſtration.

lines from B to E, and from E to C, and then beginning at the point B or at the point C there be applied into the circle ABCD right lines equall vnto EB or EC, and so continuing till ye come to the point C if you began at B, or to ỹ point B if you began at C, and there shall be described in the circle ABCD a figure of fiftene angles equilater and equiangle: which was required to be done. And in like sort as in a pentagon, if by the pointes where the circle is deuided, be drawen right lines touching the circle in the said pointes, there shall be described about ỹ circle a figure of fiftene angles equilater & equiangle. And in like sort by ỹ selfe same obseruations that were in Pentagons, we may in a figure of fiftene angles geuen being equilater and equiangle either inscribe, or circumscribe a circle.

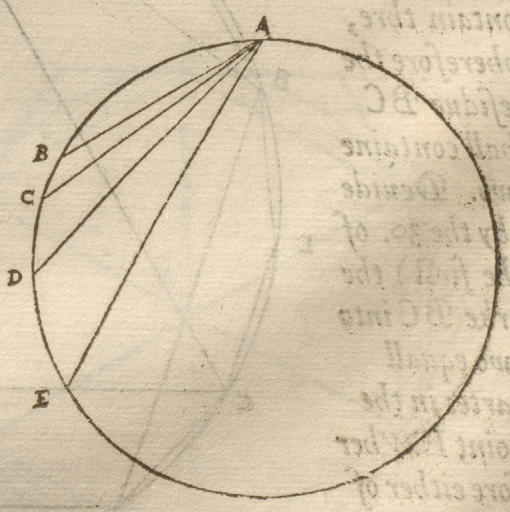
An addition of Flussates.

¶ An addition of Flussates to finde out infinite figures of many angles.

*If into a circle from one poynt be applyed the sides of two * Poligonon figures: the excesse of the greater arke aboue the lesse, shall comprehend an arke contayning so many sides of the Poligonon figure to be inscribed by how many vnities the denomination of the Poligonon figure of the lesse side excedeth the denomination of the Poligonon figure of the greater side: and the number of the sides of the Poligonon figure to be inscribed is produced of the multiplication of the denominations of the foresayd Poligonon figures the one into the other.*

** A Poligonon figure is a figure consisting of many sides.*

As for example. Suppose that into the circle ABE be applyed the side of an equilater and equiangle Hexagon figure (by the 15. of thys booke) which let be AB: and likewise the side of a Pentagon (by the 11. of this booke) which let be AC: and the side of a square (by the 6. of thys booke) which let be AD: and the side of an equilater triangle (by the 2. of this booke) which let be AE. Then I say, that the excesse of the arke AD aboue the arke AB, which excesse is the arke BD, contayneth so many sides of the Poligonon figure to be inscribed, of how many vnities the denominator of the Hexagon AB, which is sixe, excedeth the denominator of the square AD, which is foure. And forasmuch as that excesse is two vnities, therfore in BD there shall be two sides. And the denominator of the Poligonon figure which is to be inscribed shall be produced of the multiplication of the denominators of the foresayd Poligonon figures, namely, of the multiplication of 6. into 4. which maketh 24. which number is the denominator of the Poligonon figure, whose two sides shall subtend the arke BD. For of such equall partes wherof the whole circumference cõtayneth 24, of such partes I say, the circumference AB contayneth 4, and the circumference AD contayneth 6. Wherefore if from AD which subtendeth 6. partes be taken away 4. which AB subtendeth, there shall remayne vnto BD two of such partes of which the whole contayneth 24. Wherfore of an Hexagon and a square is made a Poligonon figure of 24. sides. Likewyse of the Hexagon AB and of the Pentagon AC shall be made a Poligonon figure of 30.

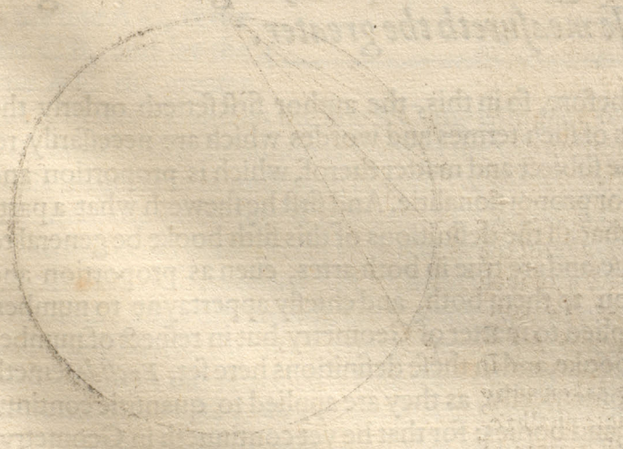


sides,

ſides, one of whoſe ſides ſhall ſubtend the arke BC. For the denomination of AB which is 6. excedeth the denomination of AC which is 5. onely by vnitie. So alſo for aſmuch as the denomination of AB which is 6. excedeth the denomination of AE which is 3. by 3. therefore the arke BE ſhall contayne 3. ſides of a Poligonon figure of 18. ſides. And obſeruing thys ſelfe ſame methode and order, a man may finde out infinite ſides of a Poligonon figure.

(.˙.)

The end of the fourth booke of Euclides Elementes.



¶ The fifth booke of Euclides Elementes.

The argument of this fift booke.

THIS FIFTH BOOKE of *Euclide* is of very great commoditie and vſe in all Geometry, and much diligence ought to be beſtowed therin. It ought of all other to be throughly and moſt perfectly and readily knowne. For nothyng in the bookes followyng can be vnderſtand without it: the knowledge of them all depende of it. And not onely they and other writinges of Geometry, but all other Sciences alſo and artes: as *Muſike*, *Aſtronomy*, *Perſpectiue*, *Arithmetique*, the arte of accomptes and reckoning, with other ſuch like. This booke therefore is as it were a chiefe treaſure, and a peculiar iuell much to be accompted of. It entreateth of proportion and Analogie, or proportionalitie, which pertayneth not onely vnto lines, figures, and bodies in Geometry: but alſo vnto ſoundes & voyces, of which Muſike entreateth, as witneſſeth *Boetius* and others which write of Muſike. Alſo the whole arte of Aſtronomy teacheth to meaſure proportions of tymes and mouinges. *Archimides* and *Iordan* with other, writing of waightes, affirme, that there is proportion betwene waight and waight, and alſo betwene place & place. Ye ſee therefore how large is the vſe of this fift booke. Wherfore the definitions alſo thereof are common, although hereof *Euclide* they be accommodate and applied onely to Geometry. The firſt author of this booke was as it is affirmed of many, one *Eudoxus* who was *Platos* ſcholer, but it was afterward framed and put in order by *Euclide*.

The firſt author of this booke Eudoxus.

Definitions.

The firſt definition.

A parte is a leſſe magnitude in reſpect of a greater magnitude, when the leſſe meaſureth the greater.

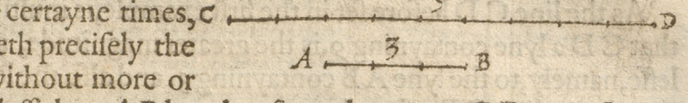
As in the other bookes before, ſo in this, the author firſt ſetteth orderly the definitions and declarations of ſuch termes and wordes which are neceſſarily required to the entreatie of the ſubiect and matter therof, which is proportion and compariſon of proportions or proportionalitie. And firſt he ſheweth what a parte is. Here is to be conſidered that all the definitions of this fifth booke be general to Geometry and Arithmetique, and are true in both artes, euen as proportion and proportionalitie are common to them both, and chiefly appertayne to number, neither can they aptly be applied to matter of Geometry, but in reſpect of number and by number. Yet in this booke, and in theſe definitions here ſet, *Euclide* ſemeth to ſpeake of them onely Geometrically, as they are applied to quantitie continuall, as to lines, ſuperficieces, and bodies: for that he yet continueth in Geometry. I wil notwithſtanding for facilitie and farther helpe of the reader, declare thē both by example in number, and alſo in lynes.

A part taken two maner of wayes.

The firſt way.

For the clearer vnderſtandyng of a parte, it is to be noted, that a part is taken in the Mathematicall Sciences two maner of wayes. One way a part is a leſſe quantitie

titie in respect of a greater, whether it measure the greater or no. The second way, a part is onely that lesse quantitie in respect of the greater, which measureth the greater. A lesse quantitie is sayd to measure or number a greater quantitie, when it, beyng oftentymes taken, maketh precisely the greater quantitie without more or lesse, or beyng as oftentymes taken from the greater as it may, there remayneth nothyng. As suppose the line A B to contayne 3. and the lyne C D to contayne 9. thē doth the line A B measure the line C D: for that if it be takē certayne times, namely, 3. tymes, it maketh precisely the lyne C D, that is 9. without more or lesse. Agayne if the sayd lesse lyne A B be taken from the greater C D, as often as it may be, namely, 3. tymes, there shall remayne nothing of the greater. So the nūber 3. is sayde to measure 12. for that beyng taken certayne tymes, namely, foure tymes, it maketh iust 12. the greater quantitie: and also beyng taken from 12. as often as it may, namely, 4. tymes, there shall remayne nothyng. And in this meaning and signification doth *Euclide* vndoubtedly here in this definea part: saying, that it is a lesse magnitude in comparison of a greater, when the lesse measureth the greater. As the lyne A B before set, contayning 3. is a lesse quantitie in comparison of the lyne C D which containeth 9. and also measureth it. For it beyng certayne tymes taken, namely, 3. tymes, precisely maketh it, or taken from it as often as it may, there remayneth nothyng. Wherfore by this definition the lyne A B is a part of the lyne C D. Likewise in numbers, the number 5. is a part of the number 15. for it is a lesse number or quantitie compared to the greater, and also it measureth the greater: for beyng taken certayne tymes, namely, 3. tymes, it maketh 15. And this kynde of part is called commonly *pars metiens* or *mensurans*, that is, a measuryng part: some call it *pars multiplicatiua:* and of the barbarous it is called *pars aliquota*, that is an aliquote part. And this kynde of parte is commonly vsed in Arithmetique.



The second way.

How a lesse quantity is sayd to measure a greater.

In what significatiō Euclide here taketh a part.

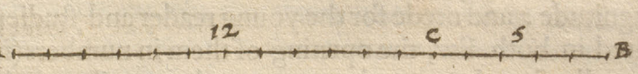
Pars metiens or mensurans.

Pars multiplicatiua.

Pars aliquota.

This kynde of part cōmonly vsed in Arithmetique.

The other kinde of a part, is any lesse quantitie in comparison of a greater, whether it be in number or magnitude, and whether it measure orno. As suppose the line A B to be 17. and let it be deuided into two partes in the poynt C, namely, into the line A C, & the line C B, and let the lyne A C the greater part containe 12. and let the line B C the lesse part contayne 5. Now eyther of these lines by this definition is a part of the whole lyne A B. For eyther of them is a lesse magnitude or quātity in cōparisō of the whole lyne A B: but neither of thē measureth the whole line A B: for the lesse lyne C B contayning 5. taken as oftē as ye list, will neuer make precisely A B which contayneth 17. If ye take it 3. tymes it maketh only 15. so lacketh it 2. of 17. which is to litle. If ye take it 4. times, so maketh it 20. thē are there thre to much, so it neuer maketh precisely 17. but either to much or to litle. Likewise the other part A C measureth not the whole lyne A B: for takē once, it maketh but 12. which is lesse then 17. and taken twise, it maketh 22. which are more then 17. by 5. So it neuer precisely maketh by takyng therof the whole A B, but either more or lesse. And this kynde of part they commonly call *pars constituens*, or *componens*: Becausse that it with some other part or partes, maketh the whole. As the lyne C B together with the line A C maketh the whole lyne A B. Of the barbarous it is called *pars aliquanta*. In this signification it is taken in *Barlaam* in the beginnyng of his booke, in the definition of a part, when he saith: *Euery*



Pars constituens, or componens.

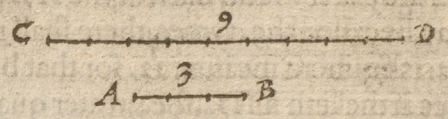
Pars aliquanta.

leſſe number compared to a greater, is ſayd to be a part of the greater, whether the leſſe meaſure the greater, or meaſure it not.

The ſecond definition.

Multiplex is a greater magnitude in reſpect of the leſſe, when the leſſe meaſureth the greater.

As the line C D before ſet in the firſt example, is multiplex to the lyne A B. For that C D a lyne contayning 9. is the greater magnitude, and is compared to the leſſe, namely, to the lyne A B contayning 3. and alſo the leſſe lyne A B meaſureth the greater line C D: for taken 3. tymes, it maketh it, as was aboue ſayde. So in numbers 12. is multiplex to 3: for 12 is the greater number, and is compared to the leſſe, namely, to 3. which 3. alſo meaſureth it: for 3 taken 4 tymes maketh 12. By this worde *multiplex* which is a terme proper to Arithmetike and number, it is eaſy to conſider that there can be no exact knowledge of proportion and proportionalitie, and ſo of this fifth booke wyth all the other bookes followyng, without the ayde and knowledge of numbers.



Numbers very neceſſary for the vnderſtanding of this booke and the other bookes following.

The third definition.

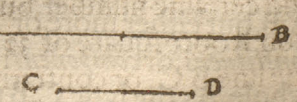
Proportion is a certaine reſpecte of two magnitudes of one kinde, according to quantitie.

Euclide as in the firſt definition, ſo in this & the other following, and likewiſe in all his Propoſitions of this booke, mentioneth onely magnitudes, and geueth his examples and demonſtrations of lines: for that hetherto in the 4. bookes before he hath entreated of lines & figures, and ſo cõtinueth in his ſixth booke following after this, comparing figure to figure, and ſides of figures to ſides of figures, without mention of number at all. Notwithſtanding as it is ſayd they are generall to all kinde of quantitie, both diſcrete and continuall, namely, number and magnitude: and neede for the young reader and ſtudient in theſe artes to be declared in both. For, the opening of them in numbers (in which they are firſt and naturally founde) geueth a great and marueilous light to their declaration in magnitudes. *Proportion* (ſayth he) *is a certaine behauiour*, that is, *a certaine reſpect or compariſon of two quantities of one kinde*: as of one line to an other, and one figure generally to an other, and one number to an other, *as touching quantitie*, that is to ſay, that the quantitie compared, is to that wherunto it is compared, eyther equall, or greater, or leſſe then it. For after theſe three maners may thinges be compared the one to the other. But quantities of diuers kindes can not be compared together. A ſuperficies can not be compared to a line: nor number to a body: nor a body to a line or number: for that they are not of one kinde. For example of this definition, take two quantities, namely, two lines A B and C D, and compare the one to the other, namely, A B to C D according to ſome certaine reſpect of greatnes, or leſſenes, or equalitie, namely, in this example, let A B be greater then C D, & containe it twiſe. Now thys compariſon, relation, or reſpect of A B to C D, and generally of any one quantitie to any other, is called proportion. Likewiſe is it of

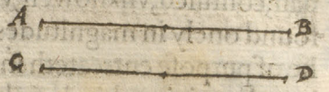
what proportion it.

Thinges compared together this wayes.

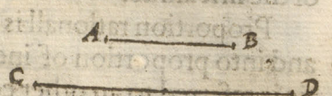
Example of this definition in magnitudes.



it, if A B be equall to C D, as in the second example: or if A B be lesse then C D, as in the third example. The like is it also in numbers comparing 5. to 5. equall to equall, or 6. to 3. the greater to the lesse, or 4. to 8. the lesse to the greater. So to the accomplishing of any Proportion there are required two quantities, and also a comparing or respect of the one to the other. The quantities compared together are commonly called the *Termes* of the Proportion. And in this booke of *Euclide*, and also in other writers of *Geometrie*, the first *Terme*, namely, that which is compared is called the antecedent, whether it be equall, greater, or lesse then the other: And the second *Terme*, namely, that wherunto the comparison is made, is called the consequent. As in the former example, the line A B compared to the line C D is antecedent: and the line C D is consequent: And contrariwise if the line C D be compared to the line A B, then is the line C D antecedent, and the line A B consequent. Albeit in Arithmeticke *Boetius* and others call the terme compared *Dux*, and the terme to whom the cõparison is made they call *Comes*. This booke hath bene accompted of all men one of the hardest and most intricate of all *Euclides* bookes. And proportion is a generall knowledge to all learninges, chiefly to the Mathematicalls. Wherefore it shall be very necessary some litle briefe instruction and induction to be here added in the beginning hereof: of the knowledge and nature of proportions and what they are, and of how many kindes: which thinges are here of *Euclide* supposed to be before knowen, and therefore maketh no mention so distinctly of them.



Example ther of in numbers.



In proportions two quantities required, which are called termes.

The i. terme called the antecedent.

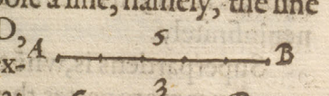
The ij. terme called the consequent.

Dux.

Comes.

This booke counted the hardest of all Euclides bookes.

Ye must vnderstand that there are of proportions two generall kindes, the one is called rationall, certaine, and knowen, and the other irrationall, vncertaine and vnknowen. Such magnitudes or quantities, which may be expressed by numbre, are called *rationall* magnitudes or quantities. As suppose a line, namely, the line A B to containe 5. inches, & compare it to the line C D, containing 3. inches: these quantities ye see may be expressed by numbers, namely, by these numbers 5. and 3: and therefore are *rationall*, and haue the same proportion, that number hath to number, namely, that the number 5. hath to the number 3.: and therefore the proportion of the one to the other, is a rationall, certaine, and knowen proportion. And generally when soeuer one number is compared to an other, or two lines or other magnitudes, both which may be expressed by number, the proportion betwene them is euer rationall, and onely the proportion of such quantities is rationall. So that in Arethmeticke all proportions are rationall, for that therein euer one number is compared to an other.

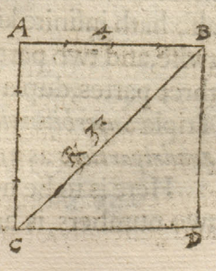


Proportion of two kindes, raticnal and irrational.

Proportion rational.

In Arithmetique all proportions are rational.

There are certaine lines magnitudes or quantities which cã not be named and expressed by number, and therefore commonly are called Surd lines or magnitudes. As suppose the square A B C D to containe 16, then the side or roote therof, namely, the line A B containeth 4, and the diameter of the same square, namely, the line B C shall be ℞ 32, which is a surd number, and can not be expressed by any determinate and certaine number, but onely by this maner of circumlocution Roote square of 32. Now if ye compare the line A B to the line B C, or contrariwise the line B C to the line A B, for that one of them is a surde quantitie, neither can ech of them be expressed by number (and therefore can not haue that proportion



Proportion irrational.

proportion that number hath to number) the proportion betwene them is irrationall, confuſed, vnknowen, vncertaine, and ſurd. And this kinde of proportion is found onely in magnitudes, as in lines and figures (and not in numbers) of which he of purpoſe entreateth in his tenth booke. Wherfore I wil here omit to ſpeake of it, and remit it to his due place. And ſomewhat will I now ſay for the elucidation of the firſt kinde.

Rational proportion deuided into two kindes.

Proportion rationall is deuided into two kindes, into proportion of equalitie, and into proportion of inequalitie. Proportion of equalitie is, when one quantitie is referred to an other equall vnto it ſelfe: as if ye compare 5 to 5, or 7 to 7, & ſo of other. And this proportion hath great vſe in the rule of *Coſſe*. For in it all the rules of equations tende to none other ende but to finde out and bring forth a nũber equall to the number ſuppoſed, which is to put the proportion of equalitie.

Proportion of equalitie.

Proportion of inequalitie.

Proportion of inequalitie is, when one vnequall quantity is compared to an other, as the greater to the leſſe, as 8. to 4: or 9. to 3: or the leſſe to the greater as 4. to 8: or 3. to 9.

Proportion of the greater to the leſſe.

Proportion of the greater to the leſſe hath fiue kindes, namely, *Multiplex*, *Superparticular*, *Superpartiens*, *Multiplex ſuperperticular*, and *Multiplex ſuperpartiens*.

Multiplex.

Multiplex, is when the antecedent containeth in it ſelfe the conſequent certayne times without more or leſſe: as twice, thrice, foure tymes, and ſo farther. And this proportion hath vnder it infinite kindes. For if the antecedent contayne the conſequent iuſtly twiſe, it is called dupla proportion, as 4 to 2. If thrice tripla, as 9. to 3. If 4. tymes quadrupla as 12. to 3. If 5. tymes quintupla as 15. to 3. And ſo infinitely after the ſame maner.

Duple proportion. Triple, quadruple. Quintuple.

Superperticular is, whẽ the antecedẽt containeth the conſequent only once, & moreouer ſome one part therof, as an halfe, a third, or fourth, &c. This kinde alſo hath vnder it infinite kindes. For if the antecedent containe the conſequent once and an halfe, therof it is called *Seſquialtera*, as 6. to 4: if once and a third part *Seſquitertia*, as 4. to 3: if once and a fourth part *Seſquiquarta*, as 5. to 4. And ſo in like maner infinitely.

Superperticular. Seſquialtera. Seſquitertia. Seſquiquarta.

Superpartiens is, whẽ the antecedẽt cõtaineth the conſequent onely once, & moreouer more partes then one of the ſame, as two thirdes, three fourthes, foure fifthes and ſo forth. This alſo hath infinite kindes vnder it. For if the antecedent containe aboue the conſequent two partes, it is called *Superbipartiens*, as 7. to 5. If 3. partes *Supertripartiens* as 7. to 4. If 4. partes *Superquadripartiens*, as 9. to 5. If 5. partes *Superquintipartiens* as 11. to 6. And ſo forth infinitely.

Superpartiens. Superbipartiens. Supertripartiens. Superquadripartiens. Superquintipartiens.

Multiplex Superperticular is when the antecedent containeth the conſequent more then once, and moreouer onely one parte of the ſame. This kinde likewiſe hath infinite kindes vnder it. For if the antecedent containe the conſequent twiſe and halfe therof, it is called dupla *Seſquialtera*, as 5. to 2. If twiſe and a third *Dupla Seſquitertia* as 7. to 3. If thrice and an halfe *Tripla ſeſquialtera* as 7. to 2. If foure times and an halfe *Quadrupla Seſquialtera*, as 9. to 2. And ſo goyng on infinitely.

Multiplex ſuperperticular. Dupla Seſquialtera. Dupla ſeſquitertia. Tripla ſeſquiquartã.

Multiplex Superpartient, is when the antecedent contayneth the conſequent more then once, and alſo more partes then one of the conſequent. And this kinde alſo hath infinite kindes vnder it. For if the antecedent containe the conſequent twiſe, and two partes ouer, it is called dupla *Superbipartiens* as 8. to 3. If twice and three partes, dupla *Supertripartiens* as 11. to 4. If thrice and two partes, it is named Tripla *Superbipartiens* as 11. to 3. If three tymes and foure partes Tripla *Superquadripartiens* as 31. to 9. And ſo forth infinitely.

Multiplex ſuperpartient. Dupla ſuperbipartiens. Dupla ſupertripartiens. Tripla ſuperquadripartiens. Tripla ſuperquintipartiens.

Here is to be noted that the denomination of the proportion betwene any two numbers, is had by deuiding of the greater by the leſſe. For the quotient or

number

number produced of that diuision is euer the denomination of the proportion. Which in the first kinde of proportion, namely, multiplex, is euer a whole number, and in all other kindes of proportion it is a broken number.

How to know the denomination of any proportion.

As if ye will know the denomination of the proportion betwene 9 and 3. Deuide 9. by 3. so shall ye haue in the quotient 3. which is a whole number, and is the denomination of the proportion: and sheweth that the proportion betwene 9. & 3. is Tripla. So the proportion betwene 12. and 3. is quadrupla, for that 12. beyng deuided by 3. the quotient is 4. and so of others in the kinde of multiplex. And although in this kinde the quotient be euer a whole number, yet properly it is referred to vnitie, and so is represented in maner of a broken number as $\frac{3}{1}$ and $\frac{4}{1}$ for vnitie is the denominaton to a whole number.

Likewise the denomination of the proportion betwene 4 and 3 is $1\frac{1}{3}$ for that 4 deuided by 3. hath in the quotient $1\frac{1}{3}$ one and a third part, of which third part, it is called *sesquitercia*: so the proportion betwene 7 and 6. is $1\frac{1}{6}$ one and a sixt, of which sixt part it is called *sesquisexta*, and so of other of that kinde. Also betwene 7 and 5 the denomination of the proportion is $1\frac{2}{5}$ one and two fifthes, which denomination cõsisteth of two parts, namely, of the munerator and denominator of the quotient of 2. and 5: of which two fifthes it is called *superbipartiens quintas*: for 2 the numerator sheweth the denomination of the number of the partes, and 5. the denominator, sheweth the denominatiõ, what parts they are, & so of others. Also the denomination betwene 5 and 2. is $2\frac{1}{2}$ two and a halfe, which consisteth of a whole number and a broken, of 2. the whole number it is dupla, and of the halfe, it is called *sesquialtera*, so is the proportion *dupla sesquialtera*.

Agayne the denomination of the proportion betwene 11. and 3. is $3\frac{2}{3}$ three and two thirdes, consisting also of a whole number and a broken, of 3. the whole numbre it is called tripla, and of $\frac{2}{3}$ the broken number, it is called *Superbipartiens tertias*, so the proportion is *tripla superbipartiens tertias*. Thus much hetherto touching proportion of the greater quantitie to the lesse.

Proportion of the lesse in the greater.

Proportion of the lesse quantitie to the greater hath as many kindes, as that of the greater to the lesse, which kindes are in the same order: and haue also the selfe same names, but that to the names afore put ye must adde here this word *sub*. As comparing the greater to the lesse, it was called *multiplex*, *superparticular*, *superpartient*, *multiplex superparticular*, and *multiplex superpartient*, now comparing the lesse quantitie to the greater, it is called *submultiplex*, *subsuperparticular*, *subsuperpartient*, *submultiplex superparticular*, and *submultiplex superpartient*. And so in like maner to all the inferior kindes of all sortes of proportion ye shall adde that worde *sub*. The examples of the former serue also here, onely transposing the termes of the proportion making the antecedent consequent, and the consequent the antecedent. As 4. to 2. is dupla proportion: so 2. to 4. is subdupla. As 9. to 3. is tripla: so is 3. to 9. subtripla. And as 9. to 6. is sesquialtera, so 6. to 9. is subsesquialtera. As 7. to 5. is superbipartiens quintas, so is 5. to 7. subsuperbipartiens quintas. As 5. to 2. is dupla sesquialtera, so is 2. to 5. subdupla sesquialtera. And also as 8. to 3. is dupla superbipartiens tertias, so is 3. to 8. subdupla superbipartiens tertias. And so may ye procede infinitely in all others. Thus much thought I good in this place for the ease of the beginner to be added touching proportion.

Submultiplex.
Subsuperparticular.
Subsuperpertient. &c.

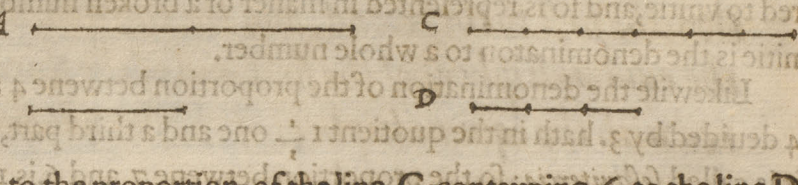
Pro-

The fourth definition.

Proportionalitie, is a similitude of proportions.

As in proportion are compared together two quantities, and proportion is nothing els but the respect and comparison of the one to the other, and these quantities are the termes of the proportion: so in proportionallitie are compared together two proportions. And proportionallitie is nothing els, but the respect & comparison of the one of them to the other. And these two proportions are the termes of this proportionallitie. He calleth it the similitude, that is, the likenes or idemptitie of proportions:

Example of this definition in magnitudes.



As if ye wil cõpare the proportion of the line A contaynyng 2. to the line B contayning 1, to the proportion of the line C contayning 6. to the line D contayning 3, either proportion is dupla. This likenes, idemptitie, or equallitie of proportion is called proportionallitie. So in number 9. to 3. and 21. to 7. either proportion is tripla. Where note that proportions compared together, are sayd to be like the one to the other: but magnitudes compared together, are said to be equall the one to the other.

Example thereof in numbers.

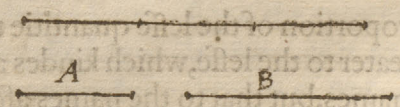
Note.

The fifth definition.

Those magnitudes are sayd to haue proportion the one to the other, which being multiplied may exceede the one the other.

Before he shewed and defined, what proportion was, now by this definition he declareth betwene what magnitudes proportion falleth, saying: That those quantities are said to haue proportion the one to the other, which being multiplyed, may excede the one the other. As for that the line A being multiplied by what soeuer multiplication or nũber, as taken twise, thrise, or foure, fiue, or more times, or once and halfe, or once and a third, & so of any other part, or partes, may excede and become greater then the line B: or contrariwise, then these two lines are said to haue proportion the one to the other. And so ye may see that betwene any two quãtities of one kinde, there is a proportion. For the one remayning vnmultiplied, & the other being certaine times multiplied, shall be greater then it. As 3. to 24. hath a proportion, for leauing 24. vnmultiplied, and multiplying 3. by 9, ye shall produce 27: which is greater then 24, and excedeth it. Here is to be noted, that *Euclide* in defining what quantities haue proportion, was compelled to vse multiplication, or els should not his definition be generall to either kinde of proportion: namely, to rationall and irrationall: to such proportion I say which may be expressed by number, and to such as cannot be expressed by any determinate number, but remaineth surd and innominable. In rationall quantities which haue one common measure, the excesse of the one aboue the other is knowen, and by it is knowen the proportion, which may be expressed by some determinate number. But in irrationall quantities which haue no cõmon measure, it is not so. For in them the excesse of the one to the other is euer vn-

An example of this definition in magnitudes.



Why Euclide in defining of Proportion vsed multiplication.

knowen,

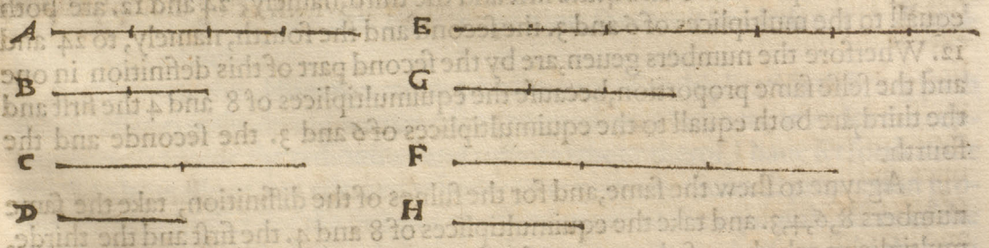
knowen,& therefore is ſurd,and innominable.As betwene the ſide of a ſquare and the diameter therof,there is vndoubtedly a proportion, for that the ſide certaine times multiplied may excede the diameter. Likewiſe betwene the diameter of a circle and the circumference therof there is certainlie,by this definition, a proportion,for that the diameter certaine times multiplied may excede the circumference of the circle: although neither of theſe proportions can be named & expreſſed by number. For this cauſe therefore vſed *Euclide* this maner of defining by multiplication.

Magnitudes are ſayd to be in one or the ſelfe ſame proportion,the firſt to the ſecond, and the third to the fourth, when the equimultiplices of the firſt and of the third beyng compared with the equimultiplices of the ſecond and of the fourth, according to any multiplication: either together exceede the one the other, or together are equall the one to the other, or together are leſſe the one then other.

The ſixth definition.

In the definition laſt going before, he ſhewed what magnitudes haue proportion the one to the other, & now this diffinition ſheweth what magnitudes are in one and the ſelfe ſame proportion,and how to know whether they be in one and the ſelf ſame proportion,or not.It is plaine that euery proportiō hath two termes, ſo that when ye compare proportion to proportion, ye muſt of neceſſitie haue 4. termes,that is,an antecedent and a conſequent,to either of the proportiōs.As ſuppoſe A,B,C,D, to be foure magnitudes, A the firſt,B the ſecond,C the third, and

An example of this definition in magnitudes.



D the fourth: now if ye take the equimultiplices of A and C the firſt & the third, that is,if ye multiply A and C by one and the ſelfe ſame number, as let the multiplex of A be E,and let the equimultiplex of C be F. Likewiſe alſo if ye take the equimultiplices of B and D, the ſecond and the fourth,that is if ye multiply them by any one number,whether it be by that number wherby ye multiplied A & C, or by any other number greater or leſſe, as let the multiplex of B be G, and the equimultiplex of D be H: now it the equimultiplices of A and C be both greatet hen the eqnimultiplices of B and D, that is if the multiplex of A be greater then the multiplex of B, and the multiplex of C be greater then the multiplex of D, or if they be both leſſe then they: or both equall to them, then are the magnitudes A,B and C,D in one and the ſelfe ſame proportion.

An example in numbers.

Likewiſe in numbers 8. to 6. hath a proportion,alſo 4. to 3. hath a proportion:

now to ſee whether they be in one and the ſelfe ſame proportion or not, ſet them in order as in the example here written, 8 the firſt, 6 the ſecond, 4 the third, and 3. the fourth. Now take the equimultiplices of 8 and 4. the firſt, and the third, that is, multiply them by one and the ſelfe ſame number, ſuppoſe it be by 3. ſo the triple of 8 is 24. & the triple of 4. is 12: likewiſe take the equimultiplices of 6 and 3. the ſecond and the fourth, multiplyeng them likewiſe by one and the ſelfe ſame number, ſuppoſe it be alſo by 3 as before ye did, the triple of 6 is 18. and the triple of 3 is 9. Now ye ſee that the triple of 8 the firſt, namely, 24. excedeth the triple of 6. the ſecond, namely, 18: likewiſe the triple of 4 the third number, namely, 12. excedeth the triple of 3. the fourth, namely, 9. Wherefore by the firſt part of this definition, the numbers 8 to 6. and 4 to 3. are in one and the ſelfe ſame proportiõ, becauſe that the equimultiplices of 8 and 4. the firſt & the third, do both exceede the equimultiplices of 6 and 3. the ſecond and the fourth.

First	24	8	Third	12 / 4
Second	6 / 18		Fourth	3 / 9

An other example in numbers.

Againe, take the ſame numbers and try the ſame after this maner. Take the equimultiplices of 8. and 4. the firſt and the third, multiplieng eche by 3. as before ye did, ſo ſhall ye haue 24 for the triple of 8. and 12 for the triple of 4. as ye had before. Then take the equimultiplices of 6 and 3. the ſecond and the fourth, multipliyng them by ſome one number, but not by 3 as before ye did: but by 4. ſo for the quadruple of 6 the ſecond number, ſhall ye haue 24. and for the quadruple of 3 the fourth number, ye ſhall haue 12. And now ye ſee that the equimultiplices of 8 and 4. the firſt and the third, namely, 24 and 12. are both equall to the multiplices of 6 and 3. the ſecond and the fourth, namely, to 24 and 12. Wherfore the numbers geuen, are by the ſecond part of this definition in one and the ſelfe ſame proportion, becauſe the equimultiplices of 8 and 4 the firſt and the third, are both equall to the equimultiplices of 6 and 3. the ſeconde and the fourth.

First	24	8	Third	12 / 4
Second	6 / 24		Fourth	3 / 12

An other example in numbers.

Agayne to ſhew the ſame, and for the fulnes of the diffinition, take the ſame numbers 8, 6, 4, 3. and take the equimultiplices of 8 and 4. the firſt and the thirde, multiplieng eche by 2. ſo haue ye 16 for the duple of 8, the firſt number, and 8 for the duple of 4 the third number: then take alſo the equimultiplices of 6 and 3, the ſecond and the fourth, multipliyng eche by 3. ſo haue ye 18 for the triple of 6 the ſecond, and 9 for the triple of 3. the fourth number. And now ye ſee that the equimultiplices of 8 and 4. the firſt and the third, namely, 16. and 8 are both leſſe then the equimultiplices of 6 and 3 the ſecond & the fourth namely 18 and 9. For 16 are leſſe then 18, and 8 are leſſe then 9. Wherefore by the third part of this diffinition, the numbers propoſed are in one and the ſelfe ſame proportion, for that the equimultiplices of 8 and 4 the firſt and the third are both leſſe then the equimultiplices of 4 and 3 the ſecond and the fourth.

First	16	8	Third	8 / 4
Second	6 / 18		Fourth	3 / 9

Further

Farther in this diffinition, this particle (according to any multiplication) is moſt diligently to be conſidered, which ſignifieth by any multiplication indifferently whatſoeuer. For whenſoeuer the quantities be in one and the ſelfe ſame proportion, then by any multiplication whatſoeuer, the equimultiplices of the firſt and the third, ſhall exceede the equimultiplices of the ſecond and the fourth, or ſhall be equall vnto them, or leſſe then them. Yet it may ſo happen by ſome one multiplication, that the equimultiplices of the firſt and the third, do exceede the equimultiplices of the ſecond and the fourth, and yet the quantities geuen ſhal not be in one and the ſelfe ſame proportion. As in this example here ſet, where the equimultiplices of 6 and 5, the firſt and the thirde, namely, 18. and 15. doo both exceede the equimultiplices of 4 and 3. the ſecond and the fourth, namely, 8 and 6. yet are not the numbers geuen in one and the ſelfe ſame proportion. For 6 hath not that proportion to 4. which 5. hath to 3. In this example 6 and 5 the firſt and the third were multiplied by 3. which made their equimultiplices 18 and 15. which exceede the equimultiplices of 4 and 3, the ſecond and the fourth beyng multiplied by 2. namely, 8 and 6: but if ye ſhall multiply 6 and 5 the firſt and the thirde by 2. ye ſhall produce 12 and 10 for their equimultiplices, and then if ye multiply 4 and 3. the ſecond and the fourth by 3. ſo ſhall ye produce for their equimultiplices 12 and 9. Now ye ſee that by this multiplication the equimultiplices of the firſt and the thirde doo not both exceede the equimultiplices of the ſecond and fourth: for 12 the multiplex of 6 doth not exceede 12 the multiplex of 4. and therfore the numbers or quantities are not in one and the ſelfe ſame proportion, for that it holdeth not in all multiplications whatſoeuer.

	18			15
First	6		Third	5
Second	4		Fourth	3
	8			6

	12			10
First	6		Third	5
Second	4		Fourth	3
	12			9

Note this particle according to any multiplication.

An example where the equimultiplices of the firſt and third exceede the equimultiplices of the ſecond and fourth, and yet the quantities geuen are not in one and the ſelfe ſame proportion.

And becauſe this diffinition requireth all maner of multiplicatiõs to bring forth the exceſſes, equalities, and wantes of the antecedents aboue, to, or vnder the conſequents, to auoide the tediouſnes and infinite labour therof, I haue ſet forth a rule much to be made of and eſtemed, wherby ye may in any rationall proportion produce equimultiplices of the firſt and the third equall to the equimultiplices of the ſecond and the fourth. The rule is this, take two numbers whatſoeuer in that proportion in which your quantities are, & by the number which is antecedent multiply the conſequents of your proportions, namely, the ſecond and the fourth: and by the number which is the conſequent multiply the antecedentes of your proportions, namely, the firſt and the third: then neceſſarily ſhalbe produced the equimultiplices of the firſt and the third equall to the equimultiplices of the ſecond & the fourth. As by example, take 6 to 2. and 3 to 1. which are in one & the ſelfe ſame proportiõ, & taking theſe two nũbers 9 & 3. which are in the ſame proportiõ, now by 9 the antecedent multiply the conſequẽts 2 & 1. and ſo ſhal ye haue 18 & 9 for the equimultiplices of the ſecond & the fourth, then by 3 the conſequent multiply the antedẽts 6 & 3, ſo ſhal ye haue 18 & 9 for the equimultiplices of the firſt & the third, which are equal to the former equimultiplices of the ſecõd & fourth. Wherof it foloweth that if ye multiply 18 & 9 the equimultiplices of the firſt and the third by any nũber greater thẽ 3. wherby they were now multiplied, they ſhal both euer exceede the equimultiplices of the ſecond & the fourth: & if ye multiply thẽ

A rule to produce equimultiplices of the firſt and third equall to the equimultiplices of the ſeconde and fourth.

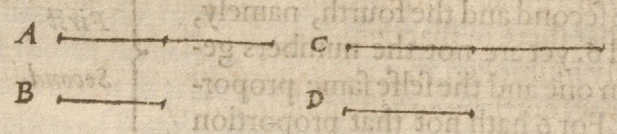
Example thereof.

by any number leſſe then 3. they ſhall euer both want of them. So that whatſoeuer multiplication it be, they ſhall euer both exceede, be equal, or want aboue, to, or from 18. and 9. the equimultiplices of the ſecond and fourth.

The ſeuenth definition

Magnitudes which are in one and the ſelfe ſame proportion, are called Proportionall.

As if the lyne A, haue the ſame proportion to the line B, that the lyne C hath to the lyne D, then are the ſaid foure magnitudes A, B, C, D, called proportionall. Alſo in numbers for that 9. to 3. hath that ſame proportiõ that 12 hath to 4: therefore theſe foure nũbers 9.3.12.4. are ſaid to be proportionall. Here is to be noted that this likenes or idemptitie of proportiõ which is called, as before was ſaid proportionalitie, is of two ſortes: the one is continual, the other is diſcontinuall. Continuall proportionalitie is, when the magnitudes ſet in lyke proportion, are ſo ioyned together, that the ſecond which is conſequent to the firſt, is antecedent to the third, and the fourth which is conſequent to the third, is antecedent to the fift, and ſo continually forth. So euery quantitie or terme in this proportionalitie, is both antecedent and conſequent (conſequent in reſpect of that which went before, & antecedent in reſpect of that which followeth) except the firſt, which is onely antecedent to that which followeth, and the laſt which is onely conſequent to that which went before. Take an example in theſe numbers, 16.8.4.2.1. In what proportion 16. is to 8, in the ſame is 8. to 4, in the ſame alſo is 4. to 2, and likewiſe 2. to 1. For they all are in duple proportion: 16. the firſt is antecedent to 8, and 8. is conſequent vnto it: and the ſelfe ſame 8. is antecedent to 4: which 4 beyng conſequent to 8. is antecedent to 2, which 2 likewiſe is conſequent to 4. and antecedent to 1: which becauſe he is the laſt, is onely conſequent, and antecedent to none, as 16. becauſe it was the firſt, was antecedent onely, and conſequent to none. Alſo in this proportionalitie all the magnitudes muſt of neceſſitie be of one kynde, by reaſon of the continuation of the proportions in this proportionalitie, becauſe there is no proportion betwene quantities of diuers kyndes. Diſcontinuall proportionalitie is, when the magnitudes which are ſet in lyke proportion, are not continually ſet, as before they were, hauyng one terme referred both to that which went before, and to that which foloweth, but haue their termes diſtinct and ſeuered aſonder: as the firſt is antecedent to the ſecond, ſo is the third antecedent to the fourth. Example in numbers, as 8 is to 4. ſo is 6. to 3. for either proportion is duple. Where ye ſee, how ech proportion hath hys owne antecedent and conſequent diſtinct from the antecedent and conſequent of the other, and no one number is antecedent and conſequent in diuers reſpectes. And by reaſon of the diſcontinuaunce of the proportions in this proportionalitie, the quantities compared, may be of diuers kyndes, becauſe the conſequent in the firſt proportion is not the antecedent in the ſecond proportion. So that ye may compare ſuperficies to ſuperficies, or body to body in the ſelfe ſame proportion that ye do lyne to lyne.



9 12
3 4

Proportionality of two ſortes, continuall and diſcontinuall.

An example of continuall proportionalitie in numbers.
16.8.4.2.1.

In continuall proportionalitie the quantities cannot be of one kinde.

Diſcontinuall proportionalitie.

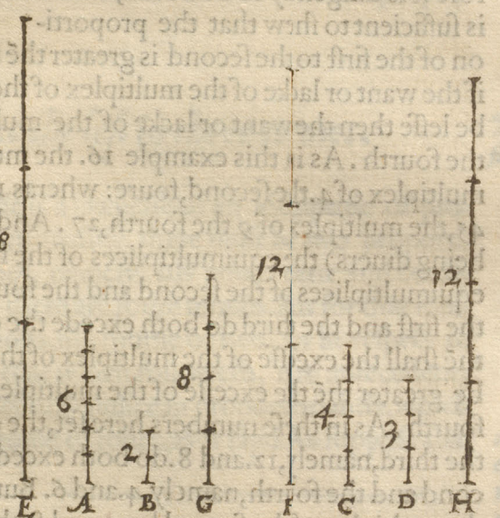
Example of diſcontinual proportionality in numbers.

In diſcõtinual proportionalitie the proportions may be of diuers kindes.

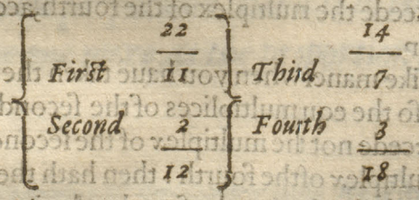
When the equemultiplices being taken, the multiplex of the first excedeth the multiplex of the second, & the multiplex of the third, excedeth not the multiplex of the fourth: then hath the first to the second a greater proportion, then hath the third to the fourth.

The eight definition.

In the sixt definition was declared what magnitudes are said to be in one and the same proportion: now he sheweth in this definition what magnitudes are said to be in a greater proportion. And here is supposed the same order of multiplication, that there in that definition was vsed: namely, that the first and the third be equally multiplied, that is, by one & the selfe same nũber: and also that the second and the fourth be equally multiplied by the same or some other number: and then if the multiplex of the first, excede the multiplex of the second: & the multiplex of the third, excede not the multiplex of the fourth, the first hath a greater proportion to the second, then hath the third to the fourth. As suppose that there be foure quantities, A, B, C, D: of which let A be the first, B the second, C the third, & D the fourth. And let A the first cõtaine 6. and let B the second containe 2. & C the third 4. & D the fourth 3: Now take the equimultiplices of A and C, the first & the third, which let be E and F, so that how multiplex E is to A so multiplex let F be to C: namely, for example sake let either of them be triple: so haue you 18. for the multiplex of A, and 12. for the multiplex of C. Likewise take the equimultiplices of B & D, the second & the fourth, multiplying them also by one and the self same number, as by 4: so haue ye for the multiplex of B the second 8, namely, the line G, and for the multiplex of D the fourth 12, namely, the line H. Now because the line E multiplex to the first, namely, 18, excedeth the line G multiplex to the second, namely, 8: And the line F multiplex to the third, namely, 12, excedeth not the line H multiplex to the fourth, namely, 12 (for that they are equall) the proportiõ of A to B the first to the second, is greater then the proportion of C to D the third to the fourth. So likewise in nũbers: take 11. to 2. & 7. to 3. and multiply 11. & 7. (the first, and the third) by 2, so shall ye haue 22. for the multiplex of the first, and 14. for the multiplex of the third: and multiply 2. and 3. the second and the fourth by 6: so shall the multiplex of the second be 12. and the multiplex of the fourth be 18:



An example of this definition in magnitudes.



An example in numbers.

Now ye ſee 22. the multiplex of the firſt, excedeth 12, the multiplex of the ſecond. But 14. the multiplex of the third, excedeth not 18. the multiplex of the fourth: Wherefore the proportion of 11. to 2. the firſt to the ſecond, is greater then the proportion of 7. to 3, the third to the fourth. And ſo of all other quantities and numbers, which are not in one and the ſelfe ſame proportion, ye may know when the firſt to the ſecond hath a greater proportion then the third to the fourth.

¶ An other example.

This example haue I ſet to declare that although the proportion of the firſt to the ſecond be greater then the proportion of the third to the fourth, yet the multiplex of the firſt excedeth not the multiplex of the ſecõd. Wherfore it is diligently to be noted, that it is ſufficient to ſhew that the proportion of the firſt to the ſecond is greater thẽ the proportion of the third to the fourth, if the want or lacke of the multiplex of the firſt from the multiplex of the ſecond, be leſſe then the want or lacke of the multiplex of the third to the multiplex of the fourth. As in this example 16. the multiplex of 8. the firſt, wanteth of 20. the multiplex of 4. the ſecond, foure: wheras 18. the multiplex of 9, the third, wãteth of 45, the multiplex of 9 the fourth, 27. And ſo of all others wheras (the proportions being diuers) the equimultiplices of the firſt and the third are both leſſe, then the equimultiplices of the ſecond and the fourth. Likewiſe if the equimultiplices of the firſt and the third do both excede the equimultiplices of the ſecond & the firſt, thẽ ſhall the exceſſe of the multiplex of the firſt aboue the multiplex of the ſecond, be greater thẽ the exceſſe of the multiplex of the third, aboue the multiplex of the fourth. As in theſe numbers here ſet, the equimultiplices of 6. and 4. the firſt and the third, namely, 12. and 8. do both excede the equimultiplices of 2. and 3. the ſecond and the fourth, namely, 4. and 6. But 12. the multiplex of the firſt excedeth 4. the multiplex of the ſecond by 4, and 8. the multiplex of the thyrd excedeth 6. the multiplex of the fourth by 2. but 8. is more then 2. Howbeit this is generally certaine that when ſoeuer the proportion of the firſt to the ſecõd is greater then the proportion of the third to the fourth, there may be found ſome multiplication, that whẽ the equimultiplices of the firſt and the third ſhall be compared to the equimultiplices of the ſecond and the fourth, the multiplex of the firſt ſhall excede the multiplex of the ſecond, & the multiplex of the third ſhall not excede the multiplex of the fourth, according to the plaine wordes of the definition.

Note.

	16			18
First	8		Third	9
Second	4		Fourth	9
	20			45

	12			8
First	6		Third	4
Second	2		Fourth	3
	4			6

In like maner when you haue taken the equimultiplices of the firſt & the third, and alſo the equimultiplices of the ſecond and the fourth, if the multiplex of the firſt excede not the multiplex of the ſecond, and the multiplex of the third excede the multiplex of the fourth: then hath the firſt to the ſecond a leſſe proportion, then hath the third to the fourth. As in the example before, if ye chaunge the termes, and make C the firſt, D the ſecond, A the third, and B the fourth: then ſhall

F, namely,

F, namely, 12. the multiplex of the first not excede H, namely, 12. the multiplex of the second: but E, namely, 18. the multiplex of the third excedeth G, namely, 8. the multiplex of the fourth. Wherefore the proportion of C to D, the first to the second, is lesse then the proportion of A to B, the third to the fourth.

Euen so in numbers. As in this example, 5. to 4. and 7. to 3. If ye multiply 5. and 7. the first and the third eche by 3, ye shall for the multiplex of 5. the first haue 15. and for the multiplex of 7. the third shall ye haue 21: againe if ye multiply 4. and 3. the second & the fourth by 6, for the multiplex of 4. the second ye shall haue 24, and for the multiplex of 3. the fourth, ye shall haue 18. So ye see that 15. the multiplex of the first, is lesse then 24, the multiplex of the second: and 21. the multiplex of the third is greater then 18. the multiplex of the fourth. Wherefore the proportion of 5. to 4. the first to the second is lesse then the proportion of 7. to 3. the third to the fourth.

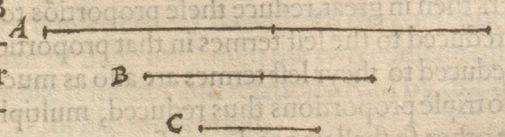
First	15	Third	21	
	5		7	
Second	4	Fourth	3	
	24		18	

Proportionallitie consisteth at the lest in three termes.

The ninth definition.

Before it was sayd, that proportionalitie is a likenesse or an idemptitie of proportions. Wherfore of necessitie in proportionalitie, there must be two proportions, and euery proportion hath two termes, namely, his antecedent and consequent. Therfore in euery proportionalitie there are foure termes. But for that somtyme, one terme supplieth by diuers relations, the roume of two, for in respect to the first it is consequent, and in respect to that which followeth, it is antecedent: therfore three termes at least and not vnder may suffice in proportionalitie, which three are in power foure, and occupy the rome of foure, as is sayd. As suppose that A hath to B that proportion, that B hath to C: then are these thre quantities A, B, C, set in the lest number of proportionality. Likewise in numbers, as 8. 4. 2. and 9. 6. 4.

An example of this definition in magnitudes.



Example in numbers.

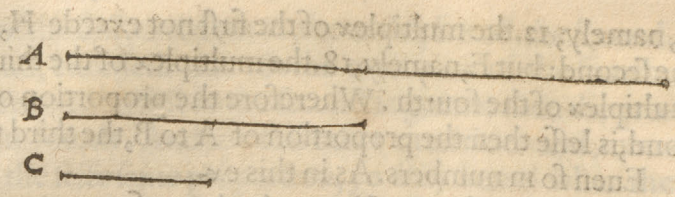
When there are three magnitudes in proportion, the first shall be vnto the third in double proportion that it is to the second. But when there are foure magnitudes in proportion the first shall be vnto the fourth in treble proportion that it is to the second. And so alwaies in order one more, as the proportion shall be extended.

The tenth definition.

This definition is also vnderstand in continuall proportionalitie. As if the thre magnitudes A, B, C, bee proportionall: then shall the proportion of A the first

Example of this definition in magnitudes.

firſt to C the thirde, bee double to the proportion which is betwene A & B the firſt and the ſeconde, that is the proportiō of A to B taken twiſe, or added to it ſelf (which is all one) ſhall make the proportiō of A to C. For the eaſier vnderſtādyng of this & the practiſe therof, it ſhall be much neceſſary ſomwhat to inſtruct the rude beginner how proportions may be added one to an other. Which is done by this rule.



A rule to adde proportions to proportions.

Multiply the antecedent of the one proportion by the antecedēt of the other, and the number produced ſhall be the antecedent of the proportion which contayneth them both. Likewyſe multiply the conſequent of the one proportion by the conſequent of the other, and the number produced ſhall be conſequent of the proportion which ſhall contayne them both.

Example thereof.

An example. If ye will adde the proportion which is betwene 4 and 2. (which is dupla) to the proportion which is betwene 9 and 3. (which is tripla) multiply 9. the antecedent of the firſt proportion by 4. the antecedent of the ſecond proportion, and ye ſhall produce 36. which reſerue and kepe for the antecedent of the proportion which ye ſeeke for. Likewiſe multiply 3 the conſequent of the firſt proportion, by 2 the conſequent of the ſecond, ſo ſhall ye haue 6. which 6. ſhall be conſequent to the former antecedēt, namely, to 36: ſo ſhal the proportiō which is betwene 36 and 6. namely, ſextupla, contayne in it the two proportions geuen, namely, tripla, and dupla. And by this meanes are they added together, & brought into one. And by this may ye adde all other kyndes of proportions whatſoeuer they be. Now for that the diffinition ſayth, that if there be three quantities in proportiō, that is, what proportiō the firſt hath to the ſecōd, the ſame hath the ſecond to the third, which for example let be triple, as in theſe nūbers, 27. 9. 3. adde triple to triple by the rule abouesaid. And foraſmuch as it is eaſier to worke in ſmall nūbers then in great, reduce theſe proportiōs to theyr leaſt denomination: So 27. to 9. reduced to the leſt termes in that proportion, is as much as 3. to 1. Likewiſe 9 to 3 reduced to theyr leſt termes are alſo as much as 3 to 1. now adde together theſe two triple proportions thus reduced, multipliyng 3 by 3. the one antecedent by the other, ſo ſhall ye produce 9 for a new antecedent, then multiply 1 by 1. the one conſequent by the other, ſo ſhall you produce 1. which let be conſequent to 9. your antecedent, ſo the proportion betwene 9 and 1. which is noncuple contayneth both the two triple proportions. And becauſe they were equal the one to the other, it is duple to eche of them. Ye ſee alſo that the proportion of 27 to 3. the firſt to the third, is alſo noncuple. Wherfore according to the definition, the proportion of the firſt to the third, is double to the proportion of the firſt to the ſecond, as 9 to 1. beyng noncuple, is double 3 to 1. which is triple, becauſe it contayneth it twiſe.

27. 9. 3.

9
3 — 3
1 — 1

So if there be 4. quantities in continuall proportion, the proportion of the firſt to the fourth, ſhall be triple to the proportion which is betwene the firſt and the ſecond, that is, it ſhall contayne it three tymes. As for example, Take 4. numbers in continual proportion 8. 4. 2. 1. Ye ſee that the proportiō of 8 to 1. the firſt to the fourth, is octupla: the proportion of 8 to 4. the firſt to the ſecond is dupla, now treble dupla proportion, that is, adde 3. dupla proportions together, by the rule before geuen, as ye ſee in the example. Multiply all the antecedentes together 2.

8. 4. 2. 1.

2 2 2
1 1 1

36	
9	4
3	2
6	

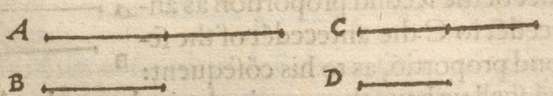
the

the antecedent of the firſt proportion, by 2. the antecedent of the ſecond, ſo haue ye 4: which 4. multiply by 2 the antecedent of the third proportiõ, ſo ſhal ye haue 8 for a new antecedent. In lyke maner multiply all the conſequentes together, 1. the conſequent of the firſt proportion by 1. the conſequent of the ſecond proportiõ, ſo ſhal ye haue 1, which 1. multiply agayne by 1. the cõſequent of the third proportion, ſo ſhall ye haue agayne 1: which 1. let be conſequent to your former antecedent 8: ſo haue ye 8 to 1. which is octupla, which was alſo the proportion of the firſt to the fourth, which octupla is alſo brought fourth of the addition of thre dupla proportions together, and contayneth it three tymes, wherefore octupla is tripla to dupla, and therfore as the diffinition ſayth: the proportion of the firſt to the fourth is tripla to the proportion of the firſt to the ſecond. And ſo conſequently forth as long as the proportionalitie continueth accordyng to the ſentence of the diffinition, the termes of the proportions exceding the number of proportions by one. As if ye haue 5. termes in proportion, the proportiõ of the firſt to the fifth ſhal be quadrupla to the proportion of the firſt to the ſecond, and if there be 6. termes, it ſhall be quintupla and ſo in order.

Magnitudes of like proportion, are ſayd to be antecedents to antecedentes, and conſequentes to conſequentes.

The eleuenth definition.

For that before it was ſayd, that proportion was a relation or a reſpect of one quantitie to an other, now ſheweth he what magnitudes are ſayd to be of like proportion, namely, theſe whoſe antecedents haue like reſpect to their conſequentes, and whoſe conſequents receyue like reſpectes of their antecedẽts. As putting 4. magnitudes A, B, C D. If A antecedent to B, be double to B, and C antecedent to D, be double alſo to D, thẽ haue the two antecedentes like reſpectes to their conſequents. Likewiſe if B the conſequent be halfe of A, and alſo D the conſequent be halfe of C, then the two conſequentes B and D receiue of their antecedentes like reſpectes and relations. And by this diffinition, are theſe magnitudes A, B, C, D, of like proportion.



Example of this deſinition in magnituds.

Alſo in numbers, 9. 3. 6. 2: becauſe 9 the antecedent is triple to 3. his conſequent, and the antecedent 6. is alſo triple to 2 his conſequent: the two antecedẽts 9 and 6 haue like reſpectes to their conſequentes, and becauſe that 3 the conſequent is the ſubtriple or third part of 9. his antecedent, and likewiſe 2 the conſequent is the ſubtriple or third part of 6. his antecedent, the two conſequentes 3 and 2 receiue alſo lyke reſpectes of their antecedentes, and therfore are numbers of like proportion.

9.	6.
3.	2.

Example in numbers.

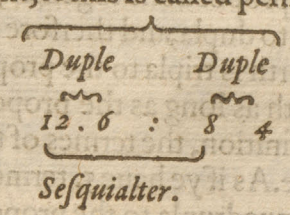
Proportion alternate, or proportion by permutation is, when the antecedent is compared to the antecedent, and the conſequent to the conſequent.

The twelfth definition.

The vnderſtanding of this definition & of all the definitions following, dependeth of the definition going before, and vſe it for a generall ſuppoſition, namely, to haue

Example of this definition in magnituds.

haue foure quantities in proportion. Suppoſe foure magnitudes A,B,C,D, to be in proportion, namely, as A is to B, ſo let C be to D. Now if ye compare A the antecedent of the firſt proportion to C the antecedent of the ſecond as to his conſequent, & likewiſe if ye compare B the cõſequent of the firſt proportion as an antecedent to D the conſequent of the ſecond as to his conſequent: then ſhall ye haue the magnitudes in this ſort: as A to C, antecedent to antecedent, ſo B to D, conſequent to conſequent, & this is called permutate proportion or alternate. In numbers as 12. to 6, ſo 8. to 4. either is dupla. Wherefore by permutation of proportion, as 12. to 8. antecedent to antecedent, ſo is 6. to 4. conſequent to conſequent, for either is ſeſquialtera.



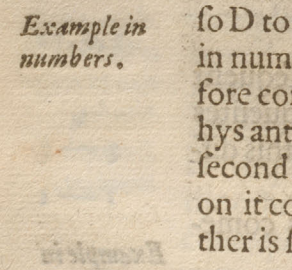
Example in numbers.

The thirtenth definition.

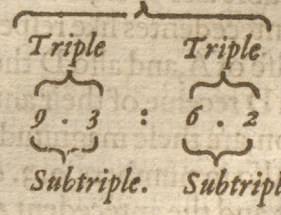
Conuerſe proportion, or propoꝛtion by conuerſion is, when the conſequent is taken as the antecedent, and ſo is compared to the antecedent as to the conſequent.

Example of this definition in magnituds.

Suppoſe as before foure magnitudes in proportion, A,B,C,D, as A to B, ſo C to D: if ye referre B the conſequent of the firſt proportion, as antecedent, to A the antecedent of the firſt, as to his conſequent: and likewiſe if ye referre D the conſequẽt of the ſecond proportion as antecedẽt to C the antecedẽt of the ſecond proportiõ, as to his cõſequent: thẽ ſhall ye haue the magnitudes in thys order. As B to A cõſequent to antecedẽt: ſo D to C conſequent to antecedẽt. And thys is called cõuerſe proportion. So alſo in numbers, 9. to 3, as 6. to 2, eyther is tripla, wherfore comparing 3. to 9, the conſequent of the firſt to hys antecedent 9, and alſo 2. the conſequent of the ſecond to hys antecedent 6, by conuerſe proportion it commeth to paſſe as 3. to 9, ſo 2. to 6: For either is ſubtripla.



Example in numbers.

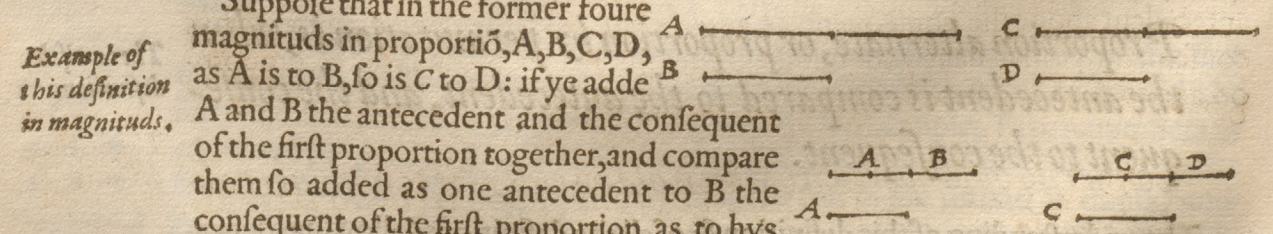


The fourtenth definition.

Proportion compoſed, or compoſition of proportion is, when the antecedent and the conſequent are both as one compared vnto the conſequent.

Example of this definition in magnituds.

Suppoſe that in the former foure magnituds in proportiõ, A,B,C,D, as A is to B, ſo is C to D: if ye adde A and B the antecedent and the conſequent of the firſt proportion together, and compare them ſo added as one antecedent to B the conſequent of the firſt proportion as to hys conſequent: and likewiſe if ye adde together



C and D,

C and D the antecedent and the conſequent of the ſecond proportion, and ſo added, compare them as one antecedent to D the conſequent of the ſecond proportion, as to his conſequent: then ſhall ye haue the magnitudes in this order. As A B to B, ſo C D to D, for either of them is tripla. And this is called compoſed proportion, or compoſition of proportion. And ſo alſo in numbers. As 8. to 4, ſo 6. to 3: 8. and 4, the antecedent and conſequent of the firſt proportion added together, make 12: which 12. as antecedent compare to 4. the conſequent of the firſt proportion as to his conſequent: ſo adde together 6. and 3, the antecedent and conſequent of the ſecond proportion, they make 9: which 9. as antecedent compare to 3. the conſequent of the ſecond proportion, as to his conſequent: ſo ſhall ye haue by compoſition of proportion, as 12. to 4, ſo 9. to 3, for either of them is tripla.

Example in numbers.

8 . 4 : 6 . 3

12 . 4 : 9 . 3

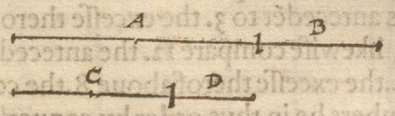
The fiftene definition.

Proportion deuided, or diuiſion of proportiõ is, when the exceſſe wherein the antecedent excedeth the conſequent, is compared to the conſequent.

This is the cõuerſe of the former definition.

Thys definition is the conuerſe of the definition going next before: in it was vſed compoſition, and in thys is vſed diuiſion. As before ſo now ſuppoſe foure magnitudes in proportion A B the firſt, B the ſecond, C D the third, and D the fourth: as A B to B: ſo C D to D: A B, the antecedent of the firſt proportion excedeth B the conſequent of the firſt proportion by the magnitude A, wherfore A is the exceſſe of the antecedent A B aboue the conſequent B: ſo likewiſe C D the antecedent of the ſecond proportion, excedeth D the conſequent of the ſame proportion, by the quantitie C, wherefore C is the exceſſe of the antecedent C D aboue the conſequent D. Now if ye compare A the exceſſe of A B the firſt antecedent, aboue the conſequent B, as antecedent to B the conſequent, as to his conſequent: alſo if ye compare D the exceſſe of the ſecond antecedent C D, aboue the conſequent D, as antecedent to D the conſequent, as to his conſequent: then ſhall your magnitudes be in this order. As A to B, ſo is C to D: which is called diuiſion of proportion, or proportion deuided.

Example in magnitudes.



Example in numbers.

And ſo in numbers, as 9. to 6, ſo 12. to 8, either proportion is ſeſquialtera: the exceſſe of 9. the antecedent of the firſt proportion aboue 6. the conſequent of the ſame is 3: the exceſſe of 12. the antecedent of the ſecond proportion aboue 8, the conſequent of the ſame, is 4: then if ye compare 3. the exceſſe of 9. the firſt antecedent aboue the conſequent, as antecedent to 6, the conſequent, as to hys conſequent: and alſo if ye compare .4 the exceſſe of 12. the ſecond antecedent aboue the conſequent, as antecedent, to 8. the conſequent, as to hys conſequent, ye ſhall haue your numbers after this maner by diuiſion of proportion, as 3. to 6: ſo 4. to 8: for either proportion is ſubdupla.

9 . 6 : 12 . 8

3 . 6 : 4 . 8

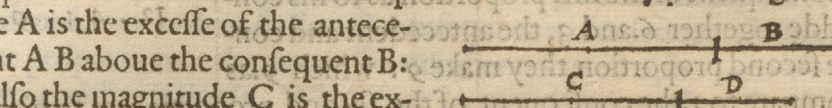
The ſixtens definition.

Conuerſiõ of proportion (which of the elders is commonly called euerſe proportion, or euerſiõ of proportion) is, whẽ the antecedent

tecedent is compared to the exceſſe, wherein the antecedent excedeth the conſequent.

An example of this definition in magnitudes.

Foure magnitudes ſuppoſed as before, A B the firſt, B the ſecond, C D the third, and D the fourth. As A B to A, ſo C D to C: A B the antecedent of the firſt proportion excedeth B the conſequent of the ſame by the magnitude A, wherefore A is the exceſſe of the antecedent A B aboue the conſequent B: ſo alſo the magnitude C is the exceſſe of C D the antecedent of the ſecond proportion aboue D the conſequent of the ſame: now if ye referre A B the antecedent of the firſt proportion, as antecedent, to A the exceſſe therof aboue the conſequent B, as to his conſequent: if ye compare alſo C D the antecedent of the ſecond proportion as antecedent to C the exceſſe therof aboue the conſequent D, as to his conſequent: then ſhall your magnitudes come to thys order. As A B to A, ſo C D to C, and thys is called conuerſion of proportion, and of ſome euerſion of proportion. Likewyſe in numbers, as 9. to 6, ſo 12. to 8. eyther proportion is ſeſquialtera: the exceſſe of 9. the antecedent of the firſt proportion aboue 6. the conſequent of the ſame is 3: the exceſſe of 12. the antecedent of the ſecond proportion aboue 8. the conſequent of the ſame, is 4: now cõpare the antecedent of the firſt proportion 9. as antecedẽt to 3. the exceſſe therof aboue 6. the conſequẽt, as to his conſequent, likewiſe compare 12. the antecedent of the ſecond proportion as antecedent to 4. the exceſſe therof aboue 8. the conſequent, as to his conſequent: ſo ſhall your numbers be in thys order by conuerſion of proportion: as 9. to 3: ſo 12. to 4: for either proportion is triple.



An example in numbers.

9 . 6 : 12 . 8

9 . 3 : 12 . 4

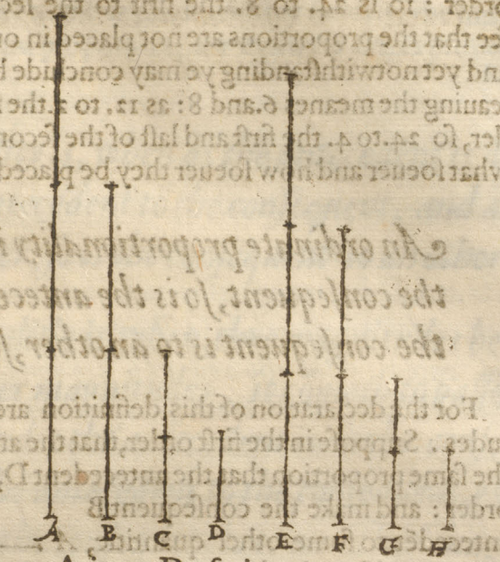
The ſeuẽtenth definition.

Proportion of equalitie is, when there are taken a number of magnitudes in one order, and alſo as many other magnitudes in an other order, comparing two to two beyng in the ſame proportion, it commeth to paſſe, that as in the firſt order of magnitudes, the firſt is to the laſt, ſo in the ſecond order of magnitudes is the firſt to the laſt. Or otherwiſe it is a compariſon of extremes together, the middle magnitudes being taken away.

An example of this definition in magnitudes.

To the declaration of thys definition are required two orders of magnitudes equall in number, and in lyke proportion: As if there be taken in ſome determinate number certayne magnitudes, namely, foure, A, B, C, D. And alſo in the ſame number be taken other quantities, namely, foure, E, F, G, H: then take the equall proportions by two and two: as A to B, ſo E to F: as B to C, ſo F to

so F to G: and as C to D, so G to H. Now according to the first definition, if A the first magnitude of the first order be to D the last magnitude of the same order, as E the first magnitude of the second order is to H the last magnitude the same, then it is called proportion of equalitie, or equall proportion.

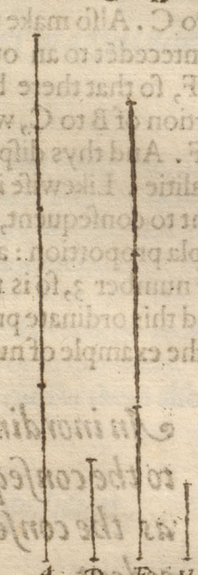


By the second definition, which is all one in substance with the first, ye leaue the meane magnitudes in eyther order, namely, B, C, on the one side, and F, G on the other side, and onely compare the extremes of ech side together, which by thys definition shall be in lyke proportion, as A is to D, so is E to H.

Euen so in numbers, take for example these two orders, 27. 9. 12. 24. 15. and 9. 3. 4. 8. 5. there are in eche order as ye see, fiue numbers, then see that all the proportions taken by two & two be like: betwene 27. & 9, numbers of the first order, and betwene 9. and 3, numbers of the second order, there is one and the self same proportiō, namely, tripla: also betwene 9. and 12, numbers of the first order, and 3. and 4, numbers of the second order, is like proportion, namely, subsesquitertia proportion: so betwene 12. and 24, numbers of the first order, and 4. and 8, numbers of the second order, is also lyke proportion, namely, subdupla: Last of all, betwene 24. and 15, numbers of the first rowe, and betwene 8. and 5, numbers of the second rowe, the proportion is one, namely, supertripartiens quintas. Wherefore by this definition, leauing out all the meane numbers of eche side, ye may compare together onely the extremes, and conclude that as 27. of the first row is to 15. the last of the same row, so is 9. the first of the second rowe to 5, the last of the same rowe: for the proportion of ech is superquadripartiens quintas.

An example in numbers.

27	9
9	3
12	4
24	8
15	5



Here is to be considered, that it is not of necessitie that all the proportions in eche rowe of numbers be set in like order, as in the one so in the other: but it shall be sufficient that the proportions be the same and in equall number in eche rowe. Whether it be in the selfe same order, or in contrary, or inuerted order, it maketh no matter. As in these numbers, 12. 6. 2. in the first row, and 24. 8. 4. in the second. As 12. is to 6, the first to the second of the first row, so is 8. to 4. the second to the third of the second row: either is duple proportion. And as 6. to 2. the second to the third in the first order:

Note.

12	24
6	8
2	4

order : ſo is 24. to 8. the firſt to the ſecond in the ſecond order. Where ye ſee that the proportions are not placed in one and the ſelfe ſame order, and courſe, and yet notwithſtanding ye may conclude by equalitie of proportion, leauing the meanes 6. and 8: as 12. to 2. the firſt to the laſt of the firſt order, ſo 24. to 4. the firſt and laſt of the ſecond order. And ſo of others what ſoeuer and how ſoeuer they be placed.

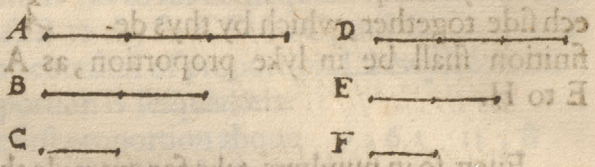
12	24
2	4

The eighteenth definition.

An ordinate proportionality is, when as the antecedent is to the conſequent, ſo is the antecedent to the conſequent, and as the conſequent is to another, ſo is the conſequent to another.

An example of this definition in magnitudes.

For the declaration of this definition are alſo required two orders of magnitudes. Suppoſe in the firſt order, that the antecedent A, to his conſequent B, haue the ſame proportion that the antecedent D, hath to his cõſequent E in the ſecond order: and make the conſequent B antecedẽt to ſome other quantitie, as to C. Alſo make the conſequent E antecedẽt to an other quãtitie, as to F, ſo that there be the ſame proportion of B to C, which is of E to to F. And thys diſpoſition of proportions is called ordinate proportionalitie.



Example in numbers.

Likewiſe in numbers, 18.9.3. and 6.3.1. As 18. to 9. antecedent to conſequent, ſo is 6. to 3. antecedent to conſequent: either is dupla proportion: and as 9. the conſequent is to an other, namely, to the number 3, ſo is the conſequent 3. to an other, namely, to vnity. And this ordinate proportionalitie may be extẽded as farre as ye liſt, as ye may ſee in the example of numbers in the definition next before.

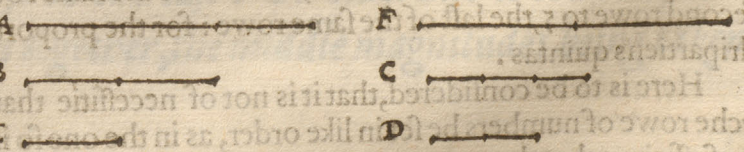
18	6
9	3
3	1

The nintenth definition.

An inordinate proportionality is, when as the antecedent is to the conſequent, ſo is the antecedent to the conſequent: and as the conſequent is to an other, ſo is an other to the antecedent.

An example of this definition in magnitudes.

This definition alſo as the other before, requireth two orders of magnitudes. Suppoſe in the firſt order that the antecedẽt A be to the cõſequẽt B, as the antecedẽt C, in the ſecond order is to the conſequent D, & let B the conſequẽt of the firſt proportiõ be to ſome other, namely, to the magnitude E, as ſome other, namely, the magnitude F, is to the antecedent C of the ſecond proportiõ: this kinde of proportionalitie is called inordinate or perturbate.



Example in numbers.

Take alſo an example in numbers, as 9 to 6. the antecedent to the conſequent, ſo is 3 to 2 the antecedent to the conſequent, either proportiõ is ſeſquialtera, and as 6 the conſequent of the firſt proportion,

9	3
6	2
3	6

proportion, is to an other, namely, to the number 3, so is another namely, the number 6. to 3. the antecedent of the second proportion, for eyther is dupla proportion.

An extended proportionality is, when as the antecedent is to the consequent, so is the antecedent to the consequent, and as the consequent is to an other, so is the consequent to an other. The 20. definition.

A pertubate proportionalitie is, when, thre magnitudes being compared to three other magnitudes, it cõmeth to passe, that as in the first magnitudes the antecedent is to the consequent, so in the second is the antecedent to the consequent, & as in the first magnitudes the consequent is to an other magnitude, so in the second magnitudes is an other magnitude to the antecedent. The 21. definition.

These two last definitions here put by Zamberte seeme all one with the other two last before set. Wherfore it is not lyke that they were written and set here by *Euclide*, for that they seeme not necessary, but rather superfluous, neither are they found in the Greeke examples commonly set forth in print, nor mentioned of any that hath written commentaries vpon *Euclide*, olde or new: Not of *Campane*, *Sceubelius*, *Pellitarius*, *Orontius*, nor *Flussates*: wherfore it is not of necessitie to adde vnto them any explanation or example either in magnitudes or in numbers. The examples of the two last definitions set before, may likewise serue for them also. *These two last definitions not found in the greeke examplers.*

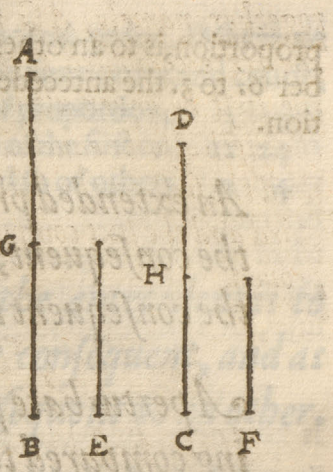
The 1. Theoreme. The 1. Proposition.

If there be a number of magnitudes how many soeuer equemultiplices to a like number of magnitudes ech to ech: how multiplex on magnitude is to one, so multiplices are all the magnitudes to all.

Vppose that there be a number of magnitudes, namely, AB and DC equimultiplices to a like number of magnitudes E and F ech to ech. Then I say, that how multiplex AB is to E, so multiplices are AB and DC to E and F. For forasmuch as how multiplex AB is to E, so multiplex is DC to F, therefore how many magnitudes there are in AB equall vnto E, so many are there in DC

Construction. equall vnto F. Deuide AB into the magnitudes which are equall vnto E, that is, into AG and GB: and likewise DC into the magnitudes which are e-

Demonstration. quall vnto F, that is, into DH and HC. Now then the multitude of these DH & HC, is equall vnto the multitude of these AG & GB. And forasmuch as AG is equall vnto E, and DH vnto F: therfore AG and DH, are equall vnto E and F: and by the same reason forasmuch as GB is equall vnto E, and HC vnto F, GB also and HC are equall vnto E and F. Wherefore how many magnitudes there are in AB equall vnto E, so many are there in AB and DC equall vnto E & F: Wherefore how multiplex AB is to E, so multiplices are AB and DC to E and F. If therefore there be a number of magnitudes how many soeuer equemultiplices to a like number of magnitudes ech to ech, how multiplex one magnitude is to one, so multiplices are all the magnitudes to all: which was required to be proued.

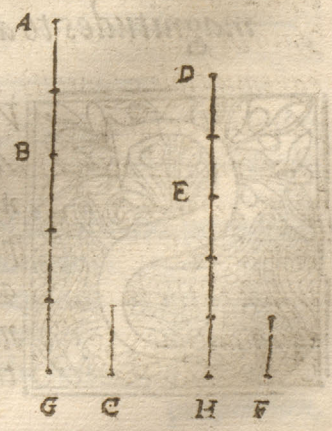


The 2. Theoreme. The 2. Propoſition.

If the first be equemultiplex to the second as the third is to the fourth, and if the fifth also be equemultiplex to the second as the sixt is to the fourth: then shall the first and the fifth composed together be equemultiplex to the second, as the third and the sixt composed together is to the fourth.

Vppose that there be sixe quantities, of which let AB be the first, C the second, DE the third, F the fourth, BG the fifth, & EH the sixt: and suppose that the first, AB, be equemultiplex vnto the second, C, as the third, DE, is to the fourth, F: and let the fift, BG, be equemultiplex vnto the second, C, as the sixt, EH, is to the fourth, F. Then I say, that the first and the fifth composed together, which let be AG, is equemultiplex vnto the second, C, as the third and sixt composed together, which let be DH, is to the fourth, F. For

Demonstration. forasmuch as AB is equemultiplex to C, as DE is to F, therefore how many magnitudes there are in AB equall vnto C, so many magnitudes are there in DE equall vnto F: and by the same reason how many there are in BG equall vnto C, so many also are there in EH equall vnto F. Wherefore how many there are in the whole AG equall vnto C, so many are there in the whole DH equall vnto F. Wherefore how multiplex AG is vnto C, so multiplex is DH vnto F. Wherefore the first and the fifth composed together,



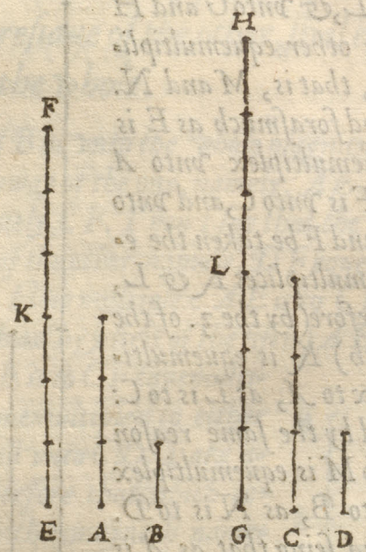
together, namely, AG is equemultiplex vnto the second C, as the third and the sixte composed together, namely, DH, is to the fourth F. If therfore the first be equemultiplex to the second as the third is to the fourth, and if the fifth also be equemultiplex to the second as the sixt is to the fourth: then shall the first & the fifth cõposed together, be equemultiplex to the second, as the third and the sixt composed together, is to the fourth: which was required to be proued.

The 3. Theoreme. The 3. Proposition.

If the first be equemultiplex to the second, as the third is to the fourth, and if there be taken equemultiplices to the first & to the third: they shall be equemultiplices to them which were first taken, the one to the second, the other to the fourth.

SVppose that there be foure magnitudes, of which let A be the first, B the second, C the third, and D the fourth. And let the first, A, be equemultiplex to the second, B, as ỹ third, C, is to the fourth, D. And vnto A and C take equemultiplices, which let be EF and GH, so that how multiplex EF is to A, so multiplex let HG be to C. Then I say, that EF is equemultiplex vnto B, as GH is vnto D. For forasmuch as EF is equemultiplex vnto A, as GH is vnto C, therefore how many magnitudes there are in EF equall vnto A, so many magnitudes also are there in GH equall vnto C. Let EF be deuided into the magnitudes that are equall vnto A, that is, into EK and KF. And likewise GH into the magnitudes equall vnto C, that is, into GL and LH. Now then the multitude of these magnitudes EK and KF, is equall vnto ỹ multitude of these magnitudes GL and LH. And forasmuch as A is equemultiplex to B, as C is to D: but EK is equall vnto A, and GL vnto C, therefore EK is equemultiplex vnto B, as GL is vnto D. And by the same reason KF is equemultiplex vnto B, as LH is to D. Now then there are sixe magnitudes whereof EK is the first: B ỹ second: GL ỹ third: D ỹ fourth: KF the fifth: LH the sixt. And forasmuch as the first EK is equemultiplex to the second B, as the third GL is to the fourth D: and the fift KF is equemultiplex to the second B, as the sixt LH is to the fourth D: therefore (by

Construction.



Demonstration.

the second of the fist) the first & the fift composed together, namely, E F is equemultiplex vnto the second B, as the third and the sixt composed together, namely, G H is to the fourth D. If therefore the first be equemultiplex to the second, as the third is to the fourth, and if there be taken the equemultiplices to the first and to the third, they shall be equemultiplices to them which were first taken, the one to the second, ther other to the fourth: which was required to be proued.

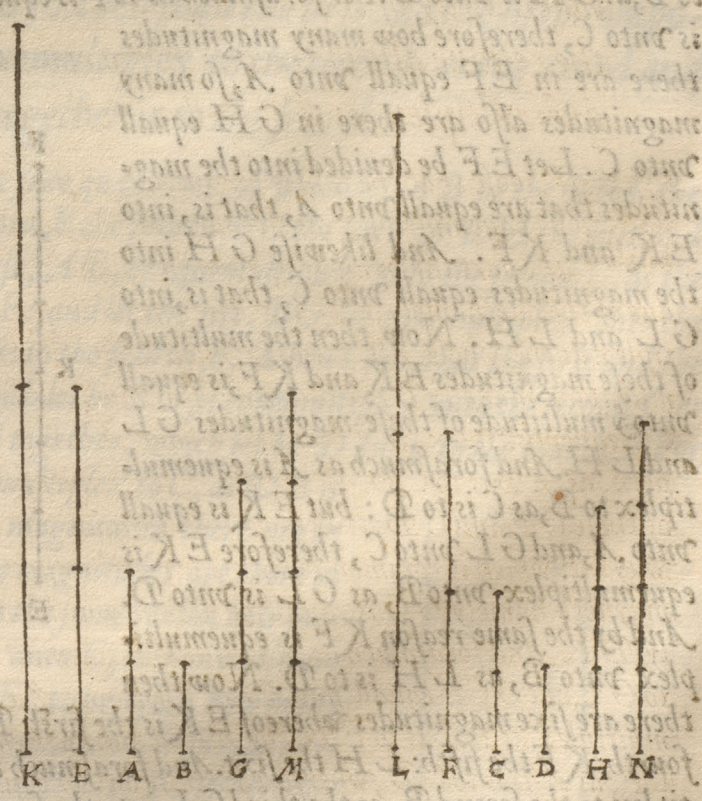
The 4. Theoreme. The 4. Proposition.

If the first be vnto the second in the same proportion that the third is to the fourth: then also the equemultiplices of the first and of the third, vnto the equemultiplices of the second and of the fourth, accordyng to any multiplication, shall haue the same proportion beyng compared together.

Suppose that there be foure magnitudes, of which let A be the first, B the second, C the third, and D the fourth. And let the first A be vnto the second B in the same proportion, that the third C is vnto the fourth D.

Construction. *And to A and C take equemultiplices E, and F, and likewise to B and D, any other equimultiplices G & H. The I say y^t as E is to G, so is F to H. Vnto E & F take equemultiplices K & L, & vnto G and H any other equemultiplices, that is, M and N.*

Demonstration. *And forasmuch as E is equemultiplex vnto A as F is vnto C, and vnto E and F be taken the equemultiplices K & L, therfore (by the 3. of the fifth) K is equemultiplex to A, as L is to C: and by the same reason also M is equemultiplex vnto B, as N is to D. And seing that as A is to B, so is C to D, and of A and C are taken equemultiplices K and L, and likewise of B & D are taken other equemultiplices, namely, M and N, therfore if K exceede M, L also*



also excedeth N: and if it be equall, it is equall, and if it be lesse, it is lesse (by the conuerse of the 6. definition of the fifth). And K and L are equemultiplices to E and F: and M and N are other equemultiplices to G and H. Wherefore as E is to G, so is F to H by the said sixt definition. If therfore the first be vnto the second in the same proportion that the third is to the fourth: then also the equemultiplices of the first and of the third, vnto the equemultiplices of the second & of the fourth, according to any multiplication, shall haue the same proportion beyng compared together: which was required to be proued.

An Assumpt. *Wherfore seing it hath bene proued that if K exceede M, L also excedeth N, and if it be equall it is equall: and if it be lesse, it is lesse: it is manifest that if M exceede K, N also excedeth L: and if it be equall it is equall: and if it be lesse it is lesse: and by this reason as G is to E, so is H to F.*

A Lemma, or an assumpt.

A Corollary.

Hereby it is manifest that if there be foure magnitudes proportional, they shal also by conuersion be proportionall: that is, if the first be vnto the second, as the thide is to the fourth: then by conuersion as the seconde is to the first, so is the fourth to the third.

A Corollary. Conuerse proportion.

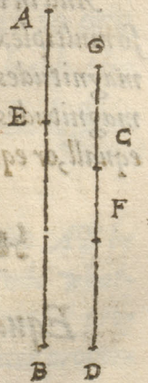
The 5. Theoreme. The 5. Proposition.

If a magnitude be equemultiplex to a magnitude, as a parte taken away of the one, is to a part taken away from the other: the residue also of the one, to the residue of the other, shal be equemultiplex, as the whole is to the whole.

Vppose that the whole magnitude AB be vnto the whole magnitude CD equemultiplex, as the part taken away of the one, namely, AE, is to the part taken away of the other, namely, CF. Then I say that the residue of the one, namely, EB, is to the residue of the other, namely, DF equemultiplex as the whole AB is to the whole CD. How multiplex AE is to CF, so multiplex make EB to CG. And forasmuch as (by ỹ first of the fifth) AE is to GF equemultiplex, as AB is to GF: but AE is to CF equemultiplex, as AB is to CD. Wherfore AB is equemultiplex to either of these GF and CD. Wherfore GF is equall vnto CD. Take away CF which is common to them both. Wherfore that which remayneth namely, GC, is equall vnto that which remayneth namely, DF. And forasmuch as AE is to CF equemultiplex as EB is to GC, but GC is equall vnto DF, therefore AE is to CF equemultiplex as EB is to FD. But AE is put to be equemultiplex to CF, as AB is to CD, wherfore EB is to FD equemultiplex, as AB is to CD. Wherfore the residue EB is to the residue FD equemultiplex, as the whole AB is to the whole CD. If therfore a

Construction.

Demonstration.



magnitude be equemultiplex to a magnitude, as a part taken away of the one is to a parte taken away of the other: the reſidue of the one alſo to the reſidue of the other, ſhalbe equemultiplex as the whole is to the whole: which was required to be proued.

The 6. Theoreme. The 6. Propoſition.

If two magnitudes be æquemultiplices to two magnitudes, & any partes taken away of them alſo, be æquemultiplices to the ſame magnitudes: the reſidues alſo of them ſhal vnto the ſame magnitudes be either equall, or equemultiplices.

Two caſes in this Propoſition.

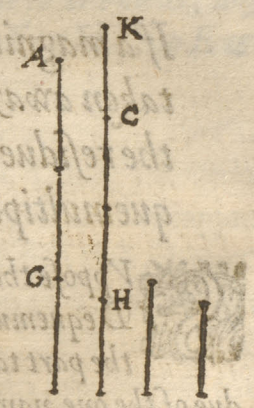
SVppoſe that there be two magnitudes AB and CD equemultiplices to two magnitudes E and F, and let the partes takē away of the magnitudes AB and CD, namely, AG and CH be equemultiplices to the ſame magnitudes E and F. Then I ſay that the reſidues GB and HD, are vnto the ſelfe ſame magnitudes E and F either equall, or els equemultiplices.

The firſt.

Conſtruction.

Demonſtration.

Suppoſe firſt that GB be equall vnto E. Then I ſay that HD is equall vnto F. Vnto F put an equall magnitude CK. And foraſmuch as AG is equemultiplex vnto E, as CH is vnto F: but GB is equall vnto E, & KC vnto F: therfore AB is equemultiplex to E, as KH is to F. But AB is put equemultiplex vnto E, as CD is to F. Wherfore KH is equimultiplex vnto F, as CD is to F. And foraſmuch as either of theſe KH and CD are equimultiplices vnto F, therfore (by the 1 common ſentence) KH is equall vnto CD. Take away CH which is common to them both. Wherefore the reſidue KC is equal vnto the reſidue HD. But KC is equal vnto F, wherfore HD is equall vnto F. Wherfore if GB be equal vnto E, DH alſo ſhall be equall vnto F.



The ſecond.

And in like ſort may we proue, yͤ if GB be multiplex to E, HD alſo ſhal be ſo multiplex vnto F. If therfore there be two magnitudes equemultiplices to two magnitudes, and any parts taken away of them be alſo equemultiplices to yͤ ſame magnitudes: the reſidues alſo of them ſhall vnto the ſame magnitudes be either equall, or equemultiplices: which was required to be proued.

The 7. Theoreme. The 7. Propoſition.

Equall magnitudes haue to one & the ſelfe ſame magnitude,

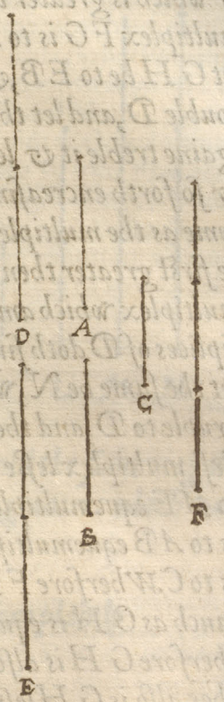
one

one and the ſame proportion. And one and the ſame magnitude hath to equall magnitudes one and the ſelfe ſame proportion.

SVppoſe that A and B be equall magnitudes, and take any other magnitude, namely, C. Then I ſay, that either of theſe A and B haue vnto C one and the ſame proportion: and that C alſo hath to either of theſe A and B one and the ſame proportion.

Take the equemultiplices of A and B, and let the ſame be D and E: and likewiſe of C, take any other multiplex, and let the ſame be F. Now foraſmuch as D is vnto A equemultiplex as E is to B, but A is equall vnto B, therfore (by the firſt common ſentence) D is equall vnto E. And of C there is taken any other multiplex F. Wherefore if D exceede F, E alſo excedeth F: and if it be equall it is equal, and if it be leſſe it is leſſe. But D and E are the equemultiplices of A & B, and F is of C an other multiplex. Wherfore as A is to C, ſo is B to C.

The firſt part of this Propoſition demonſtrated.



I ſay moreouer, that C hath to either of theſe A and B one and the ſame proportion. For the ſame order of conſtruction remaining, we may in like ſort proue, that D is equal vnto E, & there is taken an other multiplex to C, namely, F. Wherefore if F exceede D, it alſo excedeth E: and if it be equall it is equall: and if it be leſſe it is leſſe. But F is multiplex to C: and D & E are other equemultiplices to A and B. Wherfore as C is to A, ſo is C to B. Wherfore equall magnitudes haue to one and the ſame magnitude, one and the ſame proportion: and one and the ſame magnitude hath to equall magnitudes one and the ſelfe ſame proportion: which was required to be demonſtrated.

The ſecond part demonſtrated.

The 8. Theoreme. The 8. Propoſition.

VNequall magnitudes beyng taken, the greater hath to one and the ſame magnitude a greater proportion then hath the leſſe. And that one and the ſame magnitude hath to the leſſe a greater proportion then it hath to the greater.

SVppoſe that AB and C be vnequall magnitudes, of which let AB be the greater, and C the leſſe. And let there be an other magnitude whatſoeuer, namely, D. Then I ſay that AB hath vnto D a greater proportion then hath C to D: and alſo that D hath to C a greater proportion,

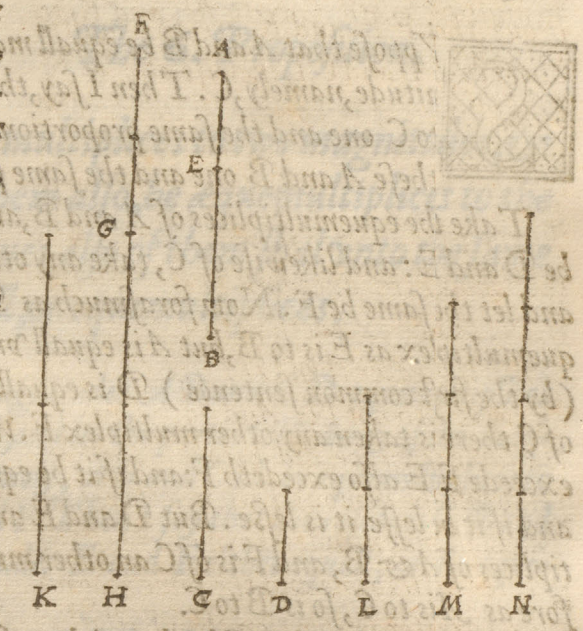
tion, then it hath to AB. For forasmuch as AB is greater then C, let there be taken a magnitude equall vnto C, namely, BE.

The first part of this Proposition demonstrated.

Now then the leſſe of theſe two magnitudes AE and EB being multiplied will at the length be greater then D.

The first case of the same.

First let AE be leſſe then EB: and let AE be ſo oft multiplieed vntill that which is produced be greater then D; and let that multiplex be FG which is greater then D. And how multiplex FG is to AE, ſo multiplex let GH be to EB, & K to C. Then double D, and let the ſame be L. And againe treble it & let the ſame be M: & ſo forth encreaſing by one till ſuch tyme as the multiplex of D ſo taken be firſt greater then K, that is, that multiplex which amongſt al the multiplices of D doth firſt exceede K, & let the ſame be N, which here is quadruple to D, and the firſt multiplex greater then K. Now forasmuch as K is y firſt multiplex leſſe then N, therfore K is not leſſe then M. And for that FG is to AE equemultiplex as GH is to EB, therfore (by the firſt of the fifth) FH is to AB equemultiplex as FG is to AE: but FG is equemultiplex to AE as K is to C. Wherfore FH and K are equemultiplices to AB and C. Againe forasmuch as GH is equemultiplex to EB as K is to C, but EB is equall vnto C, therfore GH is alſo equall vnto K. But K is not leſſe then M: wherfore neyther alſo is GH leſſe then M. But FG is greater then D. Wherefore the whole FH is greater then both theſe D and M. But both theſe D and M are equall vnto N, for M is triple to D, and M and D together are quadruple to D, and N alſo is quadruple to D: wherefore both theſe M and D are equall vnto N, but FH is greater then M and D, wherfore FH excedeth N, (that is, y multiplex of the firſt, namely, of AB excedeth the multiplex of the ſecond, namely, of D.) But K excedeth not N (that is, the multiplex of the third, namely, of C, excedeth not the multiplex of the fourth, namely, of the ſame D:) and FH and K are equemultiplices to AB, and C, and N, is a certayne other multiplex to D. Wherfore (by the 8. definition) AB hath to D a greater proportion then hath C to D.



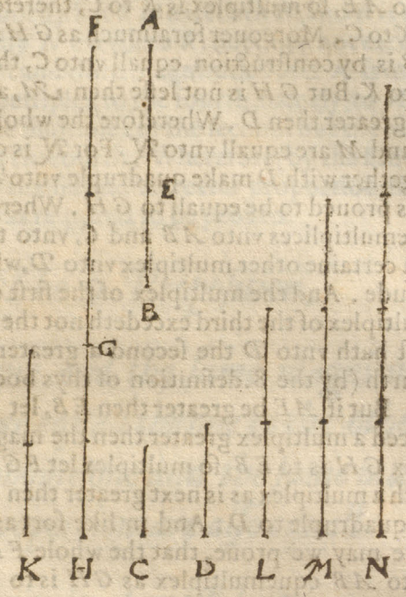
The second part of the propoſition demonſtrated.

I ſay moreouer that D hath to C a greater proportion then D hath to AB. For the ſame order of conſtruction ſtill remayning, we may in like ſort proue that N is greater then K, and that it is not greater then FH. And N is multiplex to D, and FH and K are certayne other equemultiplices to AB and C. Wherfore D hath to C a greater proportion then D hath to AB.

But

But now suppose that AE be greater then EB. Now then EB beyng the lesse, and being oftentimes multiplied, will at the length be greater then D. Let it be so multiplied, and let the multiplex of EB, namely, GH, be greater then D. And how multiplex GH is to EB so multiplex let FG be to AE and K to C. And by the former reason may we proue, that FH and K are equemultiplices to AB and C. Likewise also let N be multiplex of D, & also let it be ỹ first multiplex which is greater then FG: wherfore agayn FG is not lesse then M. But GH is greater then D: wherefore the whole FH, excedeth D & M, that is, N: but K excedeth not N. For FG which is greater then GH, that is, then K exceedeth not N. And so followyng the same order we did before, we shall performe the demonstration. Wherefore vnequall magnitudes beyng taken, the greater hath to one and the same magnitude a greater proportion then hath the lesse: and that one and the same magnitude hath to the lesse a greater proportion then it hath to the greater: which was required to be proued.

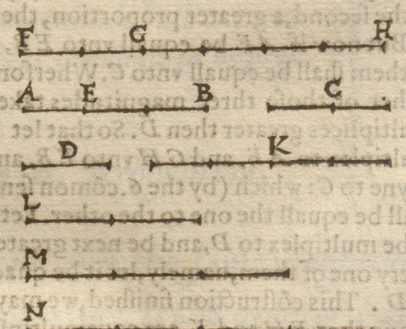
The second case of the first part.



¶ *For that Orontius seemeth to demonstrate this more plainly therefore I thought it not amisse here to set it.*

Suppose that there be two vnequall magnitudes, of which let *AB* be the greater, and *C* the lese: and let there be a certaine other magnitude, namely, *D*. Then I say first, that *AB* hath to *D* a greater proportion then hath *C* to *D*. For forasmuch as by supposition *AB* is greater then the magnitude *C*: therefore the magnitude *AB* contayneth the same magnitude *C*, and an other magnitude besides. Let *EB* be equall vnto *C*: and let *AE* be the part remayning of the same magnitude. Now *AE* and *EB* are eyther vnequall or equall the one to the other. First let them be vnequall: and let *AE* be lesse then *EB*. And vnto *AE* the lesse take an multiplex whatsoeuer, so that it be greater then the magnitude *D*: and let the same be *FG*. And how multiplex *FG* is to *AE*, so multiplex let *GH* be to *EB*, and *K* to *C*. Agayne take the duple of *D*, which let be *L*, and then the triple, and let the same be *M*. And so forward, alwayes adding one vntill there be produced such a multiplex to *D* which shall be next greater then *GH* (that is, which amongest the multiplices of *D*, by the continuall addition of

First differēce of the first part.

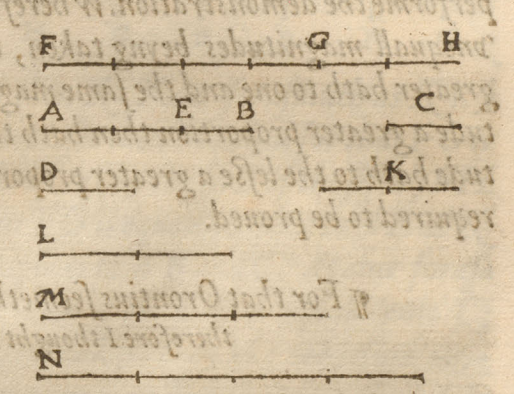


on of one, doth first beginne to exceede GH) and let the same be $\mathcal{N}$: which let be quadruple to D: Now then the multiplex GH is the next multiplex lesse then $\mathcal{N}$, and therefore is not lesse then M, that is, is either equall vnto it or greater then it. And forasmuch as FG is equemultiplex to AE, as GH is to EB, therefore how multiplex FG is to AE, so multiplex is FH to AB (by the first of the fift). But how multiplex FG is to AE, so multiplex is K to C, therefore how multiplex FH is to AB, so multiplex is K to C. Moreouer forasmuch as GH and K are equemultiplices vnto EB and C: and EB is by construction equall vnto C, therfore (by the common sentence) GH is equall vnto K. But GH is not lesse then M, as hath before bene shewed, and FG was put to be greater then D. Wherefore the whole FH is greater then these two D and M. But D and M are equall vnto $\mathcal{N}$. For $\mathcal{N}$ is quadruple to D. And M being triple to D, doth together with D make quadruple vnto D. Wherefore FH is greater then N. Farther, K is proued to be equall to GH. Wherefore K is lesse then $\mathcal{N}$. But FH and K are equemultiplices vnto AB and C, vnto the first magnitude, I say, and the third: and N is a certaine other multiplex vnto D, which representeth the second & the fourth magnitude. And the multiplex of the first excedeth the multiplex of the second: but the multiplex of the third excedeth not the multiplex of the fourth. Wherefore AB the first hath vnto D the second a greater proportion, then hath C the third to D the fourth (by the 8. definition of thys booke).

Demonstratiō of the same first differēce.

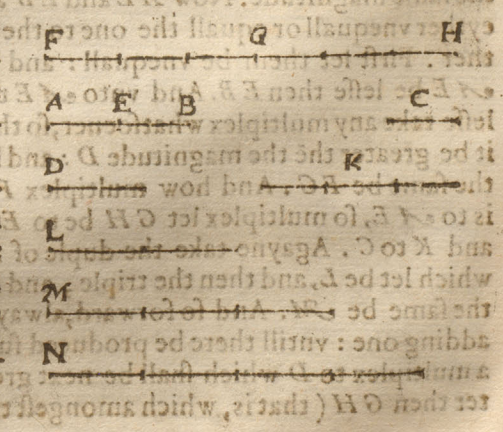
But if AE be greater then EB, let EB the lesse be multiplied vntill there be produced a multiplex greater then the magnitude D: which let be GH. And how multiplex GH is to EB, so multiplex let FG be to AE, and K also to C. Then take vnto D such a multiplex as is next greater then FG: and againe let the same be $\mathcal{N}$, which let be quadruple to D. And in like sort as before may we proue, that the whole FH is vnto AB equemultiplex as GH is to EB: and also that FH & K are equemultiplices vnto AB & C: and finally that GH is equall vnto K. And forasmuch as the multiplex $\mathcal{N}$ is next greater then FG: therefore FG is not lesse then M. But GH is greater then D by construction. Wherefore the whole FH is greater then D and M: and so consequently is greater then N. But K excedeth not N: for K is equall to GH: for how multiplex K is to EB the lesse, so multiplex is FG to AE the greater. But those magnitudes which are equemultiplices vnto vnequall magnitudes are according to the same proportion vnequall. Wherefore K is lesse then FG, and therefore is much lesse then $\mathcal{N}$. Wherefore againe the multiplex of the first exceedeth the multiplex of the second: but the multiplex of the third excedeth not the multiplex of the fourth. Wherefore (by the 8. definition of the fift) AB the first, hath to D the second, a greater proportion, then hath C the third to D the fourth.

Second difference.



But now if AE be equall vnto EB, eyther of them shall be equall vnto C. Wherfore vnto either of those three magnitudes take ecuemultiplices greater then D. So that let FG be multiplex to AE, and GH vnto EB, and K agayne to C: which (by the 6. cōmon sentence) shall be equall the one to the other. Let $\mathcal{N}$ also be multiplex to D, and be next greater then euery one of them, namely, let it be quadruple to D. This cōstruction finished, we may again proue that FH and K are equemultiplices to AB and C: and that FH the multiplex of the first magnitude exceedeth N the multiplex of the second magnitude: and that K the multi-

Third difference.



plex

plex of the third excedeth not the multiplex of the fourth. Wherfore we may conclude that *AB* hath vnto *D* a greater proportion, then hath *C* to *D*.

Now also I say, that the self same magnitude *D* hath vnto the lesse magnitude *C* a greater proportion, thē it hath to the greater *AB*. And this may plainly be gathered by the foresayd discourse, without chaunging the order of the magnitudes & of the equemultiplices. For seing that euery way it is before proued, that *FH* excedeth *N*, and *K* is exceeded of the selfe same *N*: therefore conuersedly *N* excedeth *K*, but doth not excede *FH*. But *N* is multiplex to *D* that is to the first and third magnitude: and *K* is multiplex to the second, namely, to *C*: and *FH* is multiplex to the fourth, namely, to *AB*: Wherefore the multiplex of the first excedeth the multiplex of the second: but the multiplex of the third excedeth not the multiplex of the fourth. Wherefore (by the 8. definition of this fift booke) *D* the first hath vnto *C* the second a greater proportion, then hath *D* the third to *AB* the fourth: which was required to be proued.

The second part of this proposition.

The 9. Theoreme. The 9. Proposition.

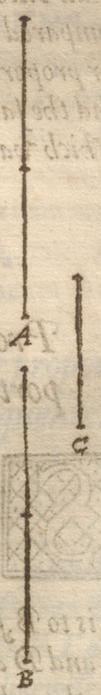
Magnitudes which haue to one and the same magnitude one and the same proportion: are equall the one to the other. And those magnitudes vnto whome one and the same magnitude hath one and the same proportion: are also equall.

Suppose that either of these two magnitudes A and B haue to C one and the same proportion. Then I say that A is equall vnto B. For if it be not, then either of these A and B should not haue to C one & the same proportiō (by the 8. of ỹ fifth) but by supposition they haue, wherefore A is equall vnto B.

The first part of this Proposition demonstrated.

Againe, suppose that the magnitude C haue to either of these magnitudes A and B one and the same proportion. Then I say that A is equall vnto B. For if it be not, C should not haue to either of these A and B one and the same proportion (by the former proposition): but by supposition it hath, wherfore A is equall vnto B. Wherfore magnitudes which haue to one and the same magnitude one and the same proportion, are equall the one to the other. And those magnitudes vnto whome one and the same magnitude hath one and the same proportion, are also equall: which was required to be proued.

The second part proued.



The 10. Theoreme. The 10. Proposition.

Of magnitudes compared to one and the same magnitude, that which hath the greater proportion, is the greater. And that magnitude wherunto one and the same magnitude hath the greater proportion, is the lesse.

The first part of this proposition proued.

SVppose that A haue to C a greater proportion then B hath to C. Then I say that A is greater then B. For if it be not, then either A is equall vnto B or lesse then it. But A cannot be equal vnto B, for then either of these A and B should haue vnto C one and the same proportion (by the 7 of the fifth): but by supposition they haue not, wherfore A is not equall vnto B. Neither also is A lesse then B, for thē should A haue to C a lesse proportion, then hath B to C (by the 8. of the fifth) but by supposition it hath not. Wherfore A is not lesse then B. And it is also proued that it is not equall, wherfore A is greater then B.

The second part demonstrated.

Agayne suppose that C haue to B a greater proportion then C hath to A. Then I say that B is lesse then A. For if it be not, then is it either equall vnto it or els greater, but B cannot be equall vnto A, for then should C haue to either of these A and B one and the same proportion (by the 7. of the fifth) but by supposition it hath not, wherfore B is not equall vnto A. Neither also is B greater then A, for then should C haue to B a lesse proportion then it hath to A (by the 8. of the fifth) but by supposition it hath not: wherefore B is not greater then A. And it was proued that it is not equall vnto A, wherfore B is lesse then A. Wherfore of magnitudes compared to one and the same magnitude, that which hath ỹ greater proportion, is the greater. And that magnitude wherunto one and the same magnitude hath the greater proportion, is the lesse. Which was required to be proued.

The 11. Theoreme. The 11. Proposition.

Proportions which are one and the selfe same to any one proportion, are also the selfe same the one to the other.

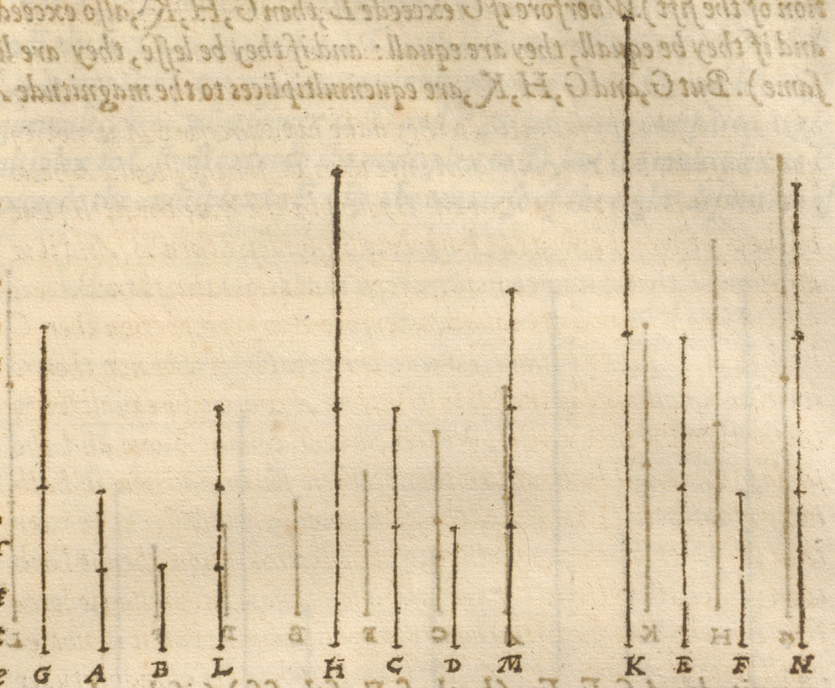
Construction.

SVppose that as A is to B, so is C to D, and as C is to D, so is E to F. Then I say that as A is to B, so is E to F. Take equemultiplices to A, C and E, which let be G, H, K. And likewise to B, D and F take any other equemultiplices, which let be L, M, and N. And because as A is to B, so is C to D: and to A and C are taken equemultiplices G & H, & to B and D are take̅ certaine other equemultiplices L & M. If therfore G exceede L, then also H excedeth M, and if it be equall it is equall, and if it be lesse it is lesse (by the conuerse of the 6. definition of the fifth). Agayne because that as C is to D, so is E to F: and to C and E are taken equemultiplices H and K: and likewise to D & F are take̅ certaine other equemultiplices M & N. If therfore H exceede M, then also K excedeth N: and if it be equall, it is equall, and if it be lesse, it is lesse (by the same conuerse) But if K exceede M, then also G exce-

Demonstration.

deth

deth L: and if it be equall, and if it be leſſe, it is leſſe (by the ſame conuerſe) Wherfore if G excede L, then K alſo excedeth N, and if it be equall it is equall, and if it be leſſe, it is leſſe. But G & K are equemultiplices of A & E. And L & N are certaine other equemultiplices of B & F. Wherfore (by the 6. definition) as A is to B, ſo is E to F. Proportions therfore which are one and the ſelfe ſame to any one proportion, are alſo the ſelfe ſame one to the other: which was required to be proued.



The 12. Theoreme. The 12. Propoſition.

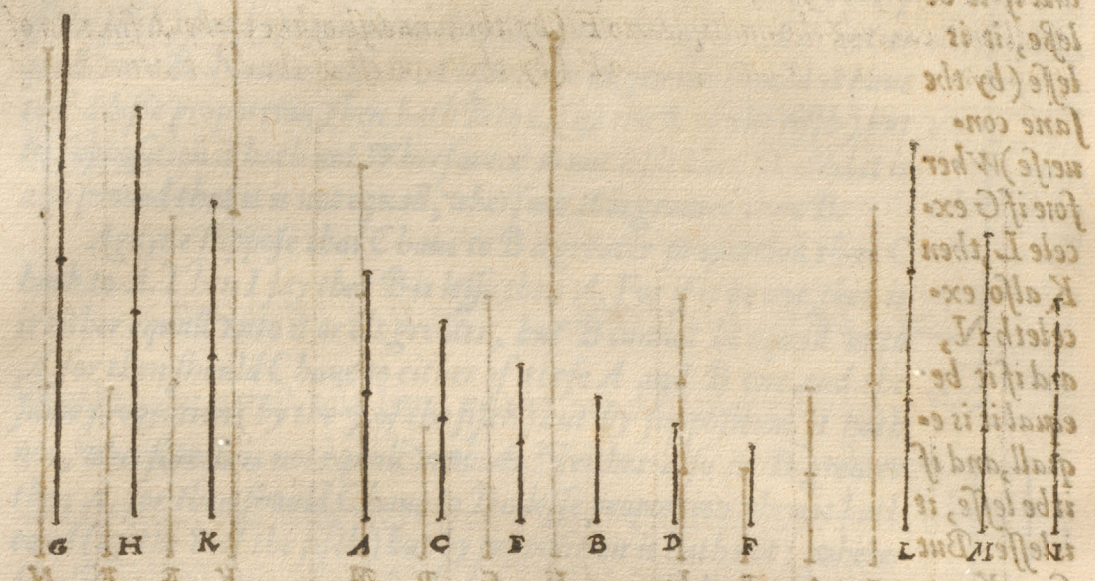
If there be a number of magnitudes how many ſoeuer proportionall: as one of the antecedentes is to one of the cõſequentes, ſo are all the antecedentes to all the conſequentes.

SVppoſe that there be a number of magnitudes how many ſoeuer, namely, A, B, C, D, E, F, in proportion: ſo that as A is to B, ſo let C be to D, and E to F. Then I ſay, that as A is to B, ſo is A, C, E, to B, D, F. Take equemultiplices to A, C, and E. And let the ſame be G, H, K. And likewiſe to B, D, and F, take any other equemultiplices, which let be L, M, N. And becauſe that as A is to B, ſo is C to D, and E to F. And to A, C, E, are taken equemultiplices G, H, K: and likewiſe to B, D, F, are taken certaine other equemultiplices L, M, N. If therefore G exceede L, H alſo exceedeth M, and K, N: and if it be equall, it is equall, and if it be leſſe, it is leſſe (by the conuerſe of the ſixt defini-

Conſtruction.

Demonſtration.

tion of the fift). Wherfore if G exceede L, then G, H, K, alſo exceede L, M, N: and if they be equall, they are equall: and if they be leſſe, they are leſſe (by the ſame). But G, and G, H, K, are equemultiplices to the magnitude A, and to the



magnitudes A, C, E. For (by the firſt of the fift) (if there be a number of magnitudes equemultiplices to a like number of magnitudes ech to ech, how multiplex one magnitude is to one, ſo multiplices are all the magnitudes, to all). And by the ſame reaſon alſo L, and L, M, N, are equemultiplices to the magnitude B, and to the magnitudes B, D, F: Wherefore as A is to B, ſo is A, C, E, to B, D, F (by the ſixt definition of the fift). If therefore there be a number of magnitudes how many ſoeuer, proportionall: as one of the antecedentes is to one of the conſequentes, ſo are all the antecedentes to all the conſequentes: which was required to be proued.

The 13. Theoreme. The 13. Propoſition.

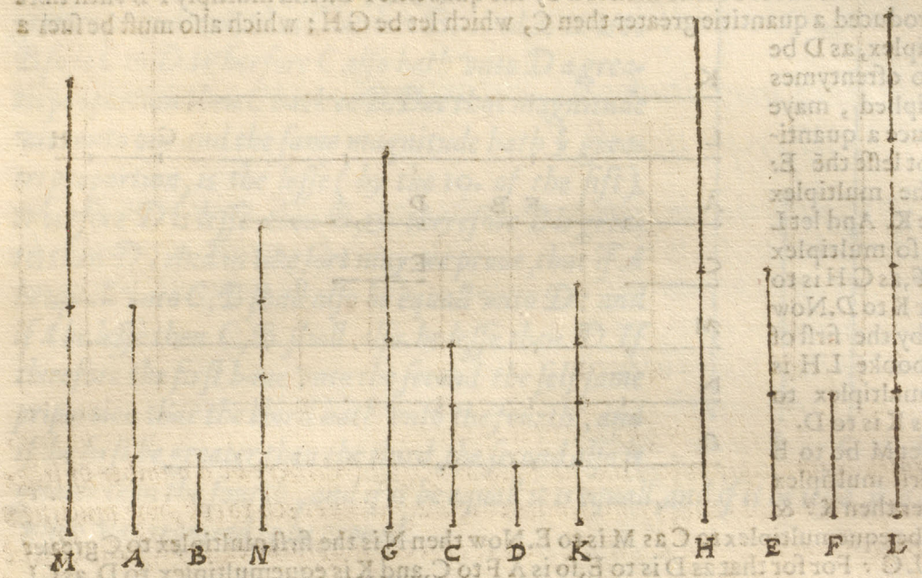
If the firſt haue vnto the ſecond the ſelf ſame proportion that the third hath to the fourth, and if the third haue vnto the fourth a greater proportiõ thẽ the fifth hath to the ſixth: thẽ ſhall the firſt alſo haue vnto the ſecond a greater proportion then hath the fifth to the ſixth.

SVppoſe that there be ſixe magnitudes, of which let A be the firſt, B the ſecond, C the third, D the fourth, E the fifth, and F the ſixth. Suppoſe that A the firſt haue vnto B the ſecond, the ſelf ſame proportion that C the third hath to D the fourth. And let C the third haue vnto D the fourth, a greater proportion then hath E the fifth to F the ſixth.

Then

Then I say that A the first hath to B the second a greater proportion, then hath E the fifth to F the sixt. For forasmuch as C hath to D a greater proportion then hath E to F, therfore there are certaine equemultiplices to C and E, and likewise any other equemultiplices whatsoeuer to D and F, which being compared together, the multiplex to C shall exceede the multiplex to D, but the multiplex to E shall not exceede the multiplex to F (by the conuerse of the eight definition of this booke). Let those multiplices be taken, and suppose that the equemultiplices to C and E, be G and H: and likewise to D and F take any other equemultiplices whatsoeuer, and let the same be K and L, so that let G exceede K, but let not H exceede L. And how multiplex G is to C, so multiplex let M be to A. And how multiplex K is to D, so multiplex also let N be to B. And because that as A is to B, so is C to D: and to A and C are taken equemultiplices M and G. And likewise to B and D are taken certayne other equemultiplices N & K: if therfore M exceede N, G also excedeth K: and if it be equall it is equall, and if it be lesse it is lesse (by the conuersion of the sixt definition of the fifth.) But by construction G excedeth K, wherfore M also excedeth N, but H excedeth not L. But M & H are equemultiplices to A & E: and N & L are certaine other equemultiplices whatsoeuer, to B and F. Wherfore A hath vnto B a greater proportion then E hath to F (by the 8. definition.) If therefore the first haue vnto the second the selfe same proportion that the third hath to the fourth, and if the third haue vnto the fourth a greater proportion then the fifth hath to the sixth, then shall the first also haue vnto the second a greater proportion then hath the fifth to the sixth: Which was required to be proued.

Construction.



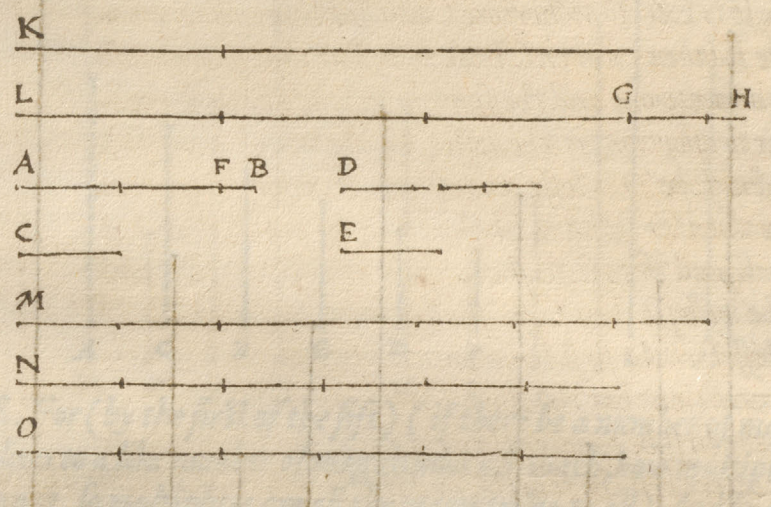
Demonstration.

¶ An addition of Campane.

An addition of Campane

If there be foure quantities, and if the firſt haue vnto the ſecond a greater proportion thē hath the third to the fourth: then ſhall there be ſome equemultiplices of the firſt and the third, which beyng compared to ſome equemultiplices of the ſecond and the fourth, the multiplex of the firſt ſhall be greater then the multiplex of the ſecond, but the multiplex of the third ſhall not be greater then the multiplex of the fourth.

Which is thus proued. Suppoſe that A B haue vnto C a greater proportion thē hath D to E. And let A F be to C as D is to E. Now then by this propoſition & the tēth, A F is leſſe then A B. Let it be leſſe then A B by the quantitie F B. And multiply F B vntil there be produced a quantitie greater then C, which let be G H: which alſo muſt be ſuch a multiplex, as D beyng ſo oftentymes multiplied, maye produce a quantitie not leſſe thē E: whiche multiplex let be K. And let L G be ſo multiplex to A F, as G H is to F B, or K to *D*. Now then by the firſt of this booke L H is equemultiplex to A B as K is to D. And let M be to E the firſt multiplex greater then K: & let N be equemultiplex to C as M is to E. Now then N is the firſt multiplex to C greater then L G: For for that as D is to E, ſo is A F to C, and K is equemultiplex to D as G L is to A F, alſo M is equemultiplex to E, as N is to C: therfore (by the 4. of this booke) as K is to M, ſo is G L to N: but K is to M the firſt multiplex leſſe then M: wherfore alſo G L is the firſt multiplex leſſe then N: and G L by ſuppoſition is not leſſe thē C. Wherfore take the greateſt multiplex of C vnder N: or a multiplex equall to N, if peraduenture N be the firſt of the multiplices of C, which let be O. Now then then N ſhall conſiſt of O and C. Wherfore foraſmuch as L G is not leſſe then O, and G H is greater then C, therfore L H ſhall be greater then N. And foraſmuch as K is leſſe then M, therfore that which was required to be proued, is manifeſt.



Although this propoſition here put by Campane nedeth no demonſtration for that it is but the conuerſe of the 8. definition of this booke, yet thought I it not worthy to be omitted, for that it teacheth the way to finde out ſuch equemultiplices, that the multiplex of the firſt ſhall excede the multiplex of the ſecond, but the multiplex of the third ſhall not exceede the multiplex of the fourth.

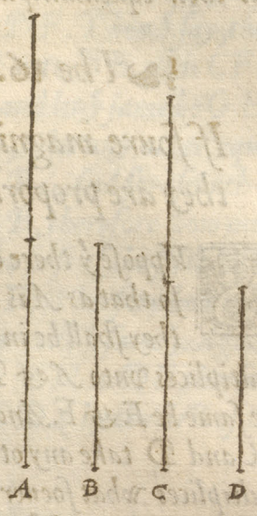
The 14. Theoreme. The 14. Propoſition.

If the firſt haue vnto the ſecond the ſelfe ſame proportion that the third hath vnto the fourth: and if the firſt be greater then the third, the ſecond alſo is greater then the fourth: and if it be equall it is equall: and if it be leſſe it is leſſe.

Suppoſe

SVppoſe that there be foure magnitudes, of which let A be the firſt, B the ſecond, C the third, and D the fourth: and let A the firſt, haue vnto B the ſecond, the ſelf ſame proportion that C the third hath vnto D the fourth. And let A be greater then C. Then I ſay, that B alſo is greater then D. For foraſmuch as A is greater then C, and there is a certaine other magnitude, namely, B, therefore (by the 8. of the fift) A hath vnto B a greater proportion then C hath to B. And as A is to B, ſo is C to D. Wherfore C alſo hath vnto D a greater proportion then C hath to B. But that magnitude wherunto one and the ſame magnitude hath ỳ greater proportion, is the leſſe (by the 10. of the fift). Wherfore D is leſſe then B, & therefore B is greater then D. And in like ſort may we proue, that if A be equall vnto C, B ſhall alſo be equall vnto D: and if A be leſſe then C, B ſhall alſo be leſſe then D. If therefore the firſt haue vnto the ſecond the ſelf ſame proportion that the third hath vnto the fourth, and if the firſt be greater then the third, the ſecond alſo is greater then the fourth, and if it be equall it is equall, and if it be leſſe it is leſſe: which was required to be proued.

Demonſtration.



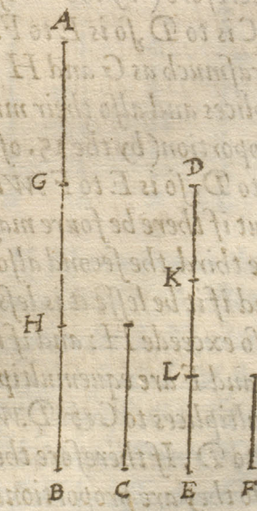
The 15. Theoreme. The 15. Propoſition.

Like partes of multiplices, and alſo their multiplices compared together, haue one and the ſame proportion.

SVppoſe that AB be equemultiplex to C, as DE is to F. Then I ſay, that as C is to F, ſo is AB to DE. For foraſmuch as how multiplex AB is to C, ſo multiplex is DE to F: therfore how many magnitudes there are in AB equall vnto C, ſo many are there in DE equall vnto F. Deuide AB into the magnitudes equall vnto C, that is, into AG, GH, and HB: and likewiſe DE into the magnitudes equall vnto F, that is, into DK, KL, and LE. Now then the multitude of theſe AG, GH, and HB, is equall to ỳ multitude of theſe DK, KL, and LE. And foraſmuch as AG, GH, and HB, are equall the one to ỳ other: and likewiſe DK, KL, and LE, are alſo equall the one to ỳ other: therfore as AG is to DK, ſo is GH to KL, and HB

Conſtruction.

Demonſtration.



to L E. Wherefore (by the 12. of the fift) as one of ỹ antecedentes is to one of the consequentes, so are all the antecedentes to all the consequentes. Wherfore as A G is to D K, so is A B to D E. But A G is equall vnto C, and likewise D K to F. Wherefore as C is to F, so is A B to D E. Like partes therefore of multiplices and also their multiplices compared together, haue one and the same proportion that their equemultiplices haue: which was required to be demonstrated.

The 16. Theoreme. The 16. Proposition.

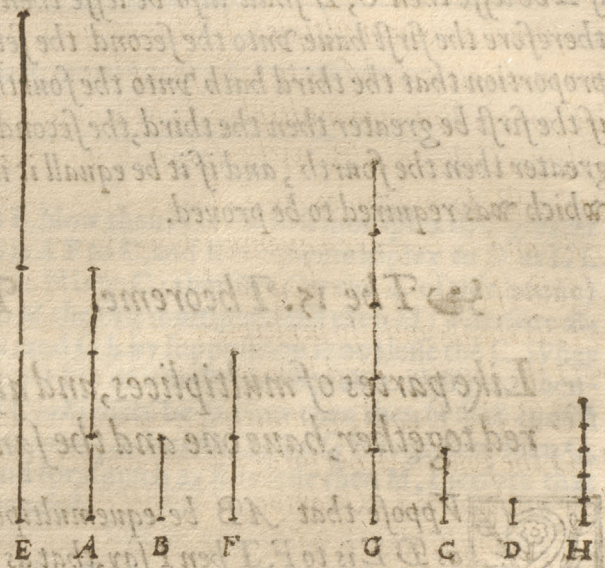
If foure magnitudes be proportionall: then alternately also they are proportionall.

Demõstration of alternate proportion.

SVppose ỹ there be foure magnitudes proportionall, namely, A, B, C, D, so that as A is to B, so let C be to D. Then I say, that alternately also they shall be in proportion, that is, as A is to C, so is B to D. Take equemultiplices vnto A & B, and let the same be E & F. And likewise to C and D take any other equemultiplices what soeuer, and let the same be G and H. And forasmuch as how multiplex E is to A, so multiplex is F to B, but like partes of multiplices & also their multiplices haue one and the selfe same proportion the one to the other (by the former Proposition). Wherefore as A is to B, so is E to F. But as A is to B, so is C to D, wherefore (by the 11. of the fift) as C is to D, so is E to F. Againe, forasmuch as G and H are equemultiplices to C and D, but like partes of multiplices and also their multiplices haue the one to the other one and the selfe same proportion (by the 15. of the fift). Wherefore as C is to D, so is G to H. But as C is to D, so is E to F. Wherefore as E is to F, so is G to H (by the 11. of the fift). But if there be foure magnitudes in proportion, and if the first be greater then the third, the second also is greater then the fourth: and if it be equall it is equall: and if it be lesse it is lesse (by the 14. of the fift). If therfore E excede G, F shall also exceede H: and if it be equall, it is equall: and if it be lesse, it is lesse. But E and F are equemultiplices to A and B: and G and H are certaine other equemultiplices to C & D. Wherfore (by the 6. definition of the fift) as A is to C, so is B to D. If therefore there be foure magnitudes proportionall, then alternately also they are proportionall: which was required to be proued.

Construction.

Demonstration.



The

The 17. Theoreme. The 17. Proposition.

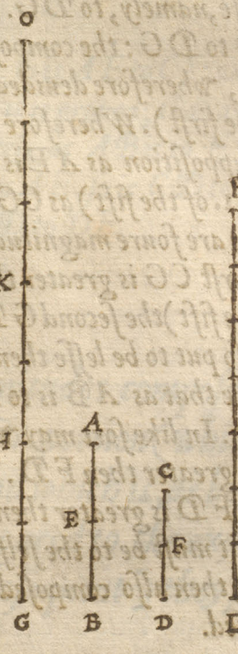
If magnitudes composed be proportionall, then also deuided they shall be proportionall.

Demonstratiõ of proportion by diuision.

Construction.

Demonstration.

SVppose that the magnitudes composed being proportionall, be AB, BE, CD, DF, so that as AB is to BE, so is CD to DF. Then I say, that deuided also they shall be proportionall, as AE is to BE, so is CF to DF. Take equemultiplices vnto AE, EB, CF, FD, and let ỹ same be GH, HK, LM, and MN. And likewise to EB, and FD, take any other equemultiplices what soeuer, and let the same be KO, and NP. And forasmuch as how multiplex GH is to AE, so multiplex is KH to EB, therefore how multiplex GH is to AE, so multiplex is GK to AB (by the first of the fifth). But how multiplex GH is to AE, so multiplex is LM to CF: Wherfore how multiplex GK is to AB, so multiplex is LM to CF (by the 11. of ỹ same). Againe forasmuch as how multiplex LM is to CF, so multiplex is MN to DF, therfore how multiplex LM is to CF, so multiplex is LN to CD (by the first of the self same). But how multiplex LM is to CF, so multiplex is GK to AB. Wherfore how multiplex GK is to AB, so multiplex is LN to CD. Wherefore GK and LN, are equemultiplices to AB & CD. Againe forasmuch as how multiplex HK the first is to EB the second, so multiplex is MN the third to FD the fourth. And how multiplex KO the fift is to EB the second, so multiplex is NP the sixt to FD the fourth. Wherfore (by the second of the same) how multiplex HO composed of the first and fift is to EB, so multiplex is MP composed of the third and sixt to FD. And for that as AB is to BE, so is CD to DF: and to AB & CD are taken equemultiplices GK and LN: and likewise to EB and FD are taken certaine other equemultiplices, that is, HO, and MP. If therefore GK exceede HO, then LN also exceedeth MP: and if it be equall, it is equall: and if it be lesse, it is lesse (by the conuersion of the sixt definition of the fift). Let GK exceede HO: Wherefore KH common to them both being taken away, the residue GH, shall exceede the residue KO. But if GK exceede HO, then doth LN exceede MP: Wherefore let LN excede MP: and MN which is cõmon to thẽ both being take away, ỹ residue LM shall exceede the residue NP. Wherefore if GH exceede KO, then shall LM exceede NP. And in like sort may we proue, that if GH be equall vnto KO, then LM shall be equall vnto NP: and if it be lesse, it shall be lesse: but GH



and

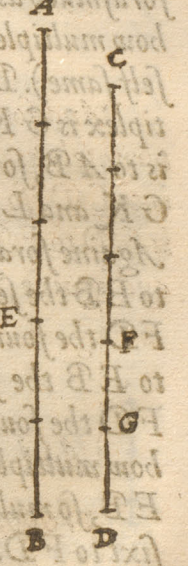
and L M are equemultiplices to A E and C F: and likewiſe K O and N P are certayne other equemultiplices to E B, and F D. Wherfore as A E is to E B, ſo is C F to F D (by the ſixt definition of the fift). If compoſed magnitudes therefore be proportionall, then alſo deuided they ſhall be proportionall: which was required to be demonſtrated.

The 18. Theoreme. The 18. Propoſition.

If magnitudes deuided be proportionall: then alſo compoſed they ſhall be proportionall.

Demonſtration of proportion by compoſition. This propoſition is the conuerſe of the former. Demonſtratiõ teading to an impoſsibility.

Vppoſe that the magnitudes deuided being proportionall, be A E, E B, C F, & F D, ſo that as A E is to E B, ſo let C F be to F D. Then I ſay, that compoſed alſo they ſhall be proportionall, that is, as A B is to B E, ſo is C D to D F. For if A B be not vnto B E, as C D is to F D, then ſhall A B be vnto B E, as C D is either vnto a magnitude leſſe then F D, or vnto a magnitude greater. Let it firſt be vnto a leſſe, namely, to D G. And foraſmuch as, as A B is to B E, ſo is C D to D G: the compoſed magnitudes therefore are proportionall, wherefore deuided alſo they ſhall be proportionall (by 17. of the firſt). Wherefore as A E is to E B, ſo is C G to G D. But by ſuppoſition as A E is to E B, ſo is C F to F D. Wherefore (by the 11. of the fift) as C G is to G D, ſo is C F to F D. Now then there are foure magnitudes, C G, G D, C F, and F D: of which the firſt C G is greater then the third C F. Wherefore (by the 14. of the fift) the ſecond G D is greater then the fourth F D. But it is alſo put to be leſſe then it: which is impoſsible. Wherfore it can not be that as A B is to B E, ſo is C D to a magnitude leſſe then F D. In like ſort may we proue, that it can not be ſo to a magnitude greater then F D. For by the ſame order of demonſtration, it would follow that F D is greater then the ſayd greater magnitude: which is impoſsible. Wherfore it muſt be to the ſelfe ſame. If therefore magnitudes deuided be proportionall, then alſo compoſed they ſhall be proportionall: which was required to be proued.

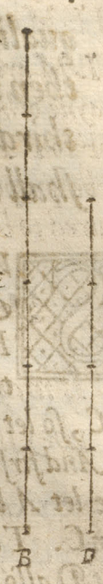


The 19. Theoreme. The 19. Propoſition.

If the whole be to the whole, as the part taken away is to the part taken away: then ſhall the reſidue be vnto the reſidue, as the whole is to the whole.

Suppoſe

SVppose that as the whole AB is to the whole CD, so is the part taken away AE to the part taken away CF. Then I say, that the residue EB shall be vnto the residue FD, as the whole AB is to the whole CD. For for that as the whole AB is to the whole CD, so is AE to CF, therfore alternately also (by the 16. of the fift) as AB is to AE, so is CD to CF. And for that when magnitudes composed are proportionall, the same deuided also are proportionall (by the 17. of the fift): therefore as BE is to EA, so is DF to FC. Wherfore alternately also (by the 16. of the fift) as BE is to DF, so is EA to FC. But as AE is to CF, so (by supposition) is the whole AB to the whole CD. Wherefore the residue EB shall be vnto the residue FD, as the whole AB is to the whole CD. If therfore the whole be to the whole, as the part taken away is to the part taken away, then shall the residue be vnto the residue, as the whole is to the whole: which was required to be proued.



That which the fift of this booke proued only touching multiplices, this proueth generally of all magnitudes.

¶ A Lemma or Assumpt.

A Lemma.

And forasmuch as by supposition as AB is to CD, so is AE to CF: and alternately as AB is to AE, so is CD to CF. And now it is proued, that as AB is to CD, so is EB to FD. Wherefore againe alternately, as AB is to EB, so is CD to FD. Wherefore it followeth, that as AB is to AE, so is CD to CF: and againe, as the same AB is to EB, so is the same CD to DF.

Corollary.

A Corollary.

Conuersion of proportion.

And hereby it is manifest, that if magnitudes composed be proportionall, then also by conuersion of Proportion (which of some is called Proportion by Euersion, and which is, as before it was defined, whē the antecedent is compared to the excesse, wherein the antecedent exceedeth the consequent) they shall be proportionall.

The 20. Theoreme. The 20. Proposition.

If there be three magnitudes in one order, and as many other magnitudes in an other order, which being taken two and two in eche order, are in one and the same proportion, and if of e-

qualitie

qualitie in the first order the first be greater then the third, then in the second order the first also shall be greater then the third: and if it be equall, it shall be equall: and if it be lesse, it shall be lesse.

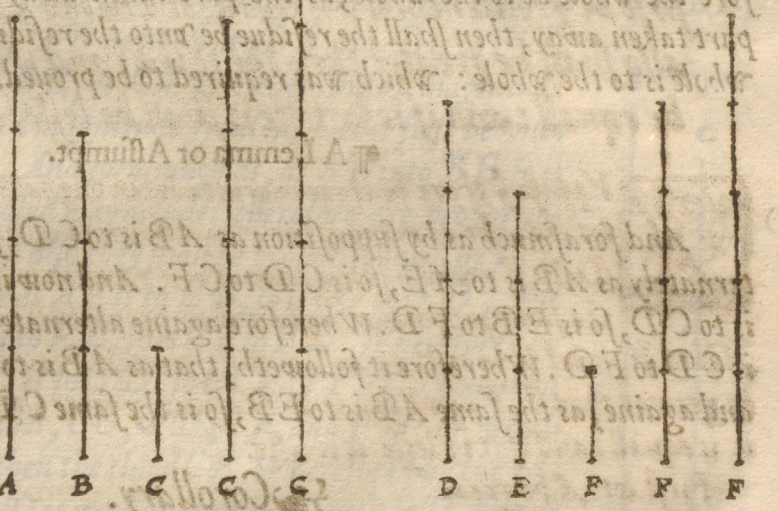
This proposition pertaineth to Proportion of equality inordinate proportionality.

SVppose that there be three magnitudes in one order, namely, A, B, C, & let there be as many magnitudes in an other order, which let be D, E, F, which being taken two and two in ech order, let be in one and the same proportion, that is, as A is to B, so let D be to E, and as B is to C, so let E be to F.

The first difference.

And first of equalitie let A be greater then C. Then I say, that D also is greater then F: and if it be equall, it is equall, and if it be lesse, it is lesse.

Demonstration.



For forasmuch as A is greater then C, and there is a certaine other magnitude, namely, B, but the greater hath to one and the same magnitude a greater proportion then hath the lesse (by the 8. of the fift). Wherefore A hath vnto B a greater proportion then C hath to B. But as A is to B, so is D to E: and as C is to B, so is F to E (by the Corellary of the fourth of the fift). Wherefore D hath vnto E a greater proportion, then F hath to E. But of magnitudes compared to one and the same magnitude that which hath the greater proportion is the greater (by the 10. of the fift). Wherefore D is greater then F.

The second difference.

But now if A be equall vnto C, D also shall be equall vnto F. For then A and C haue vnto B one and the same proportion (by the first part of the seuenth of this booke). And for that as A is to B, so is D to E, and as C is to B, so is F to E: therefore D and F haue vnto E one and the same proportion. Wherefore by the first part of the 9. of this booke D is equall vnto F.

The third difference.

But now suppose that A be lesse then C. Then also shall D be lesse then F. For by the 8. of this booke C shall haue vnto B a greater proportion then hath A to B. But as A is to B, so is D to E by supposition, and as C is to B, so haue we proued is F to E. Wherefore F hath vnto E a greater proportion then hath D to E. Wherefore by the first part of the 10. of this booke F is greater then D.

If therefore

If therefore there be three magnitudes in one order, and as many other magnitudes in an other order, which being taken two and two in ech order, are in one and the same proportion, and if of equalitie in the first order the first be greater then the third, then in the second order also the first shall be greater then ỹ third: and if it be equall, it shall be equall: and if it be lesse, it shall be lesse: which was required to be proued.

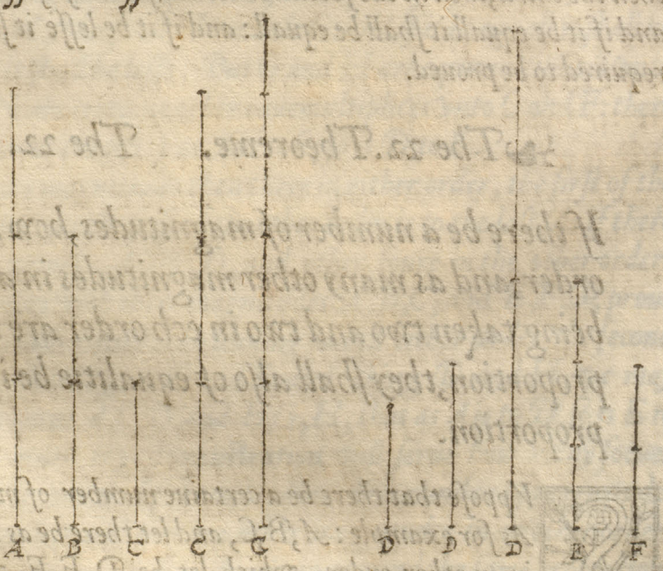
The 21. Theoreme. The 21. Proposition.

If there be three magnitudes in one order, and as many other magnitudes in an other order, which being taken two and two in eche order are in one and the same proportion, and their proportion is perturbate: if of equalitie in the first order the first be greater then the third, thē in the second order the first also shall be greater then the third: and if it be equall it shall be equall: and if it be lesse it shall be lesse.

This proposition pertaineth to Proportion of equality in perturbate proportionality.

SVppose ỹ there be three magnitudes in one order, namely, *A, B, C*, and let there be as many other magnitudes in an other order, which let be *D, E, F*: which being taken two & two in ech order, let be in one and the same proportion, and let their proportion be perturbate. So that as *A* is to *B*, so let *E* be to *F*, & as *B* is to *C*, so let *D* be to *E*, and of equalitie let *A* be greater then *C*. Then I say, that *D* also is greater then *F*: and if it be equall it is equall: and if it be lesse it is lesse.

The first difference.

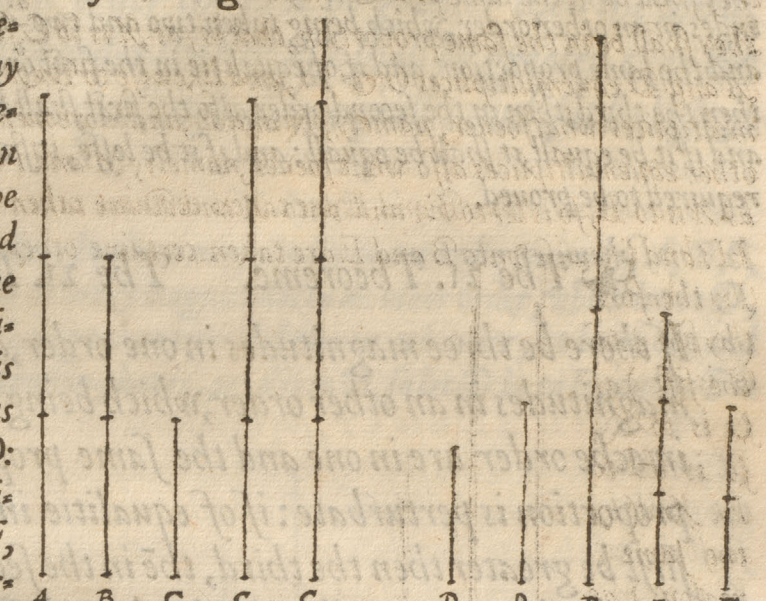


First let *A* be greater then *C*. And for asmuch as *A* is greater then *C*, & there is a certaine other magnitude, namely, *B*: therfore (by the 8. of ỹ fift) *A* hath a greater proportiō vnto *B*, then *C* hath to *B*. But as *A* is to *B*, so is *E* to *F*: and againe as *C* is to *B*, so is *E* to *D* (by the Corellary of ỹ fourth of the fift). Wherfore *E* hath vnto *F* a greater proportion, then *E* hath to *D*. But that magnitude wherunto one and the same magnitude hath the greater proportion, is the lesse (by the 10. of the fift). Where-

fore F is leſſe then D. Wherefore D is greater then F.

The ſecond difference.

But now if A be equall vnto C, we may alſo proue that D is equall vnto F. For then (by the firſt part of the 7. of this booke) A and C ſhall haue vnto B one and the ſame proportion. And for that as A is to B, ſo is E to F, & as C is to B, ſo is E to D: therefore E hath to either of theſe D and F, one and ỹ ſame proportion. Wherefore by the ſecond part of the 9. of the fift D is equall vnto F.



The third difference.

Likewiſe, if A be leſſe then C, D alſo is leſſe then F. For then C ſhall haue vnto B a greater proportion, then hath A to B (by the 8. of the fift. Wherefore E alſo hath vnto D a greater proportion then it hath to F. Wherefore by the ſecond part of the 10. of this booke D is leſſe then F. If therefore there be three magnitudes in one order, & as many other magnitudes in an other order, which being taken two and two in ech order are in one & the ſame proportion, & their proportion is perturbate, and if of equalitie in the firſt order the firſt be greater then the third, then in the ſecond order the firſt alſo ſhall be greater then ỹ third: and if it be equall it ſhall be equall: and if it be leſſe it ſhall be leſſe: which was required to be proued.

The 22. Theoreme. The 22. Propoſition.

If there be a number of magnitudes, how many ſoeuer in one order, and as many other magnitudes in an other order, which being taken two and two in ech order are in one and the ſame proportion, they ſhall alſo of equalitie be in one and the ſame proportion.

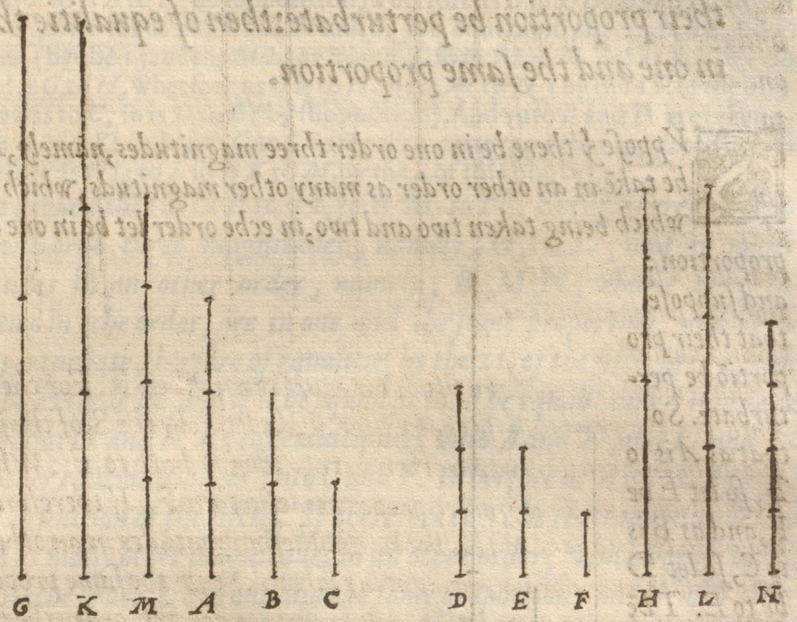
Proportion of equality in ordinate proportionality.

SVppoſe that there be a certaine number of magnitudes in one order. As for example: A, B, C, and let there be as many other magnitudes in an other order, which let be D, E, F, which being taken two and two let be in one and the ſame proportion. So that as A is to B, ſo let D be to E, and as B is to C, ſo let E be to F. Then I ſay, that of equalitie they

they shall be in the same proportion, that is, as A is to C, so is D to F. Take vnto A and D equemultiplices G & H, and likewise to B & E take any other equemultiplices whatsoeuer, namely, K and L, and moreouer vnto C and F take any other equemultiplices also whatsoeuer, namely, M and N. And forasmuch as, as A is to B, so is D to E: and vnto A and D are taken equemultiplices G and H: and likewise vnto B and E are taken certaine other equemultiplices K and L: therfore (by the 4. of the fift) as G is to K, so is H to L. And (by the same reason) as K is to M, so is L to N. Seing therefore ỹ there be in order three magnitudes G, K, M, & as many other magnitudes in an other order, namely, H, L, N, which being compared two to two are in one and the same proportion, therefore of equalitie (by the 20. of the fift) if N exceede M, then shall H exceede G: and if it be equall it shall be equall: and if it be lesse it shall be lesse. But G and H are equemultiplices vnto A and D, and M and N are certaine other equemultiplices vnto C and F: therfore (by the 6. definition of the fift) as A is to C, so is D to F.

Construction.

Demonstration.



So also if there be more magnitudes then three in either order, the first of the one order shall be to the last, as the first of the other order is to the last. As if there were foure in one order, namely, A B C D, and other foure in the other order, namely, E F G H, we may with three magnitudes A, B, C, and E, F, G, proue that as A is to C, so is E to G: And then leauing out in either order the second and taking the fourth, as leauing out B and F, and taking D and H, we may proue by these three and three A, C, D, and E, G, H, that as A is to D, so is E to H. And obseruing this order, thys demonstration will serue how many soeuer the magnitudes be in either order. If therefore there be a number of magnitudes how many soeuer in one order, and as many other magnitudes in an other order, which being taken two and two in eche order are in one and the same proportion, they shall also of equalitie be in one and the same proportion: which was required to be demonstrated.

When there are more then three magnitudes in either order.

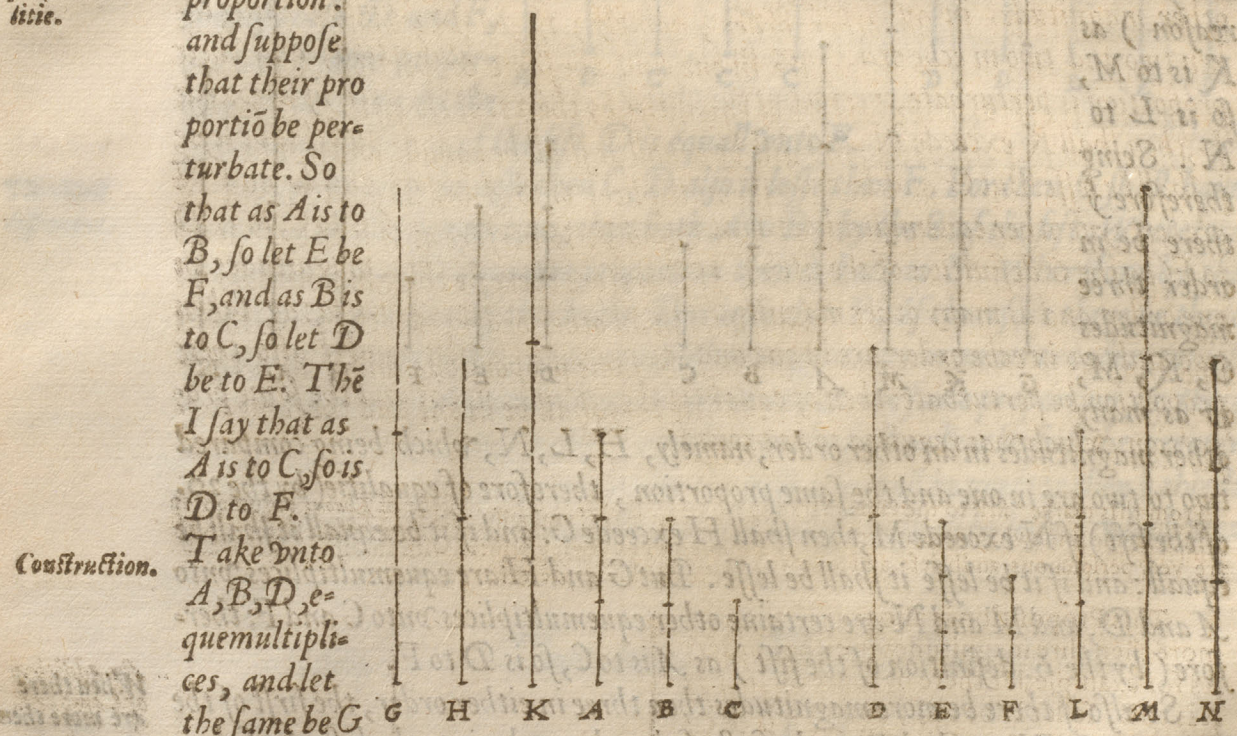
A B C D
E F G H

The 23. Theoreme. The 23. Propoſition.

If there be three magnitudes in one order, and as many other magnitudes in an other order, which beyng taken two & two in eche order are in one and the ſame proportion, and if alſo their proportion be perturbate: then of equalitie they ſhall be in one and the ſame proportion.

Proportion of eqnality in perturbate proprotionalitie.

Suppoſe ẏ there be in one order three magnitudes, namely, A, B, C, & let be takē in an other order as many other magnituds, which let be D, E, F which being taken two and two, in eche order let be in one and the ſame proportion: and ſuppoſe that their proportiō be perturbate. So that as A is to B, ſo let E be F, and as B is to C, ſo let D be to E. Thē I ſay that as A is to C, ſo is D to F.



Construction.

Take vnto A, B, D, equemultiplices, and let the ſame be G H K: and likewiſe vnto C, E, F, take any other equemultiplices whatſoeuer, and let ẏ ſame be L M N.

Demonſtration.

And foraſmuch as G and H are equemultiplices vnto A and B, but the partes of equemultiplices are in the ſame proportion that their equemultiplices are (by ẏ 15. of the fift) wherfore as A is to B, ſo is G to H. And by ẏ ſame reaſon alſo as E is to F, ſo is M to N. But as A is to B, ſo is E to F. Wherfore (by ẏ 11. of ẏ fift) as G is to H, ſo is M to N. And foraſmuch as, as B is to C, ſo is D to E, and vnto B & D are taken equemultiplices H & K: and likewiſe vnto C and E are taken certayne other equemultiplices L and M: therfore (by the 4. of the fifth) as H is to L, ſo is K to M, and alternately alſo (by the 16. of the fift) as B is to D, ſo is C to E. And foraſmuch as H and K are the equemultiplices of B and D, but the partes of equemultiplices are in the ſame proportion that*

their

*their equemultiplices are (by the 15. of the fift) therfore as B is to D, so is H to K. But as B is to D, so is C to E, therfore (by the 11. of the first) as H is to K, so is C to E. Agayne forasmuch as L and M are the equemultiplices of C and E, therfore as C is to E, so is L to M. But as C is to E, so is H to K: therfore as H it to K, so is L to M, and alternately (by the 16. of the fift) as H is to L, so is K to M.**

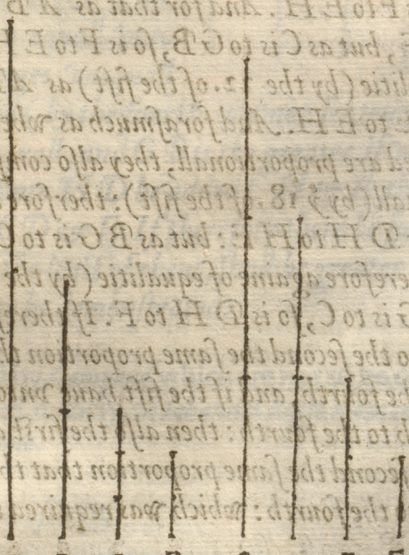
(But as *A* is to *B* so is *E* to *F* (by supposition) wherfore as *G* is to *H*, so is *E F*, (by the 11. of the fift.) Agayne for as much as *M* and *N* are equemultiplices vnto *E* aud *F*, therefore agayne (by the 15. of the fifth) as *E* is to *F*, so is *M* to *N*. But as *E* is to *F*, so haue we proued is *G* to *H*, wherfore as *G* is to *H*, so is *M* to *N* (by the 11. of the fift) And for that as *B* is to *C*, so is *D* to *E* (by supposition). And vnto *B* and *D* are taken equemultiplices *H* and *K*: and vnto *C* and *E* are taken certayne other equemultiplices *L* and *M*, therefore as *H* is to *L* so is *K* to *M*, by the 4 of this booke).

But it is proued that as G is to H, so is M to N. Seyng therefore that there are in one orde three magnitudes, namely, G, H, L, and as many other magnitudes in an other order, namely, K, M, N, whiche being taken two and two in eche order, are in one and the same proportion, and their proportion is perturbate, therfore of equalitie (by the 21. of the fifth) if G excede L, then shall K excede N, and if it be equall, it shall be equall: and if it be lesse it shall be lesse. But G and K are equemultiplices vnto A and D, and L and N are certayne other equemultiplices vnto C and F. Wherfore as A is to C, so is D to F (by the 6. definition of the fifth). If therefore there be three magnitudes in one order, and as many other magnitudes in an other order, which beyng taken two and two in eche order, are in one and the same proportion, and if also their proportion be perturbate: then of equalitie they shall be in one and the same proportion: which was required to be proued.

From this marke * first to the same marke agayne, you may if you will in stede of *Theons* argumentes which seeme somewhat intricate, read those argumentes following printed with an other letter, which are very perspicuous and briefe, and followed of the most interpreters.

This Proposition is also true if there be more then thre magnitudes in either order: As for example, let there be foure. So that as A is to B, so let E be to F, and as B is to C, so let D be to E, & as C is to P, so let Q be to D. Then I say that as A is to P, so is Q to F. For forasmuch as it is before proued, that A is to C, as D is to F taking away B & E, there shall be three magnitudes A, C, P, in one order, and as many other in an other order, namely, Q, D, F, which being take in ech order two and two, are in one and the same proportion, and their proportiõ is perturbate, wherfore by the former propositiõ A is to P as Q is to F, which was required to be demonstrated.

An addition of Campane.



Note.

And euen as by the demonstration in three magnitudes is taken the proofe in foure magnitudes by leauing out one of

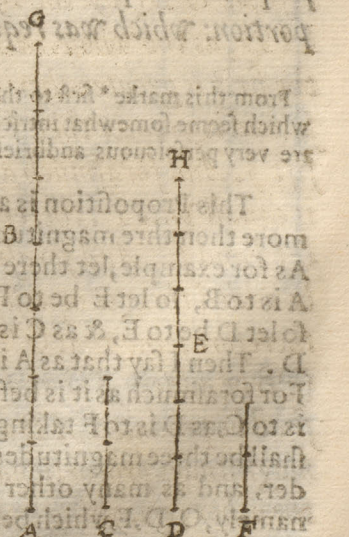
the meanes: ſo by the demonſtration in foure magnitudes is taken the proofe in fiue magnitudes by leauyng out two of the meanes: and by the demonſtration in fiue, the proofe in ſixe, by leauing out three meanes. And ſo forward cõtinually, which is alſo to be vnderſtanded in the former kynde of proportion of equalitie, which is in ordinate proportion.

The 24. Theoreme. The 24. Propoſition.

If the firſt haue vnto the ſecond the ſame proportion that the third hath to the fourth, and if the fift haue vnto the ſecond the ſame proportion that the ſixt hath to the fourth: then alſo the firſt and fift compoſed together ſhall haue vnto the ſecond the ſame proportion that the third and ſixt compoſed together haue vnto the fourth.

That which the ſecond propoſitiõ of this booke proued only touching multiplices, is here proued generally touching magnitudes.

SVppoſe that there be ſixe magnitudes AB, C, DE, F, BG, & EH: of which let AB be the firſt, C the ſecond, DE the third, F the fourth, BG the fift, and EH the ſixt. And ſuppoſe that AB the firſt, haue vnto C the ſecond, the ſame proportion that DE the third hath to F the fourth, and let BG the fift haue vnto C the ſecond the ſame proportion that EH the ſixt hath vnto F the fourth. Then I ſay, that the firſt and fift compoſed together, namely, AG, hath vnto C the ſecond the ſame proportion, that the third and ſixt compoſed together, namely, DH, hath vnto F the fourth. For, for that as BG is to C, ſo is EH to F: then alſo by conuerſion (by the Corollary of the 4. of the fift) as C is to BG, ſo is F to EH. And for that as AB is to C, ſo is DE to F, but as C is to GB, ſo is F to EH: therefore of equalitie (by the 22. of the fift) as AB is to BG, ſo is DE to EH. And foraſmuch as when magnitudes deuided are proportionall, they alſo compoſed are proportionall (by the 18. of the fift): therfore as AG is to GB, ſo is DH to HE: but as BG is to C, ſo is EH to F. Wherefore againe of equalitie (by the 22. of the fift) as AG is to C, ſo is DH to F. If therefore the firſt haue vnto the ſecond the ſame proportion that the third hath to the fourth, and if the fift haue vnto the ſecond the ſame proportion that the ſixt hath to the fourth: then alſo the firſt and fift compoſed together ſhall haue vnto the ſecond the ſame proportion that the third and ſixt compoſed together haue vnto the fourth: which was required to be proued.

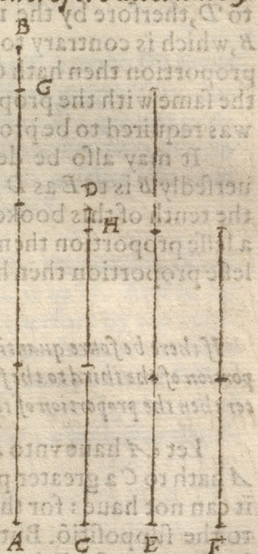


The

The 25. Theoreme. The 25. Propoſition.

If there be foure magnitudes proportionall: the greateſt and the leaſt of them, ſhall be greater then the other remayning.

SVppoſe that there be foure magnitudes proportionall AB, CD, E and F. So that as AB is to CD, ſo let E be to F. And let the greateſt of them be AB, & the leſt of them be F. Then I ſay, that theſe two magnitudes AB and F, are greater then the two magnitudes CD & E. Foraſmuch as AB is ſuppoſed to be the greateſt of all foure, therefore it is greater then E. Therefore from the greater AB cut of (by the 3. of the firſt) vnto E an equall magnitude AG. and likewiſe (by the ſame) from CD cut of vnto F an equall magnitude CH. (Which may be done, for that the magnitude CD is greater then the magnitude E: for that as AB is to CD, ſo is E to F, therefore alternately as AB is to E, ſo is CD to F (by the 16. of the fift). But AB is greater then E: Wherfore alſo CD is greater then F: Which thing may alſo be proued by ſ. 14. of the ſame.) Now for that as AB is to CD, ſo is E to F: but E is equall vnto AG, and F is equall vnto CH: therefore as AB is to CD, ſo is AG to CH: and foraſmuch, as the whole AB is to the whole CD, ſo is the part taken away AG, to the part taken away CH: therefore the reſidue GB (by the 19. of the fift) is vnto the reſidue HD, as the whole AB is to the whole CD. But AB the firſt is greater then CD the third: Wherfore GB the ſecond is greater then HD the fourth (by the 14. of the fift). And foraſmuch as AG is equall vnto E, & CH is equall vnto F: therefore AG and F are equall vnto CH and E. And foraſmuch as if vnto thinges vnequall be added thinges equall, all ſhall be vnequall (by the fourth common ſentence): therefore ſeing that GB and DH are vnequall, and GB is the greater, if vnto GB be added AG and F: and likewiſe if vnto HD be added CH & E, there ſhall be produced AB and F greater then CD & E. If therefore there be foure magnitudes proportionall, the greateſt and the leaſt of them, ſhall be greater the the other remayning: which was required to be demonſtrated.



Here follow certayne propoſitions added by *Campane*, which are not to be contemned, and are cited euen of the beſt learned, namely, of *Iohannes Regiomontanus*, in the Epitome which he writeth vpon *Ptolome*.

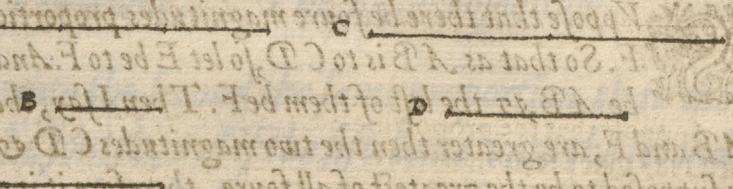
¶ The firſt Propoſition.

If there be foure quantities, and if the proportion of the firſt to the ſecond, be greater

then the proportion of the third to the fourth: then contrariwise by conuersion, the proportion of the second to the first, shall be lesse then the proportion of the fourth to the third.

Demonstratiō leading to an impossibilitie.

Suppose that the proportion of *A* to *B* be greater then the proportion of *C* to *D*. Then I say contrariwise by conuersion, the proportion of *B* to *A* is lesse then the proportion of *D* to *C*. For if the proportion of *B* to *A* be one and the same with the proportion of *D* to *C*, then conuersedly the proportion of *A* to *B* is one and the same with the proportion of *C* to *D*, which is contrary to suppositiō. But if the proportion of *B* to *A* be greater then the proportion of *D* to *C*. Then let *E* be vnto *A*, as *D* is to *C*. And by the 13. of this booke, the proportion of *E* to *A* shall be lesse then the proportion of *B* to *A*. Wherfore (by the first part of the tenth of the same) *E* is lesse thē *B*. And therfore by the secōd part of the 8. of the same, *A* hath vnto *E* a greater proportiō then *A* hath to *B*. And forasmuch as by conuerse proportionalitie *A* is vnto *E* as *C* is to *D*, therfore by the 13. of the same, *C* hath to *D* a greater proportion then hath *A* to *B*, which is contrary to the suppositiō. For *A* was supposed to haue vnto *B* a greater proportion then hath *C* to *D*. Wherefore the proportion of *B* to *A* is neither one and the same with the proportion of *C* to *D*, nor greater then it. Wherfore it is lesse: which was required to be proued.



An other demonstration of the same affirmatiuely.

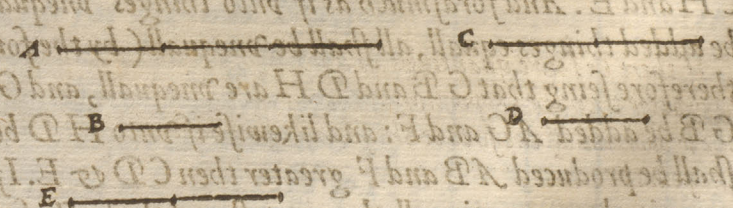
It may also be demonstrated directly. For let *E* be vnto *B* as *C* is to *D*. Then cōuersedly *B* is to *E* as *D* is to *C*. And forasmuch as *A* is greater then *E* by the first part of the tenth of this booke, therfore by the second part of the 8 of the same *B* hath vnto *A* a lesse proportion then hath *B* to *E*. Wherfore by the 13. of the same *B* hath vnto *A* a lesse proportion then hath *D* to *C*: which was required to be proued.

¶ The second Proposition.

If there be foure quantities, and if the proportion of the first to the second be greater then the proportion of the third to the fourth, then alternately the proportion of the first to the third, shall be greater then the proportion of the second to the fourth.

Demonstratiō leading to an impossibilitie.

Let *A* haue vnto *B* a greater proportion then hath *C* to *D*. Then I say alternately *A* hath to *C* a greater proportion then hath *B* to *D*. For one and the same proportion it can not haue: for then alternately *A* should be to *B* as *C* is to *D*, which is contrary to the suppositiō. But if it haue a lesse proportiō, let *E* be vnto *C* as *B* is to *D*. Now thē by the 13. of this booke *E* hath vnto *C* a greater proportion then hath *A* to *C*. Wherefore (by the first part of the tenth of the same) *E* is greater thē *A*. Wherfore by the first part of the 8. of the same, *E* hath to *B* a greater proportion then hath *A* to *B*. And forasmuch as by suppositiō *E* is vnto *C*, as *B* is to *D*, therfore alternately *E* is to *B* as *C* is to *D*. Wherfore by the 13. of the same, *C* hath to *D* a greater proportion then hath *A* to *B*, which is contrary to the suppositiō. Wherfore the proportion of *A* to *C* is neither one and the same with the proportion of *B* to *D*, nor lesse then it, wherefore it is greater. Which was required to be proued.



An other demonstration of the same affirmatiuely.

This may also be demonstrated affirmatiuely, let *E* be vnto *B* as *C* is to *D*. Now thē by the first part of the tenth of this booke, *E* is lesse then *A*: wherfore by the first parte of the 8. of the same, the proportion of *A* to *C* is greater then the proportion of *E* to *C*. But alternately *E* is to *C* as *B* is to *D*. Wherfore (by the 13. of the same) *A* hath to *C* a greater

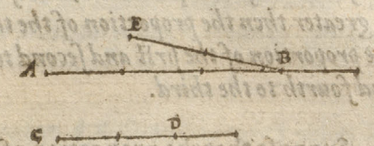
greater proportion then hath *B* to *D*: which was required to be proued.

¶ The third Proposition.

If there be foure quantities, and if the proportion of the first to the second be greater then the proportion of the third to the fourth: then by composition also the proportiō of the first and second to the second, shall be greater then the proportion of the third and fourth to the fourth.

Suppose that the proportion of *A* to *B* be greater then the proportion of *C* to *D*. Then I say that the proportion of the whole *AB* to *B* is greater then the proportion of the whole *CD* to *D*. For the proportion of *AB* to *B* can not be one and the same with the proportion of *CD* to *D*: for then by diuision also *A* should be vnto *B* as *C* is *D*: which is contrary to the suppositiō. Neither also can it be lesse. For if it be possible, let it be: & let *EB* be vnto *B* as *CD* is to *D*. Now then (by the 12. of this boke *EB* hath vnto *B* a greater proportion then hath *AB* to *B*. Wherefore by the first part of the 10. of the same *EB* is greater then the whole *AB*. And by the common sentence *E* is greater then *A*. Wherefore by the first part of the 8. of this booke *E* hath to *B* a greater proportiō then hath *A* to *B*. But *E* is to *B* as *C* is to *D* by diuisiō of proportion: for *EB* is to *B* as *CD* is to *D*. Wherfore (by the 12. of the same) *C* hath to *D* a greater proportiō then hath *A* to *B*, which is cōtrary to the suppositiō. Wherfore the proportion of *AB* to *B* is not one and the same with the proportion of *CD* to *D*, neither also is it lesse then it. Wherfore it is greater: which was required to be proued.

Demonstratiō leading to an impossibilitie.



This may also be demonstrated affirmatiuely. Forasmuch as the proportion of *A* to *B* is greater then the proportion of *C* to *D*: let *E* be vnto *B* as *C* is to *D*. And so by the first part of the 10. of this booke, *E* shall be lesse then *A*. And therfore by the common sentence *EB* shall be lesse then *AB*. Wherfore by the first part of the 8. of the same *AB* hath vnto *B* a greater proportion then hath *EB* to *B*. But by composition *EB* is to *B*, as *CD* is to *D*. For by suppositiō *E* is vnto *B* as is to *D*. Wherfore (by the 12. of this booke) *AB* hath to *B* a greater proportion then hath *CD* to *D*: which was required to be proued.

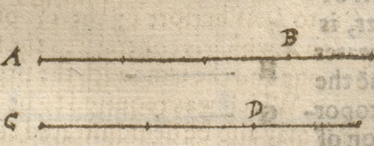
An other demonstration of the same.

¶ The fourth Proposition.

If there be foure quantities, and if the proportion of the first and the second to the second be greater then the proportion of the third and fourth to the fourth: then by diuision also the proportion of the first to the second, shall be greater then the proportion of the thirde to the fourth.

Suppose that the proportion of *AB* to *B* be greater then the proportion of *CD* to *D*. Then I say that by diuision also the proportion of *A* to *B* is greater then the proportion of *C* to *D*. For it cannot be the same. For then by composition *AB* should be to *B* as *CD* is to *D*. Neither also can it be lesse: for if the proportion of *C* to *D* be greater then the proportion of *A* to *B*, then by the former proposition, the proportion of *CD* to *D* should be greater then the proportion of *AB* to *B*: which is contrary also to the suppositiō. Wherfore the proportion of *A* to *B* is neither one and the same with the proportion of *C* to *D*, nor is lesse then it. Wherefore it is greater then it: which was required to be proued.

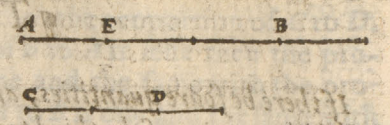
Demonstratiō leading to an impossibilitie.



The

An other demonstration of the same affirmatiuely.

The same may also be proued affirmatiuely. Suppose that *E B* be vnto *B* as *C D* is to *D*. Now then (by the first part of the 10. of the fifth) *E B* shall be lesse then *A B*: and therefore by the common sentence, *E* is lesse then *A*, wherfore by the first part of the 8. of this booke, the proportion of *E* to *B*, is lesse then the proportion of *A* to *B*, but as *E* is to *B*, so is *C* to *D*: wherfore the proportion of *C* to *D*, is lesse then the proportion of *A* to *B*. Wherfore the proportion of *A* to *B* is greater then the proportion of *C* to *D*: which was required to be proued.

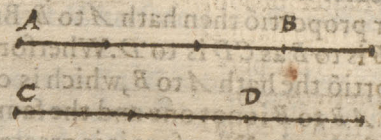


¶ The fifth Proposition.

If there be foure quantities, and if the proportion of the first and the second to the second be greater then the proportion of the third and the fourth to the fourth: then by euersion the proportion of the first and second to the first, shall be lesse then the proportion of the third and fourth to the third.

Demonstration.

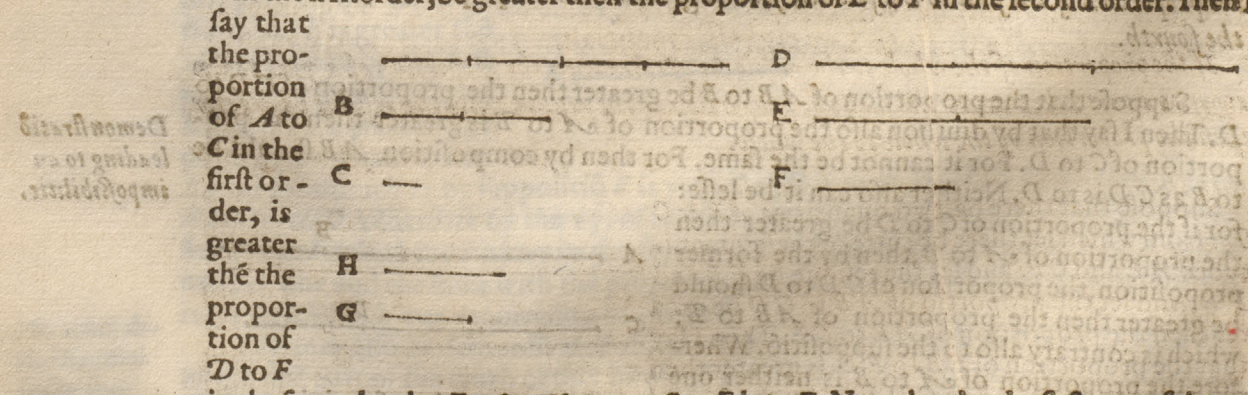
Suppose that the proportion of *A B* to *B* be greater then the proportion of *C D* to *D*. Then I say that by euersion the proportion of *A B* to *A* is lesse then the proportion of *C D* to *C*. For by diuision by the former proposition the proportion of *A* to *B* is greater then the proportion of *C* to *D*. Wherefore by the first of these propositions conuersedly, *B* hath vnto *A* a lesse proportiō thē hath *D* to *C*. Wherfore by the 3. of the same by composition, the proportion of *A B* to *A* is lesse thē the proportion of *C D* to *C*: which was required to be proued.



¶ The sixt Proposition.

If there be taken three quantities in one order, and as many in an other order, and if the proportion of the first to the second in the first order, be greater then the proportion of the first to the second in the latter order: then also the proportion of the first to the third in the first order, shall be greater then the proportion of the first to the third in the latter order.

Suppose that there be three quātities in one order *A, B, C*, & as many other quātities in an other order *D, E, F*. And let the proportion of *A* to *B* in the first order be greater then the proportion of *D* to *E* in the second order, and let also the proportion of *B* to *C* in the first order, be greater then the proportion of *E* to *F* in the second order. Then I say that the proportion of *A* to *C* in the first order, is greater thē the proportion of *D* to *F* in the second order.



Demonstration.

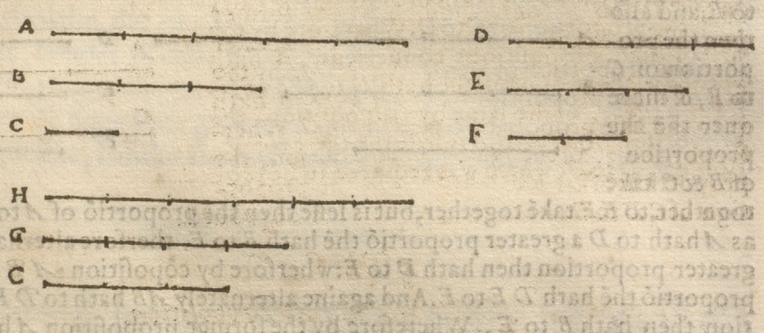
For let *G* be vnto *C* as *E* is to *F*. Now then by the first part of the 10 of this booke *G* shall be lesse then *B*. And therefore by the second parte of the 8. of the same,

ſame, the proportion of *A* to *G* is greater then the proportion of *A* to *B*. Wherfore the proportion of *A* to *G* is muche greater then the proportion of *D* to *E*. Now then let *H* be vnto *G* as *D* is to *E*. Wherfore by the firſt part of the 10. of the ſame, *A* is greater thē *H*. And therfore by the firſt part of the 8. of the ſame, the proportion of *A* to *C* is greater then the proportion of *H* to *C*. But by proportion of equality *H* is to *C* as *D* is to *F* (for *H* is to *G* as *D* is to *E*, and *G* is to *C* as *E* is to *F*. Wherfore by the 12. of the ſame *A* hath to *C* a greater proportion then hath *D* to *F*: which was required to be proued.

¶ The ſeuenth Propoſition.

If there be taken three quantities in one order, and as many other in an other order, and if the proportion of the ſecond to the third in the firſt order be greater then the proportion of the firſt to the ſecond in the latter order, if alſo the proportion of the firſt to the ſecond in the firſt order be greater then the proportion of the ſecond to the third in the latter order: then ſhall the proportion of the firſt to the third in the firſt order be greater, then the proportion of the firſt to the third in the latter order.

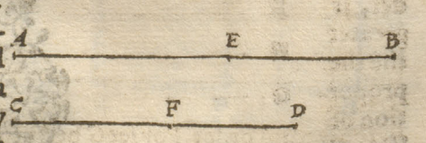
Suppoſe that there be three quātities in one order *A*, *B*, *C*, and as many other in an other order *D*, *E*, *F*. And let the proportion of *B* to *C* in the firſt order, be greater then the proportion of *D* to *E* in the ſecond order, and let alſo the proportion of *A* to *B* in the firſt order, be greater then the proportion of *E* to *F* in the ſecond order. Then I ſay that *A* hath to *C* a greater proportion then hath *D* to *F*. This pertaineth to proportion of equalitie. For let *G* be vnto *C*, as *D* is to *E*. And by the firſt part of the 10. of this boke, *G* ſhal be leſſe thē *B*. And therfore by the ſecond part of the 8. of the ſame, the proportiō of *A* to *G* is greater then the proportion of *A* to *B*. Wherfore *A* hath vnto *G* a much greater proportiō then hath *E* to *F*. Now then let *H* be vnto *G* as *E* is to *F*. And by the firſt part of the 10. of the ſame, *A* ſhalbe greater then *H*. And by the firſt part of the 8. of the ſame, the proportion of *A* to *C* is greater then the proportion of *H* to *C*. But by the 23. of the ſame the proportion of *H* to *C* is as the proportion of *D* to *F* (for *G* is to *C* as *D* is to *E*, and *H* is to *G* as *E* is to *F*.) Wherfore (by the 12. of the ſame) the proportion of *A* to *C* is greater then the proportion of *D* to *F*, which was required to be proued.



¶ The eight Propoſition.

If the proportion of the whole to the whole, be greater then the proportion of a part taken away, to a part taken away: thē ſhall the proportion of the reſidue vnto the reſidue be greater then the proportion of the whole to the whole.

Suppoſe that there be two quantities *AB* & *CD*: from which let there be cutte of theſe magnitudes *AE* and *CF*: and let the reſidue be *EB* and *FD*. And let the proportiō of *AB* to *CD* be greater then the proportion of *AE* to *CF*. Then I ſay that the proportion of *EB* to *FD* is greater then the proportion of *AB* to *CD*. For (by the ſecond



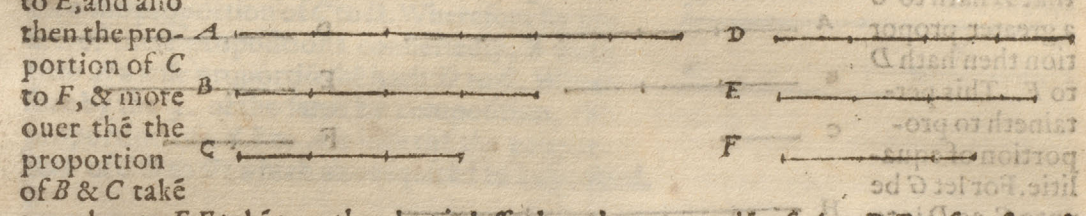
of

of these propositions now added) alternately the proportion of *A B* to *A E* is greater then the proportion of *C D* to *C F*. And therfore by euersion of proportion (by the 5. of the same) the proportion of *A B* to *E B* is lesse then the proportion of *C D* to *F D*. Wherfore agayne alternately the proportion of *A B* to *C D* is lesse then the proportiō of *E B* to *F D*: which was required to be proued.

¶ The ninth Proposition.

If quantities how many soeuer in one order be compared to as many other in an other order, and if there be a greater proportion of euery one that goeth before to that wherunto it is referred, then of any that followeth to that wherunto it is referred: the proportion of them all taken together vnto all the other taken together, shall be greater, then the proportion of any that followeth to that wherunto it is compared, and also then the proportion of all them taken together to all the other taken together, but shall be lesse then the proportion of the first to the first.

Suppose that there be three quantities in one order, *A, B, C*, & as many other in an other order *D, E, F*. And let the proportiō of *A* to *D* be greater thē the proportiō of *B* to *E*, let also the proportiō of *B* to *E*, be greater then the proportiō of *C* to *F*. Thē I say that the proportiō of *A B C* takē al together, to *D E F* takē altogether, is greater thē the proportion of *B* to *E*, and also then the proportion of *C* to *F*, & more ouer thē the proportion of *B* & *C* takē together, to *E F* takē together, but is lesse then the proportiō of *A* to *D*.



Demonstration.

For forasmuch as *A* hath to *D* a greater proportiō thē hath *B* to *E*, therfore alternately *A* hath to *B* a greater proportion then hath *D* to *E*: wherfore by cōposition *A B* hath to *B* a greater proportiō thē hath *D E* to *E*. And againe alternately *A B* hath to *D E* a greater proportion then hath *B* to *E*. Wherefore by the former proposition *A* hath to *D* a greater proportion then hath *A B* to *D E*. And by the same reason may it be proued that hath to *E* a greater proportion then hath *B C* to *E F*. Wherefore *A* hath to *D* a greater proportion then hath *B C* to *E F*. Wherfore alternately *A* hath to *B C* a greater proportion then hath *D* to *E F*, wherfore by composition *A B C* hath to *B C* a greater proportion then hath *D E F* to *E F*. Wherfore agayne alternately *A B C* hath to *D E F* a greater proportion then hath *B C* to *E F*. Wherefore (by the former proposition) the proportion of *A* to *D* is greater then the proportion of *A B C* to *D E F*: Which was required to be proued.

The end of the fifth booke of Euclides Elementes.

¶ The sixth booke of Euclides Elementes.

THIS SIXTH BOOKE, is for vse and practise, a most speciall booke. In it are taught the proportions of one figure to an other figure, & of their sides the one to the other, and of the sides of one to the sides of an other, likewise of the angles of the one to the angles of the other. Moreouer it teacheth the description of figures like to figures geuen, and marueilous applications of figures to lines, euenly, or with decrease or excesse, with many other Theoremes, not onely of the proportions of right lined figures, but also of sectors of circles, with their angles. On the Theoremes and Problemes of this Booke depend for the most part, the compositions of all instrumentes of measuring length, breadth, or deepenes, and also the reason of the vse of the same instrumentes, as of the Geometricall square, the Scale of the Astrolabe, the quadrant, the staffe, and such other. The vse of which instrnmentes, besides all other mechanicall instrumentes of rayfing vp, of mouing, and drawing huge thinges incredible to the ignorant, and infinite other ginnes (which likewise haue their groundes out of this Booke) are of wonderfull and vnspeakeable profite, besides the inestimable pleasure which is in them.

The argument of this sixth booke.

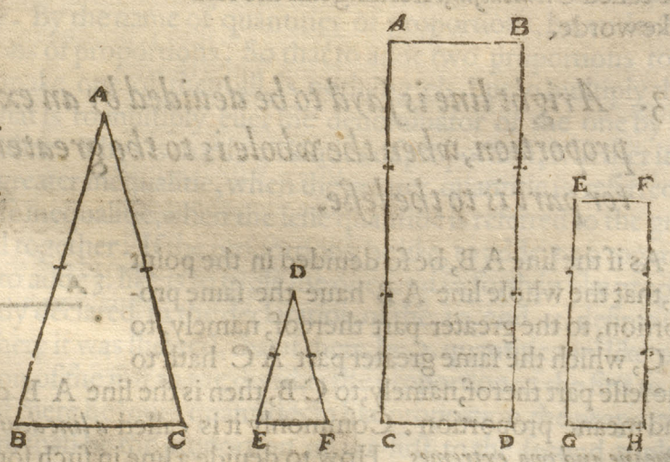
This booke necessary for the vse of instrumentes of Geometry.

Definitions.

1. *Like rectiline figures are such, whose angles are equall the one to the other, and whose sides about the equall angles are proportionall.*

The first definition.

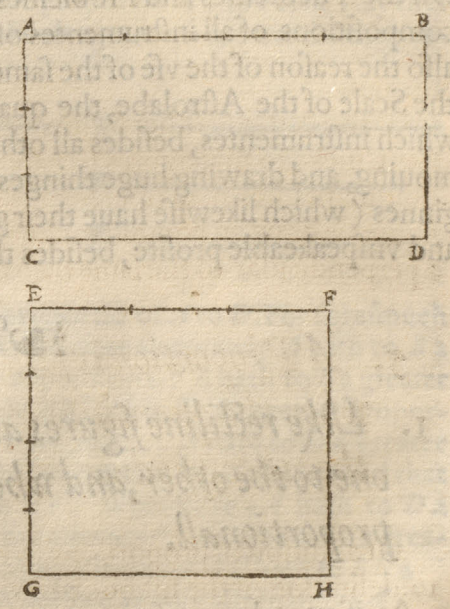
As if ye take any two rectiline figures. As for example, two triangles A B C, and D E F: if the angles of the one triangle be equall to the angles of the other, namely, if the angle A be equall to the angle D, and the angle B equall to the angle E, & also the angle C equall to the angle F. And moreouer, if the sides which containe the equall angles be proportionall. As if the side A B haue that proportion to



the side B C, which the side D E hath to the side E F, and also if the side B C be vnto the side C A, as the side E F is to the side F D, and moreouer, if the side C A be to the side A B, as the side F D is to the side D E, then are these two triangles sayd to be like: and so iudge ye of any other kinde of figures. As if in the parallelogrammes A B C D and E F G H, the angle A be equall to the angle E, and the angle B equall to the angle F, and the angle C equall to the angle G, and the angle D equall to the angle H. And farthermore, if the side A C haue that proportion to the side C D which the side E G hath to the side G H, and if also the side C D be to the side D B as the side G H is to the side H F, and moreouer, if the side D B be to the side B A as the side H F is to the side F E, and finally, if the side B A be to the side A C as the side F E is to the side E G, then are these parallelogrammes like.

The second definition.

2. *Reciprocall figures are those, when the termes of proportion are both antecedentes and consequentes in either figure.*



As if ye haue two parallelogrammes A B C D and E F G H. If the side A B to the side E F, an antecedent of the first figure to a consequent of the second figure, haue mutually the same proportion, which the side E G hath to the side A C an antecedent of the second figure to a consequent of the first figure: then are these two figures Reciprocall. They are called of some, figures of mutuall sides, and that vndoubtedly not amisse nor vnaptly. And to make thys definition more plaine, *Campane* and *Pellitarius*, and others, thus put it: *Reciprocall figures are, when the sides of either be mutually proportionall, as in the example and declaration before geuen*. Among the barbarous they are called *Mutekesia*, reseruing still the Arabike worde.

Reciprocall figures called mutuall figures.

The third definition.

3. *A right line is sayd to be deuided by an extreme and meane proportion, when the whole is to the greater part, as the greater part is to the lesse.*

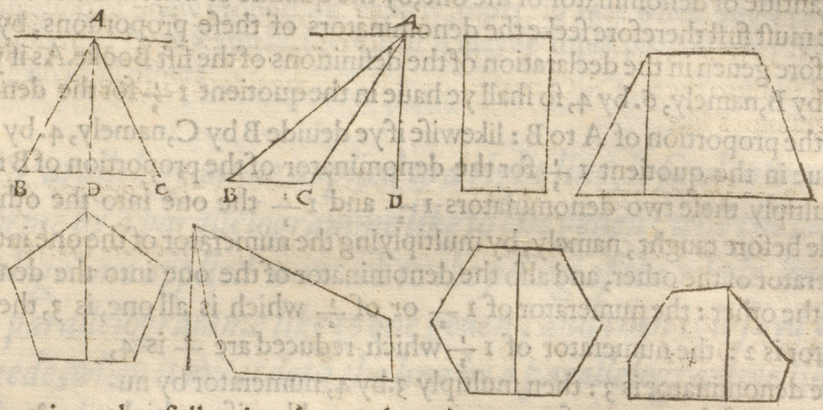
As if the line A B, be so deuided in the point C, that the whole line A B haue the same proportion, to the greater part thereof, namely, to A C, which the same greater part A C hath to the lesse part therof, namely, to C B, then is the line A B deuided by an extreme and meane proportion. Commonly it is called *a line deuided by proportion hauing a meane and two extremes*. How to deuide a line in such sort was taught in the 11. Proposition of the second Booke, but not vnder this forme of proportion.

4. *The*

4. *The alitude of a figure is a perpendicular line drawen from the toppe to the baſe.*

The fourth definition.

As the altitude or hight of the triangle A B C, is the line A D being drawen perpendicularly from the poynt A, being the toppe or higheſt part of the triangle to the baſe therof B C. So likewiſe in other figures as ye ſee in the examples here ſet. That which here hee calleth the altitude or height of a figure, in the firſt booke in the 35. Propoſition and certaine other following, he taught to be contayned within two equidiſtant lines: ſo that figures to haue one altitude and to be contayned within two equidiſtant lines, is all one. So in all theſe examples, if from the higheſt point of the figure ye draw an equidiſtant line to the baſe therof, and then frō that poynt draw a perpendicular to the ſame baſe that perpendicular is the altitude of the figure.



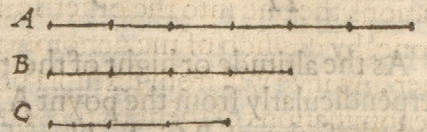
5. *A Proportion is ſaid to be made of two proportions or more, when the quantities of the proportions multiplied the one into the other, produce an other quantitie.*

The fifth definition.

Of addition of proportions, hath bene ſomewhat ſayd in the declaration of the 10. definition of the fift booke: which in ſubſtance is all one with that which is here taught by *Euclide*. By the name of quantities of proportions, he vnderſtandeth the denominations of proportions. So that to adde two proportions together, or more, and to make one of them all, is nothyng els, but to multiply their quantities together, that is to multiply euer the denominator of the one by the denominator of the other. Thys is true in all kindes of proportion, whether it be of equalitie, or of the greater inequalitie, when the greater quantitie is referred to the leſſe: or of the leſſe inequalitie, when the leſſe quantitie is referred to the greater: or of them mixed together. If the proportions be like, to adde two together is to double the one, to adde 3. like is to triple the one, and ſo forth in like proportions, as was ſufficiently declared in the declaration of the 10. and 11. definitions of the fift Booke. Where it was ſhewed, that if there be 3. quantities in like proportion, the proportion of the firſt to the thyrd, is the proportion of the firſt to the ſecond doubled: and if there be foure quantities in like proportion, the proportion of the firſt to the fourth ſhall be the proportion of the firſt to the ſecond tripled: which thing how to do was there taught. Likewyſe in proportions vnlike, the proportion of the firſt extreme to the laſt is made of all the meane proportions ſet

By the name of quantities is vnderſtanded the denominations of proportions.

betwene them. Suppose three quantities A,B,C, so that let A haue to B sesquialtera proportion, namely, 6. to 4. And let B to C haue sesquitertia proportion, namely, 4. to 3. Now the proportion of A to C, the first to the thyrd, is made of the proportion of A to B, and of the proportion of B to to C added together. If ye will adde them together, ye must by this definition multiply the quantitie or denominator of the one, by the quãtitie or denominator of the other. Ye must first therefore seeke the denominators of these proportions, by the rule before geuen in the declaration of the definitions of the fift Booke. As if ye deuide A by B, namely, 6. by 4, so shall ye haue in the quotient $1\frac{1}{2}$ for the denominator of the proportion of A to B: likewise if ye deuide B by C, namely, 4. by 3. ye shall haue in the quotient $1\frac{1}{3}$ for the denominator of the proportion of B to C, now multiply these two denominators $1\frac{1}{2}$ and $1\frac{1}{3}$ the one into the other, by the rule before taught, namely, by multiplying the numerator of the one into the numerator of the other, and also the denominator of the one into the denominator of the other: the numerator of $1\frac{1}{2}$ or of $\frac{3}{2}$ which is all one, is 3, the denominator is 2: the numerator of $1\frac{1}{3}$ which reduced are $\frac{4}{3}$ is 4, the denominator is 3: then multiply 3. by 4, numerator by numerator, so haue ye 12. for a new numerator: likewise multiply 2. by 3. denominator by denominator, ye shall produce 6. for a new denominator: so haue you produced 12. and 6, betwene which there is dupla proportion. Which proportion is also betwene A and C, namely, 6. to 3, the first quantitie to the third. Wherfore the proportion of A to C is sayd to be made of the proportion of A to B and of the proportion of B to C, for that it is produced of the multiplication of the quantitie or denominator of the one, into the quantitie or denominator of the other. And so of all others be they neuer so many. As in these examples in numbers here set 2. 3. 15. 18. In this example the lesse numbers are compared to the greater, as in the former the greater were compared to the lesse: the denominator of the proportion of 2. to 3. is $\frac{2}{3}$ that is, subsesquialtera, the denomination betwene 3. and 15. is $\frac{3}{15}$ or $\frac{1}{5}$ which is all one, that is, subquintupla, betwene 15. and 18. the denomination of the proportion is $\frac{5}{6}$ that is, subsesquiquinta, multiply all these denominations together: first the numerators: 2. into 1. produce 2, then 2. into 5. produce 10: which shall be a new numerator. Then the denominators: 3. into 5. produce 15: and 15. into 6. produce 90: which shall be a new denominator. So haue you brought forth $\frac{10}{90}$ or $\frac{1}{9}$ which is proportion subnoncupla: which is also the proportion of 2. to 18. Wherefore the proportion of 2. to 18. that is, of the extremes, namely, subnoncupla, is made of the proportions of 2. to 3: of 3. to 15: and of 15. to 18: namely, of subsesquialtera, subquintupla, and subsesquiquinta.



$$\frac{2}{3}\ \frac{1}{5}\ \frac{5}{6}\ \Big|\ \frac{1}{9} \qquad \begin{matrix} 12 \\ 10 \\ 90 \end{matrix}$$

$$\frac{3}{2}\ \frac{4}{3} \qquad 6$$

Example of this definitiõ.

Example in numbers. 2. 3. 15. 18.

An other example, where the greater inequalitie and the lesse inequalitie are mixed together 6. 4. 2. 3. the denomination of the proportion of 6. to 4, is $1\frac{1}{2}$, of 4. to 2, is $\frac{2}{1}$, and of 2. to 3, is $\frac{2}{3}$: now if ye multiply as you ought, all these denominations together, ye shall produce 12. to 6, namely, dupla proportion.

An other example.

Forasmuch as so much hath hetherto bene spoken of addition of proportions it shall

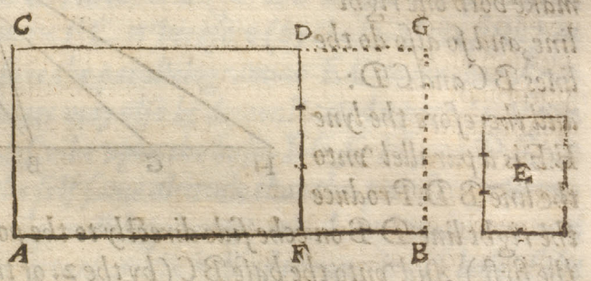
it shall not be vnnecessary somewhat also to say of substraction of them. Where it is to be noted, that as addition of them, is made by multiplicatiō of their denominations the one into the other: so is the substraction of the one from the other done, by diuision of the denomination of the one by the denomination of the other. As if ye will from sextupla proportion subtrahe dupla proportion, take the denominations of them both. The denomination of sextupla proportion, is 6, the denomination of dupla proportion, is 2. Now deuide 6. the denomination of the one by 2. the denomination of the other: the quotient shall be 3: which is the denomination of a new proportion, namely, tripla: so that when dupla proportion is subtrahed from sextupla, there shall remayne tripla proportion. And thus may ye do in all others.

Of substraction of proportion.

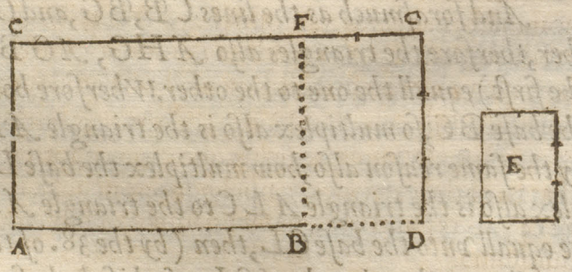
6. *A Parallelogramme applied to a right line, is sayd to want in forme by a parallelogramme like to one geuen: whē the parallelogrāme applied wanteth to the filling of the whole line, by a parallelogramme like to one geuen: and then is it sayd to exceede, when it exceedeth the line by a parallelogramme like to that which was geuen.*

The sixth definition.

As let E be a Parallelogrāme geuen, and let A B be a right line, to whom is applied the parallelogramme A C D F. Now if it want of the filling of the line A B, by the parallelogrāme D F G B being like to the parallelogramme geuen E, then is the parallelogramme sayd to want in forme by a parallelogramme like vnto a parallelogramme geuen.



Likewise if it exceede, as the parallelogramme A C G D applyed to the line A B, if it exceede it by the parallelogramme F G B D being like to the parallelogramme E which was geuen, then is the parallelogramme A B G D, sayd to exceede in forme by a parallelogramme like to a parallelogramme geuen.



This definition is added by *Flussates* as it seemeth, it is not in any cōmon Greke booke abroad, nor in any Commentary. It is for many Theoremes following very necessary.

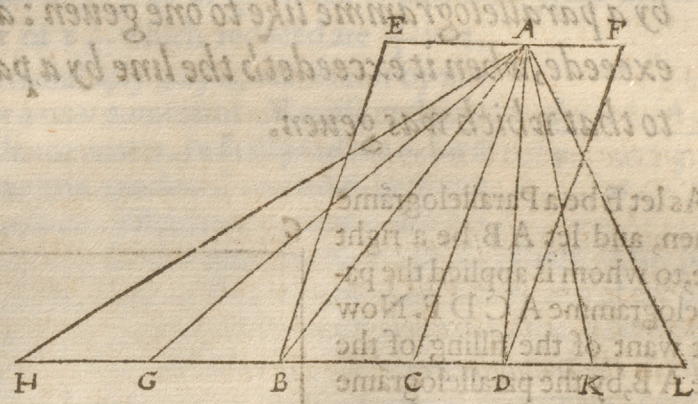
The 1. Theoreme. The 1. Proposition.

Triangles & parallelogrammes which are vnder one & the selfsame altitude: are in proportion as the base of the one is to the base of the other.

Vppose that there be two triangles *A B C* and *A C D*, & two paralleiogrammes *E C* and *C F*. Which let be set vnder one and the selfe same altitude, or perpendicular line drawen from the toppe *A* to the base *B D*. Then I say that as the base *B C* is to the base *C D* so is the triangle *A B C* to the triangle *A C D*: and the parallelogramme *E C* to the parallelogramme *C F*. For forasmuch as the

Construction.

parallelogrammes *E C* and *C F* are vnder one and the selfe same altitude, therfore the lines *E A* and *A F* make both one right line, and soalso do the lines *B C* and *C D*: and therefore the lyne *E F* is a parallel vnto the line *B D*. Produce



the right line *D B* on eche side directly to the pointes *H, L* (by the 2. petition of the first) And vnto the base *B C* (by the 2. of the first) put as many equall lines as you will, as for example, two, namely, *B G*, and *G H* and vnto the base *C D* on the other side put as many equall as you did to the other base, which let be *D K*, and *K L*. Then draw these right lines *A G*, *A H*, *A K*, and *A L*.

Demonstration of the first part.

And forasmuch as the lines *C B*, *B G*, and *G H* are equall the one to the other, therfore the triangles also *A H G*, *A G B* and *A B C*, are (by the 38. of the first) euuall the one to the other. Wherfore how multiplex the base *H C* is to the base *B C*, so multiplex also is the triangle *A H C* to the triangle *A B C*. And by the same reason also, how multiplex the base *L C* is to the base *D C*, so multiplex also is the triangle *A L C* to the triangle *A D C*. Wherfore if the base *H C* be equall vnto the base *C L*, then (by the 38. of the first) the triangle *A H C* is equall vnto the triangle *A C L*. And if the base *H C* exceede the base *C L*, then also the triangle *A H C* excedeth the triangle *A C L*, and if the base be lesse, the triangle also shall be lesse. Now then there are foure magnitudes, namely, the two bases *B C* and *C D*, and the two triangles *A B C*, and *A C D*, and to the base *B C*, and to the triangle *A B C*, namely, to the first and the third, are taken eque-

mul-

multiplices, namely, the base HC and the triangle AHC, and likewise to ỹ base CD and to the triangle ADC, namely, to the second and the fourth, are taken certaine other equemultiplices, that is, the base CL, and the triangle ALC. And it hath bene proued that if the multiplex of the first magnitude, that is, the base HC, do exceede the multiplex of the second, that is, the base CL, the multiplex also of the third, that is, the triangle AHC excedeth the multiplex of ỹ fourth, that is, the triangle ALC, and if the said base HC be equall to the said base CL, the triangle also AHC is equall to the triangle ALC, and if it be lesse it is lesse. Wherfore by the sixt definition of ỹ fifth, as the first of the foresaid magnitudes is to the second, so is the third to the fourth. Wherfore as the base BC is to the base CD, so is the triangle ABC to the triangle ACD.

Demonstration of the second part.

And because (by the 41. of the first) the parallelogramme EC is double to the triangle ABC, and (by the same) the parallelogramme FC is duble to the triangle ACD, therfore the parallelogrammes EC and FC are equemultiplices vnto the triangles ABC and ACD. But the partes of equemultiplices (by the 15. of the fifth) haue one and the same proportion with their equenultiplices. Wherfore as the triangle ABC is to the triangle ACD, so is the parallelograme EC to the parallelogramme FC. And forasmuch as it hath bene denonstrated, that as the base BC is to the base CD, so is the triangle ABC, to the triangle ACD, and as the triangle ABC is to the triangle ACD so is the parallelogramme EC to the parallelogramme FC. Wherefore (by the 11. of the fifth) as the base BC is to the base CD, so is the parallelogramme EC to the parallelogramme FC. The parallelogrammes may also be demonstrated apart by themselues as the triangles are, if we describe vpon the bases BG, GH, and DK & KL parallelogrammes vnder the self same altitude that the parallelogrammes geuen are. Wherfore triangles and parallelogrammes which are vnder one and the selfe same altitude, are in proportion, as the base of the one is to ỹ base of the other: which was required to be demonstrated.

Here Flussates addeth this Corollary.

A Corollary added by Flussates.

If two right lines being geuen, the one of them be deuided how so euer: the rectangle figures contayned vnder the whole line vndeuided, and eche of the segmentes of the line deuided, are in proportion the one to the other, as the segmentes are the one to the other. For imaginyng the figures BA and AD in the former description, to be rectangled, the rectangle figures contayned vnder the whole right lyne AC, and the segments of the right line BD, which is cut in the poynt C, namely, the parallelogrammes BA and AD, are in proportion the one to the other, as the segmētes BC ard CD are.

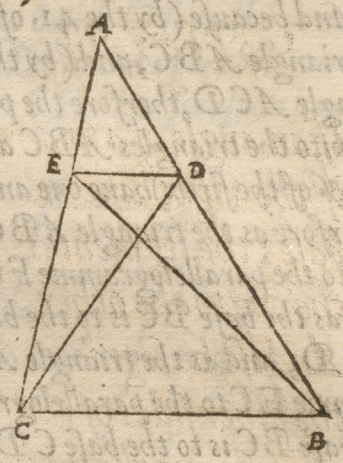
The 2. Theoreme. The 2. Proposition.

If to any one of the sides of a triangle be drawen a parallel

right line, it shall cut the sides of the same triangle proportionally. And if the sides of a triangle be cut proportionally, a right lyne drawn from section to section is a parallel to the other side of the triangle.

The first part of this Theoreme.

SVppose that there be a triangle A B C, vnto one of the sides whereof, namely, vnto B C, let there be drawen a parallel line D E cuttyng the sides A C and A B in the pointes E and D. Then I say first that as B D is to D A, so is C E to E A. Draw a line from B to E, & also from C to D. Wherfore (by the 37. of the first) the triangle B D E is equall vnto the triangle C D E: for they are set vpon one and the same base D E, and are contained within the selfe same parallels D E and B C. Consider also a certaine other triangle A D E. Now thinges equall (by the 7. of the fifth) haue to one selfe thing one and the same proportion. Wherfore as ỹ triangle B D E is to ỹ triangle A D E, so is ỹ triangle C D E to the triangle A D E. But as ỹ triangle B D E is to ỹ triangle A D E, so is ỹ base B D to ỹ base D A (by the first of this booke.) For they are vnder one and the selfe same toppe, namely, E, and therfore are vnder one and the same altitude. And by the same reason as the triangle C D E is to the triangle A D E, so is the lyne C E to the lyne E A. Wherfore (by the 11. of the fifth) as the line B D is to the line D A, so is the line C E to the line E A.



Demonstration of the second part.

But now suppose that in ỹ triangle A B C the sides A B & A C be cut proportionally, so ỹ as B D is to D A, so let C E be to E A, & draw a line from D to E. Then secondly I say ỹ the line D E is a parallel to ỹ lyne B C. For the same order of construction being kept, for ỹ as B D is to D A, so is C E to E A, but as B D is to D A, so is ỹ triangle B D E to ỹ triangle A D E (by the 1. of the sixt) & as C E is to E A, so (by ỹ same) is the triangle C D E to ỹ triangle A D E: therfore (by the 11. of the fifth) as the triangle B D E is to the triangle A D E, so is the triangle C D E to the triangle A D E. Wherfore either of these triangles B D E and C D E haue to the triangle A D E one and the same proportion. Wherefore (by the 9. of the fifth) the triangle B D E is equall vnto the triangle C D E, and they are vpon one and the selfe base, namely, D E. But triangles equall and set vpon one base, are also contained within the same parallel lines (by the 39. of the first.) Wherfore the line D E is vnto the line B C a parallel. If therfore to any one of the sides of a triangle be drawn a parallel line, it cutteth the other sides of the same triangle proportionally. And if the sides of a triangle be cut proportionally, a right lyne drawen from section to section, is parallel to the other side of the triangle: which thing was required to be demonstrated.

Here

¶ *Here also Flussates addeth a Corollary.*

If a line parallel to one of the sides of a triangle do cut the triangle, it shall cut off from the whole triangle a triangle like to the whole triangle. For as it hath bene proued it deuideth the sides proportionally. So that as E C is to E A, so is B D to D A, wherfore by the 18. of the fifth, as A C is to A E, so is A B to A D. Wherfore alternately by the 16. of the fifth as A C is to A B, so is A E to A D: wherefore in the two triangles E A D and C A B the sides about the common angle A are proportional. The sayd triangles also are equiangle. For forasmuch as the right lynes A E C and A D B do fall vpon the parallel lynes E D and C B, therefore by the 29. of the first they make the angles A E D and A D E in the triangle A D E equall to the angles A C B and A B C in the triangle A C B. Wherfore by the first definition of this booke the whole triangle A B C is like vnto the triangle cut of A D E.

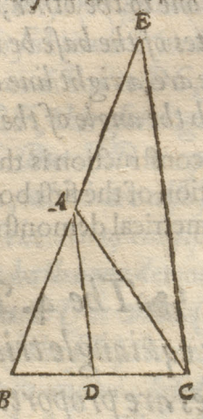
A Corollary added by Flussates.

The 3. Theoreme. The 3. Proposition.

If an angle of a triangle be deuided into two equall partes, and if the right line which deuideth the angle deuide also the base: the segmentes of the base shall be in the same proportion the one to the other, that the other sides of the triangle are. And if the segmẽtes of the base be in the same proportion that the other sides of the sayd triangle are: a right drawen from the toppe of the triangle vnto the section, shall deuide the angle of the triangle into two equall partes.

Suppose that there be a triangle A B C, and (by the 9. of the first) let the angle B A C be deuided into two equall partes by the right lyne A D, which let cut also the base B C in the point D. Then I say that as the segment B D is to ỹ segment D C, so is the side B A to the side A C. For by the point C (by the 31. of the first) draw vnto the line D A a parallel line C E and extende the line B A till it concurre with the line C E in the point E, and do make the triangle B E C. But the lyne B A shall concurre with the line C E (by the 5. petition) for that the angles E B C, and B C E are lesse then two right angles. For the angle E C B is equall to the outwarde and opposite angle A D B (by the 29. of the first.) And the two angles A D B and D B A of the triangle B A D are lesse then two right angles (by the 17. of the first) Now forasmuch as vpon the parallels A D and E C falleth the right line A C, therefore

Construction.



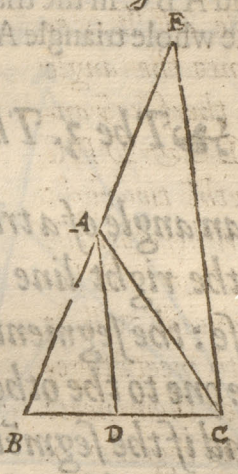
Demonstration of the first part.

(by

by the 29. of the first) the angle A C E is equall vnto the angle C A D. But vnto the angle C A D is the angle B A D supposed to be equall. Wherfore the angle B A D is also equall vnto the angle A C E. Againe because vpon the parallels A D and E C falleth the right line B A E, the outward angle B A D (by the 28. of the first) is equall vnto the inward angle A E C. But before it was proued that ỹ angle A C E is equall vnto ỹ angle B A D, wherfore ỹ angle A C E is equall vnto ỹ angle A E C. Wherefore (by ỹ 6. of ỹ first) ỹ side A E is equall vnto the side A C. And because to one of ỹ sides of ỹ triangle B C E, namely, to E C is drawen a parallel line A D, therfore (by ỹ 2. of ỹ sixt) as B D is to D C, so is B A to A E. But A E is equall vnto A C, therfore as B D is to D C, so is B A to A C.

Demonstration of the second part, which is the conuerse of the first.

But now suppose that as the segment B D is to the segment D C, so is the side B A to the side A C, & draw a line from A to D. Then I say that the angle B A C is by ỹ right line A D deuided into two equall partes. For the same order of construction remayning, for that as B D is to D C, so is B A to A C, but as B D is to D C, so is B A to A E (by the 2. of the sixt) for vnto one of the sides of the triangle B C E, namely, vnto the side E C is drawn a parallel line A D. Wherefore also as B A is to A C, so is B A to A E (by the 11. of the fifth) Wherfore (by the 9. of the fifth) A C is equall vnto A E. Wherfore also (by the 5. of the first) the angle A E C is equall vnto the angle A C E, but the angle A E C (by the 29. of the first) is equall vnto the outward angle B A D: and the angle A C E is equall vnto the angle C A D which is alternate vnto him: wherefore the angle B A D is equall vnto the angle C A D. Wherfore the angle B A C is by the right line A D deuided into two equall partes. Wherefore if an angle of a triangle be deuided into two equall partes, and if the right line which deuideth the angle cut also the base, the segmentes of the base shall be in the same proportion the one to the other, that the other sides of the said triangle are. And if the segmentes of the base be in the same proportion that the other sides of the sayd triangle are, a right line drawen from the toppe of the triangle vnto the section deuideth the angle of the triangle into two equall partes.



This construction is the halfe part of that Gnomical figure described in the 43. proposition of the first booke, which Gnomical figure is of great vse in a maner in all Geometrical demonstrations.

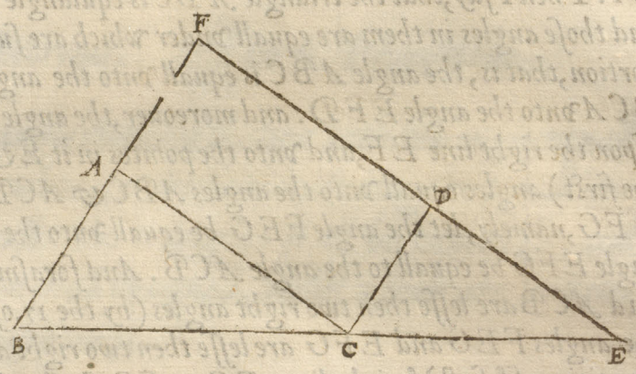
The 4. Theoreme. The 4. Proposition.

In equiangle triangles, the sides which cõtaine the equall angles are proportionall, and the sides which are subtended vnder the equall angles are of like proportion.

Suppose

SVppose that there be two equiangle triangles ABC and DCE: and let the angle ABC of the one triangle, be equall vnto ỹ angle DCE of the other triangle, and the angle BAC equall vnto ỹ angle CDE, and moreouer, the angle ACB equall vnto the angle DEC. Then I say, that those sides of ỹ triangles ABC, & DCE, which include the equall angles, are proportionall, and the sides which are subtended vnder the equall angles are of like proportion. For let two sides of the sayd triangles, namely, two of those sides which are subtended vnder equall angles: as for example the sides BC and CE, be so set that they both make one right line. And because the angles ABC & ACB are lesse then two right angles (by the 17. of the first): but the angle ACB is equall vnto the angle DEC: therfore ỹ angles ABC & DEC are lesse thē two right angles. Wherefore the lines BA & ED being produced, will at the length meete together. Let them meete and ioyne together in the poynt F. And becaus by supposition the angle DCE is equall vnto the angle ABC, therfore the line BF is (by the 28. of the first) a parallell vnto the line CD. And forasmuch as by supposition the angle ACB is equall vnto the angle DEC, therefore againe (by the 28. of the first) the line AC is a parallell vnto the line FE. Wherefore FADC is a parallelogramme. Wherfore the side FA is equall vnto the side DC: and the side AC vnto the side FD (by the 34. of the first). And because vnto one of the sides of the triangle BFE, namely, to FE is drawen a parallell line AC, therefore as BA is to AF, so is BC to CE (by the 2. of the sixt). But AF is equall vnto CD. Wherfore (by the 11. of the fift) as BA is to CD, so is BC to CE, which are sides subtended vnder equall angles. Wherefore alternately (by the 16. of the fift) as AB is to BC, so is DC to CE. Againe forasmuch as CD is a parallell vnto BF, therefore againe (by the 2. of the sixt) as BC is to CE, so is FD to DE. But FD is equall vnto AC. Wherefore as BC is to CE, so is AC to DE, which are also sides subtended vnder equall angles. Wherfore alternately (by the 16. of the fift) as BC is to CA, so is CE to ED. Wherfore forasmuch as it hath bene demonstrated, that as AB is vnto BC, so is DC vnto CE: but as BC is vnto CA, so is CE vnto ED: it followeth of equalitie (by the 22. of the fift) that as BA is vnto AC, so is CD vnto DE. Wherfore in equiangle triangles, ỹ sides which include ỹ equall angles are proportionall: and ỹ sides which are subtended vnder the equall angles are of like proportion: which was required to be demonstrated.

Construction.



Demonstration.

The

The 5. Theoreme. The 5. Proposition.

If two triangles haue their sides proportionall, the triangles are equiangle, and those angles in thē are equall, vnder which are subtended sides of like proportion.

This is the conuerse of the former proposition.

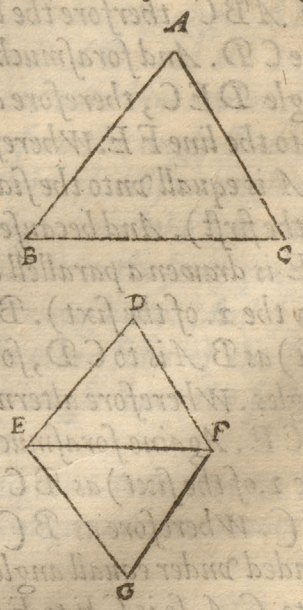
Suppose that there be two triangles ABC, & DEF, hauing their sides proportionall, as AB is to BC, so let DE be to EF: & as BC is to AC, so let EF be to DF: and moreouer, as BA is to AC, so let ED be to DF. Then I say, that the triangle ABC is equiangle vnto the triangle DEF: and those angles in them are equall vnder which are subtended sides of like proportion, that is, the angle ABC is equall vnto the angle DEF: and the angle BCA vnto the angle EFD: and moreouer, the angle BAC to ỹ angle EDF.

Construction.

Vpon the right line EF, and vnto the pointes in it E & F, describe (by the 23. of the first) angles equall vnto the angles ABC & ACB, which let be FEG and EFG, namely, let the angle FEG be equall vnto the angle ABC, and let the angle EFG be equall to the angle ACB.

Demonstration.

And forasmuch as the angles ABC and ACB are lesse then two right angles (by the 17. of the first): therefore also the angles FEG and EFG are lesse then two right angles. Wherefore (by the 5. petition of ỹ first) ỹ right lines EG & FG shall at ỹ length concurre. Let thē concurre in the poynt G. Wherefore EFG is a triangle. Wherefore the angle remayning BAC is equall vnto the angle remayning EGF (by the first Corollary of the 32. of the first). Wherfore the triangle ABC is equiangle vnto the triangle GEF. Wherefore in the triangles ABC and EGF the sides, which include the equall angles (by the 4. of the sixt) are proportionall, and the sides which are subtended vnder the equall angles are of like proportion. Wherefore as AB is to BC, so is GE to EF. But as AB is to BC, so by supposition is DE to EF. Wherefore as DE is to EF, so is GE to EF (by the 11. of the fift). Wherefore either of these DE and EG haue to EF one and the same proportion. Wherefore (by the 9. of the fift) DE is equall vnto EG. And by the same reason also DF is equall vnto FG. Now forasmuch as DE is equall to EG and EF is common vnto them both, therefore these two sides DE & EF, are equall vnto these two sides GE and EF, and ỹ base DF is equall vnto the base FG. Wherefore the angle DEF (by the 8. of the first) is equall vnto the angle GEF: and the triangle DEF (by the 4. of the first) is equall vnto the triangle GEF: and the rest of the angles of the one triangle



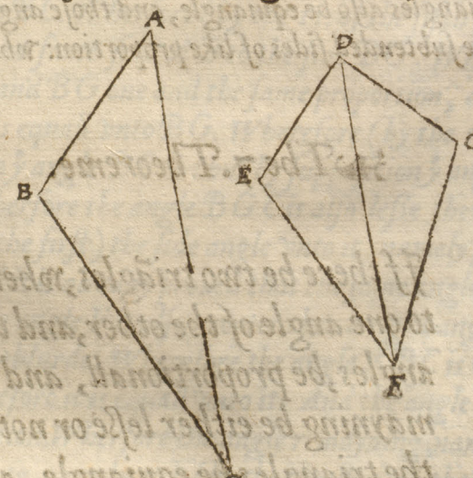
are

are equall vnto the rest of the angles of the other triangle the one to ỹ other, vnder which are ſubtended equall ſides. Wherefore the angle DFE is equall vnto the angle GFE: and the angle EDF vnto the angle EGF. And becauſe the angle FED is equall vnto the angle GEF: but the angle GEF is equall vnto the angle ABC: therefore the angle ABC is alſo equall vnto the angle FED. And by the ſame reaſon the angle ACB is equall vnto ỹ angle DFE: and moreouer, the angle BAC vnto the angle EDF. Wherefore the triangle ABC is equiangle vnto the triangle DEF. If two triangles therefore haue their ſides proportionall, the triangles ſhall be equiangle, & thoſe angles in them ſhall be equall, vnder which are ſubtended ſides of like proportion: which was required to be demonſtrated.

The 6. Theoreme. The 6. Propoſition.

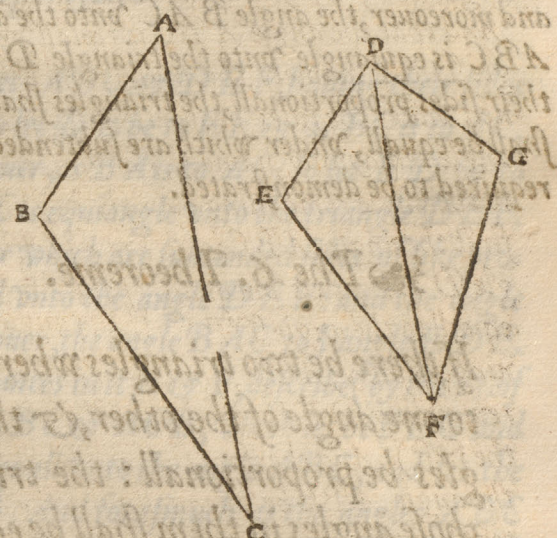
If there be two triangles wherof the one hath one angle equall to one angle of the other, & the ſides including the equall angles be proportionall: the triangles ſhall be equiangle, and thoſe angles in them ſhall be equall, vnder which are ſubtended ſides of like proportion.

Vppoſe that there be two triangles ABC, and DEF, which let haue the angle BAC of the one triangle equall vnto the angle EDF of the other triangle, and let the ſides including the equall angles be proportionall, that is, as BA is to AC, ſo let ED be to DF. Then I ſay, that ỹ triangle ABC is equiangle vnto the triangle DEF: and the angle ABC is equall vnto the angle DEF, and the angle ACB equall vnto the angle DFE, which angles are ſubtẽded to ſides of like proportion. Vnto the right line DF, and to the poynt in it D (by the 23. of the firſt) deſcribe vnto either of ỹ angles BAC and EDF, an equall angle FDG. And vnto the right line DF, and vnto the point in it F (by ỹ ſame) deſcribe vnto ỹ angle ACB an equall angle DFG. And foraſmuch as the two angles BAC and ACB, are (by the 17. of the firſt) leſſe then two right angles: therefore alſo the two angles FDG and DFG, are leſſe then two right angles. Wherfore ỹ lines DG & FG being produced, ſhall cõcurre (by the 5. petition). Let thẽ concurre in the point G. Wherefore DFG is a triangle.



Conſtruction.

Wherefore the angle remaining ABC is equall vnto the angle remaining DGF (by the 32. of the first). Wherefore the triangle ABC is equiangle vnto the triangle DGF. Wherefore as BA is in proportion to AC, so is GD to DF (by the 4. of the sixt). But it is supposed, that as BA is to AC, so is ED to DF. Wherefore (by the 11. of the fift) as ED is to DF, so is GD to DF. Wherefore (by the 9. of the fift) ED is equall vnto DG. And DF is common vnto them both. Now then there are two sides ED and DF equall vnto two sides GD and DF: and the angle EDF (by supposition) is equall vnto the angle GDF. Wherefore (by the 4. of the first) the base EF is equall vnto the base GF, and the triangle DEF is (by the same) equall vnto the triangle GDF, and the other angles remayning in them are equall the one to the other, vnder which are subtended equall sides. Wherefore the angle DFG is equall vnto the angle DFE: and the angle DGF vnto the angle DEF. But the angle DFG is (by construction) equall vnto ỹ angle ACB. And the angle DGF is as it hath bene proued, equall to ỹ angle ABC. Wherfore also ỹ angle ACB is equall vnto the angle DFE. And ỹ angle ABC is equall to the angle DEF. But by supposition the angle BAC is equall vnto the angle EDF. Wherefore the triangle ABC is equiangle vnto the triangle DEF. If therefore there be two triangles, whereof the one hath one angle equall to one angle of the other, and if also the sides including the equall angles be proportionall: then shall the triangles also be equiangle, and those angles in them shalbe equall, vnder which are subtended sides of like proportion: which was required to be proued.

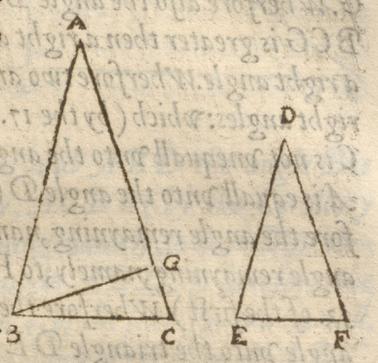


The 7. Theoreme. The 7. Proposition.

If there be two triãgles, wherof the one hath one angle equal to one angle of the other, and the sides which include the other angles, be proportionall, and if either of the other angles remayning be either lesse or not lesse then a right angle: thẽ shal the triangles be equiangle, and those angles in them shall be equall, which are contayned vnder the sides proportionall.

Suppose

SVppose that there be two triangles ABC and DEF, which let haue one angle of the one, equall to one angle of the other, namely, the angle BAC equall vnto the angle EDF. And let the sides which include the other angles, namely, the angles ABC and DEF be proportionall, so that as AB is to BC, so let DE be to EF. And let the other angles remayning, namely, ACD and DFE be first either of them lesse then a right angle. Then I say that the triangle ABC is equiangle vnto the triangle DEF. And that the angle ABC is equall vnto the angle DEF, namely, the angles which are contained vnder the sides proportionall, and that the angle remayning, namely, ỹ angle C is equall vnto the angle remayning, namely, to ỹ angle F. For first the angle ABC is either equall to the angle DEF, or els vnequall. If the angle ABC be equall to the angle DEF, then the angle remaining, namely, ACB, shall be equall to the angle remayning DFE (by the corollary of the 32. of the first) And therfore the triangles ABC and DEF are equiangle. But if the angle ABC be vnequall vnto the angle DEF, then is the one of them greater then the other. Let the angle ABC be the greater, and vnto the right line AB and vnto the point in it B (by the 23. of the first) describe vnto the angle DEF an equall angle ABG. And forasmuch as the angle A is equall vnto the angle D, and the angle ABG is equall vnto the angle DEF, therfore the angle remayning AGB is equall vnto the angle remayning DFE (by the corollary of the 32. of the first. Wherfore the triangle ABG is equiangle vnto the triangle DEF. Wherfore (by the 4. of the sixth) as the side AB is to the side BG, so is the side DE to the side EF. But by suppposition the side DE is to the side EF, as the side AB is to the side BC. Wherfore (by the 11. of the fifth) as the side AB is to the side BC so is the same side AB to the side BG. Wherfore AB hath to either of these BC and BG one and the same proportion, and therfore (by the 9. of the fifth) BC is equall vnto BG. Wherefore (by the 5. of the first) ỹ angle BGC is equall vnto ỹ angle BCG: but by supposition ỹ angle BCG is lesse then a right angle. Wherfore the angle BGC is also lesse then a right angle. Wherfore (by the 13. of the first) the side angle vnto it, namely, AGB is greater then a right angle, and it is already proued that the same angle is equall vnto the angle F. Wherfore the angle F is also greater then a right angle. But it is supposed to be lesse which is absurde. Wherefore the angle ABC is not vnequall vnto the angle DEF, wherfore it is equall vnto it. And the angle A is equall vnto the angle D by supposition. Wherfore the angle remayning, namely, C, is equall vnto the angle remayning, namely, to F (by the corollary of the 32. of the first) Wherfore the triangle ABC is equiangle vnto the triangle DEF.



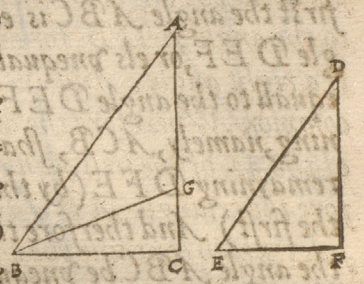
The first part of this proposition.

Demonstration leading to an impossibilitie.

The second part of this proposition.

But now suppose that either of the angles ACB and DFE be not lesse then a right

a right angle. That is, let either of them be a right angle, or either of them greater then a right angle. Then I say againe that in that case also the triangle ABC is equiangle vnto the triangle DEF. For if either of them be a right angle, forasmuch as all right angles are (by the 4. petition) equall the one to the other, straight way will follow the intent of the proposition. But if either of them be greater then a right angle, then the same order of construction that was before being kept, we may in like sort proue that the side BC is equall vnto the side BG. Wherfore also the angle BCG is equall vnto the angle BGC. But the angle BCG is greater then a right angle. Wherfore also the angle BGC is greater thē a right angle. Wherfore two angles of the triangle BGC are greater then two right angles: which (by the 17. of the first) is impossible. Wherfore the angle ABC is not vnequall vnto the angle DEF. And therfore is it equall: but the angle A is equall vnto the angle D (by supposition) Wherfore the angle remayning, namely, C is equal vnto the angle remayning, namely, to F (by the corollary of the 32. of the first). Wherfore the triangle ABC is equiangle vnto the triangle DEF. If therefore there be two triangles wherof the one hath one angle equall to one angle of the other, and the sides which include the other angle be proportionall, and if either of the other angles remaining be either lesse or not lesse then a right angle, the triangle shall be equiangle, and those angles in them shall be equall which are contained vnder sides proportionall: which was required to be proued.

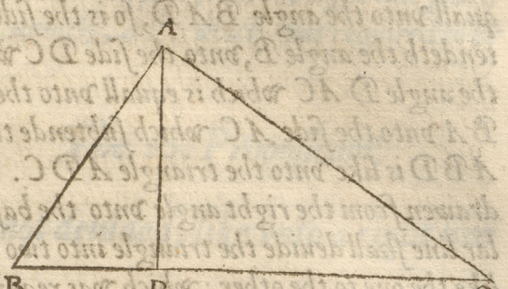


The 8. Theoreme. The 8. Proposition.

If in a rectangle triangle be drawen from the right angle vnto the base a perpendicular line, the perpendicular line shall deuide the triangle into two triangles like vnto the whole, and also like the one to the other.

Construction. Suppose that there be a rectangle triangle ABC, whose right angle let be BAC: and (by the 12. of the first) from the point A to the line BC let there be drawen a perpendicular line AD, which perpendicular line let deuide the whole triangle ABC into these two triangles ABD and ADC. (Note that this perpendicular line AD, drawen from the right angle to the base, must needes fall within the triangle ABC, & so deuide the triangle into two triangles. For if it should fall without, then producing the side BC vnto the perpendicular line, there should be made a triangle, whose outward angle being an acute angle, should be lesse then the inward and opposite angle which is a right angle: which is cōtrary to the 16. of the first. Neither

ther can it fall vpon any of the sides AB or AC: for then two angles of one and the selfe same triangle should not be lesse then two right angles, contrary to the selfe same 17. of the first. Wherefore it falleth within the triangle ABC). Then I say, that either of these triangles ABD and ADC, are like vnto the whole triangle ABC: and moreouer, that they are like the one to the other. First that the triangle ABD is like vnto the whole triangle ABC is thus proued. Forasmuch as (by the 4. petition) the angle BAC is equall vnto the angle ADB, for either of them is a right angle. And in the two triangles ABC and ABD the angle B is common. Wherefore the angle remayning, namely, ACB is (by the Corollary of the 32. of the first) equall vnto the angle remayning, namely, to BAD. Wherfore the triangle ABC is equiangle vnto the triangle ABD. Wherefore the sides which containe the equall angles, are (by the 4. of the sixt) proportionall. Wherfore as the side CB which subtendeth ỹ right angle of the triangle ABC, is vnto the side BA which subtendeth the right angle of the triangle ABD, so is the same side AB which subtendeth the angle C of the triangle ABC, vnto the side BD which subtendeth ỹ angle BAD of the triangle ABD, which is equall vnto the angle C: and moreouer, the side AC vnto the side AD which subtend the angle B common to both the triangles. Wherfore the triangle ABC is like vnto the triangle ABD (by the 1. definition of the sixt). In like maner also may we proue, that the triangle ADC is like vnto the triangle ABC. For the right angle ADC is equall to the right angle BAC, & the angle at the point C is common to either of those triangles. Wherefore the angle remayning, namely, DAC is equall to the angle remaining, namely, to ABC (by ỹ Corollary of the 32. of the first). Wherefore the triangles ABC & ADC are equiangle. And therefore (by the 4. of the sixt) the sides which are about the equall angles are proportionall. Wherefore as in the triangle ABC the side BC is to the side CA, so in the triangle ADC is the side AC to the side DC: and againe, as in the triangle ABC the side CA is to the side AB, so in the triangle ADC is the side CD to the side DA. And moreouer, as in the triangle ABC the side CB is to the side BA, so in the triangle ADC is the side CA to the side AD. Wherefore the triangle ADC is like vnto the whole triangle ABC. Wherfore either of these triangles ABD, & ADC is like vnto ỹ whole triangle ABC.



Demonstration.

I say also, that the triangles ABD and ADC are like the one to the other. For forasmuch as the right angle BDA is equall vnto ỹ right angle ADC (by the 4. petition) and as it hath already bene proued, the angle BAD is equall vnto the angle C: therefore the angle remayning, namely, B is equall vnto the angle remayning, namely, to DAC (by the Corollary of the 32. of the first). Wher-

fore the triangle A B D is equiangle vnto the triangle A D C. Wherefore as the side B D which in the triangle A B D subtendeth the angle B A D is vnto the side D A which in the triangle A D C subtendeth the angle C which is equall vnto the angle B A D, so is the side A D which in ỹ triangle A B D subtendeth the angle B, vnto the side D C which in the triangle A D C subtendeth the angle D A C which is equall vnto the angle B: and moreouer, so is the side B A vnto the side A C which subtende the right angles. Wherefore the triangle A B D is like vnto the triangle A D C. If therefore in a rectangle triangle be drawen from the right angle vnto the base a perpendicular line, the perpendicular line shall deuide the triangle into two triangles like vnto the whole, and also like the one to the other: which was required to be proued.

Corollary.

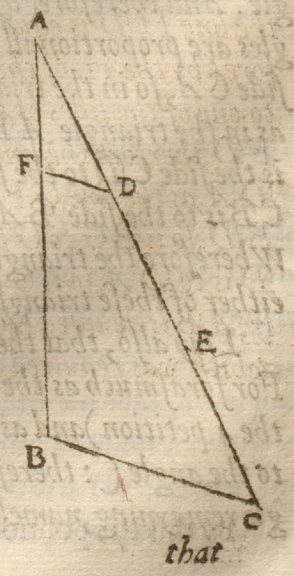
Hereby it is manifest, that if in a rectangle triangle be drawen from the right angle vnto the base a perpendicular line, the same line drawen is a meane proportionall betwene the sections of the base: and moreouer, betwene the whole base and either of the sections, the side annext to the sayd section is the meane proportionall. For is was proued, that as C D is to D A, so is D A to D B: and moreouer, as C B is to B A, so is B A to B D: and finally, as B C is to C A, so is C A to C D.

The 1. Probleme. The 9. Proposition.

A right line being geuen, to cut off frō it any part appointed.

Construction. *LEt the right line geuen be A B. It is required that from the same line A B be cut of any part appointed. Suppose that a thyrd part be appointed to be cut of. From ỹ point A draw a right line A C making with the line A B an angle: and in the line A C take a poynt at all aduentures, which let be D. And beginning at D put vnto A D two equall lines D E & E C (by the 2. of the first). And draw a right line from B to C: and by the point D (by the 31. of the first) draw vnto B C a parallell line D F. Now forasmuch as vnto one of ỹ sides of the triangle A B C, namely, vnto ỹ side B C is drawen a parallell line F D, it followeth by ỹ 2. of this booke,*

Demonstration.



that

that as C D is in proportion vnto D A, so is B F to F A. But (by construction) C D is double to D A. Wherefore ỹ line B F is also double to the line F A. Wherfore the line B A is treble vnto the line A F. Wherfore from the right line geuen A B, is cut of a third part appoynted, namely, A F: which was required to be done.

The 2. Probleme. The 10. Proposition.

To deuide a right line geuẽ not deuided, like vnto a right line geuen beyng deuided.

SVppose that the right line geuen not deuided be A B, and the right lyne geuen being deuided, let be A C. It is required to deuide ỹ line A B which is not deuided like vnto the line A C which is deuided. Suppose the lyne A C be deuided in the pointes D and E, & let ỹ lines A B & A C so be put, that they make an angle at all aduentures, and draw a line from B to C, and by the pointes D and E draw vnto the line B C (by the 31. of the first) two parallel lines D F and E G: and by the point D vnto the line A B (by the same) draw a parallel line D H K. Wherfore either of these figures F H and H B are parallelogrammes. Wherfore the line D H is equall vnto the line F G, and the line H K is equall vnto the line G B. And because to one of the sides of the triangle D K C, namely, to the side K C is drawn a parallel line H E, therefore the line C E (by the 2. of the sixt) is in proportion vnto the line E D as the line K H is to the line H D: but the line K H is equall vnto the line B G, and the line H D is equal vnto the line G F. Wherfore (by the 11. of the fift) as C E is vnto E D, so is B G to G F. Agayne because to one of the sides of the triangle A G E, namely, to G E is drawn a parallel lyne F D, therfore the line E D (by the 2. of the sixth) is in proportion vnto the lyne D A, as the line G F is to the line F A. And it is already proued that as C E is to E D, so is B G to G F. Wherfore as C E is to E D, so is B G to G F, and as E D is to D A, so is G F to F A. Wherfore the right line geuen not deuided, namely, A B is deuided like vnto the right line geuen being deuided, which is A C: which was required to be done.

Construction.

Demonstration.

¶ A Corollary out of Flussates.

A Corollary out of Flussates.

By this Proposition we may deuide any right line geuen, accordyng to the pro-

portion of any right lynes geuen. For let those right lynes hauyng proportion be ioyned together directly, that they may make all one right lyne, and then ioyne them to the lyne geuen anglewise. And so proceede as in the proposition, where you see that the right line geuen A B is deuided into the right lynes A F, F G and G B which haue the selfe same proportion that the right lines A D, D E, and E C haue.

By this and the former proposition may a right line be deuided into what partes soeuer you will.

By this and the former proposition also may a right line geuen be easily deuided into what partes so euer you will name. As if you will deuide the line A B into three equall partes, let the lyne D E be made equall to the lyne A D, and the lyne E C made equall to the same by the third of the first. And then vsing the selfe same maner of construction that was before: the lyne A B shall be deuided into three equall partes. And so of any kynde of partes whatsoeuer.

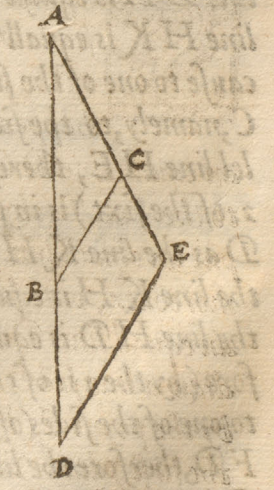
The 3. Probleme. The 11. Proposition.

Vnto two right lines geuen, to finde a third in proportion with them.

Construction.

Demonstration.

Suppose that there be two right lines geuen B A and A C, and let them be so put that they comprehend an angle howsoeuer it be. It is required to finde vnto B A and vnto A C a third line in proportion. Produce ye lynes A B and A C vnto the pointes D and E. And vnto the line A C (by the 2. of the first) put an equall line B D, and draw a lyne from B to C. And by the pointe D (by the 31. of the first) draw vnto the lyne B C a parallel lyne D E, which let concurre with the line A C in the point E. Now forasmuch as vnto one of the sides of the triangle A D E, namely, to D E is drawne a parallel line B C: therfore as A B is in proportion vnto B D, so (by the 2. of the sixt) is A C vnto C E. But the lyne B D is equall vnto the line A C. Wherfore as the lyne A B is to the line A C, so is the line A C to the line C E. Wherfore vnto the two right lines geuen A B and A C is found a third line C E in proportiõ with them: which was required to be done.

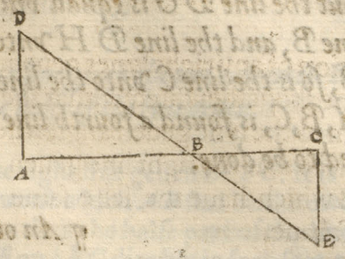


¶ An other way after Pelitarius.

An other way after Pelitarius.

Let the lines A B and B C be set directly in such sort that they both make one righte line. Then frõ the point A erect the lyne A D makyng with the lyne A B an angle at all aduentures. And put the lyne A D equall to the lyne B C. And draw a right line from D to B which produce beyond the poynt B vnto the point E. And by the point C draw vnto the lyne D A a parallel lyne C E concurring with the lyne D E in the point E. Then I say that the line C E is the third line proportionall with the lines A B and B C. For for asmuch

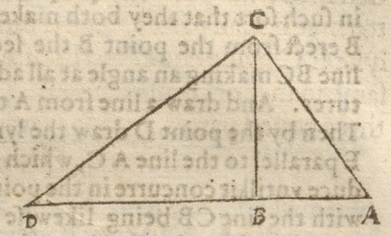
aſmuch as by the 15. of the firſt, the angle B of the triangle A B D is equall to the angle B of the triangle C B E, and by the 29. of the ſame, the angle A is equall to the angle C, and the angle D to the angle E: therefore by the 4. of this booke A B is to D A, as B C is to C E. Wherfore (by the 11. of the fifth) A B is to B C as B C is to C E: which was required to be done.



¶ *An other way alſo after Pelitarius.*

An other way after Pelitarius.

Let the lines A B and B C be ſo ioyned together, that they may make a right angle, namely, A B C. And drawe a line from A to C, and from the point C drawe vnto the line A C a perpendicular C D (by the 11. of the firſt) And produce the lyne C D till it concurre with the line A B produced vnto the pointe D. Then I ſay that the line B D is a third lyne proportionall with the lines A B and B C: which thing is manifeſt by the corollary of the 8. of this booke.

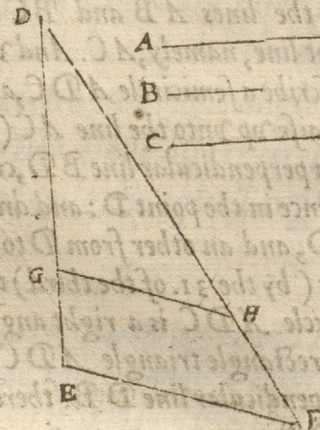


¶ The 4. Probleme. The 12. Propoſition.

Vnto three right lines geuen to finde a fourth in proportion with them.

Vppoſe that the three right lines geuen be A, B, C. It is required to finde vnto A, B, C, a fourth line in proportiõ with them. Let there be taken two right lines D E & D F comprehending an angle as it ſhall happen, namely, E D F. And (by the 2. of the firſt) vnto the line A put an equall line D G. And vnto the line B (by the ſame) put an equall line G E. And moreouer, vnto ẏ line C put an equall line D H. Then draw a line frõ G to H. And by the poynt E (by the 31. of the firſt) draw vnto the line G H a parallell line E F. Now foraſmuch as vnto one of the ſides of the triangle D E F, namely, vnto ẏ ſide E F is drawen a parallell line G H: therefore (by the 2. of the ſixt) as the line D G is to the line G E, ſo is the line D H to the line H F.

Conſtruction.



Demonſtration.

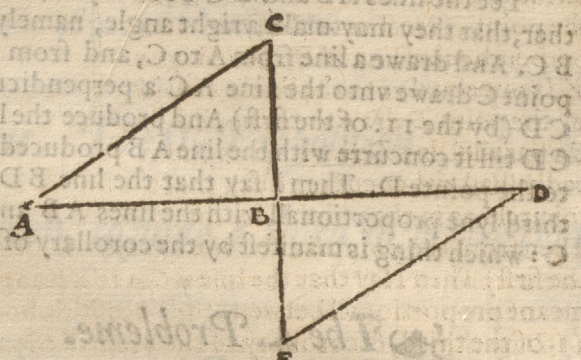
But

But the line D G is equall vnto the line A, and the line G E is equall vnto the line B, and the line D H vnto the line C. Wherfore as the line A is vnto the line B, so is the line C vnto the line H F. Wherfore vnto the three right lines geuen A, B, C, is found a fourth line H F in proportion with them: which was required to be done.

¶ *An other way after Campane.*

An other way after Campane.

Suppose that there be three right lines A B, B C, and B D. It is required to adde vnto them a fourth line in proportion with them. Ioyne A B the first, with B D the third, in such sort that they both make one right line, namely, A D. And vpon the said lyne A B erect from the point B the second line B C making an angle at all aduentures. And draw a line from A to C. Then by the point D draw the lyne D E parallel to the line A C, which produce vntill it concurre in the point E, with the line C B being likewise produced to the point E. Then I say that the line B E is the fourth line in proportion with the lines A B, B C, and B D: so that as A B is to B C, so is B D to B E. For forasmuch as by the 15 and 29. of the first the two triangles A B C and D B E are equiangle, therfore (by the 4. of this booke) A B is to B C, as B D is to B E: which was required to be done.



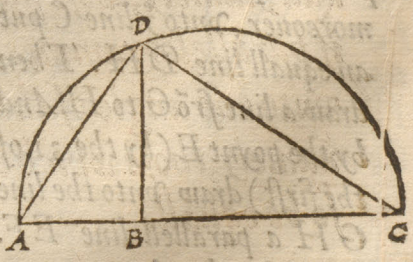
The 5. Probleme. The 13. Proposition.

Vnto two right lines geuen, to finde out a meane proportionall.

Suppose the two right lines geuen to be A B and B C. It is required betwene these two lines A B and B C to finde out a meane line proportionall. Let the lines A B and B C be so ioyned together that they both make one right line, namely, A C. And vpon the line A C describe a semicircle A D C, and from the poynt B raise vp vnto the line A C (by the 11. of the first) a perpendicular line B D, cutting the circumference in the point D: and draw a line from A to D, and an other from D to C. Now forasmuch as (by the 31. of the third) the angle in the semicircle A D C is a right angle, and for that in the rectangle triangle A D C is drawen from the right angle vnto the base a perpendicular line D B: therefore (by the Corollary of the 8. of the sixt) the line D B is a meane proportionall betwene the segmẽtes of the base A B & B C. Wherefore betwene the two right lines geuen, A B & B C, is found a meane proportionall D B: which was required to be done.

Construction.

Demonstration.



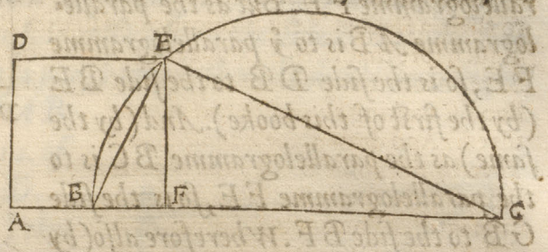
¶ *A Pro-*

¶ A Propoſition added by Pelitarius.

A meane proportionall beyng geuen, to finde out in a line geuen the two extremes. Now it behoueth that the meane geuen be not greater then the halfe of the lyne geuen.

A propoſition added by Pelitarius.

Suppoſe that the meane geuen be *AB*, and let the right line geuen be *BC*. It is required in the line *BC* to finde out two extremes, betwene which *AB* ſhal be the meane proportionall. So that yet the lyne AB be not greater then the halfe part of the line *BC*. For ſo could it not be a meane. Ioyne the lines *AB* and *BC* directly in ſuch ſort, that they both make one right line, namely, *AC*. Then vppon the line *BC* deſcribe the ſemicircle *BEC*. And from the point *A* erect vnto the lyne *AC* a perpendicular line *AD*: which lyne *AD* put equal vnto the line *AB*. And by the point *D* draw vnto the line *AC* a parallel line *DE*, which vndoubtedly ſhall either cut or touch the ſemicircle, as in the point E, for that the line *AD* is not greater then the ſemidiameter. Then from the point *E* draw vnto the line *BC* a perpendicular line *EF* (by the 12. of the firſt) Then I ſay that the line *BC* is ſo deuided in the point *F*, that the lyne *AB* is a meane proportionall betwene the lines *BF* and *FC*. Which thing is manifeſt (by the 31. of the third) & corollary of the 8. of this booke. For the line *FE* is equal to the line *AD* by the 34. of the firſt, and ſo is equall to the line *AB*: then if we draw the ryght lines *BE* and *CE*, there ſhall be made a rectangle triangle *BEC*. And ſo by the ſayd corollary, the line *BF* ſhall be to the lyne *FE* (and therfore to the line *AB*) as the line *FE* is to the lyne *FC*: which was required to be done.



Fluſſates putteth this Propoſition added by Pelitarius as a corollary following of this 13. propoſition.

The 9. Theoreme: The 14. Propoſition.

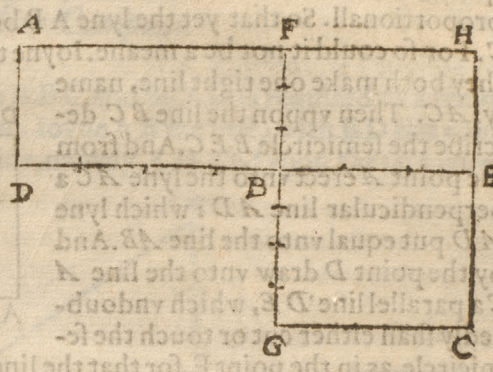
In equall parallelogrammes which haue one angle of the one equall vnto one angle of the other, the ſides ſhall be reciprokall, namely, thoſe ſides which containe the equal angles. And if parallelogrammes which hauing one angle of the one, equal vnto one angle of the other, haue alſo their ſides reciprokal, namely, thoſe which contayne the equall angles, they ſhall alſo be equall.

Vppoſe that there be two equall Parallelogrammes AB and BC, hauing the angle B of the one equall vnto the angle B of the other. And let the lines DB and BE be ſet directly in ſuch ſort that they both make one right line, namely, DE. And then (by the 14. of the firſt) ſhall the lines FB and BG be ſo ſet that they ſhall make alſo one right line, namely, GF. Then I ſay, that the ſides of the parallelogrammes AB and BC, which containe the equall angles,

The firſt part of this propoſition.

angles, are reciprocally proportionall: that is, as BD is to BE, so is GB to BF. Make complete the parallelogramme FE by producing the sides AF and CE, till they concurre in the poynt H. Now forasmuch as the parallelogramme AB is (by supposition) equall vnto the parallelogramme BC, and there is a certaine other parallelogramme FE: therfore (by the 7. of the fift) as the parallelogrāme AB is to the parallelogramme FE, so is the parallelogramme BC to the parallelogramme FE. But as the parallelogramme AB is to ỹ parallelogramme FE, so is the side DB to the side BE (by the first of this booke). And (by the same) as the parallelogramme BC is to the parallelogramme FE, so is the side GB to the side BF. Wherefore also (by the 11. of the fift) as the side DB is to the side BE, so is the side GB to the side BF. Wherefore in the parallelogrammes AB and BC the sides which containe the equall angles, are reciprokally proportionall: which was first required to be proued.

Demonstration of the of the same.



The second part which is the conuerse of the first.

But now suppose that the sides about the equall angles be reciprokally proportionall, so that as the side DB is to the side BE, so let the side GB be to the side BF. Then I say, ỹ the parallelogramme AB is equall vnto ỹ parallelogramme BC. For, for that as the side DB is to the side BE, so is the side GB to the side BF: but as the side DB is to the side BE, so (by the 1. of the sixt) is the parallelogramme AB to the parallelogramme FE: and as the side GB is to the side BF, so is the parallelogramme BC to the parallelogramme FE. Wherefore also (by the 11. of the fift) as the parallelogramme AB is to the parallelogrāme FE, so is the parallelogramme BC to the same parallelogramme FE. Wherefore the parallelogramme AB is equall vnto the parallelogramme BC (by the 9. of the fift). Wherefore in equall and equiangle parallelogrammes the sides which containe the equall angles are reciprokall: and if in equiangle parallelogrammes the sides which containe the equall angles be reciprokall, the parallelogrammes also shall be equall: which was required to be proued.

The 10. Theoreme. The 15. Proposition.

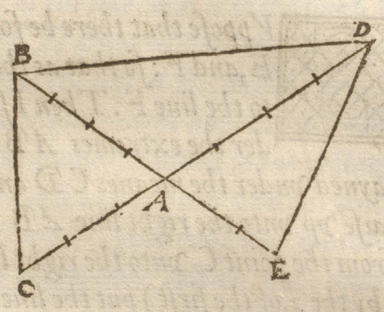
In equal triangles which haue one angle of the one equall vnto one angle of the other, those sides are reciprokal, which include the equall angles. And those triāgles which hauyng one angle of the one equall vnto one angle of the other, haue also

their

their ſides which include the equall angles reciprokal, are alſo equall.

SVppoſe that there be two equall triangles A B C, and A D E hauing one angle of the one equall vnto one angle of the other, namely, the angle B A C equall vnto the angle D A E. Then I ſay that in thoſe triangles A B C and A D E, the ſides which include ỹ equal angles, are reciprokallie proporionall, that is, as the ſide C A is to the ſide A D, ſo is the ſide E A to the ſide A B. For let ỹ lines C A and A D be ſo put, ỹ they both make directly one right line. And ſo alſo the lines E A and A B ſhal both make one right line (by the 14. of the firſt) And draw a line from B to D. Now foraſmuch as (by ſuppoſition) the triangle A B C is equall vnto the triangle A D E. And there is a certaine other triangle B A D, vnto which the two equall triangles being compared, it will follow by the 7. of the fifth, that as the triangle A B C is vnto the triangle B A D, ſo is ỹ triangle E A D to the ſame triangle B A D. But as the triangle A B C is to the triangle B A D, ſo by the 1. of the ſixth, is the baſe C A to the baſe A D: and as the triangle E A D is to the triangle B A D, ſo (by the ſame) is the baſe E A to the baſe A B. Wherfore (by the 11. of the fifth) as the ſide C A is to the ſide A D, ſo is the ſide E A to the ſide A B. Wherefore in the triangles A B C and A D E the ſides which include the equall angles are reciprokally proportionall.

The firſt part of this propoſition.



Demonſtration of the ſame.

But now ſuppoſe that in the triangles A B C and A D E, the ſides which include the equall angles, be reciprokally proportionall, ſo that as the ſide C A is to the ſide A D, ſo let the ſide E A be to the ſide A B. Then I ſay that the triangle A B C is equall vnto the triangle A D E. For agayne draw a line from B to D. And for that as the line C A is to the line A D, ſo is the line E A to the line A B, but as the line C A is to the line A D, ſo is the triangle A B C to the triangle B A D, and as the line E A is to the line A B ſo is the triangle E A D to the triangle B A D. Wherfore as the triangle A B C is to the triangle B A D ſo is the triangle E A D to ỹ ſame triangle B A D. Wherfore either of theſe triangles A B C and E A D haue vnto ỹ triangle B A D one and ỹ ſelfe ſame proportion. Wherfore (by the 9. of the fifth) the triangle A B C is equal vnto the triangle E A D. If therfore there be taken equall triangles hauyng one angle of the one equal vnto one angle of the other, thoſe ſides in them ſhal be reciprokal, which include the equal angles: and thoſe triangles which hauing one angle of the one equall vnto one angle of the other, haue alſo their ſides which include the equall angles reciprokal, ſhal alſo be equall: which was required to be proued.

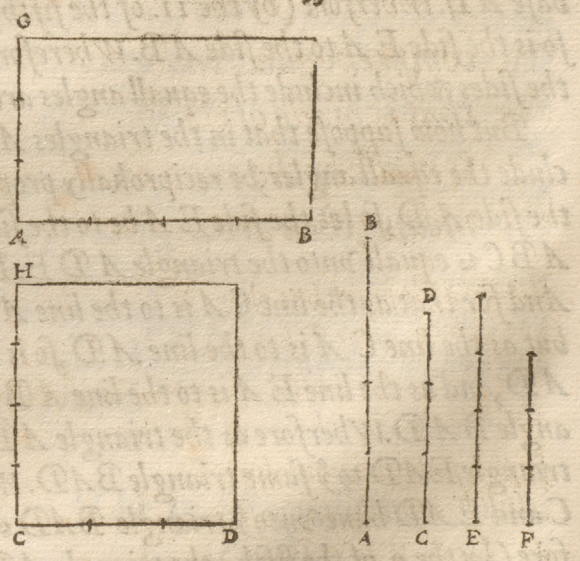
The ſecond part which is the conuerſe of the firſt.

The 11. Theoreme. The 16. Proposition.

If there be foure right lines in proportion, the rectangle figure comprehended vnder the extremes: is equall to the rectangle figure contayned vnder the meanes. And if the rectangle figure which is contained vnder the extremes, be equall vnto the rectangle figure which is contayned vnder the meanes: then are those foure lines in proportion.

Demonstration of the first part.

SVppose that there be foure right lines in proportiõ, namely, AB, CD, E, and F: so that as the line AB is to the line CD, so let the line E be to the line F. Then I say, that the rectangle figure comprehended vnder the extremes AB and F, is equall vnto the rectangle figure contayned vnder the meanes CD and E. From the poynt A (by the 11. of the first) raise vp vnto the right line AB a perpendicular line AG. And (by the same) from the point C vnto the right line CD raise vp a perpendicular line CH. And (by the 2. of the first) put the line AG equall vnto the line F, and put also ỹ line CH equall vnto the line E, and make complete the parallelogrammes GB and HD. Now for that by supposition as the line AB is to the line CD, so is the line E to the line F. But the line E is equall vnto the line CH, & the line F vnto the line AG, therefore as the line AB is to the line CD, so is the line CH to the line AG. Wherefore in the



parallelogrãmes BG and DH the sides which include ỹ equall angles, are reciprokally proportionall. But equiangle parallelogrammes whose sides which include the equall angles, are reciprokall, are also equall (by the 14. of the sixt). Wherefore the parallelogramme BG is equall vnto the parallelogrãme DH. But the parallelogramme BG is that which is contayned vnder the lines AB and F, for the line AG is put equall vnto the line F. And the parallelogramme DH is that which is contained vnder the lines CD and E, for the line CH is put equall vnto the line E. Wherfore the rectangle figure contained vnder the lines AB and F, is equall vnto the rectangle figure contayned vnder the lines CD and E.

The second part which is the conuerse of the first.

But now suppose that the rectangle figure comprehended vnder the lines AB and F, be equall vnto the rectangle figure cõprehended vnder ỹ lines CD & E.

Then

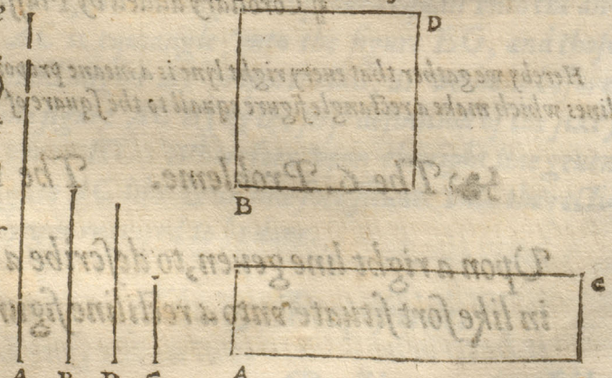
Then I say, that the foure right lines AB, C D, E and F, are proportionall, that is, as the line A B is to the line C D, so is the line E to the line F. The same order of construction that was before being kept, forasmuch as that which is contained vnder the lines A B and F is equall vnto that which is contained vnder the lines C D and E, but that which is contayned vnder the lines A B and F is the parallelogramme B G, for the line A G is equall vnto the line F. And that also which is contained vnder the lines C D & E is the parallelogramme D H, for the line C H is equall vnto the line E. Wherefore the parallelogramme B G is equall vnto the parallelogramme D H, & they are also equiangle. But in parallelogrammes equall & equiangle the sides which include the equall angles are reciprokall (by the 14. of the sixt). Wherfore as the line A B is to the line C D, so is the line C H to the line A G, but the line C H is equall vnto the line E, and the line A G is equall vnto ỹ line F. Wherefore as the line A B is to the line C D, so is the line E to the line F. If therefore there be foure right lines in proportion, the rectangle figure comprehended vnder the extremes, is equall to the rectangle figure contayned vnder the meanes. And if the rectangle figure which is contained vnder the extremes, be equall vnto the rectangle figure which is contained vnder the meanes, then are those foure lines in proportion: which was required to be proued.

The 12. Theoreme. The 17. Proposition.

If there be three right lines in proportion, the rectangle figure comprehended vnder the extremes, is equall vnto the square that is made of the meane. And if the rectangle figure which is made of the extremes, be equal vnto the square made of the meane, then are those three right lines proportional.

Vppose that there be three lines in proportion A, B, C, so that as A is to B, so let B be to C. Then I say that the rectangle figure comprehended vnder ỹ lines A and C is equall vnto ỹ square made of the line B. Vnto the line B (by the 2. of the first) put an equall line D. And because by supposition) as A is to B, so is B to C, but B is equall vnto D, wherefore (by the 7. of the fifth) as A is to B, so is D to C, but if there be foure right lines proportionall, the rectangle figure comprehended vnder

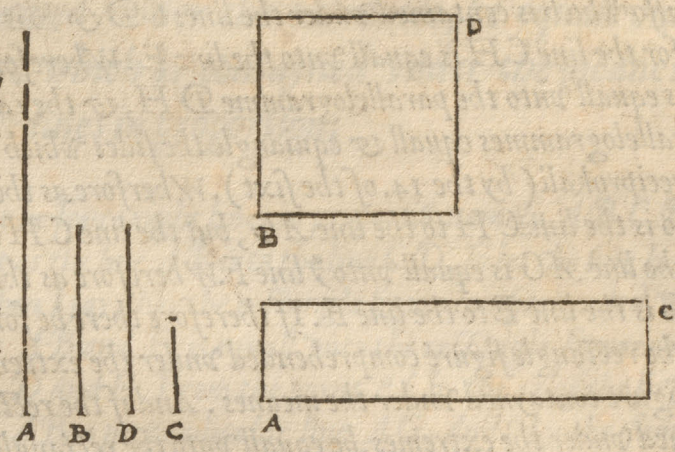
The first part of this Theoreme.



the extremes is equall vnto the rectangle figure comprehended vnder the meanes (by the 16. of the sixt). Wherfore that which is contained vnder the lines A and C is equall vnto that which is comprehended vnder the lines B and D. But that which is contained vnder the lines B and D, is the square of the line B, for the line B is equall vnto the line D. Wherfore the rectangle figure comprehẽded vnder the lines A and C is equall vnto the square made of the lyne B.

The second part which is the conuerse of the first.

But now suppose that that which is comprehended vnder the lines A & C be equal vnto the square made of the line B. Then also I say, that as the line A is to the line B, so is the line B to the lyne C. The same order of construction that was before, beyng kept, forasmuch as ẏ which is contained vnder the lynes A and C is equall vnto the square which is made of the line B. But the square which is made of the line B is that which is contained vnder ẏ lines B & D, for the line B is put equall vnto the line D. Wherefore that which is contayned vnder the lines A and C is equall vnto that which is contayned vnder the lines B and D. But if the rectangle figure comprehended vnder the extremes, be equall vnto the rectangle figure comprehended vnder the meane lynes, the foure right lines shall be proportionall (by the 16. of the sixth) Wherfore as the line A is to the line B, so is the line D to the line C. But the line B is equall vnto the lyne D. Wherfore as the line A is to the lyne B, so is B to the line C. If therefore there be three right lynes in proportion, the rectangle figure comprehended vnder the extremes, is equall vnto the square that is made of the meane. And if the rectangle figure which is contayned vnder the extremes, be equall vnto the square made of the meane, then are those three right lines proportional: which was required to be demonstrated.



¶ Corollary added by Flussates.

A Corollary.

Hereby we gather that euery right lyne is a meane proportionall betwene euery two right lines which make a rectangle figure equall to the square of the same right lyne.

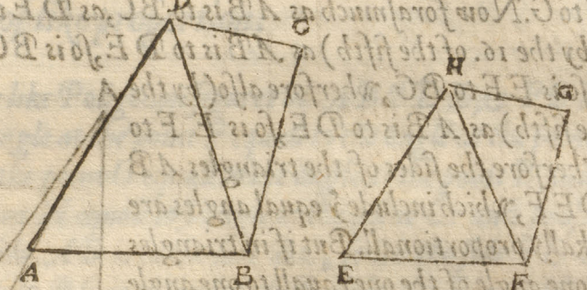
The 6. Probleme. The 18. Proposition.

Upon a right line geuen, to describe a rectiline figure like, and in like sort situate vnto a rectiline figure geuen.

Suppose

SVppose that the right line geuen be AB, and let the rectiline figure geuen be EG. It is required vpon the right line geuen AB to describe a rectiline figure like, and in like sort situate vnto the rectiline figure geuen GE. Drawe a line from H to F, and vnto the right line AB and to ỹ point in it A, make vnto the angle E an equall angle DAB (by the 23. of the first) and vnto the right line AB and vnto the point in it B (by the same) make vnto the angle EFH an equall angle ABD. Wherefore ỹ angle remayning EHF is equall vnto the angle remayning ADB. Wherefore the triangle HEF is equiangle vnto the triangle DAB. Wherefore (by the 4. of the sixt) as the side HF is in proportion to the side DB, so is the side HE to the side DA, and ỹ side EF to the side AB. Againe (by the 23. of the first) vnto the right line BD and vnto ỹ point in it D, make vnto the angle FHG an equall angle BDC, and (by the same) vnto the right line BD and vnto the point in it B, make vnto the angle HFG an equall angle DBC. Wherefore the angle remayning, namely, G, is equall vnto the angle remayning, namely, to C. Wherefore the triangle HFG is equiangle vnto the triangle DBC. Wherefore (by the 4. of the sixt) as the side HF is in proportion to the side DB, so is the side HG to the side DC, and the side GF to the side CB. And it is already proued that as HF is to DB, so is HE to DA, and EF to AB. Wherefore (by the 11. of the fift) as EH is to AD, so is EF to AB, and HG to DC, and moreouer, GF to CB. And forasmuch as the angle EHF is equall vnto the angle ADB, and the angle FHG is equall vnto the angle BDC: therefore the whole angle EHG is equall vnto the whole angle ADC, and by the same reason the angle EFG is equall vnto the angle ABC. But (by construction) the angle E is equall vnto the angle A, and the angle G is proued equall vnto the angle C. Wherefore the figure AC is equiangle vnto the figure EG, and those sides which in it include the equall angles are proportionall, as we haue before proued. Wherefore the rectiline figure AC is (by the first definition of the sixt) like vnto the rectiline figure geuen EG. Wherefore vpon the right line geuen AB is described a rectiline figure AC like & in like sort situate vnto the rectiline figure geuen EG: which was required to be done.

Description of the rectiline figure required.



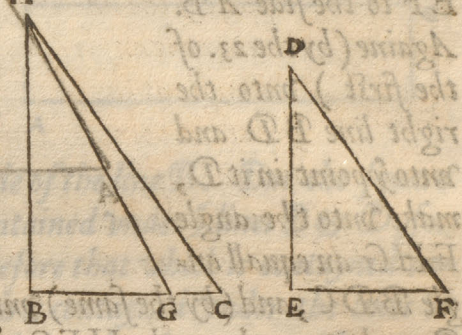
Demonstration.

The 13. Theoreme. The 19. Propoſition.

Like triangles are one to the other in double proportion that the sides of lyke proportion are.

SVppose the triangles like to be ABC and DEF, hauing the angle B of the one triangle, equal vnto the angle E of the other triangle, & as AB is to BC, so let DE be to EF, so that let BC & EF be sides of like proportion. Then I say that the proportion of the triangle ABC vnto the triangle DEF is double to the proportion of the side BC to the side EF. Vnto the two lines BC and EF (by the 10. of the sixth) make a third lyne in proportion BG, so that as BC is to EF, so let EF be to BG, and draw a lyne from A to G. Now forasmuch as AB is to BC, as DE is to EF, therfore alternately (by the 16. of the fifth) as AB is to DE, so is BC to EF. But as BC is to EF, so is EF to BG, wherfore also (by the 11. of the fifth) as AB is to DE, so is EF to BG. Wherfore the sides of the triangles ABG & DEF, which include ỹ equal angles are reciprokally proportionall. But if in triangles hauing one angle of the one equall to one angle of ỹ other, the sides which include ỹ equall angles, be reciprokal, the triangles also (by ỹ 15. the sixth) shal be equall. Wherfore the triangle ABG is equall vnto ỹ triangle DEF. And for ỹ as ỹ line BC is to ỹ line EF, so is the line EF to ỹ line BG: but if there be three lines in proportion, the first shall haue to the third double proportion that it hath to the second (by the 10. definition of the fifth) therfore the line BC hath vnto the line BG double proportion that it hath to the line EF. But as BC is to BG, so (by the 1. of the sixth) is the triangle ABC to the triangle ABG. Wherfore the tiangle ABC is vnto the triangle ABG in double proportion that the side BC is to the side EF. But the triangle ABG is equall to the triangle DEF. Wherfore also the triangle ABC is vnto the triangle DEF in double proportion that the side BC is to the side EF. Wherfore lyke triangles are one to the other in double proportion that the sides of like proportion are: which was required to be proued.

Demonstration.



Corollary.

A Corollary.

Hereby it is manifest that if there be three right lines in proportion, as the first is to the third, so is the triangle described vpon the first, vnto the triangle described vpon the second, so that the sayd triangles be like, and in lyke sort described, for it hath bene proued that as the lyne CB is to the line BG, so

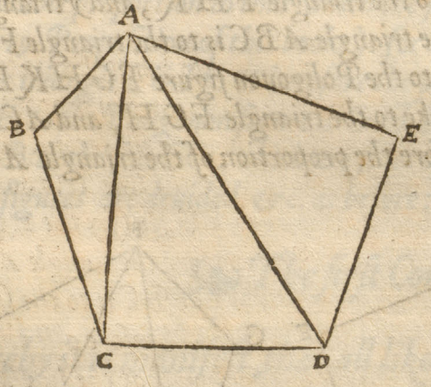
is

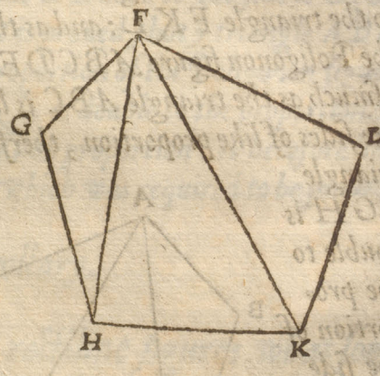
is the triangle ABC to the triangle DEF: which was required to be demonstrated.

The 14. Theoreme. The 20. Proposition.

Like Poligonon figures, are deuided into like triangles and equall in number, and of like proportion to the whole. And the one Poligonon figure is to the other Poligonon figure in double proportion that one of the sides of like proportion is to one of the sides of like proportion.

Vppose ẏ the like Poligonon figures be ABCDE, & FGHKL, hauing the angle at the point F equall to the angle at the point A, and the angle at the point G equall to the angle at the point B, and the angle at the point H equall to ẏ angle at the point C: and so of the rest. And moreouer, as the side AB is to the side BC, so let the side FG be to the side GH, and as the side BC is to the side CD, so let the side GH be to the side HK, and so forth. And let the sides AB & FG be sides of like proportion. Then I say first, that these Poligonon figures ABCDE & FGHKL, are deuided into like triangles and equall in number. For draw these right lines, AC, AD, FH, & FK. And forasmuch as (by supposition, that is, by reason the figure ABCDE is like vnto the figure FGHKL) the angle B is equall vnto the angle G, and as the side AB is to the side BC, so is the side FG to the side GH, it followeth that the two triangles ABC and FGH haue one angle of the one equall to one angle of the other, and haue also the sides about the equall angles proportionall. Wherefore (by the 6. of the sixt) the triangle ABC is equiangle vnto the triangle FGH. And those angles in thē are equall, vnder which are subtended sides of like proportion: namely, the angle BAC is equall to the angle GFH, and the angle BCA to the angle GHF. Wherefore (by the 4. of the sixt) the sides which are about the equall angles are proportionall: and the sides which are sub-



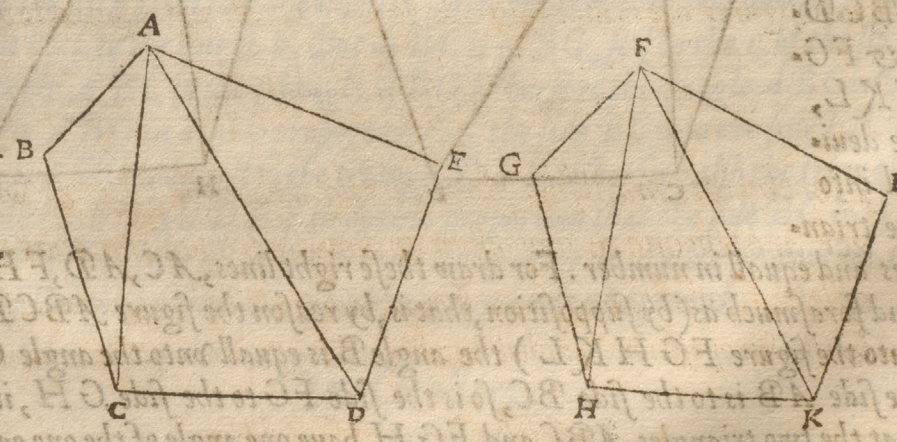


The first part of this Theoreme.

tended vnder the equall angles are of like proportion. Wherfore as AC is to BC, ſo is FH to GH. But by ſuppoſition as BC is to CD, ſo is GH to HK. Wherefore of equalitie (by the 22. of the fift) as AC is to CD, ſo is FH to HK. And foraſmuch as by ſuppoſition the whole angle BCD is equall to the whole angle GHK, and it is proued that the angle BCA is equall to the angle GHF: therefore the angle remayning ACD is equall to the angle remayning FHK (by the 3. common ſentence). Wherefore the triangles ACD, and FHK, haue againe one angle of the one equall to one angle of the other, and the ſides which are about the equall ſides are proportionall: Wherefore (by the ſame ſixt of this booke) the triangles ACD & FHK are equiangle. And (by the 4. of this booke) the ſides which are about the equall angles are proportionall. And by the ſame reaſon may we proue that the triangle ADE is equiangle vnto the triangle FKL. And that the ſides which are about the equall angles are proportionall. Wherefore the triangle ABC is like to ỹ triangle FGH, and the triangle ACD to the triangle FHK, and alſo the triangle ADE to the triangle FKL (by the firſt definition of this ſixt booke). Wherfore the Poligonon figures geuen $ABCDE$, and $FGHKL$, are deuided into triangles like and equall in number.

The ſecond part demonſtrated.

I ſay moreouer, that the triangles are the one to the other, and to the whole Poligonon figures proportionall: that is, as the triangle ABC is to the triangle FGH, ſo is the triangle ACD to the triangle FHK, and ỹ triangle ADE to the triangle FKL: and as the triangle ABC is to the triangle FGH, ſo is the Poligonon figure $ABCDE$ to the Poligonon figure $FGHKL$. For foraſmuch as the triangle ABC is like to the triangle FGH, and AC and FH are ſides of like proportion, therfore the proportion of the triangle ABC to the triangle FGH is double to the proportion of the ſide AC to the ſide FH (by ỹ former Propoſition).



And therefore alſo the proportion of the triangle ACD to the triangle FKH, is double to the proportion that the ſame ſide AC hath to the ſide FH. Wherefore (by the 11. of the fift) as the triangle ABC is to the triangle FGH, ſo is the triangle ACD to the triangle FHK. Againe, foraſmuch as the triangle ACD is like to the triangle FHK, and the ſides AD & FK are of like proportion: therefore the proportion of the triangle ACD to the triangle FHK is double

is double to the proportion of the side AD to the side FK (by the foresayd 19. of the sixt). And by the same reason the proportion of the triangle ADE to ỹ triangle FKL, is double to the proportion of the same side AD to the side FK. Wherfore (by the 11. of the fift) as the triangle ACD is to the triangle FHK, so is the triangle ADE to the triangle FKL. But as the triangle ACD is to the triangle FHK, so is it proued that the triangle ABC is to the triangle FGH. Wherefore also (by the 11. of the fift) as the triangle ABC is to the triangle FGH, so is the triangle ADE to the triangle FKL. Wherefore the foresayd triangles are proportionall: namely, as ABC is to FGH, so is ACD to FHK, and ADE to FKL. Wherefore (by the 12. of the fift) as one of the antecedentes is to one of the consequentes, so are all the antecedentes to all the consequentes. Wherefore as the triangle ABC is to the triangle FGH, so is the Poligonon figure ABCDE to the Poligonon figure FGHKL. Wherfore the triangles are proportionall both the one to the other, & also to the whole Poligonon figures.

Lastly I say, that the Poligonon figure ABCDE hath to the Poligonon figure FGHKL a double proportion to that which the side AB hath to the side FG: which are sides of like proportion. For it is proued, that as the triangle ABC is to the triangle FGH, so is the Poligonon figure ABCDE to the Poligonon figure FGHKL. But the triangle ABC hath to the triangle FGH a double proportion to that which the side AB hath to the side FG (by the former 19. Proposition of this booke): for it is proued, that ỹ triangle ABC is like to the triangle FGH. Wherefore the proportion of the Poligonon figure ABCDE to the Poligonon figure FGHKL is double to the proportion of the side AB to the side FG: which are sides of like proportion. Wherefore like Poligonon figures are deuided. &c. as before: which was required to be proued. The third part.

The first Corollary.

Hereby it is manifest, that all like rectiline figures what soeuer, are the one to the other in double proportion that the sides of like proportion are. For any like rectiline figures whatsoeuer are by this Proposition deuided into like triangles and equall in number. The first Corollary.

The second Corollary.

Hereby also it is manifest, that if there be three right lines proportionall, as the first is to the third, so is the figure described vppon the first to the figure described vpon the second, so that the sayd figures be like and in like sort described. The second Corollary.

For

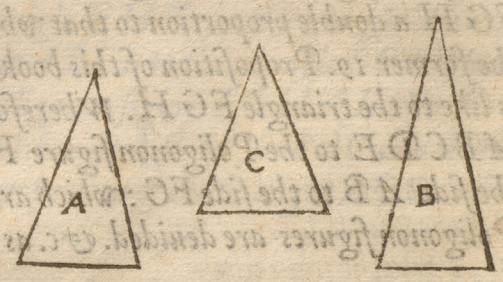
For it is proued, that the proportion of the Poligonon figure A B C D E to the Poligonon figure F G H K L is double to ỹ proportion of the side A B to the side F G. And if (by the 11. of the sixt) vnto the lines A B and F G we take a third line in proportion, namely, M N, the first line, namely, A B shall haue vnto the third line, namely, to M N, double proportion that it hath to the second line, namely, to F G (by the 10. definition of the fift). Wherfore as ỹ line A B is to the line M N, so is the rectiline figure A B C to the rectiline figure F G H, the sayd rectiline figures being like & in like sort described.

The 15. Theoreme. The 21. Proposition.

Rectiline figures which are like vnto one and the same rectiline figure, are also like the one to the other.

Demonstration.

Vppose there be two rectiline figures A and B like vnto the rectiline figure C. Then I say that the figure A is also like vnto the figure B. For forasmuch as the figure A is like vnto the figure C, it is also equiangle vnto it (by the conuersion of the first definition of the sixth) & the sides including the equall angles shall be proportionall. Agayne forasmuch as the figure B is like vnto the figure C, it is also (by the same definition) equiangle vnto it, and the sides about the equall angles are proportionall. Wherfore both these figures A and B are equiangle vnto the figure C, and the sides about the equall angles are proportionall. Wherfore (by the first common sentence) the figure A is equiangle vnto the figure B, and the sides about the equall angles are proportionall, wherfore the figure B is like vnto the figure A, which was required to be proued.



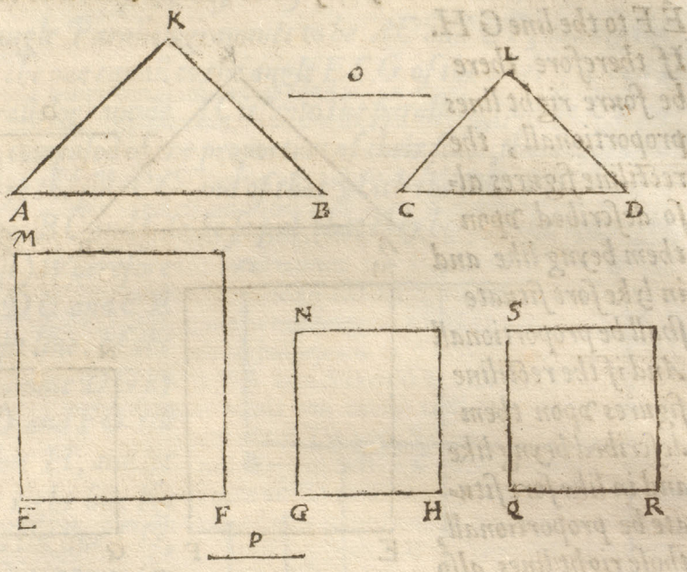
The 16. Theoreme. The 22. Proposition.

If there be foure right lines proportionall, the rectiline figures also described vpon them beyng lyke, and in like sorte situate, shall be proportional. And if the rectiline figures vppon them described be proportional, those right lynes also shall be proportionall.

Suppose

SVppose there be foure right lines AB, CD, EF, and GH, and as AB is to CD, so let EF be to GH. And vpon the lines AB and CD (by the 18. of the sixth) let there be described two rectiline figures KAB, and LCD like the one to the other, and in like sort situate. And vpon the lynes EF and GH (by the same) let there be described also two rectiline figures MF and NH like the one to the other, and in like sorte situate. Then I say that as the figure KAB is to the figure LCD, so is the figure MF to the figure NH. Vnto the lines AB and CD (by the 11. of the sixth) make a third lyne in proportion, namely, O: and vnto the lines EF and GH in like sort make a third lyne in a line proportion, namely, P. And for that as the line AB is to the line CD, so is the line EF to the line GH, but as the line CD is to the line O, so is the line GH to the lyne P. Wherfore of equality (by the 22. of the fifth) as the lyne AB is vnto the line O, so is the lyne EF to the line P. But as the line AB is to the line O, so is the figure KAB to the figure LCD (by the second corollary of the 20. of the sixth). And as the line EF is to the lyne P, so is the figure MF to the figure NH. Wherefore (by the 11. of the fifth) as the figure KAB is to the figure LCD, so is the figure MF to the figure NH.

The first part of this proposition.



The second part which is the conuerse of the first.

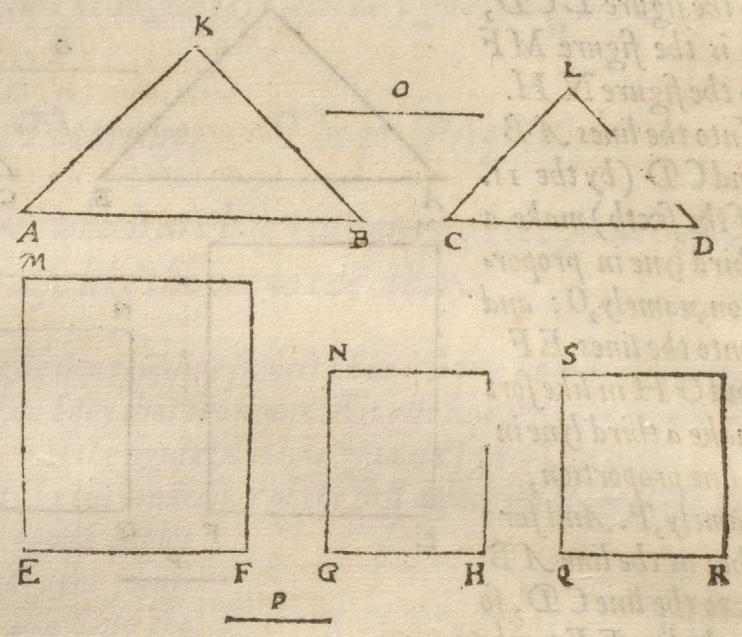
But now suppose that as the figure KAB is to the figure LCD, so is the figure MF to the figure NH, then I say that as the line AB is to the line CD, so is the line EF to the line GH. As the line AB is to the lyne CD, so (by the 12. of the sixth) let the lyne EF be to the lyne QR, and vpon the lyne QR (by the 18. of the sixth) describe vnto either of these figures MF and NH a like figure, and in like sort situate SR. Now forasmuch as the lyne AB is to the lyne CD, so is the lyne EF to the line QR, and vpon the lines AB and CD are described two figures lyke, and in like sort situate KAB and LCD, and vpon the lines EF and QR are described also two figures like, and in like sort situate MF and SR, therfore as the figure KAB is to the figure LCD, so is the figure MF to the figure SR: wherfore also (by the 11. of the fifth) as the figure MF is to the figure SR, so is the figure MF to the figure NH, wherfore the figure

MF

M F hath to either of these figures N H, and S R one and the same proportion, wherfore by the 9. of the fifth, the figure N H is equal vnto the figure S R. And it is vnto it like, and in like sort situate. * But in like and equall rectiline figures beyng in like sort situate, the sides of like proportion on which they are described are equall. Wherfore ỹ line G H is equall vnto the line Q R. And because as the lyne A B is to the line C D, so is the line E F to the line Q R, but the line Q R is equall vnto the line G H, therfore as the line A B is to the line C D, so is the line E F to the line G H.

* Note that this is proued in the assumpt following.

If therefore there be foure right lines proportionall, the rectiline figures also described vpon them beyng like and in lyke sort situate shall be proportionall And if the rectiline figures vpon them described beyng like and in like sort situate be proportionall, those right lines also shall be proportional: which was required to be proued.



An Assumpt.

An assumpt.

And now that in like and equall figures, being in like sort situate, the sides of like proportion are also equall (which thing was before in this proposition taken as graunted) may thus be proued. Suppose ỹ the rectiline figures N H and S R be equall and like, and as H G is to G N, so let R Q be to Q S, and let G H and Q R be sides of like proportion. Then I say that the side R Q is equall vnto the side G H. For if they be vnequall, the one of them is greater then the other, let the side R Q be greater then the side H G. And for that as the line R Q is to the line Q S, so is the line H G to the line G N, and alternately also (by the 16. of the fifth) as the line R Q is to the line H G, so is the line Q S, to the lyne G N, but the line R Q is greater then the line H G. Wherfore also the line Q S is greater then ỹ line G N. Wherefore also ỹ figure R S is greater then the figure H N but (by supposition) it is equall vnto it, which is impossible. Wherfore ỹ line Q R is not greater then ỹ line G H. In like sorte also may we proue that it is not lesse then it, wherfore it is equall vnto it: which was required to be proued.

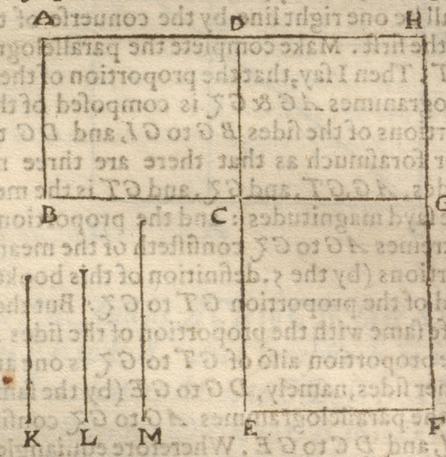
The

Flussates demonstrateth this second part more briefly, by the first corollary of the 20. of this boke, thus. Forasmuch as the rectiline figures are by supposition in one and the same proportion, and the same proportion is double to the proportion of the sides A B to C D, and E F to G H (by the foresaid corollary) the proportion also of the sides shall be one and the selfe same (by the 7. common sentence) namely, the line A B shall be vnto the line C D as the line E F is to the line G H. *An other demonstration of the second part after Flussates.*

The 17. Theoreme. The 23. Proposition.

Equiangle Parallelogrammes haue the one to the other that proportion which is composed of the sides.

Suppose the equiangle Parallelogrammes to be AC and CF, hauing the angle BCD of the one equall to the angle ECG of the other. Then I say, that the parallelogramme AC is vnto the parallelogramme CF, in that proportion which is composed of the proportion of their sides, that is, of that which the side BC hath to the side CG, and of that which the side DC hath to the side CE. Let the lines BC and CG be so put that they both make one right line (by the 14. of the first). Wherefore (by the same) the lines DC and CE shall make also one right line. Make complete the parallelogramme DG by producing the sides AD and FG till they concurre in the point H, and let there be put a certaine right line K. And as the line BC is to the line CG, so (by the 12. of the sixt) put vnto the line K a line in the same proportion which let be L: and as DC is to CE, so vnto L put a line in the same proportion, namely, M. Wherefore the proportions of the lines K to L, and L to M, are one and ỹ same with the proportions of the sides BC to CG, and DC to CE: but the proportion of K to M, is composed of the proportions of K to L, & L to M: Wherefore the proportion of K to M, is composed of the proportions of the sides BC to CG, & EC to CD. And for that as the line BC is to the line CG, so is the parallelogramme AC to the parallelogrãme CH (by the 1. of the sixt). But as the line BC is to the line CG, so is the line K to the line L. Wherefore also (by the 11. of the fift) as the line K is to the line L, so is the parallelogramme AC to the parallelogramme CH. Againe, for that as the line DC is to the line CE, so is the parallelogramme CH to the parallelogramme CF: but as the line DC is to the line CE, so is the line L to the line M: Wherefore also (by ỹ same) as the line L is to the line M, so is the parallelogramme CH to the parallelogramme CF. And forasmuch as it is proued, that as the line K is to the line L, so is the parallelogramme AC to the parallelogramme CH, and as the line L is to the line M, so is the parallelogramme CH to the parallelogramme CF: therefore of equalitie (by the 22. of the fift) as the line K is to the line M, so is the parallelogramme AC to the parallelogramme CF. But as it hath before bene proued, the proportion of the line K to the line M, is composed of the proportions of



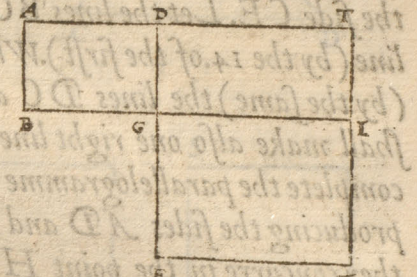
Demonstration.

the sides BC to CG, and DC to CE. Wherefore also the proportion of the parallelogramme AC to the parallelogramme CF, is composed of the proportions of the sides BC to CG, and DC to CE. Wherefore equiangle parallelogrammes haue the one to the other that proportion which is composed of the proportions of the sides: which was required to be proued.

Flussates demonstrateth this Theoreme without taking of these three lines, K, L, M, after this maner.

An other demonstration after flussates.

Forasmuch as (sayth he) it hath bene declared vpon the 10. definition of the fift booke, and fift definition of this booke, that the proportions of the extremes consist of the proportions of the meanes, let vs suppose two equiangle parallelogrãmes *A B G D*, and *G E Z I*, and let the angles at the poynt *G* in eyther be equall. And let the lines *B G* and *G I* be set directly that they both make one right line, namely, *B G I*. Wherefore *E G D* also shall be one right line by the conuerse of the 15. of the first. Make complete the parallelogramme *G T*. Then I say, that the proportion of the parallelogrammes *A G* & *G Z* is composed of the proportions of the sides *B G* to *G I*, and *D G* to *G E*. For forasmuch as that there are three magnitudes, *A G*, *G T*, and *G Z*, and *G T* is the meane of the sayd magnitudes: and the proportion of the extremes *A G* to *G Z* consisteth of the meane proportions (by the 5. definition of this booke) namely, of the proportion of *A G* to *G T*, and of the proportion *G T* to *G Z*: But the proportion of *A G* to *G T* is one and the selfe same with the proportion of the sides *B G* to *G I* (by the first of this booke). And the proportion also of *G T* to *G Z* is one and the selfe same with the proportion of the other sides, namely, *D G* to *G E* (by the same Proposition). Wherefore the proportion of the parallelogrammes *A G* to *G Z* consisteth of the proportions of the sides *B G* to *G I*, and *D C* to *G E*. Wherefore equiangle parallelogrammes are the one to the other in that proportion which is composed of theyr sides: which was required to be proued.



The 18. Theoreme. The 24. Proposition.

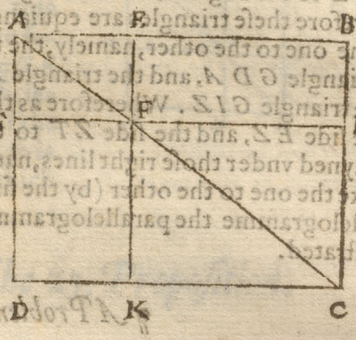
In euery parallelogramme, the parallelogrammes about the dimecient are lyke vnto the whole, and also lyke the one to the other.

Demonstration of this propositiõ wherein is first proued that the parallegramme EG is like to the whole parallelogrãme ABCD.

Vppose ỹ there be a parallelogramme A B C D, and let the dimecient therof be A C: and let the parallelogrammes about the dimecient A C, be E G and H K. Then I say that either of these parallelogrames E G and H K is like vnto the whole parallelogramme A B C D, and also are lyke the one to the other. For forasmuch as to one of the sides of the triangle A B C, namely, to B C is drawen a parallel lyne E F, therfore as B E is to E A, so (by the 2. of the sixt) is C F to F A. Agayne forasmuch as to one of ỹ sides

of the

of the triangle A D C, namely, to C D is drawen a parallel lyne F G, therefore (by the same) as C F is to F A, so is D G to G A. But as C F is to F A, so is it proued that B E is to E A. Wherfore as B E is to E A, so (by the 11. of the fifth) is D G to G A. Wherfore by composition (by the 18. of the fifth) as B A is to A E, so is D A to A G. And alternately (by the 16. of the fifth) as B A is to A D, so is E A to A G. Wherfore in the parallelogrammes A B C D and E G ỹ sides which are about the common angle B A D are proportionall. And because ỹ line G F is a parallel vnto the lyne D C, therfore the angle A G F (by the 29. of the first) is equall vnto ỹ angle A D C, & ỹ angle G F A equall vnto ỹ angle D C A and the angle D A C is common to the two triangles A D C and A F G. Wherefore the triangle D A C is equiangle vnto the triangle A G F. And by the same reason the triangle A B C is equiangle vnto the triangle A E F. Wherefore the whole parallelogramme A B C D is equiangle vnto the parallelogrãme E G. Wherfore as A D is in proportion to D C, so (by the 4. of the sixth) is A G to G F, and as D C is to C A, so is G F to F A. And as A C is to C B, so is A F to F E. And moreouer as C B is to B A, so is F E to E A. And forasmuch as it is proued that as D C is to C A, so is G F to F A: but as A C is to C B, so is A F to F E. Wherfore of equalitie (by the 22. of the fifth) as D C is to C B, so is G F to F E. Wherefore in the parallelogrammes A B C D and E G, the sides which include the equall angles are proportionall. Wherefore the parallelogramme A B C D is (by the first definition of the sixth) like vnto the parallelogramme E G.



That the parallelogrãme K H is like to the whole parallelogramme A B C D.

And by the same reason also the parallelogramme A B C D is like to the parallelogramme K H: wherefore either of these parallelogrammes E G and K H is like vnto the parallelogramme A B C D. But rectiline figures which are like to one and the same rectiline figure are also (by the 21. of the sixth) like the one to the other. Wherefore the parallelogramme E G is like to the parallelogramme H K. Wherfore in euery parallelogramme, the parallelogrammes about the dimecient are like vnto the whole, and also like the one to the other. Which was required to be proued.

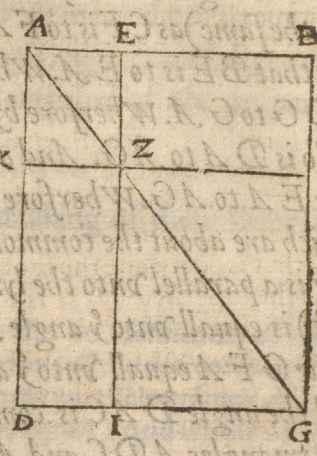
That the parallelogrammes E G and K H are like the one to the other.

¶ An other more briefe demonstration after Flussates.

An other Demonstration after Flussates.

Suppose that there be a parallelogrãme A B G D, whose dimetient let be A G, about which let consist these parallelogrammes E K and T I, hauing the angles at the pointes A and G common with the whole parallelogramme A B G D. Then I say, that those parallelogrammes E K and T I are like to the whole parallelogramme D B and also are

like the one to the other. For forasmuch as BD, EK, and TI are parallelogrammes, therefore the right line AZG falling vpon these parallell lines AEB, KZT, and DIG, or vpon these parallell lines AKD, EZI, and BTG, maketh these angles equall the one to the other, namely, the angle EAZ to the angle KZA, & the angle EZA to the angle KAZ, and the angle TZG to the angle ZGI, and the angle TGZ to the angle IZG, and the angle BAG to the angle AGD: and finally, the angle BGA to the angle DAG. Wherefore (by the first Corollary of the 32. of the first, and by the 34. of the first) the angles remayning are equall the one to the other, namely, the angle B to the angle D, and the angle E to the angle K, and the angle T to the angle I. Wherefore these triangles are equiangle and therefore like the one to the other, namely, the triangle ABG to the triangle GDA, and the triangle AEZ to the triangle ZKA, & the triangle ZTG to the triangle GIZ. Wherefore as the side AB is to the side BG, so is the side AE to the side EZ, and the side ZT to the side TG. Wherefore the parallelogrammes contayned vnder those right lines, namely, the parallelogrammes $ABGD, EK$, & TI, are like the one to the other (by the first definition of this booke). Wherefore in euery parallelogramme the parallelogrammes. &c. as before: which was required to be demonstrated.

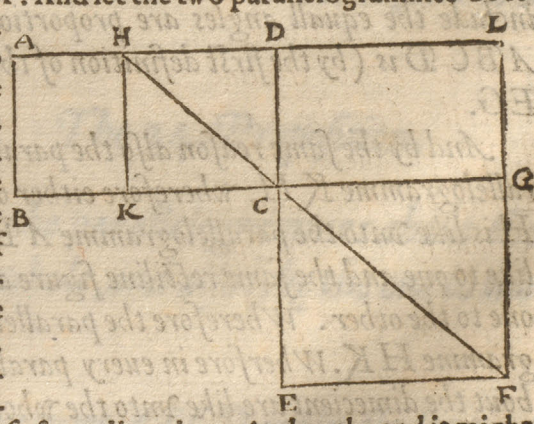


¶ A Probleme added by Pelitarius.

Two equiangle Parallelogrammes being geuen, so that they be not like, to cut of from one of them a parallelogramme like vnto the other.

An addition of Pelitarius.

Suppose that the two equiangle parallelogrammes be $ABCD$ and $CEFG$, which let not be like the one to the other. It is required from the Parallelogramme $ABCD$, to cut of a parallelogramme like vnto the parallelogramme $CEFG$. Let the angle C of the one be equall to the angle C of the other. And let the two parallelogrammes be so set, that the lines BC & CG may make both one right line, namely, BG. Wherefore also the right lines DC and CE shall both make one right line, namely, DE. And drawe a line from the poynt F to the poynt C, and produce the line FC till it cōcurre with the line AD in the poynt H. And draw the line HK parallell to the line CD (by the 31. of the first). Then I say, that from the parallelogramme AC is cut of the parallelogrāme $CDHK$, like vnto the parallelogrāme EG. Which thing is manifest by thys 24. Proposition. For that both the sayd parallelogrammes are described about one & the selfe same dimetient. And to the end it might the more plainly be seene, I haue made complete the Parallelogramme $ABGL$.



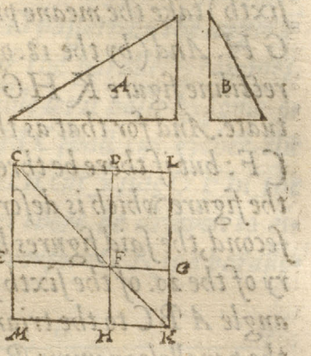
¶ An other Probleme added by Pelitarius.

Betwene two rectiline Superficieces, to finde out a meane superficies proportionall.

An other addition of Pelitarius.

Suppose that the two superficieces be A and B, betwene which it is required to place a meane superficies proportionall. Reduce the sayd two rectiline figures A and B vnto

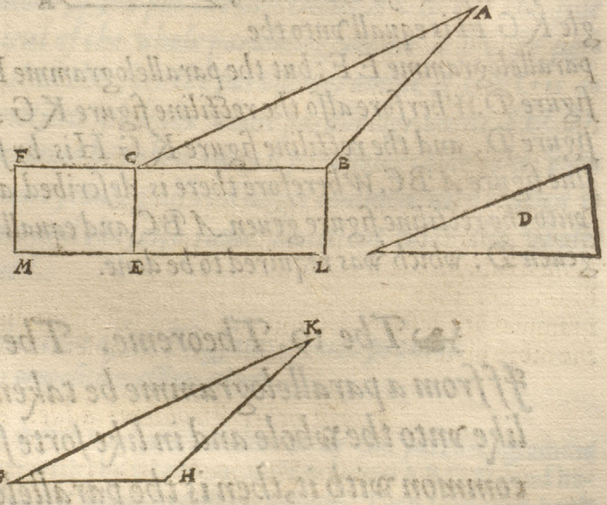
vnto two like parallelogrāmes (by the 18. of this booke) or if you thinke good reduce eyther of them to a square, (by the last of the second). And let the said two parallelogrammes like the one to the other and equall to the superficieces *A* and *B*, be *CDEF* and *FGHK*. And let the angles *F* in either of them be equall, which two angles let be placed in such sort, that the two parallelogrammes *ED* and *HG* may be about one and the selfe same dimetient *CK* (which is done by putting the right lines *EF* and *FG* in such sort that they both make one right line, namely, *EG*). And make cōplete the parallelogrāme *CLKM*. Then I say, that either of the supplements *FL* & *FM* is a meane proportionall betwene the superficieces *CF* & *FK*, that is, betwene the superficieces *A* and *B*: namely, as the superficies *HG* is to the superficies *FL*, so is the same superficies *FL* to the superficies *ED*. For by this 24. Proposition the line *HF* is to the line *FD*, as the line *GF* is to the line *FE*. But (by the first of this booke) as the line *HF* is to the line *FD*, so is the superficies *HG* to the superficies *FL*: and as the line *GF* is to the line *FE*, so also (by the same) is the superficies *FL* to the superficies *ED*. Wherfore (by the 11. of the fift) as the superficies *HG* is to the superficies *FL*, so is the same superficies *FL* to the superficies *ED*: which was required to be done.



The 7. Probleme. The 25. Proposition.

Vnto a rectiline figure geuen to describe an other figure lyke, which shal also be equall vnto an other rectiline figure geuen.

SVppose ỹ the rectiline figure geuē, wherunto is required an other to be made like be ABC, and let the other rectiline figure whereunto the same is required to be made, equal be D. Now it is required to describe a rectiline figure like vnto the figure ABC, and equall vnto the figure D. Vppon the line BC describe (by the 44. of the first) a parallelogramme BE equall vnto the triangle ABC, and by the same vpon the line CE, describe the parallelogramme CM equall vnto the rectiline figure D, and in the said parallelogramme let the angle FCE, be equall vnto the angle CBL. And forasmuch as the angle FCE is by construction equall to the angle CBL, adde the angle BCE common to them both. Wherefore the angles LBC and BCE are e-

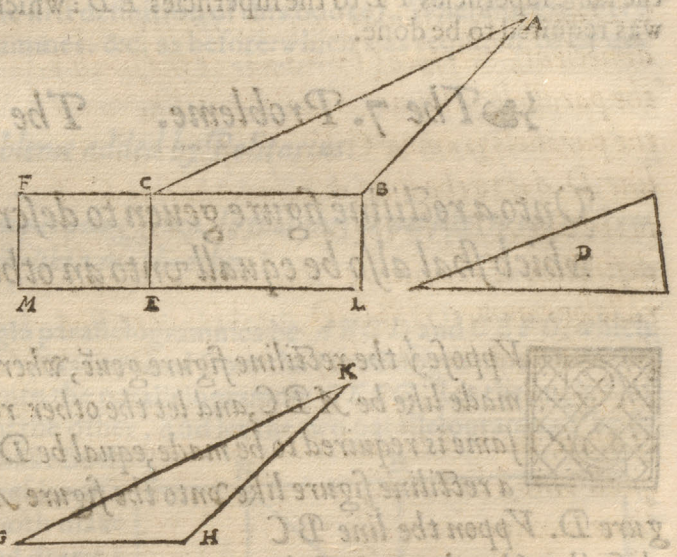


Construction.

Demonstration.

quall vnto the angles BCE and ECF, but the angles LBC and BCE are equall to two right angles (by the 29. of the first) wherfore also the angles BCE and ECF are equall to two right angles. Wherfore the lines BC and CF (by the 14. of the first) make both one right line, namely, BF, and in like sort do the lines LE and EM make both one right line, namely, LM. Then (by the 13. of the sixth) take the meane proportionall betwene the lines BC and CF, which let be GH. And (by the 18. of the sixth) vpon the line GH, let there be described a rectiline figure KHG like vnto the rectiline figure ABC, and in like sorte situate. And for that as the line BC, is to the line GH, so is the line GH to the line CF: but if there be thre right lines proportional, as the first is to the third, so is the figure which is described of the first vnto the figure which is described of the second, the said figures being like and in like sort situate (by the second correllary of the 20. of the sixth) wherfore as the line BC is to the line CF, so is the triangle ABC to the triangle KGH. But as the line BC is to the lyne CF, so is the parallelogramme BE to the parallelogramme EF (by the 1. of the sixth). Wherfore as the triangle ABC, is to the triangle KGH so is the parallelogramme BE to the parallelogramme EF. Wherfore alternately also (by the 16. of the fifth) as the triangle ABC is to the parallelogramme BE, so is the triangle KGH, to the parallelogramme EF: but the triangle ABC is equal vnto the parallelogrãme BE, wherfore also the triangle KGH is equall vnto the parallelogramme EF: but the parallelogramme FE is equall vnto the rectiline figure D. Wherfore also the rectiline figure KGH is equall vnto the rectiline figure D, and the rectiline figure KGH is by supposition like vnto the rectiline figure ABC. Wherefore there is described a rectiline figure KGH lyke vnto the rectiline figure geuen ABC, and equall vnto the other rectiline figure geuen D: which was required to be done.

Demonstration.

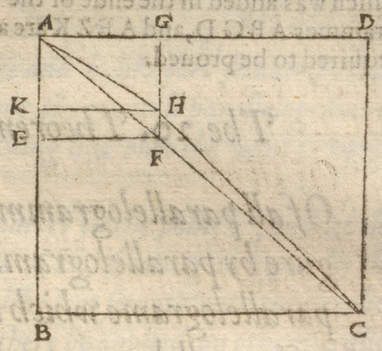


The 19. Theoreme. The 26. Proposition.

If from a parallelogramme be taken away a parallelograme like vnto the whole and in like sorte set, hauing also an angle common with it, then is the parallelogramme about one and the selfe same dimecient with the whole.

Suppose

SVppose that there be a parallelogramme ABCD, and from the parallelogramme ABCD, take away a parallelogramme AF like vnto the parallelogramme ABCD, and in like sort situate, hauing also the angle DAB common with it. Then I say, that the parallelogrammes ABCD and AF are both about one and the self same * dimecient AFC, that is, that the dimecient AFC of the whole parallelogramme ABCD passeth by the angle F of the parallelogramme AF, and is common to either of the parallelogrammes. For if AC do not passe by the point F, then if it be possible let it passe by some other point, as AHC doth. Now then the dimetient AHC shall cut eyther the side GF or the side EF of ỹ parallelogramme AF. Let it cut ỹ side GF in the point H. And (by the 31. of the first) by the point H let there be drawen to either of these lines AD and BC a parallel line HK, wherfore GK is a parallelogramme, and is about one and the selfe same dimetient with ỹ parallelogramme ABCD. And forasmuch as ỹ parallelogrammes ABCD and GK are about one and the self same dimecient, therfore (by the 24. of the sixth) the parallelogramme ABCD is like vnto the parallelogramme GK. Wherfore as the line DA is to the line AB, so is the line GA to the line AK (by the conuersion of the first definition of the sixth) And for that the parallelogrammes ABCD, and EG are (by supposition) like, therfore as the line DA is to the lyne AB so is the line GA to the line AE. Wherfore the line GA hath one and the selfe proportion to either of these lines AK and AE. Wherfore (by the 9. of the fifth) the line AK is equall vnto ỹ line AE, namely, ỹ lesse to ỹ greater, which is impossible. The selfe same inconuenience also will follow, if you put the dimetient AC to cut the side FE. Wherfore AC the dimetient of the whole parallelogramme ABCD passeth by the angle and poynt F. And therfore the parallelogramme AEFG is about one and the selfe same dimetient with the whole parallelogramme ABCD. Wherfore if from a parallelogramme be taken away a parallelograme lyke vnto the whole, and in lyke sorte situate, hauing also an angle common with it, then is that parallelogramme about one and the selfe same dimetient with the whole: which was required to be proued.



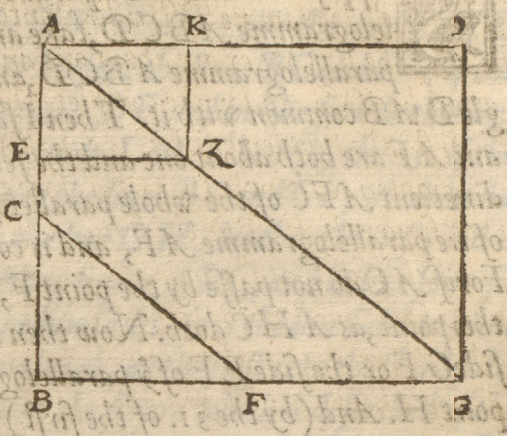
* By the dimetiẽt is vnderstand here the dimetient which is drawen from the angle which is common to them both to the opposite angle. Demonstration leading to an absurditie.

¶ *An other demonstration after Flussates, which proueth this proposition affirmatiuely.*

From the parallelogramme ABGD let there be taken away the parallelogramme AEZK like and in like sorte situate with the whole parallelogramme ABGD, and hauing also the angle A common with the whole parallelogramme. Then I say that both their diameters, namely, AZ and AZG do make one and the selfe same right line. Deuide the sides AB and BG into two equall partes in the pointes C and F (by the 10. of

An other way after Flussates.

the first.) And drawe a line from C to F. Wherefore the line C F is a parallel to the right line A G (by the corollary added by Campane after the 29. of the first) Wherfore the angles B A G and B C F are equall (by the 29. of the first): but the angle E A-Z is equall vnto the angle B A G (by reaſo the parallelogrammes are ſuppoſed to be like) wherefore the ſame angle E A Z is equall to the angle B C F, namely, the outward angle to the inward and oppoſite angle. Wherfore (by the 28. of the first) the lines A Z and C F are parallel lynes. Now then the lines A Z and A G being parallels to one and the ſelfe ſame lyne, namely, to C F do concurre in the point A. Wherefore they are ſet directly the one to the other, ſo that they both make one right line (by that which was added in the ende of the 30. propoſition of the first) wherfore the parallelogrammes A B G D, and A E Z K are about one and the ſelfe ſame dimetient: which was required to be proued.

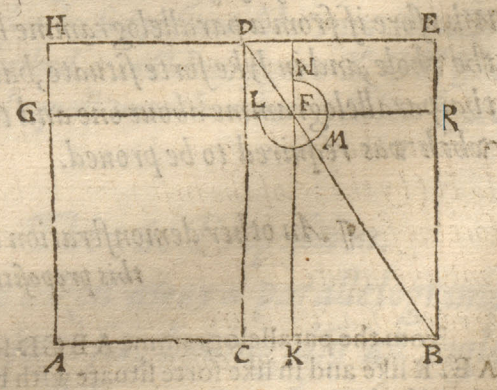


The 20. Theoreme. The 27. Propoſition.

Of all parallelogrammes applied to a right line wanting in figure by parallelogrammes like and in like ſort ſituate to that parallelograme which is deſcribed of the halfe line: the greateſt parallelogramme is that which is deſcribed of the halfe line being like vnto the want.

In this propoſitiõ are two caſes, in the firſt the parallelogramme compared to the parallelogrãme deſcribed of the halfe line is deſcribed vpon a line greater thẽ the halfe line: In the ſicond vpõ a line leſſe. The firſt caſe where the parellelogramme compared namely A F is deſcribed vpon the line A K which is greater then the halfe line A C.

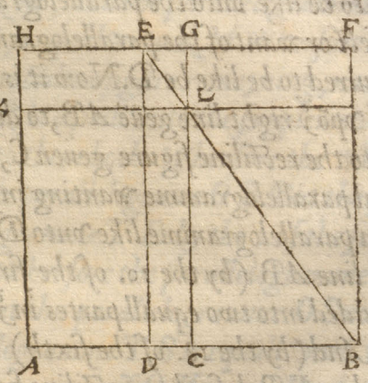
LEt there be a right line A B, and (by the 10. of the firſt) deuideit in two equall partes in the point C. And vnto the right line A B apply a parallelogramme A D wanting in figure by the parallelogramme D B, which let be like and in like ſort deſcribed vnto the parallelogramme deſcribed of halfe the line A B, which is, B C. Then I ſay, that of all the parallelogrammes which may be applied vnto the line A B and which wãt in figure by parallelogrãmes like and in like ſort ſituate vnto the parallelogramme D B the greateſt is the parallelogramme A D. For vnto ẏ right line A B let there be applied a parallelogramme A F wanting in figure by the parallelogramme F B, which let be like and in like ſort ſituate vnto the parallelogramme D B. Then I ſay, that the parallelogramme A D is greater then the parallelogramme A F. For foraſmuch



as the parallelogramme DB is like vnto the parallelogramme FB, therfore (by the 26. of the sixt) they are about one and the selfe same dimetient. Let their dimetient be DB, and make complete the figure. Now forasmuch as (by the 43. of ỹ first) the supplement FC is equall vnto the supplement FE: adde the figure FB common to them both. Wherefore the whole figure CR is equall vnto the whole figure KE. But the figure CR is equall vnto the figure CG (by the 36. of the first) for that the base AC is equall vnto the base CB. Wherefore the figure GC is equall vnto the figure KE. Adde the figure CF common vnto them both. Wherfore the whole figure AF is equall vnto the whole Gnomon LMN. But the whole parallelogramme DB is greater then the Gnomon LMN (by the 3. common sentence). Wherefore also it is greater then the parallelogramme AF. But the parallelogramme AD is equall vnto the parallelogramme DB (by the 36. of the first). Wherefore the parallelogramme AD is greater then the parallelogramme AF. Wherefore of all parallelogrammes applied to a right line vanting in figure by parallelogrammes like and in like sort situate, to that parallelogramme which is described of ỹ halfe line, the greatest parallelogrãme is that which is described of the halfe of the line, being like vnto the want: which was required to be proued.

Demonstration of this case.

Againe, let AB be deuided into two equall partes in the point C, and let the parallelogramme applied vpon the halfe line be AL, wanting in figure by the parallelogramme LB, which let be like and in like sort situate vnto the parallelogramme AL. Againe vnto the line AB let there be applied an other parallelogramme AE wanting in figure by ỹ parallelogrãme EB being like and in like sort situate vnto the parallelogrãme LB which is described vpon halfe of the line AB. The I say, that the parallelogramme AL applied vnto halfe the line is greater then the parallelogramme AE. For forasmuch as the parallelogramme EB is like vnto the parallelogramme LB, they are (by the 26. of the sixt) about one and the same dimetient. Let their dimetient be EB, and make complete the whole figure, and for that the figure LF is equall vnto the figure LH (by the 36. of the first) for the base FG is equall vnto the base GH, therfore the figure LF is greater then the figure KE. But the figure LF is equall vnto the figure DL (by the 43. of the first). Wherefore the figure DL is greater then the figure KE: put the figure KD common to them both. Wherfore the whole parallelogramme AL is greater then the whole parallelogramme AE: which was required to be proued.



The second case where the parallelogramme compared namely AE is described vpon the line AD which is lesse then the line AC. Demonstration of the second case.

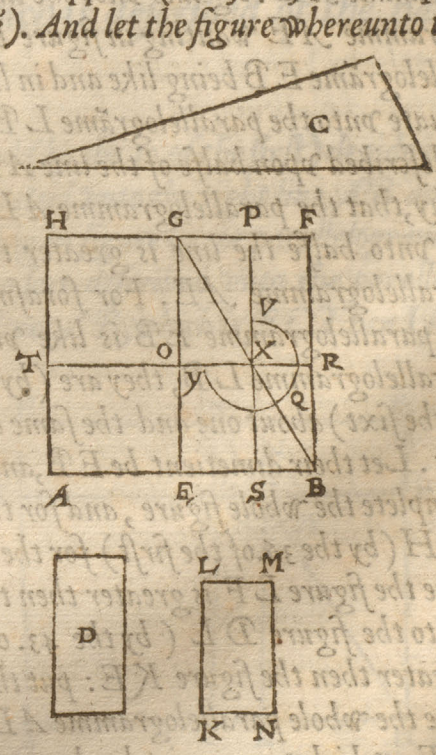
The

The 8. Probleme. The 28. Proposition.

Declaration of this case.

Vpon a right line geuen, to apply a parallelogramme equall to a rectiline figure geuen, & wanting in figure by a parallelogramme like vnto a parallelogrãme geuen. Now it behoueth that the rectiline figure geuen, whereunto the parallelogrãme applied must be equall, be not greater thẽ that parallelogramme, which so is applied vpon the halfe lyne, that the defectes shall be like, namely, the defect of the parallelogrãme applied vpon the halfe line, and the defect of the parallelogramme to be applied (whose defect is required to be like vnto the parallelogramme geuen).

VPpose the right line geuen to be AB, and let the rectiline figure geuen wherunto is required to apply vpon the right line AB an equall rectiline figure be C, which figure C, let not be greater then that parallelogrãme which is so applied vpon the halfe line, that the defectes shall be like, namely, the defect of the parallelogramme applied vpon the halfe line, and the defect of the parallelogramme to be applied (whose defect is required to be like vnto the parallelogramme geuẽ). And let the figure whereunto the defect or want of the parallelogramme is required to be like be D. Now it is required vpõ ỹ right line geuẽ AB, to describe vnto the rectiline figure geuen C, an equal parallelogramme wanting in figure by a parallelogramme like vnto D. Let the line AB (by the 10. of the first) be deuided into two equall partes in ỹ point E. And (by the 18. of the sixth) vppon the line EB describe a rectiline figure EBFG like vnto the parallelogramme D and in like sort situate, which shall also be a parallelograme. And make complete the parallelogramme AG. Now then the parallelogramme AG is either equal vnto the rectiline figure C, or greater then it by suppositiõ. If the parallelogramme AG be equal vnto the rectiline figure C, then is that done which we sought for. For then vpõ the right line AB is described vnto the rectiline figure geuen



Construction.

Two cases in this Proposition.

The first case.

geuen C an equal parallelogramme A G wanting in figure by the parallelogrãme G B, which is like vnto the parallelogramme D. But if A G be not equal vnto C then is A G greater then C, but A G is equall vnto G B (by the first of the sixt). Wherfore also G B is greater then C. Take the excesse of the rectiline figure B G aboue the rectiline figure C (by that which Pelitarius addeth after the 45. of the first) And vnto that excesse (by the 15. of the sixt) describe an equall rectiline figure K L M N like and in like sort situate vnto the rectiline figure D. But the rectiline figure D is like vnto the rectiline G B, wherfore also the rectiline figure K L M N is like vnto the rectiline figure G B (by the 21. of the sixt) Now then let the sides K L and G E be sides of like proportion, let also ỹ sides L M and G F be sides of like proportion. And forasmuch as the parallelogrãme G B is equal vnto the figures C and K M, therfore the parallelogramme G B is greater then the parallelogramme K M. Wherefore also the side G E is greater then the side K L, and the side G F is greater then the side L M, vnto the side K L put an equall line G O (by the 2. of the first) and likewise vnto the side L M put an equall line G P. And make perfect the parallelogramme O G P X. Wherfore the parallelogramme G X is equal & like vnto the parallelogramme K M. But the parallelogramme K M is lyke vnto the parallelogramme G B. Wherfore also the parallelogramme G X is like vnto the parallelogramme G B. Wherfore the parallelogrammes G X and G B are (by the 26. of the sixt) about one and the self same dimecient. Let their dimecient be G B, and make complete the figure. Now forasmuch as the parallelogramme B G is equall vnto the rectiline figure C, and vnto the parallelogramme K M, and the parallelogramme G X, which is part of the parallelogramme G B, is equal vnto K M. Wherfore the Gnomon remayning Y Q V is equall vnto the rectiline figure remayning, namely, to C. And forasmuch as the supplement P R is equall vnto the supplement O S, put the parallelogramme X B common vnto them both. Wherfore the whole parallelogramme P B is equall vnto the whole parallelograme O B. But the parallelogramme O B is equal vnto the parallelogramme T E by the 1. of the sixt, (for the side A E is equal vnto the side E B) Wherfore the parallelogramme T E is equal vnto the parallelogramme P B. Put the parallelogramme O S common to them both. Wherfore the whole parallelogramme T S is equall vnto the whole gnomon Y Q V. But it is proued that the gnomon Y Q V is equal vnto the rectiline figure C. Wherfore also the parallelogramme T S is equal vnto the rectiline figure C. Wherfore vpon the right line geuen A B is applied a parallelogramme T S equal vnto the rectiline figure geuen C, and wanting in figure by a parallelogramme X B which is like vnto the parallelogramme geuen D, for the parallelogramme X B is like vnto the parallelogramme G X: which was required to be done.

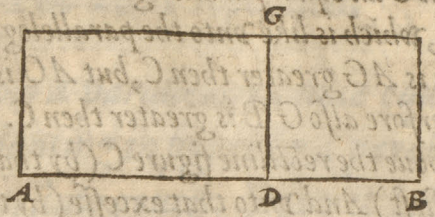
The second case.

¶ A Corollary added by Flussates.

Hereby it is manifest, that if vpon a right line be applied a parallelogramme wantyng in

A Corollary added by Flussates, and is put of Theon as an assumpt before the 17. proposition of the 6th booke: which for that it followeth of this proposition I thought it not amisse here to place.

in figure by a square, the parallelogramme applied shall be equall to the rectangle figure which is contayned vnder the segments of the line geuen which are made by the application. For the rest of the line is equall to the other side of the parallelogramme applied. For that they are sides of one & the selfe same square. As the parallelogramme A G is contained vnder the lines A D and D B, or D G which is equall to D B.

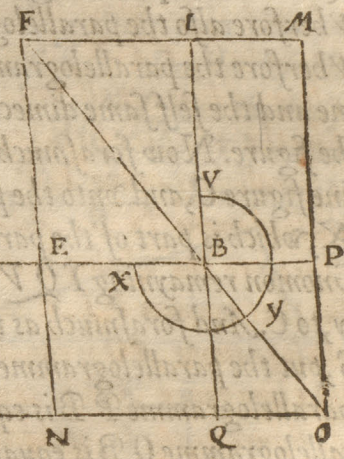


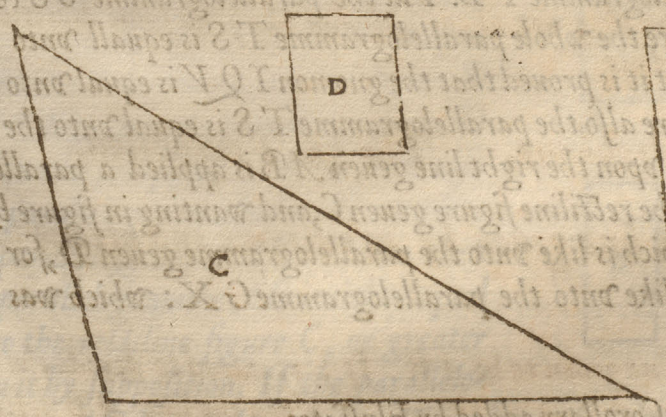
The 9. Probleme. The 29. Proposition.

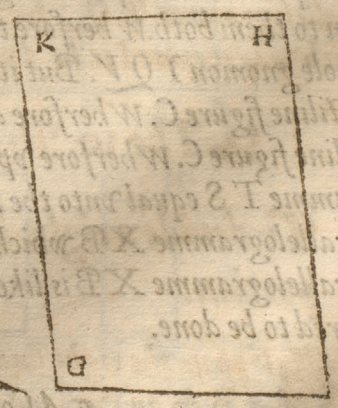
Vpon a right line geuen to apply a parallelogramme equall vnto a rectiline figure geuen, and exceeding in figure by a parallelogramme like vnto a parallelogramme geuen.

SVppose the right line geuen to be A B, and let the rectiline figure geuen whereunto is required vpon the line A B to apply an equall parallelogramme be C: let also the parallelogramme wherunto ỹ excesse is required to be like, be D. Now it is required vpon the right line A B to apply a parallelogramme equall vnto the rectiline figure C, and exceeding in figure by a parallelogramme like vnto the parallelogramme D. Let the line A B be (by the 10. of the first) deuided into two equall partes in the point E. And vpon the line E B (by the 18. of the sixt) describe a parallelogramme B F like vnto the figure D, and in like sort situate. And vnto both

Construction.







these

these figures BF and C, describe an equall rectiline figure GH like vnto the figure D and in like sort situate (by the 25. of the sixt). Wherefore the parallelogramme GH is (by the 21. of the sixt) like vnto the parallelogramme BF. Let the sides KH and FL be sides of like proportion, and so also let the sides KG and FE be. And forasmuch as the parallelogramme GH is (by construction) greater then the parallelogramme FB, therefore the line KH is greater then the line FL, and the line KG is greater then the line FE. Extend the lines FL and FE to the pointes M & N, and vnto the line KH put an equall line FLM, and likewise vnto the line KG put an equall line FEN: and make perfect the figure MN. Wherefore the parallelogramme MN is equall and like vnto the parallelogramme GH. But the parallelogramme GH is like vnto the parallelogramme EL. Wherefore also the parallelogramme MN is like vnto the parallelogramme EL. Wherefore the parallelogrammes EL and MN are (by the 26. of the sixt) about one and the same dimetient. Let the sayd dimetient be FO, and make perfect the figure. Now forasmuch as the parallelogramme GH is equall vnto the figures EL and C. But by construction the parallelogramme GH is equall vnto the parallelogramme MN. Wherfore the parallelogramme MN is equall vnto the figures EL and C. Take away the figure EL which is common to them both. Wherefore the Gnomon remayning, namely, VYX, is equall vnto the rectiline figure C. And forasmuch as the line AE is equall vnto the line EB, therefore the parallelogramme AN is (by the 36. of the first) equall vnto the parallelogramme NB, that is, vnto the parallelogramme LP, which (by the 43. of the first) is equall vnto the parallelogramme NB. Adde the parallelogramme BO common to them both. Wherefore the whole parallelogramme AO is equall vnto the Gnomon VYX. But the Gnomon VYX is equall vnto the rectiline figure C. Wherefore the parallelogramme AO is equall vnto the rectiline figure C. Wherefore vpon the right line geuen AB is applied the parallelogramme AO equall vnto the rectiline figure geuen C, and exceeding in figure by the parallelogramme QP which is like vnto the parallelogramme geuen D. For the parallelogramme D is like vnto the parallelogramme BF: and the parallelogramme BF is like vnto the parallelogramme PQ: for they are about one and the selfe same dimetient: which was required to be done.

Demonstration.

The 10. Probleme. The 30. Propositition.

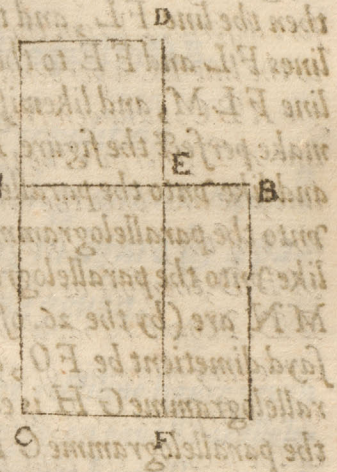
To deuide a right line geuen by an extreme and meane proportion.

Vppose the right line geuen to be AB. It is required to deuide the line AB by an extreme and meane proportion. Vpon the line AB describe (by the 46. of the first) a square BC. And vpon the line AC (by the

Construction.

Demonstration.

29. of the sixt) applie a parallelogramme C D equall vnto the square BC, and exceding in figure by the figure AD like vnto the figure BC. Now BC is a square. Wherefore also AD is a square. And forasmuch as BC is equall vnto CD, take away the figure CE which is common to them both. Wherefore the figure remayning, namely, BF, is equall to the figure remayning, namely, to AD, and the angle E of the one is equall vnto the angle E of the other. Wherefore (by the 2. definition of the sixt, and by the 14. of the sixt) the sides of the figures BF and DA, which containe the equall angles, are reciprokall. Wherefore as the side FE is to the side ED, so is the side AE to the side EB. But the side FE is equall vnto the line AC, that is, vnto the line AB, and the side ED is equall vnto the line AE (by the 34. of the first). Wherefore as the line BA is to the line AE, so is the line AE to the line EB. But the line AB is greater then the line AE. Wherfore also the line AE is greater then the line EB. Wherefore the right line AB is deuided by an extreme and meane proportion in the point E: and the greater segment thereof is AE: which was required to be done.



*An other way.

An other way.

Suppose the right line geuen to be AB. It is required to deuide the line AB by an extreme and meane proportion. Deuide the line AB in the point C (by the 11. of the second) in such sort that the rectangle figure comprehended vnder the lines AB and BC may be equall vnto the square described of the line CA. And forasmuch as that which is comprehended vnder the lines AB & BC, is equall vnto the square made of the line AC, therefore as the line BA is to the line AC, so (by the 17. of the sixt) is the line AC to the line CB. Wherfore the line AB is deuided by an extreme & meane proportion in the point C. which was required to be done.

The 21. Theoreme. The 21. Proposition.

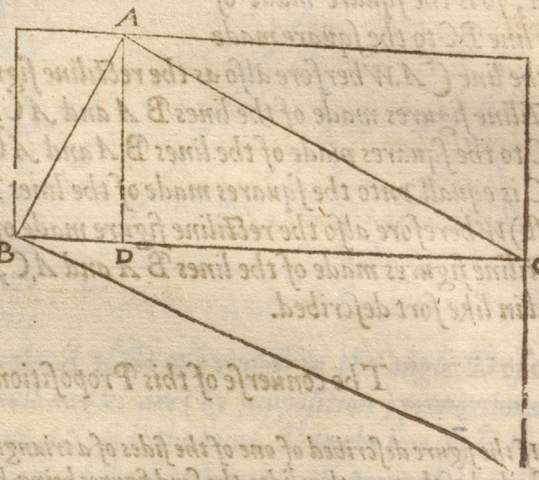
In rectangle triangles the figure made of the side subtending the right angle, is equal vnto the figures made of the sides comprehending the right angle, so that the sayd three figures be like

be like and in like sort described.

Vppose that there be a triangle ABC, whose angle BAC let be a right angle. The I say that the figure which is described of the lyne BC is equall vnto the two figures which are described of the lines BA & AC, the said thre figures being like the one to the other, and in like sort described. From the point A (by the 12. of the first) let there be drawne vnto the line BC a perpendiculer line AD. Now forasmuch as in ỹ rectangle triangle ABC is drawen from the right angle A vnto the base BC a perpendicular line AD, therfore the triangles ABD, and ADC set vpon the perpendiculer line, are like vnto the whole triangle ABC, and also like the one to the other (by the 8. of the sixt). And forasmuch as the triangle ABC is like vnto the triangle ABD, therfore as the line CB is to the line BA, so is the line AB to the lyne BD. Now for that there are three right lines proportional, therfore (by the 2. correllary of the 20. of the sixth) as the first is to the third, so is the figure made of the first, to the figure made of the second, the said figures being like and in like sorte described. Wherefore as the lyne BC is to the line BD, so is the figure made of the line BC to the figure made of the line BA, they beyng like and in lyke sort described. And by ỹ same reason as the line BC is to the lyne CD, so is the figure made of the line BC, to the figure made of the line CA, they being like and in like sort described. Wherfore as the line BC is to the lines BD and DC, so is the figure made of the line BC to the figures made of the lines BA and AC, they being like and in like sort described. But the line BC is equall vnto the lines BD and DC, wherfore the figure made of the lyne BC is equall vnto the figures made of the lines BA and AC, they being like and in like sort described. Wherfore in rectangle triangles the figure made of the side subtending the right angle is equall vnto the figures made of the sides comprehending the right angle, so that the said thre figures be like and in like sort described: which was required to be proued.

Construction.

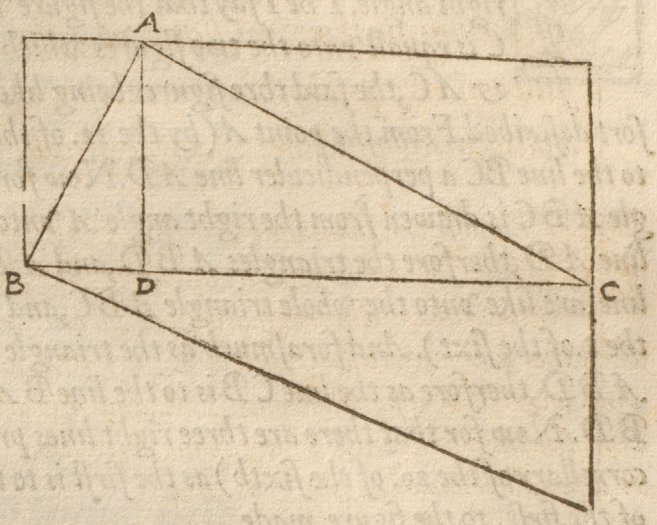
Demonstration.



An other way.

Forasmuch as (by the first correllary of the 20. of the sixth) like rectiline figures are in double proportion to that that the sides of like proportion are, therfore

fore the rectiline figure made of the line B C is vnto the rectiline figure made of the line B A in double proportion to that that the line C B is to the line B A, and (by the same) the square also made of the line B C is vnto the square made of the line B A in double proportion to that that the line C B is vnto the line B A. Wherfore also as the rectiline figure made of the line C B is to the rectiline figure made of the line B A, so is the square made of the lyne C B to the square made of the line B A. And by the same reason also, as the rectiline figure made of the line B C is to the rectiline figure made of ỹ line C A, so is the square made of the line B C to the square made of the line C A. Wherfore also as the rectiline figure made of the line B C is to the rectiline figures made of the lines B A and A C, so is the square made of the line B C to the squares made of the lines B A and A C. But the square made of ỹ line B C is equall vnto the squares made of the lines B A and A C (by the 47. of the first) Wherefore also the rectiline figure made of the line B C is equall vnto the rectiline figures made of the lines B A and A C, the said three figures beyng lyke and in like sort described.

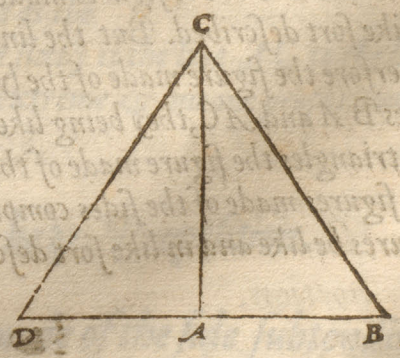


The conuerse of this Proposition after Campane.

The conuerse of the former proposition.

If the figure described of one of the sides of a triangle be equall to the figures which are described of the two other sides, the sayd figures being like and in like sort described, the triangle shall be a rectangle triangle.

Suppose that A B C be a triangle, and let the figure described of the side B C be equall to the two figures described of the sides A B and A C, the said figures being like, and in like sort described. Then I say that the angle A is a right angle. Let the angle C A D be a right angle, and put the line A D equall to the line A B, and drawe a lyne from D to C. Now then by this 31. proposition, the figure made of the line C D is equall to the two figures made of the lines A C and A D, the said figures being like and in like sort described. Wherefore also it is equall vnto the figure made of the line B C, which is by supposition equal to the two figures made of the lines A C and A D (for the line A D is put equall to the line A B) wherfore the line D C is equall to the line B C. Wherfore (by the 8. of the first) the angle B A C is a right angle, which was required to be proued.



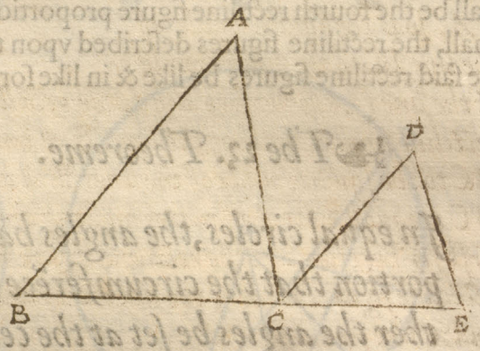
The

The 22. Theoreme. The 32. Proposition.

If two triangles be set together at one angle, hauing two sides of the one proportionall to two sides of the other, so that their sides of like proportion be also parallels: then the other sides remayning of those triangles shall be in one right line.

SVppose the two triangles to be *ABC*, and *DCE*, and let two of their sides *AC* & *DC* make an angle *ACD*, and let the said triangles haue two sides of the one, namely, *BA* and *AC* proportionall to two sides of the other, namely, to *DC* and *DE*, so ỹ as *AB* is to *AC*, so let *DC* be to *DE*. And let *AB* be a parallell vnto *DC*, and *AC* a parallell vnto *DE*. Then I say, that the lines *BC* and *CE* are in one right line. For forasmuch as the line *AB* is a parallell vnto the line *DC*, and vpon thẽ lighteth a right line *AC*: therefore (by the 29. of the first) the alternate angles *BAC* and *ACD* are equall the one to the other. And by the same reason the angle *CDE* is equall vnto ỹ same angle *ACD*. Wherefore the angle *BAC* is equall vnto the angle *CDE*. And forasmuch as there are two triangles *ABC* and *DCE*, hauing the angle *A* of the one equall to the angle *D* of the other, and the sides about the equall angles are (by supposition) proportionall, that is, as the line *BA* is to the line *AC*, so is the line *CD* to the line *DE*, therefore the triangle *ABC* is (by the 6. of the sixt) equiangle vnto the triangle *DCE*. Wherefore the angle *ABC* is equall vnto the angle *DCE*. And it is proued, that the angle *ACD* is equall vnto the angle *BAC*. Wherefore the whole angle *ACE* is equall vnto the two angles *ABC* and *BAC*. Put the angle *ACB* common to them both. Wherefore the angles *ACE* and *ACB* are equall vnto the angles *CAB*, *ACB*, & *CBA*. But the angles *CAB*, *ACB*, and *CBA*, are (by the 32. of the first) equall vnto two right angles. Wherefore also the angles *ACE* and *ACB* are equall to two right angles. Now then vnto the right line *AC*, and vnto the point in it *C*, are drawen two right lines *BC* and *CE*, not on one and the same side, making the side angles *ACE* & *ACB* equall to two right angles. Wherefore the lines *BC* and *CE* (by the 14. of the first) are set directly and do make one right line. If therefore two triangles be set together at one angle hauing two sides of the one proportionall to two sides of ỹ other, so ỹ their sides of like proportion be also parallels: then ỹ sides remayning of those triangles shall be in one right line: which was required to be proued.

Demonstration.



Although *Euclide* doth not diſtnictly ſet forth the maner of proportion of like rectiline figures, as he did of lines in the 10. Propoſitiõ of this Booke, and in the 3. following it, yet as *Fluſſates* noteth, is that not hard to be done by the 22. of thys Booke. For two like rectiline figures being geuen to finde out a third proportionall: alſo betwene two rectiline ſuperficieces geuen to finde out a meane proportionall (which we before taught to do by *Pelitarius* after the 24. Propoſition of this booke): and moreouer three like rectiline figures being geuen to finde out a fourth proportionall like and in like ſort deſcribed, and ſuch kinde of proportions, are eaſie to be found out by the proportions of lines. As thus. If vnto two ſides of like proportion we ſhould find out a third proportionall by the 11. of this boke: the rectiline figure deſcribed vpon that line ſhall be the third rectiline figure proportionall with the two firſt figures geuen by the 22. of thys booke. And if betwene two ſides of like proportion be taken a meane proportionall by the 13. of thys Booke: the rectiline figure deſcribed vpon the ſayd meane ſhall likewiſe be a meane proportionall betwene the two rectiline figures geuẽ by the ſame 22. of the ſixt. And ſo if vnto three ſides geuen be found out the fourth ſide proportionall (by the 12. of this booke) the rectiline figure deſcribed vpon the ſayd fourth line ſhall be the fourth rectiline figure proportionall. For if the right lines be proportionall, the rectiline figures deſcribed vpon them ſhall alſo be proportionall, ſo that the ſaid rectiline figures be like & in like ſort deſcribed by the ſaid 22. of the ſixt.

The 23. Theoreme. The 33. Propoſition.

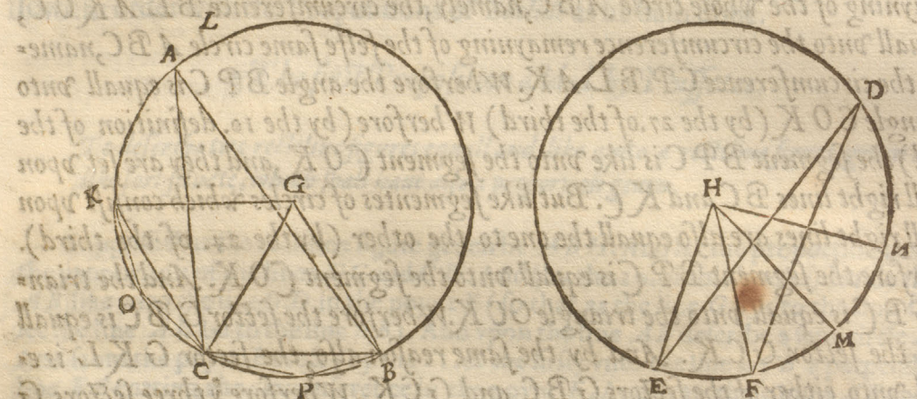
In equal circles, the angles haue one and the ſelfe ſame proportion that the circumferẽce haue, wherin they conſiſt, whether the angles be ſet at the centres or at the circumferences. And in like ſort are the ſectors which are deſcribed vppon the centres.

Vppoſe the equall circles to be A B C, and D E F, whoſe centres let be G and H, and let the angles ſet at their centres G and H, be B G C, and E H F: and let the angles ſet at their circumferences be B A C and E D F. Then I ſay that as the circumference B C is to the circumference E F ſo is the angle B G C to the angle E H F: and the angle B A C to the angle E D F, and moreouer the ſector G B C to the ſector H E F. Vnto the circumference B C (by the 28. of the third) put as many equall circumferences in order as you will, namely, C K and K L, and vnto the circumference E F put alſo as many equall circumferences in number as you will, namely, F M, and M N. And drawe theſe right lines G K, G L, H M, and H N. Now foraſmuch as the circumferences B C, C K, and K L are equall the one to the other, the angles alſo B G C, and C G K, and K G L are (by the 27. of the third) equall the one to the other. Therfore how multiplex the circumference B L is to the circumference B C, ſo multiplex is ỹ angle B G L to the angle B G C. And by ỹ ſame reaſon alſo, how multiplex the circumference N E is to the circumference E F, ſo multiplex is the

That the angles at the cẽter are in proportiõ the one to the other, as the circumferences wheron they are.

angle

angle NHE to the angle EHF. Wherfore if the circumference BL be equal vnto the circumference EN, the angle BGL is equall vnto the angle EHN, and if the circumference BL be greater then the circumference EN, the angle BGL is greater then the angle NHE, and if the circumference be lesse, the angle is lesse. Now then there are foure magnitudes, namely, the two circumferences BC and EF, and the two angles that is, BGC, and EHF, and to the circumference BC and to the angle BGC, that is, to the first and third are take equemultiplices, namely, the circumference BL, and the angle BGL, and likewise to the circumference EF, and to the angle EHF, that is, to the second and fourth, are taken certayne other equemultiplices, namely, the circumference EN and the angle EHN. And it is proued, that if the circumference BL exceede the circumference EN, the angle also BGL exceedeth the angle EHN. And if the circumference be equall, the angle is equall, and if the circumference be lesse, the angle also is lesse. Wherfore (by the 6. definition of the fifth) as the circumference BC is to the circumference EF, so is the angle BGC to the angle EHF.

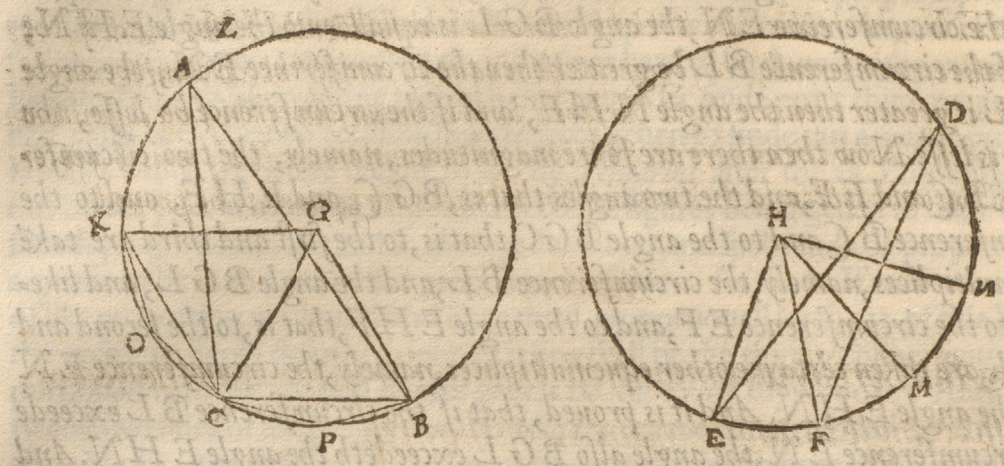


But as the angle BGC is to the angle EHF, so is the angle BAC to the angle EDF, for the angle BGC is double to the angle BAC, and the angle EHF is also double to the angle EDF (by the 20. of the third) Wherfore as the circumference BC is to the circumference EF, so is the angle BGC to the angle EHF, and the angle BAC to the angle EDF. Wherfore in equall circles the angles are in one and the selfe same proportion that their circumferences are, whether the angles be set at the centres, or at the circumferences: which was required to be proued.

That the angles at the circumferences are so also.

I say moreouer that as the circumference BC is to the circumference EF, so is the sector GBC to the sector HEF. Draw these lines BC and CK. And in the circumferences BC and CK take pointes at all aduentures, namely, P and O. And draw lines from B to P, and from P to C, from C to O, and from O to K. And forasmuch as (by the 15. definition of the first) the two lines BG and GC are equall vnto the two lines CG and GK, and they also comprehend equall angles

That the sectors are so also.



gles, therfore (by the 4. of the first) the base B C is equall vnto the base C K, & the triangle G B C is equall vnto the triangle G C K. And seing that the circumference B C is equall vnto the circumference C K, therefore the circumference remayning of the whole circle A B C, namely, the circumference B L A K O C, is equall vnto the circumference remayning of the selfe same circle A B C, namely, to the circumference C P B L A K. Wherfore the angle B P C is equall vnto the angle C O K (by the 27. of the third) Wherfore (by the 10. definition of the third) the segment B P C is like vnto the segment C O K, and they are set vpon equall right lines B C and K C. But like segmentes of circles which consist vpon equall right lines are also equall the one to the other (by the 24. of the third). Wherfore the segment B P C is equall vnto the segment C O K. And the triangle G B C is equall vnto the triangle G C K. Wherfore the sector G B C is equall vnto the sector G C K. And by the same reason also, the sector G K L is equall vnto either of the sectors G B C and G C K. Wherfore ỹ three sectors G B C and G C K, and G K L, are equall ỹ one to ỹ other. And by the same reason also the sectors H E F and H F M, and H M N are equall the one to the other. Wherefore how multiplex the circumference B L is to the circumference B C so multiplex is the sector G L B to the sector G B C. And by the same reason how multiplex the circumference N E is to the circumference E F, so multiplex is ỹ sector H E N to the sector H E F. If therfore the circumference B L be equall vnto ỹ circumference E N, ỹ sector also B G L is equall vnto ỹ sector E H N. And if the circumferẽce B L excede ỹ circumference E N, ỹ sector also B G L excedeth the sector E H N. And if the circumference be lesse, the sector also is lesse. Now thẽ there are foure magnitudes, namely, the two circumferences B C and E F, and the two sectors G B C & H E F, and to the circumference B C, & to the sector G B C, namely, to the first and the third, are taken equemultiplices, that is, the circumference B L, and the sector G B L, and likewise to the circumference E F, and to the sector H E F, namely, to the second and fourth, are taken certayne other equimultiplices, namely, the circumference E N and the sector

tor HEN. And it is proued that if the circumference BL excede the circumference EN, the ſector alſo BGL excedeth the ſector EHN. And if the circumference be equall, the ſegment alſo is equall, and if the circumference be leſſe, the ſegment alſo is leſſe. Wherfore (by the conuerſion of the ſixt definition of the fifth) as the circumference BC is to the circumference EF, ſo is ỹ ſector GBC vnto the ſector HEF: which is all that was required to be proued.

Corollary.

And hereby it is manifeſt, that as the ſector is to the ſector, ſo is angle to angle by the 11. of the fifth.

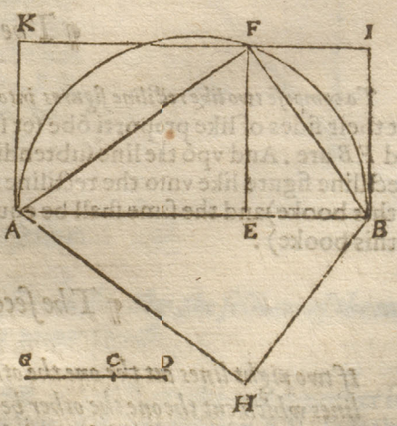
Fluſſates here addeth fiue Propoſitions wherof one is a Probleme hauing three Corollaryes following of it, and the reſt are Theoremes: which for that they are both witty, & alſo ſerue to great vſe, as we ſhall afterward ſee, I thought not good to omitte, but haue here placed them: but onely that I haue not put them to followe in order with the Propoſitions of *Euclide* as he hath done.

¶ The firſt Propoſition added by Fluſſates.

To deſcribe two rectiline figures equall and like vnto a rectiline figure geuen and in like ſort ſituate, which ſhall haue alſo a proportion geuen.

Conſtruction of the Probleme.

Suppoſe that the rectiline figure geuen be *ABH*. And let the proportion geuen be the proportion of the lines *GC* and *CD*. And (by the 10. of this booke) deuide the line *AB* like vnto the line *GD* in the poynt *E* (ſo that as the line *GC* is to the line *CD*, ſo let the line *AE* be to the line *EB*). And vpon the line *AB* deſcribe a ſemicircle *AFB*. And from the poynt *E* erect (by the 11. of the firſt) vnto the line *AB* a perpendicular line *EF* cutting the circumference in the poynt *F*. And draw theſe lines *AF* and *FB*. And vpon either of theſe lines deſcribe rectiline fignres like vnto the rectiline figure *AHB* and in like ſort ſituate (by the 18. of the ſixt): which let be *AKF*, & *FIB*. Then I ſay, that the rectiline figures *AKF*, and *FIB*, haue the proportion geuē (namely, the proportion of the line *GC* to the line *CD*) and are equall to the rectiline figure geuen *ABH* vnto which they are deſcribed like and in like ſort ſituate. For foraſmuch as *AFB* is a ſemicircle, therefore the angle *AFB* is a right angle (by the 31. of the third) and *FE* is a perpendicular line. Wherefore (by the 8. of this booke) the triangles *AFE* and *FBE* are like both to the whole triangle *AFB* and alſo the one to the other. Wherefore (by the 4. of this booke) as the line *AF* is to the line *FB*, ſo is the line *AE* to the line *EF*, and the line *EF* to the line *EB*, which are ſides cōtayning equall angles. Wherfore (by the 22. of this booke) as the rectiline figure deſcribed of the line *AF* is to the rectiline figure deſcribed of the line *FB*, ſo is the rectiline figure deſcribed of the line *AE* to the rectiline figure deſcribed of the line *EF*, the ſayd rectiline figures being like and in like ſort ſituate.



Demonſtration of the ſame.

situate. But as the rectiline figure described of the line *AE* being the first, is to the rectiline figure described of the line *EF* being the second, so is the line *AE* the first, to the line *EB* the third (by the 2. Corollary of the 20. of thys booke). Wherfore the rectiline figure described of the line *AF* is to the rectiline figure described of the line *FB*, as the line *AE* is to the line *EB*. But the line *AE* is to the line *EB* (by construction) as the line *GC* is to the line *CD*. Wherefore (by the 11. of the fift) as the line *GC* is to the line *CD*, so is the rectiline figure described of the line *AF* to the rectiline figure described of the line *FB*, the sayd rectiline figures being like and in like sort described. But the rectiline figures described of the lines *AF* and *FB*, are equall to the rectiline figure described of the line *AB*, vnto which they are (by construction) described lyke and in like sort situate. Wherefore there are described two rectiline figures *AKF* and *FIB* equall and like vnto the rectiline figure geuen *ABH* and in like sort situate, and they haue also the one to the other the proportion geuen, namely, the proportion of the line *GC* to the line *CD*: which was required to be done.

¶ The first Corollary.

The first Corollary.

To resolue a rectiline figure geuẽ into two like rectiline figures which shall haue also a proportiõ geuẽ. For if there be put three right lines in the proportiõ geuẽ, and if the line *AB* be cut in the same proportiõ that the first line is to the third, the rectiline figures described of the lines *AF* and *FB* (which figures haue the same proportion that the lines *AE* and *EB* haue) shall be in double proportion to that which the lines *AF* and *FB* are (by the first Corollary of the 20. of this booke). Wherefore the right lines *AF* and *FB* are the one to the other in the same proportion that the first of the three lines put is to the second. For the first line to the third, namely, the line *AE* to the line *EB* is in double proportion that it is to the second, by the 10. definition of the fift.

¶ The second Corollary.

The second Corollary.

Hereby may we learne, how from a rectiline figure geuen to take away a part appointed, leauing the rest of the rectiline figure like vnto the whole. For if frõ the right line *AB* be cut of a part appoynted, namely, *EB* (by the 9. of this booke) as the line *AE* is to the line *EB*, so is the rectiline figure described of the line *AF* to the rectiline figure described of the line *FB* (the sayd figures being supposed to be like both the one to the other and also to the rectiline figure described of the line *AB*, and being also in like sort situate). Wherfore taking away from the rectiline figure described of the line *AB*, the rectiline figure described of the line *FB*, the residue, namely, the rectiline figure described of the line *AF* shall be both like vnto the whole rectiline figure geuen described of the line *AB*, and in like sort situate.

¶ The third Corollary.

The third Corollary.

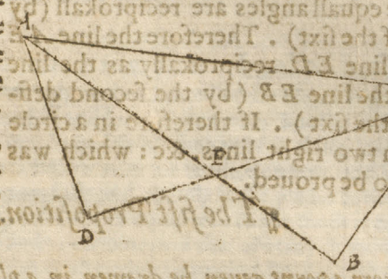
To compose two like rectiline figures into one rectiline figure like and equall to the same figures. Let their sides of like proportiõ be set so that they make a right angle, as the lines *AF* and *FB* are. And vpõ the line subtending the said angle, namely, the line *AB*, describe a rectiline figure like vnto the rectiline figures geuen and in like sort situate (by the 18. of this booke) and the same shall be equall to the two rectiline figures geuen (by the 31. of this booke).

¶ The second Proposition.

If two right lines cut the one the other obtuseangled wise, and from the endes of the lines which cut the one the other be drawen perpendicular lines to either line: the lines which are betwene the endes and the perpendicular lines are cut reciprokally.

Suppose

Suppoſe that there be two right lines *AB* and *GD* cutting the one the other in the point *E*, and making an obtuſe angle in the ſection *E*. And from the endes of the lines, namely, *A* and *G*, let there be drawen to either line perpendicular lines, namely, from the point *A* to the line *GD*, which let be *AD*, and from the point *G* to the right line *AB*: which let be *GB*. Then I ſay, that the right lines *AB* and *GD* do, betwene the end *A* and the perpendicular *B*, and the end *G* and the perpendicular *D*, cut the one the other reciprokally in the point *E*: ſo that as the line *AE* is to the line *ED*, ſo is the line *GE* to the line *EB*. For foraſmuch as the angles *ADE* and *GBE*, are right angles, therfore they are equall. But the angles *AED* and *GEB* are alſo equall (by the 15. of the firſt). Wherefore the angles remayning, namely, *EAD*, & *EGB*, are equall (by the Corollary of the 32. of the firſt). Wherefore the triangles *AED* and *GEB*, are equiangle. Wherfore the ſides about the equall angles ſhall be proportionall (by the 4. of the ſixt). Wherfore as the line *AE* is to the line *ED*, ſo is the line *GE* to the line *EB*. If therefore two right lines cut the one the other obtuſeangled wiſe. &c: which was required to be proued.

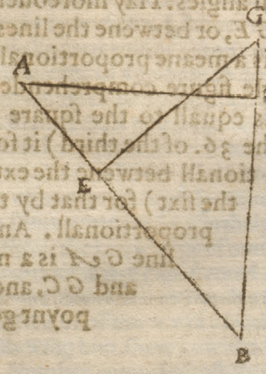


Demonſtration of this propoſition.

¶ The third Propoſition.

If two right lines make an acute angle, and from their endes be drawen to ech line perpendicular lines cutting them: the two right lines geuen ſhall be reciprokally proportionall as the ſegmentes which are about the angle.

Suppoſe that there be two right lines *AB* and *GB*, making an acute angle *ABG*. And from the poyntes *A* and *G* let there be drawen vnto the lines *AB* and *GB* perpendicular lines *AC* and *GE*, cutting the lines *AB* and *GB* in the poyntes *E* and *C*. Then I ſay, that the lines, namely, *AB* to *GB*, are reciprokally proportionall, as the ſegmentes, namely, *CB* to *EB* which are about the acute angle *B*. For foraſmuch as the right angles *AGB* and *GEB* are equall, and the angle *ABG* is common to the triangles *ABC*, and *GBE*, therefore the angles remayning *BAC* and *EGB* are equall (by the Corollary of the 32. of the firſt). Wherfore the triangles *ABC* and *GBE* are equiangle. Wherefore the ſides about the equall angles are proportionall (by the 4. of the ſixt): ſo that, as the line *AB* is to the line *BG*, ſo is the line *GB* to the line *BE*. Wherefore alternately as the line *AB* is to the line *GB*, ſo is the line *CB* to the line *BE*. If therefore two right lines make an acute angle. &c. which was required to be proued.



Demonſtration of this propoſition.

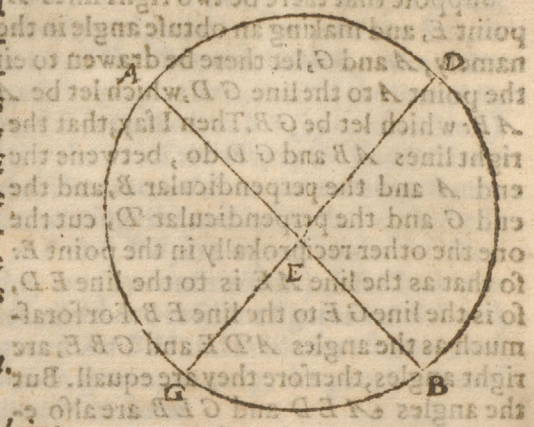
¶ The fourth Propoſition.

If in a circle be drawen two right lines cutting the one the other, the ſections of the one to the ſections of the other ſhall be reciprokally proportionall.

In the circle *AGB* let theſe two right lines *AB* and *GD* cut the one the other in the poynt *E*. Then I ſay, that reciprokally as the line *AE* is to the line *ED*, ſo is the line *GE* to the line *EB*. For foraſmuch as (by the 35. of the third) the rectangle figure

Demonſtration of this propoſition.

gure contayned vnder the lines *A E* and *E B* is equall to the rectangle figure contayned vnder the lines *G E* and *E D*, but in equall rectangle parallelogrammes the sides about the equall angles are reciprokall (by the 14. of the sixt). Therefore the line *A E* is to the line *E D* reciprokally as the line *G E* is to the line *E B* (by the second definition of the sixt). If therefore in a circle be drawen two right lines, &c: which was required to be proued.



Demonstration of this proposition.

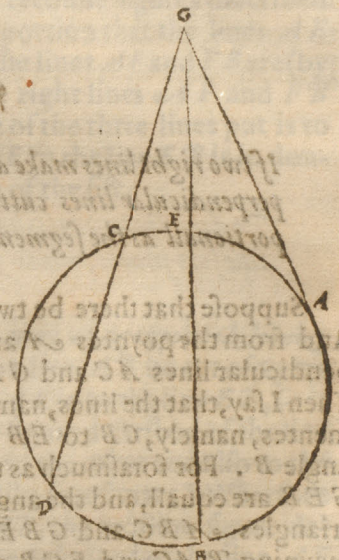
¶ The fift Proposition.

If from a poynt geuen be drawen in a plaine superficies two right lines to the concaue circumference of a circle: they shall be reciprokally proportionall with their partes take without the circle. And moreouer a right line drawen from the sayd poynt & touching the circle, shall be a meane proportionall betwene the whole line and the vtter segment.

Suppose that there be a circle *A B D*, and without it take a certayne poynt, namely, *G*, And from the point *G* drawe vnto the concaue circumference two right lines *G B* and *G D*, cutting the circle in the poyntes *C* and *E*. And let the line *G A* touch the circle in the point *A*. The I say, that the lines, namely, *G B* to *G D* are reciprokally as their parts taken without the circle, namely, as *G C* to *G E*. For forasmuch as (by the Corollary of the 36. of the third) the rectangle figure contayned vnder the lines *G B* and *G E* is equall to the rectangle figure contayned vnder the lines *G D* and *G C*, therefore (by the 14. of the sixt) reciprokally as the line *G B* is to the line *G D*, so is the line *G C* to the line *G E*, for they are sides contayning equall angles. I say moreouer, that betwene the lines *G B* and *G E*, or betwene the lines *G D* and *G C* the touch line *G A* is a meane proportionall. For forasmuch as the rectangle figure comprehended vnder the lines *G B* and *G E* is equall to the square made of the line *A G* (by the 36. of the third) it followeth that the touch line *G A* is a meane proportionall betwene the extremes *G B* and *G E* (by the second part of the 17. of the sixt) for that by that Proposition the lines *G B*, *G A*, and *G E* are proportionall. And by the same reason may it be proued that the line *G A* is a meane proportionall betwene the lines *G D* and *G C*, and so of all others. If therefore from a poynt geuen, &c: which was required to be demonstrated.

Demonstration of the first part of this proposition.

Demonstration of the second part.



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The end of the sixth booke of Euclides Elementes.

¶ The seuenth booke of Euclides Elementes.

HETHERTO IN THE SIXE bookes before hath *Euclide* passed through, and entreated of the Elementes of Geometrie without the ayde and succor of nomber. But the matters which remayne to be taught and to be spoken of in these his Geometricall bookes which follow as in the tenth, eleuenth, and so forth, he could by no meanes fully and clearely make plaine & demonstrate, without the helpe and ayd of nombers. In the tenth is entreated of lines irrationall and vncertaine, and that of many & sondry kindes: and in the eleuenth & the other following he teacheth the natures of bodyes, and compareth theyr sides and lines together. All which for the most part are also irrationall. And as rationall quantites, and the comparisons and proportions of them, cannot be knowen, nor exactly tried, but by the meane of nomber, in which they are first sene and perceiued: euen so likewise cannot irrationall quantities be knowen and found out without nomber. As straightnes is the triall of crokednes, and inequalitie is tried by equalitie: so are quantities irrationall perceiued and knowen by quantities rationall: which are first and chiefely found among nombers. Wherefore in these three bookes following, being as it were in the middest of his Elementes, he is compelled of necessitie to entreate of nombers, although not so fully, as the nature of nombers requireth, yet so much as shall seme to be fit, and sufficiently to serue for his purpose. Wherby is sene the necessitie, that the one Arte, namely, Geometrie, hath of the other, namely, of Arithmeticke. And also of what excellécy and worthines Arirhmeticke is aboue Geometrie: in that, Geometrie boroweth of it principles, ayd, and succor, and is as it were maymed with out it. Whereas Arithmeticke is of it selfe, sufficient and neadeth not at all any ayde of Geometrie, but is absolute and perfitin it selfe, and may well be taught and attayned vnto without it. Agayne the matter or subiect where about Geometrie is occupied, which are lines, figures, and bodyes, are such as offer them selues to the sences, as triangles, squares, circles, cubes, and other are sene & iudged to be such as they are, by the sight: but nomber, which is the subiect and matter of Arithmeticke, falleth vnder no sence, nor is represented by any shape, forme, or figure: and therefore cannot be iudged by any sence, but only by consideration of the minde, and vnderstanding. Now thinges sensible are farre vnder in degree then are thinges intellectuall: and are of nature much more grosse then they. Wherefore nomber, as being only intellectuall, is more pure, more immateriall, and more subtile, farre then is magnitude: and extédeth it selfe further. For Arithmeticke, not onely aydeth Geometrie: but ministreth principles, and groundes to many other, nay rather to all other sciences and artes. As to musicke, Astronomy, naturall philosophy, perspectiue, with others. What other thing is in musicke entreated of, then nomber contracted to sound and voyce? In Astronomie, who without the knowledge of nomber can doo any thing, either in searching out of the motions of the heauens, and their courses, either in iudging and foreshewing the effectes of them? In natural philosophie, it is of no small force. The wisest and best learned philosophers tha: haue bene, as *Pithagoras*, *Timeus*, *Plato*, and their followers, found out & taught most pithely and purely, the secret and hidden knowledge of the nature and condicion of all thinges, by nombers, and by the proprieties and passions of them. Of what force nomber is in perspectiue, let him declare and iudge, who hath any thing traueled therein. Yea to be short, what can be worthely and with prayse practised in common life of any man of any condition, without the knowledge of nomber. Yea it hath bene taught of the chiefest amongst philosophers, that all naturall thinges are framed, and haue their constitucion of nomber. *Boetius* sayth *Hoc fuit principale in animo conditoris exemplar*: Nomber (sayth he) was the principall example and patron in the minde of the creator of the world. Doth not that great philopher *Timeus* in his booke, & also *Plato* in his *Timeo*, following him, shew how the soule is composed of harmonicall nombers, and consonantes of musicke. Nomber compaseth all thinges, and is (after these men) the being and very essence of all thinges. And ministreth ayde and helpe, as to all other knowledges so also no small to Geometrie. Which thing causeth *Euclide* in the midest of his booke of Geometrie, to inserte and place these three bookes of Arithmeticke: as without the ayde of which, he could not well passe any father.

Why Euclide in the middest of his workes was compelled to adde these three bookes of numbers.

Arithmetike of more excellency then Geometry.

Things intellectuall of more worthines then things sensible.

Arithmetike ministreth principles and groundes in a maner to all sciences.

Boetius: Cap. 2. Lib. prim. Arithmet.

Timaus.

In this seuenth booke, he first placeth the generall principles, and first groundes of Arithmeticke, and setteth the diffinitions of nomber or multitude and the kinds therof: as in the first boke, he did of magnitude and the kindes and partes thereof. After that he entreateth of nombers generally, and of theyr partes: and searcheth and demonstrateth in generall the most common passions and proprieties of the same, and chiefely of nombers prime or incommensurable, and of nombers composed or commensurable, and of their proprieties and partely also of the comparison or proportion of one nomber to an other.

The argument of the seuenth booke.

The seuenth Booke

¶ Definitions.

1 Vnitie is that, whereby euery thing that is, is sayd to be on.

The first definition.

As a point in magnitude, is the least thing in magnitude, and no magnitude at all, & yet the ground and beginning of all magnitudes: euen so is vnitie in multitude or nomber, the least thing in nomber, and no nomber at all, and yet the ground and beginning of all nombers. And therefore it is here in this place, of *Euclide* first defined: as in the first booke, for the like reason and cause was a point first defined. *Vnitie*, sayth *Euclide*, *is that whereby euery thing is sayd to be one:* that is, vnitie is that, whereby euery thing is deuided and seperated from an other, and remayneth on in it selfe pure and distinct frō all others. Otherwise, were not this vnitie, whereby all thinges are seioyned the on from the other, all things should suffer mixtion and be in confusion. And where confusion is, there is no order, nor any thing can be exactly knowen, either what it is, or what is the nature, and what are the properties thereof. Vnitie therefore is that which maketh euery thing to be that which it is. *Boetius* sayth very aptly: *Vnum quodq; ideo est, quia vnum numero est*, that is euery thing therfore is (that is, therefore hath his being in nature, and is that it is) for that it is on in nomber. According whereunto *Iordane* (in that most excellent and absolute worke of Arithmeticke which he wrote) defineth vnitie after this maner.

Without vnity should be confusion of thinges.

Boetius in his booke de vnitate & vno.

Vnitas, est rei per se discretio: that is, vnitie is properly, and of it selfe the difference of any thing. That is, vnitie is that whereby euery thing doth properly and essentially differ, and is an other thing from all others. Certainely a very apt definition and it maketh playne the definition here set of *Euclide*.

An other definition of vnity.

2 Number is a multitude composed of vnities.

The second definition.

As the number of three, is a multitude composed and made of three vnities. Likewise the number of fiue is nothing ells, but the composition & putting together of fiue vnities. Although as was before sayde, betwene a poynt in magnitude, and vnitie in multitude, there is great agreement and many thinges are common to them both, (for as a poynt is the beginning of magnitude, so is vnitie the beginning of nomber. And as a poynt in magnitude is indiuisible, so is also vnitie in number indiuisible) yet in this they differ and disagree. There is no line or magnitude made of pointes, as of his partes. So that although a point be the beginning of a lyne, yet is it no part therof. But vnitie, as it is the beginning of number, so is it also a part therof, which is somewhat more manifestly set of *Boetius* in an other diffinition of number which he geueth in his Arithmetike, which is thus.

Difference betwene a point and vnity. Boetius.

Numerus, est quantitatis aceruus ex vnitatibus profusus, that is. Number is a masse or heape of quantities produced of vnities: which diffinition in substance is all one with the first, wherin is said most plainly, that the heape or masse, that is, the whole substance of the quantitie of number is produced & made of vnities. So that vnitie is as it were the very matter of number. As foure vnities added together are the matter wherof the number 4. is made, & eche of these vnities is a part of the number foure, namely, a fourth part, or a quarter. Vnto this diffinition agreeth also the definition geuen of *Iordane*, which is thus. *Number is a quantitie which gathereth together thinges seuered a sonder.* As fiue men beyng in themselues seuered aud distincte, are by the number fiue brought together, as it were into one masse, and so of others. And although vnitie be no number, yet it contayneth in it the vertue and power of all numbers, and is set and taken for them.

An other definition of number. Iordane. An other definition of numbers. Vnity hath in it the vertue and power of all numbers.

In this place (for the farther elucidation of thinges, partly before set, and chiefly hereafter to be set, because *Euclide* here doth make mention of diuers kyndes of numbers, and also defineth the same) is to be noted, that number may be considered three maner of wayes. First, number may be considered absolutely, without comparyng it to any other number, or without applieng it to any other thing, onely vewing and paysing what it is in it selfe, and in his owne nature onely, and what partes it hath, and what proprieties and passions. As this number fixe, may be considered absolutely in his owne nature, that it is an euen number, and that it is a perfect number, and hath many mo conditions and proprieties. And so conceiue ye of all other numbers, whatsoeuer, of 9. 12. and so forth.

Number considered three maner of wayes.

An other way, number may be cōsidered by way of cōparison, and in respect of some other number either as equall to it selfe, or as greater thē it selfe, or as lesse thē it selfe. As 12. may be cōsidered, as cōpared to 12. which is equall vnto it, or as to 24. which is greater then it, for 12 is the halfe thereof, or as to 6. which is lesse then it, as beyng the double therof. And of this consideration of numbers ariseth and springeth all kyndes and varieties of proportiō: as hath before bene declared in the explanation of the principles of the fift booke, so that of that matter it is needelesse any more to be sayd in this place.

The third way may numbers be considered as they are applied to formes and figures of Geometry. And numbers so considered are not represented by figures or characters of number commonly vsed in Arithmetique, but are signified by certayne pointes or prickes, which represent the vnities which they contayne: which, accordyng to the diuersitie of the disposition and placing of them, may represent diuers formes and figures of Geometry: and accordyng to the nature of the figure which it representeth, it taketh his name, and is called a trianguler number, a square number, a cube number or after any other figure. As if the figure of 10. be so described by his vni-

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ties

ties that they be orderly set in a straight course, so that they represent the forme of a line, then is the number 10 called a lineall number. And if the same number 10 be so described by his vnities, that it shew forth the forme of a triangle, then it is called a trianguler nũber: as ye here see. Likewise if 12 be in such sort described by his vnities, that it represeteth that forme or figure which in Geometry is called a figure on the one side longer: then shall the number 12 be called a number hauyng the one side longer, and so may you conceaue of all others.

Thus much of this for the declaration of the thinges following.

3 *A part is a lesse number in comparison to the greater when the lesse measureth the greater.*

The third definition.

As the number 3 compared to the number 12. is a part. For 3 is a lesse number then is 12. and moreouer it measureth 12 the greater number. For 3 taken (or added to it selfe) certayne times (namely, 4 tymes) maketh 12. For 3 foure tymes is 12. Likewise is 2 a part of 8: 2 is lesse then 8, and taken 4 tymes it maketh 8. For the better vnderstandyng of this diffinition, and how this worde Parte, is diuersly taken in Arithmetique and in Geomety, read the declaration of the first diffinition of the 5. booke.

4 *Partes are a lesse number in respect of the greater, when the lesse measureth not the greater.*

The fourth definition.

As the number 3 compared to 5, is partes of 5 and not a part. For the number 3 is lesse then the nũber 5, and doth not measure 5. For taken once it maketh but 3. once 3 is 3, which is lesse then 5. and 3 taken twise maketh 6, which is more then 5. Wherfore it is no part of 5 but partes, namely, three fifth partes of 5. For in the number 3 there are 3 vnities, and euery vnitie is the fifth part of 5. Wherfore 3 is three fifth partes of 5, and so of others.

5 *Multiplex is a greater number in comparison of the lesse, when the lesse measureth the greater.*

The fifth definition.

As 9 compared to 3 is multiplex, the number 9 is greater then the number 3. And moreouer 3 the lesse number measureth 9 the greater number. For 3 taken certaine tymes, namely, 3 tymes maketh 9. three tymes three is 9. For the more ample and full knowledge of this definition, read what is sayd in the explanation of the second definition of the 5 booke, where multiplex is sufficiently entreated of with all his kyndes.

6 *An euen number is that, which may be deuided into two equal partes.*

The sixth definition.

As the number 6 may be deuided into 3 and 3 which are his partes, and they are equall, the one not exceding the other. This definition of Euclide is to be vnderstand of two such equall partes, which ioyned together, make the whole number: as 3 and 3 (the equall partes of 6) ioyned together, make 6, for otherwise many numbers both euen and odde may be deuided into many equall partes, as into 4. 5. 6. or mo, and therfore into 2. As 9 may be deuided into 3 and 3 which are his partes, and are also equall, for the one of them excedeth not the other: yet is not therfore this number 9 an euen number, for that 3 and 3 (these equall partes of 9) added together make not 9 but onely 6. Likewise taking the definition so generally, euery number whatsoeuer should be an euen number: for in that sort of vnderstãding there is no number, but that it may be deuided into two equall partes: as this number 7 may be deuided into 3 partes, namely, 3. 1. and 3. of which two, namely, 3 and 3 are equal, yet is not 7 an euen number, becausse 3 and 3 added together, make not 7. *Boetius* therfore in the first booke of his Arithmetike, for the more playnes, after this maner defineth an euen number: *Numerus par est, qui potest in æqualia duo diuidi, & no medio non intercedente*, that is:

Boetius.

An euen number is that which may be deuided into two equall partes, without an vnitie comming betwene them. As 8 is deuided into 4 and 4 two equall partes without an vnitie comming betwene them, which added together, make 8, so that the sence of this definition is, that an euen number is that which is deuided into two such equall partes, which are his two halfe partes.

An other definition of an euẽ number.

Here is also to be noted, that a part is taken in this definition, and in certaine definitions following, not in that signification as it was before defined, namely, for such a part as measureth the whole number, but for any part which helpeth to the making of the whole, and into which the whole may be resolued, so are 3 and 5 partes of 8 in this sence but not in the other sence. For neyther 3 nor 5 measureth 8. Pithagoras and his scholers gaue an other definition of an euen number (which definition *Boetius* also hath) after this maner.

Note.

Pithagoras.

An other definition.

An euen number is that which in one and the ſame diuiſion is deuided into the greateſt, and into the leaſt: into the greateſt, as touching ſpace, and into the leaſt as touching quantitie. As 10. is deuided into 5 & 5. which are his greateſt partes (which greatnes of partes he calleth ſpace) and in the ſame diuiſion the number 10. is deuided but into two partes: but into leſſe thé two partes nothing can be deuided, which thing he calleth quantitie, ſo that 10 deuided into 5 and 5. in that one deuiſion, is deuided into the greateſt, namely, into his halues, and into the leaſt, namely, into two partes, and no mo.

There is alſo another definition more auncient, which is thus.

An other definition.

An euen number is that which may be deuided into two equall partes, and into two Vnequall partes, but in neyther diuiſion (to the conſtitucion of the whole) to the euen part is added the odde, neither to the odde is added the euen. As 8 may be deuided diuerſly, partly into euen partes, as into 4 and 4. likewiſe into 6 and 2. and partly into odde partes, as into 5 and 3. alſo into 7 and 1. In all which deuiſions, ye ſee no odde parte ioyned to an euen, nor an euen part ioyned to an odde: but if the one be euen, the other is euen, and if the one be odde, the other is odde. In the two firſt deuiſions, both partes were euen, and in the two laſt, both partes were odde. It is to be conſidered that the two partes added together muſt make the whole.

This diffinition is generall and common to all euen numbers, except to the number 2. which can not be deuided into two vnequall partes, but onely into two vnities, which are equall.

An other definition.

There is yet geué an other diffinition of an euen number, namely, thus. *An euen number is that which onely by an Vnitie either aboue it or Vnder it, differeth from an odde.* As 8 being an euen number differeth from 9 an odde number, being aboue it but by one. And alſo from 7. vnder it, it differeth likewiſe by one, and ſo of others.

The ſeuenth definition.

7 *An odde number is that which cannot be deuided into two equal partes: or that which onely by an Vnitie differeth from an euen number.*

As the number 5 can be by no meanes deuided into two equall partes, namely, ſuch two which added together, ſhall make 5. Or by the ſecond definition, 5 an odde number, differeth from 6 an euen number aboue it, by 1. And the ſame 5 differeth from 4 an euen number vnder it likewiſe by 1.

Boetius defineth an odde number after this maner.

An odde number is that which cannot be deuided into two equall partes, but that an Vnitie ſhall be betwene them. As if ye deuide 5 into 2 and 2. which are two equall partes, there remaineth one or an vnitie betwene them to make the whole number 5.

An other definition of an od number.

There is yet an other definition of an odde number. *An odde number is that, which being deuided into two Vnequall partes howſoeuer, the one is euer euen, and the other odde.* As if 9 be deuided into two partes, which added together, maketh the whole, namely, into 4 and 5. which are vnequall: ye ſee the one is euen, namely, 4. and the other is odde, namely, 5. ſo if ye deuide 9 into 6 and 3. or into 8 and 1. the one part is euer euen, and the other odde.

An other definition.

The eight definition.

8 *A number euenly euen (called in latine* pariter par*) is that number, which an euen number meaſureth by an euen number.*

As 8 is a number euenly euen. For 4 an euen number meaſureth 8 by 2, which is alſo an euen number. This definition hath much troubled many, and ſeemeth not a true definition, for that there are many numbers which euen numbers do meaſure, and that by euen numbers, which yet are not euenly euē numbers, after moſt mens minds: as 24. which 6 an euē number doth meaſure by foure, which is alſo an euen number, and yet as they thinke is not 24 an euenly euen number, for that 8 an euen nūber doth meaſure alſo 24 by 3. an odde number. Wherfore *Campane* to make his ſentence plaine, after this maner ſetteth forth this definition. *Pariter par eſt quem cuncti pares eum numerantes, paribus vicibus numerant*, that is:

Campane.

An other definitiō of an euēly euen number.

An euenly euen number is, when all the euen numbers which meaſure it, doo meaſure it by euen times, that is, by euen numbers, as 16. All the euen numbers whith meaſure 16, as are 8. 4. and 2. do meaſure it by euen numbers. As 8 by 2, twiſe 8 is 16: 4 by 4, foure times 4 is 16: and 2 by 8, 8 times 2 is 16. Which particle *(all euen numbers)* added by *Campane* maketh 24 to be no euenly euen number. For that ſome one euen number meaſureth it by an odde number as 8 by 3. *Fluſſates* alſo is plainly of this minde, that *Euclide* gaue not this definition in ſuch maner as it is by *Theon* written, for the largenes & generalitie therof, for that it extendeth it to infinite numbers which are not euenly euen as he thinketh, for which cauſe in place therof, he geueth this definition.

Fluſſates.

An other definition.

A number euenly euen, is that which onely euen numbers do meaſure. As 16 is meaſured of none but of euen numbers, and therfore is euenly euen. There is alſo of *Boetius* geuen an other definition of more facilitie, including in it no doubt at all, which is moſt commonly vſed of all writers, and is thus.

Boetius.

An other definition.

A number euenly euen is that which may be deuided into two euen partes, and that part agayne into two euen partes, and ſo continually deuiding without ſtay till ye come to Vnitie. As by example, 64. may be deuided into

into 32 and 32. And either of these partes may be deuided into two euen partes, for 32 may be deuided into 16 and 16. Againe, 16 may be deuided into 8 and 8 which are euen partes, and 8 into 4 and 4. Againe 4 into 2 and 2, and last of all may 2 be deuided into one and one.

9 *A number euenly odde (called in latine pariter impar) is that which an euen number measureth by an odde number.*

The ninth definition.

As the number 6 which 2 an euen number measureth by 3 an odde number, thre times 2 is 6. Likewise 10. which 2. an euen number measureth by 5 an odde number. In this diffinition also is found by all the expositors of *Euclide*, the same want that was found in the diffinition next before. And for that it extendeth it selfe to large, for there are infinite numbers which euen numbers do measure by odde numbers, which yet after their mindes are not euẽly odde nũbers, as for example 12. For 4 an euẽ nũber, measureth 12 by 3 an odde number: three times 4 is 12. yet is not 12 as they thinke an euenly odde number. Wherfore *Campane* amendeth it after his thinking, by adding of this worde *all*, as he did in the first, and defineth it after this maner.

Campane.

A number euenly odde is, when all the euen numbers which measure it, do measure it by vneuen tymes, that is, by an odde number.

An other definition.

As 10. is a number euenly odde, for no euen number but onely 2 measureth 10. and that is by 5 an odde number. But not all the euen numbers which measure 12. do measure it by odde numbers. For 6 an euen number measureth 12 by 2 which is also euen. Wherfore 12 is not by this definition a number euenly odde. *Flussates* also offended with the ouer large generalitie of this definition to make the definition agree with the thing defined putteth it after this maner.

Flussates.

A number euenly odde, is that which an odde number doth measure onely by an euen number.

An other definition.

As 14. which 7. an odde number doth measure onely by 2. which is an euen number. There is also an other definition of this kinde of number commonly geuen of more plaines, which is this.

A number euenly odde is that which may be deuided into two equall partes, but that part cannot agayne be deuided into two equall partes: as 6. may be deuided into two equall partes into 3. and 3. but neither of them can be deuided into two equall partes: for that 3. is an odde number and suffereth no such diuision.

An other definition.

10 *A number oddly euen (called im lattin in pariter par) is that which an odde number measureth by an euen number.*

The tenth definition.

As the number 12: for 3. an odde number measureth 12. by 4. which is an euen number: three times 4. is 12.

This definition is not founde in the greeke neither was it doubtles euer in this maner written by *Euclide*: which thing the slendernes and the imperfection thereof and the absurdities following also of the same declare most manifestly. The definition next before geuen is in substance all one with this. For what number soeuer an euen number doth measure by an odde, the selfe same number doth an odde number measure by an euen. As 2. an euẽ number measureth 6. by 3. an odde number. Wherfore 3. an odde number doth also measure the same number 6. by 2. an euẽ nũber. Now if these two definitions be definitions of two distinct kindes of numbers, then is this number 6. both euenly euen, and also euenly odde and so is contayned vnder two diuers kindes of numbers. Which is directly agaynst the authoritie of *Euclide* who playnely proueth here after in the 9. booke, that euery nomber whose halfe is an odde number, is a number euenly odde onely. Flussates hath here very well noted, that these two euenly odde, and oddely euen, were taken of *Euclide* for on and the selfe same kinde of nomber. But the number which here ought to haue bene placed is called of the best interpreters of *Euclide*, *numerus pariter par & impar*, that is a number euẽly euẽ, and euẽly odde. Yea and it is so called of *Euclide* him selfe in the 34. proposition of his 9. booke: which kinde of number *Campanus* and *Flussates* in steade of the insufficient and vnapt definition before geuen, assigne this definition.

This definition not found in the Greeke.

A number euenly euen, and euenly odde, is that which an euen number doth measure sometime by an euen number, and sometime by an odde.

An other definition

As the number 12. for 2. an euen number, measureth 12. by 6. an euen number: two times 6. is 12. Also 4. an euen number measureth the same number 12. by 3. an odde number. And therefore is 12. a number euenly euen, and euenly odde, and so of such others.

The cause why that *Campanus* and *Flussates* were so scrupelous in amending (as they supposed) the two definitions before, namely, of a number euẽly euen and of a number euenly odde, the one by adding this word *all*, and the other by adding this word *onely*, was for that they were offended with the largeues and generalitie of them. For that by them, on and the selfe same number might be comprehended vnder either definition. And so, the selfe number should be both euenly euen, and also euenly odde: which they tooke for an absurditie. For that they are two distinct and diuers kindes of numbers. But all things well and iustly conceiued, it shall not be hard nor amisse to thinke, that these definitiõs were set and written by *Euclide* in such forme and maner, as they are deliuered vnto vs by *Theon*: and

Campane and Flussates with other, run in an error.

that they neede not these correctiõs and amendementes by adding these wordes *all* and *onely*, for admit that they be distinct kinds of numbers, may not contraries be attributed in diuerse respectes to one thing? May not one line be sayd to be great and little, compared to diuers? Great in comparison of a lesse, and lesse in comparison of a greater? Euen so one number in diuers respects may be of diuers and contrary kindes of numbers. What are more diuers them a square number and a cube number. And yet is 64. in diuers respectes a number both square and cube. In respect of 8. to be his roote, it is a square number: for 8. times 8. is 64. and in respect of 4. to be his roote, it is a cube number for 4. times 4. tower times is 64: so in diuers respectes it is both, without any absurditie at all. Likewise this number 6. in diuers respectes, is a number on the on side longer, and also a trianguler number: which yet are diuers and distinct kindes of numbers. For 6. described by his vnities resembling a figure of lengthe and breadthe hauing two sides, namely 2. and 3. is a plaine or superficiall number of one side longer. And if the same 6. be so described by his vnities, that it representeth the figure of a triangle, then is it and beareth it the name of a triangler figure: as here ye may see the forme of either. And if ye extende in description therof all his vnities in length onely, so is 6 also a lineall number. So you see 6 in diuers respectes is a lineall number, a number on the one side longer, and also a trigonall or trianguler number, and yet therby no inconuenience at all. And why may not likewise one and the selfe number in diuers respectes be accompted a number both euenly euen, and euenly odde? Yea *Euclide* him selfe doth most manifestly proue the same, and in the same wordes, if it be diligently wayed, in his ninth booke. For he sayth, that all numbers being double in continuall course from the number 2. be euenly euen numbers only: and agayne all numbers whose halues are odde, are euenly odde numbers only: and that number which neither is duple from the number two nor hath to his halfe an odde number is a number euenly euen and a number euenly odde. What in this can be spokē more playnely? So that by *Euclide* it is no inconuenience that on number, as 12, for example, in diuers respectes should be both a number euenly euen, and also a number euenly odde. In respect that 6. an euen number measureth 12. by 2. an euen number, 12 is a number euenly euen: and in respect that 4. an euen number measureth 12. by 3. an odde number, 12. is a number euenly odde. And thus iudge ye of all others like.

One number in diuers respectes may be of diuers kindes of numbers.

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There is also an other definition geuen of this kinde of number by *Boetius* and others commonly which is thus.

Boetius definition of a number euenly euē, and euenly od.

A number euenly euen, and euenly odde is that, which may be deuided into two equall partes, and eche of them may agayne be deuided into two equall partes: and so forth. But this diuision is at lenghth stayd, and continueth not till it come to Vnitie. As for example 48: which may be deuided into two equall partes, namely, into 24. and 24. Agayne 24. which is on of the partes may be deuided into two equall partes 12. and 12. Agayne 12. into 6. and 6. And agayne 6 may be deuided into two equall partes, into 3. and 3: but 3. cannot be deuided into two equall partes. Wherefore the deuision there stayeth: and continueth not till it come to vnitie as it did in these numbers which are euenly euen only.

The eleuenth definition.

11 *A number odly odde is that, which an odde number doth measure by an odde number.*

As 25, which 5. an odde number, measureth by an odde number, namely, by 5. Fiue times fiue is 25: Likewise 21. whom 7. an odde number doth measure by 3, which is likewise an odde number. Three times 7. is 21.

Flussates.

Flussatus geueth this definition following of this kinde of number, which is all one in substance with the former definition.

An other definition.

A number odly odde, is that, which onely an odde number doth measure.

As 15. for no number measureth 15. but onely 5. and 3: also 25: none measureth it but onely 5. which is an odde number, and so of others.

The twelfth

12 *A prime (or first) number is that, which onely Vnitie doth measure.*

Prime numbers called incomposed numbers.

As 5. 7. 11. 13. For no number measureth 5, but onely vnitie. For v. vnities make the number 5. So no number measureth 7, but onely vnitie. 2. taken 3. times maketh 6. which is lesse then 7: and 2. taken 4. times is 8, which is more then 7. And so of 11. 13. and such others. So that all prime numbers, which also are called first numbers, and numbers vncomposed, haue no part to measure thē, but onely vnitie.

The thirteenth definition.

13 *Numbers prime the one to the other are they, which onely Vnitie doth measure, being a common measure to them.*

As 15. and 22. be numbers prime the one to the other. 15. of it selfe is no prime number, for not one-

ly

ly vnitie doth measure it, but also the numbers 5. and 3, for 3. times 5. is 15. Likewise 22. is of it selfe no prime number: for it is measured by 2. and 11, besides vnitie. For 11. twise, or 2. eleuen times, make 22. So that although neither of these two numbers 15. and 22. be a prime or incomposed number, but eyther haue partes of his owne, whereby it may be measured beside vnitie: yet compared together, they are prime the one to the other: for no one number doth as a common measure, measure both of them but onely vnitie, which is a common measure to all numbers. The numbers 5. and 3. which measure 15. will not measure 22: againe, the numbers 2. and 11. which measure 22, do not measure 15.

14 A number composed, is that which some one number measureth.

The fourteenth definition.

A number composed is not measured onely by vnitie, as was a prime number, but hath some number which measureth it. As 15: the number 3. measureth 15, namely, taken 5. times. Also the number 5. measureth 15, namely, taken 3. times. 5. times 3, and 3. times 5, is 15. Likewise 18. is a composed number, it is measured by these numbers 6. 3. 9. 2. and so of others. These numbers are also called commonly second numbers, as contrary to prime or first numbers.

15 Numbers composed the one to the other, are they, which some one number, being a common measure to them both, doth measure.

The fifteenth definition.

As 12. and 8. are two composed numbers the one to the other. For the number 4, is a common measure to them both: 4. taken three times maketh 12: and the same 4. taken two tymes maketh 8. So are 9. and 15: 3. measureth them both. Also 10. and 25: for 5. measureth both of them: and so infinitely of others. In thys do numbers composed the one to the other or second numbers, differre from numbers prime the one to the other: for that two numbers being composed the one to the other, ech of them seuerally is of necessitie a composed number. As in the examples before 8. and 12. are composed numbers: likewise 9. and 15: also 10. and 25: but if they be two numbers prime the one to the other, it is not of necessitie, that ech of them seuerally be a prime number. As 9. and 22. are two numbers prime the one to the other: no one number measureth both of them: and yet neither of them in it selfe and in his owne nature is a prime number, but ech of them is a composed number. For 3. measureth 9, and 11. and 2. measure 22.

16 A number is sayd to multiply a number, when the number multiplyed, is so oftentimes added to it selfe, as there are in the number multiplying vnities: and an other number is produced.

The sixteenth definition.

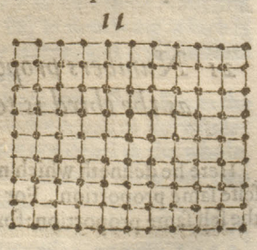
In multiplication are euer required two numbers, the one is whereby ye multiply, commonly called the multiplier or multiplicant, the other is that which is multiplied. The number by which an other is multiplied, namely, the multiplyer, is sayd to multiply. As if ye will multiply 4. by 3, then is three sayd to multiply 4: therefore according to this definition, because in 3. there are three vnities: adde 4, 3. times to it selfe, saying 3. times 4: so shall ye bring forth an other number, namely, 12, which is the summe produced of that multiplication: and so of all other multiplications.

Two numbers required in multiplication.

17 When two numbers multiplying them selues the one the other, produce an other: the number produced is called a plaine or superficiall number. And the numbers which multiply them selues the one by the other, are the sides of that number.

The seuententh definition.

As let these two numbers 3. and 6, multiply the one the other, saying, 3. times 6, or sixe tymes 3, they shall produce 18. Thys nnmber 18. thus produced, is called a plaine number, or a superficiall number. And the two multiplying numbers which produced 12, namely, 3. and 6, are the sides of the same superficiall or plaine number, that is, the length and breadth thereof. Likewise if 9. multiply 11. or eleuen nine, there shal be produced 99. a plaine number, whose sides are the two numbers 9. and 11: as the length and breadth of the same. They are called plaine and superficiall numbers, because being described by their vnities on a plaine superficies, they represent some superficiall forme or figure Geometricall, hauing length and breadth. As ye see of this example: and so of others. And all such plaine or superficiall numbers do euer represent right angled figures as appeareth in the example.

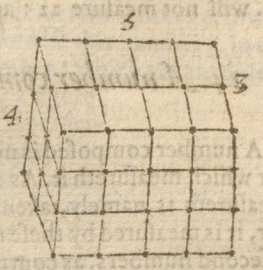


Why they are called superficiall numbers.

The eightenth definition.

18 *When three numbers multiplyed together & one into the other, produce any number, the number produced, is called a ſolide number: and the numbers multiplying them ſelues the one into & other, are the ſides therof.*

As taking theſe three nũbers 3.4.5. multiply the one into the other. Firſt 4. into 5. ſaying, foure times 5. is 20: then multiply that number produced, namely, 20. into 3: which is the third number, ſo ſhall ye produce 60. which is a ſolide number: and the three numbers which produced the number, namely, 3.4. and 5. are the ſides of the ſame. And they are called ſolide numbers, becauſe being deſcribed by their vnities, they repreſent ſolide and bodylicke figures of Geometry, which haue length, breadth, and thicknes. As ye ſee this number 60. expreſſed here by hys vnities. Whoſe length is hys ſide 5, his breadth is 3, and thicknes 4. And thus may ye do of all other three nũbers multiplying the one the other.

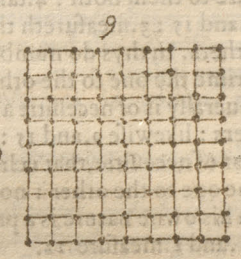


Why they are called ſolid numbers.

The ninetenth definition.

19 *A ſquare number is that which is equally equall: or that which is contayned vnder two equall numbers.*

As multiply two equall numbers the one into the other. As 9. by 9. ye ſhall produce 81, which is a ſquare number. *Euclide* calleth it a number equally equall, becauſe it is produced of the multiplication of two equall numbers the one into the other. Which numbers are alſo ſayd in the ſecond definition to contayne a ſquare number. As in the definitions of the ſecond booke two lines are ſayd to containe a ſquare or a parallelogramme figure. It is called a ſquare number, becauſe being deſcribed by his vnities it repreſenteth the figure of a ſquare in Geometry. As ye here ſee doth the number 81. whoſe ſides, that is to ſay, whoſe length and breadth, are 9. and 9, equall numbers: which alſo are ſayd to contayne the ſquare number 81: and ſo of others.

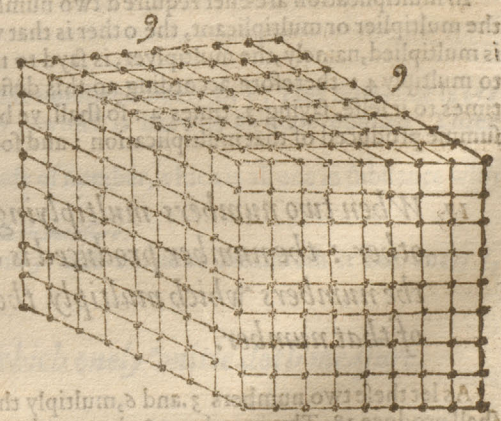


Why it is called a ſquare number.

The twenteth definition.

20 *A cube number is that which is equally equall equally: or that which is contayned vnder three equall numbers.*

As multiply three equall numbers the one into the other, as 9, 9, and 9: firſt 9. by 9, ſo ſhall ye haue 81: which agayne multiply by 9, ſo ſhall ye produce 729. which is a cube number. And *Euclide* calleth it a number equally equall equally, becauſe it is produced of the multiplication of three equall numbers the one into the other: which three numbers are ſayd in the ſecond definition (wherein he ſpeaketh more applying to Geometry) to contayne the cube number. It is therefore called a cube number, becauſe being deſcribed by hys vnities, it repreſenteth the forme of a cube in Geometry, whoſe ſides, that is to ſay, whoſe length, breadth, and thicknes, are the three equall numbers 9, 9, and 9, of which he was produced: which three ſides alſo are ſayd to containe the cube number 729: beholde here the deſcription therof.



Why it is called a cube number.

The twenty one definition.

21 *Numbers proportionall are, when the firſt is to the ſecond equemultiplex, as the third is to the fourth, or the ſelfe ſame part, or the ſelfe ſame partes.*

Here he defineth which numbers are called proportionall, that is, what numbers haue one and the ſelfe ſame proportion. For example 6. to 3: and 4. to 2, are numbers proportionall, and haue one and the ſelf ſame proportion: for 6. the firſt is to 3. the ſecõd equemultiplex, as 4. the third is to 2. the fourth:

4.is double to 3: and ſo is 4.double to 2.Likewiſe theſe foure numbers are in like proportion 3.9.4.12. for what part 3.is of 9,ſuch part is 4.of 12 : 3.of 9.is a third part, ſo is alſo 4. of 12. a third part. So are theſe foure numbers alſo in proportion 2,5 : 4.10 : what partes 2.are of 5,ſuch partes are 4.of 10 : 2, of 5,are two fift partes, likewiſe 4.of 10 are two fift partes. Moreouer,theſe numbers 8.6: 12.9: be in proportion,for what and how many parts 8.are of 6,ſuch & ſo many parts are 12.of 9: 8.of 6,is foure third partes,for one third part of 6.is 2, which taken foure times maketh 8 : ſo 12.of 9,is alſo foure thyrd partes : for one third part of 9.is 3, which taken foure times make 12. And ſo conceaue ye of all other proportionall numbers.

In the fixt definition of the v.booke, *Euclide* gaue a farre other definition of magnitudes proportionall,and much vnlike to this which he here geueth of numbers proportionall: the reaſon is as there alſo was partly noted,for that there he gaue a definition common to all quantities diſcrete and continuall,rationall,and irrationall : and therefore was conſtrayned to geue the definition by the exceſſe,equalitie or want of their equemultiplices,and that generally onely : for that irrationall quantities haue no certayne part or common meaſure to be meaſured by or knowen,neyther can they be expreſſed by any certayne numbers. But here in this place becauſe in numbers there are no irrationall quantities, but all are certaynly knowen,ſo that both they and the proportions betwene them may be expreſſed by numbers certayne and knowen, by reaſon of their partes certayne, and for that they haue ſome common meaſure to meaſure them(at the leſt vnitie which is a common meaſure to all numbers)he geueth here this definition of proportionall numbers,by that the one is like equemultiplex to the other,or the ſame part,or the ſame partes : which definition is much eaſier then was the other : and is not ſo large, as is the other,neither extendeth it ſelf generally to all kinde of quantities rationall and irrationall,but contayneth it ſelfe within the limites and bondes of rationall quantitie and numbers.

Why the definition of proportionall magnitudes is vnlike to the definitiõ of proportionall numbers.

22 *Like plaine numbers,and like ſolide numbers,are ſuch,which haue their ſides proportionall.*

The twenty two definition.

Before he ſhewed that a plaine number hath two ſides, and a ſolide number three ſides. Now he ſheweth by this definition which plaine numbers, and which ſolide numbers are like the one to the other. The likenes of which numbers dependeth altogether of the proportions of the ſides of theſe numbers. So that if the two ſides of one plaine number,haue the ſame proportion the one to the other,that the two ſides of the other plaine number haue the one to the other,then are ſuch two plaine numbers like.For an example 6 and 24 be two plaine numbers, the ſides of 6 be 2 and 3, two tymes 3 make 6: the ſides of 24 be 4 and 6, foure times 6 makes 24.Againe the ſame proportion that is betwene 3 and 2 the ſides of 6, is alſo betwene 6 and 4 the ſides of 24. Wherfore 24 and 6 be two like plaine and ſuperficiall numbers. And ſo of other plaine numbers. After the ſame manner is it in ſolide numbers. If three ſides of the one be in like proportion together,as are the three ſides of the other,then is the one ſolide number like to the other. As 24 and 192 be ſolide numbers, the ſides of 24.are 2. 3. and 4, two tymes three taken 4 times are 24, the ſides of 192 are 4.6. and 8: for foure tymes 6. 8 times make 192. Againe the proportion of 4 to 3 is ſeſquitercia,the proportion of 3 to 2 is ſeſquialtera, which are the proportions of the ſides of the one ſolide number,namely,of 24 : the proportion betwene 8 and 6 is ſeſquetercia,the proportion betwene 6 and 4 is ſeſquialtera, which are the proportions of the ſides of the other ſolide number,namely,of 192. And they are one and the ſame with the proportions of the ſides of the other,wherfore theſe two ſolide numbers 24 and 192 be like,and ſo of other ſolide nũbers.

23 *A perfect number is that,which is equall to all his partes.*

The twenty three definitiõ.

As the partes of 6 are 1. 2. 3. three is the halfe of 6, two the third part,and 1. the ſixth part, and no partes 6 hath not : which three partes 1.2.3. added together,make 6 the whole number, whoſe partes they are. Wherfore 6 is a perfect number. So likewiſe is 28 a perfect number, the partes whereof are theſe numbers 14. 7. 2 and 1: 14 is the halfe therof, 7 is the quarter,4 is the ſeuenth part, 2 is a fourtenth part,and 1 an 28 part,and theſe are all the partes of 28. all which, namely, 1, 2, 4, 7 and 14 added together,make iuſtly without more or leſſe 28. Wherfore 28 is a perfect number, and ſo of others the like.This kinde of numbers is very rare and ſeldome found. From 1 to 10,there is but one perfect number,namely,6.From 10 to an 100,there is alſo but one,that is,28. Alſo from 100 to 1000 there is but one which is 496. From 1000 to 10000 likewiſe but one. So that betwene euery ſtay in numbring,which is euer in the tenth place,there is found but one perfect number And for their rarenes and great perfection,they are of maruelous vſe in magike,and in the ſecret part of philoſophy.

Perfect numbers rare & of great vſe in magike & in ſecret philoſophy.

This kinde of number is called perfect,in reſpect of other numbers which are imperfect. For as the nature of a perfect number ſtandeth in this , that all his partes added together are equall to the whole: and make the whole: ſo in an imperfect nũber all the parts added together are not equal to the whole, nor make the whole,but make either more or leſſe . Wherefore of imperfect numbers there are two kindes,the one is called *abundans* or abunding,the other *diminutus*,or wanting.

In what reſpect a number is perfect.

Two kinds of imperfect numbers.

A number abunding is that whoſe partes being all added together make more then the whole number whoſe partes they are,as 12.is an abundant number. For all the partes of 12. namely, 6.4.3.2.

and

and 1.added together make 16:which are more then 12.Likewise 18.is a number abunding,all his parts namely,9.6.3.2.and 1.added together make 20.which are more then 18:and so of others.

A number wanting.

A number diminute,or wanting is that whose partes being all added together, make lesse then the whole,or number whose partes they are.

As 9.is a diminute,or wanting number,for all his partes,namely,3.and 1.(moe partes he hath not) added together make onely 4:which are lesse then 9.Also 26.is a diminute nũber,all his partes, namoly,13.2.1.added together make onely 16:which is a number much lesse then 26.And so of such like.

Common sentences.

CAmpane and *Flussates* here adde certayne common sentences, some of which, for that they are in these three bookes following somtimes alledged,I thought good here to annexe.

Firſt common sentence.

1 The lesse part is that which hath the greater denomination: and the greater part is that, which hath the lesse denomination.

As the numbers 6.and 8.are either of them a part of the number 24:6.is a fourth part,4.times 6.is 24:and 8.is a third part,3.times 8.is 24.Now forasmuch as 4(which denominateth what part 6.is of 24) is greater then 3.(which denominateth what part 8.is of 24.)therefore is 6.a lesse part of 24,then is 8.and so is 8.a greater part of 24. thẽ 6.is.And so in others.

Second common sentence.

2 Whatsoeuer numbers are equemultiplices to one & the selfe same nũber, or to equall numbers,are also equall the one to the other.

As if vnto the number 3 be taken two numbers containing the same number foure times, that is being equemultiplices to the same number three: the sayd two numbers shalbe equall.For 4.times 3.will euer be 12.So also will it be if vnto the two equal numbers 3.& 3.be taken two numbers,the one cõtaining the one number 3.foure times, the other containing the other number 3.also foure times,that is,being equemultiplices to the equall numbers 3..and 3.

Third common sentence.

3 Those numbers to whome one and the selfe same number is equmultiplex,or whose equemultiplices are equall: are also equall the on to the other.

As if the number 18.be equemultiplex to any two numbers, that is, contayne any two numbers twise,thrise, fower times &c? As for example 3.times: then are the sayd two numbers equall.For 18.deuided by 3.will euer bring forth 6.So that that diuision made twise will bring forth 6.and 6.two equall numbers. So also would it follow if the two numbers had equall equemultiplices,namely, if 18.and 18. which are equall numbers contayned any two numbers 3.times.

Fourth common sentence.

4 If a number measure the whole,and a part taken away:it shall also measure the residue.

As if from 24.be taken away 9,there remaineth 15.And for as much as the number 3 measureth the whole number 24,& also the number takẽ away,namely,9. it shall also measure the residue,which is 15.For 3.measureth 15 by fiue,fiue times 3.is 15.And so of others.

Fifth common sentence.

5 If a number measure any number:it also measureth euery number that the sayd number measureth.

As the number 6.measuring the number 12.shall also measure all the numbers that 12.measureth:as the numbers 24.36.48.60.and so forth:which the number 12. doth measure

measure by the numbers 2.3.4.and 5.And for as much as the number 12.doth measure the numbers 24.36.48.and 60:And the nũber 6.doth measure the number 12.(namely by 2.) It followeth by this commõ sentence,that the number 6.measureth eche of these numbers 24.36.48.and 60.And so of others.

6 If a number measure two numbers, it shall also measure ỹ number composed of them.

Sixth common sentence.

As the number 3 measureth these two numbers 6.and 9:it measureth 6.by 2.and 9. by 3. And therefore by this common sentence it measureth the number 15. which is composed of the numbers 6.and 9:namely it measureth it by 5.

7 If in numbers there be proportions how manysoeuer equall or the selfe same to one proportion: they shall also be equall or the selfe same the one to the other.

Seuenth common sentence.

As yf the proportion of the number 6. to the number 3. be as the proportion of the number 8. to the number 4, if also the proportion of the number 10. to the number 5.be as the proportion of the number 8.to the number 4:then shall the proportion of the number 6. to the number 3.be as the proportion of the number 10. is to the number 5: namely,eche proportion is duple.And so of others. *Euclide* in his 5. booke the 11.proposition demonstrated this also in continuall quantitie: which although as touching that kinde of quantitie it might haue bene put also as a principle (as in numbers he taketh it)yet for that in all magnitudes theyr proportion can not be expressed, (as hath before bene noted & shalbe afterward in the tenth booke more at large made manifest:)therefore he demonstrateth it there in that place, and proueth that it is true as touching all proportions generally whither they be rational or irrationall.

¶ The first Proposition. The first Theoreme.

If there be geuen two vnequall numbers,and if in taking the lesse continually from the greater, the number remayning do not measure the number going before, vntill it shall come to vnitie: then are those numbers which were at the beginning geuen,prime the one to the other.

Vppose that there be two vnequal numbers A B *the greater, and* C D *the lesse, and from* A B *the greater, take away* C D *the lesse as often as you can leauing* F A, *and from* C D *take away* F A *as often as you can,leauing the number* G C. *And from* F A *take away* G C *as often as you can, and so do continually till there remayne onely vnitie, which let be* H A. *Then I say that no number measureth the numbers* A B *and* C D. *For if it be possible let some number measure them,and let the same be* E. *Now* C D *measuring* A B *leaueth a lesse number then it selfe,which let be* F A. *And* F A *measuring* D C *leaueth also a lesse then it selfe,namely,* G C. *And* G C *measuring* F A *leaueth vnitie* H A. *And forasmuch as the number* E *measureth* D C, *and the number* C D *measureth the number* B F,*therfore the number* E *also measureth* B F, *and it measureth the whole*

Construction.

E..
D...G..C
B.....F..H.A

Demonstration leading to an absurditie.

whole number B A, *wherfore it also measureth that which remayneth, namely, the number* F A *(by the 4. cõmon sentence of the seuenth). But the number* A F *measureth the number* D G, *wherfore* E *also measureth* D G. *And it measureth also the whole* D C, *wherfore it also measureth that which remayneth, namely, the number* G C *(by the same common sentence): but* G C *measureth the number* F H, *wherfore also* E *measureth* F H, *and it measureth the whole number* F A, *wherfore (by the former common sentence) it also measureth that which remayneth* H A, *which is vnitie, it selfe being a number, which is impossible. Wherfore no number doth measure the numbers* A B *and* C D, *wherfore the numbers* A B *and* C D *are prime numbers the one to the other: which was required to be proued.*

The conuerse of this proposition after *Campane*.

The conuerse of this proposition.

And if the two numbers, namely A B and C D be prime the one to the other. Then the lesse being continually taken from the greater there shalbe no stay of that sustraction, till that you come to vnitie. For if in the continuall substraction there be a stay before you come to vnitie. Suppose that H A be the number whereat the stay is made, which also being subtrahed out of G C leaueth nothing. Wherfore H A measureth G C wherfore also it measureth F H by the 5. common sentence of the seuenth. And for as much as it also measureth it selfe, therefore it also measureth the whole A F by the sixth common sentence of the seuenth, wherfore also it measureth D G by the 5. common sentence. But it is before proued that it measureth G C, wherfore it measureth the whole C D, by the sixth common sentence of the seuenth: wherfore also it measureth B F by the 5. common sentence of the seuenth. And it is also proued that it measureth F A, wherfore also it measureth the whole number A B by the sixth common sentence of the seuenth. Now for as much as the number H A measureth the numbers A B and C D, therfore the numbers A B and C D are numbers composed: wherfore they are not prime the one to the other: which is contrary to the supposition.

D . . . G . C
B . . . . . F . . H . A

How to know whether two numbers geuen be prime the one to the other.

And by this proposition if there be two numbers geuen. It is easy to finde out, whether they be prime the one to the other or no. For if by such continual substraction of the lesse from the greater, you come at the length to vnitie. Then are those numbers geuen prime the one to the other. But if there be a stay before you come to vnitie, then are the numbers geuen, numbers composed the one to the other.

¶ The 1. Probleme. The 2. Proposition.

Two numbers being geuen not prime the one to the other, to finde out their greatest common measure.

Two cases in this probleme. The first case.

Vppose the two numbers geuen not prime the one to the other to be A B *and* C D. *It is required to finde out the greatest common measure of the said numbers* A B *and* C D. *Now the number* C D *either measureth the number* A B *or not. If* C D *measure* A B *it also measureth it selfe. Wherefore* C D *is a common measure to the numbers* C D *and* A B. *And it is manifest also that it is the greatest common measure: for there is no number greater then* C D *that will measure* C D.

A . . . . . B
C . . . . D

The second case.

But if C D *do not measure* A B, *then if of the numbers* A B *and* C D, *the lesse be continually taken away from the greater, there will before you come to vnitie, be left a number, which will measure the number going before (by the 1. of the seuenth). For if there should not, then should the numbers* A B *and* C D *be prime the one to the other, which is contrary to the supposition. Let the sayd number left by the continuall substraction of the lesse number out of the greater be* F C. *So that let the number* C D *measuring* A B, *and substrahed out of it as often as you can leue a lesse number then it selfe, namely* A E. *And let* A E *measuring* C D, *and substrahed out of it*

A . . . . E . . . . . B
C . . F . . . D

as often as you can leaue a lesse then it selfe namely, CF. And suppose that CF do so measure AE that there remayne nothing. Then I say that CF is a common measure to the numbers AB and CD. For forasmuch as CF measureth AE, and AE measureth DF, therefore CF also measureth DF (by the fifth common sentence of the seuenth) and it likewise measureth it selfe, wherfore it also measureth the whole CD (by the sixth common sentence of the seuenth) but CD measureth BE, wherefore CF also measureth BE (by the fifte common sentence of the seuenth). And it measureth also EA: wherefore it also measureth the whole BA (by the sixth common sentence of the seuenth): and it also measureth CD as we haue before proued: wherefore the number CF measureth the numbers AB & CD: wherfore the number CF is a commō measure to the numbers AB & CD.

Demonstratiō of the second case.

That CF is a common measure to the numbers AB and CD.

I say also that it is the greateste common measure. For if CF be not the greateste commō measure to AB and CD, let there be a number greater then CF, which measureth AB and CD: which let be G. And forasmuch as G measureth CD, and CD measureth BE, therefore G also measureth BE (by the fift common sentence of the seuenth). And it measureth the whole AB, wherefore also it measureth the residue, namely, AE (by the 4. common sentence of the seuenth). But AE measureth DF, wherefore G also measureth DF (by the foresayd 5. common sentence of the seuenth). And it measureth the whole CD. Wherefore it also measureth the residue FC: namely, the greater number the lesse: which is impossible. No number therefore greater then CF shall measure those numbers AB and CD: wherefore CF is the greateste common measure to AB and CD: which was required to be done.

A.... E..... B
G...
C...F....D

That CF is the greateste common measure to AB and CD.

Corrolary.

Hereby it is manifest, that if a number measure two numbers it shall also measure their greateste common measure. For if it measure the whole & the part taken away, it shall alwayes measure the residue also, which residue is at the length, the greateste common measure of the two numbers geuen.

¶ The 2. Probleme. The 3. Proposition.

Thre numbers being geuē, not prime the one to the other: to finde out their greateste common measure.

SVppose the three numbers geuen not prime the one to the other to be A, B, C. Now it is required vnto the sayd numbers A, B, C to finde out the greateste common measure. Take the greateste common measure of the two numbers A and B (by the 2 of the seuenth) which let be D: which number D either measureth the number C or not.

A.....
B.....
C....
D..
E...

First let D measure C. And it also measureth the numbers A and B, wherfore D measureth the numbers A, B, C. Wherefore D is a common measure vnto the numbers A, B, C. Then I say also, that it is the greateste common measure vnto them. For if D be not the greateste common measure vnto the numbers A, B, C, let some number greater then D measure the numbers A, B, C. And let the same number be E. Now forasmuch as E measureth the numbers A, B, C, it measureth also the numbers A, B. Wherefore it measureth also

Two cases in this Proposition. The first case.

the greatest common measure of the numbers A,B (by the Corollary of the second of the seuenth). But the greatest common measure of the numbers A, B, is the number D (by construction). Wherefore the number E measureth the number D, namely, the greater the lesse: which is impossible. Wherefore no number greater thē D measureth the nūbers A,B,C. Wherefore D is the greatest common measure to the numbers A,B,C.

A .....
B .....
C .....
D .....
E ..
F ...

The second case.

But now suppose that D do not measure C. First I say that D & C are not prime numbers the one to the other. For forasmuch as the numbers A, B, C, are not prime the one to the other (by supposition) some one number will measure them: but that number that measureth the numbers A,B,C, shall also measure the numbers A,B, and shall likewise measure the greatest cōmon measure of A B, namely, D (by the Corollary of the second of the seuēth). And the sayd number measureth also C. Wherfore some one number measureth the numbers D and C. Wherefore D and C are not prime the one to the other.

Now then let there be taken (by the 2. of the seuenth) the greatest common measure vnto the numbers D and C, which let be the number E. And forasmuch as E measureth D, and D measureth the numbers A,B, therefore E also measureth the numbers A, B (by the sixt common sentēce): and it measureth also C. Wherfore E measureth the nūbers A,B,C. Wherefore E is a common measure vnto the numbers A,B,C. I say also that it is the greatest. For if E be not the greatest common measure vnto the numbers A, B, C, let there be some number greater then E, which measureth the nūbers A,B,C. And let the same number be F. And forasmuch as F measureth the numbers A,B,C: it measureth also the numbers A,B. Wherefore also it measureth the greatest common measure of the numbers A,B (by the Corollary of the 2. of the seuenth). But the greatest common measure of the numbers A,B, is D. Wherefore E measureth D. And it measureth also the number C. Wherefore F measureth the numbers D,C. Wherefore also (by the same Corollary) it measureth the greatest common measure of the numbers D, C. But the greatest common measure of the numbers D,C, is E. Wherfore F measureth E, namely, the greater number the lesse: which is impossible. Wherefore no number greater then E shall measure the nūbers A,B,C. Wherefore E is the greatest common measure to the numbers A,B,C: which was required to be done.

¶ Corollary.

Wherefore it is manifest, that if a number measure three numbers, it shall also measure their greatest common measure. And in like sort more numbers being geuē not prime the one to the other may be found out their greatest common measure, and the Corollary will followe.

¶ The 2. Theoreme. The 4. Proposition.

Euery lesse number is of euery greater number, either a part, or partes.

Two cases in this Proposition.

The first case.

Suppose there be two numbers A and B C. of which let B C be the lesse. Then I say, that B C is either a part or partes of A. For the numbers A and B C are either prime the one to the other, or not. First let A and B C be prime the one to the other. And deuide the number B C into those vnities which are in it. Now euery one of the

the vnities which are in B C is some certaine part of A. Wherefore B C are partes of A.

But now suppose that the numbers A and B C be not prime the one to the other. Then B C either measureth A or not measureth it. If B C measure A, then is B C a part of A. But if not, take (by the 2. of the seuenth) the greatest common measure of A and B C, and let the same be D. And let B C be deuided into as many partes as it hath equall vnto D, that is, into B E, E F, and F C. And forasmuch as D measureth A, therefore D is a part of A. But D is equall vnto euery one of these partes B E, E F, and F C. Wherfore also euery one of these partes B E, E F, and F C, is a part of A. Wherfore the number B C is partes of A. Wherefore euery lesse number is of euery greater nũber, either a part or partes: which was required to be proued.

The second case.

A.....
B.....C
B.....C
B...E...F...C
D...

¶ The 3. Theoreme. The 5. Proposition.

If a number be a part of a number, and an other nũber the selfe same part of an other number, then both the numbers added together shall be the selfe same part of both the numbers added together, which one number was of one number.

Vppose the number A to be a part of the number B C, and let an other number, namely, D, be the selfe same part of an other number, namely, of E F. The I say, that the numbers A and D added together, are the selfe same part of the nũbers B C and E F added together, that A is of B C. For forasmuch as what part the number A is of the number B C, the selfe same part is the number D of the number E F, therfore how many numbers there are in B C equall vnto A, so many numbers are there in E F equall vnto D. Deuide I say, B C into the numbers that are equall vnto A, that is, into B G, and G C, and likewise deuide E F into the numbers that are equall vnto D, that is, into E H and H F. Now then the multitude of these B G and G C, is equall vnto the multitude of these E H and H F. And forasmuch as B G is equall vnto A, and E H vnto D, therefore B G and E H are equall vnto A & D. And by the same reason forasmuch as G C is equall vnto A, and H F vnto D: therefore G C and H F are also equall vnto A and D. Wherefore how many numbers there are in B C equall vnto A, so many are there in B C and E F equall vnto A and D. Wherefore how multiplex B C is to A, so multiplex are both the numbers B C and E F, to both the numbers A and D. Wherefore what part A is of B C, the selfe part also are A & D added together, of B C and E F added together: which was required to be proued.

This propositiõ, and the 6. proposition in discrete quantitie, answer to the first of the fifth in continuat quantitie.

A.....
B.....G.....C
D....
E....H....F

Demonstration.

¶ The 4. Theoreme. The 6. Proposition.

If a number be partes of a number, and an other number the selfe same partes of an other number: then both numbers added together shall be of both numbers added together the selfe same partes, that one number was of one number.

SVppose that the number AB be partes of the number C, and let an other number, namely, DE be the selfe same partes of an other number, namely, of F. Then I say, that the numbers AB and DE added together, are of the numbers C and F added together the selfe same partes, that AB is of C. For forasmuch as what partes AB is of C, the selfe same partes is DE of F: therefore how many partes of C there are in AB, so many partes of F are there in DE. Deuide AB into the partes of C, that is, into AG and GB, and likewise DE into the partes of F, that is, into DH and HE. Now then the multitude of these AG and GB is equall to the multitude of these DH and HE. And forasmuch as what part AG is of C, the selfe same part is DH of F: therefore what part AG is of C, the selfe same part is AG and DH added together of C and F added together. And by the same reason also what part GB is of C, the selfe same part is GB and HE added together of C and F added together. Wherefore what partes AB is of C, the selfe same partes are AB and DE added together of C and F added together: which was required to be demonstrated.

Construction.

Demonstration.

¶ The 5. Theoreme. The 7. Proposition.

If a number be the selfe same part of a number, that a part taken away is of a part taken away: then shall the residue be the selfe same part of the residue, that the whole was of the whole.

SVppose that the number AB be of the number CD the selfe same part, that the part taken away AE is of the part taken away CF. Then I say, that the residue EB is of the residue FD the self same part that the whole AB is of the whole CD. What part AE is of CF, the selfe same part let EB be of GC. And for that what part AE is of CF, the same part is EB of CG, therefore what part AE is of CF, the same part (by the 5. of the seuenth) is AB of FG. But what part AE is of CF, the same part (by supposition) is AB of CD. Wherefore what part AB is of FG, the self same part is AB of CD. Wherefore AB is one & the selfe same part of both these numbers GF and CD. Wherefore GF is equall vnto CD (by the second common sentence of the seuenth). Take away CF which is common to them both. Wherefore the residue GC is equall vnto the residue FD. And forasmuch as what part AE is of CF, the same part is EB of GC: but GC is equall vnto FD: therefore what part AE is of FC, the selfe same part is EB of FD. But what part AE is of CF, the same part is AB of CD. Wherefore what part EB is of FD, the same part is AB of CD. Wherefore the residue EB is of the residue FD the selfe same part that the whole AB is of the whole CD: which was required to be demonstrated.

This proposition and the next following, in discret quãtitie answereth to the fifth propositiõ of the fifth boke in continuall quãtity. Construction. Demoustration.

¶ The 6. Theoreme. The 8. Proposition.

If a number be of a number the selfe same partes, that a part taken away is of a part taken away, the residue also shall be of the residue the selfe same partes that the whole is of the whole.

Suppose

Suppose that the number AB be of the number CD the selfe same partes that the part taken away AE is of the part taken CF. Then I say, that the residue EB is of the residue FD the selfe same partes that the whole AB is of the whole CD. Vnto AB put an equall nũber GH. Wherefore what partes GH is of CD, the selfe same partes is AE of CF. Deuide GH into the partes of CD, that is, GK, and KH, and likewise AE into the partes of CF, that is, into AL and LE. Now then the multitude of these GK and KH, is equall vnto the multitude of these AL and LE. And forasmuch as what part GK is of CD, the selfe same part is AL of CF: but CD is greater then CF. Wherefore GK is greater then AL. Put vnto AL an equall number MG. Wherefore what part GK is of CD, the same part is GM of CF. Wherefore the residue MK is (by the 7. of the seuenth) of the residue FD, the selfe same part that the whole GK is of the whole CD. Againe, forasmuch as what part KH is of CD, the selfe same part is EL of CF: but CD is greater then CF. Wherefore HK is greater then EL. Put vnto EL an equall number KN. Wherefore what part KH is of CD, the selfe same part is KN of CF. Wherefore the residue also NH is (by the 7. of the seuenth) of the residue FD, the selfe same part that the whole KH is of the whole DC. Wherefore both these MK and NH added together are (by the 5. of the seuenth) of DF the self same partes that the whole HG is of the whole CD. But both these MK and NH added together, are equall vnto EB. And HG is equall vnto BA. Wherefore the residue EB is of the residue FD the selfe same partes that the whole AB is of the whole CD: which was required to be proued.

Construction.

Demonstration.

A . . . L . . . E . . B

C . . . . . F . . . D

G . . . M . K . . . N . H

¶ An other demonstration after *Flussates*.

Suppose that the number AB be of the number CD the selfe same partes that the part taken away AE is of the part taken away CF. Then I say, that the residue EB is of the residue FD the selfe same partes that the whole AB is of the whole CD. Let EB be of CI the self same partes that AB is of CD, or AE of CF. Now forasmuch as EB is of CI the selfe same partes that AE is of CF: therefore both these AE and EB added together are of both these CF and CI added together (that is, the whole AB is of the whole FI) the selfe same partes that AE is of CF (by the sixt of this booke). But what partes AE is of CF, the selfe same partes is the number AB of the number CD (by supposition). Wherefore what partes the number AB is of the nũber FI, the selfe same partes is the same number AB of the number CD. Wherefore the numbers FI and CD are equall. Take away the number CF which is common to them both. Wherefore the numbers remayning CI and FD are equall. Wherefore what partes the number EB is of the number CI, the selfe same partes is the same number EB of the number FD. But what partes EB is of CI, the selfe same partes (by construction) is AB of CD. Wherefore what partes the residue EB is of the residue FD, the selfe same partes is the whole AB of the whole CD: which was required to be proued.

An other demonstration after Flussates.

B . . E . . . . . A

D . . F . . . . . C . . I

¶ *The 7. Theoreme.* *The 9. Proposition.*

If a number be a part of a number, and if an other number be the self same part of an other nũber: then alternately what part or partes the first is of the third, the self same part or partes shall the second be of the fourth.

Suppose that the number A be of the number BC the selfe same part, that an other number D is of an other number EF. And let A be lesse then D. Then I say, that alternately what part or partes A is of D, the selfe same part or partes is BC of EF. For forasmuchas what part A is of BC, the selfe same part is D of EF, therefore how many

Construction.

A . . .

B . . . G . . . C

D . . .

E . . . H . . . F

numbers there are in B C equall vnto A, so many are there in E F equall vnto D. Deuide B C into the numbers equall vnto A, that is, into B G & G C: and likewise E F into the numbers equall vnto D, that is, into E H and H F. Now then the multitude of these B G and G C, is equall vnto the multitude of these E H & H F. And forasmuch as the numbers B G and G C are equall the one to the other, the numbers also E H and H F are equall the one to the other: and the multitude of these B G & G C is equall vnto the multitude of these E H and H F. Wherefore what part or partes B G is of E H, the selfsame part or partes is G C of H F. Wherefore what part or partes B G is of E H, the selfe same part or partes (by the fift & sixt of the seuenth) are B G and G C added together, of E H and H F added together. But B G is equall vnto A, and E H vnto D. Wherefore what part or partes A is of D, the selfe same part or partes is B C of E F: which was required to be demonstrated.

A....
B....G....C
D.....
E.....H.....F

Demonstration.

¶ The 8. Theoreme. The 10. Proposition.

If a number be partes of a number, and an other nũber the selfsame partes of an other number, then alternately what partes or part the first is of the third, the selfe same partes or part is the second of the fourth.

SVppose that the number A B be of the number C the selfe same partes, that an other number D E is of an other nũber F, and let A B be leße then D E. Then I say, that alternately also what part or partes A B is of D E, the selfe same partes or part is C of F. Forasmuch as what partes A B is of C, the selfe same partes is D E of F: therefore how many partes of C there are in A B, so many partes of F also are there in D E. Deuide A B into the partes of C, that is, into A G and G B. And likewise D E into the partes of F, that is, D H and H E. Now then the multitude of these A G and G B, is equall vnto the multitude of these D H and H E. And forasmuch as what part A G is of C, the selfe same part is D H of F, therefore alternately also (by the former) what part or partes A G is of D H, the selfe same part or partes is C of F. And by the same reason also what part or partes G B is of H E, the same part or partes is C of F. Wherefore what part or partes A G is of D H, the selfe same part or partes is A B of D E (by the 6. of the seuenth). But what part or partes A G is of D H, the selfe same part or partes is it proued that C is of F. Wherefore what partes or part A B is of D E, the selfe same partes or part is C of F: which was required to be proued.

Construction.

C.....
A..G..B
F.....
D.....H.....E

Demonstration.

¶ The 9. Theoreme. The 11. Proposition.

If the whole be to the whole, as a part taken away is to a part taken away: then shall the residue be vnto the residue, as the whole is to the whole.

This proposition in discret quãtitie answereth to the ninth propositiõ of the fifth boke in continual quãtitie.

SVppose that the whole number A B be vnto the whole number C D, as the part takẽ away A E, is to the part takẽ away C F. Thẽ I say that the residue E B, is to the residue F D, as the whole A B is to the whole C D. For forasmuch as, A B is to C D, as A E is to C F: therfore what part or partes A B is of C D, the selfe same part or partes is A E of C F. Wherfore also the residue E B is of the residue F D (by the 8. of the seuenth) the selfe same parte

C.....F.....D
A...E...B

or partes

or partes that A B is of C D. Wherefore alſo (by the 21. definition of this booke) as E B is to F D, ſo is A B to C D: which was required to be proued. Demonſtration.

¶ The 10. Theoreme. The 12. Propoſition.

If there be a multitude of numbers how many ſoeuer proportionall: as one of the antecedentes is to one of the conſequentes, ſo are all the antecedentes to all the conſequentes.

Vppoſe that there be a multitude of nũbers how many ſoeuer proportional, namely, A, B, C, D, ſo that as A is to B, ſo let C be to D. Then I ſay that as one of the antecedentes, namely, A is to one of the conſequentes, namely, to B, or as C is to D, ſo are all the antecedentes: namely, A and C to all the conſequentes, namely, to B and D. For foraſmuch as (by ſuppoſition) as A is to B, ſo is C to D, therfore what parte or partes A is of B, the ſelfe ſame part or partes is C of D (by the 21. definition of this booke) wherefore alternately what part or partes A is of C the ſelfe ſame parte or partes is B of D (by the ninth and tenth of the ſeuenth) wherefore both theſe numbers added together, A and C, are of both theſe numbers B and D added together, the ſelfe ſame part or partes that A is of B (by the 5. and 6. of the ſeuenth) wherfore (by the 21. definition of the ſeuenth) as one of the antecedents, namely, A, is to one of the conſequentes, namely, to B, ſo are all the antecedentes A and C to all the conſequentes B & D. Which was required to be proued.

This in diſcret quãtity anſwereth to the twelfe propoſition of the fifth in continual quãtity. Demonſtration.

A ..
B ...
C .....
D .....

¶ The 11. Theoreme. The 13. Propoſition.

If there be foure numbers proportionall: then alternately alſo they ſhall be proportionall.

Vppoſe that there be foure numbers proportional, A, B, C, D, ſo that as A is to B, ſo let C be to D. Then I ſay that alternately alſo they ſhalbe proportional, that is. as A is to C, ſo is B to D. For foraſmuch as (by ſuppoſition) as A is to B, ſo is C to D, therfore (by the 21. definition of this booke) what part or partes A is of B the ſelfe ſame part or partes is C of D. Therfore alternately what part or partes A is of C the ſelfe ſame part or partes is B of D (by the 9. of the ſeuenth) & alſo (by the 10. of the ſame) wherfore as A is to C, ſo is B to D (by the 21. definition of this booke): which was required to be proued.

This in diſcrete quantity anſwereth to the ſixteenth propoſition of the fifth booke in continual quantitie.

A ...
B .....
C ...
D .....

Here is to be noted, that although in the foreſayd example and demonſtration the number A be ſuppoſed to be leſſe then the number B, and ſo the number C is leſſe then the number D: yet will the ſame ſerue alſo though A be ſuppoſed to be greater then B, wherby alſo C ſhall be greater then D, as in this example here put: For for that (by ſuppoſition) as A is to B, ſo is C to D, and A is ſuppoſed to be greater then B, and C greater then D: therefore (by the 21. definition of this Booke) how multiplex A is to B, ſo multiplex is C to D, and therefore what part or partes B is of A, the ſelfe ſame part or partes is D of C. Wherefore alternately what part or partes B is of D, the ſelfe ſame part or partes is A of C, and therefore by the ſame definition, B is to D, as A is to C. And ſo muſt you vnderſtand of the former Propoſition next going before. Note.

A .....
B ...
C .....
D ....

¶ The 12. Theoreme. The 14. Propoſition.

If there be a multitude of nnmbers how many ſoeuer, and alſo other numbers equall vnto them in multitude, which being compared two and two are in one and the ſame proportion: they ſhall alſo of equalitie be in one and the ſame proportion.

This in diſcrete quantity anſwereth to the twẽty one propoſition of the fifth booke in continuall quantitie.

Demonſtration.

VPpoſe that there be a multitude of numbers how many ſoeuer: namely, A, B, C, and let the other numbers equall vnto them in multitude be D, E, F: which being compared two and two, let be in one and the ſame proportion: that is, as A to B, ſo let D be to E: and as B is to C, ſo let E be to F. Then I ſay, that of equalitie, as A is to C, ſo is D to F. For foraſmuch as by ſuppoſition as A is to B, ſo is D to E: therefore alternately alſo (by the 13 of the ſeuenth) as A is to D, ſo is B to E. Againe for that as B is to C, ſo is E to F: therfore alternately alſo (by the ſelf ſame) as B is to E, ſo is C to F. But as B is to E, ſo is A to D. Wherfore (by the ſeuenth common ſentence of the ſeuenth) as A is to D, ſo is C to F. Wherfore alternately (by the 13. of the ſeuenth) as A is to C, ſo is D to F: which was required to be demonſtrated.

A . . . . . .
B . . . . .
C . . . .
D . . . . . .
E . . . .
F . .

Certaine additions of Campane.

After this Propoſition, *Campane* demonſtrateth in numbers theſe foure kindes of proportionalitie, namely, proportion conuerſe, compoſed, deuided, and euerſe: which were in continual quantitie, demonſtrated in the 4. 17. 18. and 19. propoſitions of the fift booke. And firſt he demonſtrateth conuerſe proportion in this maner.

Two caſes in this propoſitiõ. The firſt caſe.

Suppoſe that the number A be to the number B, as the number C is to the number D. Then I ſay, that conuerſedly B is to A, as D is to C. For if A be leſſe then B, C alſo ſhall be leſſe then D, and what part or partes A is of B, the ſelfe ſame part or partes is C of D. Wherefore B is equemultiplex to A, as D is to C. Wherefore (by the 21. definition of this booke) as B is to A, ſo is D to C.

A . . . . . .
B . . . . .
C . . . .
D . . . .

The ſecond caſe.

But if A be greater then B, C alſo is greater then D: and what part or partes B is of A, the ſelfe ſame part or partes is D of C. Wherefore (by the ſame definition) as B is to A, ſo is D to C: which was required to be proued.

Proportionalitie deuided, is thus demonſtrated.

Proportionalitie deuided.

Suppoſe that the number A B be to the number B, as the number C D is to the number D. Then I ſay, that deuided alſo, as A is to B, ſo is C to D. For for that as A B is to B, ſo is C D to D: therefore alternately (by the 14. of this booke) as A B is to C D, ſo is B to D. Wherefore (by the 11. of this booke) as A B is to C D, ſo is A to C. Wherefore as B is to D, ſo is A to C: and for that as A is to C, ſo is B to D, therefore alternately as A is to B, ſo is C to D.

A . . . . . B . .
C . . . . D . . .

Proportionalitie compoſed, is thus demonſtrated.

Proportionalitie compoſed.

If A be vnto B, as C is to D: then ſhall A B be to B, as C D is to D. For alternately A is to C, as B is to D. Wherefore (by the 13. of this booke) as A B, namely all the antecedentes are to C D, namely, to all the conſequentes, ſo is B to D, namely, one of the antecedentes to one of the conſequentes. Wherfore alternately as A B is to B, ſo is C D to D.

A . . . . . B . .
C . . . D . .

Euerſe proportionalitie, is thus proued.

Euerſe proportionalitie.

Suppoſe that A B be to B, as C D is to D: then ſhall A B be to A, as C D is to C. For alternately A B is

A B is to C D, as E is to D. Wherefore (by the 13. of this booke) A E is to C D, as A is to C. Wherefore alternately A B is to A, as C D is to C : which was required to be proued.

A.....B...
C....D..

¶ A proportion here added by *Campane*.

If the proportion of the first to the second, be as the proportion of the third to the fourth, and if the proportion of the fift to the second be as the proportion of the sixt to the fourth : then the proportion of the first and the fifth taken together, shall be to the second, as the proportion of the third and the sixt taken together to the fourth.

As if A be vnto B, as C is to D : and if also E be to B, as F is to D. Then shall A & E taken together, be vnto B, as C and F taken together, are vnto D. For by conuerse proportionalitie, B is to E, as D is to F. Wherefore by proportion of equalitie, as A is to E, so is C to F. Wherefore (by composition) as A and E are to E, so are C and F to F. But (by supposition) as E is to B, so is F to D. Wherefore againe by proportion of equalitie, as A and E are to B, so are C and F to D : which was required to be proued.

A.....E....C.....,F...B....D...

This proposition in discret quantity answereth to the 24. propositiō of the fifth boke in continual quantity.

And after the same maner may you proue the conuerse of this Proposition. If B be to A, as D is to C: and if also B be vnto E, as D is to F : Then shall B be to A E, as D is to C F. For by conuerse proportionalitie, A is to B, as C is to D. Wherefore of equalitie, A is to E, as C is to F. Wherefore by composition A and E are to E, as C and F are to F. Wherefore conuersedly, E is to A and E, as F is to C and F. But by supposition, B is to E, as D is to F. Wherefore agayne by Proportion of equalitie, B is to A and E, as D is to C and F : which was required to be proued.

The conuerse of the same propposition. Demonstration.

A Corollary.

By this also it is manifest that if the proportion of numbers how many soeuer vnto the first, be as the proportion of as many other numbers vnto the second, then shall the proportion of the numbers composed of all the numbers that were antecedentes to the first, be to the first, as the number composed of all the numbers that were antecedentes to the second is to the second. And also conuersedly if the proportion of the first to nūbers how many soeuer, be as the proportion of the second to as many other numbers: then shall the proportion of the first to the number composed of all the numbers that were consequentes to it selfe, be as the proportion of the second to the number composed of all the numbers that were consequentes to it selfe.

A Corollary following those propositions added by Campane.

¶ *The 13. Theoreme.* *The 15. Proposition.*

If vnitie measure any number, and an other number do so many times measure an other number : vnitie also shall alternately so many times measure the third number, as the second doth the fourth.

Vppose that vnitie A do measure the number E C : and let an other nūber D so many times measure some other nūber, namely, E F. Then I say, that alternately, vnitie A shall so many times measure the number D, as the number B C doth measure the number E F. For forasmuch as vnitie A doth so many times measure B C, as D doth E F : therefore how many vnities there are in B C, so many numbers are there in E F equall vnto D. Deuide (I say) B C into the vnities which are in it, that is, into B G, G H, and H C. And deuide likewise E F into the numbers equall vnto D, that is, into E K, K L, and L F. Now then the multitude of these B G, G H, and H C, is equall vnto the multitude of these E K, K L, L F. And forasmuch as these vnities B G, G H, and H C, are equall the one to the other, and these numbers E K, K L, & L F, are also equall the one to the other, and the multitude of the vnities B G, G H, and H C, are equall vnto the multitude of the numbers E K, K L, & L F : therefore as vnitie B G is to the number E K, so is vnitie G H to the number K L, and also vnitie H C to the number L F. Wherefore

Construction.

A.
B.G.H.C
D..
E..K..L..F

Demonstration.

fore (by the 12. of the seuenth) as one of the antecedentes is to one of the consequentes, so are all the antecedentes to all the consequentes. Wherfore as vnitie B G is to the number E K, so is the number B C to the number E F. But vnitie B G is equall vnto vnitie A, and the number E K to the number D. VVherefore (by the 7. common sentence) as vnitie A is to the number D, so is the number B C to the number E F. VVherefore vnitie A measureth the nũber D, so many times, as B C measureth E F (by the 21. definition of this booke): which was required to be proued.

A.
B. G. H. C
D..
E..K..L..F

¶ The 14. Theoreme. The 16. Proposition.

If two numbers multiplying them selues the one into the other, produce any numbers: the numbers produced shall be equall the one into the other.

Demonstration.

SVppose that there be two numbers A and B: and let A multiplying B produce C, and let B multiplying A produce D. Then I say, that the number C is equall vnto the number D. Take any vnitie, namely, E. And forasmuch as A multiplying B produced C, therefore B measureth C by the vnities which are in A. And vnitie E measureth the number A by those vnities which are in the number A. VVherefore vnitie E so many times measureth A, as B measureth C. VVherefore alternately (by the 15. of the seuenth) vnitie E measureth the number B, so many times as A measureth C. Againe, for that B multiplying A produced D, therefore A measureth D by the vnities which are in B. And vnitie E measureth B by the vnities which are in B. VVherefore vnitie E so many times measureth the number B, as A measureth D. But vnitie E so many times measureth the number B, as A measureth C. VVherefore A measureth either of these numbers C and D alike. VVherefore (by the 3. common sentence of this booke) C is equall vnto D: which was required to be demonstrated.

E.
A...
B....
C.....
D.....

The 15. Theoreme. The 17. Proposition.

If one number multiply two numbers, and produce other numbers, the numbers produced of them, shall be in the selfe same proportion, that the numbers multiplied are.

Demonstration.

SVppose that the number A multiplieng two numbers B and C, do produce the numbers D and E. Then I say that as B is to C, so is D to E. Take vnitie, namely, F. And forasmuch as A multiplieng B produced D, therfore B measureth D by those vnities that are in A. And vnitie F measureth A by those vnities whih are in A. Wherfore vnitie F so many times measureth the number A, as B measureth D. VVherfore as vnitie F is to the number A, so is the number B to the number D (by the 21. definition of this booke) And by the same reason, as vnitie F is to the number A, so is the number C to the number E: wherefore also (by the 7. common sentence of this booke) as B is to D, so is C to E. VVherfore alternately (by the 15. of the seuenth) as B is to C, so is D to E. If therfore one number multiply two numbers, and produce other numbers: the numbers produced of them, shall be in the selfe same proportion, that the numbers multiplied are: which was required to be proued.

F.
A...
B....
C.....
D.....
E.....

Here

Here *Flussates* addeth this Corollary.

If two numbers hauing one and the same proportion with two other numbers do multiply the one the other alternately, and produce any numbers, the numbers produced of them shall be equall the one to the other.

A Corollary added by Flussates.

Suppose that there be two numbers *A* and *B*, and also two other numbers *C* and *D*, hauing the same proportion that the numbers *A* and *B* haue: and let the numbers *A* and *B* multiply the numbers *C* & *D* alternately, that is, let *A* multiplieng *D* produce *F*, and let *B* multiplieng *C* produce *E*. Then I say that the numbers produced namely, *E* & *F* are equall. Let *A* and *B* multiply the one the other in such sort, that let *A* multiplieng *B* produce *G*, and let *B* multiplieng *A* produce *H*. Now then the numbers *G* and *H* are equal by the 16. of this booke. And forasmuch as *A* multiplieng the two numbers *B* and *D*, produced the numbers *G* and *F*, therfore *G* is to *F*, as *B* is to *D* by this proposition. So likewise *B* multiplieng the two numbers *A* and *C* produced the two numbers *H* and *E*. Wherfore by the same *H* is to *E* as *A* is to *C*. But alternately (by the 13. of this booke) *A* is to *C* as *B* is to *D*, but as *A* is to *C* so is *H* to *E*, and as *B* is to *D*, so is *G* to *F*. Wherfore by the seuenth common sentence, as *H* is to *E*, so is *G* to *F*. Wherfore alternately (by the 13. of this booke) *H* is to *G* as *E* is to *F*. But it is proued that *G* & *H* are equall: wherfore *E* and *F* (which haue the same proportion that *A* and *B* haue) are equall. If therefore there be two numbers, &c. Which was required to be proued.

H .....
A ..... *C* ..... *F* .....
B .. *D* ... *E* .....
G .....

¶ The 16. Theoreme. The 18. Proposition.

If two numbers multiply any number, & produce other numbers: the numbers of them produced, shall be in the same proportion that the numbers multiplying are.

Demonstration.

Svppose that two numbers A and B multiplieng the number C, doo produce the numbers D and E. Then I say that as A is to B, so is D to E. For forasmuch as A multiplieng C produced D, therfore C multiplieng A produceth also D (by the 16. of this booke.) And by the same reason C multiplieng B produceth E. Now then one number C multiplieng two numbers A and B, produceth the numbers D and E. Wherfore by the 17. of the seuenth, as A is to B, so is D to E: which was required to be demonstrated.

A ....
B .....
C ...
D .....
E .....

This proposition and the former may be extended to numbers how many soeuer.

This Proposition, and the former touching two numbers, may be extended to numbers how many soeuer. So that if one number multiply numbers how many soeuer, and produce any numbers, the proportion of the numbers produced, and of the numbers multiplied, shall be one and the selfe same. Likewise if numbers how many soeuer multiply one number, and produce any numbers, the proportion of the numbers produced, and of the numbers multiplieng shall be one and the selfe same: which thing by this and the former proposition repeted as often as is needefull, is not hard to proue.

¶ The 17. Theoreme. The 19. Proposition.

If there be foure numbers in proportion: the number produced of the first and the fourth, is equall to that number which is produced of the second and the third. And if the number which is produced of the first and the fourth be equall to that which is produced of the second & the third: those foure numbers shall be in proportion.

Suppose

This proposition in numbers demonstrateth that which the 16. of the sixth booke demonstrateth in lines. Construction. Demonstration.

VPpose that there be foure numbers in proportion A, B, C, D, as A is to B, so let C be to D. And let A multiplieng D produce E: and let B multiplieng C produce F. Then I say that the number E is equall vnto the number F. Let A multiplieng C produce G. Now forasmuch as A multiplieng C, produceth G, & multiplieng D it produceth E: it followeth that the number A multiplieng two numbers C and D, produceth G and E. VVherfore by the 17 of the seuenth, as C is to D, so is G to E. But as C is to D, so is A to B, wherfore as A is to B, so is G to E. Againe, forasmuch as A multiplieng C produced G, and B multiplieng C produced F: therfore two numbers A and B, multiplieng one nũber C, do produce G & F. VVherfore by the 18. of the seuenth, as A is to B, so is G to F. But as A is to B, so is G to E: wherfore as G is to E, so is G to F. VVherfore G hath to either of these E and F one & the same proportion (But if one number haue to two numbers one and the same proportion, the saide two numbers shall be equall). VVherfore E is equall vnto F.

A .....
B ....
C ...
D ..
E .....
F .....
G .....

The second part of this proposition which is the conuerse of the first. Demonstration.

But now againe, suppose that E be equall vnto F. Then I say that as A is to B, so is C to D. For the same order of construction remayning still, forasmuch as A multiplieng C & D produced G and E, therfore by the 17. of the seuenth, as C is to D so is G to E, but E is equall vnto F (But if two numbers be equall, one number shall haue vnto them one and the same proportion) wherfore as G is to E, so is G to F. But as G is to E, so is C to D. Wherefore as C is to D, so is G to F, but as G is to F, so is A to B by the 18. of the seuenth, wherfore as A is to B, so is C to D: which was required to be proued.

An assumpt added by Campane.

Here *Campane* addeth, that it is needeles to demonstrate, that if one number haue to two numbers one and the same proportion, the said two numbers shall be equall: or that if they be equal, one number hath to them one and the same proportion. For (saith he) if G haue vnto E and F one and the same proportion, thẽ either, what part or partes G is of E, the same part or parts is G also of F: or how multiplex G is to E, so multiplex is G to F (by the 21. definition) And therfore by the 2 and 3 common sentence, the said numbers shall be equall. And so conuersedly, if the two numbers E and F be equal, then shall the numbers E and F be either the selfe same parte or partes of the number G, or they shall be equemultiplices vnto it. And therfore by the same definition the number G shall haue to the numbers E and F one and the same proportion.

¶ The 18. Theoreme. The 20. Proposition.

If there be three numbers in proportion, the number produced of the extremes, is equall to the square made of the middle number. And if that nũber which is produced of the extremes, be equall to the square made of the middle number, those three numbers shall be in proportion.

This proposition in numbers demonstrateth that which the 17. of the sixth demonstrateth in lines. Demonstration.

VPpose there be three numbers in proportion, A, B, C, as A is to B, so let B be to C. Then I say that the number produced of A and C is equall to the square number which is made of B. Put vnto B an equall number D. Wherfore as A is to B, so is D to C. Wherfore that which is produced of A into C, is equal vnto that which is produced of B into D. But that which is produced of B into D is equal to that which is made of B (for B is equal vnto D) wherfore that which is produced of A into C is equal to that which is made of B.

A .....
B .....
C ....
D .....

But now suppose that that which is produced of A into C, be equall

to that which is made of B. Then I say that as A is to B, so is B to C. For (the same order of construction remayning) forasmuch as that which is produced of A into C, is equall to that which is made of B, but that which is made of B, is equal to that which is produced of B into D. (For B and D are by supposition equall): therfore that which is produced of A into C is equall to that which is produced of B into D: wherfore (by the second part of the former proposition) as A is to B, so is D to C: but D is equal to B. Wherefore as A is to B, so is the same B to C: which was required to be proued.

The second part which is the conuerse of the first. Demonstration.

¶ The 19. Theoreme. The 21. Proposition.

The least numbers in any proportion, measure any other nũbers hauing the same proportion equally, the greater the greater, & the lesse the lesse.

SVppose that C D & E F be the least numbers that haue one & the same proportion with the numbers A and B. Then I say, that the number C D so many times measureth the number A, as the number E F measureth the uumber B. For forasmuch as by supposition C D is to E F, as A is to B, and C D and E F are also supposed to be lesse then A and B: therefore C D and E F are either part or partes of A and B (by the 4. of this booke, and by the 21. definition of the same). But they are not partes. For if it be possible, let C D be partes of A. Wherfore E F is the selfe same partes of B, that C D is of A. Wherefore how many partes of A there are in C D, so many partes are there of B in E F. Deuide C D into the partes of A, that is, into C G and G D. And likewise deuide E F into the partes of B, that is, into E H and H F. Now then the multitude of these C G and G D, is equall vnto the multitude of these E H & H F. And forasmuch as C G and G D are numbers equall the one to the other, and these numbers E H and H F are also equall the one to the other, and the multitude of these C G and G D, is equall to the multitude of these E H and H F: therefore as C G is to E H, so is G D to H F. Wherefore (by the 12. of the seuenth) as one of the antecedentes is to one of the consequentes, so are all the antecedentes to all the consequentes. Wherefore as C G is to E H, so is C D to E F. Wherfore C G and E H are in the selfe same proportion that C D and E F are, being also lesse then C D and E F: which is impossible. For C D and E F are supposed to be the least that haue one and the same proportion with them. Wherefore C D is not partes of A: wherefore it is a part. Wherefore E F is of B the selfe same part, that C D is of A. Wherefore C D so many times measureth A, as E F doth B: which was required to be demonstrated.

Demonstration leading to an impossibility.

C . . G . . D
E . . H . F
A . . . . . .
B . . . . . .

¶ The 20. Theoreme. The 22. Proposition.

If there be three numbers, and other numbers equall vnto thẽ in multitude, which being compared two and two are in the selfe same proportion, and if also the proportion of them be perturbate, then of equalitie they shall be in one and the same proportion.

SVppose that there be three numbers A, B, and C: and let the other numbers equall vnto them in multitude be D, E, and F. And let two and two compared together be in one and the same proportion: and let the proportion of them be perturbate, so that as A is to B, so let E be to F, and as B is to C, so let D be to E. Then I say, that of equalitie as A is to C, so is D to F. For, for that as A is to B, so is E to F,

A . . . . .
B . . . .
C . . .
D . . . . . .
E . . . . . .
F . . . . . .

This proposition in discret quãtitie answereth to the 23. propositiõ of the fifth boke in continual quãtitie.

therefore that which is produced of A into F, is (by the 19. of the seuẽth) equall to that which is produced of B into E. Againe for that as B is to C, so is D to E, therefore that which is produced of D into C, is equall to that which is produced of B into E. And it is proued, that that which is produced of A into F, is equal to that which is produced of B into E. Wherfore that which is produced of A into F, is equal to that which is produced of D into C. Wherefore (by the second part of the 19. of the seuenth) as A is to C, so is D to F: which was required to be proued.

A .....
B ....
C ...
D .....
E .....
F .....

The same may also be proued if in either order be more then three numbers: as it was proued in the 23. of the fift touching more magnitudes then three.

¶ The 21. Theoreme. The 23. Proposition.

Numbers prime the one to the other: are ỹ least of any numbers, that haue one and the same proportion with them.

This and the eleuen propositions following, declare the passions and properties of prime nũbers.

SVppose that A and B be numbers prime the one to the other. Then I say that A and B are the least of any numbers that haue one and the same proportion with them. For if A and B be not the least of any numbers that haue one and the same proportion with them, then are there some numbers lesse then A and B, being in the selfe same proportion that A and B are. Let the same be C and D. Now forasmuch as the least numbers in any proportion measure any other numbers hauing the same proportion equally, the greater the greater, and the lesse the lesse (by the 21. of the seuenth) that is, the antecedent the antecedent, and the consequent the consequent: therefore C so many times measureth A, as D measureth B. How many times C measureth A, so many vnities let there be in E. Wherefore D measureth B by those vnities which are in E. And forasmuch as C measureth A by those vnities which are in E, therefore E also measureth A by those vnities which are in C. And by the same reason E measureth B, by those vnities which are in D. Wherefore E measureth A and B being prime numbers the one to the other: which (by the 13. definition of the seuenth) is impossible. Wherefore there are no other numbers lesse then A and B, which are in the selfe same proportion that A and B are. Wherefore A and B are the least numbers that haue one and the same proportion with them: which was required to be demonstrated.

Demonstration leading to an impossibility.

A .....
B ....
E ..
C ....
D ...

¶ The 22. Theoreme. The 24. Proposition.

The least numbers that haue one and the same proportion with them: are prime the one to the other.

This is the cõuerse of the former proposition.

SVppose that A and B be the least numbers that haue one and the same proportion with them. Then I say that A and B be prime the one to the other. For if A & B be not prime the one to the other, then shall some one nũber measure A & B. Let the same be C. And how oftentimes C measureth A, so many vnities let there be in D: and how oftẽtimes C measureth B, so many vnities let there be in E. And forasmuch as C measureth A by

A .....
B .....
C ....
D ...
E ..

those

those vnities that are in D, therefore C multiplying D produceth A. And by the same reason C multiplying E produceth B. Wherefore the number C multiplying two numbers D and E, produceth A & B. Wherefore (by the 17. of the seuenth) as D is to E, so is A to B. But the numbers D and E are lesse then A and B, and are also in the selfe same proportion with thē, which is impossible. Wherefore no number measureth these numbers A and B. Wherefore A and B are prime the on to the other: which was required to be demonstrated.

Demonstration leading to an absurditie.

¶ The 23. Theoreme. The 25. Proposition.

If two numbers be prime the on to the other: any number measuring one of them shalbe prime to the other number.

SVppose that A and B be two prime numbers the on to the other. And let some number, namely, C, measure A. Then I say that C and B are prime numbers the on to the other. For if C and B be not prime the one to the other, let some number measure C and B, and let the same be D. And forasmuch as D measureth C, and C measureth A, therfore D also measureth A (by the fift common sentence) and D measureth B. Wherefore D measureth A and B being numbers prime the on to the other, which is impossible (by the 13. definition of the seuenth): wherefore no number measureth these numbers B and C. Wherefore B and C are numbers prime the on to the other. Which was required to be proued.

A .....
B .....
C ...
D ....

Demonstration leading to an absurditie.

¶ The 24. Theoreme. The 26. Proposition.

If two numbers be prime to any one number, the number also produced of them shall be prime to the selfe same.

SVppose that the two numbers A and B be prime to any one number, namely, to C, and let A multiplieng B produce D. Then I say that C and D are prime numbers the one to the other. For if C and D be not prime the one to the other then some number shall measure them. Let there be a number that measureth them, and let the same be E. And forasmuch as A and C are prime numbers the one to the other, and the number E measureth C. Therfore E and A are (by the 25. of the seuenth) prime the one to the other. And forasmuch as E measureth D. How many times E measureth D, so many vnities let there be in F. Wherfore F also measureth D by those vnities which are in E. Wherfore E multiplieng F produceth D. But A also multiplieng B produced D. Wherfore that which is produced of E into F is equall to that which is produced of A into B. But if that which is produced of the extremes be equall to that which is produced of the meanes, then are those foure numbers in proportion (by the 19. of the seuenth) wherefore as E is to A, so is B to F. But A and E are prime the one to the other. Yea they are prime and the least in the same proportion (by the 23 of the seuenth.) But the least numbers in any proportion measure any other numbers hauing the same proportion equally, the greater the greater, and the lesse the lesse, that is, the antecedent the antecedent, and the consequent the consequent by the 21. of the seuenth: wherfore E measureth B, and it also measureth C: wherfore E measureth C and B, which are by supposition numbers prime the one to the other, which is impossible by the 13. definition of the

A .. B ...
C .....
D .....
E ...
F ..

Demonstration leading to an absurditie.

seuenth. Wherfore no number measureth those numbers C and D. Wherfore C and D are prime the one to the other: which was required to be proued.

¶ The 25. Theoreme. The 27. Propoſition.

If two numbers be prime the one to the other, that which is produced of the one into him ſelfe, is prime to the other.

Demonſtration.

SVppoſe that there be two numbers prime the one to the other, A and B, and let A multiplieng himſelfe produce C. Then I ſay that B and C are prime the one to the other. For put vnto A an equall number, namely, D. And foraſmuch as A & B are prime the one to the other, and A is equall vnto D: therefore D and B alſo are prime the one to the other. Wherfore either of theſe numbers D and A is prime to B. Wherfore that which is produced of D into A is prime vnto B by the former propoſition. But that number which is produced of D into A is the number C. Wherefore C and B are prime numbers the one to the other: which was required to be proued.

A ... B....
C .....
D ...

¶ The 26. Theoreme. The 28. Propoſition.

If two numbers be prime to two numbers, eche to either of both: the numbers produced of them ſhall be prime the one to the other.

Demonſtration.

SVppoſe that there be two numbers A and B prime to two numbers C and D, either of both, to either of both, namely: let either of theſe A and B be prime to C, and alſo to D. And let A multiplieng B produce E, and let C multiplieng D produce F. Then I ſay that E and F are prime numbers the one to the other. For foraſmuch as either of theſe A & are B prime vnto C, therfore that which is produced of A into B is prime vnto C by the 26. of the ſeuenth. But that which is produced of A into B is the number E, therefore E and C are prime the one to the other. And by the ſame reaſon alſo E and D are prime the one to the other. Wherfore either of theſe numbers C and D are prime vnto E: wherefore that alſo which is produced of C into D is prime vnto E by the ſame. But that which is produced of C into D, is the number F. Wherfore E and F are numbers prime the one to the other: which was required to be demonſtrated.

A ...
B .....
E .....
C ..
D ....
F .....

¶ The 27. Theoreme. The 29. Propoſition.

If two numbers be prime the one to the other, and ech multiplying him ſelfe bring forth certaine numbers: the numbers of them produced ſhall be prime the one to the other. And if thoſe numbers geuen at the beginning multiplying the ſayd numbers produced, produce any numbers: they alſo ſhall be prime the one to the other: and ſo ſhall it be continuing infinitely.

Suppoſe

SVppose that there be two numbers A and B prime the one to the other. And let A multiplying him selfe produce C: and multiplying C let it produce E. Likewise let B multiplying him selfe produce D, and multiplying D let it produce F. The I say, that C and D are numbers prime the one to the other. And likewise that E and F are numbers prime the one to the other. For forasmuch as A and B, are prime the one to the other, and A multiplying him selfe produced C, therefore C and B are prime the one to the other (by the 27. of the seuenth). And by the same reason forasmuch as C and B

Demonstration.

A...
C.....
E.....
B....
D.....
F.....

are prime the one to the other, and B multiplying him selfe produced D, therefore C and D are prime the one to the other. Againe forasmuch as A and B are prime the one to the other, and B multiplying him selfe produced D. Therefore (by the 27. of the seuenth) A and D are prime the one to the other. Now then forasmuch as two numbers A and C are prime to two numbers B and D, either of both to either of both: therefore (by the 28. of the seuenth) that which is produced of A into C is prime to that which is produced of B into D. But that which is produced of A into C is the number E, and that which is produced of B into D is the number F. Wherefore E and F are numbers prime the one to the other. And so alwayes if A & B multiplying the numbers E and F do produce any numbers, the numbers produced, may by the former Proposition, be proued to be prime the one to the other: which was required to be proued.

¶ The 28. Theoreme. The 30. Proposition.

If two numbers be prime the one to the other: then both of them added together, shall be prime to either of them. And if both of them added together be prime to any one of them, then also those numbers geuen at the beginning, are prime the one to the other.

SVppose that these two numbers AB and BC being prime numbers be added together. Then I say, that both these added together, namely, the number ABC, is prime to either of these AB, and BC. For if CA and AB be not prime the one to the other, some number then shall measure them. Let some number measure them, and let the same be D. Now then forasmuch as D measureth the whole CA and the part taken away AB, it measureth also the residue CB (by the 4. common sentence). And it measureth BA. Wherfore D measureth these numbers AB and BC, being prime the one to the other: which is impossible (by the 13. definition of the seuenth). Wherefore no number measureth these numbers CA and AB. Wherefore CA and AB are prime the one to the other. And by the same reason also may it be proued, that CA and BC are prime the one to the other. Wherefore the number AC is to either of these numbers AB and BC, prime.

Demonstration of the first part leading to an absurditie.

A.....B.....C
D....

But now suppose that the numbers CA and AB be prime the one to the other. Then I say, that the numbers AB and BC are also prime the one to the other. For if AB & BC be not prime the one to the other: some one number measureth these numbers AB and BC:

Demonstration of the second part which is the conuerse of the first, leading also to an absurditie.

Let some one number measure them, and let the same be D. And forasmuch as D measureth either of these numbers A B and B C, it shall also measure the whole C A (by the 6. common sentence). And it also measureth A B. Wherefore D measureth these numbers C A and A B being prime the one to the other: which is impossible (by the 13. definition of the seuenth). Wherefore no number measureth these numbers A B and B C. Wherefore A B and B C are prime the one to the other, which was required to be proued.

¶ The 29. Theoreme. The 31. Proposition.

Euery prime number is to euery number which it measureth not, prime.

Demonstration leading to an absurditie.

SVppose that there be geuen a prime number, namely, A, and let B be an other nũber, which it measureth not. Then I say, that the numbers B & A are prime the one to the other. For if A and B be not prime the one to the other, then some number measureth them. Let there be a number that measureth them, and let the same be C. Now C is no vnitie. And forasmuch as C measureth B, but A measureth not B, therefore C is not one and the same number with A. And forasmuch as C measureth A & B, it also measureth A being a prime number, and being not one and the same with it: which is impossible (by the 13. definition of the seuenth). Wherfore no number measureth these numbers A and B. And therefore A and B are prime the one to the other: which was required to be proued.

A ..... B .....
C .....

¶ The 30. Theoreme. The 32. Proposition.

If two numbers multiplying the one the other produce any number, and if also some prime number measure that which is produced of them: then shall it also measure one of those numbers which were put at the beginning.

Demonstration.

SVppose that two numbers A and B multiplying the one to the other do produce the nũber C: and let some prime number, namely, D measure C. Thẽ I say that D measureth one of these numbers either A or B. Suppose that it measure not A: now D is a prime number. Wherefore A and D are prime the one to the other (by the proposition next going before). And how often D measureth C, so many vnities let there be in E. And forasmuch as D measureth C by those vnities which are in E, therfore D multiplying E produceth C: but A also multiplying B produced C wherfore that which is produced of D into E is equal to that which is produced of A into B. VVherfore (by the 19. of the seuenth) as A is to D, so is E to B: but D and A are prime numbers: and therefore the least numbers in that proportion: but the least in any proportion measure the numbers hauing the selfe same proportion with them equally, the greater, the greater, and the lesse, the lesse, that is the antecedent, the antecedent, and the consequent, the consequent (by the 21. of the seuẽth). wherefore the consequent D measureth the consequent B. In like sort may we proue that if D measure not B it measureth A. VVherfore D measureth one of these nũbers A or B: which was required to to be proued.

E ....
D ...
C .....
B .....
A ..

A Corollary.

A Corollary added by Campane.

Hereby it is manifest that if a number measure a number produced of two nũbers multiplied the one into the other, or be commensurable to the same, it shall also either measure

measure one of the two numbers multiplied, or be commensurable with one of them.

The 31. Theoreme. The 33. Proposition.

Euery composed number, is measured by some prime number.

SVppose that A be a composed number. Then I say that A is measured by some prime number. For forasmuch as A is a composed number, some number must needes measure it (by the 14. definition of the seuenth). Let there be a number that measureth it, and let the same be B. Now if B be a prime number then is that manifest which we seeke for: but if it be a composed number some number must needes measure it (by the selfe same definition) Let there be a number that measureth it, and let the same be C. And forasmuch as C measureth B, and B measureth A: therfore C also measureth A (by the 5. common sentẽce): and if C be a prime number then is that manifest which we sought for. But if it be a composed number some number shall measure it: and the like consideration being had there shall at the length be found some prime number which measureth the number going before, which shall also measure A. For if there be not found any such prime number then shall infinite numbers decresing measure the sayd number A of which the one is lesse thẽ the other, which is impossible in numbers. VVherfore some prime number shall at the length be found which shall measure the number going before and which shall also measure the number A (by the 5. common sentence). Euery cõposed number therefore is measured by some prime number: which was required to be proued.

Demonstration leading to an impossibilitie.

C . .
B . . . . . .
A . . . . . .

¶ An other way.

Suppose that A be a composed number. Then I say that some prime number measureth it. For forasmuch as A is a composed number, some number shall measure it (by the 14. definition of the seuenth) Let the least number that measureth it be B. Then I say that B is a prime number. For if B be not a prime number some number shall measure it. Let C measure it. Wherefore C is lesse then B. And forasmuch as C measureth B, and B measureth A, therfore C also measureth A being lesse then B, which by suppo-sition is the least number that measureth A, which is absurd. Wherfore B is not a composed number, but a prime number, which was required to be proued.

An other demonstration.

A . . . . . .
B . . .
C . .

The 32. Theoreme. The 34. Proposition.

Euery number is either a prime number, or els some prime number measureth it.

SVppose that there be a number A. Then I say that A is either a prime number, or els some prime number measureth it. For if A be a prime number then is that had which is required. But if it be a composed number, some prime number shall measure it (by the 33. of the seuenth). Euery number therfore is either a prime number, or els some prime number measureth it: which was required to be demonstrated.

Demonstration.

A . . .
A . . . . .

¶ The 3. Probleme. The 35. Proposition.

How many numbers soeuer being geuen, to find out the least numbers that haue one and the same proportion with them.

Two cases in this Proposition.

The first case.

The second case.

Demonstration.

Demonstration leading to an absurditie.

SVppose that there be a multitude of numbers geuen, namely, A,B, and C. It is required to finde out the least numbers that haue one and the same proportion with these numbers A,B,C. These numbers A, B,C, are either prime the one to the other, or not prime. If A,B, C, be prime the one to the other, then are they the least that haue one and the same proportion with them (by the 23. of the seuenth).

But if they be not prime, take by the 3. of the seuenth, vnto A, B,C, the greatest common measure, which let be the number D. And how often D measureth euery one of these A,B,C, so many vnities let there be in euery one of these numbers E F G. Wherfore these numbers E,F,G, do measure these numbers A, B,C, by those vnities which are in D. Wherfore these numbers E,F,G, measure these numbers A,B,C, equally. Wherfore E, F,G, are by the 18. of the seuenth, in the selfe same proportion that A,B,C, are. Now then I say that they also are the least. For if E,F,G, be not the least that haue one and the same proportion, with A,B,C, there shall thē be some numbers lesse then E,F,G, being in the selfe same proportion that A,B,C, are. Suppose that the same nūbers be H,K,L, which shall measure the numbers A,B,C, equally. How many times H measureth A, so many vnities let there be in M. Wherfore either of these K and L measureth either of these B and C by those vnities which are in M (by the 21. of the seuenth). And foraſmuch as H measureth A by those vnities which are in M, therfore M also measureth A by those vnities which are in H. And by the same reason M measureth either of these B and C by those vnities which are in either of these K and L. Wherefore M measureth these numbers A,B,C. And foraſmuch as H measureth A by those vnities which are in M, therfore H multiplieng M, produceth A. And by the same reason E multiplieng D, produceth A. Wherfore that which is produced of E into D is equall to that which is produced of H into M. Wherfore (by the 19. of the seuenth) as E is to H, so is M to D. But E is greater then H, wherfore M also is greater then D, and it measureth these numbers A,B,C, which is impossible. For D is supposed to be the greatest common measure vnto A,B,C. Wherfore there shall be no other numbers lesse then E,F,G, and in the selfe same proportion with A,B,C. Wherfore E,F,G, are the lest numbers which haue one and the same proportion with A,B,C: which was required to be done.

A Corollary added by Campane.

❋ A Corollary.

Hereby it is manifest that the greatest common measure to numbers how many soeuer: measureth the sayd numbers by the numbers in the least proportiō that the numbers geuen are.

¶ The 4. Probleme. The 36. Proposition.

Two numbers being geuen, to finde out the lest nūber which they measure.

Suppose

SVppose that the two numbers geuen be A and B. It is required to finde out the leſt number which they meaſure. Now A and B are either prime the one to the other, or not. Suppoſe firſt that A and B be prime the one to the other: and let A multiplying B produce C: wherefore B, multiplying A produceth alſo C (by the 16. of the ſeuenth). Wherefore A and B meaſure C. Now alſo I ſay, that C is the leſt nũber which they meaſure. For if it be not, thoſe numbers A and B meaſure ſome number leſſe then C: let them meaſure ſome number leſſe then C, and let the ſame be D: and how often A meaſureth D, ſo many vnities let there be in E: and how often B meaſureth D, ſo many vnities let there be in F. Wherefore A multiplying E produceth D, and B multiplying F produceth alſo D. Wherefore that which is produced of A into E, is equall to that which is produced of B into F: wherefore (by the 19. of the ſeuenth) as A is to B, ſo is F to E. But A and B are prime: yea they are prime and alſo the leſt in that proportion (by the 23. of the ſeuenth): but the leſt numbers in any proportion meaſure thoſe numbers that haue one and the ſame proportion with them equally: the greater the greater: and the leſſe the leſſe (by the 21. of the ſeuenth). Wherfore B meaſureth E, namely, the conſequent the conſequent. And foraſmuch as A multiplying B and E produced C and D: therefore (by the 17. of the ſeuenth) as B is to E, ſo is C to D. But B meaſureth E. Wherefore C alſo meaſureth D, the greater, the leſſe: which is impoſſible. Wherefore if thoſe numbers A and B be prime, they ſhall meaſure no number leſſe then C. Wherefore C is the leſt number which A and B meaſure.

Two caſes in this propoſitiõ. The firſt caſe.

Demonſtration leading to an abſurditie.

A ....
B ...
C .....
D .....
E ....
F .....

But now ſuppoſe that A and B be not prime the one to the other, and take (by the 35. of the ſeuenth) the leſt numbers that haue one and the ſame proportion with A and B, and let the ſame be F and E. Wherefore that which is produced of A into E, is equall to that which is produced of B into F (by the 19. of the ſeuenth). Let A multiplying E produce C: wherfore B multiplying F produceth alſo C. Wherefore A and B meaſure C. Then I ſay, that C is the leſt number that they meaſure. For if it be not, thoſe numbers A and B ſhall meaſure ſome number leſſe then C: let them meaſure a number leſſe then C, and let the ſame be D. And how often A meaſureth D, ſo many vnities let there be in G. And how often B meaſureth D, ſo many vnities let there be in H. Now then A multiplying G produceth D. And B multiplying H produceth alſo D. Wherefore that which is produced of A into G, is equall to that which is produced of B into H. Wherefore (by the 19. of the ſeuenth) as A is to B, ſo is H to G. But as A is to B, ſo is F to E. Wherefore as F is to E, ſo is H to G: but the leſt numbers in any proportion meaſure the numbers that haue the ſame proportion with them equally, the greater the greater, & the leſſe the leſſe (by the 21. of the ſeuẽth). Wherefore E meaſureth G. And foraſmuch as A multiplying G and E produced C and D, therefore (by the 17. of the fift) as E is to G, ſo is C to D. But E meaſureth G. Wherefore C alſo meaſureth D, the greater the leſſe: which is impoſſible. Wherefore thoſe numbers A and B do not meaſure any number leſſe then C. Wherefore C is the leſt number that is meaſured by A and B: which was required to be done.

The ſecond caſe.

Demonſtration leading to an abſurditie.

F ... A .....
E .. B ....
C .....
D .....
G ..
H ...

The 33. Theoreme. The 37. Propoſition.

If two numbers meaſure any number, the leaſt nũber alſo which they meaſure, meaſureth the ſelfe ſame number.

Suppoſe

Demonstration leading to an impossibilitie.

SVppose that there be two numbers geuen A and B: and let them measure the number CD: and let the least number that they measure be E. The I say that E also measureth the number CD. For if E do not measure CD, le E measuring CD, that is subtrahed out of CD as ofte as you can, as for example, once, leaue a lesse then it selfe, namely, CF. And let the number subtrahed which E measureth be FD, and forasmuch as A and B measure E, and E measureth DF, therefore A and B also measure DF. And they measure the whole CD, wherefore by the 4. common sentence of the seuenth they measure also that which remayneth CF being lesse then E: which is impossible. Wherefore E of necessitie measureth CD, which was required to be proued.

A ..
B ...
E .....
C ....F ..... D

¶ The 5. Probleme. The 38. Proposition.

Three numbers being geuen, to finde out the least number which they measure.

Two cases in this propositiō. The first case.

SVppose that there be three numbers geuen A, B, C. It is required to finde out the least number which they measure. Take (by the 36. of the seuenth) the least number which A and B measure, and let the same be D. Now then C either measureth D or els measureth it not. First let it measure it. And the numbers also A and B measure D: wherefore A, B, C, measure D. Now then I say that D is the least number which they measure. For

Demonstration leading to an absurditie.

if not, let the numbers A, B, C, measure some number lesse then D, and let the same be E. And forasmuch as A, B, C, measure E therefore also A and B measure E, wherefore (by the 37. of the seuenth) the least number which those numbers A and B measure shal also measure E. But the least number which A and B measure is D. Wherfore D measureth E, the greater the lesse: which is impossible. Wherefore these numbers A, B, C, shall not measure any number lesse then D. Wherefore D is the least number that A, B, C, doo measure.

A ...
B ....
C .....
D .....
E .....

The second case.

But nov suppose that C measure not D. And take (by the 36. of the seuenth) the least number which those two numbers C and D do measure, and let the same be E. And forasmuch as A and B measure D, and D measureth E, therefore A and B also measure E, and C also measureth E, wherefore A, B, C, also measure E. I say moreouer that E is the least number which A, B, C measure. For if it be not, let there be some lesse number then E which they measure, and let the same be F. And forasmuch as A, B, C, measure F. Therefore A and B also measure F: wherefore the least number which these numbers A and B do measure doth also measure F (by the 37. of the seuenth). But the least number which A and B doo measure is D. Wherefore D measureth F. And C also measureth F. Wherfore D and C measure F. Wherefore the least number which C and D doo measure, shall also (by the selfe same) measure F. But the least number which C & D measure is E. Wherfore E measureth F, namely, the greater, the lesse, which is impossible. Wherefore these numbers A, B, C, do not measure any number lesse then E. Wherefore E is the least number which A, B, C, doo measure: which was required to be demonstrated.

Demonstration leading to an absurditie.

A ...
B ....
C .....
D .....
E .....
F .....

In like maner also how many numbers soeuer being geuen, may be found out the least

leaſt number which they meaſure. For if vnto the three numbers A, B, C, be added a forth, then if the ſayd forth number meaſure the number E, then is E the leaſt number which the fower numbers geuen meaſure. But if it doo not meaſure E, tlē by the 37. of this booke muſt you finde out the leaſt number which E and the forth number meaſure. Which ſhall be the number ſought for. And ſo likewiſe if there be fiue, ſixe, or how manyſoeuer geuen.

Corollary.

Hereby it is manifeſt that the leaſt commō meaſure to numbers howmanyſoeuer, meaſureth euery number which the ſayd numbers howmanyſoeuer meaſure. *A Corollary.*

¶ The 34. Theoreme. The 39. Propoſition

If a number meaſure any number: the number meaſured ſhal haue a part after the denomination of the number meaſuring.

Vppoſe that there be a number B, which let meaſure the number A. Then I ſay, that A hath a part taking his denomination of the number B. For how often B meaſureth A, ſo many vnities let there be in C. And let D be vntie. And foraſmuch as B meaſureth A, by thoſe vnities which are in C, and vnitie D meaſureth C by thoſe vnities which are in C, therefore vnitie D, ſo many times meaſureth the number C, as B doth meaſure A. Wherefore alternately (by the 15. of the ſeuenth) vnitie D, ſo many times meaſureth B, as C doth meaſure A. Wherfore what part vnitie D is of the number B, the ſame part is C of A. But vnitie D is a part of B hauing his denomination of B. Wherfore C alſo is a part of A hauing his denomination of B. Wherfore A hath C a part taking his denomination of B: which was required to be proued.

A .....
B ....
C ...
D.

Demonſtration.

The meaning of this Propoſition is, that if three meaſure any number, that number hath a third part, and if foure meaſure any number the ſayd number hath a fourth part. And ſo forth.

¶ The 35. Theoreme. The 40. Propoſition.

If a number haue any part: the number wherof the part taketh his denomination ſhall meaſure it.

Vppoſe that the number A haue a part, namely, B: and let the part B haue his denomination of the number C. Then I ſay, that C meaſureth A. Let D be vnitie. And foraſmuch as B is a part of A, hauing his denomination of C: and D being vnitie is alſo a part of the number C, hauing his denominatiō of C: therefore what part vnitie D is of the number C, the ſame part is alſo B of A: wherefore vnitie D ſo many times meaſureth the number C, as B meaſureth A. Wherefore alternately (by the 15. of the ſeuenth) vnitie D ſo many times meaſureth the number B, as C meaſureth A. Wherefore C meaſureth A: which was required to be proued.

A .....
B ....
C ..
D.

The conuerſe of the former propoſition. Demonſtration.

This Propoſition is the conuerſe of the former: and the meaning therof is, that euery number hauing a third part is meaſured of three, and hauing a fourth part is meaſured of foure. And ſo forth.

¶ The

¶ The 6. Probleme. Th 41. Proposition.

To finde out the least number, that containeth the partes geuen.

Construction.

SVppose that the partes geuen be A,B,C, namely, let A be an halfe part, B a third part, & C a fourth part. Now it is required to finde out the leaſt nũber which cõtaineth the partes A,B,C. Let the ſaid partes A,B,C, haue their denominations of the numbers D,E,F. And take (by the 38. of the ſeuenth) the leaſt number which the numbers D,E,F, meaſure, and let the ſame be G. And foraſmuch as the numbers D,E,F meaſure the number G, therfore the number G hath partes denominated of the nũbers D,E,F (by the 39. of the ſeuẽth). But the parts A,B,C, haue their denominatiõ of the numbers D,E,F. Wherfore G hath thoſe partes A,B,C. I ſay alſo that it is the leaſt number which hath theſe partes. For if G be not the leaſt number which containeth thoſe partes A, B, C, then let there be ſome number leſſe then G which containeth the ſaide partes A,B,C. And ſuppoſe the ſame to be the number H. And foraſmuch as H hath the ſaid partes A,B,C, therfore the numbers that the partes A,B,C, take their denominations of, ſhall meaſure H (by the 40. of the ſeuenth) But the numbers wherof the partes A,B,C, take their denominations of, are D,E,F. Wherfore the numbers D,E,F, meaſure the uumber H which is leſſe then G, which is impoſſible. For G is ſuppoſed to be the leaſt number that the numbers D,E,F, do meaſure. Wherfore there is no number leſſe then G, which containeth theſe partes A,B,C: which was required to be done.

A an halfe
B a third
C a fourth
D ..
E ...
F ....
G .....
H .....

Demonſtratiõ leading to an abſurditie.

Corrolary.

A Corollary added by Campane.

Hereby it is manifeſt that if there be taken the leaſt number, that numbers how many ſoeuer do meaſure, the ſayd number ſhall be the leaſt which hath the partes denominated of the ſayd numbers how many ſoeuer.

How to finde out the ſeconde leaſt number and the third, and ſo forth infinitly.

Campane after he hath taught to finde out the firſt leaſt number that contayneth the partes geuen, teacheth alſo to finde out the ſecond leaſt number, that is, which except the leaſt of all is leſſe then all other, and alſo the third leaſt, and the fourth &c. The ſecond is found out by doubling the number *G*. For the numbers which meaſure the nũber *G* ſhall alſo meaſure the double therof (by the 5. common ſentence of the ſeuenth). But there cannot be geuen a number greater then the number *G*, & leſſe then the double therof, whom the partes geuen ſhall meaſure. For foraſmuch as the partes geuen do meaſure the whole, namely, which is leſſe then the double, and they alſo meaſure the part taken away, namely, the number *G*, they ſhould alſo meaſure the reſidue, namely, a number leſſe then *G*, which is proued to be the leſt number that they do meaſure, which is impoſſible: wherefore the ſecond number which the ſaid partes geuen do meaſure, muſt, exceeding G, needes reache to the double of G, and the third to the treble, and the fourth to the quadruple, and ſo infinitely, for thoſe partes can neuer meaſure any number leſſe then the number G.

How to find out the leaſt number containing the partes of parts.

By this Propoſition alſo it is eaſie to find out the leaſt number containing the partes geuen of partes. As if we would finde out the leaſt number which contayneth one third part of an halfe part, and one fourth part of a third part, reduce the ſaid diuers fractiõs into ſimple fraction (by the common rule of reducing of fractions) namely, the third of an halfe into a ſixt part of an whole, and the fourth of third into a twelfth part of an whole. And then by this Probleme ſearch out the leaſt number which contayneth a ſixt part and a twelfth part, and ſo haue you done.

The end of the ſeuenth booke of Euclides Elementes,

The

¶ The eight the booke of Euclides Elementes.

AFter that *Euclide* hath in the seuenth booke entreated of the proprieties of numbers in generall, and of certayne kindes thereof more specially, and of prime and composed numbers with others: now in this eight booke he prosecuteth farther, and findeth out and demonstrateth the properties and passions of certayne other kindes of numbers: as of the least numbers in proportion, and how such may be found out infinitely in whatsoeuer proportion: which thing is both delectable, and to great vse. Also here is entreated of playne numbers, and solide: and of theyr sides, and proportion of them. Likewise of the passions of numbers square and cube, and of the natures and conditions of their sides, and of the meane proportionall numbers of playne, solide, square, and cube numbers, with many other thinges very requisite and necessary to be knowne.

The Argumẽt of the eight booke.

¶ *The first Theoreme.* *The first Proposition.*

If there be numbers in continuall proportion howmanysoeuer, and if their extremes be prime the one to the other: they are the least of all numbers that haue one and the same proportion with them.

SVppose that the numbers in continuall proportion be A, B, C, D. And let their extreames namely, A and D be prime the one to the other. Then I say that the numbers A, B, C, D, are the least of all numbers that haue one and the same proportion with thẽ. For if they be not, let E, F, G, H being lesse numbers then A, B, C, D, be in the selfe same proportion that A B C D are. And forasmuch as the numbers A, B, C, D, are in the selfe same proportion that the numbers E, F, G, H, are, & the multitude of these numbers E, F, G, H, is equall to the multitude of these numbers A, B, C, D, therefore of equalitie (by the 14. of the seuenth) as A is to D, so is E to H. But A and D are prime the one to the other, yea they are prime and the least that haue the same proportion with them. But the least numbers in any proportion measure the numbers that haue the same proportion with them equally, the antecedent the antecedent, and the consequẽt the consequent (by the 21. of the seuenth) wherfore A measureth E, the greater the lesse: which is impossible. Wherefore the numbers E, F, G, H, being lesse then A, B, C, D, are not in the same proportion that A, B, C, D, are, wherfore A, B, C, D, are the least of all numbers which haue one and the same proportion with them

A .....
B .....
C .....
D .....
E .....
F .....
G .....
H .....

Demonstration leading to an absurditie.

them. which was required to be demonſtrated.

¶ The 1. Probleme. The 2. Propoſition.

To finde out the leaſt numbers in continuall proportion, as many as ſhall be required, in any proportion geuen.

Conſtruction.

Demonſtration.

SVppoſe that the proportion geuen in the leſt numbers be A to B. It is required to finde out the leſt numbers in continuall proportion, as many as ſhall be required, in the ſame proportion that A is to B. Let there be required foure. And let A multiplying him ſelfe produce C: and multiplying B let it produce D: and likewiſe let B multiplying him ſelfe produce E. And moreouer let A multiplying thoſe numbers C, D, E, produce F, G, H: and let B multiplying E produce K. And foraſmuch as A multiplying him ſelfe produced C, and multiplying B produced D, now then the number A multiplying two numbers A and B produced C & D. Wherefore (by the 17. of the ſeuenth) as A is to B, ſo is C to D. Againe, foraſmuch as A multiplying B produced D, and B multiplying him ſelfe produced E, therefore ech of thoſe numbers A and B multiplying B, bringeth forth theſe numbers D and E. VVherefore (by the 18. of the ſeuenth) as A is to B, ſo D to E. But as A is to B, ſo is C to D. Wherefore as C is to D, ſo is D to E. And foraſmuch as A multiplying C and D produced F and G, therefore (by the 17. of the ſeuenth) as C is to D, ſo is F to G. But as C is to D, ſo is A to B. Wherefore as A is to B, ſo is F to G. Againe foraſmuch as A multiplying D and E produced G and H, therefore (by the 17. of the ſeuenth) as D is to E, ſo is G to H. But as D is to E, ſo is A to B. Wherefore as A is to B, ſo is G to H.

A ...
B ....

C .....
D .....
E .....

F .....
G .....
H .....
K .....

And foraſmuch as thoſe numbers A and B multiplying E produced H and K, therefore (by the 18. of the ſeuenth) as A is to B, ſo is H to K. And it is proued, that as A is to B, ſo is F to G, and G to H: wherefore as F is to G, ſo is G to H, and H to K. Wherefore theſe numbers C, D, E, and F, G, H, K, are proportionall in the ſame proportion, that A is to B. Now I ſay, that they are alſo the leſt. For foraſmuch as A and B are the leſt of all numbers that haue the ſame proportion with them: but the leſt numbers that haue one & the ſame proportion with them are prime the one to the other (by the 24. of the ſeuenth): therefore A and B are prime the one to the other: and ech of theſe numbers A & B multiplying him ſelfe produced theſe numbers C and E, and likewiſe multiplying ech of theſe numbers C and E they produced F and K. Wherefore (by the 29. of the ſeuenth) C, E, and F, K, are prime the one to the other. But if there be numbers in continuall proportion how many ſoeuer, and if their extremes be prime the one to the other, they are the leſt of all nũbers that haue the ſame proportion with them (by the firſt of the eight). Wherefore theſe numbers C, D, E, and F, G, H, K, are the leſt of all

of all numbers that haue the ſame proportion with A and B. And forasmuch as (by the 29. of the ſeuenth) that alwaies happeneth touching the extremes, namely, that A and B multiplying the numbers produced F and K ſhall produce other prime numbers, namely, the extremes of fiue numbers in continuall proportion, therefore (by the firſt of this booke) all fiue are the leſt of that proportion. And ſo infinitely: which was required to be done.

¶ *Corollary.*

Hereby it is manifeſt, that if three numbers being in continuall proportion, be the leſt of all numbers that haue the ſame proportion with them, their extremes are ſquares: and if there be foure their extremes are cubes. For the extremes of three are produced of the multiplying of the nũbers A and B into them ſelues. And the extremes of foure are produced of the multiplying of the rootes A and B into the ſquares C and E, whereby are made the cubes F and K.

The 2. Theoreme. The 3. Propoſition.

If there be numbers in continuall proportion how many ſoeuer, and if they be the leſt of all numbers that haue one and the ſame proportion with thẽ: their extremes ſhall be prime the one to the other.

Vppoſe that the numbers in continuall proportion being the leaſt of all numbers that haue the ſame proportion with them be A,B,C,D. Then I ſay that their extremes A and D are prime the one to the other. Take (by the 2. of the eight, or by the 35. of the ſeuenth) the two leaſt numbers that are in the ſame proportion that A,B,C,D, are, and let the ſame be the numbers E,F. And after that take thre numbers *This propoſition is the cõuerſe of the firſt.*

A .....
B .....
C .....
D .....

E ...
F ...
G .....
H .....
K .....
L .....
M .....
N .....
O .....

G,H,K, and ſo alwayes forward on more (by the former propoſition) vntill the multitude taken be equall to the multitude of the numbers geuen A,B,C,D. And let thoſe numbers be L,M,N,O. Wherfore (by the 29. of the ſeuenth) their extremes L,O, are prime the one to the other. For forasmuch as E and F are prime the one to the other, and eche of them mul- *Demonſtration.*

tiplieng himselfe produced G and K, & likewise ech of these G & K multiplieng himself produced L & O, therfore (by the 29 of the seuenth) G & K are prime the one to the other, & so likewise are L and O prime the one to the other. And forasmuch as A, B, C, D, are the least of

A .....
B .....
C .....
D .....

E ...
F ....

G .....
H .....
K .....

L .....
M .....
N .....
O .....

all numbers that haue the same proportion with them, and likewise L, M, N, O, are the least of all numbers that are in the same proportion that A, B, C, D, are, and the multitude of these numbers A, B, C, D, is equall to the multitude of these L, M, N, O: therfore euery one of these A, B, C, D, is equall vnto euery one of these L, M, N, O. Wherefore A is equall vnto L, and D is equall vnto O. And forasmuch as L and O are prime the one to the other, and L is equall vnto A, and O is equall vnto D: therfore A and D are prime the one to the other: which was required to be proued.

The 2. Probleme. The 4. Proposition.

Proportions in the least numbers how many soeuer beyng geuen, to finde out the least numbers in continuall proportion in the said proportions geuē.

Two cases in this propositiō. The first case.

Demonstration leading to an absurditie.

SVppose that the proportions in the least numbers geuen, be A to B, C to D, and E to F. It is required to finde out the least numbers in continuall proportion, in the same proportion that A is to B, and that C is to D, and that E is to F. Take the least number whom B and C do measure, and let the same be G. And how often B measureth G, so many times let A measure H. And how oftē C measureth G, so many times let D measure K. Now E either measureth K or measureth it not. First let it measure it. And how often E measureth K, so many times let F measure L. And forasmuch as how often A measureth H, so many times doth B measure G: therfore by the 17. of the seuenth, as A is to B, so is H to G. And by the same reasō as G is to D, so is G to K, and moreouer as E is to F, so is K to L. Wherfore these numbers H, G, K, L, are in continuall proportion, and in the same proportion that A is to B, and that C is to D, and moreouer that E is to F. I say also that they are the least in those proportions. For if H, G, K, L, be not the least numbers in continuall proportion, and in the same proportions that A is to B, and C to D, and E to F, then are there some numbers lesse then H, G, K, L, in the same proportions that A is to B and C to D, and E to F, let those numbers be N, X, M, O. And forasmuch as A is to

B, so

B, so is N to X: and A and B are the leſt, but the leaſt meaſure thoſe numbers that haue one and the ſame proportion with them equally, the greater the greater, and the leſſe the leſſe, that is, the antecedent, the antecedent, & the conſequent the conſequent (by the 21. of the ſeuenth) therfore B meaſureth X. And by the ſame reaſon C alſo meaſureth X, wherfore B and C meaſure X. Wherfore the leaſt number whom B and C meaſure, ſhall alſo by the 37 of the ſeuenth meaſure X. But the leaſt number whome B and C meaſure is G. Wherefore G meaſureth X, the greater, the leſſe, which is impoſſible. Wherfore there ſhall not be any leſſe numbers then H, G, K, L, in continuall proportion, and in the ſame proportions that A is to B, and C to D, and E to F.

But now ſuppoſe that E meaſure not K. And by the 36. of the ſeuenth, take the leaſt number whome E and K meaſure, and let the ſame be M. And how often K meaſureth M, ſo often let either of theſe G and H meaſure either of theſe N and X. And how often E meaſureth M, ſo often let F meaſure O. And foraſmuch as how often G meaſureth N, ſo often doth H meaſure X, therfore as H is to G, ſo is X to N. But as H is to G, ſo is A to B Wherfore as A is to B, ſo is X to N. And by the ſame reaſon as C is to D, ſo is N to M. Againe, foraſmuch as how often E meaſureth M, ſo often F meaſureth O, therfore as E is to F ſo is M to O. Wherfore X, N, M, O, are in continuall proportion, and in the ſame proportions that A is to B, and C to D, and E to F. I ſay alſo that they are the leaſt in that proportion. For if X, N, M, O, be not the leaſt in continuall proportion, and in the ſame proportions that A is to B, and C to D, and E to F, then ſhall there be ſome numbers leſſe then X, N, M, O, in continuall proportion, and in the ſame proportions that A is to B, and C to D, and E to F. Let the ſame be the numbers P, R, S, T. And for that as P is to R, ſo is A to B, and A and B are the leaſt, but the leaſt numbers meaſure thoſe numbers that haue one & the ſame proportiõ with them equally, the greater the greater, and the leſſe the leſſe, that is, the antecedent the antecedent, and the conſequent the conſequent by the 21. of the ſeuenth, therfore B meaſureth R. And by the ſame reaſon alſo C meaſureth R. Wherefore B and C meaſure R. Wherfore the leaſt number whom B and C meaſure ſhall alſo meaſure R (by the 37. of the ſeuenth). But the leſt number whom B and C meaſure is G, wherfore G meaſureth R. And as G is to R, ſo is K to S. Wherfore K meaſureth S. And E alſo meaſureth S. Wherefore E and K meaſure S. Wherfore the leaſt number whom E and K meaſure, ſhall (by the ſelfe ſame) meaſure S. But the leaſt number whom E and K meaſure is M. Wherefore M meaſureth S the greater the leſſe, which is impoſſible. Wherfore there are no numbers leſſe then X, N, M, O, in continuall proportion, & in the ſame proportions that A is to B, and C to D, and E to F. Wherfore X, N, M, O, are the leaſt numbers in continuall proportion, and in the ſame proportions that A is to B, and C to D, and E to F: which was required to be done.

A ..
B ....
C ..
D ...
E .
F ..
H ...
G ....
K .....
L .....
N ...
X .....
M .....
O .....

The ſecond caſe.

Demonſtration.

A ...
B .....
C ..
D ...
E ....
F .....
H ...
G ....
K .....
X .....
N .....
M .....
O .....
P ....
R .....
S .....
T .....,

¶ The 3. Theoreme. The 5. Proposition.

Playne or superficiall numbers are in that proportion the one to the other which is composed of the sides.

This proposition in numbers answereth to the of the sixth touching parelelogrammes.

Construction.

Demonstration.

Vppose th at A *and* B *be playne or superficiall numbers, and let the sides of* A *be the numbers* C *and* D, *and let the sides of* B, *be the numbers* E *and* F. *Then I say that* A *is to* B *in that proportion that is composed of the sides. Take (by the fourth of the eight) the least numbers in continuall proportion, and in the same proportions that* C *is to* E, *and* D *to* F. *And let the same be the numbers* G, H, K: *so that as* C *is to* E, *so let* G *be to* H, *& as* D *is to* F, *so let* H *be to* K. *Wherefore those numbers* G, H, K, *haue the proportions of the sides: but the proportion of* G *to* K *is composed of that which* G *hath to* H *and of that which* H *hath to* K: *wherefore* G *is vnto* K *in that proportion which is composed of the sides. Now I say that as* A *is to* B, *so is* G *to* K. *For let* D *multiplying* E *produce* L. *And forasmuch as* D *multiplying* C *produced A, and multiplying* E *produced* L: *therefore (by the 17. of the seuenth) as* C *is to* E, *so is* A *to* L. *But as* C *is to* E, *so is* G *to* H, *wherefore as* G *is to* H, *so is* A *to* L. *Agayne forasmuch as* E *multiplying* D *produced* L, *& multiplying* F *produced* B: *therefore (by the 17. of the seuenth) as* D *is to* F, *so is* L *to* B. *But as* D *is to* F, *so is* H *to* K, *wherefore as* H *is to* K, *so is* L *to* B. *And it is proued that as* G *is* H, *so is* A *to* L. *Wherefore of equalitie (by the 14. of the seuenth) as* G *is to* K, *so is* A *to* B. *But* G *is vnto* K *in that proportion which is composed of the sides, wherefore* A *is vnto* B *in that proportion which is composed of the sides: which was required to be demonstrated.*

A ....
L .....
B .....
C ...
D .....
E ....
F .....
G .....
H .....
K .....

¶ An other demonstration of the same after *Campane*.

An other demonstratiõ after Campane.

Suppose that A and B be plaine numbers: and let the sides of A be the numbers C and D: and let the numbers E and F be the sides of the number B. And let D multiplying E produce the number G. Thẽ I say that the proportiõ of A to B is cõposed of the propor tiõs of C to E & D to F that is, of the sides of the superficial nũ ber A to the sides of the super- ficiall number B. For forasmuch as D multiplying E produced G, and multiplying C it produced A, therefore by (the 17. of the seuenth) A is to G as C is to E: agayne forasmuch as E multiplying D produced G and multiplying F it produceth B, therefore by the same G is to B as D is to F. Wherefore the proportions of the sides namely, of C to E and of D to F are one and the same with the proportions of A to G and G to B. But (by the fifth definition of the sixth) the proportion of the extremes A to B is composed of the proportions of the meanes, namely, of A to G and G to B, which are proued to be one and the same with the proportions of the sides C to E, and D to F. Wherefore the proportion of the superficiall numbers A to B is cõposed of the proportions of the sides C to E, and D to F. Wherefore playne. &c. which was required to be proued.

A .....
G .....
B .....
C ...
D .....
E ....
F .....

¶ The

¶ The 4. Theoreme. The 6. Proposition.

If there be numbers in continuall proportion how many soeuer, and if the first measure not the second, neither shall any one of the other measure any one of the other.

SVppose that there be numbers how many soeuer in continuall proportion, namely, fiue, A,B,C,D,E. And suppose that A measure not B. Then I say, that neither shall any other of the numbers A,B,C,D,E, measure any one of the other. That A,B,C,D,E, do not in continuall order measure one the other, it is manifest: for A measureth not B. Now I say, that neither shall any other of them measure any other of them. I say that A shall not measure C. For how many in multitude A,B,C, are, take so many of the left numbers that haue one and the same proportion with A, B, C (by the 35. of the seuenth) and let the same be F,G,H. And forasmuch as F,G,H are in the selfe same

Demonstration.

A .....
B .....
C .....
D .....
E .....

F ....
G .....
H .....

proportion that A,B,C,are: and the multitude of these numbers A,B,C,is equall to the multitude of those numbers F,G,H, therefore of equalitie (by the 14. of the seuenth) as A is to C, so is F to H. And for that as A is to B, so is F to G, but A measureth not B, therefore neither doth F measure G. Wherefore F is not vnitie. For if F were vnitie, it should measure any number. But F and H are prime the one to the other (by the 3. of the eight). Wherefore F measureth not H: & as F is to H, so is A to C, wherefore neither doth A measure C. In like sort may we proue that neither shall any other of the numbers A,B,C,D,E, measure any other of the numbers A,B,C,D,E: which was required to be demonstrated.

¶ The 5. Theoreme. The 7. Proposition.

If there be numbers in continuall proportion how many soeuer, and if the first measure the last, it shall also measure the second.

SVppose that there be a multitude of numbers in continuall proportion, namely, A,B, C,D. And let A the first measure D the last. Then I say, that A the first measureth B the second. For if A do not measure B, neither shall any other measure any other (by the 7. of the eight): which (by supposition) is not true. For A is supposed to measure D. Now then A measuring D, shall also measure B: which was required to be proued.

Demonstration leading to an impossibilitie.

A ...
B .....
C .....
D .....

¶ The 6. Theoreme. The 8. Proposition.

If betwene two numbers there fall numbers in continuall proportion: how many numbers fall betwene them, so many also shall fall in continuall proportion betwene other numbers which haue the selfe same proportion.

Suppose that betwene the two numbers A and B, do fall in continuall proportion the numbers C and D. And as A is to B, so let E be to F. Then I say, that how many numbers in continuall proportion do fall betwene A and B, so many numbers also in continuall proportion shall there fall betwene E and F. How many A,B,C,D, are in multitude, take (by the 35. of the seuenth) so many of the least numbers that haue one and the same proportion with A,B,C,D, and let the same be G,H,K,L. Wherefore their extremes G and L are prime the one to the other (by the 3. of the eight). And forasmuch as A and C, and D and B, are in the selfe same proportion that G & H, and K and L are, and the multitude of these numbers A,C, D, B, is equall to the multitude of these numbers G, H, K, L: therefore of equalitie (by the 14. of the seuenth) as A is to B, so is

Demonstration.

A ...
C .....
D .....
B .....

G .
H ...
K .....
L .....

E ..
M .....
N .....
F .....

G to L. But as A is to B, so is E to F. Wherefore as G is to L, so is E to F. But G and L are prime the one to the other: yea they are prime and the least. But the least numbers measure those numbers that haue the same proportion with them equally, the greater the greater, and the lesse the lesse (by the 21. of the seuenth) that is, the antecedent, the antecedent, & the consequent, the consequent. Wherefore how many times G measureth E, so many times L measureth F. How often G measureth E, so often let H measure M, and K measure N. Wherefore these numbers G,H,K,L, equally measure these numbers E,M,N,F. Wherefore (by the 18. of the seuēth) these nūbers G,H,K,L, are in the selfe same proportion that E,M,N,F, are. But G,H,K,L, are in the selfe same proportion that A, C, D, B, are: wherefure those numbers A,C,D,B, are in the selfe same proportion that E,M,N,F, are. But A,C,D,B, are in continuall proportion: wherefore also E,M,N,F, are in continuall proportion. Wherfore how many numbers in continuall proportion fall betwene A and B, so many also in continuall proportion fall there betwene E and F: which was required to be demonstrated.

❋ A Corollary added by *Flussates*.

A Corollary added by Flussates.

Betwene two numbers whose proportion is superparticular, or superbipartient, there falleth no meane proportionall. For the least numbers of that proportion differ the one from the other onely by vnitie or by two.

But

But if betwene the greater numbers of that proportion there ſhould fall a meane proportionall, then ſhould there fall alſo a meane proportionall betwene the leaſt numbers which haue the ſame proportion by this Propoſition. But betwene numbers differing onely by vnitie or by two, there falleth no meane proportionall.

¶ The 7. Theoreme. The 9. Propoſition.

If two numbers be prime the one to the other, and if betwene them ſhall fall numbers in continuall proportion: how many numbers in continuall proportion fall betwene them, ſo many alſo ſhall fall in continuall proportion betwene either of thoſe numbers and vnitie.

Vppoſe that there be two numbers prime the one to the other A and B: and let there fall betwene them in continuall proportion theſe numbers C and D: and let E be vnitie. Then I ſay, that how many numbers in continuall proportion fall betwene A and B, ſo many alſo ſhall fall in continuall proportion betwene A and vnitie E: and likewiſe betwene B and vnitie E. Take (by the 35. of the ſeuenth) the two leaſt numbers that are in the ſame proportion that A, C, D, B, are: and let the ſame be F and G: and then take three of the leaſt nũbers that are in the ſame proportion that A, C, D, B, are: and let the ſame be H, K, L: and ſo alwaies in order one more, vntill the multitude of them be equall to the multitude of theſe numbers A, C, D, B: and thoſe being ſo taken let them be M, N, X, O. Now it is manifeſt, that F multiplying him ſelfe produced H, and multiplying H produced M. And G multiplying him ſelfe produced L, and multiplying L produced O. And foraſmuch as M, N, X, O, are (by ſuppoſition) the leaſt of all numbers that haue the ſame proportion with G, F: and A, C, D, B, are (by the firſt of the eight) the leaſt of all numbers that haue the ſame proportion with G, F: and the multitude of theſe numbers M, N, X, O, is equall to the multitude of theſe numbers A, C, D, B: therefore euery one of theſe numbers M, N, X, O, is equall to euery one of theſe numbers A, C, D, B. Wherefore M is equall vnto

Construction.

Demonſtration.

	M 27	A 27	
	H		
F ...	N 36	C 36	
E vnitie	K		
G ...	X 48	D 48	
	L		
	O 64	B 64	

A, and O is equall vnto B. And foraſmuch as F multiplying him ſelfe produced H: therfore F meaſureth H by thoſe vnities which are in F: and vnitie E meaſureth F by thoſe vnities which are in F: wherfore (by the 15. of the ſeuenth) vnitie E, ſo many times meaſureth the number F, as F meaſureth H: wherefore as vnitie E is to the number F, ſo is F to H. Againe foraſmuch as F multiplying H produced M, therfore H meaſureth M by thoſe vnities which are in F. And vnitie E meaſureth F by thoſe vnities which are in F: wherefore (by the ſelf ſame) vnitie E ſo many times meaſureth F, as H meaſureth M. Wherefore as vnitie E is to the number F, ſo is H to M. But it is proued, that as vnitie E is to the number F, ſo is F to H: wherefore as vnitie E is to the number F, ſo is F to H, and H to M. But M is equall vnto A: wherefore as vnitie E is to the number F, ſo is F to H, & H to A. And by the ſame reaſon as vnitie E is to the number G, ſo is G to L and L to B. Wherefore how many numbers fall in continuall proportion betwene A and B: ſo many numbers alſo in continuall proportion fall there betwene vnitie E and the number A, and likewiſe betwene vnitie E

and

and the number B: which was required to be demonstrated.

¶ The 8. Theoreme. The 10. Proposition.

If betwene two numbers and vnitie fall numbers in continuall proportion: how many numbers in continuall proportion fal betwene either of them & vnitie, so many also shall there fall in continuall proportion betwene them.

This proposition is the conuerse of the former.

Vppose that betwene the two numbers A, B, *and vnitie* C, *do fall these numbers in continuall proportion* D, E, *and* F, G. *Then I say that how many numbers in continuall proportion there are betwene either of these* A, B, *and vnitie* C, *so many numbers also in continuall proportion shall there fall betwene* A *and* B. *Let* D *multiplying* F *produce* H, *and let* D *multiplying* H *produce* K, *and like wise let* F *multiplying* H *produce* L. *And for that by supposition as vnitie* C *is to the number* D, *so is* D *to* E, *therefore how many times vnitie* C *measureth the number* D, *so many times doth* D *measure* E. *But vnitie* C *measureth* D *by those vnities which are in* D: *wherefore* D *measureth* E *by those vnities which are in* D. *Wherefore* D *multiplying himselfe produceth* E. *Againe for that as vnitie* C *is to the number* D, *so is* E *to* A, *therefore how many times vnitie* C *measureth the number* D, *so many times* E *measureth* A. *But vnitie* C *measureth* D, *by those vnities which are in* D, *therefore* E *measureth* A *by those vnities which are in* D. *Wherefore* D *multilying* E *produced* A. *And by the same reason* F *multiplying himselfe produced* G, *and multiplying* G *produced* B. *And forasmuch as* D *multiplying himselfe produced* E, *and multiplying* F *produced* H, *therefore (by the 17. of the seuenth) as* D *is to* F, *so is* E *to* H. *And by the same reason as* D *is to* F, *so is* H *to* G. *Wherefore as* E *is to* H, *so is* H *to* G. *Agayne forasmuch as* D *multiplying* E *produced* A, *and multiplying* H *produced* K, *therefore (by the 17. of the seuēth) as* E *is to* H, *so is* A *to* K. *But as* E *is to* H, *so is* D *to* F, *therefore as* D *is to* F, *so is* A *to* K. *Againe forasmuch as* D *multiplying* H *produced* K, *and* F *multiplying* H *produced* L, *therefore (by the 17. of the seuenth) as* D *is to* F, *so is* K *to* L. *But as* D *is to* F, *so is* A *to* K, *wherfore as* A *is to* K, *so is* K *to* L. *Againe forasmuch as* F *multiplying* H *produced* L *and multiplying* G *produced* B, *therefore (by the 17. of the seuenth) as* H *is to* G, *so is* L *to* B. *But as* H *is to* G *so is* D *to* F, *wherefore as* D *is to* F *so is* L *to* B. *And it is proued that as* D *is to* F, *so is* A *to* K, *and* K *to* L, *and* L *to* B. *Wherfore the numbers* A, K, L, B, *are continuall proportion. Wherefore how many numbers in continuall proportion fall betwene either of these numbers* A, B, *&* C, *so many also in continuall proportion fall there betwene the numbers* A *and* B: *which was required to be proued.*

Construction.

Demonstration.

¶ The 9. Theoreme. The 11. Proposition.

Betwene two square numbers there is one meane proportional number. And a square

a ſquare number to a ſquare, is in double proportion of that which the ſide of the one is to the ſide of the other.

Vppoſe that there be two ſquare numbers A and B, and let the ſide of A be C, & let the ſide of B be D. Then I ſay that betwene theſe ſquare numbers A and B, there is one meane proportionall number, and alſo that A is vnto B in double proportion of that which C is to D. Let C multiplieng D produce E. And foraſmuch as A is a ſquare nũber, & the ſide thereof is C, therfore C multiplieng himſelfe produced A. And by the ſame reaſon D multiplieng himſelfe produced B. Now foraſmuch as C multiplieng C produced A, and multiplieng D produced E, therfore (by the 17. of the ſeuenth) as C is to D, ſo is A to E. Againe foraſmuch as C multiplieng D produced E, and D multiplieng himſelfe produced B, therefore theſe two numbers C and D multiplieng one number, namely, D, produce E and B. Wherfore (by the 18. of the ſeuenth) as C is to D, ſo is E to B. But as C is to D, ſo is A to E. Wherefore as A is to E, ſo is E to B. Wherefore betwene theſe ſquare numbers A and B, there is one meane proportionall number, namely, E. Now alſo I ſay that A is vnto B in double proportion of that which C is to D. For foraſmuch as there are three numbers in continuall proportion, A, E, B, therfore (by the 10. definition of the fift) A is vnto B in double proportiõ of that which A is to E. But as A is to E, ſo is C to D. Wherefore A is vnto B in double proportion of that which the ſide C is vnto the ſide D: which was required to be proued.

A ..... C ... E ..... D .... B .....

The firſt part of this propoſition demonſtrated.

The ſecond part demonſtrated.

¶ The 10. Theoreme. The 12. Propoſition.

Betwene two cube numbers there are two meane proportionall numbers. And the one cube is to the other cube in treble proportion of that which the ſide of the one is to the ſide of the other.

Vppoſe that there be two cube numbers A and B, and let the ſide of A be C, and let the ſide of B be D. Then I ſay that betwene thoſe cube numbers A and B, there are two meane proportionall numbers, and that A is vnto B in treble proportion of that which C is to D. Let C multiplieng himſelfe produce E, and multiplieng D let it produce F, and let D multiplieng himſelfe produce G. And let C multiplieng F produce H, and let D multiplieng F produce K. And foraſmuch as A is a cube number, and the ſide therof is C, & C multiplieng himſelfe produceth E, therfore C multiplieng E produceth A. And by the ſame reaſon

Conſtruction.

A .....
H .....
K .....
B .....

C ...
D ....
E .....
F .....
G .....

The firſt part of this propoſition demonſtrated.

reason for that D multiplieng himselfe, produced G, therfore D multiplieng G produceth B. And forasmuch as C multiplieng C and D produced E and F: therfore by the 17. of the fift, as C is to D, so is E to F. And by the same reason also, as C is to D, so is F to G. Againe forasmuch as C multiplieng E and F produced A and H, therfore as E is to F, so is A to H. But as E is to F, so is C to D. Wherfore as C is to D: so is A to H. Againe forasmuch as eche of these

A .....
H .....
K .....
B .....

C ...
D ....

E .....
F .....
G .....

numbers C and D multiplieng F produced H and K, therfore (by the 18. of the seuenth) as C is to D, so is H to K. Againe forasmuch as D multiplieng F and G produced K & B: therfore (by the 17. of the seuenth) as F is to G, so is K to B. But as F is to G, so is C to D, wherefore as C is to D, so is K to B. And it is proued that as C is to D, so is A to H, and H to K, and K to B: wherfore betwene these cube numbers A and B, there are two meane proportionall numbers, that is, H and K.

The second part demonstrated.

Now also I say, that A is vnto B in treble proportion of that which C is to D. For forasmuch as there are foure numbers proportionall A, H, K, B, therfore (by the 10. definition of the fift) A is vnto B in treble proportion of that which A is vnto H. But as A is vnto H, so is C to D, wherfore A is vnto B in treble proportion of that which C is to D: which was required to be proued.

¶ The 11. Theoreme. The 13. Proposition.

If there be numbers in continuall proportion how many so euer, and ech multiplying himselfe produce certayne numbers, the numbers of them produced shall be proportinall. And if those numbers geuen at the beginning multiplying the numbers produced, produce other numbers, they also shalbe proportionall: and so shall it be continuing infinitely.

Construction.

Supposse that there be a multitude of nũbers in cõtinuall proportiõ, namely, A, B, C, as A is to B, so let B be to C. And let A, B, C, multiplying ech himself bring forth the nũbers D, E, F, & multiplying the nũbers D, E, F, let thẽ bring forth the nũbers G, H, K. Thẽ I say that D, E, F, are in cõtinuall proportiõ, and also that G, H, K, are in cõtinuall proportiõ. For it is manifest that the nũbers D, E, F, are square numbers, & that the nũbers G, H, K, are cube nũbers. Let A multiplying B produce L. And let A & B multiplying L produce M and N. And againe let B multiplying C produce X: and let B and C multiplying X produce O and P. Now by the discourse of the proposition going before we

Demonstration.

may

may proue that D, L, E, and also G, M, N, H, are in continuall proportion and in the same proportion that A is to B: and likewise that E, X, F, and also H, O, P, K, are in continuall proportion and in the same proportion that B is to C. But as A is to B, so is B to C. Wherefore D, L, E, are in one and the same proportion with E, X, F: and moreouer G, M, N, H, are in one and the same proportion with H, O, P, K,: and the multitude of these numbers D, L, E, is equall to the multitude of these numbers E, X, F, and likewise the multitude of these numbers G, M, N, H, is equall to the multitude of these numbers H, O, P, K, wherefore of equality (by the 14. of the seuenth) as D is to E, so is E to F. And as G is to H, so is H to K: which was required to be proued.

A 2 B 4 C 8
D 4
L 8
E 16
X 32
F 64
G 8
M 16
N 32
H 64
O 128
P 256
K 512

¶ The 12. Theoreme. The 14. Proposition.

If a square number measure a square number, the side also of the one shall measure the side of the other. And if the side of the one measure the side of the other, the square number also shall measure the square number.

Vppose that there be two square numbers A and B, and let the sides of them be C and D: and let A measure B. Wherefore C also shall measure D. Let C multiplying D produce E. Wherefore (by the 17. and 18. of the seuenth, and 13. of the eight) those numbers A, E, B, are in continuall proportion, and are in the same proportion that C is to D. And forasmuch as A, E, B, are in continuall proportion, and A measureth B, therefore (by the 7. of the eight) A measureth E. But as A is to E, so is C to D: wherefore C measureth D. The first part of this proposition.

A .....
E .....
B .....
C ....
D .....

But now suppose that the side C do measure the side D. Then I say, that the square number A also measureth the square number B. For the same order of construction remayning, we may in like sort proue, that the numbers A, E, B, are in continuall proportion, & in the same proportion, that C is to D. And for that as C is to D, so is A to E, but C measureth D: therefore A measureth E: and A, E, B, are in continuall proportion: wherefore A measureth B. If therefore a square number measure a square number, the side also of the one shall measure the side of the other. And if the side of the one measure the side of the other, the square number also shall measure the square number: which was required to be demonstrated. The second part is the conuerse of the first.

¶ The 13. Theoreme. The 15. Proposition.

If a cube number measure a cube number, the side also of the one shall mea-

sure the side of the other. And if the side of the one measure the side of the other, the cube number also shall measure the cube number.

SVppose that the cube number A do measure the cube number B, and let the side of A be C, and the side of B be D. Then I say, that C measureth D. Let C multiplying him selfe produce E, & multiplying D let it produce F. And let D multiplying him selfe produce G. And moreouer let C and D multiplying F produce H and K. Now it is manifest (by the 17. and 18. of the seuenth, and 12. of the eight) that those numbers E,F,G, and also A,H,K,B, are in continuall proportion, & in the same proportion that C is to D. And forasmuch as A,H,K,B, are in continuall proportion, and A measureth B, therefore (by the 7. of the eight) A also measureth H. But as A is to H, so is C to D. Wherefore C also measureth D.

The first part of this proposition.

A .....
H .....
K .....
B .....

C ..
D ....
E
F
G .....

But now suppose that the side C do measure the side D. Then I say, that the cube number A also measureth the cube number B. For the same order of construction being kept, in like sort may we proue, that A,H,K,B, are in continuall proportion, and in the same proportion that C is to D. And forasmuch as C measureth D, but as C is to D, so is A to H, therefore A measureth H: wherefore A also measureth B. If therefore a cube number measure a cube number, the side also of the one shall measure the side of the other. And if the side of the one measure the side of the other, the cube number also shall measure the cube number: which was required to be proued.

The second part is the conuerse of the first.

¶ The 14. Theoreme. The 16. Proposition.

If a square number measure not a square number, neither shall the side of the one measure the side of the other. And if the side of the one measure not the side of the other, neither shall the square number measure the square number.

SVppose that A and B be two square numbers, and let the side of A be C: and let the side of B be D. And be it that A measureth not B. Then I say, that neither shall C measure D. For if C do measure D, then (by the 14. of the eight) A also measureth B. But A by supposition measureth not B: wherefore neither doth C measure D.

A .....
B .....
C ...
D ....

A negatiue proportion. The first part of this proposition.

But now againe suppose that the side C measure not the side D. Then I say, that neither shall the square number A measure the square number B. For if A do measure B, then shall C (by

The second part is the cõuerse of the first.

C (by the 14. of the eight) measure D. But (by supposition) C measureth not D. Wherefore neither doth A measure B: which was required to be proued.

¶ The 15. Theoreme. The 17. Proposition.

If a cube number measure not a cube number, neither shall the side of the one measure the side of the other. And if the side of the one measure not the side of the other, neither shall the cube nũber measure the cube number.

A negatiue proposition.

SVppose that the cube number A do not measure the cube number B: and let the side of A be C: and the side of B be D. Then I say, that C shall not measure D. For if C do measure D, then (by the 15. of the eight) A also shall measure B. But (by supposition) A measureth not B: wherefore neither shall C measure D.

The first part of this proposition.

A .....
B .....
C ..
D ...

The second part is the cõuerse of the first.

But now suppose that the side C measure not the side D. Then I say, that neither shall the cube number A measure the cube number B. For if A do measure B, then also (by the 15. of the eight) shall C measure D. But (by supposition) C measureth not D. Wherefore neither shall A measure B: which was required to be proued.

¶ The 16. Theoreme. The 18. Proposition.

Betwene two like plaine or superficiall numbers there is one meane proportionall number. And the one like plaine number is to the other like plaine number in double proportion of that which the side of like proportion, is to the side of like proportion.

Demonstration of the first part of this proposition.

SVppose that there be two like plaine or superficiall numbers A & B. And let the sides of A be the nũbers C, D: and the sides of B be the numbers E, F. And forasmuch as like plaine numbers are those which haue their sides proportionall (by the 22. definition of the seuenth) therefore as C is to D, so is E to F. Then I say that betwene A and B there is one meane proportionall number, and that A is vnto B in double proportiõ of that which C is vnto E, or of that which D is vnto F, that is, of that which side of like proportion is to side of like proportion. For for that as C is to D, so is E to F, therefore alternately (by the 13. of the seuenth) as C is to E, so is D to F. And forasmuch as A is a plaine or superficiall number, and the sides thereof are C and D: therefore D multiplying C produced A.

A .....
G .....
B .....
C ..
D .....
E ...
F .....

And by the same reason also E multiplying F produced B. Let D multiplying E produce G. And forasmuch as D multiplying C produced A, and multiplying E produced G, therefore (by the 17. of the seuenth) as C is to E, so is A to G. But as C is to E, so is D to F, wherefore as D is to F, so is A to G. Againe forasmuch as E multiplying D produced G, and multiplying F produced B, therefore (by the 17. of the seuenth) as D is to F, so is G to B. But it is proued

that as D is to F, so is A to G: wherfore as A is to G, so is G to B. Wherefore these numbers A, G, B, are in continuall prorortion. Wherefore betwene A and B there is one meane proportionall number.

Demonstration of the second part.

Now also I say that A is vnto B in dooble proportiõ of that which side of like proportion is to side of like proportion, that is, of that which C is vnto E, or of that which D is vnto F. For forasmuch as A, G, B, are in continuall proportion, therefore (by the 10. definition of the fift) A is vnto B in double proportion of that which A is vnto G. But as A is to G, so is C to E, and D to F: wherefore A is vnto B in double proportion of that which C is to E, or D to F: which was required to be demonstrated.

¶ The 17. Theoreme. Th 19. Proposition.

Betwene two like solide numbers, there are two meane proportionall numbers. And the one like solide number, is to the other like solide number in treble proportion of that which side of like proportion is to side of lyke proportion.

Demonstration of the first part of this proposition.

Vppose that there be two like solide numbers A and B. And let the sides of the number A, be the numbers C, D, E. And let the sides of the number B, be the numbers F, G, H. And forasmuch as (by the 22. definition of the seuenth) lyke solide numbers haue their sides proportionall, therfore as C is to D, so is F to G, and as D is to E, so is G to H. Then I say that betwene A and B, there are two meane proportionall numbers. And that A is vnto B in treble proportion of that which C is to F, or of that which D is to G, or moreouer of that which E is vnto H.

For let C multiplieng D produce K. And let F multiplieng G, produce L. And forasmuch as C, D, are in the self same proportiõ that F, G, are, & of C & D is produced K, and of F and G is produced L, therefore K and L are like plaine numbers. And therfore betwene those numbers K and L, there is one meane proportionall number (by the 18. of the seuenth) Let the same be M. Wherfore M is produced of D and F, as it is manifest by the proposition goyng before. Wherfore as K is to M, so is M to L. And forasmuch as D multiplieng C produced K, and multiplieng F produced M: therfore (by the 17. of the seuenth) as C is to F, so is K to M, but as K is to M, so is M to L. Wherfore these numbers K M, L, are in continuall proportion and in the same proportion that C is to D. And for that as C is to D, so is F to G, therfore alternately (by the 13. of the seuenth) as C is to F, so is D to G. Againe, for that as D is to E, so is G to H, therfore alternately also as D is to G, so is E to H. Wherfore these numbers K, M, L, are in continuall proportion, and in the same proportion that C is to F, and that D is to G, and moreouer that E is to H. Now let E and H multiplieng M produce N and X. And forasmuch as A is a solide number, and the sides thereof are C, D, E, therefore E multiplieng that which is produced of C and D, produceth A. But that which is produced of C and D is K. Wherfore E multiplieng K produceth A. And (by the same reason H multiplieng that which is produced of F and G, that is multiplieng L produceth

A .....
N .....
X .....
B .....
C ..
D ..
E ..
F ...
G ...
H ...
K ....
M .....
L .....

duceth B. And forasmuch as E multiplieng K produced A, and multiplieng M produced N, therfore (by the 17. of the seuēth), as K is to M, so is A to N. But as K is to M, so is C to F, & D to G, and moreouer E to H, therfore as C is to F, and D to G, and E to H, so is A to N. Agayne, forasmuch as E multiplieng M produced N, and H multiplieng M, produced X, therfore (by the 18. of the seuenth) as E is to H, so is N to X. But as E is to H, so is C to F, and D to G. Wherfore as C is to F, and D to G, and E to H, so is A to N, and N to X. Againe forasmuch as H multiplieng M, produced X, and multiplieng L produced B, therefore (by the 17. of the seuenth) as M is to L, so is X to B. But as M is to L, so is C to F, and D to G, and E to H therfore as C is to F, and D to G, and E to H, so is not onely X to B, but also A to N, and N to X. Wherfore these numbers A, N, X, B, are in continuall proportion, and that in the proportions of the sides. I say moreouer that A is vnto B in treble proportion of that, which side of like proportion, is to side of like proportion, that is, of that which the number C hath to the number F, or of that which D hath to G, or moreouer of that which E hath to H. For forasmuch as there are foure numbers in continual proportion, that is, A, N, X, B, therfore (by the 10. definition of the fift) A is vnto B in treble proportion of that which A is vnto N. But as A is to N, so is it proued that C is to F, and D to G, and moreouer E to H. Wherefore A is vnto B in treble proportion of that which side of like proportion is vnto side of like proportion, that is of that which the number C is to the number F, and of that which D is to G, and moreouer of that which E is to H: Which was required to be proued.

The second part.

¶ The 18. Theoreme. The 20. Proposition.

If betwene two numbers there be one meane proportionall number: those numbers are like plaine numbers.

SVppose that betwene the two numbers A and B there be one meane proportionall number, and let the same be C. Then I say, that those numbers A and B are like plaine numbers. Take (by the 35. of the seuenth) two of the least numbers that haue one & the same proportion with A, C, B: and let the same be the numbers D, E. Wherefore as D is to E, so is A to C, but as A is to C, so is C to B, wherefore as D is to E, so is C to B. Wherefore how many times D measureth A, so many times doth E measure C. How many times D measureth A, so many vnities let there be in F. Wherefore F multiplying D produceth A, and multiplying E it produceth C: wherefore A is a plaine number: and the sides therof are D and F (by the 17. definition of the seuenth). Againe forasmuch as D and E are the lest numbers that haue one & the same proportion with C, B, therefore (by the 21. of the seuenth) how many times D measureth C, so many times doth E measure B. How often E measureth B, so many vnities let there be in G. Wherefore E measureth B by those vnities which are in G: wherefore G multiplying E produceth B: wherefore B is a plaine number (by the 17. definition of the seuenth). And the sides thereof are E and G. Wherefore those two numbers A and B are two plaine numbers. I say moreouer that they are like. For forasmuch as F multiplying E produced C. and G multiplying E produced B: therefore (by the 17. of the seuenth) as F is to G, so is C to B, but as C is to B, so is D to E, wherefore as D is to E, so is F to G. Wherefore A and B are like plaine numbers, for their sides are proportionall: which was required to be proued.

This proposition is the conuerse of the 18. proposition.

Construction.

A .....
C .....
B .....
D ...
E ....
F .....
G .....

Demonstration.

¶ The 19. Theoreme. The 21. Proposition.

If betwene two numbers, there be two meane proportionall numbers, those numbers are like solide numbers.

This proposition is the conuerse of the 19. proposition.

Construction.

Suppose that betwene two numbers A and B, there be two meane proportionall numbers C, D. Then I say that A and B are like solide numbers. Take (by the 35 of the seuenth, or 2. of the eight) three of the least numbers that haue one and the same proportion with A, C, D, B, and let the same be E, F, G. Wherefore (by the 3. of the eight) their extremes E, G are prime the one to the other. And forasmuch as betwene the numbers E and G there is one meane proportionall number: therfore (by the 20 of the eight) they are like plaine numbers. Suppose that the sides of E, be H and K. And let the sides of G, be L and M. Now it is manifest that these numbers E, F, G, are in continuall

A .....
C .....
D .....
B .....

E .....
F .....
G .....

H ...
K ...
N ...

L ....
M ....
X ....

Demonstration.

proportion, and in the same proportion that H is to L, and that K is to M. And forasmuch as E, F, G are the least numbers that haue one and the same proportion with A, C, D, therefore of equalitie (by the 14. of the seuenth) as E is to G, so is A to D. But E, G, are (by the 3. of the eight) prime numbers, yea they are prime and the least, but the least numbers (by the 21. of the seuenth) measure those numbers that haue one & the same proportion with them equally, the greater the greater, and the lesse the lesse, that is, the antecedent the antecedent, & the consequent the consequent: therfore how many times E measureth A, so many times G measureth D. How many times E measureth A, so many vnities let there be in N. Wherefore N multiplieng E, produceth A. But E is produced of the numbers H, K. Wherfore N multiplieng that which is produced of H, K, produceth A. Wherefore A is a solide number, and the sides therof are H, K, N. Agayne, forasmuch as E, F, G, are the least numbers that haue one and the same proportion with C, D, B, therefore how many times E measureth C, so many times G measureth B. How oftetimes G measureth B, so many vnities let there be in X. Wherfore G measureth B by those vnities which are in X. Wherfore X multiplieng G produceth B. But G is produced of the numbers L, M. Wherefore X multiplieng that number which is produced of L and M, produceth B. Wherfore B is a solide number, and the sides therof are L, M X. Wherfore A, B are solide numbers. I say moreouer that they are like solide numbers. For forasmuch as N and X multiplieng E produced A and C: therfore by the 18. of the seuenth,

as

as N is to X, ſo is A to C, that is E to F. But as E is to F, ſo is H to L, and K to M: therefore as H is to L, ſo is K to M, and N to X. And H, K, N, are the ſides of A, and likewiſe L, M, X, are the ſides of B: wherfore A, B, are like ſolide numbers: which was required to be proued.

¶ The 20. Theoreme. The 22. Propoſition.

If three numbers be in continuall proportion, and if the firſt be a ſquare number, the third alſo ſhall be a ſquare number.

Vppoſe that there be three numbers in continuall proportion A, B, C, and let the firſt be a ſquare number. Then I ſay that the third is alſo a ſquare number. For foraſmuch as betwene A and C there is one meane proportionall number, namely B, therefore (by the 20. of the eight) A and C are like playne numbers. But A is a ſquare number. Wherefore C alſo is a ſquare number: which was required to be proued.

A .....
B .....
C .....

Demonſtration.

¶ The 21. Theoreme. The 23. Propoſition.

If foure numbers be in continuall proportion, and if the firſt be a cube nũber, the fourth alſo ſhall be a cube number.

Vppoſe that there be foure numbers in continuall proportion A, B, C, D. And let A be a cube number. Thẽ I ſay that D alſo is a cube number. For foraſmuch as betwene A and D there are two meane proportionall numbers B, C. Therfore A, D are like ſolide numbers (by the 21. of this booke) But A is a cube number, wherfore D alſo is a cube number: which was required to be demonſtrated.

A .....
B .....
C .....
D .....

Demonſtration.

¶ The 22. Theoreme. The 24. Propoſition.

If two numbers be in the ſame proportiõ that a ſquare number is to a ſquare number, and if the firſt be a ſquare number, the ſecond alſo ſhall be a ſquare number.

Vppoſe that two numbers A and B be in the ſame proportion, that the ſquare number C is vnto the ſquare nũber D. And let A be a ſquare nũber. Then I ſay that B alſo is a ſquare number. For foraſmuch as C and D are ſquare numbers. Therfore C and D are like plaine numbers. Wherfore (by the 18. of the eight) betwene C and D there is one meane proportionall number. But as C is to D, ſo is A to B. Wherfore betwene A and B there is one meane proportionall number (by the 8. of the eight) But A is a ſquare number. Wherfore (by the 22. of the eight) B alſo is a ſquare number which was required to be proued.

A ....
.....
B .....
C .....
.....
D .....

Demonſtration.

¶ The 23. Theoreme. The 25. Propoſition.

If two numbers be in the ſame proportion the one to the other, that a cube number is to a cube number, and if the firſt be a cube number, the ſecond alſo ſhall be a cube number.

Demonſtration.

Vppoſe that two numbers A and B be in the ſame proportiõ the one to the other, that the cube nũber C is vnto the cube number D. And let A be a cube number. Then I ſay that B alſo is a cube nũber. For foraſmuch as C, D, are cube nũbers, therfore C, D are like ſolide numbers, wherfore (by the 19. of the eight) betwene

A .....
E .....
F .....
B .....

C .....
—96
—144
D 216

C and D there are two proportionall numbers. But how many numbers fall in continual proportion betwene C and D, ſo many (by the 8. of the eight) fal there betwene the numbers that haue the ſame proportion with them. Wherefore betwene A and B there are two meane proportionall numbers which let be E and F. And foraſmuch as there are foure numbers in continuall proportion, namely, ◡A, E, F, B, and ◡A is a cube number, therefore (by the 23. of the eight) B alſo is a cube number: which was required to be demonſtrated.

❊ A Corollary added by Fluſſates.

A Corollary added by Fluſſates.

Betwene a ſquare number and a number that is not a ſquare number, falleth not the proportion of one ſquare number to an other. For if the firſt be a ſquare number, the ſecond alſo ſhould be a ſquare number which is contrary to the ſuppoſition. Likewiſe betwene a cube number, and a number that is no cube number falleth not the proportion of one cube number to an other. For if the firſt be a cube number, the ſecond alſo ſhould be a cube number, which is contrary to the ſuppoſition, & therfore impoſſible.

¶ The 24. Theoreme. The 26. Propoſition.

Like playne numbers, are in the ſame proportion the one to the other, that a ſquare number is to a ſquare number.

Conſtruction.

Vppoſe that A and B be like plaine numbers. Then I ſay that A is vnto B in the ſame proportiõ that a ſquare number is to a ſquare number. For foraſmuch as A, B, are like plaine numbers, therefore betwene A and B there falleth one meane proportional nũber (by the 18. of the eight). Let there fal ſuch a number, and let the ſame be C. And (by the 35. of the ſeuenth) take the three leaſt numbers that haue one and the ſame proportion with A, C, B, and let the ſame be

A .....
C .....
B .....

D .....
E .....
F .....

D, E, F,

D,E,F: wherefore (by the corollary of the 2. of the eight) their extremes, that is D, F, are ſquare numbers. And for that as D is to F, ſo is A to B, (by the 14. of the ſeuẽth): and D, F, ar ſquare numbers. Therfore A is vnto B in that proportion, that a ſquare nũber is vnto a ſquare number: which was required to be proued.

The 25. Theoreme. The 27. Propoſition.

Like ſolide numbers are in the ſame proportion the one to the other, that a cube number is to a cube number.

VPpoſe that A and B be like ſolide numbers. Then I ſay that A is vnto B, in the ſame proportion, that a cube numbe is to to a cube number. For foraſmuch as A, B, are like ſolide numbers. Therefore (by the 19. of the eight) betwene A and B there Conſtruction.

A .....
C .....
D .....
B .....

E .....
F .....
G .....
H .....

fal two meane proportionall numbers. Let there fall two ſuch numbers, and let the ſame be C and D. And take (by the 35. of the ſeuenth) the leaſt numbers that haue one and the ſame proportion with A, C, D, B, and equall alſo with thẽ in multitude, and let the ſame be E, F, G, H. Wherfore (by the corollary of the 2. of the eight) their extreames, that is, EH, are cube numbers. But as E is to H, ſo is A to B. Wherefore A is vnto B in the ſame proportion, that a cule number is to a cube number: which was required to be demonſtrated. Demonſtration.

¶ A Corollary added by *Fluſates.*

If two mmbers be in the ſame proportion the one to the other that a ſquare number is to a ſquare number: thoſe two numbers ſhall be like ſuperficiall numbers. And if they be in the ſame proportion the one to the other that a cube number is to a cube number, they ſhall be like ſolide nũbers. A Corollary added by Fluſates.

Firſt let the number A haue vnto the number B the ſame proportion, that the ſquare number C hath to the ſquare number D. Then I ſay, that A and B are like ſuperficiall nũbers. For foraſmuch as betwene the ſquare numbers C and D there falleth a meane proportionall (by the 11. of this booke) there ſhall

E ..... D .....
A ..... C .....

alſo betwene A and B (which haue one and the ſame proportion with C and D) fall a meane proportionall (by the 8. of this booke). Wherefore A and B are like ſuperficiall nũbers (by the 20. of this booke).

But if A be vnto B, as the cube number C, is to the cube number D. Then are A & B like ſolide number. For foraſmuch as C and D are cube numbers, there falleth betwene them two meane proportio-

A .....
E ..... C .....
D .....

nall numbers (by the 12. of this booke). And therefore (by the 8. of the ſame) betwene A and B (which are in the ſame proportion that C is to D) there falleth alſo two meane proportionall numbers. Wherfore (by the 21. of this booke) A and B are like ſolide numbers.

* An

❧ An other Corollary added alſo by *Fluſſates*.

Another Corollary added by Fluſſates.

If a number multiplying a ſquare number, produce not a ſquare number: the ſayd number multiplying ſhall be no ſquare number. For if it ſhould be a ſquare number, then ſhould it and the number multiplied being like ſuperficiall numbers (by reaſon they are ſquare numbers) haue a meane proportionall (by the 18. of thys booke). And the number produced of the ſayd meane ſhould be equall to the number contayned vnder the extremes, which are ſquare numbers (by the 20. of the ſeuenth). Wherefore the number produced of the extremes being equall to the ſquare number produced of the meane, ſhould be a ſquare number. But the ſayd number by ſuppoſition, is no ſquare number. Wherefore neither is the number multiplying the ſquare number, a ſquare number.

The firſt part of the firſt Corollary is the conuerſe of the 26. Propoſition of this booke, and hath ſome vſe in the tenth booke. The ſecond part of the ſame alſo is the conuerſe of the 27. Propoſition of the ſame.

❧ The end of the eighth booke of Euclides Elementes.

¶ The ninth booke of Euclides Elementes.

IN THIS NINTH BOOKE *Euclide* continueth his purpose touching numbers: partly prosecuting thynges more fully, which were before somewhat spoken of, as of square and cube numbers: and partly setting out the natures and proprieties of such kindes of number, as haue not yet bene entreated of: which yet are most necessary to be knowen. As are numbers euen, and odde: whose passions and conditions are in this booke largely taught, with their compositions, and subductions of the one from the other: with many other generall and speciall thinges to be noted, worthy the knowledge.

The Argument of the ninth booke.

¶ The 1. Theoreme. The 1. Proposition.

If two like plaine numbers multiplying the one the other produce any number: the number of them produced shall be a square number.

Vppose that A and B be two like plaine numbers. And let A multiplying B produce the number C. Then I say, that C is a square number. For let A multiplying him selfe produce D. Wherefore D is a square number. And forasmuch as A multiplying him selfe produced D, and multiplying B produced C, therefore (by the 17. of the seuenth) as A is to B, so is D to C. And forasmuch as A, B, are like plaine numbers, therefore (by the 18. of the eight) betwene A and B there falleth a meane proportionall number. But if betwene two numbers fall numbers in continuall proportion, how many numbers fall betwene them, so many also (by the 8. of the eight) shall there fall betwene the numbers that haue the same proportion with them. Wherfore betwene C and D also there falleth a meane proportionall number. But D is a square number. Wherfore (by the 22. of the eight) C also is a square number: which was required to be proued.

Demonstration.

A ....

.....

B .....

D .....

.....

C .....

¶ The 2. Theoreme. The 2. Proposition.

If two numbers multiplying the one the other produce a square number: those numbers are like plaine numbers.

Suppose

This proposition is the converse of the former.

SVppose that two numbers A and B multiplying the one the other do produce C a square number. Then I say, that A and B are like plaine numbers. For let A multiplying him selfe produce D. Wherefore D is a square number. And forasmuch as A multiplying him selfe produced D, and multiplying B produced C, therefore (by the 17. of the seuenth) as A is to B, so is D to C. And forasmuch as D is a square number, and so likewise is C, therefore D and C are like plaine numbers. Wherefore betwene D and C there is (by the 18. of the eight) one meane proportionall number. But as D is to C, so is A to B. Wherefore (by the 8. of the eight) betwene A and B there is one meane proportionall number. But if betwene two numbers there be one meane proportionall number, those numbers are (by the 20. of the eight) like plaine numbers. Wherefore A and B are like plaine numbers: which was required to be proued.

A .....
.....
B .....
D .....
.....
C .....

Demonstration.

❋ A Corollary added by Campane.

A Corollary added by Campane.

Hereby it is manifest, that two square numbers multiplyed the one into the other do alwayes produce a square number. For they are like superficiall numbers, and therefore the number produced of them, is (by the first of this booke) a square number. *But a square number multiplyed into a number not square, produceth a number not square.* For if they should produce a square number, they should be like superficiall numbers (by this Proposition). But they are not. Wherefore they produce a number not square. *But if a square number multiplyed into an other number produce a square number, that other number shall be a square number.* For by this Proposition that other number is like vnto the square number which multiplyeth it, and therefore is a square number. *But if a square number multiplyed into an other number produce a number not square, neither shall that other number also be a square number.* For if it should be a square number, then being multiplyed into the square number it should produce a square number, by the first part of this Corollary.

The 3. Theoreme. The 3. Proposition.

If a cube number multiplying himselfe produce a number, the number produced shall be a cube number.

SVppose that A being a cube number multiplieng himselfe, do produce the number B. Then I say that B is a cube number. Take the side of A, and let the same be the number C, and let C multiplieng himselfe produce the number D. Now it is manifest that C multiplieng D produceth A (by the 20. definition of the seuēth)

B .....
.....
.....
A .....
D ....
C ..
Vnitie.

Demonstration.

And forasmuch as C multiplieng himselfe produced D, therfore C measureth D by those vnities

nities which are in C. But vnitie also measureth C by those vnities which are in C. Wherfore as vnitie is to C, so is C to D. Againe forasmuch as C multiplieng D produceth A: therefore D measureth A by those vnities which are in C. But vnitie measureth C by those vnities which are in C: wherefore as vnitie is to C, so is D to A. But as vnitie is to C, so is C to D, wherfore as vnitie is to C, so is C to D & D to A. Wherefore betwene vnitie & A there are two meane proportionall numbers, namely, C, D. Againe forasmuch as A multiplieng himselfe produced B, therefore A measureth B by those vnities which are in A. But vnitie also measureth A by those vnities which are in A. Wherfore as vnitie is to A, so is A to B. But betwene A and vnitie, there are two meane proportionall numbers. Wherfore betwene A and B also there are two meane proportionall numbers by the 8. of the eight. But if betwene two numbers, there be two meane proportionall numbers, and if the first be a cube number, the fourth also shall be a cube number by the 21. of the eight. But A is a cube number, wherefore B also is a cube number: which was required to be proued.

¶ The 4. Theoreme. The 4. Proposition.

If a cube number multiplieng a cube number, produce any number, the number produced shall be a cube number.

Demonstration.

Supposse that the cube number A multiplieng the cube number B, do produce the nũber C. Then I say that C is a cube number. For let A multiplieng himselfe produce D. Wherefore D is a cube number (by the proposition going before). And forasmuch as A multiplieng himselfe produced D, and multiplieng B, it produced C: therfore (by the 17. of the seuenth) as A is to B, so is D to C. And forasmuch as A and B are cube numbers, therfore A and B are like solide numbers. Wherfore betwene A and B (by the 19. of the eight) there are two meane proportionall numbers. Wherefore also (by the 8. of the same) betwene D and C there are two meane proportionall numbers. But D is a cube number. Wherfore C also is a cube number (by the 23. of the eight) which was required to be demonstrated.

A ..... D 64
..... 96
..... 144
B ..... C 216

¶ The 5. Theoreme. The 5. Proposition.

If a cube number multiplying any number produce a cube nũber: the number multiplyed is a cube number.

Demonstration.

Supposse that the cube number A, multiplying the number B, do produce a cube number, namely, C. Then I say, that B is a cube number. For let A multiplying himselfe produce D. Wherefore (by the 3. of the ninth) D is a cube nũber. And forasmuch as A multiplying himselfe produced D, and multiplying B, it produced C: therefore (by the 17. of the seuenth) as A is to B, so is D to C. And forasmuch as D

A .....
B .....

D 729
C 1728

and C are cube numbers, they are also like solide nũbers. Wherefore (by the 19. of the eight) betwene D and C there are two meane proportionall numbers. But as D is to C, so is A to B.

Wherefore (by the 8. of the eight) betwene A and B there are two meane proportionall numbers. But A is a cube number. Wherefore B also is a cube number (by the 23. of the eight): which was required to be proued.

¶ A Corollary added by *Campane.*

A Corollary added by Campane.

Hereby it is manifest, that if a cube number multiply a number not cube, it shall produce a number not cube. For if it should produce a cube number, then the number multiplyed should also be a cube number (by this Proposition) which is contrary to the supposition. For it is supposed to be no cube number. *And if a cube number multiplying a number produce a number not cube, the number multiplyed shall be no cube number.* For if the number multiplied should be a cube number, the number produced should also be a cube number (by the 4. of this booke): which is contrary to the supposition, and impossible.

¶ *The 6. Theoreme.* *The 6. Proposition.*

If a number multiplieng himselfe produce a cube number: then is that number also a cube number.

Demonstration.

Svppose that the number A multiplieng himself, do produce B a cube nũber. Then I say that A also is a cube number. For let A multiplieng B produce C. And forasmuch as A multiplieng himselfe produced B, & multiplieng B it produced C: therfore C is a cube number. And for that A multiplieng himselfe produced B, and multiplieng B it produced C, therfore (by the 17. of the seuenth) as A is to B, so is B to C. And for that B and C are cube numbers, they are also like solide numbers. Wherfore (by the 19. of the eight) betwene C and B there are two meane proportional numbers. But as B is to C, so is A to B: wherfore (by the 8. of the eight) betwene A and B there are two meane proportional nũbers. But B is a cube number. Wherefore A also is a cube number by the 23. of the eight: which was required to be demonstrated.

A .....
B 729
C 19683

¶ *The 7. Theoreme.* *The 7. Proposition.*

If a composed nnmber multiplieng any number, produce a number: the nũber produced shall be a solide number.

Svppose that the composed number A multiplieng the nũber B, do produce the number C. Then I say that C is a solide number. For forasmuch as A is a composed nũber, therfore some number measureth it (by the 14. definition). Let D measure it.

D ... A ..... C .....
E .. B .....

Demonstration.

And how often D measureth A, so many vnities let there be in E. Wherefore E multiplieng D produceth A. And forasmuch as two numbers D and E, multiplieng themselues, produce A, which A againe multiplieng B produceth C: therfore C produced of three numbers multiplieng the one the other, namely, D, E, and B is (by the 18. definition of the seuenth) a solide number. And the sides therof are the numbers D, E, B. If therefore a composed number &c, which was required to be proued.

¶ *The 8. Theoreme.* *The 8. Proposition.*

If from vnitie there be numbers in continuall proportion how many soeuer: the third number from vnitie is a square number, and so are all forwarde leauing one betwene. And the fourth number is a cube number, and so are all forward leauing two betwene. And the seuenth is both a cube number

and

and also a square number, and so are all forward leauing fiue betwene.

SVppose that from vnitie there be these numbers in continuall proportion A, B, C, D, E, F. Then I say that the third number from vnitie, namely, B is a square number, and so are all forward leauing one betwene, namely, D and F. And that C the fourth number is a cube number, and so are all forwarde leauyng two betwene. And that F the seuenth number is both a cube number and also a square number, and so are all forward leauing fiue betwene. For for that as vnitie is to A, so is A to B. Therefore how many times vnitie measureth A, so many times A measureth B. But vnitie measureth A by those vnities which are in A, wherefore A measureth B by those vnities which are in A. And forasmuch as A measureth B by those vnities which are in A. Therfore A multiplieng himselfe produceth B. Wherfore B is a square number. And forasmuch as these numbers B, C, D, are in continuall proportion, and B is a square number, therfore by the 22. of the eight, D also is a square number. And by the same reason also F is a square number. And in like sort may we proue that leauing alwayes one betwene, all the rest forward are square numbers.

Demonstration of the first part.

F 729
E 243
D .....
C .....
B .....
A ...
Vnitie

Now also I say that the fourth number from vnitie, that is, C, is a cube number, and so are all forward leauing two betwene. For for that as vnitie is to the number A, so is B to C, therefore how many times vnitie measureth the number A, so many times B measureth C. But vnitie measureth A by those vnities which are in A, wherfore B measureth C by those vnities which are in A. Wherfore A multiplieng B produceth C. And forasmuch as A multiplieng himselfe produced B, and multiplieng B it produced C, therefore C is a cube number. And forasmuch as C, D, E, F, are in continuall proportion. But C is a cube number, therefore (by the 23. of the eight) F also is a cube number.

The second part demonstrated.

And it is proued, that F being the seuenth number from vnitie is also a square number. Wherfore F is both a cube number, and also a square number. In like sort may we proue, that leauing alwaies fiue betwene, all the rest forwarde, are numbers both cube and also square: which was required to be proued.

Demonstratiõ of the third part.

¶ The 9. Theoreme. The 9. Proposition.

If from vnitie be numbers in continuall proportion how many soeuer: and if that number which followeth next after vnitie be a square number, then all the rest following also be square numbers. And if that number which followeth next after vnitie be a cube number, then all the rest following shall be cube numbers.

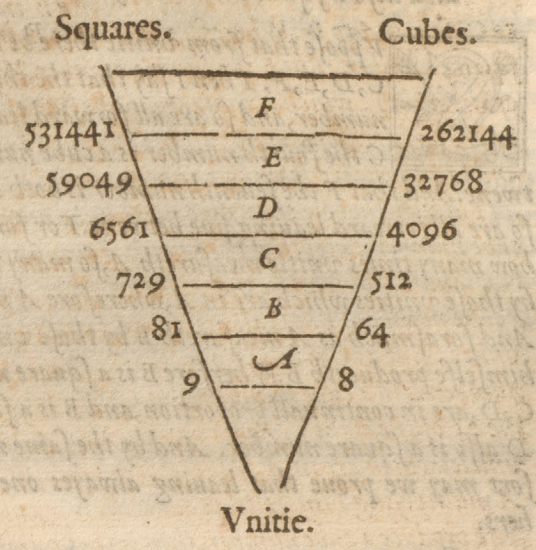
SVppose that from vnitie there be these numbers in continuall proportion A, B, C, D, E, F. And let A which followeth next vnto vnitie be a square number. Then I say, that all the rest following also are square numbers. That the third number, namely, B, is a square number, & so all forward leauing one betwene, it is plaine by the Proposition next going before. I say also that all the rest are square numbers. For, forasmuch as A, B, C, are in continuall proportion, and A is a square number, therfore (by the 22. of the eight) C also is a square number. Againe forasmuch as B, C, D, are in con-

Demonstration of the first part of this proposition.

tinuall proportion, and B is a ſquare number, therfore D alſo (by the 22. of the eight) is a ſquare number. In like ſort may we proue, that all the reſt are ſquare numbers.

The ſecond part demonſtrated.

But now ſuppoſe that A be a cube number. Then I ſay, that all the reſt following are cube numbers. That the fourth from vnitie, that is, C is a cube number, and ſo all forward leauing two betwene, it is plaine (by the Propoſition going before). Now I ſay, that all the reſt alſo are cube numbers. For, for that as vnitie is to A, ſo is A to B: therefore how many times vnitie meaſureth A, ſo many times A meaſureth B. But vnitie meaſureth A by thoſe vnities which are in A. Wherefore A alſo meaſureth B by thoſe vnities which are in A. Wherefore A multiplying him ſelfe produceth B. But A is a cube number. But if a cube number multiplying him ſelfe produce any number, the number produced, is (by the 3. of the ninth) a cube number. Wherefore B is a cube number. And foraſmuch as there are foure numbers in continuall proportion A, B, C, D, and A is a cube number, therefore D alſo (by the 23. of the eight) is a cube number. And by the ſame reaſon E alſo is a cube number, and in like ſort are all the reſt following: which was required to be proued.



¶ The 10. Theoreme. The 10. Propoſition.

If from vnitie be numbers in continuall proportion how many ſoeuer, and if that number which followeth next after vnitie be not a ſquare number, then is none of the reſt following a ſquare number, excepting the third from vnitie, and ſo all forward leauing one betwene. And if that number which followeth next after vnitie be not a cube number, neither is any of the reſt following a cube number, excepting the fourth from vnitie, and ſo all forward leauing two betwene.

Demonſtration of the firſt part leauing to an abſurditie.

Vppoſe that from vnitie be theſe numbers in continuall proportion A, B, C, D, E, F. And let A which followeth next after vnitie be no ſquare number. Then I ſay, that neither is any of the reſt a ſquare number, excepting the third from vnitie, & ſo all forward leauing one betwene, namely, B, D, F, which are ſquare numbers (by the 8. of this booke). For if it be poſsible, let C be a ſquare number. But B alſo is a ſquare number. Wherfore B is vnto C in that proportion that a ſquare number is to a ſquare number. But as B is

Vnitie .
A ...
B .....
C .....
D .....
E 243
F 729

to C,

to C, so is A to B. Wherefore A is vnto B in that proportion that a square number is to a square number. But B is a square number. Wherefore A also is a square number (by the 24. of the eight): which is contrary to the supposition. Wherefore C is not a square number. And by the same reason none of all the other is a square number, excepting the third from vnitie, and so all forward leauing one betwene.

But now suppose that A be not a cube number. Then I say, that none of all the rest is a cube number, excepting the fourth from vnitie, & so all forward leauing two betwene, namely, C, and F, which (by the 8. of this booke) are cube numbers. For if it be possible, let D be a cube number. But C also is a cube number (by the 8. of the ninth). For it is the fourth from vnitie. But as C is to D, so is B to C. Wherefore B is vnto C, in that proportion that a cube number is to a cube number. But C is a cube number. Wherefore B also is a cube number (by the 25. of the eight). And as vnitie is to A, so is A to B. But vnitie measureth A by those vnities which are in A. Wherefore A measureth B by those vnities which are in A. Wherfore A multiplying him selfe produceth B a cube number. But if a number multiplying him selfe produce a cube number, then is that number also a cube number (by the 6. of the ninth). Wherefore A is a cube number: which is contrary to the supposition. Wherefore D is not a cube number. In like sort may we proue, that neither is any of the rest a cube number, excepting the fourth from vnitie, and so all forward leauing two betwene: which was required to be proued.

Demonstration of the second part leading also to an absurditie.

¶ The 11. Theoreme. The 11. Proposition.

If from vnitie be numbers in continuall proportion how many soeuer, the lesse measureth the greater by some one of them which are before in the said proportionall numbers.

Suppose that from vnitie A be these numbers in continuall proportion B, C, D, E. Then I say that of these numbers B, C, D, E: B being the lesse, measureth E the greater by one of these numbers C or D. For for that as vnitie A is vnto the number B, so is D to E, therfore how many times vnitie A measureth the number B, so many times D measureth E: wherefore alternately (by the 15. of the seuenth) how many times vnitie A measureth the number D, so many times B measureth E. But vnitie A measureth D by those vnities which are in D. Wherefore B also measureth E by those vnities which are in D. Wherefore B the lesse, measureth E the greater by some one of the numbers which went before E in the proportionall numbers. And so likewise may we proue that B measureth D by some one of the numbers B, C, D, namely, by C. And so of the rest. If therfore from vnitie &c. Which was required to be proued.

Demonstration.

E .....
D .....
C ....
B ..
A.

¶ The 12. Theoreme. The 12. Proposition.

If from vnitie be numbers in continuall proportion how many soeuer, how many prime numbers measure the least, so many also shal measure the number which followeth next after vnitie.

Demonstration leading to an absurditie.

Svppose that from vnitie be these numbers in continuall proportion A,B,C,D. Thē I say that how many prime nūbers measure D, so many also do measure A. Suppose that some prime number namely, E, do measure D. Thē I say that E also measureth A, which is next vnto vnitie. For if E do not measure A, and E is a prime number, but euery number is to euery number which it measureth not a prime number (by the 31. of the seuenth). Wherefore A and E are prime numbers the one to the other. And forasmuch as E measureth D, let it measure D by the number F. Wherefore E multiplieng F produceth D. Againe forasmuch as A measureth D by those vnities which are in C, therefore A multiplieng C produceth D. But E also multiplieng F produced D, wherfore that which is produced of the numbers A,C is equall to that which is produced of the numbers E,F. Wherfore as A is to E, so is F to C. But A,E, are prime numbers, yea they are prime and the least. But the left numbers measure the numbers that haue one and the same proportion with them equally by the 21. of the seuenth, namely, the antecedent the antecedent, and the consequent the consequent. Wherfore E measureth C. Let it measure it by G. Wherefore E multiplieng G produceth C. But A also multiplieng B produceth C. Wherfore that which is produced of the num-

Vnitie.
A....
B.....
C.....
L 256
E..
Il.....
G.....
F 128

bers A,B, is equall to that which is produced of the numbers E,G. Wherfore as A is to E, so is G to B. But A,E are prime numbers, yea they are prime and the least. But the least numbers (by the 21. of the seuenth) measure the numbers that haue one and the same proportion with thē equally, namely, the antecedēt the antecedēt, & the cōsequēt the conseqēt. Wherfore E measureth B. Let it measure it by H. Wherefore E multiplieng H produceth B. But A also multiplieng himselfe produceth B, wherfore that which is produced of the numbers E,H, is equall to that which is produced of the number A. Wherfore as E is to A, so is A to H. But A E are prime nūbers, yea they are prime & the least, but the least numbers (by the 21. of the seuenth) measure the numbers that haue one and the same proportion with thē equally, namely, the antecedēt the antecedent, and the cōsequent the consequent. Wherfore E measureth A and it also doth not measure it by supposition, which is impossible. Wherfore A and E are not prime the one to the other, wherfore they are composed. But all composed numbers are measured of some prime number, wherfore A and E are measured by some prime number. And forasmuch as E is supposed to be a prime number. But a prime number is not (by the definition) measured by any other number but of himselfe. Wherfore E measureth A and E, wherfore E measureth A, and it also measureth D. Wherfore E measureth these numbers A and D. And in like sort may we proue that how many prime numbers measure D, so many also shall measure A: which was required to be proued.

✻ An other more briefe demonstration after *Flussates*.

An other demonstratiō after Flussates.

Suppose that from vnitie be nūbers in cōtinuall proportion how many so euer, namely, A,B,C,D. And let some prime nūber, namely, E measure the last nūber which is D. Thē I say that the same E measureth A which is the next number vnto vnitie. For if E doo not measure A, then are they prime, the one to the other by the 31. of the seuenth. And forasmuch as A,B,C,D, are proportionall from vnitie, ther-

therefore A multiplying himselfe produceth B. Wherfore B and E are prime numbers (by the 27. of the seuenth). And forasmuch as A multiplying B produceth C, therefore C is to E also a prime number by the 26. of the seuenth. And likewise infinitely A multiplying C produceth D: wherefore D and E are prime numbers the one to the other (by the same 26. of the seuenth). Wherefore E measureth not D as it was supposed, which is absurd, wherefore the prime number E measureth A, whiche is nexte vnto vnitie: which was required to be proued.

Vnitie		
A	4	3
B	16	9
C	64	27
D	256	81
E	2	3

¶ The 13. Theoreme. The 13. Proposition.

If from vnitie be numbers in continuall proportion how many soeuer, and if that which followeth next after vnitie be a prime number: then shall no other number measure the greatest number, but those onely which are before in the sayd proportionall numbers.

Suppose that from vnitie be these numbers in continuall proportion A,B,C,D, and let that which followeth next after vnitie, that is, A, be a prime number. Then I say, that no other number besides these numbers A,B,C, measureth the greatest number of them which is D. For if it be possible, let E measure D. And let E be none of these numbers A,B,C,D. Now it is manifest that E is not a prime number. For if E be a prime number, & do also measure D, it shall likewise measure A being a prime number and not being one and the same with A, by the former Proposition: which is impossible. Wherefore E is not a prime number. Wherefore it is a composed number. But euery composed number (by the 33. of the seuenth) is measured by some prime number.

Demonstration leading to an absurditie.

D .....
C .....
B .....
A ...
Vnitie.

E ———
H ———
C ———
F ———

Now I say, that no other prime nũber besides A shall measure E. For if any other prime nũber do measure E, & E measureth D, therfore that number also shall measure D (by the 5. common sentence of the seuenth). Wherfore it shal also measure A (by the proposition next going before) being a prime number and not being one and the same with A: which is impossible. Wherefore onely the prime number A measureth E which measureth the greatest number D.

And forasmuch as E measureth D, let it measure it by F. Now I say, that F is none of these numbers A,B,C. For if F be one and the same with any of these numbers A,B,C, and it measureth D by E, therefore one of these numbers A,B,C, measureth D by E. But one of these numbers A,B,C, measureth D by some one of these numbers A,B,C, therefore E is one and the same with one of these numbers A,B,C: which is contrary to the supposition. Wherfore F is not one and the same with any of these numbers A,B,C.

In like sort may we proue, that onely the prime number A measureth F, prouing first that

F is not a prime number: For if F be a prime number, and it measureth D, therefore it also measureth A being a prime number, and not being one and the same with A, by the former Proposition: which is impossible. Wherefore F is not a prime number: wherefore it is a composed number, and therefore some prime number shall measure it. Now I say, that no other prime number besides A shall measure it. For if any other prime number do measure F, and F measureth D, therefore that number shall measure D (by the 5. common sentence of the seuenth). Wherefore it shall also measure A (by the former Proposition) being a prime number and not being one and the same with A: which is impossible. Wherefore onely the prime number A measureth F. And forasmuch as E measureth D by F, therefore E multiplying F produceth D. But A also multiplying C produceth D, therefore that which is produced

D .....
C .....
B .....
A ...
Vnitie .

E _____
H _____
G _____
F _____

of A into C, is equall to that which is produced of E into F. Wherfore proportionally as A is to E, so is F to C. But A measureth E. Wherefore F measureth C. Let F measure C by G. And in like sort may we proue, that G is not one and the same with any of these numbers A, B, C, and that G is a composed number, and also that onely the prime number A measureth it. And forasmuch as F measureth C by G, therefore G multiplying F produced C. But A also multiplying B produced C. Wherefore that which is produced of A into B, is equall to that which is produced of F into G. Wherefore proportionally as A is to F, so is G to B. But A measureth F. Wherefore G also measureth B. Let G measure B by H. Now in like sort as before may we proue, that H is not one and the same with A, and that H is a composed number, and measured onely of the prime number A. And forasmuch as G measureth B by those vnities which are in H, therfore G multiplying H produced B. But A multiplying him selfe produced B. Wherfore that which is produced of H into G, is equall to the square number which is produced of A. Wherefore as H is to A, so is A to G. But A measureth G. Wherfore H measureth A being a prime number and not being one and the same with it: which is absurde. Wherfore no other number besides these numbers A, B, C, measureth the greatest number D: which was required to be demonstrated.

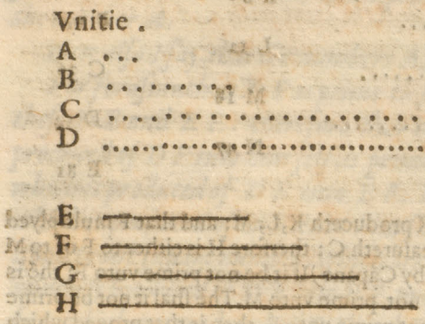
An other demonstration of the same after Campane.

An other demonstratiõ after Campane.

Suppose that E not being one and the same with the numbers A, B, C, D doo measure the number D. And let it measure it by the number F. And forasmuch as A being a prime number measureth the number D, which is produced of E into F: therefore by the 32. of the seuẽth, A measureth either E or F. Let it measure E. Now forasmuch as D is produced of A into C, and also of E into F: therefore by the second part of the 19. of the seuenth, A is to E, as F is to C. But A measureth E: wherefore F measureth C. Let it measure it by G. Wherefore by the 32. of the seuenth A shall measure either F or G. Let it measure F. Wherefore as before by the second part of the 19. of the seuenth G shall measure B. Let it measure it by H. Now then as before it followeth by the 32. of the seuenth that A shall measure either G or H: suppose that it measure G. Wherefore by the second part of the 20. of the seuenth H shall measure A (for of A into himselfe is produced B, and of H into G also is produced B) If therefore H be not

equall

equall vnto A, A ſhall be no prime number. Which is contrary to the ſuppoſition. But if it be equall vnto A, then euery one of theſe numbers G, F, E, ſhall be ſome one of the numbers A, B, C, D, by the 11.

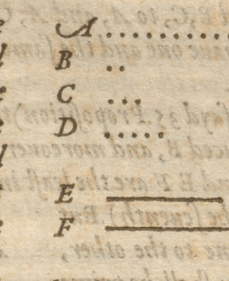


poſition of the ninth repeted as often as neede requireth . Wherefore E is not a number diuerſe from them, but is one and the ſame with ſome one of them: which is contrary to the ſuppoſition, wherfore that is manifeſt which was required to be proued.

¶ The 14. Theoreme. The 14. Propoſition.

If there be geuen the leaſt number, whom certayne prime numbers geuen, do meaſure: no other prime number ſhall meaſure that nũber, beſides thoſe prime numbers geuen.

Vppoſe that the leaſt number whom theſe prime numbers B, C, D, do meaſure, be A. Then I ſay that no other prime number beſides B, C, D, meaſureth A. For if it be poſſible, let E being a prime number meaſure A, and let E be none of theſe numbers B, C, D. And foraſmuch as E meaſureth A, let it meaſure it by F. Wherfore E multiplieng F produceth A. And theſe prime numbers B, C, D, meaſure A. But if two numbers multiplieng the one the other produce any number. And if ſome prime number meaſure that which is produced, it ſhall alſo meaſure one of thoſe numbers which were put at the beginning (by the 32. of the ſeuenth) Wherfore thoſe numbers B, C, D, meaſure one of theſe numbers E or F. But they meaſure not E, for E is a prime number, and is not one and the ſame with any one of theſe numbers B, C, D. Wherfore they meaſure F being leſſe then A which is impoſſible. For A is ſuppoſed to be the leaſt whom B, C, D, meaſure. Wherefore no prime number beſides B, C, D, meaſureth A: which was required to be demonſtrated.



Demonſtration leading to an abſurditie.

A propoſition added by Campane.

If there be numbers how many ſoeuer in continuall proportion being the leaſt in that proportion: a number meaſuring one of them, ſhall be a number not prime to one of the two leaſt numbers in that proportion.

Suppoſe that there be numbers in continuall proportion how many ſoeuer namely A, B, C, D, E which let be the leaſt that haue the ſame proportion with them: and let the two leaſt numbers in that proportion be F and G . And let ſome number as H meaſure ſome one of the numbers A, B, C, D, E, namely, C. Then I ſay that H is a number not prime either to F or G. Take (by the 2. of the eight) the three

A propoſition added by Campane.

three least numbers in the proportion of *A* to *B*: which let be P, Q, R. And afterward fower (by the same) which let be K,L,M,N: & so forward till you come to the multitude of the numbers geuen *A,B,C,D,E*. Now it is manifest (by the demonstration of the second of the eight) that F multiplyed by P,Q, and R produceth K,L, M: and that F multiplyed by K,L,M,N produceth *A,B,C,D*. And forasmuch as H measureth C: therfore H is either to F or to M not prime (by the corollary of the 32. of the seuenth added by Cãpane.) If it be not prime vnto F: thẽ is that manifest which was required to be proued. But if H be not prime vnto M. Thẽ shal it not be prime either to F or to R (by the same corollary). If agayne it be not prime vnto F, then is that proued which was required. But if it be not prime vnto R, then (by the same corollary) shall it be a number not prime vnto G (which produceth R by the 2. of the eight) but G is one of the two least numbers F or G which are in the proportion of the numbers geuen at the beginning *A*, B, C, D, E. If therefore there be numbers how many soeuer. &c. which was required to be proued.

H F.. G... P.... Q..... R..... K 8 L 12 M 18 N 47 *A* 16 *B* 24 C 36 D 54 E 81

¶ *The 15. Theoreme.* *The 15. Proposition.*

If three numbers in continuall proportion be the least of all numbers that haue one and the same proportion with them: euery two of them added together shall be prime to the third.

Construction.

SVppose that there be three numbers in continuall proportion A,B,C, being the lest of all numbers that haue one and the same proportion with them. Then I say, that euery two of these numbers A,B,C, added together, are prime to the third: namely, that A,B, is prime to C, and B,C, to A, and A, C, to B. Take (by the 35. of the seuenth) two of the least numbers that haue one and the same proportion with A,B,C, & let the same be the numbers D E, and E F.

Demonstration.

Now it is manifest (by the sayd 35. Proposition) that D E multiplying him selfe produced A, and multiplying E F produced B, and moreouer E F multiplying him selfe produced C.

And forasmuch as D E and E F are the least in that proportion, they are also prime the one to the other (by the 24. of the seuenth). But if two numbers be prime the one to the other, then both of thẽ added together, shall be prime to either of them (by the 30. of the seuenth). Wherefore the whole number D F is prime to either of these nũbers D E & E F. But D E also is prime vnto E F. Wherfore D F, & D E are prime vnto E F. Wherfore that which is produced of D F into D E, is (by the 26. of the seũeth) prime vnto E F. But if two nũbers be prime the one to the other, that which is produced of the one of thẽ into himselfe, is prime to the other (by the 27. of the seũeth). Wherfore that which is produced of D F into D E, is prime to that which is produced of E F into himselfe. But that which is produced of F D into D E, is the square nũber which is produced of D E into himselfe together with that which is produced of D E into E F (by the 3. of the second). Wherfore the square nũber which is produced of D E together with that which is produced of D E into E F, is prime to that which is produced of E F into himself. But that which is produced of D E into him selfe, is the number A, & that which is produced of D E into E F, is the number B: and that which is produced of E F into himselfe, is the number C. Wherefore the numbers A,B, added together are prime vnto C.

A ..... B ..... C ..... D ... E .... F

By the

By the like demonſtration alſo may we proue, that the numbers B,C, are prime vnto the number A.

B ..... C .....
A .....

Now alſo I ſay, that the numbers A,C, are prime vnto the number B.

For foraſmuch as D F is prime to either of theſe D E and E F: therefore that which is produced of D F into him ſelf, is prime to that which is produced of D E into E F. But that which is produced of D F into him ſelfe, is equall to the ſquare numbers which are produced of D E and E F together with that number which is produced of D E into E F, twiſe (by the 4. of the ſecond). Wherefore the ſquare nũbers which are produced of D E and E F together with that which is produced of D E into E F twiſe are prime to that which is produced of D E into E F. And by diuiſion alſo (by the 30. of the ſeuenth) the ſquare numbers produced of D E and E F, together with that which is produced of D E into E F once are prime to that which is produced of D E into E F. Againe (by the ſame 30. of the ſeuenth) the ſquare nũbers produced of D E and E F, are prime to that which is produced of D E into E F. But that which is produced of D E into him ſelfe is A, and that which is produced of E F into him ſelfe is C, and that which is produced of D E into E F, is B. Wherefore the numbers A,C, added together are prime vnto the number B: which was required to be demonſtrated.

A ..... C .....
B .....
D ... E ....F

This latter part of the demonſtration, which proueth that the numbers A & C are prime vnto B, is ſomewhat obſcurely put of *Theon*. And therefore I will here make it playner.

Demonſtration to proue that the numbers A and C are prime to B.

Foraſmuch as either of the numbers D E, and E F is prime to the whole D F: (as hath before bene proued) therefore that which is produced of D E into E F (which is the number B) is prime vnto D F, by the 26. of the ſeuenth. Wherefore by the 27. of the ſame that which is produced of D F into himſelfe (which is the number compoſed of A and C and of the double of B by the 4. of the ſecond) ſhall be prime vnto B. Wherefore it followeth that the number compoſed of A and C is prime vnto B. For if a number compoſed of two numbers, be prime to one of the ſaid two numbers, as here the number compoſed of A and C taken as one number and of the double of B, is prime vnto the double of B: then the two numbers whereof the number is compoſed, namely, the number compoſed of A and C, and the double of B ſhall be prime the one to the other (by the 30 of the ſeuenth). And therefore the number compoſed of A and C ſhall be prime to B taken once. For if any number ſhould meaſure the two numbers, namely the number compoſed of A and C, and the number B, it ſhould alſo meaſure the number compoſed of A and C, and the double of B (by the 5. common ſentence of the ſeuenth): which is not poſſible, for that they are proued to be prime numbers.

Here haue I added an other demonſtration of the former Propoſition after *Campane*, which proueth that in nũbers how many ſoeuer, which is there proued onely touching three numbers: and the demonſtration ſeemeth ſomwhat more perſpicous then *Theons* demonſtration. And thus he putteth the propoſition.

If numbers how many ſoeuer being in continuall proportion be the leaſt that haue one & the ſame proportion with them: euery one of them ſhalbe to the number compoſed of the reſt prime.

Demonſtration leading to an abſurditie.

Suppoſe that there be numbers in continuall proportion how many ſoeuer, and the leaſt in their proportion: namely, A,B,C,D. Then I ſay that euery one of them, as for example firſt D, is prime to the number compoſed of the reſt, namely, of A,B,C. For if it be not, let ſome number, namely E meaſure D, and the number compoſed of A,B,C. Take the two leaſt numbers in the ſame proportion that A,B,C, D are (by the 35. of the ſeuenth) which let be F,G. And foraſmuch as E meaſureth one of theſe number A,B,C, D, the ſame E ſhalbe a number not prime either to F or to G (by the propoſition before added by Campane after the 14. propoſition) wherefore ſome number ſhall meaſure E and one of theſe numbers F or G: which let be H. And foraſmuch as H meaſureth E, it ſhall alſo meaſure D, which number D the number E alſo meaſureth (by the 5. common ſentence of the ſeuenth). Moreouer foraſmuch as H (by ſuppoſition) meaſureth one of theſe num-

A 8
L ....
F ... B 12
I .....
G ... C 18
K .....
D 27
E ——— A B C 38
H ——— A B D 47

bers

bers F or G, the same H shall measure all the meanes betwene A and D by the same cõmon sentẽce. For either of these numbers F or G produceth all the meanes by the next numbers in continuall proportion and in the same proportion with them (as by L,I,K) by the second of the eight. Agayne forasmuch as H measureth E, which (by supposition) measureth the whole A, B, C: the same H shall also measure the whole A,B, C (by the foresayd common sentence) and it measureth the part taken away, namely, the meanes B,C (as it hath bene proued) wherefore it also measureth the residue A (by the 4. common sentence of the seuenth) wherefore H measureth the extreames D and A, which are prime the one to the other (by the 3. of the eight) which were absurd. Wherefore D is a number prime to the number composed of the rest, namely, of A,B,C.

Secondly I say that this is so in euery one of them: namely that C is a prime number to the number composed of A,B,D. For if not, then as before let E measure C, and the number composed of A,B, D: which E shalbe a number not prime either to F or to G (by the former proposition added by Campane) wherefore let H measure them. And forasmuch as H measureth E, it shall also measure the whole A, B, C, D whom E measureth. And forasmuch as H measureth one of these numbers F or G, it shall measure one of the extreames A or D: which are produced of F or G (by the second of the eight) if they be multipl ed into the meanes L or K. And moreouer the same H shall measure the meames, B C (by the 5. common sentence of the seuenth) when as by supposition it measureth either F or G, which measure B,C (by the second of the eight). But the same H measureth the whole A,B,C, D as we haue proued, for that it measureth E. Wherefore it shall also measure the residue, namely, the number composed of the extreames A and D (by the 4. common sentence of the seuenth). And it measureth one of these A or D (for it measureth one of these F or G which produce A and D) wherefore the same H shall measure one of these A or D and also the other of them (by the former common sentence) which numbers A and D are by the 3. of the eight prime the one to the other. Which were absurd. This may also be proued in euery one of these numbers A,B,C,D. Wherefore no number shall measure one of these numbers A,B,C, D and the number composed of the rest. Wherefore they are prime the one to the other: If therefore numbers how many soeuer. &c: which was required to be proued.

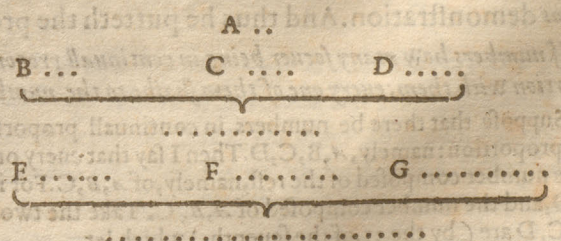
Here as I promised, I haue added *Campanes* demonstrations of those Propositions in numbers, which *Euclide* in the second booke demonstrated in lines. And that in thys place so much the rather, for that *Theon* as we see in the demonstration of the 15. Proposition seemeth to alledge the 3. & 4. Proposition of the second boke: which although they concerne lines onely, yet as we there declared and proued, are they true also in numbers.

¶ The first Proposition added by *Campane*.

That number which is produced of the multiplication of one number into numbers how many soeuer: is equall to that number which is produced of the multiplication of the same number into the number composed of them.

Demonstration.

This proueth that in numbers which the first of the second proued touching lines. Suppose that the number A being multiplyed into the number B, and into the number C, and into the number D, doo produce the numbers E, F and G. Then I say that the number produced of A multiplyed into the number composed of B,C, and D is equall to the number composed of E,F, and G. For by the conuerse of the definition of a number multiplyed, what part vnitie is of A, the selfe same part is B of E, and C of F, and also D of G. Wherefore by the 5. of the seuenth what part vnitie is of A, the selfe same part is the number composed of B,C, and D, of the number composed of E,F, and G. Wherfore by the definition that which is produced of A into the number composed of B,C,D, is equall to the number composed of E, F, G: which was required to be proued.



✻ The second Proposition.

That number which is produced of the multiplication of numbers how many soeuer into one nũber: is equall to that number which is produced of the multiplication of the number composed of them into the same number.

Thys

This is the conuerſe of the former, As if the numbers B and C and D multiplyed into the number A doo produce the numbers E and F and G. Then the number compoſed of B,C,D, multiplyed into the number A ſhall produce the number compoſed of the numbers E, F, G. Which thing is eaſly proued by the 16. of the ſeuenth and by the former propoſition.

B ... C .... D .....
A ..
E ..... F ..... G .....

This propoſition is the cõuerſe of the former.

¶ The third Propoſition.

That number which is produced of the multiplication of numbers how many ſoeuer into other numbers how many ſoeuer, is equall to that number which is produced of the multiplication of the number compoſed of thoſe firſt numbers, into the number compoſed of theſe latter numbers.

As if the numbers A,B,C doo multiply the numbers D,E,F, ech one eche other, and if the numbers produced be added together. Then I ſay that the number compoſed of the numbers produced is equall to the number produced of the number compoſed of the numbers A, B, C into the number compoſed of the numbers D,E,F. For by the former propoſitiõ that which is produced of the number compoſed of A,B,C into D is equall to that which is produced of euery one of the ſayd numbers into D: and by the ſame reaſon that which is produced of the number compoſed of A,B,C into E, is equal to that which is produced of euery one of the ſayd numbers into E: and ſo likewiſe that which is produced of the number compoſed of A,B,C into F is equall to that which is produced of euery one of the ſayd numbers into F. But by the firſt of theſe propoſitions that which is produced of the number compoſed of theſe numbers A,B,C into euery one of theſe numbers D, E,F is equall to that which is produced of the number compoſed into the number compoſed: wherefore that is manifeſt which was required to be proued.

A .. B ... C ....
D .... E ..... F .....

Demonſtration.

¶ The fourth Propoſition.

If a number be deuided into partes how many ſoeuer: that nũber which is produced of the whole into him ſelfe, is equall to that number which is produced of the ſame number into all his partes.

This proueth in numbers that which the ſecond of the ſecond proued in lines. As if the number A, be deuided into the numbers B and C, and D. Then I ſay, that that which is produced of A into himſelfe, is equall to that which is produced of A into all the ſayd numbers B, C, and D. For putting the number E equall to the number A, it is manifeſt by the firſt of theſe propoſitions that that which is produced of E into A, is equall to that which is produced of E into all the partes of A. But by the common ſentence that which is produced of E into A is equal to that which is produced of A into himſelfe: and that which is produced of E into the partes of A is equall to that which is produced of A into the ſelfe ſame partes. Wherefore that is manifeſt which was required to be proued.

A .....
B .. C ... D ....
E .....

This anſwereth to the 2. of the ſecond.

Demonſtration.

¶ The fift Propoſition.

If a number be deuided into two partes: that which is produced of the whole into one of the partes, is equall to that which is produced of the ſelfe ſame part into him ſelfe, and into the other part.

This proueth in numbers that which in the 3. of the ſecond was proued in lines. For let the number A be deuided into the numbers B and C. Then I ſay that that which is produced of A into C, is equall to that which is produced of C into himſelfe and into B. For by the 16. of the ſeuenth, that which is produced of A into C is equall to that which is produced of C into A. Now then put the number D equall to the number C. Wherefore that which is produced of A into C is equall to that which is produced of D into A. But by the firſt of theſe propoſitions that which is produced of D into A is equall to that which is produced of D into B and of D into C. Wherfore foraſmuch as that which is produced of D into A and into B and into C is equall to that which is produced of C into A, and into B, and into himſelfe, by reaſon of the equalitie of C and D: that is manifeſt which was required to be proued.

A .....
B ..... C ...
D ...

This anſwereth to the 3. of the thirde.

Demonſtration.

¶ The sixt Proposition.

If a number be deuided into two partes: that which is produced of the multiplication of the whole into him selfe, is equall to that which is produced of the multiplication of either of the partes into him selfe, and of the one into the other twise.

This proueth in numbers that which the fourth of the second proued touching lines. As if the number A be deuided into the numbers B and C. Then I say that that which is produced of A into himself is equal to that which is produced of B into himselfe, and of C into himselfe, and of B into C twise. For by the 4 of these propositions, that which is produced of A into himselfe, is equall to that which is produced of A into B, and into C. But that which is produced of A into B, is equall to that which is produced of B into himselfe and into C (by the former proposition). And by the same that which is produced of A into C is equall to that which is produced of C into himselfe and into B. And forasmuch as that which is produced of C into B is equall to that which is produced of B into C by the 16. of the seuenth, it is manifest that that is true which was required to be proued.

A .....
B ..... C ....

This answereth to the 4. of the second.

Demonstration.

¶ The seuenth Proposition.

If a number be deuided into two equall partes, & into two vnequall partes: that which is produced of the greater of the vnequall partes into the lesse, together with the square nũber of the number set betwene, is equall to the square number produced of the halfe of the whole.

This proueth in numbers that which the 5. of the second proued in lines. As if the number A B be deuided into two equall numbers, which let be A C, and C B: and also in two vnequal numbers namely, A D and D B, of which let A D be the greater, and D B the lesse. Then I say that that which is produced of the whole A D into D B together with the square number of C D, is equall to the square number of C B. For by the former propositiõ the square of C B is equall to the square of C D and to the square of D B, and to that which is produced of B D into C D twise. But that which is produced of B D into himselfe and into C D is equall to that which is produced of B D into C B by the first of these propositions, and therefore vnto that which is produced of B D into AC. Wherefore that which is produced of B D into himselfe and into C D twise is equall to that which is produced of B D into A D. Wherefore by the same the square of C B exceedeth that which is produced of B D into A D by the square of C D: wherefore that is manifest which was required to be proued.

A ..... C .... D .. B

This answereth to the 5. of the second.

Demonstration.

¶ The 8. Proposition.

If a number be deuided into two equall partes, and if vnto it be added an other number: that which is produced of the multiplication of the whole being composed into the number added, together with the square of the halfe, is equall to the square of the number composed of the halfe and the number added.

This proueth in numbers that which the 6. of the second proued touching lines. For suppose that the number *A B* be deuided into equall numbers, which let be *A C* and *C B*: and vnto it adde the number *B D*. Then I say, that that which is produced of the whole *A D* into *D B* together with the square of *B C*, is equall to the square of *C D*. For by the 6. of these propositiõs the square of *C D* is equal to the square of *D B*, & to the square of *B C*, and to that which is produced of *D B* into *B C* twise. But by the 1. of these propositions, that which is produced of *B D* into himselfe and into *B C* twise is equall to that which is produced of *B D* into *D A* (for *A C* and *C B* are equall) wherefore the square of *C D* exceedeth that which is produced of *B D* into *D A* by the square of *C B*. Wherefore that is manifest which was required to be proued.

A .... C .... B ..... D

This answereth to the 6. of the second.

Demonstration.

¶ The 9. Proposition.

If a number be deuided into two partes: that which is produced of the whole number into him selfe together with that which is produced of one of the partes into him self, is equall to that which is produced of the whole into the sayd part twise together with that which is produced of the other part into him selfe.

This

This proueth in numbers that which the 7. of the ſecond proued in lines. For let the number *A* be deuided into the numbers *B* and *D*. Then I ſay that the ſquare of *A* together with the ſquare of *D* is equall to that which is produced of *A* into *D* twiſe together with the ſquare of *B*. For it is manifeſt by the 6. of theſe propoſitions that the ſquare of *A* is equall to the ſquares of *B* and D together with that which is produced of *B* into *D* twiſe. Wherefore the ſquare of *A* together with the ſquare of *D*, is equall to two ſquares of *D*, and to that which is produced of *D* into *B* twiſe together with the ſquare of *B*. But by the firſt of theſe propoſitions two ſquares of *D*, and that which is produced of *D* into *B* twiſe is equall to that which is produced of *D* into *A* twiſe. Wherfore that which is produced of *D* into *A* twiſe together with the ſquare of *B*, is equall to the ſquare of *A* together with the ſquare of *D*: wherfore that is manifeſt which was required to be proued.

A
B ..... D ....

This anſwereth to the 7. of the ſecond.

Demonſtration.

¶ The 10. propoſition.

If a number be deuided into two partes, and vnto it be added a number equall to one of the parts: the ſquare of the whole number compoſed, is equall to the quadruple of that which is produced of the firſt number into the number added, together with the ſquare of the other part.

This proueth in numbers, that which the 8. of the ſecond proued in lines. Suppoſe that the number *A B* be deuided into the numbers *A C* and *C B*, vnto which adde the number *B D*, which let be equall to the number *C B*. Then I ſay that the ſquare of the whole number compoſed, namely, *A D*, is equall to that which is produced of *A B* into *B D* fower times together with the ſquare of *A C*. For by the 6. of theſe propoſitions the ſquare of *A D*, is equall to the ſquare *A B* and to the ſquare of *B D* together with that which is produced of *A B* into *B D* twiſe. And foraſmuch as the ſquare of *B D* is equal to the ſquare *C B*: therfore the ſquare of *A D* ſhall be equall to the ſquare of *A B* and to the ſquare of *C B* together with that which is produced of *A B* into *B D* twiſe. But by the former propoſition the ſquare of *A B* together with the ſquare of *C B*, is equall to the ſquare of *A C* together with that which is produced of *A B* into *B C* twiſe wherfore the ſquare of *A D* is equal to that which is produced of *A B* into *B D* twiſe, and to that which is produced of *A B* into *B C* twiſe together with the ſquare of *A C*. And for that that which is produced of *A B* into *B C* is equall to that which is produced of *A B* into *B D*, therefore is that manifeſt which was required to be proued.

A ... C ..... B ..... D

This anſwereth to the 8. of the ſecond.

Demonſtration.

✻ The 11. propoſition.

If a number be deuided into two equall partes, and into two vnequall partes: the ſquares of the two vnequall partes taken together, are double to the ſquare of the halfe, and to the ſquare of the exceſſe of the greater part aboue the leſſe, the ſayd two ſquares being added together.

This proueth in numbers that which the 9. of the ſecond proued in lines. For ſuppoſe that the number *A B* be deuided into two equall partes: which let be *A C* and *C B*, and into two vnequall parts, which let be *A D* and D *B*. Then I ſay that the ſquares of the two numbers *A D* & *D B*, taken together, are double to the two ſquares of the two numbers *A C* and *C D*, taken together. For by the 6. of theſe propoſitions the ſquare of *A D* is equall to the ſquares of *A C* and *C D*, and to that which is produced of *A C* into *C D* twiſe. And foraſmuch as the ſquare of *A C* is equal to the ſquare of *C B*, the ſquare of *A D* ſhal be equall to the ſquare of *B C* & to the ſquare of *C D* together with that which is produced of *B C* into *C D* twiſe. Wherefore the ſquare of *A D* together with the ſquare of *B D*, is equall to the ſquare of *B C*, and to the ſquare of *C D* and to that which is produced of *B C* into *C D* twiſe together with the ſquare of *B D*. But that which is produced of *B C* into *C D* twiſe together with the ſquare of *B D*, is equall to the ſquare of *B C* and to the ſquare of *C D* by the 9. of theſe propoſitions. Wherfore the ſquares of the two numbers *A D* and *D B* are equall to the ſquares of the two numbers *B C* and *C D*, taken twiſe. And therefore the ſquares of the two numbers *A D* and *D B* are double to the ſquares of the two numbers *B C* and *C D*, that is *A C* and *C D* (for the numbers *A C* and *B C* are by ſuppoſition equall) wherfore that is manifeſt which was required to be proued.

A ..... C ... D ... B

This anſwereth to the 9. of the ſecond.

Demonſtration.

✻ The 12. propoſition.

If a number be deuided into two equall partes, and vnto it be added an other number: the ſquare of the whole number compoſed together with the ſquare of the number added, is double to the ſquare of the halfe, together with the ſquare of the number compoſed of the halfe and the number added.

This answereth to the 10. of the second.

This proueth in numbers that which the 10. of the second proued in lines. Suppose that the number AB be deuided into two equall numbers AC and CB: and vnto it adde the number BD. Then I say that the square of AD together with the square of BD, is double to the square of AC together with the square of CD. For forasmuch as the number CD is deuided into two partes, and vnto it is added the number AC which is equall to one of the partes (namely, to CB) therefore by the 10. of these propositions, the square of AD is equall to that which is produced of CD into CA foure times together with the square of BD. And forasmuch as AC is equall to CB, therefore the square of AD, is equall to that which is produced of DC into CB fower times together with the square of BD. Wherefore the square of AD together with the square of DB, is equall to that which is produced of DC into CB fower times together with two squares of BD. But by the 9. of these propositions, that which is produced of DC into CB fower times together with two squares of BD is double to the square of CD together with the square of CB (for the square of CD together with the square of CB is equal to that which is produced of DC into CB twise together with one square of CB). Wherefore forasmuch as the square of CB is equall to the square of AC, that is manifest: which was required to be proued.

Demonstration.

A ..... C ..... B ... D

❧ The 13. proposition.

It is impossible to deuide a number in such sorte: that that which is contayned vnder the whole and one of the partes, shall be equall to the square of the other part.

A negatiue proposition.

That which the 11. of the second taught to be doone in lines is here proued to be impossible to be doone in numbers. For suppose that there be a number whatsoeuer namely AB. Then I say, that it is impossible to deuide it in such sort as is required in the proposition. For so should it be deuided according to a proportion hauing a meane and two extreames. But if it be possible, let the number AB be so deuided in C. And as AB is to BC, so let BC be to CA. Wherefore AC shall be lesse then CB. Now then take away from CB a number equall to AC which let be CD. And forasmuch as the proportion of the whole AB to the whole BC, is as the proportion of the part taken away from AB, namely, BC to the part taken away from BC, namely, CD: therefore the proportion of the residue of AB, namely, AC, to the residue of BC, namely, to BD, is as the proportion of the whole AB to the whole BC (by the 11. of the seuenth). Wherefore BC is to CD, as CD is to DB. Wherfore CD is greater then DB. Wherefore subtrahing DE out of CD, so that let DE be equall to DB: the proportion of BC to CD is as the proportiõ of CD to DE. Wherfore the residue of CB, namely, DB, shal be to the residue of CD, namely, to CE, as the proportion of the whole BC to the whole CD. Wherfore CE may be subtrahed out of ED: wherfore there shalbe no end of this subtraction: which is impossible.

Demonstration leading to an impossibilitie.

A ..... C .... E .. D .. B

¶ The 16. Theoreme. The 16. Proposition.

If two numbers be prime the one to the other, the second shall not be to any other uumber, as the first is to the second.

Demonstration leading to an absurditie.

Svppose that these two numbers A and B be prime the one to the other. Then I say that B is not to any other nũber as A is to B. For if it be possible, as A is to B, so let B be to C. Now A and B are prime numbers, yea they are prime and the lest by the 23. of the seuenth. But (by the 21. of the seuenth) the least measure the numbers that haue one and the same proportion with them equally, the antecedent the antecedent, and the consequent the consequent. Wherefore the antecedent A, measureth the antecedent B, and it measureth also it selfe. Wherfore A measureth these numbers A & B being prime the one to the other, which is impossible. Wherfore as A is to B, so is not B to C: which was required to be proued.

A .....
B .....
C ——

¶ The 17. Theoreme. The 17. Proposition.

If there be numbers in continuall proportion how many soeuer, and if theyr extremes

extremes be prime the one to the other, the leſſe ſhall not be to any other number, as the firſt is to the ſecond.

SVppoſe that there be theſe numbers in continuall proportion A,B,C,D, and let their extremes A and D be prime the one to the other. Then I ſay that D is not to any other number as A is to B. For if it be poſſible, as A is to B, ſo let D be to E. Wherfore alternately by the 13. of the ſeuenth, as A is to D, ſo is B to E. But A and D are prime, yea they are prime and the leaſt. But the leaſt numbers (by the 21. of the ſeuenth) meaſure the numbers that haue one and the ſame proportiõ with them equally, the antecedent the antecedent, and the conſequent. Wherefore the antecedent A meaſureth the antecedent B: but as A is to B, ſo is B to C. Wherfore B alſo meaſureth C. Wherfore A alſo meaſureth C (by the 5. commõ ſentence of the ſeuenth) and foraſmuch as B is to C, ſo is C to D, but B meaſureth C. Wherfore C meaſureth D. But A meaſureth C. Wherfore A alſo meaſureth D by the ſame common ſentence, and it alſo meaſureth it ſelfe. Wherefore A meaſureth theſe numbers A and D being prime the one to the other, which is impoſſible. Wherfore D is not to any other number as A is to B: which was required to be proued.

A .....
B .....
C .....
D .....
E ———

Demonſtration leading to an abſurditie.

¶ The 18. Theoreme. The 18. Propoſition.

Two numbers being geuen, to ſearche out if it be poſſible a third number in proportion with them.

SVppoſe that the two numbers geuen be A and B. It is required to ſearche out if it be poſſible a third number proportionall with them. Now A,B are either prime the one to the other or not prime. If they be prime, then (by the 16. of the ninth) it is manifeſt that it is impoſſible to finde out a third number proportional with them. But now ſuppoſe that A B be not prime the one to the other. And let B multiplieng himſelfe produce C. Now A either meaſureth C, or meaſureth it not. Firſt, let it meaſure it and that by D. Wherfore A multiplieng D produceth C. But B alſo multiplieng himſelf produced C. Wherfore that which is produced of A into D, is equall to that which is produced of B into himſelfe. Wherefore (by the ſecond part of the 19. of the ſeuẽth) as A is to B, ſo is B to D. Wherfore vnto theſe numbers A,B is found out a third number in proportion, namely, D.

But now ſuppoſe that A do not meaſure C. Then I ſay that it is impoſſible to finde out a third nũber in proportion with theſe numbers A,B. For if it be poſſible, let there be found out ſuch a number, and let the ſame be D.

Three caſes in this propoſition.
The firſt caſe.
The ſecond caſe.
The third caſe.

A ....
B .....
———
A ....
B .....
D .....
C .....
A .....
B ....
D ———
C .....

Wherfore that which is produced of A into D, is equall to that which is produced of B into himselfe, but that which is produced of B into himselfe is C. Wherfore that which is produced of A into D is equall vnto C. Wherfore A multiplieng D produced C. Wherefore A measureth C by D. But it is supposed also not to measure it, which is impossible. Wherefore it is not possible to finde out a third number in proportion with A & B, whensoeuer A measureth not C: which was required to be proued.

¶ The 19. Theoreme. The 19. Proposition.

Three numbers beyng geuen, to search out if it be possible the fourth number proportionall with them.

Diuers cases in this proposition.

Vppose that the three numbers geuen be A,B,C. It is required to search out if it be possible a fourth number proportionall with them. Now A,B,C, are either in continuall proportion, and their extremes A,C are prime the one to the other: or they are not in continuall proportion, and their extremes are yet prime the one to the other: or they are in continuall proportion, and their extremes are not prime the one to the other: or they are neither in continuall proportion, nor their extremes are prime the one to the other.

The first case. *If A,B,C, be in continuall proportion, and their extremes be prime the one to the other, it is manifest (by the 17. of the ninth) that it is impossible to finde out a fourth number proportionall with them.*

The second case. *But now suppose that A,B,C, be not in continuall proportion, and yet let their extremes be prime the one to the other. Then I say that so also it is impossible to finde out a fourth number proportional with thē. For if it be possible, let there be found such a number, and let the same be D. So that as A is to B, so let C be to D, and as B is to C, so let D be to E. And for that as A is to B, so is C to D, and as B is C, so is D to E, therfore of equallitie (by the 14. of the seuenth) as A is to C, so is C to E. But A and C are prime the one to the other, yea they are prime, and the least: but the least measure the numbers that haue one & the same proportion with them equally, the antecedent, the antecedent, and the consequent the consequent (by the 21. of the seuenth). Wherfore A measureth C, namely, the antecedent the antecedent, and it also measureth it selfe. Wherfore A measureth these numbers A and C being prime the one to the other, which is impossible. Wherfore it is not posible to finde out a fourth number proportionall with these numbers A,B,C.*

A .....
B .....
C .....
D ——————
E ——————

The third case. *But now againe suppose that A,B,C, be in continuall proportion, and let A and C not be prime the one to the other. Then I say that it is possible to finde out a fourth number proportionall with them. For let B multiplieng C produce D. Now A either measureth D, or measureth it not. First let it measure it, and that by E. Wherfore A multiplieng E produced D. But B also multiplieng C produced D. Wherfore that which is produced of AE is equal to that which is produced of BC: wherfore in what proportiō A is to B, in ỹ same is C to E. Wherfore there is found out a fourth number, namely, E, proportionall with these nūbers A,B,C.*

A .....
B .....
C .....
E .....
D 216

But

But now suppose that A do not measure D. Then I say that it is not possible to finde out a fourth number proportionall with these numbers A, B, C. For if it be possible, let there be found such a number, and let the same be E. Wherfore that which is produced of A into E is equall to that which is produced of B into C. But that which is produced of B into C is D. Wherfore that which is produced of A into E is equall vnto D. Wherefore A multiplieng E produced D, wherfore A measureth D, but it also measureth it not, which is impossible. Wherfore it is impossible to finde out a fourth number proportionall, with these numbers A, B, C, whensoeuer A measureth not D.

The fourth case.

But now suppose that A, B, C be neither in continuall proportiõ, neither also their extremes be prime the one to the other. And let B multiplieng C produce D.

A .....
B .....
C .....
E ——————————————
D 1350

And in like sorte may we proue that if A do measure D, it is possible to finde out a fourth number proportionall with them. But if it do not measure D, thẽ is it impossible: which was required to be proued.

¶ The 20. Theoreme. The 20. Proposition.

Prime numbers being geuen how many soeuer, there may be geuen more prime numbers.

Two cases in this Proposition.

Suppose that the prime numbers geuen be A, B, C. Then I say, that there are yet more prime numbers besides A, B, C. Take (by the 38. of the seuenth) the lest number whom these numbers A, B, C do measure, and let the same be D E. And vnto D E adde vnitie D F. Now E F is either a prime number or not.

The first case.

First let it be a prime number, then are there found these prime numbers A, B, C, and E F more in multitude then the prime numbers first geuen A, B, C.

A ..
B ...
C .....
E 114 D F
G .....

The second case.

But now suppose that E F be not prime. Wherefore some prime number measureth it (by the 24. of the seuenth). Let a prime number measure it, namely, G. Then I say, that G is none of these numbers A, B, C. For if G be one and the same with any of these A, B, C. But A, B, C, measure the nũber D E: wherfore G also measureth D E: and it also measureth the whole E F. Wherefore G being a number shall measure the residue D F being vnitie: which is impossible. Wherefore G is not one and the same with any of these prime numbers A, B, C: and it is also supposed to be a prime number. Wherefore there are found these prime numbers A, B, C, G, being more in multitude then the prime numbers geuen A, B, C: which was required to be demonstrated.

❋ A Corollary.

By thys Proposition it is manifest, that the multitude of prime numbers is infinite.

¶ The 21. Theoreme. The 21. Proposition.

If euen nũbers how many soeuer be added together: the whole shall be euẽ.

Demonſtration.

Vppoſe that theſe euen numbers A B, B C, C D, and D E, be added together. Then I ſay, that the whole number, namely, A E, is an euen number. For foraſmuch as euery one of theſe numbers A B, B C, C D, and D E, is an euen number, therefore euery one of them hath an halfe. Wherefore the whole A E alſo hath an halfe. But an euen number (by the definition) is that which may be deuided into two equall partes. Wherefore A E is an euen number: which was required to be proued.

A .... B ..... C ..... D ..... E

¶ The 22. Theoreme. The 22. Propoſition.

If odde numbers how many ſoeuer be added together, & if their multitude be euen, the whole alſo ſhall be euen.

Demonſtration.

Vppoſe that theſe odde numbers A B, B C, C D, and D E, being euen in multitude, be added together. Then I ſay, that the whole A E is an euen number. For foraſmuch as euery one of theſe numbers A B, B C, C D, and D E, is an odde number, if ye take away vnitie from euery one of them, that which remayneth of euery one of thē is an euen number. Wherefore they all added together, are (by the 21. of the ninth) an euen number: and the multitude of the vnities taken away is euen. Wherefore the whole A E is an euen number: which was required to be proued.

A ..... B ..... C ..... D ..... E

¶ The 23. Theoreme. The 23. Propoſition.

If odde numbers how many ſoeuer be added together, and if the multitude of them be odde, the whole alſo ſhall be odde.

Demonſtration.

Vppoſe that theſe odde numbers, A B, B C, and C D being odde in multitude be added together. Then I ſay that the whole A D is an odde number. Take away from C D, vnitie D E, wherefore that which remaineth C E is an euen number. But A C alſo (by the 22. of the ninth) is an euen number. Wherfore the whole A E is an euen number.

A ..... B ..... C ..... E . D

But D E which is vnitie being added to the euen number A E, maketh the whole A D an odde number: which was required to be proued.

¶ The 24. Theoreme. The 24. Propoſition.

If from an euen number be takē away an euen number, that which remaineth ſhall be an euen number.

Demonſtration.

Vppoſe that A B be an euen number, and from it take away an euen number C B. Then I ſay that that which remayneth, namely, A C is an euen number. For foraſmuch as A B is an euen

A ..... C .... B

euen

euen number, it hath an halfe, and by the ſame reaſon alſo B C hath an halfe. Wherfore the reſidue C A hath an halfe. Wherfore A C is an euen number: which was required to be demonſtrated.

¶ The 25. Theoreme. The 25. Propoſition.

If from an euen number be taken away an odde number, that which remaineth ſhall be an odde number.

SVppoſe that A B be an euen number, and take away from it B C an odde number. Then I ſay that the reſidue C A is an odde number. Take away from B C vnitie C D. Wherfore D B is an euen number. And A B alſo is an euen number, wherefore the reſidue A D is an euen number (by the former propoſition) But C D which is vnitie, being taken away from the euen nũber A D maketh the reſidue A C an odde number: which was required to be proued.

A ..... C. D ..... B

Demonſtration.

¶ The 26. Theoreme. The 26. Propoſition.

If from an odde number be taken away an odde number, that which remayneth ſhall be an euen number.

SVppoſe that A B be an odde number, and from it take away an odde number B C. Thẽ I ſay that the reſidue C A is an euen number. For foraſmuch as A B is an odde number, take away from it vnitie B D. Wherfore the reſidue A D is euen. And by the ſame reaſon C D is an euen number: wherfore the reſidue C A is an euen number (by the 24. of this booke): which was required to be proued.

A .... C ..... D . B

Demonſtration.

¶ The 27. Theoreme. The 27. Propoſition.

If from an odde number be taken away an euen number, the reſidue ſhall be an odde number.

SVppoſe that A B be an odde number, and from it take away an euen number B C. Then I ſay that the reſidue C A is an odde number. Take away frõ A B vnitie A D. Wherfore the reſidue D B is an euẽ number, & B C is (by ſuppoſition) euen. Wherfore the reſidue C D is an euen number. Wherefore D A which is vnitie, beyng added vnto C D which is an euen number maketh the whole A C an odde number: which was required to be proued.

A . D .... C .... B

Demonſtration.

¶ The 28. Theoreme. The 28. Propoſition.

If an odde number multiplieng an euen number produce any number, the number produced ſhall be an euen number.

Suppoſe

Demonstration.

Vppoſe that A being an odde number multiplieng B being an euen number, do produce the number C. Then I ſay that C is an euen number. For foraſmuch as A multiplieng B produced C, therfore C is compoſed of ſo many numbers equall vnto B as there be in vnities in A. But B is an euen nũber: wherfore C is compoſed of ſo many euen numbers, as there are vnities in A. But if euẽ numbers how many ſoeuer be added together, the whole (by the 21. of the ninth) is an euen number: wherfore C is an euen number: which was required to be demonſtrated.

C .....
B ....
A ...

¶ The 29. Theoreme. The 29. Propoſition.

If an odde number multiplying an odde number produce any number, the number produced ſhalbe an odde number

Demonſtration.

Vppoſe that A being an odde number multiplying B being alſo an odde number, doo produce the number C. Then I ſay that C is an odde number. For foraſmuch as A multiplying B produced C, therefore C is compoſed of ſo many numbers equall vnto B as there be vnities in A. But either of theſe numbers A and B is an odde number. Wherefore C is compoſed of odde numbers, whoſe multitude alſo is odde. Wherfore (by the 23. of the ninth) C is an odde nũber: which was required to be demonſtrated.

C .....
B ...
A .....

✻ A propoſition added by Campane.

A propoſition added by Campane.

If an odde number meaſure an euen number, it ſhall meaſure it by an euen number.

For if it ſhould meaſure it by an odde number, then of an odde number multiplyed into an odde number ſhould be produced an odde number, which by the former propoſition is impoſſible.

An other propoſition added by him.

An other added by him.

If an odde number meaſure an odde number, it ſhall meaſure it by an odde number.

For if it ſhould meaſure it by an euen nnmber, then of an odde number multiplyed into an euen number ſhould be produced an odde number which by the 28. of this booke is impoſſible.

¶ The 30. Theoreme. The 30. Propoſition.

If an odde number meaſure an euen number, it ſhall alſo meaſure the halfe thereof.

Demonſtration leading to an abſurditie.

Vppoſe that A being an odde number doo meaſure B being an euen number. Then I ſay that it ſhall meaſure the halfe thereof. For foraſmuch as A meaſureth B let it meaſure it by C. Thẽ I ſay that C is an euen number. For if not then, if it be poſſible let it be odde. And foraſmuch as A meaſureth B by C: therfore A multiplying C produceth B. Wherfore B is compoſed of odde numbers whoſe multitude alſo is odde. Wherfore B is an odde number (by the 29. of this booke) which is abſurd, for it is ſuppoſed to be euen: wherefore C is an euen number. Wherefore A meaſureth B by an euen number; and C meaſureth B by A. But either of

A ...
C .....
B .....

of these numbers C and B hath an halfe part: wherfore as C is to B, so is the halfe to the halfe. But C measureth B by A. Wherefore the halfe of C measureth the halfe of B by A: wherfore A multiplying the halfe of C produceth the halfe of B. Wherfore A measureth the halfe of B: and it measureth it by the halfe of C. Wherefore A measureth the halfe of the number B: which was required to be demonstrated.

¶ The 31. Theoreme. The 31. Proposition.

If an odde number be prime to any number, it shal also be prime to the double thereof.

Demonstration.

Vppose that A being an odde number be prime vnto the number B: and let the double of B be C. Then I say, that A is prime vnto C. For if A and C be not prime the one to the other, some one number measureth them both. Let there be such a number which measureth them both, and let the same be D.

But A is an odde number. Wherefore D also is an odde number. (For if D which measureth A should be an euen number, then should A also be an euen number (by the 21. of this booke): which is cõtrary to the suppositi-on. For A is supposed to be an odde nũber: & therefore D also is an odde number). And forasmuch as D being an odde number measureth C, but C is an euẽ number (for that it hath an halfe, namely, B). Wherfore (by the Proposition next going before) D measureth the halfe of C. But the halfe of C is B. Wherefore D measureth B: and it also measureth A. Wherefore D measureth A and B being prime the one to the other: which is absurde. Wherefore no number measureth the numbers A & C. VVherfore A is a prime number vnto C. VVherefore these numbers A and C are prime the one to the other: which was required to be proued.

A .....
B .....
C .....
D _____

¶ The 32. Theoreme. The 32. Proposition.

Euery nũber produced by the doubling of two vpward, is euenly euen onely.

Demonstration.

Vppose that A be the number two: and from A vpward double numbers how many soeuer, as B, C, D. Then I say, that B, C, D, are numbers euenly euen onely. That euery one of them is euenly euen, it is manifest: for euery one of them is produced by the doubling of two. I say also, that euery one of them is euenly euen onely: Take vnitie E. And forasmuch as from vnitie are certaine numbers in continuall proportion, & A which followeth next after vnitie is a prime number, therefore (by the 13. of the third) no number measureth D being the greatest number of these numbers A, B, C, D, besides the selfe same numbers in proportion. But euery one of these numbers A, B, C, is euenly euen. VVherefore D is euenly euen onely. In like sort may we proue, that euery one of these numbers A, B, C, is euenly euen onely: which was required to be proued.

D .....
C .....
B ....
A ..
E vnitie

¶ The 33. Theoreme. The 33. Proposition.

A number whose halfe part is odde, is euenly odde onely.

Suppose

Demonſtration leading to an abſurditie.

Vppoſe that A be a number whoſe halfe part is odde. Then I ſay that A is euenly od onely. That it is euenly odde it is manifeſt: for his halfe being odde meaſureth him by an euē number, namely, by 2. (by the definition). I ſay alſo that it is euenly odde onely. For if A be euenly euen, his halfe alſo is euen. For (by the definition) an euen number meaſureth him by an euen number. Wherefore that euen number which meaſureth him by an euen number ſhall alſo meaſure the halfe thereof being an odde number by the 4. common ſentence of the ſeuenth: which is abſurd. Wherfore A is a number euenly odde onely: which was required to be proued.

A .....

An other demonſtration to proue the ſame.

An other demonſtration.

Suppoſe that the number A haue to his halfe an od nūber, namely, B. Thē I ſay that A is euēly od onely. That it is euenly odde needeth no profe: foraſmuch as the number 2. an euen number meaſureth it by the halfe thereof which is an odde number. Let C be the number 2. by which B meaſureth A

B ..... C ..

A .....

D ..... F .....

(for that A is ſuppoſed to be double vnto B). And let an euen number, namely, D meaſure A (which is poſſible for that A is an euen number by the definition) by F. And foraſmuch as that which is produced of C into B is equall to that which is produced of D into F, therefore by the 19. of the ſeuenth, as C is to D, ſo is B to F. But C the number two meaſureth D being an euen number: wherfore F alſo meaſureth B which is the halfe of A. Wherfore F is an odde number. For if F were an euen number then ſhould it make B whome it meaſureth an odde number alſo by the 21. of this booke, which is contrary to the ſuppoſition. And in like maner may we proue that all the euē nūbers which meaſure the number A, do meaſure it by odde numbers. Wherefore A is a number euenly odde onely: which was required to be proued.

¶ The 34. Theoreme. The 34. Propoſition.

If a number be neither doubled from two, nor hath to his half part an odde number, it ſhall be a number both euenly euen, and euenly odde.

Demonſtration.

Vppoſe that the nūber A be a nūber neither doubled frō the nūber two, neither alſo let it haue to his halfe part an odde nūber. Then I ſay that A is a nūber both euenly euen, and euenly odde. That A is euenly euen it is manifeſt, for the halfe therof is not odde, and is meaſured by the number 2. which is an euen number. Now I ſay that it is euenly odde alſo. For if we deuide A into two equall partes, and ſo continuing ſtill, we ſhall at the length light vpon a certaine odde number which ſhall meaſure A by an euen number. For if we ſhould not light vpon ſuch an odde nūber, which meaſureth A by an euen number, we ſhould at the length come vnto the number two, and ſo ſhould A be one of thoſe numbers which are doubled from two vpward, which is contrary to the ſuppoſition. Wherfore A is euenly odde. And it is proued that it is euenly euē: wherfore A is a number both euenly euen and euenly odde: whiche was required to be demonſtrated.

A .....

This propoſition and the two former manifeſtly declare that which we noted vpon the tenth definition of the ſeuenth booke namely, that *Campane* and *Fluſſates* and diuers other interpreters of *Euclide* (onely *Theon* except) did not rightly vnderſtand the 8. and 9. definitions of the ſame booke concerning a number euenly euen, and a number euenly odde. For in the one definition they adde vnto *Euclides* wordes extant in the

the Greeke this word *onely* (as we there noted) and in the other this word *all*. So that after their definitions a number can not be euenly euen vnleſſe it be meaſured onely by euen numbers: likewiſe a number can not be euenly odde vnleſſe all the euen numbers which doo meaſure it, doo meaſure it by an odde number. The contrary whereof in this propoſition we manifeſtly ſee. For here *Euclide* proueth that one number may be both euenly euen and euenly odde. And in the two former propoſitions he proued that ſome numbers are euenly euen onely, and ſome euenly odde onely: which word onely had bene in vaine of him added, if no number euenly euen could be meaſured by an odde number, or if all the numbers that meaſure a number euenly odde muſt needes meaſure it by an odde number. Although *Campane* and *Fluſsates* to auoyde this abſurdity haue wreaſted the 32. propoſition of this booke frō the true ſence of the Greeke and as it is interpreted of *Theon*. So alſo hath *Fluſſates* wreaſted the 33. propoſition. For wheras *Euclide* ſayth, *Euery nūber produced by the doubling of two vpward, is euēly euē only*: they ſay, *onely the numbers produced by the doubling of two, are euenly euen*. Likewiſe whereas *Euclide* ſaith, *A number whoſe halfe part is odde, is euenly odde onely*, *Fluſſates* ſayth, *onely a number whoſe halfe part is od, Is euēly od*. Which their interpretatiō is not true, neither can be applyed to the propoſitions as they are extāt in the Greeke. In dede the ſayd 32. and 33. propoſitions as they put thē are true touching thoſe numbers which are euenly euen onely, or euēly od onely. For no number is euenly euen onely, but thoſe onely which are doubled from two vpward. Likewiſe no numbers are euenly odde onely, but thoſe onely whoſe halfe is an odde number. But this letteth not, but that a number may be euenly euen although it be not doubled from two vpward & alſo that a number may be euēly odde although it haue not to his halfe an odde number. As in this 34. propoſitiō *Euclide* hath plainly proued. Which thing could by no meanes be true, if the foreſayd 32. & 33. propoſitons of this booke ſhould haue that ſence and meaning wherein they take it.

¶ *The 35. Theoreme.* *The 35. Propoſition.*

If there be numbers in continuall proportion how many ſoeuer, and if from the ſecond and laſt be taken away numbers equall vnto the firſt, as the exceſſe of the ſecond is to the firſt, ſo is the exceſſe of the laſt to all the nūbers going before the laſt.

Vppoſe that theſe numbers A, B C, D, and E F, be in continuall proportion beginning at A the leaſt. And from B C, which is the ſecond, take away C G equall vnto the firſt, namely, to A, and likewiſe from E F the laſt take away F H equall alſo vnto the firſt, namely, to A. Then I ſay, that as the exceſſe B G is to A the firſt, ſo is H E the exceſſe, to all the numbers D, B C, and A, which go before the laſt number, namely, E F. Foraſmuch as E F is the greater (for the ſecond is ſuppoſed greater then the firſt) put the number F L equall to the number D, and likewiſe the number F K equall to the number B C. And foraſmuch as F K is equall vnto C B, of which F H is equall vnto G C, therefore the reſidue H K is equall vnto the reſidue G B. And for that as the whole F E, is to the whole F L, ſo is the part taken away F L, to the part taken away F K, therefore the reſidue L E is to the reſidue K L, as the whole F E is to the whole F L (by the 11. of the ſeuenth). So likewiſe for that F L is to F K, as F K is to F H, K L ſhall be to H K, as the whole F L is to the whole F K (by the ſame Propoſition). But as F B is to F L, and as F L is to F K, and F K to F H, ſo were F E to D, and D to B C, and B C to A. Wherefore as L E is to K L, and as K L is to H K, ſo is D to B C. Wherefore alternately (by the 13. of the ſeuenth) as L E is to D, ſo is K L to be B C, and as K L is to B C, ſo is H K to A. Wherefore alſo as one of the antecedentes is to one of the conſequentes, ſo are

A ....
B .... G C
D .....
E ..... L ..... K .... H .... F

Demonſtration.

CC 5. *all the*

all the antecedentes to all the consequentes. Wherefore as K H is to A, so are H K, K L, and L E, to D, B C, and A (by the 12. of the seuenth). But it is proued, that K H is equall vnto B G. Wherefore as B G, which is the excesse of the second, is to A, so is E H the excesse of the last vnto the numbers going before D, B C, and A. Wherefore as the excesse of the second is vnto the first, so is the excesse of the last to all the numbers going before the last: which was required to be proued.

¶ The 36. Theoreme. The 36. Proposition.

If from vnitie be taken numbers how many soeuer in double proportion continually, vntill the whole added together be a prime number, and if the whole multiplying the last produce any number, that which is produced is a perfecte number.

This proposition teacheth how to finde out a perfect number.

Suppose that from vnitie be taken these numbers A, B, C, D, in double proportion continually, so that all those numbers A, B, C, D, & vnitie added together, make a prime number: and let E be the number composed of all those numbers A, B, C, D, & vnitie added together: and let E multiplying D, which is the last number, produce the number F G. Then I say, that F G is a perfect number.

Vnitie.
A ..
B ....
C ....
D ....
O ———
P ———
E ——— 31
H ——— 62
31 *N* 31 *K*
L ——— 124
M ——— 248
F ——— 496
31 *X* 465 *G*

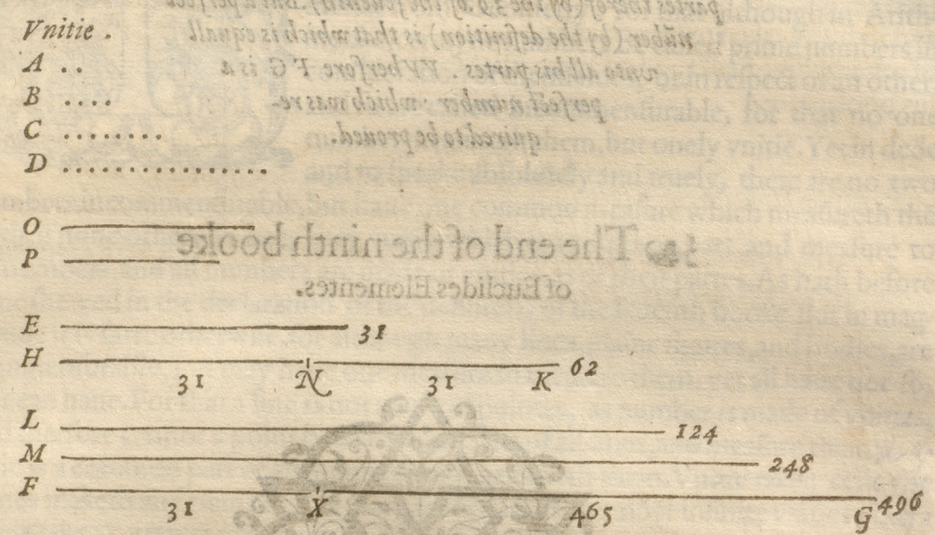
Construction. *How many in multitude A, B, C, D, are, so many in continuall double proportion take beginning at E, which let be the numbers E, H K, L, and M. Wherefore of equalitie (by the 13. of the seuenth) as A is to D, so is E to M. Wherefore that which is produced of E into D, is equall to that which is produced of A into M. But that which is produced of E into D, is the number F G. Wherefore that which is produced of A into M, is equall vnto F G. Wherefore A multiplying M produceth F G. Wherefore M measureth F G by those vnities which are in A. But A is the number two. Wherefore F G is double to M. And the numbers M, L, H K, and E, are also in continuall double proportion. Wherefore all the numbers E, H K, L, M, and F G, are continually proportionall in double proportion. Take from the second number K H, and from the last F G a number equall vnto the first, namely, to E: and let those numbers taken be H N, & F X. Wherefore (by the Proposition going before) as the excesse of the second number is to the first number, so is the excesse of the last to all the*

Demonstration.

numbers

numbers going before it. VVherefore as N K is to E, so is X G to these numbers M, L, K H, and E. But N K is equall vnto E (for it is the halfe of H K, which is supposed to be double to E). VVherefore X G is equall vnto these numbers M, L, H K, and E. But X F is equall vnto E, and E is equall vnto these numbers A, B, C, D, and vnto vnitie. Wherfore the whole number F G is equall vnto these numbers E, H K, L, M, and also vnto these numbers A, B, C, D, and vnto vnitie. Moreouer I say, that vnitie and all the numbers A, B, C, D, E, H K, L, and M, do measure the number F G. That vnitie measureth it needeth no proufe. And forasmuch as F G is produced of D into E, therefore D and E do measure it. And forasmuch as the double from vnitie, namely, the nũbers A, B, C, do measure the number D (by the 13. of this booke) therefore they shall also measure the number F G (whom D measureth) by the 3. common sentence. By the same reason forasmuch as the nũbers E, H K,

Demonstration leading to an absurditie.



L, and M, are vnto F G, as vnitie and the numbers A, B, C, are vnto D (namely, in subduple proportion) and vnitie and the numbers A, B, C, do measure D, therefore also the numbers E, H K, L, and M, shall measure the number F G. Now I say also, that no other number measureth F G besides these numbers A, B, C, D, E, H K, L, M, and vnitie. For if it be possible, let O measure F G. And let O not be any of these numbers A, B, C, D, E, H K, L, and M. And how often O measureth F G, so many vnities let there be in P. Wherefore O multiplying P produceth F G. But E also multiplying D produced F G. Wherefore (by the 19. of the seuenth) as E is to O, so is P to D. Wherefore alternately (by the 9. of the seuenth) as E is to P, so is O to D. And forasmuch as from vnitie are these numbers in continuall proportion A, B, C, D, and the number A which is next after vnitie is a prime number, therfore (by the 13. of the ninth) no other number measureth D besides the numbers A, B, C. And it is supposed that O is not one and the same with any of these nũbers A, B, C. Wherefore O measureth not D. But as O is to D, so is E to P. Wherefore neither doth E measure P. And E is a prime number. But (by the 31. of the seuenth) euery prime number, is to euery number that it measureth not, a prime number. Wherefore E and P are prime the one to the other: yea they are prime and the least. But (by the 21. of the seuenth) the least measure the numbers that haue one and the same proportion with them equally, the antecedent the antecedent, and the consequent the consequent. And as E is to P, so is O to D. Wherefore how many times E measureth O, so many times P measureth D. But no other number measureth D besides the numbers A, B, C (by the 13. of this booke). Wherefore P is one and the same with one of these numbers A, B, C. Suppose that P be one and the same with B, & how many B, C, D, are

in multitude, ſo many take from E vpward, namely, E, H K, and L. But E, H K, and L, are in the ſame proportion that B, C, D, are. VVherefore of equalitie, as B is to D, ſo is E to L. VVherefore that which is produced of B into L, is equall to that which is produced of D into E. But that which is produced of D into E, is equall to that which is produced of P into O. VVherefore that which is produced of P into O, is equall to that which is produced of B into L. VVherefore as P is to B, ſo is L to O: and P is one & the ſame with B: wherefore L alſo is one and the ſame with O: which is impoſsible. For O is ſuppoſed not to be one and the ſame with any of the numbers geuen. VVherefore no number meaſureth F G beſides theſe numbers A, B, C, D, E, H K, L, M, and vnitie. And it is proued, that F G is equall vnto theſe numbers A, B, C, D, E, H K, L, M, and vnitie, which are the partes therof (by the 39. of the ſeuenth). But a perfect nũber (by the definition) is that which is equall vnto all his partes. VVherfore F G is a perfect number: which was required to be proued.

The end of the ninth booke of Euclides Elementes.

¶ The tenth booke of Euclides Elementes.

IN THIS TENTH BOOKE doth *Euclide* entreat of lines and other magnitudes rationall & irrationall, but chiefly of irrationall magnitudes, commensurable and incommensurable: of which hitherto, in al his former 9. bokes he hath made no mention at all. And herein differeth number from magnitude, or Arithmetike from Geometry: for that although in Arithmetike, certayne numbers be called prime numbers in consideration of themselues, or in respect of an other, and so are called incommensurable, for that no one number measureth them, but onely vnitie. Yet in dede and to speake absolutely and truely, there are no two numbers incommensurable, but haue one common measure which measureth thē both, if none other, yet haue they vnitie, which is a common part and measure to all numbers, and all numbers are made of vnities, as of their partes. As hath before bene shewed in the declaration of the definitiōs of the seuenth booke. But in magnitude it is farre otherwise, for although many lines, plaine figures, and bodies, are commensurable, and may haue one measure to measure them, yet all haue not so, nor can haue. For that a line is not made of pointes, as number is made of vnities, and therfore cannot a point be a common part of all lines, and measure them, as vnitie is a common part of all numbers and measureth them. Vnitie taken certayne tymes maketh any number. For there are not in any number infinite vnities: but a point taken certayne tymes, yea as often as ye list, neuer maketh any line, for that in euery line there are infinite pointes. Wherfore lines, figures, and bodies in Geometry, are oftentymes incommensurable and irrationall. Now which are rationall, and which irrationall, which commensurable, and which incommensurable, how many and how sundry sortes and kindes there are of them, what are their natures, passions, and properties, doth Euclide most manifestly shew in this booke, and demonstrate them most exactly.

The Argumēt of the tenth booke.

Difference betweene number and magnitude.

A line is not made of points as number is made of vnities.

This tenth booke hath euer hitherto of all men, and is yet thought & accompted, to be the hardest booke to vnderstand of all the bookes of Euclide. Which cōmon receiued opinion, hath caused many to shrinke, and hath (as it were) deterred them from the handeling and treatie thereof. There haue bene in deede in times past, and are presently in these our dayes, many which haue delt, and haue taken great and good diligence in commenting, amending, and restoryng of the sixe first bookes of Euclide, and there haue stayed themselues and gone no farther, beyng deterred and made afrayde (as it seemeth, by the opinion of the hardnes of this booke) to passe forth to the bookes following. Truth it is that this booke hath in it somewhat an other & straūger maner of matter entreated of, thē the other bokes before had, and the demonstrations also thereof, & the order seeme likewise at the first somewhat straunge and vnaccustomed, which thinges may seeme also to cause the obscuritie therof, and to feare away many from the reading and diligent study of the same, so much that many of the well learned haue much complayned of the darkenes and difficultie thereof, and haue thought it a very hard thing, and in ma-

This booke the hardest to vnderstand of all the bookes of Euclide.

In this booke is entreated of a straunger maner of matter then in the former.

Many euen of the well learned haue thought that this booke can not well be vnderſtanded without Algebra.

ner impoſſible to attayne to the right and full vnderſtanding of this booke, without the ayde and helpe of ſome other knowledge and learnyng, and chiefly without the knowledge of that more ſecret and ſubtill part of Arithmetike, commonly called Algebra, which vndoubtedly firſt well had and knowne, would geue great light therunto: yet certainly may this booke very well be entred into, and fully vnderſtand without any ſtraunge helpe or ſuccour, onely by diligent obſeruation of the order, and courſe of Euclides writinges. So that he which diligently hath peruſed and fully vnderſtandeth the 9. bookes goyng before, and marketh alſo earneſtly the principles and definitions of this tenth booke, he ſhal well perceiue that Euclide is of himſelfe a ſufficient teacher and inſtructer, and needeth not the helpe of any other, and ſhall ſoone ſee that this tenth booke is not of ſuch hardnes and obſcuritie, as it hath bene hetherto thought. Yea, I doubt not, but that by the trauell and induſtry taken in this tranſlation, and by addicions and emendations gotten of others, there ſhall appeare in it no hardnes at all, but ſhall be as eaſie as the reſt of his bookes are.

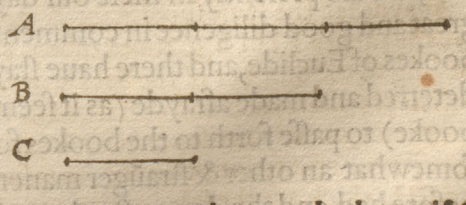
The nine former bookes & the principles of this booke well vnderſtoode, this booke will not be hard to vnderſtand.

Definitions.

The firſt definition.

1 *Magnitudes commenſurable are ſuch, which one and the ſelfe ſame meaſure doth meaſure.*

Firſt he ſheweth what magnitudes are commenſurable one to an other. To the better and more cleare vnderſtanding of this definition, note that that meaſure whereby any magnitude is meaſured, is leſſe then the magnitude which it meaſureth, or at leaſt equall vnto it. For the greater can by no meanes meaſure the leſſe. Farther it behoueth, that that meaſure if it be equall to that which is meaſured, taken once make the magnitude which is meaſured: if it be leſſe, then oftentimes taken and repeted, it muſt preciſely render and make the magnitude which it meaſureth. Which thing in numbers is eaſely ſene, for that (as was before ſaid) all numbers are commenſurable one to an other. And although Euclide in this definition comprehendeth purpoſedly, onely magnitudes which are continuall quantities, as are lines, ſuperficieces, and bodies, yet vndoubtedly the explication of this and ſuch like places, is aptly to be ſought of numbers as well rationall as irrationall. For that all quantities commenſurable haue that proportion the one to the other, which number hath to numbers. In numbers therfore, 9 and 12 are commenſurable, becauſe there is one common meaſure which meaſureth them both, namely, the number 3. Firſt it meaſureth 12, for it is leſſe then 12. and being taken certaine times, namely, 4 times, it maketh exactly 12: 3 times 4 is 12, it alſo meaſureth 9, for it is leſſe then 9. and alſo taken certaine times, namely, 3 times, it maketh preciſely 9: 3 times 3 is 9. Likewiſe is it in magnitudes, if one magnitude meaſure two other magnitudes, thoſe two magnitudes ſo meaſured, are ſaid to be commenſurable. As for example, if the line C being doubled, do make the line B, and the ſame lyne C tripled, do make the line A, then are the two lines A and B, lines or magnitudes commenſurable. For that one meaſure, namely, the line C meaſureth thē both. Firſt, the line C is leſſe thē the line A, and alſo leſſe thē the line B, alſo the line C taken or repeted certaine times, namely, 3 times, maketh preciſely the line A, and the ſame line C taken alſo certain times, namely, two times, maketh preciſely the line B. So that the line C is a common meaſure to them both, and doth meaſure them both. And therfore are the two lines A and B lines commenſurable. And ſo imagine ye of magnitudes of other kyndes, as of ſuperficiall figures, and alſo of bodies,



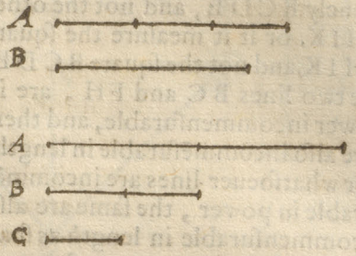
Incom-

2 Incommenſurable magnitudes are ſuch, which no one common meaſure doth meaſure.

The ſecond definition.

This diffinition neadeth no explanation at all, it is eaſely vnderſtanded by the diffinition going before of lines commenſurable. For contraries are made manifeſt by comparing of the one to the other: as if the line C, or any other line oftentimes iterated, doo not render preciſely the line A, nor the line B, thē are the lines A and B incommenſurable: Alſo if the line C, or any other line certayne times repeted, doo exactly render the line A, and doo not meaſure the line B: or if it meaſure the line B, and meaſureth not alſo the line A, the lines A and B, are yet lines incōmenſurable: & ſo of other magnitudes, as of ſuperficieces, and bodyes.

Contraryes made manifeſt by the comparing of the one to the other.



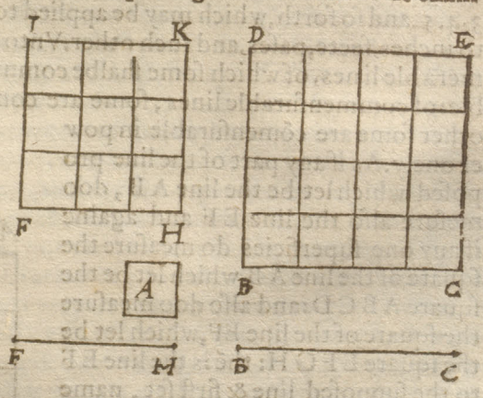
3 Right lines commenſurable in power are ſuch, whoſe ſquares one and the ſelfe ſame ſuperficies, area, *or* plat *doth meaſure.*

The thirde definition.

To the declaration of this diffinition we muſt firſt call to minde what is vnderſtanded & ment by the power of a line: which as we haue before in the former bookes noted is nothing ells but the ſquare thereof, or any other plaine figure equall to the ſquare therof. And ſo great power & habilitie is a line ſaid to haue, as is the quantitie of the ſquare, which it is able to deſcribe, or a figure ſuperficial equal to the ſquare therof.

What the power of a line is.

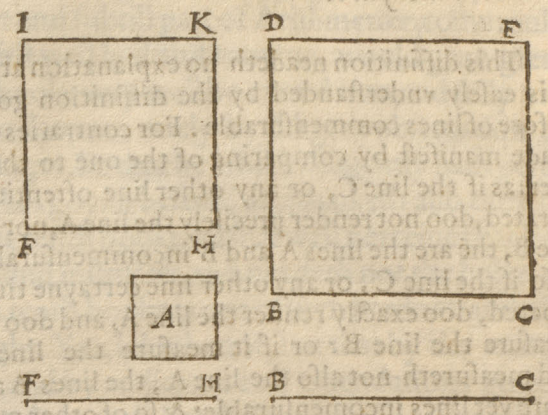
This is alſo to be noted that of lines, ſome are commenſurable in length, the one to the other, and ſome are commenſurable the one to the other in power. Of lines commenſurable in length the one to the other, was geuen an example in the declaration of the firſt diffinitiō, namely, the lines A and B, which were commenſurable in length, one and the ſelfe meaſure, namely, the line C meaſured the length of either of them. Of the other kinde is geuen this diffinition here ſet: for the opening of which take this example. Let there be a certaine line, namely, the line B C, and let the ſquare of that line be the ſquare B C D E. Suppoſe alſo an other line, namely, the line F H, & let the ſquare thereof be the ſquare F H I K, and let a certayne ſuperficies, namely, the ſuperficies A, meaſure the ſquare B C D E, taken 16. times: which is the number of the litle areas, ſquares, plats, or ſuperficieces cōtained and deſcribed within the ſayd ſquares ech of which is equall to the ſuperficie: A. Agayne let the ſame ſuperficies A meaſure the ſquare F H I K 9. times taken, according to the number of the fieldes or ſuperficieces contayned and deſcribed in the ſame. Ye ſee thē that one and the ſelfe ſame ſuperficies, namely, the ſuperficies A, is a common meaſure to both theſe ſquares, and by certayne repeticions thereof, meaſureth them both. Wherefore the two lines B C and F H, which are the ſides or lines producing theſe ſquares, and whoſe powers theſe ſquares are, are by this diffinition lines commenſurable in power.



4 Lines incommenſurable are ſuch, whoſe ſquares no one plat or ſuperficies doth meaſure.

The fourth definition.

This diffinition is eaſy to be vnderſtanded by that which was ſayd in the diffinition laſt ſet before this, and neadeth no farther declaration. And thereof take this example. If neither the ſuperficies A, nor any other ſuperficies doo meaſure the two ſquares BCDE, and FHIK: or if it meaſure the one, namely BCDE, and not the other FHIK, or if it meaſure the ſquare FHIK, and not the ſquare BCDE: the two lines BC and FH, are in power incommenſurable, and therfore alſo incommẽſurable in length. For whatſoeuer lines are incommẽſurable in power, the ſame are alſo incommenſurable in length as ſhall afterward in the 9. propoſition of this booke be proued. And therfore ſuch lines are here defined to be abſolutely incommenſurable. Theſe thinges thus ſtanding it may eaſely appeare, that if a line be aſſigned and layd before vs, there may be innumerable other lines commenſurable vnto it, and other incommenſurable vnto it: of commenſurable lines ſome are commenſurable in length and power, and ſome in power onely.

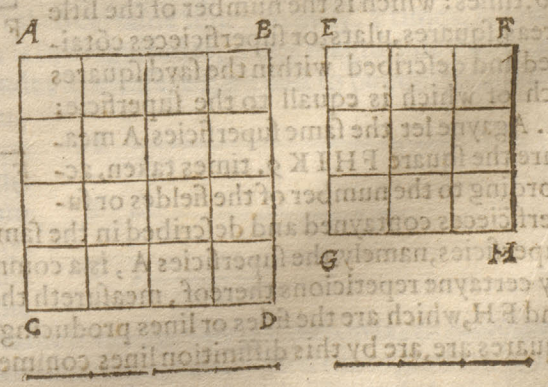


5. And that right line ſo ſet forth is called a rationall line.

The fifth definition.

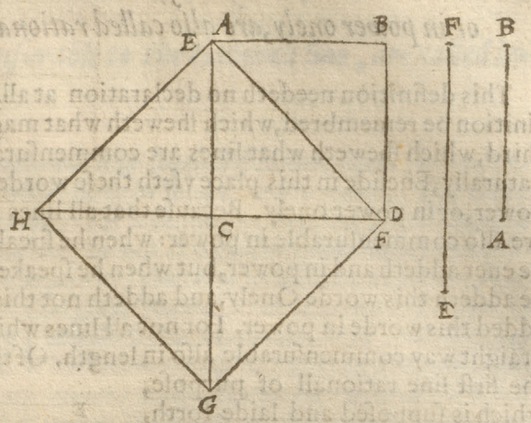
The principles of this booke ought well to be peiſed, for that they are more ſtraunge then the principles of the former bokes.

Theſe principles, diffinitions and groundes of this tenth booke ought well to be payſed, they are ſomewhat more ſtrange and more hard, then are the diffinitions and principles of the other bookes of *Euclide* going before, and therefore at the firſt ſight or reading are not ſtraight way conceiued, but ought often to be repeted, and by vſe to be confirmed. For the propoſitions following, bring vnto them much light, and facilitie of vnderſtanding. Firſt there is a line ſuppoſed, and layd before vs, which may be any line whatſoeuer, of what length, or ſhortnes ye will: this line thus firſt ſuppoſed is imagined to haue ſuch diuiſions and ſo many partes as ye liſt to conceiue in minde, as 3. 4. 5. and ſo forth, which may be applied to any kinde of meaſure, as it ſhall happen, as to inches, feete, paſes, and ſuch other. Vnto this line ſaith *Euclide* may be cõpared innumerable lines, of which ſome ſhalbe commenſurable, and other ſome incommenſurable: of commenſurable lines, ſome are commenſurable both in length and in power, other ſome are cõmenſurable in power onely. As if any part of the line propoſed which let be the line AB, doo meaſure alſo the line EF and againe if any one ſuperficies do meaſure the ſquare of the line AB, which let be the ſquare ABCD: and alſo doo meaſure the ſquare of the line EF, which let be the ſquare EFGH: thẽ is the line EF to the ſuppoſed line & firſt ſet, namely, to the line AB, a line commenſurable both in length and power, as ye may ſee in the firſt example here ſet. And if it ſo be that one & the ſelfe ſame ſuperficies do meaſure both the ſquares of theſe two lines AB and EF, namely the ſquares ABCD and EFGH: and no one line do meaſure the lines AB and EF: then is the line

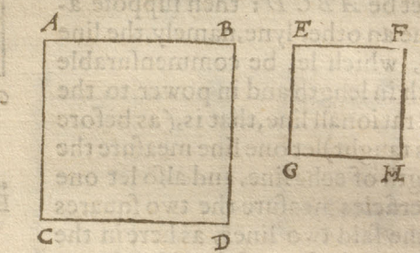


EF

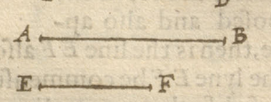
EF (compared to the ſuppoſed and firſt line AB) commenſurable in power onely. As in this example ye may eaſely perceiue. For the triangle or ſuperficies A C D, twiſe taken, meaſureth the ſquare A B C D, namely, the ſquare of the line A B: and the ſelfe ſame triangle A C D taken foure times meaſureth the other ſquare, namely, the ſquare of the line E F. But no one meaſure or line can be aſſigned to meaſure both the lines A B and E F, becauſe the ſides of a ſquare and the diameter are incommenſurable in length the one to the other, as afterward ſhalbe ſhewed. Wherefore they are in length incommenſurable, & commenſurable in power onely, that is by reaſon of their ſquares, which are commenſurable the one to the other.



Agayne if it ſo be that no one line may be found to be a cõmon meaſure, to meaſure both the firſt line, namely, A B, and alſo the line E F: nor yet any one ſuperficies to meaſure the ſquare or powers of theſe two lines, then is the line E F, to the firſt line ſet and ſuppoſed, incommenſurable both in length and in power. As is ſuppoſed to be in this example.



Thus may ye ſee, how to the ſuppoſed line firſt ſet may be compared infinite lines, ſome commenſurable both in length & power, and ſome commenſurable in power onely, and incommenſurable in length, and ſome incommenſurable both in power & in length. And this firſt line ſo ſet, whereunto, and to whoſe ſquares the other lines and their ſquares are compared, is called a rationall line, commonly of the moſt part of writers. But ſome there are, which miſſlike that it ſhould be called a rationall line, & that not without iuſt cauſe. In the Greeke copy it is called ῥητὴ, *rete*, which ſignifieth a thing that may be ſpokẽ, & expreſſed by word, a thing certayne, graunted and appoynted. Wherefore *Fluſſates*, a man which beſtowed great trauell and diligence in reſtoring of theſe elementes of *Euclide*, leauing this word rationall, calleth this line ſuppoſed and firſt ſet, a line certaine, becauſe the partes thereof into which it is deuided are certaine, and known, and may be expreſſed by voyce, and alſo be coumpted by number: other lines, being to this line incommenſurable, whoſe parts are not diſtinctly known, but are vncertayne, nor can be expreſſed by name nor aſſignd by number, which are of other men called irrationall, he calleth vncertaine and ſurd lines. *Petrus Montaureus* although he doth not very wel like of the name, yet he altereth it not, but vſeth it in al his booke. Likewiſe wil we doo here, for that the word hath bene and is ſo vniuerſally receiued. And therefore will we vſe the ſame name, and call it a rationall line. For it is not ſo great a matter what names we geue to thinges, ſo that we fully vnderſtand the thinges which the names ſignifie.



Vnto the ſuppoſed line firſt ſet may be compared infinite lines.

Why ſome miſlike that the line firſt ſet ſhould be called a rationall line.

Fluſſates calleth this line, a line certaine.

This rationall line thus here defined, is the ground and foundation of all the propoſitions almoſt of this whole tenth booke. And chiefly from the tenth propoſition forwardes. So that vnleſſe ye firſt place this rationall line, and haue a ſpeciall and continuall regard vnto it before ye begin any demonſtration, ye ſhall not eaſely vnderſtand it. For it is as it were the touch and triall of all other lines, by which it is known whether any of them be rationall or not. And this may be called the firſt rationall line, the line rationall of purpoſe or a rationall line ſet in the firſt place, and ſo made diſtinct and ſeuered from other rationall lines, of which ſhall be ſpoken afterwarde. And this muſt ye well commit to memory.

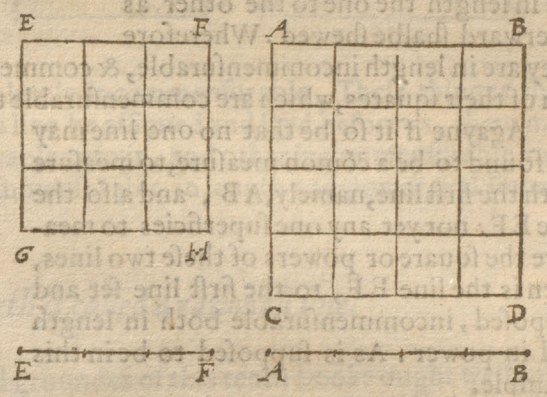
This rational line the groũd in a maner of all the propoſitions in this tenth booke.

Note. The line Rationall of purpoſe.

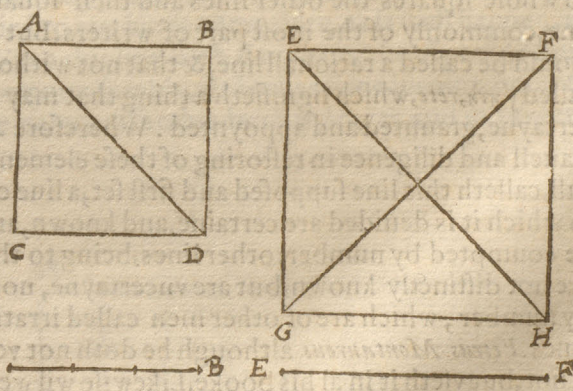
The ſixth definition.

6 Lines which are commenſurable to this line, whether in length and power, or in power onely, are alſo called rationall.

This definition needeth no declaration at all, but is eaſily perceiued, if the firſt definition be remembred, which ſheweth what magnitudes are commenſurable, and the third, which ſheweth what lines are commenſurable in power. Here note, how aptly & naturally, Euclide in this place vſeth theſe wordes commenſurable either in length and power, or in power onely. Becauſe that all lines which are commenſurable in length, are alſo commenſurable in power: when he ſpeaketh of lines commenſurable in lēgth, he euer addeth and in power, but when he ſpeaketh of lines commenſurable in power, he addeth this worde Onely, and addeth not this worde in length, as he in the other added this worde in power. For not all lines which are commenſurable in power, are ſtraight way commenſurable alſo in length. Of this definition, take this example. Let the firſt line rationall of purpoſe, which is ſuppoſed and laide forth, whoſe partes are certaine & known, and may be expreſſed, named, and nūbred be *AB*, the quadrate wherof let be *ABCD*: then ſuppoſe againe an other lyne, namely, the line *EF*, which let be commenſurable both in length and in power to the firſt rationall line, that is, (as before was taught) let one line meaſure the length of eche line, and alſo let one ſuperficies meaſure the two ſquares of the ſaid two lines, as here in the example is ſuppoſed and alſo appeareth to the eie, then is the line *EF* alſo a rationall line.



Moreouer if the lyne *EF* be commenſurable in power onely to the rationall line *AB* firſt ſet and ſuppoſed, ſo that no one line do meaſure the two lines *AB* and *EF*: As in example ye ſee to be (for that the line *EF*, is made equall to the line AD, which is the dianeter of the ſquare *ABCD*, of which ſquare the line *AB* is a ſide, it is certayne that the ſide of a ſquare is incōmēſurable in lēgth to the diameter of the ſame ſquare: if there be yet founde any one ſuperficies, which meaſureth the two ſquares *ABCD*, and *EFGH* (as here doth the triangle *ABD*, or the triangle *ACD* noted in the ſquare *ABCD*, or any of the foure triangles noted in the ſquare *EFGH*, as appeareth ſomwhat more manifeſtly in the ſecond example,



in the declaration of the laſt definition going before) the line *EF* is alſo a rational line. Note that theſe lines which here are called rationall lines, are not rational lines of purpoſe, or by ſuppoſition, as was the firſt rationall line, but are rationall onely by reaſon of relation and compariſon which they haue vnto it, becauſe they are commenſurable vnto it either in length and power, or in power onely. Farther here is to be noted, that theſe wordes length, and power, and power onely, are ioyned onely with theſe wordes commenſurable or incommenſurable, and are neuer ioyned with theſe woordes rationall or irrationall. So that no lines can be called rational in length, or in power, nor like wiſe can they be called irrationall in length, or in power. Wherin vndoubtedly Campanus was deceiued, who vſing thoſe wordes & ſpeaches indifferently, cauſed & brought in great obſcuritie to the propoſitions and demonſtrations of this boke, which he ſhall

Campanus hath cauſed much obſcuritie in this tenth booke.

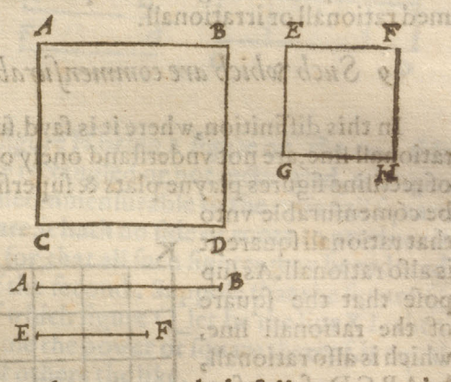
eaſily

easily see which marketh with diligence the demonstrations of *Campanus* in this booke.

7 *Lines which are incommensurable to the rationall line, are called irrationall.*

The seuenth definition.

By lines incommensurable to the rationall line supposed in this place, he vnderstandeth such as be incommensurable vnto it both in length and in power. For there are no lines incommensurable in power onely: for it cannot be that any lines should so be incommensurable in power onely, that they be not also incommensurable in length. What so euer lines be incommensurable in power, the same be also incommensurable in length. Neither can *Euclide* here in this place meane lines incommensurable in length onely, for in the diffinition before, he called them rationall lines, neither may they be placed amongst irrationall lines. Wherfore it remayneth that in this diffintion he speaketh onely of those lines which are incommensurable to the rationall line first geuen and supposed, both in length, and in power. Which by all meanes are incommensurable to the rationall line, & therfore most aptly are they called irrationall lines. This diffinition is easy to be vnderstanded by that which hath bene sayd before. Yet for the more plainenes see this example. Let the first rationall line supposed, be the line AB, whose square or quadrate, let be ABCD. And let there be geuen an other line EF which let be to the rationall line incommensurable in length and power, so that let no one line measure the length of the two lines, AB and EF: and let the square of the line EF be EFGH. Now if also there be no one superficies which measureth the two squares ABCD, and EFGH, as is supposed to be in this example, thē is the line EF an irrationall line, which word irrational (As before did this word rational) misliketh many learned in this knowledge of Geometry. *Flussates*, as he left the word rationall, and in steade thereof vsed this word certaine, so here he leaueth the word irrationall, and vseth in place thereof this word *vncertaine*, and euer nameth these lines vncertaine lines. *Petrus Montaureus* also misliking the word irrationall, would rather haue them to be called surd lines, yet becawse this word irrationall hath euer by custome and long vse, so generally bene receiued, he vseth continually the same. In Greeke such lines are called ἄλογοι, *alogoi*, which signifieth nameles, vnspeakeable, vncertayne, indeterminate, and with out proportion: not that these irrationall lines haue no proportion at all, either to the first rationall line, or betwene them selues: but are so named, for that theyr proportions to the rationall line cannot be expressed in number. That is vndoubtedly very vntrue, which many write, that their proportions are vnknowne both to vs and to nature. Is it not thinke you a thing very absurd to say that there is any thing in nature, and produced by nature, to be hidde from nature, and not to be knowne of nature? it can not be sayd that their proportions are vtterly hidde and vnknowne to vs (much lesse vnto nature) although we cannot geue them their names, and distinctly expresse them by numbers: otherwise should *Euclide* haue taken all this trauell and wonderfull diligence bestowed in this booke, in vaine and to no vse: in which he doth nothing ells but teach the proprieties and passions of these irrationall lines, and sheweth the proportions which they haue the one to the other.



Flussates in steede of this word irrationall vseth this word vncertayne.

Why they are called irrationall lines.

Here is also to be noted, which thing also *Tartalea* hath before diligently noted, that *Campanus* and many other writers of Geometry, ouer much erred and were deceiued in that they wrote and taught, that all these lines whose squares were not signified and mought be expressed by a square number (although they might by any other number: as by 11. 12. 14. and such others not square numbers) are irrationall lines. Which is manifestly repugnant to the groundes and principles of *Euclide*, who wil, that all lines which are commensurable to the rationall line, whether it be in length and

power

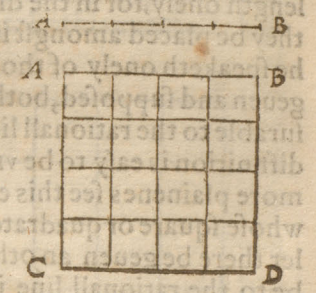
The cauſe of the obſcurity and confuſednes in this booke.

power, or in power onely, ſhould be rationall. Vndoubtedly this hath bene one of the chiefeſt and greateſt cauſes of the wonderfull confuſion and darkenes of this booke, which ſo hath toſſed, and tormoyled the wittes of all both writers and readers, maſters and ſcholers, and ſo ouerwhelmed them, that they could not without infinite trauell and ſweate, attayne to the truth and perfect vnderſtanding thereof.

The eighth definition.

8 *The ſquare which is deſcribed of the rationall right line ſuppoſed, is rationall.*

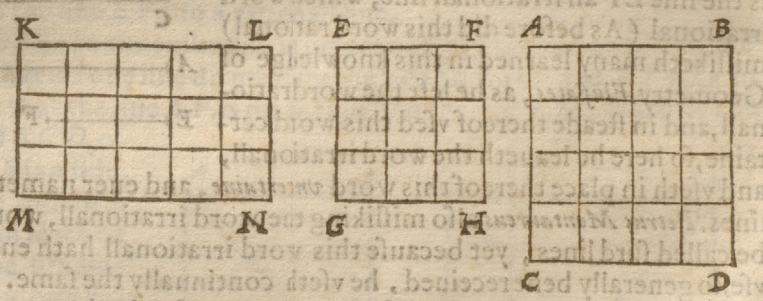
Vntill this diffinition hath *Euclide* ſet forth the nature and proprietie of the firſt kinde of magnitude, namely, of lines how they are rationall or irrationall, now he beginneth to ſhew how the ſecond kinde of magnitudes, namely ſuperficies are one to the other rationall or irrationall. This diffinition is very playne. Suppoſe the line A B, to be the rationall line, hauing his parts and diuiſions certaynely knowne: the ſquare of which line let be the ſquare A B C D. Now becauſe it is the ſquare of the rationall line A B, it is alſo called rationall: and as the line A B, is the firſt rationall line, vnto which other lines compared are coumpted rationall, or irrationall, ſo is the quadrat or ſquare thereof, the firſt rationall ſuperficies vnto which all other ſquares or figures compared, are coumpted and named rationall or irrationall.



The ninth definition.

9 *Such which are commenſurable vnto it, are rationall.*

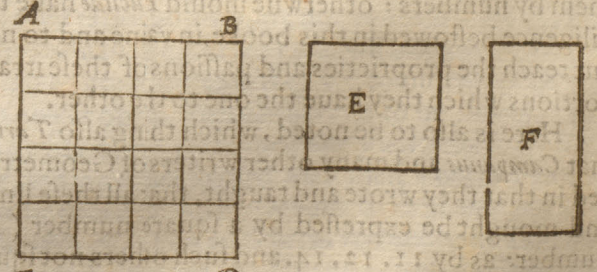
In this diffinition, where it is ſayd, ſuch as are commenſurable to the ſquare of the rationall line, are not vnderſtand onely other ſquares or quadrates, but all other kindes of rectiline figures playne plats & ſuperficieſes. What ſo euer ſo that if any ſuch figure be cõmenſurable vnto that rationall ſquare, it is alſo rationall. As ſuppoſe that the ſquare of the rationall line, which is alſo rationall, be A B C D: ſuppoſe alſo ſome other ſquare as the ſquare E F G H, to be commenſurable to the ſame: thẽ is the ſquare E F G H alſo rational. So alſo if the rectiline figure K L M N, which is a figure on the one ſide longer, be commenſurable vnto the ſayd ſquare as is ſuppoſed in this example, it is alſo a rational ſuperficies and ſo of all other ſuperficieſes.



The tenth definition.

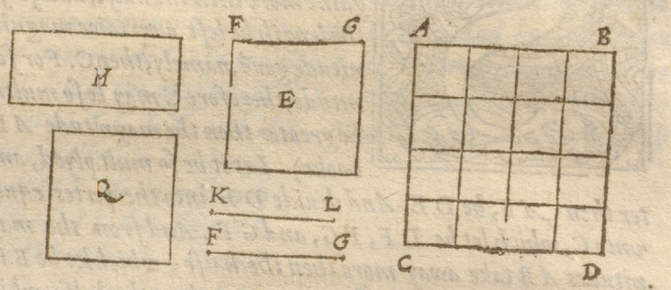
10 *Such which are incommenſurable vnto it, are irrationall.*

Where it is ſayd in this diffinition ſuch which are incommenſurable, it is generally to be taken: as was this word cõmenſurable in the diffinitiõ before. For al ſuperficieſes, whether they be ſquares or figures on the one ſide longer, or otherwiſe what maner of right lined figure ſo euer it be, if they be incommenſurable vnto the rationall ſquare ſuppoſed, thẽ are they irra-



irrationall. As let the ſquare A B C D be the ſquare of the ſuppoſed rationall line which ſquare therefore is alſo rationall: ſuppoſe alſo alſo an other ſquare, namely the ſquare E, ſuppoſe alſo any other figure, as for example ſake a figure of one ſide longer, which let be *F*. Now if the ſquare E and the figure F, be both incommenſurable to the rationall ſquare A B C D, then is ech of theſe figures E & F irrationall. And ſo of other.

11 And theſe lines whoſe poweres they are, are irrationall. If they be ſquares, then are their ſides irrationall. If they be not ſquares, but ſome other rectiline figures, then ſhall the lines, whoſe ſquares are equall to theſe rectiline figures, be irrationall. The eleuenth diffinition.



Suppoſe that the rationall ſquare be ABCD. Suppoſe alſo an other ſquare, namely the ſquare E, which let be incōmēſurable to the rationall ſquare, & therefore is it irrationall: and let the ſide or line which produceth this ſquare be the line F G: then ſhall the line F G by this diffinition be an irrationall line: becauſe it is the ſide of an irrationall ſquare. Let alſo the figure H being a figure on the one ſide longer (which may be any other rectiline figure rectangled or not rectangled, triangle, pentagone, trapezite, or what ſo euer elſs) be incommenſurable to the rationall ſquare A B C D, then becauſe the figure H is not a ſquare, it hath no ſide or roote to produce it yet may there be a ſquare made equall vnto it: for that all ſuch figures may be reduced into triangles, and ſo into ſquares, by the 14. of the ſecond. Suppoſe that the ſquare Q be equall to the irrationall figure H. The ſide of which figure Q let be the line K L: then ſhall the line K L be alſo an irrational line, becauſe the power or ſquare thereof, is equal to the irrationall figure H: and thus conceiue of others the like.

Theſe irrationall lines and figures are the chiefeſt matter and ſubiect, which is entreated of in all this tenth booke: the knowledge, of which is deepe, and ſecret, and pertaineth to the higheſt and moſt worthy part of Geometrie, wherein ſtandeth the pith and mary of the hole ſcience: the knowlede hereof bringeth light to all the bookes following, with out which they are hard and cannot be at all vnderſtoode. And for the more plainenes, ye ſhall note, that of irrationall lines there be diuers ſortes and kindes. But they, whoſe names are ſet in a table here following, and are in number 13. are the chiefe, and in this tēth boke ſufficiently for Euclides principall purpoſe, diſcourſed on.

1. A mediall line.
2. A binomiall line.
3. A firſt bimediall line.
4. A ſecond bimediall line.
5. A greater line.
6. A line containing in power a rational ſuperficies and a mediall ſuperficies.
7. A line containing in power two mediall ſuperficieces.
8. A reſiduall line.
9. A firſt mediall reſiduall line.
10. A ſecond mediall reſiduall line.
11. A leſſe line.
12. A line making with a rationall ſuperficies the whole ſuperficies mediall.
13. A line making with a mediall ſuperficies the whole ſuperficies mediall.

Of all which kindes the diffinitions together with there declarations ſhalbe ſet here after in their due places.

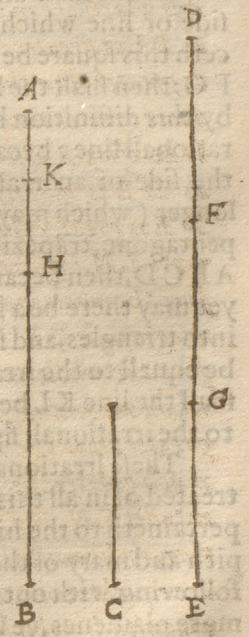
¶ The 1. Theoreme. The 1. Proposition.

Two vnequall magnitudes being geuen, if from the greater be taken away more then the halfe, and from the residue be againe taken away more then the halfe, and so be done still continually, there shall at length be left a certaine magnitude lesser then the lesse of the magnitudes first geuen.

Construction.

Demonstration.

Vppose that there be two vnequall magnitudes A B, and C, of which let A B be the greater. Then I say, that if from A B, be taken away more then the halfe, and from the residue be taken againe more then the halfe, and so still continually, there shall at the length be left a certaine magnitude, lesser then the lesse magnitude geuē, namely, then C. For forasmuch as C is the lesse magnitude, therefore C may be so multiplyed, that at the length it will be greater then the magnitude A B (by the 5. definition of the fift booke). Let it be so multiplyed, and let the multiplex of C greater then A B, be D E. And deuide D E into the partes equall vnto C, which let be D F, F G, and G E. And from the magnitudes A B take away more then the halfe, which let be B H: and againe from A H, take away more then the halfe, which let be H K: And so do continually vntill the diuisions which are in the magnitude A B, be equall in multitude vnto the diuisions which are in the magnitude D E. So that let the diuisions A K, K H, and H B, be equall in multitude vnto the diuisions D F, F G, and G E. And forasmuch as the magnitude D E is greater then the magnitude A B, and from D E is taken away lesse then the halfe, that is, E G (which detraction or taking away is vnderstand to be done by the former diuision of the magnitude D E into the partes equall vnto C: for as a magnitude is by multiplication increased, so is it by diuision diminished) and from A B is taken away more then the halfe, that is, B H: therefore the residue G D is greater then the residue H A (which thing is most true and most easie to conceaue, if we remēber this principle, that the residue of a greater magnitude, after the taking away of the halfe or lesse then the halfe, is euer greater then the residue of a lesse magnitude, after the taking away of more then the halfe). And forasmuch as the magnitude G D is greater then the magnitude H A, and from G D is taken away the halfe, that is, G F: and from A H is taken away more then the halfe, that is, H K: therefore the residue D F is greater then the residue A K (by the foresayd principle). But the magnitude D F is equall vnto the magnitude C (by supposition). Wherefore also the magnitude C is greater then the magnitude A K. Wherefore the magnitude A K is lesse then the magnitude C. Wherefore of the magnitude A B is left a magnitude A K lesse then the lesse magnitude geuen, namely, then C: which was required to be proued. In like sort also may it be proued if the halfes be taken away.



❊ A Corollary.

A Corollary.

Of this Proposition it followeth, that any magnitude being geuen how litle soeuer it be, there may be geuen a magnitude lesse then it: so that it is impossible that any magnitude

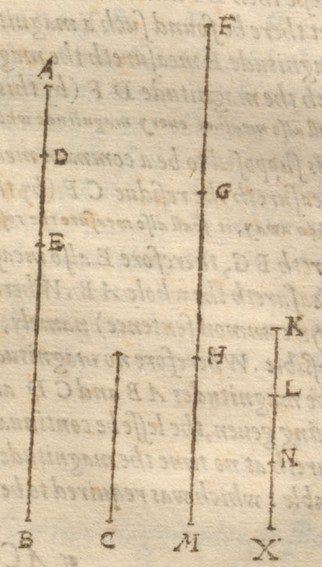
nitude ſhould be geuen then which can be geuen no leſſe.

¶ An other demonſtration of the ſame.

Suppoſe that the two vnequall magnitudes geuen be A B and C. And let C be the leſſe. And foraſmuch as C is the leſſe, therefore C may ſo be multiplyed, that it ſhall at the length be greater then A B. Let it be ſo multiplyed, and let the multiplex of C exceding A B be the magnitude F M. And deuide F M into his partes equall vnto C, that is, into the magnitudes M H, H G, and G F. And from A B take away more then the halfe, which let be the magnitude B E: and likewiſe from E A take away againe more then the halfe, namely, the magnitude E D. And thus do continually vntill the diuiſions which are in the magnitude F M, be equall in multitude to the diuiſions which are in the magnitude A B: and let thoſe diuiſions be the magnitudes B E, E D, and D A. And how multiplex the magnitude F M is to the magnitude C, ſo multiplex let the magnitude K X be to the magnitude D A. And deuide the magnitude K X into the magnitudes equall to the magnitude D A: which let be K L, L N, and N X. Now then the diuiſions which are in the magnitude K X, are equall vnto the diuiſions which are in the magnitude M F. And foraſmuch as B E is greater then the halfe of A B, therefore B E is greater then the reſidue E A. Wherefore B E is much more greater then D A. But D A is equall vnto X N. Wherefore B E is greater then X N. Againe foraſmuch as D E is greater then the halfe of E A, therefore D E is greater then the reſidue D A: but D A is equall vnto L N: wherefore D E is greater then L N. Wherfore the whole magnitude D B is greater thē the whole magnitude X L. But D A is equall vnto L K. Wherefore the whole magnitude A B is greater then the whole magnitude K X. And the magnitude M F is greater then the magnitude B A: wherefore M F is much greater thē K X. And foraſmuch as thoſe magnitudes X N, N L, & L K, are equall the one to the other, & likewiſe theſe magnitudes M H, H G, and G F, are equal the one to the other, & the multitude of thoſe magnitudes which are in M F, is equall to the multitude of thoſe magnitudes which are in K X: therfore as K L is to F G, ſo is L N to G H, and N X to H M. Wherefore (by the 12. of the fift) as one of the antecedentes, namely, K L, is to one of the conſequentes, namely, to F G, ſo are all the antecedentes, namely, the whole K X to all the conſequentes, namely, to the whole F M. But F M is greater then K X. Wherefore F G is greater then L K. But F G is equall vnto C: and K L vnto D A (by ſuppoſition). Wherefore the magnitude C is greater then the magnitude A D: which was required to be proued.

Conſtruction.

Demonſtration.

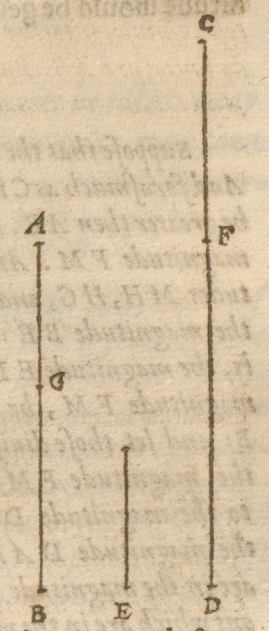


¶ The 2. Theoreme. The 2. Propoſition.

Two vnequall magnitudes being geuen, if the leſſe be continually taken from the greater, & that which remayneth meaſureth at no time the magnitude going before: then are the magnitudes geuen incommenſurable.

This proposition teacheth that incontinuall quantitie which the first of the seuenth taught in discrete quantity.

Supposse that there be two vnequall magnitudes A B, and C D, and let A B be the lesse: and taking away continually by a certaine alternate detraction the lesse from the greater, let not the residue measure the magnitude going before. Then I say, that those two magnitudes A B and C D, are incommensurable. For if they be commensurable, then (by the first definition of the tenth) some one magnitude shall measure them both: Let there be such a magnitude, if it be possible, and let the same be E. And let A B measuring D F, leaue a lesse then it selfe, namely, C F (that is, from the greater magnitude C D, take away a certayne part as D F, which let be equall to A B, or if it be not equall vnto it, yet let it be such, that that lesse magnitude A B being more then once repeated may make the magnitude D F: For this is the meaning of this *let A B measuring D F, &c.* And this detraction made, of the lesse I say out of the greater, let there be left of the greater a certaine portion C F, lesse then the magnitude A B. And this is the meaning of that which in the Theoreme was said, *And that which remaineth measureth at no time the magnitude going before*). Likewise let C F measuring B G leaue a lesse then it selfe, namely, A G: and do this continually as often as neede requireth, vntill there be found such a magnitude that is lesse then E: which must needes at the length happen (by the Proposition going before). Let there be found such a magnitude lesse then E, which let be A G. And forasmuch as the magnitude E measureth the magnitude A B, but A B measureth D F: therefore E measureth the magnitude D F (by this common sentence, *If a magnitude measure an other magnitude it shall also measure euery magnitude whom it measureth*). And it measureth also the whole C D (for it is supposed to be a common measure to the magnitudes A B and C D). Wherefore also it measureth the residue C F (by this common sentence, *If a maguitude measure an whole and a part taken away, it shall also measure the residue*). And forasmuch as E measureth C F, but C F measureth B G, therefore E also measureth B G (by the first foresaid common sentence): and it measureth the whole A B. Wherefore it shall also measure the residue A G (by the other foresayd common sentence) namely, the greater magnitude shall measure the lesse: which is impossible. Wherefore no magnitude measureth these magnitudes A B and C D. Wherefore the magnitudes A B and C D are incommensurable. If therefore two vnequall magnitudes being geuen, the lesse be continually taken from the greater, and that which remayneth measureth at no time the magnitude going before, then are the magnitudes geuen incommensurable: which was required to be demonstrated.



Construction.

Demonstration leading to an absurditie.

¶ A Corollary added by Montaureus.

By this proposition it is manifest, that if the two vnequall magnitudes geuen be not incommensurable, but commensurable, then the lesse being continually subtrahed out of the greater, the residue shall of necessitie measure that which went before.

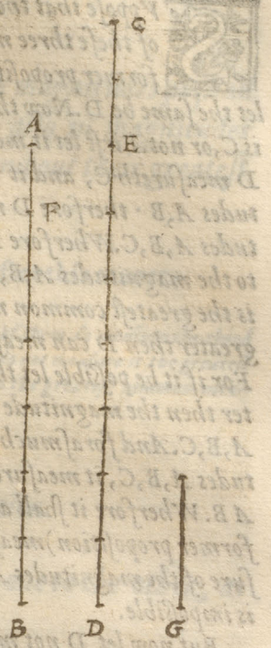
¶ The 1. Probleme. The 3. Proposition.

Two magnitudes commensurable being geuen, to finde out their greatest common measure.

Two cases in this propositiõ. The first case.

Supposse that the two commensurable magnitudes geuen be A B & C D, of which let A B be the lesse. It is required to finde out the greatest common measure of the magnitudes A B and C D. Now A B either measureth C D, or not. If it measure it, and seing it also measureth it selfe, wherfore A B is a common measure vnto the

the magnitudes A B and C D. And it is manifest that it is the greatest common measure to them. For no magnitude greater then A B can measure A B. But now suppose that A B do not measure C D. And taking continually the lesse from the greater that which remaineth shall at length measure that which goeth before (by the corollary before added), for that A B and C D are cõmensurable. Now then let A B measuring E D which is a part of the magnitude C D, leaue a magnitude lesse then it selfe, namely, E C. And let E C measuring the magnitude F B, which is a part of the magnitude A B, leaue a lesse thẽ it selfe, namely, F A, and let F A precisely measure the magnitude C E (And this is the meaning of this, *That which remayneth shall at the length measure that which goeth before*, when there is nothing left after the measuring made) And forasmuch as A F measureth the magnitude C E, but C E measureth F B: wherfore A F also measureth F B, & it measureth it selfe: wherfore A F measureth also the whole A B. But A B measureth D E, wherfore A F also measureth D E. And A F also measureth C E, wherfore it measureth the whole magnitude C D. Wherfore the magnitude A F measureth both the magnitudes A B and C D. Wherfore A F is a common measure vnto A B and C D. I say also that it is the greatest cõmon measure vnto them. For if not, then is there some magnitude greater thẽ the magnitude A F which measureth both the magnitudes A B and C D. Let there be such a one if it be possible, and let the same be the magnitude G. And forasmuch as G measureth A B, and A B measureth E D: therefore G also measureth E D, and by supposition, it measureth the whole C D, wherfore G measureth also the residue C E. But C E measureth F B, wherfore G also measureth F B. And by supposition it measureth the whole A B. Wherfore it measureth the residue A F, namely, the greater magnitude the lesse, which is impossible. Wherefore no magnitude greater then A F, measureth these magnitudes A B and C D. Wherefore A F is the greatest common measure vnto the magnitudes A B and C D. Wherfore vnto two commensurable magnitudes geuen, namely, A B and C D is found out their greatest common measure, namely, the magnitude A F: which was required to be done.



This proposition teacheth, that in continuat quantity, which the 2. of the seuẽth taught in numbers. The second case.

Demonstration leading to an absurditie.

¶ Corollary.

Hereby it is manifest that if a magnitude measure two magnitudes, it shall also measure their greatest common measure. For if it measure the wholes and the partes taken away, it shal also measure the residues, of which one is the greatest common measure, as we may see by the latter ende of the former demonstration.

A Corollary.

Montaureus reduceth this Probleme into a Theoreme after this maner.

Two vnequall and commensurable magnitudes being geuen, if the lesse do measure the greater, then is it the greatest common measure to them both. But if not, then the lesse being continually by a mutuall detraction (as before hath bene taught) taken out of the greater, whensoeuer the residue precisely measureth that which went before, leauing nothing, the said residue shall be the greatest common measure to both the magnitudes geuen.

This Probleme reduced to a Theoreme.

¶ The 2. Probleme. The 4. Proposition.

Three magnitudes commensurable beyng geuen, to finde out their greatest common measure.

This proposition teacheth, that in continual quantity, which the 3. of the second taught in numbers.

Construction.

Two cases in this Proposition.

The first case.

Demonstration leading to an absurditie.

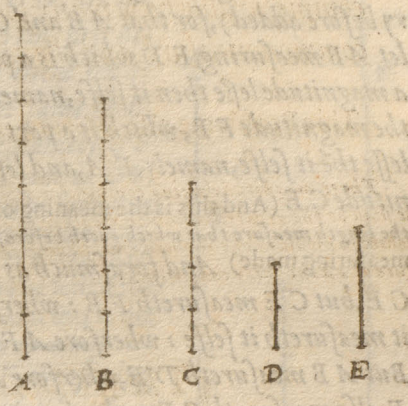
The second case.

A Lemma necessary to be proued before he fall to the demōstration.

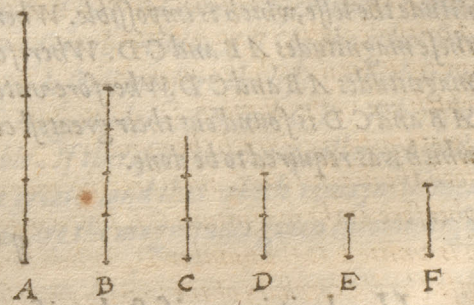
Construction.

Demonstration leading to an absurditie.

Suppose that the three commensurable magnitudes geuen be A,B,C. It is required of these three magnitudes to finde out the greatest common measure. Take (by the former proposition) the greatest common measure of the two magnitudes A,B and let the same be D. Now then this magnitude D either measureth the third magnitude, which is C, or not. First let it measure C. And forasmuch as D measureth C, and it measureth also the magnitudes A,B · therfore D measureth the three magnitudes A,B,C. Wherfore D is a common measure vnto the magnitudes A,B,C. And it is manifest, that it is the greatest common measure. For no magnitude greater then D can measure the magnitudes A,B,C. For if it be possible let the magnitude E being greater then the magnitude D, measure the magnitude A,B,C. And forasmuch as E measureth the magnitudes A,B,C, it measureth the two first magnitudes A B. Wherfore it shall also (by the Corollary of the former proposition) measure the greatest cōmon measure of the magnitudes A,B which is D, namely, the greater shall measure the lesse, which is impossible.



But now let D not measure the magnitude C. First I say that the magnitudes C,D, are commensurable. For forasmuch as the magnitudes A,B,C are commensurable, some magnitude shall measure them, which shall also measure the magnitudes A B taken a part. Wherfore it shall also (by the corollary going before) measure the greatest common measure of A,B, that is, D: and (by supposition) it measureth the magnitude C. Wherfore that said magnitude shall measure the magnitudes C and D. Wherfore the magnitudes C,D are commēsurable. Take by the third of this tenth, their greatest common measure, & let the same be the magnitude E. And forasmuch as E measureth D, but D measureth the magnitudes A,B. Wherfore E also measureth the magnitudes A,B, and it also measureth the magnitude C. Wherfore E is a common measure to the three magnitudes A,B,C. I say also that it is the greatest commō measure. For if it be possible, let there be a magnitude, namely, F, greater then the magnitude E. And let F measure the three magnitudes A,B,C. And forasmuch as F measureth the magnitudes A,B,C, it also measureth the two first magnitudes A,B. Wherfore (by the corollary going before) it shall also measure the greatest common measure of the magnitudes A,B. But the greatest common measure of the magnitudes A,B, is D. Wherfore F measureth D, and it also measureth C. Wherfore F measureth the magnitudes C D. Wherfore F shall also measure the greatest common measure of the magnitudes C,D. But the greatest common measure of the magnitudes C,D, is E. Wherfore F measureth E, namely, the greater, the lesse, which is impossible. Wherfore no magnitude greater then E measureth the magnitudes A,B,C. Wherfore E is the greatest common measure of the magnitudes A B C. If D do not measure the magnitude C. But if D do measure C, then is D the greatest commō measure. Wherfore three magnitudes commensurable being geuen, there is found their greatest common measure: which was required to be done.



Corollary.

¶ Corollary.

Hereby it is manifeſt, that if a magnitude meaſure three magnitudes, it ſhall alſo meaſure their greateſt common meaſure. In like ſort alſo in magnitudes commẽſurable how many ſoeuer being geuẽ, may be found out their greateſt common meaſure, and the corollary will euer be true.

A Corollary.

This Probleme alſo *Montaureus* reduceth into a Theoreme after this maner.

This Probleme reduced to a Theoreme.

Three magnitudes being commenſurable, if the greateſt common meaſure to two of them, do meaſure the third it ſhall be the greateſt common meaſure to all the three magnitudes geuen. But if it do not meaſure it, the greateſt common meaſure of the third and of the greateſt common meaſure of the two firſt, is the greateſt common meaſure of all the three magnitudes.

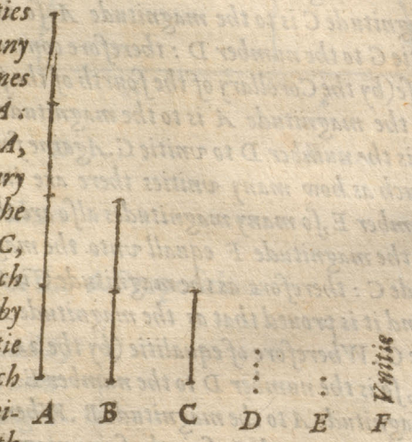
¶ *The 3. Theoreme.* *The 5. Propoſition.*

Magnitudes commenſurable, haue ſuch proportion the one to the other, as number hath to number.

Vppoſe that A and B be magnitudes cõmenſurable. Then I ſay that A hath vnto B ſuch proportion as nũber hath to number. For foraſmuch as A and B are cõmenſurable, therfore ſome magnitude meaſureth them, let there be a magnitude that meaſureth them, and let the ſame be C. And how often C meaſureth A, ſo many vnities let there be in the number D. And how often C meaſureth B, ſo many vnities let there be in the number E, and let F be vnitie. And foraſmuch as the magnitude C meaſureth the magnitude A by thoſe vnities which are in the number D, and vnitie F meaſureth the number D by thoſe vnities which are in the number D: therefore how many times vnitie meaſureth the number D, ſo many times doth the magnitude C meaſure the magnitude A: wherfore as the magnitude C is to the magnitude A, ſo is vnitie F to the number D. Wherefore contrary wiſe (by the corollary of the fourth propoſition of the fift booke) as the magnitude A is to the magnitude C, ſo is the number D to vnitie F. Againe foraſmuch as the magnitude C meaſureth the magnitude B by thoſe vnities which are in the number E, and vnitie F meaſureth the number E by thoſe vnities which are in the number E: therfore how many times vnitie F meaſureth the number E, ſo many tymes doth the magnitude C meaſure the magnitude B. Wherfore as the magnitude C is to the magnitude B, ſo is vnitie F to the number E, and it is proued that as the magnitude A is to the magnitude C, ſo is the number D to vnitie F. Wherfore of equalitie (by the 22. of the fifte) as the magnitude A is to the magnitude B, ſo is the number D to the number E. Wherefore the commenſurable magnitudes A and B haue that proportion the one to the other that the number D hath to the number E. Magnitudes therfore commenſurable, haue ſuch proportion the one to the other, as number hath to number: which was required to be proued.

Conſtruction.

Demonſtration.



How magnitudes are sayd to be in proportion the one to the other, as number is to number.

Magnitudes are sayd to haue such proportion the one to the other, as nũber hath to number, when as what soeuer proportion is betwene those magnitudes, the same is found betwene some certaine numbers: as if a magnitude be vnto a magnitude either equall, as the number 2. is to the number 2, or double, as the number 4. to the number 2, or triple, as 6. to 2, or in any other multiplex proportion. And so also touching the other kindes of proportion, either superparticular, or superpartient.

¶ The 4. Theoreme. The 6. Proposition.

If two magnitudes haue such proportion the one to the other, as number hath to number: those magnitudes are commensurable.

This proposition is the conuerse of former.

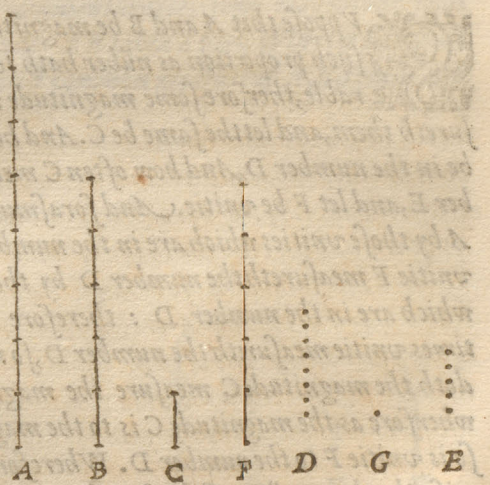
Supposse that these two magnitudes A and B haue that proportion the one to the other, that the number D hath to the number E. Then I say, that the magnitudes A, B, are commensurable. How many vnities there are in the number D, into so many equall partes deuide the magnitude A (by the 9. of the sixt) and let the magnitude C be equall to one of the partes therof. And how many vnities there are in the number E, of so many magnitudes equall vnto the magnitude C let the magnitude F be cõposed.

Construction.

And let G be vnitie.

Demonstration.

Now forasmuch as how many vnities there are in the nũber D, so many magnitudes also are there in the magnitude A equall vnto the magnitude C: therfore what part vniti G is of the nũber D, the same part is the magnitude C of the magnitude A. Wherefore as the magnitude C is to the magnitude A, so is vnitie G to the number D. But vnitie G measureth the number D. Wherefore the magnitude C also measureth the magnitude A. And for that as the magnitude C is to the magnitude A, so is vnitie G to the number D: therefore contrarywise (by the Corollary of the fourth of the fift) as the magnitude A is to the magnitude C, so is the number D to vnitie G. Againe forasmuch as how many vnities there are in the number E, so many magnitudes also are there in the magnitude F equall vnto the magnitude C: therefore as the magnitude C is to the magnitude F, so is vnitie G to the number E. And it is proued that as the magnitude A is to the magnitude C, so is the number D to vnitie G. Wherefore of equalitie (by the 22. of the fift) as the magnitude A is to the magnitude F, so is the number D to the number E. But as the number D is to the number E, so is the magnitude A to the magnitude B. Wherefore (by the 11. of the fift) as the magnitude A is to the magnitude B, so is the same magnitude A to the magnitude F: wherefore A hath vnto either of these magnitudes B and F one and the same proportion. Wherefore (by the 9. of the fift) the magnitude B is equall vnto the magnitude F. But the magnitude C measureth the magnitude F: wherefore it also measureth the magnitude B: and it likewise measureth the magnitude A. Wherefore the magnitude C measureth the magnitudes A and B. Wherfore the magnitudes A & B are commensurable. If therefore two magnitudes haue such proportion the one to the other, as number hath to number, those magnitudes are cõmensurable: which was required to be proued.

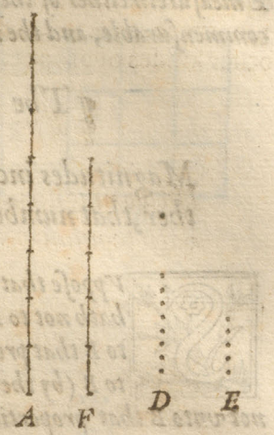


* Corollary.

❋ Corollary.

A Corollary.

Hereby it is manifeſt, that if there be two nũbers, as D and E, and a right line, as A. it is poſsible to geue an other line, vnto which the line A ſhall haue the ſame proportion, that the number D hath to the number E. For deuide the line A into ſo many equall partes as there are vnities in the number D (by the 9. of the ſixt). And take an other line, as F, which let be compoſed of ſo many partes equall to the partes of the line A, as there be vnities in the number E. Wherefore the line A ſhall be to the line F, as the number D is to the number E. And by this meanes you may vnto any line geuẽ geue an other line commenſurable in length. For if two lines be in proportion the one to the other, as number is to number, they ſhall alſo be commenſurable in length, by this 6. Theoreme.



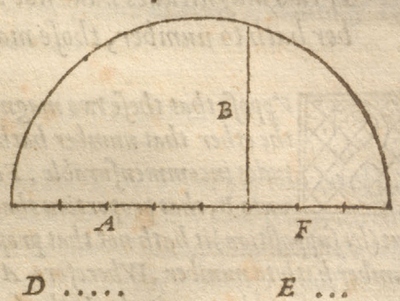
¶ An Aſſumpt.

Two numbers being geuen, and alſo a right line: as the one number is to the other, ſo to make the ſquare of ỹ line geuen to be to the ſquare of an other line.

Suppoſe that the numbers geuen be D and E: and let the right line geuen be A. It is required, as the number D is to the number E, ſo to make the ſquare of the line A to be to the ſquare of an other line. As the number D is to the number E, ſo let the line A be to the line F (by the former Corollary). And take betwene thoſe two lines A and F the meane proportionall (by the 13. of the ſixt) which let be the line B. Now for that as the number D is to the number E, ſo is the line A to the line F: and as the line A is to the line F, ſo is the ſquare of the line A to the ſquare of the line B (by the ſecond Corollary of the 20. of the ſixt). Wherefore as the number D is to the number E, ſo is the ſquare of the line A to the ſquare of the line B: which was required to be done.

Conſtruction.

Demonſtration.

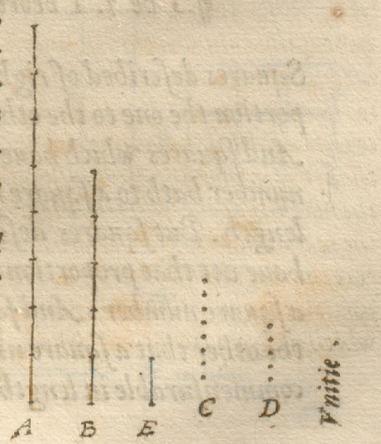


¶ An other demonſtration of the 6. Propoſition.

Suppoſe that theſe two magnitudes geuen A and B, haue that proportion the one to the other, that the number C hath to the nũber D. Thẽ I ſay, that thoſe magnitudes are commẽſurable. How many vnities there are in the number C, into ſo many equall partes let the magnitude A be deuided, & let the magnitude E be equall vnto one of thoſe partes. Wherefore as vnitie is to the number C, ſo is the magnitude E to the magnitude A. And as the number C is to the number D, ſo is the magnitude A to the magnitude B. Wherefore of equalitie (by the 22. of the fift) as vnitie is to the number D, ſo is the magnitude E to the magnitude B. But vnitie meaſureth the number

Conſtruction.

Demonſtration.



number D. Wherefore the magnitude E measureth the magnitude B. And it also measureth the magnitude A (for that vnitie measureth the number C). Wherefore the magnitude E measureth either of these magnitudes A and B. Wherefore the magnitudes A and B are commensurable, and the magnitude E is their common measure.

¶ The 5. Theoreme. The 7. Proposition.

Magnitudes incommensurable, haue not that proportion the one to the other, that number hath to number.

Demonstration leading to an absurditie.

Vppose that the magnitudes A and B be incommensurable. Then I say, that A hath not to B, that proportion that number hath to number. For if A haue vnto B that proportion that number hath to number, then is A cõmensurable vnto B (by the 6. of this tenth). But (by supposition) it is not. Wherefore A hath not vnto B that proportion that number hath to number. Magnitudes incommunsurable therfore haue not that proportion the one to the other that number hath to number: which was required to be demonstrated.

A ——————————

B ——————————————

¶ The 6. Theoreme. The 8. Proposition.

If two magnitudes haue not that proportion the one to the other that number hath to number, those magnitudes are incommensurable.

This is the conuerse of the former, and is proued by an indirect demonstration.

Vppose that these two magnitudes A and B, haue not that proportion the one to the other that number hath to number. Then I say, that A and B are magnitudes incommensurable. For if A and B be commensurable, then shall A haue vnto B, that proportion that number hath to number (by the 5. of this tenth). But (by supposition) it hath not that proportion that number hath to number. Wherefore A and B are incommensurable magnitudes. If therfore two magnitudes haue not that proportion the one to the other that number hath to nũber, those magnitudes are incommensurable: which was required to be proued.

A ——————————

B ——————————————

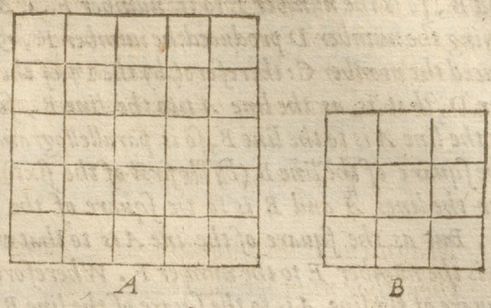
¶ The 7. Theoreme. The 9. Proposition.

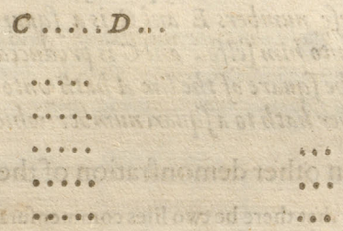
1. *Squares described of right lines commensurable in length, haue that proportion the one to the other, that a square number hath to a square number.*
2. *And squares which haue that proportion the one to the other that a square number hath to a square nũber, shall also haue their sides cõmensurable in length.*
3. *But squares described of right lines incommensurable in length, haue not that proportion the one to the other, that a square number hath to a square number.*
4. *And squares which haue not that proportion the one to the other that a square nũber hath to a square number, haue not their sides commensurable in length.*

Suppose

The first part demonstrated.

Svppose that A and B be lines comensurable in length. Then I say that the square of the line A hath vnto the square of the line B, that proportion that a square number hath to a square number. For forasmuch as the lines A and B are commensurable in length: therefore the line A hath vnto the line B that proportion that number hath to number (by the 5. of this tenth). Let the line A haue vnto the line B that proportion, that the number C hath to the number D, Now for that as the line A is to the line B, so is the number C to the number D: but the square of the line A is vnto the square of the line B in double proportiõ of that which the line A is vnto the line B (for like rectilinefigures (by the first corollary of the 20. of the sixt) are in double proportion of that which the sides of like proportion are) and likewise the square number produced of C is to the squre number produced of D, in double proportion of that which the number C is to the numter D (for by the 11. of the eight betwene two square numbers there is one meane proportional number & a square number is to a square number in double proportion of that which side is vnto side). Wherefore as the square of the line A is to the square of the line B, so is the square number produced of the number C, to the square number produced of the number D.

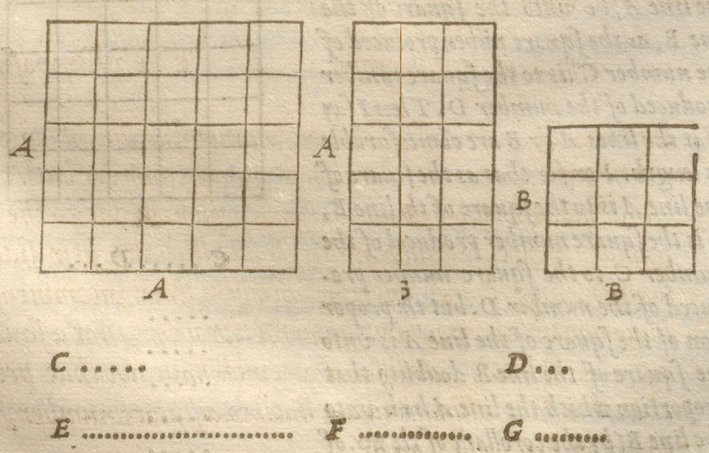




An other demonstration to proue the same.

An other demonstration of the first part.

Forasmuch as the lines A and B are commensurable, therefore (by the 5 of this tenth) A hath vnto B the same proportion that number hath to numbr. Let them haue that proportiõ that the number C hath to the number D. And let the nunner C multiplying himselfe produce the number E, and multiplying the number D, let it produce the number F: and let the number D multiplying himselfe produce the number G. And forasmuch as the number C multiplying himselfe produced the number E, and multipling the number D it produced the number F: therefore (by the 17. of the seuenth) as the number C is to the number D, that is, the line A to the line B so is the number E to the nũber F. But as the line A is to the line B, so is the square of the line A to the parallelograme contained vnder the lines A and B (by the first of the sixt) Wherfore as the square of the line A is to that which is contayned vnder the lines A



and

and B, ſo is the number E to the number F. Agayne for as muche as the number C multiplying the number D produced the number F, & the number D multiplying himſelfe produced the number G: therefore (by the 17. of the ſeuenth) as the number C is to the number D, that is, as the line A is to the line B, ſo is the number F to the number G. But as the line A is to the line B, ſo is parallellograme contained vnder the lines A and B to the ſquare of the line B (by the firſt of the ſixt). Wherefore as that which is contained vnder the lines A and B is to the ſquare of the line B, ſo is the number F to the number G. But as the ſquare of the line A is to that which is contayned vnder the lines A and B, ſo is the number E to the number F. Wherefore of equalitie (by the 22. of the fifte) as the ſquare of the line A is to the ſquare of the line B, ſo is the number E to the number G. But either of theſe numbers E and G is a ſquare number. For E is produced of the number C multiplyed into him ſelfe, and G is produced of the number D multiplyed into him ſelfe. Wherefore the ſquare of the line A hath vnto the ſquare of the line B that proportion that a ſquare number hath to a ſquare number: which was required to be demonſtrated.

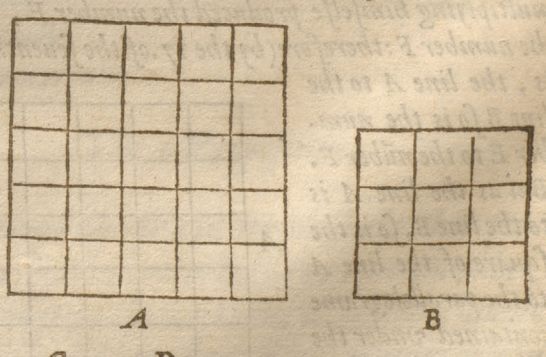
An other demonſtration of the ſame firſt part after *Montaureus*.

An other demonſtration of the ſame firſt part after Montaureus.

Suppoſe that there be two lines commenſurable in length A and B. Then I ſay that the ſquares deſcribed of thoſe lines ſhalbe in proportion the one to the other as a ſquare number is to a ſquare nũber. For foraſmuch as the lines A and B are commẽſurable in length, they ſhalbe in proportion the one to the other as number is to number (by the 5. of this booke). Let A be to B in duple proportion, which is in ſuch proportion as number is to number, namely, as 4. is to 2, and 6. to 3, and ſo of many other. And (by the ſecond of the eight) take the three leaſt numbers in continuall proportion, and in duple proportion, & let the ſame be the numbers 4.2.1: wherfore by the corrollary of the ſecond of the eight, the numbers 4. and 1. ſhalbe ſquare numbers. (For as 4. is a ſquare number produced of 2. multiplied into him ſelfe, ſo is 1. alſo a ſquare number, for it is produced of vnitie multiplied into him ſelfe.) I ſay moreouer that thoſe are the ſquare numbers, whoſe proportion the ſquares of the lines A and B haue the one to the other. For as the number 4. is to the number 2. ſo is the line A to the line B (For either proportion is double by ſuppoſition): but as the line A is to the line B, ſo is the ſquare of the line A to the parallelograme contained vnder the lines A and B (by the firſt of the ſixt). Wherefore as the number 4. is to the number 2: ſo ſhall the ſquare of the line A be to the parallelograme contained vnder the lines A and B. Likewiſe as the number 2. is to the number 1. ſo is the line A to the line B (For either proportion is duple by ſuppoſition): but as the line A is to the line B, ſo is the parallelograme contained vnder the lines A and B to the ſquare of the line B (by the ſelfe ſame firſt of the ſixt). Wherefore as the number 2. is to 1. ſo is the parallelograme contained vnder the lines A and B to the ſquare of the line B: wherefore of equallitie (by the 22. of the fifth) as the ſquare of the line A is to the ſquare of the line B, ſo is the number 4. to 1. which are proued to be ſquare numbers.

Demonſtration of the ſeconde part which is the conuerſe of the former.

But now ſuppoſe that the ſquare of the line A, be vnto the ſquare of the line B, as the ſquare nũber produced of the number C is to the ſquare number produced of the number D. Then I ſay that the lines A & B are cõmenſurable in length. For for that as the ſquare of the line A is to the ſquare of the line B, ſo is the ſquare number produced of the number C to the ſquare number produced of the number D: but the proportion of the ſquare of the line A, is vnto the ſquare of the line B double to that proportion which the line A hath vnto the line B (by the corollary of the 20. of



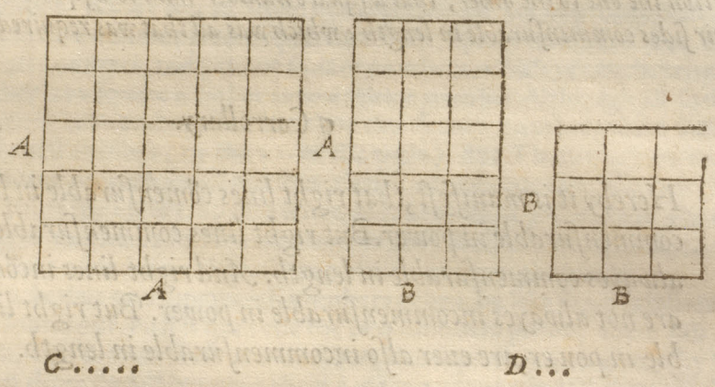
the

the sixt). And the proportion of the square number which is produced of the number C to the square number produced of the number D is (by the 11. of the eight) double to that proportion which the number C hath vnto the number D. Wherefore as the line A is to the line B, so is the number C to the number D. Wherefore the line A hath vnto the line B the same proportion that the number C hath to the number D. Wherefore (by the 6. of this booke) the lines A and B are commensurable in length: which was required to be proued.

An other demonstration to proue the same.

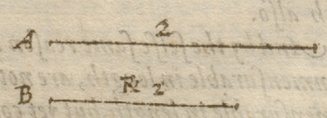
An other demonstration of the second part.

Vppose againe that the square of the line A haue vnto the square of the line B, the same proportion, that the square number E hath to the square number G. Then I say that the lines A and B are commensurable in length. For suppose that the side of the square number E be the nũber C, & let the side of the square number G be the nũber D: and let the number C multiplying the number D produce the number F. Wherefore these numbers E, F, G, are in continuall proportion, aud in the same proportion that the number C is to the number D (by the 17. and 18. of the seuenth). And forasmuch as the meane proportionall betwene the squares of the lines A and B is that which is contained vnder the lines A and B. (Which though it might briefely be proued: yet we take it as now). And likewise the meane proportionall betwene the numbers E and G is the number F (by the 20. of the seuenth): therefore as the square of the line A is to that which is contained vnder the lines A and B, so is the number E to the number F, and as that which is contained vnder the lines A and B is to the square of the line B, so is the number F to the number G. But as the square of the line A is to that which is contained vnder the lines A and B, so is the line A to the line B (by the first of the sixt). Wherefore the lines A and B are in the same proportion that the number E is to the number F, that is, that the number C is to the number D. Wherefore the lines A and B are commensurable in length (by the 6. of this tenth): which was required to be proued.



This Assumpt followeth as a Corollary of the 25: but (so as it might also be here in Methode, placed) you shall finde it after the 53. of this booke, absolutely demonstrated: for there it serueth to the 54. his demonstration.

But now suppose that the lines A and B be incommensurable in length. Then I say that the square of the line A hath not vnto the square of the line B that proportion that a square number hath to a square nũber. For if the square of the line A haue vnto the square of the line B, the same proportion that a square number hath to a square number, thẽ shall the lines A and B be cõmensurable in length (by the second part of this proposition). But by supposition they are not. Wherefore the square of the line A, hath not vnto the square of the lyne B that proportion that a square number hath to a square number: which was required to be proued.

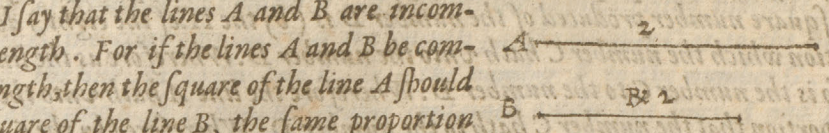


Demõstratiõ of the third part.

Againe suppose that the square of the line A haue not vnto the square of the line B, the same

Demõstratiõ of the fourth part which is the cõuerse of the 3.

same proportion that a square number hath to a square number. Then I say that the lines A and B are incommensurable in length. For if the lines A and B be commensurable in length, then the square of the line A should haue vnto the square of the line B, the same proportion that a square number hath to a square number, by the first part of this proposition, but by supposition it hath not, wherfore the lines A and B are not commensurable in length. Wherfore they are nicõmensurable in length. Wherfore squares made of right lines commensurable in length, haue that proportion the one to the other, that a square number hath to a square number. And squares which haue that proportion the one to the other, that a square number hath to a square number, shall also haue the sides commensurable in length. But squares described of right lines incommensurable in length, haue not that proportion the one to the other that a square number hath to a square number. And squares which haue not that proportion the one to the other, that a square number hath to a square number, haue not also their sides commensurable in length: which was all that was required to be proued.



Conclusion of the whole proposition.

¶ Corrollary.

Hereby it is manifest, that right lines cõmensurable in length, are also euer commensurable in power. But right lines commensurable in power, are not alwayes commensurable in length. And right lines incõmensurable in lẽgth are not alwayes incommensurable in power. But right lines incommensurable in power, are euer also incommensurable in length.

A Corollary.

For forasmuch as squares made of right lines commensurable in length, haue that proportion the one to the other, that a square number hath to a square number (by the first part of this proposition), but magnitudes which haue that proportion the one to the other, that number simply hath to number, are (by the sixt of the tenth) commensurable. Wherfore right lines commensurable in length, are commensurable not onely in length, but also in power.

Prose of the first part of the Corollary.

Againe forasmuch as there are certaine squares which haue not that proportion the one to the other that a square number hath to a square number, but yet haue that proportion the one to the other which number simply hath to number: their sides in dede are in power commensurable, for that they describe squares which haue that proportion which number simply hath to number, which squares are therfore commensurable (by the 6. of this booke): but the said sides are incommensurable in length by the latter part of this proposition. Wherefore it is true that lines commensurable in power, are not straight way commensurable in length also.

Prose of the second part.

And by the selfe same reason is proued also that third part of the corollary, that lines incommensurable in length, are not alwayes incommensurable in power. For they may be incommensurable in length, but yet commensurable in power. As in those squares which are in proportion the one to the other, as number is to number, but not as a square number is to a square number.

Prose of the third part.

But right lines incommensurable in power, are alwayes also incommensurable in length. For if they be commensurable in length, they shal also be commensurable in power by the first part of this Corollary. But they are supposed to be incommensurable in length, which is absurde. Wherfore right lines incommensurable in power, are euer incommensurable in length.

Prose of the fourth part.

For

For the better vnderſtanding of this propoſition and the other following, I haue here added certayne annotacions taken out of *Montaureus*. And firſt as touching the ſignification of wordes and termes herein vſed, which are ſuch, that vnleſſe they be well marked and peyſed, the matter will be obſcure and hard, and in a maner inexplicable. *Certayne annotations out of Montaureus.*

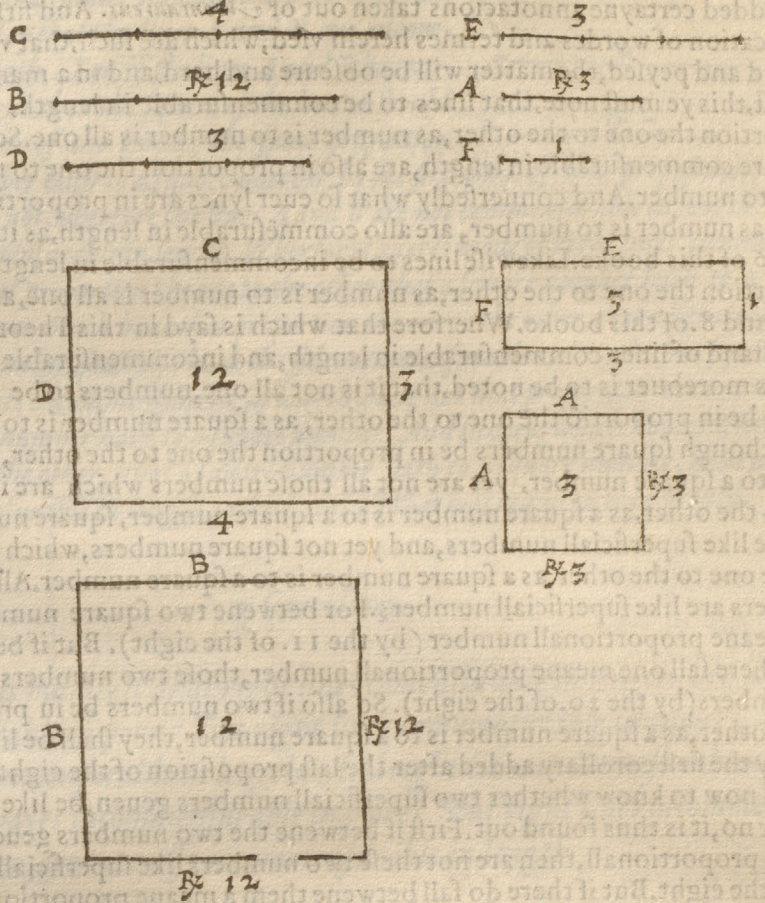
Firſt, this ye muſt note, that lines to be commenſurable in length, and lines to be in proportion the one to the other, as number is to number is all one. So that whatſoeuer lines are commenſurable in length, are alſo in proportion the one to the other, as number is to number. And conuerſedly what ſo euer lynes are in proportion the one to the other, as number is to number, are alſo commēſurable in length, as it is manifeſt by the 5 and 6 of this booke. Likewiſe lines to be incommenſurable in length, and not to be in proportion the one to the other, as number is to number is all one, as it is manifeſt by the 7. and 8. of this booke. Wherfore that which is ſayd in this Theoreme, ought to be vnderſtand of lines commenſurable in length, and incommenſurable in length.

This moreouer is to be noted, that it is not all one, numbers to be ſquare numbers, and to be in proportiō the one to the other, as a ſquare number is to a ſquare number. For although ſquare numbers be in proportion the one to the other, as a ſquare number is to a ſquare number, yet are not all thoſe numbers which are in proportion the one to the other, as a ſquare number is to a ſquare number, ſquare numbers. For they may be like ſuperficiall numbers, and yet not ſquare numbers, which yet are in proportiō the one to the other, as a ſquare number is to a ſquare number. Although all ſquare numbers are like ſuperficiall numbers. For betwene two ſquare numbers there falleth one meane proportionall number (by the 11. of the eight). But if betwene two numbers, there fall one meane proportionall number, thoſe two numbers are like ſuperficiall numbers (by the 20. of the eight). So alſo if two numbers be in proportion the one to the other, as a ſquare number is to a ſquare number, they ſhall be like ſuperficiall nūbers by the firſt corollary added after the laſt propoſition of the eight booke.

And now to know whether two ſuperficiall numbers geuen, be like ſuperficiall numbers or no, it is thus found out. Firſt if betwene the two numbers geuen, there fall no meane proportionall, then are not theſe two numbers like ſuperficiall numbers (by the 18. of the eight. But if there do fall betwene them a meane proportionall, then are they like ſuperficiall numbers (by the 20. of the eight) Moreouer two like ſuperficiall numbers multiplied the one into the other, do produce a ſquare number (by the firſt of the ninth). Wherfore if they do not produce a ſquare number, then are they not like ſuperficiall numbers. And if the one being multiplied into the other, they produce a ſquare number, then are they like ſuperficiall (by the 2. of the ninth). Moreouer if the ſaid two ſuperficial numbers be in ſuperperticular, or ſuperbipartient proportion, then are they not like ſuperficiall numbers. For if they ſhould be like, then ſhould there be a meane proportionall betwene them (by the 20. of the eight). But that is contrary to the Corollary of the 20. of the eight. *Rules to know whether two ſuperficiall numbers be like or no.*

And the eaſilier to conceiue the demonſtrations following, take this example of that which we haue ſayd.

Suppoſe that there be a line, namely, C, which imagine to be foure fote long: and let there be an other line D, which let be three foote long. And (by the 13. of the ſixt) take the meane proportionall betwene the lines A, D, which let be the line B. Wherefore the ſquare of the line B ſhall be equall to the rectangle parallelogramme contayned vnder the line C and D (by the 17. of the ſixt). Which ſquare ſhall contayne 12. foote, & ſo much alſo ſhall the parallelogramme deſcribed of the lines C & D containe. Take alſo two other lines E and F, of which let E be 3. foote long, and let F be a foote long. And let the meane proportionall betwene the lines E and F, be the line A. Now then the ſquare of the line A ſhall containe 3. foote, as alſo doth the parallelogrāme deſcribed of the lines E, F. Thē I ſay, that the ſquare of the line B, which cōtaineth 12. foote, is to the ſquare of the line A, which contayneth 3. foote, in that proportion that a ſquare number is to a ſquare number. For as the number 12. is to the number 3, ſo is the ſquare of the line B, which containeth 12. foote, to the ſquare of the line A, which contayneth 3 foote. But the numbers 12. and 3. are like ſuperficiall numbers, for the ſides of 12. which are 2. and 6, are proportionall with the ſides of 3. which are 1. and 3. Wherefore the ſquare of the line B, which contayneth 12. foote, ſhall be vnto the ſquare of the line A, which contayneth 3. foote, in that proportion that a like ſuperficiall number is to a like ſuperficiall number. But like ſuperficiall numbers are in proportion the one to the other, as a ſquare number is



to a ſquare nũber, which ſquare numbers are 4. and 1. (by the 26. of the eight). Wherefore the ſquare of the line B, which contayneth 12. foote, is to the ſquare of the line A, which cõtaineth 3. foote, in that proportion that a ſquare number is to a ſquare number, namely, that the number 4. is to the number 1: which proportion is quadruple. For the greater ſquare whiche is 12, contayneth the leſſe ſquare which is 3, foure times. Wherefore the ſide of the ſquare 12, which is the line B, is double to the ſide of the ſquare 3, which is the line A. Wherfore the line B is to the line A, in that proportion that number is to number. Wherfore (by the 5. of this booke) the lines B & A are commenſurable in length. Which is a ſuppoſition neceſſary to conclude the firſt part of this Theoreme, namely, that the ſquares of ſuch lines are in proportion the one to the other, that a ſquare number is to a ſquare number.

So alſo the nũber which denominateth the greater terme of the proportion of the line B to the line A, which is 2, if it be multiplyed into it ſelfe, it maketh a ſquare number, namely, 4. Likewiſe the number which denominateth the leſſe terme, namely, 1. if it be multiplyed into it ſelfe, it maketh no more but 1. Which vnitie is alſo in power a ſquare nũber. Wherfore the ſquare of the line B, is to the ſquare of the line A, in that proportion that a ſquare number is to a ſquare number, namely, that 4. is to 1. By this you ſee (which thing was before noted) that it is not all one, numbers to be ſquare numbers, and to be in proportion the one to the other, as a ſquare number is to a ſquare number. For it is manifeſt, that the numbers 12. and 3. are not ſquare numbers, when yet the ſquares expreſſed by thoſe numbers are in that proportion. But the ſide of the ſquare 12. although it can not of it ſelfe be expreſſed by number diſtinctly, to ſay that the ſide thereof is ſo many foote long, which feete ſquare taken, make the whole ſquare 12: yet being referred or compared to an other thyng, namely, to the ſide of the ſquare 3, which ſide alſo of it ſelfe can not be expreſſed by number, it is vnto the ſayde ſide of the ſquare 3, in double proportion. For the one ſquare being quadruple to the other ſquare (as is the ſquare of the line B, which contayneth 12. foote, to the ſquare of the line A, which contayneth 3. foote) hath his ſide double to the ſide of the other ſquare, by this generall Corollary of the 20. of the ſixt, *like rectiline figures are in double proportion the one to the other that their ſides of like proportion are*. Now if a man will ſay, that the ſide of the ſquare 12. may be meaſured, for that hys proportion which it hath to the ſide of the ſquare 3, is meaſured by 2 (foraſmuch as it is dupla proportion: this is to be conſidered, that in ſo ſaying, you ſay not, that that magnitude can of it ſelfe be meaſured, but the proportion therof. For, that magnitude, namely, the ſide of the ſquare 12, ſhould by it ſelfe be meaſured, when without any reſpect of the proportion of it to an other thing, we may ſay that the ſide of the ſquare, which contayneth 12. foote, is ſo many foote long, the number of which foote multiplyed into it ſelfe ſhould make that number 12. But this is not poſſible, for that 12. is not a ſquare number. Wherefore thus you may ſay: In aſmuch as that ſquare 12. is conſidered by it ſelf, without hauing any reſpect of the proportion of it to any other thing, but onely as it is 12. foote, it hath no ſide which of it ſelfe can be expreſſed by number. But if it be compared to any other thing, namely, to the ſquare of 3. foote, then may you ſay that the ſide of the ſquare 12. is 2, and the ſide of the ſquare 3. is 1. But thys is the denomination of that proportion which is called duple, which proportion can not be or conſidered

in

in fewer termes then two, when as it is a relation of one thing to an other thing: wherefore 2. is not the number of ſuch feete, of which there are 12. in the ſquare. Agayne, if the number 2. ſhoulde be the ſide of the ſquare 12, ſo that that ſide ſhould be 2, then of the multiplication of 2. into it ſelfe, ſhoulde not be made that ſquare 12, but an other ſquare which ſhould be 4. foote: as of the number 2. multiplyed into him ſelfe is produced the ſquare number 4. Neither alſo if any other number meaſure the ſide of the ſquare 12, and the ſayd number be multiplyed into him ſelfe ſhall it euer make the number 12. When yet all numbers denominating the ſide of any ſquare nnmber, if they be multiplyed into them ſelues, they make the number which denominateth the ſquare, whoſe ſides they denominate. As 2. multiplyed into him ſelfe maketh 4: 3. maketh 9: 4. maketh 16: and ſo likewiſe of all others. Wherefore it is not all one, magnitudes to be in proportion the one to the other, as number is to number, and euery one of them to be meaſured b him ſelfe without any reſpect had of the proportion. As here the ſide of the ſquare 12. can of it ſelfe by no meanes be meaſured, but being compared to any other magnitude, namely, to the ſide of the ſquare 3, the proportion thereof is expreſſed by number. So alſo the ſide of the ſquare 3, and of all other ſquare figures, whoſe areas yet can not be expreſſed by ſquare numbers. And that which we here ſay, is manifeſt euen by the wordes of *Euclide* in the 5. 6. 7. and 8. Theoremes of this booke. Where he ſayth not, that magnitudes commenſurable and incommenſurable are of thẽ ſelues or of their owne nature expreſſed by numbers, but that either they haue or haue not that proportion which number hath to number. Which thing not being well conſidered, it ſhould ſeme hath cauſed many to erre as hereafter ſhall be made manifeſt. And in deede they which haue demonſtrated this Theoreme, may ſeme to ſome rather to haue demõſtrated it particularly & not vniuerſally. And doubtles I iudge there are ſome which vnderſtand their ſayinges otherwiſe then they ment: when as they thinke, that they ſuppoſe certayne lines not onely commenſurable in length, as they are ſuppoſed to be in the Propoſitiõ, but alſo ſuch, that ech of them apart may be expreſſed by ſome certayne number. Wherfore for want of right vnderſtãding, this mought they ſay of their demonſtrations: that wheras they thought that they had concluded that generally, which is in this theoreme of *Euclide*, *Squares deſcribed of lines cõmenſurable in length, are in proportiõ the one to the other that a ſquare nũber is to a ſquare number:* they conclude particularly, thys onely: *Squares deſcribed of lines which may by them ſelues be expreſed by ſome certaine number, are in proportion. &c.* which yet is otherwiſe, and their demõſtrations are right & agreable with the Theoreme. Onely the picture of the figures which the Greeke boke hath, may ſeeme to bring ſome doubt. For the ſquares are ſo deſcribed with certayn litle areas, that the number of them may be denominated by a ſquare number: whereby it mought ſeeme that the lines A & B which deſcribe the ſquares, ought to be ſuch that they may be expreſſed by ſome certaine nũber. As the line A to be 5. foote, and the line B 3. foote. As the two former figures here ſet declare. Which thyng yet *Euclide* ſuppoſeth not, but only requireth that they be commenſurable in length, as in the former example of the two ſquares, the whole area of one of which is 12, and the whole area of the other is 3. For although their ſides cã not by them ſelues be expreſſed by ſome certayne number, yet are they commenſurable in length. Moreouer thys deſcribing of the ſquares of the liues A and B, deuided by certaine litle areas, may cauſe this error, that a man ſhould thinke that it is all one two numbers to be ſquare numbers, and to be in proportion the one to the other as a ſquare number is to a ſquare number. For the number of the areas in the ſquare of the line A is a ſquare number, namely, 25. produced of the roote 5, which is the length of the line A. Likewiſe the number of the areas of the ſquare of the line B, is a ſquare nũber, namely, 9, which is produced of the roote 3, which is the length of the line B. But we haue before declared that it is not all one, numbers to be called ſquare numbers, and to be in proportion the one to the other, as a ſquare nũber is to a ſquare number. Wherefore as touching thoſe areas contayned in the greater ſquare, which is of the line A, and which are in number 25, they do expreſſe that ſquare number 25, which is produced of the number 5. multiplyed into him ſelfe, which number 5. is the greater extreme of the proportion betwene 5. and 3, which is the proportion of the lines A and B. And this proportion, namely, of 5. to 3. cauſeth that the lines A and B are commenſurable in length (by the 6. of this booke). The ſame may be ſayd alſo of the areas of the leſſe ſquare. Neither is it of neceſſitie that you vnderſtand thoſe areas to be ſquares, as either feete ſquare or paſes ſquare which make the whole ſquare, although in deede they

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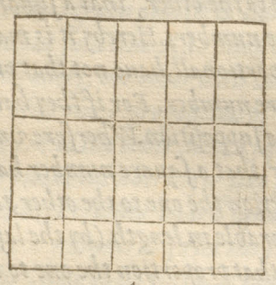
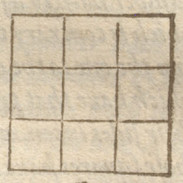


A



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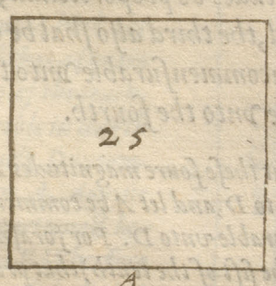

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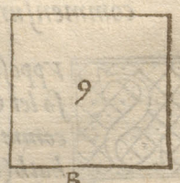
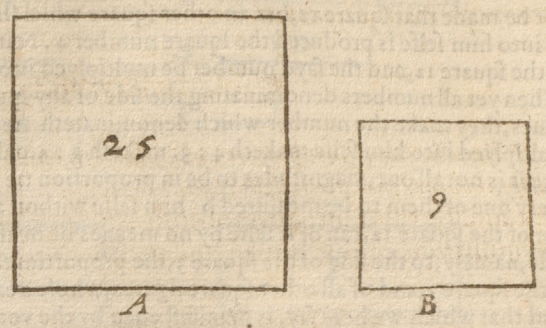


B

may be ſuch, ſo that the ſides of thoſe ſquares be ſo many foote long, as 5. foote or 3. foote. Howbeit thys is of neceſſitie that the numbers which expreſe the number of the feete ſquare or paſes ſquare, cõtayned in the ſquares, be either both of thẽ ſquare numbers, as in theſe ſquare figures of the lines A, B, or that both of them be like ſuperficiall numbers, as in the former ſquares which were 12. and 3: of which nũbers it is manifeſt by that which hath before bene ſaid, that they are like ſuperficiall numbers, and therefore haue that proportion the one to the other, that a ſquare nũber hath to a ſquare number. And therfore you may deſcribe the ſquares of the lines A, B, without any diſtinction of ſuch litle areas, ſo that the ſquares may be voyde and emptie, and contayned onely of foure right lines, as in the figure here put.



¶ An Aſſumpt.

Foraſmuch as in the eight booke in the 26. propoſition it was proued, that like playne numbers haue that proportion the one to the other, that a ſquare number hath to a ſquare number: and likewiſe in the 24. of the ſame booke it was proued, that if two numbers haue that proportion the one to the other, that a ſquare number hath to a ſquare number, thoſe numbers are like plaine numbers. Hereby it is manifeſt, that vnlike plaine numbers, that is, whoſe ſides are not proportionall, haue not that proportion the one to the other, that a ſquare number hath to a ſquare number. For if they haue, then ſhould they be like plaine numbers, which is contrary to the ſuppoſition. Wherfore vnlike plaine numbers haue not that proportion the one to the other, that a ſquare numbor hath to a ſquare nũber. And therfore ſquares which haue that proportion the one to the other, that vnlike plaine numbers haue, ſhall haue their ſides incommenſurable in length (by the laſt part of the former propoſition) for that thoſe ſquares haue not that proportion the one to the other that a ſquare number hath to a ſquare number.

This aſſumpt is the conuerſe of the 26. of the eight.

¶ *The 8. Theoreme.* *The 10. Propoſition.*

If foure magnitudes be proportionall, and if the firſt be commenſurable vnto the ſecond, the third alſo ſhal be commenſurable vnto the fourth. And if the firſt be incommenſurable vnto the ſecond, the third ſhall alſo be incommenſurable vnto the fourth.

Demonſtratiõ of the firſt part.

Suppoſe that theſe foure magnitudes A, B, C, D, be proportionall. As A is to B, ſo let C be to D, and let A be commenſurable vnto B. Then I ſay that C is alſo commenſurable vnto D. For foraſmuch as A is commenſurable vnto B, it hath (by the fift of the tenth) that proportion that number hath to number. But as A is to B, ſo is C to D. Wherfore C alſo hath vnto D that proportion that number hath to number. Wherfore C is commenſurable vnto D (by the 6. of the tenth).

Demonſtration of the ſecond part.

But now ſuppoſe that the magnitude A be incommenſurable vnto the magnitude B. Then I ſay that the magnitude C alſo is incommenſurable vnto the magnitude D. For foraſmuch as A is incommenſurable vnto B, therfore (by the 7. of this booke) A hath not vnto B ſuch proportion as number hath to number. But as A is to B, ſo is C to D. Wherefore C hath not vnto D ſuch proportion as number hath to number. Wherfore (by the 8. of the tenth) C is incommenſurable vnto D. If therefore there be foure magnitudes proportionall, and if the firſt be commenſurable vnto the

ſecond,

ſecond, the third alſo ſhall be commenſurable vnto the fourth. And if the firſt be incommenſurable vnto the ſecond, the third ſhall alſo be incommenſurable vnto the fourth: which was required to be proued.

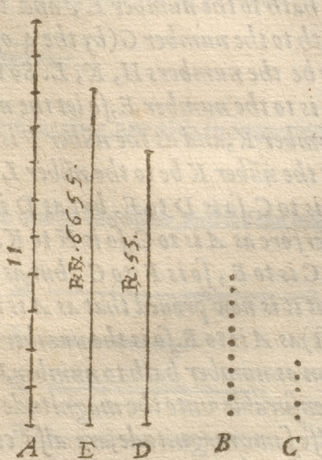
¶ A Corollary added by Montaureus.

If there be foure lines proportionall, and if the two firſt, or the two laſt be commenſurable in power onely, the other two alſo ſhall be commenſurable in power onely. This is proued by the 22. of the ſixt, and by this tenth propoſition. And this Corollary Euclide vſeth in the 27. and 28. propoſitions of this booke, and in other propoſitions alſo. *A Corollary.*

¶ The 3. Probleme. The 11. Propoſition.

Vnto a right line firſt ſet and geuen (which is called a rationall line) to finde out two right lines incommenſurable, the one in length onely, and the other in length and alſo in power.

Vppoſe that the right line firſt ſet and geuen, which is called a rationall line of purpoſe be A. It is required vnto the ſaid line A, to finde out two right lines incommenſurable, the one in length onely, the other both in length and in power. Take (by that which was added after the 9. propoſition of this booke) two numbers B and C, not hauing that proportion the one to the other, that a ſquare number hath to a ſquare number, that is, let them not be like plaine numbers (for like plaine numbers by the 26. of the eight haue that proportion the one to the other that a ſquare number hath to a ſquare number). And as the number B is to the number C, ſo let the ſquare of the line A be vnto the ſquare of an other line, namely, of D (how to do this was taught in the aſſumpt put before the 6. propoſition of this booke.) Wherfore the ſquare of the line A, is vnto the ſquare of the line D commenſurable (by the ſixt of the tenth.) *To finde out the firſt line incommenſurable in length onely to the line geuen.*



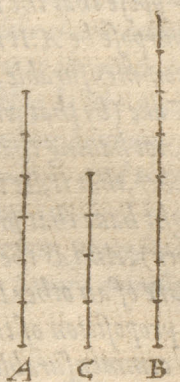
And foraſmuch as the number B hath not vnto the number C, that proportion that a ſquare number hath to a ſquare nũber, therfore the ſquare of the line A hath not vnto the ſquare of ỹ line D, that proportiõ that a ſquare number hath to a nũber. Wherfore by the 9. of the tenth, the line A is vnto the line D incommenſurable in length onely. And ſo is found out the firſt line, namely, D incommenſurable in length onely to the line geuen A. Agayne take (by the 13. of the ſixt) the meane proportionall betwene the lines A and D, and let the ſame be E. Wherfore as the line A is to the line D, ſo is the ſquare of the line A to the ſquare of the line E (by the Corollary of the 20. of the ſixt). But the line A is vnto the line D incommenſurable in length. Wherfore alſo the ſquare of the line A is vnto the ſquare of the line E incommenſurable by the ſecond part of the former propoſition. Now foraſmuch as the ſquare of the line A is incõmẽſurable to the ſquare of the line E, it followeth (by the definition of incommenſurable lynes) that the line A is incommenſurable in power to the line E. Wherfore vnto the right line geuen, and firſt ſet, A, which is a rationall line, and which is ſuppoſed to haue ſuch diuiſions and ſo many partes as ye liſt to conceyue in minde, as in this example 11, whereunto, as was declared in the 5. definition of this booke, may be compared infinite other lines, either commen- *To finde out the ſecond line incommenſurable both in length and in power to the line geuen.*

ſurable or incommenſurable, is found out the line D incommenſurable in length onely. Wherfore the line D is rationall (by the ſixt definitiõ of this booke) for that it is incommenſurable in length onely to the line A, which is the firſt line ſet, and is by ſuppoſitiõ rational. There is alſo found out the line E, which is vnto the ſame line A incommenſurable, not onely in length but alſo in power, which line E compared to the rationall line A, is by the definition irrationall. For Euclide alwayes calleth thoſe lines irrationall, which are incommenſurable both in length and in power to the line firſt ſet, and by ſuppoſition rationall.

¶ The 9. Theoreme. The 12. Propoſition.

Magnitudes commenſurable to one and the ſelfe ſame magnitude: are alſo commenſurable the one to the other.

Conſtruction. *Vppoſe that either of theſe magnitudes A and B, be commenſurable vnto the magnitude C. Then I ſay that the magnitude A is commenſurable vnto the magnitude B. For foraſmuch as the magnitude A is commenſurable vnto the magnitude C, therefore (by the 5. of the tenth) A hath vnto C ſuch proportion as number hath to number. Let A haue vnto C that proportion that the number D hath to the number E. Againe foraſmuch as B is commenſurable vnto C, therefore (by the ſelfe ſame) C hath vnto B that proportion that number hath to number. Let C haue vnto B that proportion that the number F hath vnto the number G. Now then take the leaſt numbers in continuall proportion and in theſe proportions geuen, namely, that the number D hath to the number E, and that the number F hath to the number G (by the 4. of the eight): which let be the numbers H, K, L. So that as the number D is to the number E, ſo let the number H be to the number K, and as the nũber F is to the nũber G, ſo let the nũber K be to the nũber L. Now for that as A is to C, ſo is D to E, but as D is to E ſo is H to K, therfore as A is to C, ſo is H to K. Againe for that as C is to B, ſo is F to G, but as F is to G ſo is K to L: therefore as C is to B, ſo is K to L. But it is now proued that as A is to C, ſo is H to K. Wherefore of equalitie (by the 22. of the firſt) as A is to B, ſo is the number H to the number L. Wherefore A hath vnto B ſuch proportion as number hath to number. Wherefore (by the ſixt of the tenth) the magnitude A is commenſurable vnto the magnitude B. Magnitudes therefore commenſurable to one and the ſelfe ſame magnitude, are alſo commenſurable the one to the other: which was required to be proued.*



Demonſtration.

D .....
E ....
F ....
G .....
H ...
K ..
L ....

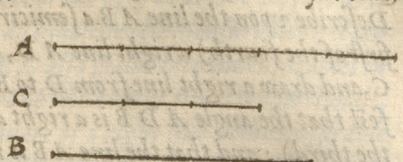
This is with Zambert an Aßumpt, but vtterly improperly: Fluſſates maketh it a Corollary, but the Greeke and Montaureus make it a propoſition: but euery way an infallible truth it is.

¶ An Aſſumpt.

If there be two magnitudes compared to one and the ſelfe ſame magnitude, and if the one of them be commenſurable vnto it, and the other incommenſurable: thoſe magnitudes are incommenſurable the one to the other.

Suppoſe

SVppose that there be two magnitudes, namely, A and B and let C be a certayne other magnitude. And let A be commensurable vnto C, and let B be commẽsurable vnto the selfe same C. Then I say that the magnitude A is incommensurable vnto B. For if A be commensurable vnto B, forasmuch as A is also commẽsurable vnto C therefore (by the 12. of the tenth) B is also commẽsurable vnto C: which is contrary to the supposition.

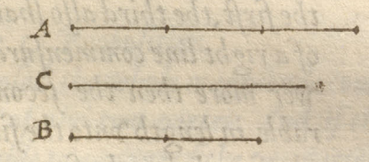


Demonstration leading to an absurditie.

¶ The 10. Theoreme. The 13. Proposition.

If there be two magnitudes commensurable, and if the one of them be incommensurable to any other magnitude: the other also shall be incommensurable vnto the same.

SVppose that these two magnitudes A, B be commensurable the one to the other, and let the one of them, namely A, be incommensurable vnto an other magnitude, namely, vnto C. Then I say that the other magnitude also, namely B, is incommensurable vnto C. For if B be commensurable vnto C, then forasmuch as A is commensurable vnto B, therefore (by the 12. of the tenth) the magnitude A also is commensurable vnto the magnitude C. But it is supposed to be incommensurable vnto it, which is impossible. Wherefore the magnitudes B and C are not commensurable. Wherefore they are incommensurable. If therefore there be two magnitudes commensurable, and if the one of them be incommensurable to any other magnitude, the other also shalbe incommensurable vnto the same: which was required to be proued.



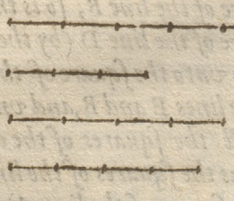
Demonstration leading to an absurditie.

¶ A Corollary added by Montaureus.

Magnitudes commensurable to magnitudes incommẽsurable, are also incommensurable the one to the other.

A Corollary.

Suppose that the magnitudes A and B be incommensurable the one to the other, and let the magnitude C be cõmensurable to A, and let the magnitude D be cõmensurable vnto B. Then I say that the magnitudes C and D are incommensurable the one to the other. For A and C are commensurable, of which the magnitude A is incommensurable vnto B, wherefore by this 13. proposition the magnitudes C and B are also incommensurable: but the magnitudes B and D are cõmensurable wherefore by the same, or by the former assumpt, the magnitudes C and D are incommensurable the one to the other. This corollary, *Theon* vseth often times as in the 22. 26. and 36 propositions of this booke, and in other propositions also.



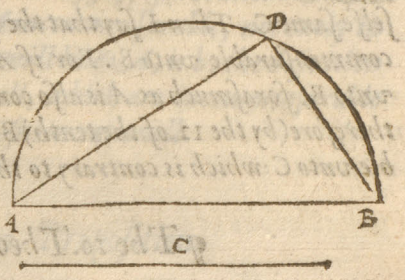
¶ An Assumpt.

Two vnequall right lines being geuen, to finde out how much the greater is in power more then the lesse.

Suppose that the two vnequall right lines geuen, be A B and C, of which let A B be the greater.

greater. It is required to finde out how much more in power the line A B is then the line C.

Construction. Describe vpon the line A B a semicircle A D B. And vnto it from the point A apply (by the first of the fourth) a right line A D, equall vnto the line C, and draw a right line from D to B.



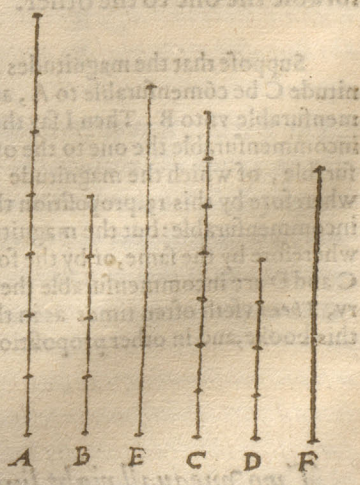
Demonstration. Now it is manifest that the angle A D B is a right angle (by the 31. of the third): and that the line A B is in power more then the line A D, that is, then the line C, by the line D B, by the 47. of the first.

A Corollary. And like in sorte, *two right lines being geuen, by this meanes may be founde out a right lyne which contayneth them both in power.* Suppose that the two right lines geuen be A D and D B. It is required to finde out a right lyne that contayneth them both in power. Let the lines A B and D B be so put, that they comprehend a right angle A D B, and draw a right line from A to B. Now agayne it is manifest (by the 47. of the first) that the line A B contayneth in power the lines A D and D B.

¶ The 11. Theoreme. The 14. Proposition.

If there be fower right lines proportionall, and if the first be in power more then the second by the square of a right line commensurable in length vnto the first, the third also shalbe in power more then the fourth, by the square of a right line commensurable vnto the third. And if the first be in power more then the second by the square of a right line incommensurable in length vnto the first, the third also shall be in power more then the fourth by the square of a right line incommensurable in length to the third.

Demonstration. Svppose that these foure right lines A, B, C, D, be proportionall. As A is to B, so let C be to D. And let A be in power more then B, by the square of the line E. And likewise let C be in power more then D, by the square of the line F. Then I say that if A be commensurable in length vnto the line E, C also shall be commensurable in length vnto the line F. And if A be incommensurable in length to the line E, C also shall be incommensurable in length to the line F. For for that as A is to B, so is C to D, therefore as the square of the line A is to the square of the line B, so is the square of the line C to the square of the line D (by the 22. of the sixt). But by supposition vnto the square of the line A are equall the squares of the lines E and B, and vnto the square of the line C are equall the squares of the of the lines D and F: Wherefore as the squares of the lines E and B (which are equall to the square of the line A) are to the square of the line B, so are the squares of the lines D and F (which are equall to the square of the line C) to the square of the line D (by the seuenth of the fift). Wherfore (by the 17. of the fift) as the square of the line E is to the square of the line B, so is the square of the line F to the square of the line D. Wherfore also as the line E is to the line B, so is the line F to the line D (by the second parte of the 22. of the sixt) wherefore contrariwise (by the Corollary of the fourth of the fift) as B is to E, so is D to F. But (by supposition)



as A is to B, so is C to D, Wherfore of equallitie (by the 22. of the fift) as A is to E, so is C to F. If therfore A be commensurable in length vnto E, C also shall be commensurable in legth vnto F: and if it ba incommensurable in length vnto E, C also shalbe incommensurablel in length vnto F, by the 10. of this booke. If therfore there be foure right tlines proportionall, and if the first be in power more then the secondby the square of a right line commensurable in length vnto the first, the third also shall be in power more then the fourth, by the square of a right line commensurable in length vnto the third: and if the first be in power more thē the second, by the square of a right line incomensurable in length vnto the first, the third also shall be in power more thē the fourth, by the square of a right line incommensurable in length to the third: which was required to be proued.

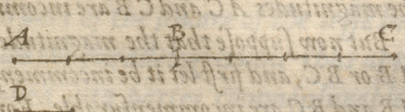
Note that the line A may be proued to be in proportion to the line E, as the line C is to the line F, by an other way, namely, by conuersion of proportion (of some, as we haue before noted, called inuerse proportion) by the 19. of the fift. For, forasmuch as the foure lines A, B, C, D, are proportionall: therfore (by the 22. of the sixt) their squares also are proportionall. And forasmuch as the antecedent, namely, the square of the line A excedeth the consequent, namely, the square of the line B, by the square of the line E: and the other antecedent, namely, the square of the line C, excedeth the other consequent, namely, the square of the line D, by the square of the line F, therefore as the square of the line A is to the excesse, namely, to the square of the line E, so is the square of the line C to the excesse, namely, to the square of the line F. Wherefore (by the second part of the 22. of the sixt) as the line A is to the line E, so is the line C to the line F.

An other way to proue that the lines A, E, C, F, are proportionall.

¶ The 12. Theoreme. The 15. Proposition.

If two magnitudes commensurable be composed, the whole magnitude composed also shall be commensurable to either of the two partes. And if the whole magnitude composed be commensurable to any one of the two partes, those two partes shall also be commensurable.

LEt these two commensurable magnitudes A B and B C, be composed or added together. Then I say, that the whole magnitude A C is cōmensurable to either of these partes A B and B C. For forasmuch as A B and B C are commensurable, therfore (by the first definition of the tenth) some one magnitude measureth them both. Let there be a magnitude that measureth them, and let the same be D. Now forasmuch as D measureth A B and B C, it shall also measure the whole magnitude composed A C, by this common sentence, what soeuer magnitude measureth two other magnitudes, shall also measure the magnitude composed of them. But the same D measureth A B and B C (by supposition). Wherefore D measureth A B, B C, and A C. Wherefore A C is commensurable to either of these magnitudes A B and B C.



Demonstration of the first part.

But now suppose that the whole composed magnitude A C be commensurable to any one of these two magnitudes A B or B C, let it be commensurable I say vnto A B. Then I say, that the two magnitudes A B and B C are commensurable. For forasmuch as A B and A C are commensurable, some one magnitude measureth them (by the first definition of the tenth). Let some magnitude measure them, and let the same be D. Now forasmuch as D measureth A B and A C, it also measureth the residue B C, by this common sentence, what soeuer measureth the whole and the part taken away, shall also measure the residue. But the same D measureth the magnitude A B (by supposition). Wherefore D measureth either of these magnitudes A B and B C. Wherefore the magnitudes A B and B C are cominensurable. If therfore two magnitudes commensurable be composed, the whole magnitude composed also shall be commensurable to either of the two partes. And if the whole magnitude composed be commensurable

Demonstration of the second patt which is the conuerse of the first.

menſurable to any one of the two partes, thoſe two partes ſhall alſo be commenſurable: which was required to be demonſtrated.

¶ A Corollary added by *Montaureus.*

A Corollary.

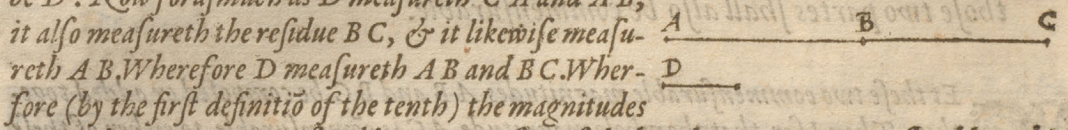
If an whole magnitude be commenſurable to one of the two magnitudes which make the whole magnitude, it ſhall alſo be commenſurable to the other of the two magnitudes. For if the whole magnitude A C be commenſurable vnto the magnitude B C, then by the 2. part of thys 15 Propoſition: the magnitudes A B and B C are commenſurable. Wherefore (by the firſt part of the ſame) the magnitude A C ſhall be commenſurable to either of theſe magnitudes A B and B C. This Corollary *Theon* vſeth in the demonſtration of the 17. Propoſition and alſo of other Propoſitions. Howbeit *Euclide* left it out, for that it ſeemed eaſie as in a maner do all other Corollaryes.

¶ The 13. Theoreme. The 16. Propoſition.

If two magnitudes incommenſurable be compoſed, the whole magnitude alſo ſhall be incommenſurable vnto either of the two partes cõponentes. And if the whole be incommenſurable to one of the partes componentes, thoſe firſt magnitudes alſo ſhall be incommenſurable.

Demonſtration of the firſt part by an argument leadindg to an abſurditie.

LEt theſe two incommenſurable magnitudes A B & B C, be compoſed, or added together. Then I ſay, that the whole magnitude A C, is incommenſurable to either of theſe magnitudes A B and B C. For if A C and A B be not incommenſurable, then ſome one magnitude meaſureth them (by the firſt definition of the tenth). Let there be ſuch a magnitude, if it be poſſible, and let the ſame be D. Now foraſmuch as D meaſureth C A and A B, it alſo meaſureth the reſidue B C, & it likewiſe meaſureth A B. Wherefore D meaſureth A B and B C. Wherefore (by the firſt definitiõ of the tenth) the magnitudes A B and B C are commenſurable. But it is ſuppoſed that they are incommenſurable: which is impoſſible. Wherefore no magnitude doth meaſure the magnitudes A B and A C. Wherefore the magnitudes C A and A B are incommenſurable. In like ſort alſo may we proue, that the magnitudes A C and C B are incommenſurable.



Demonſtration of the ſecond patt leading alſo to an impoſſibilitie. And this ſecond part is the conuerſe of the firſt.

But now ſuppoſe that the magnitude A C be incommenſurable to one of theſe magnitudes A B or B C, and firſt let it be incommenſurable vnto A B. Then I ſay, that the magnitudes A B and B C are incommenſurable. For if they be commenſurable ſome one magnitude meaſureth them. Let ſome one magnitude meaſure them, & let the ſame be D. Now foraſmuch as D meaſureth A B and B C, it alſo meaſureth the whole magnitude A C. And it meaſureth A B: Wherefore D meaſureth theſe magnitudes C A and A B. Wherefore C A & A B are commenſurable. And they are ſuppoſed to be incõmenſurable: which is impoſſible. Wherfore no magnitude meaſureth A B and B C. Wherefore the magnitudes A B and B C are incommenſurable. And in like ſort may they be proued to be incommenſurable, if the magnitude A C be ſuppoſed to be incommenſurable vnto B C. If therefore there be two magnitudes incommenſurable compoſed, the whole alſo ſhall be incommenſurable vnto either of the two partes component, and if the whole be incommenſurable to one of the partes component, thoſe firſt magnitudes ſhall be incommenſurable: which was required to be proued.

¶ A Corollary added by *Montaureus.*

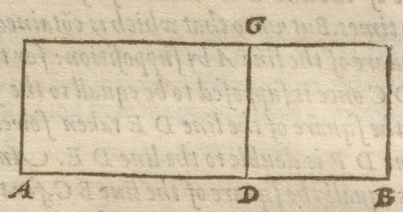
If an whole magnitude bee incommenſurable to one of the two magnitudes which make the whole magnitude, it ſhall alſo be incommenſurable to the other of the two magnitudes. For if the whole

whole magnitude A C be incōmēsurable vnto the magnitude B C, then by the 2. part of this 16. Theoreme, the magnitudes A B and B C shall be incommensurable. Wherefore by the first part of the same Theoreme, the magnitude A C shall be incommensurable to either of these magnitudes A B and B C. This Corollary *Theon* vseth in the demonstration of the 73. Theoreme, & also of other Propositions.

¶ An Assumpt.

If vpon a right line be applied a parallelogramme wanting in figure by a square: the parallelogramme so applied, is equall to that parallelogramme which is contayned vnder the segmentes of the right line, which segmentes are made by reason of that application.

Suppose that vpon a right line A B be applied a parallelogrāme A G, wanting in forme by the square G B. Then I say, that A G is equall vnto that which is contayned vnder A D and D B, which thing is of it self manifest. For forasmuch as G B is a square, therefore the line D G is equall vnto the line D B: and the parallelogramme A G is that which is contayned vnder the lines A D and D G, that is, vnder the lines A D and D B. If therefore vpon a right line be applied a parallelogramme wanting in figure by a square: the parallelogrāme applied is equall to the parallelogrāme which is contayned vnder the segmentes of the right line, which are made by reason of that application. which was required to be demonstrated.



This Assumpt I before added as a Corollary out of *Flussates* after the 28. Proposition of the sixt booke.

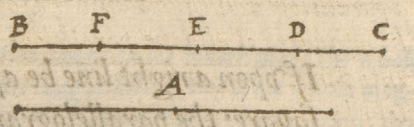
¶ *The 14. Theoreme.* *The 17. Proposition.*

If there be two right lines vnequall, and if vpon the greater be applied a parallelogramme equall vnto the fourth part of the square of the lesse line, and wanting in figure by a square, if also the parallelogramme thus applied deuide the line where vpon it is applied into partes commensurable in length: then shall the greater line be in power more then the lesse, by the square of a line commensurable in length vnto the greater. And if the greater be in power more then the lesse by the square of a right line commensurable in length vnto the greater, and if also vpon the greater be applied a parallelogrāme equall vnto the fourth part of the square of the lesse line, and wanting in figure by a square: then shall it deuide the greater line into partes commensurable.

Vppose that these two right lines A and B C, be vnequall: of which let B C be the greater. And vpon the line B C let there be applied (by the 28. of the sixt) a parallelogramme equall vnto the fourth part of the square of the line A being the lesse (that is, equall vnto the square described vpon halfe of

the line A) and wanting in figure by a ſquare. And let the ſame parallelogramme be that which is contained vnder the lines B D and D C. And (by ſuppoſition) let the lines B D and D C be commenſurable in length. Then I ſay, that the line B C, is in power more then the line A, by the ſquare of a line commenſurable in length vnto the ſayd line B C. Deuide (by the 10. of the firſt) the line B C into two equall partes in the point E. And (by the third of the firſt) vnto the line D E put an equall line E F. Wherefore the reſidue D C is equall vnto the reſidue B F. And foraſmuch as the right line B C is deuided into two equall partes in the point E, and into two vnequall partes in the point D, therefore (by the 5. of the ſecond) the rectangle figure comprehended vnder the lines B D and D C together with the ſquare of the line E D, is equall to the ſquare of the line E C. And in the ſame proportion are they eche being taken fower times by the 15. of the fifth. Wherfore that which is contained vnder the lines B D and D C taken fower times together with the ſquare of the line E D taken alſo fower times, is equall to the ſquare of the line E C taken fower times. But vnto that which is cõtained vnder the lines B D & D C foure times is equal the ſquare of the line A by ſuppoſition: for the parallelograme contained vnder the lines B D and D C once is ſuppoſed to be equall to the fourth part of the ſquare of the line A. And vnto the ſquare of the line D E taken fower times is equall the ſquare of the line D F, for the line D F is double to the line D E. And vnto the ſquare of the line E C fower times taken, is equall the ſquare of the line B C, for the line B C is alſo double to the line C E. Wherefore the ſquares of the lines A and D F are equall vnto the ſquare of the line B C. Wherefore the ſquare of the line B C, is greater then the ſquare of the line A, by the ſquare of the line D F. Wherefore the greater line B C is in power more then the leſſe line A, by the ſquare of the line D F. Now reſteth to proue that the line B C is commenſurable in length vnto the line D F. Foraſmuch as by ſupoſition the line B D is commenſurable in length vnto D C, therefore (by the 15. of the tenth) the whole line B C is commenſurable in length vnto the line D C: but the line D C equall to the line B F. Wherefore the whole line B C is commenſurable in length vnto the lines B F & C D. Let the two lines B F and C D be imagined to be ſo compoſed that they make one line. Now foraſmuch as the whole line B C is commenſurable in length to the two lines B F and C D taken as one line: therefore the lines B F and C D taken as one line are commenſurable in length to the line F D (by the 2. part of the 15. of the tenth). Wherfore alſo the whole line B C is commenſurable in length to the line F D by the firſt part of the ſame: this may alſo be proued by the corollary put after the 16. propoſition of this booke. Wherefore the line B C is in power more then the line A by the ſquare of a line commmenſurable in length vnto the line B C.

Conſtruction.



Demonſtratiõ of the firſt part.

But now ſuppoſe that the line B C be in power more then the line A, by the ſquare of a line commenſurable in length vnto the line B C. And vpon the line B C let there be applied a rectangle parallelograme equall vnto the fourth part of the ſquare of the line A, and wanting in figure by a ſquare, and let the ſayd parallelograme be that which is contained vnder the lines B D and D C. Then muſt we proue that the line B D is vnto the line D C commenſurable in length. The ſame conſtructions and ſuppoſitions, that were before, remayning, we may in like ſort proue that the line B C is in power more then the line A, by the ſquare of the line F D. But by ſuppoſitiõ the line B C is in power more thẽ the line A by the ſquare of a line cõmenſurable vnto it in length. Wherfore the line B C is vnto the line F D cõmenſurable in length. Wherefore the line compoſed of the two lines B F and D C is cõmenſurable in length vnto the line F D (by the ſecond part of the 15. of the tenth). Wherefore (by the 12. of the tenth or by the firſt part of the 15. of the tenth) the line B C is commenſurable in length to the line compoſed of B F and D C. But the whole line conpoſed of B F and D C is commenſurable in length vnto D C. For B F (as before hath bene proued) is equall to D C. Wherefore the

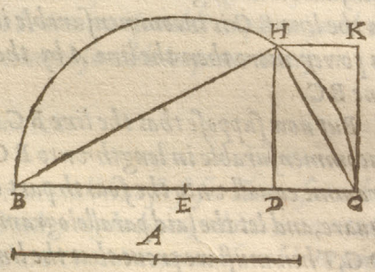
Demonſtration of the ſecond part which is the conuerſe of the firſt.

line B C is commenſurable in length vnto the line D C (by the 12. of the tenth). Wherefore alſo the line B D is commenſurable in length vnto the line D C (by the ſecond part of the 15. of the teuth). If therfore there be two right lines vnequall, and if vpon the greater be applied a parallelograme equall vnto the fourth part of the ſquare of the leſſe and wanting in figure by a ſquare, if alſo the parallelograme thus applied deuide the line whereupon it is applied into partes commenſurable in length: then ſhall the greater line be in power more then the leſſe by the ſquare of a line commenſurable in length vnto the greater. And if the greater be in power more then the leſſe by the ſquare of a line commẽſurable in length vnto the greater, and if alſo vpon the greater be applied a pvrallelograme equall vnto the fourth part of the ſquare made of the leſſe and wanting in figure by a ſquare: then ſhall it deuide the greater line into partes commenſurable in length: which was required to be proued.

Campane after this propoſition teacheth how we may redily apply vpon the line B C a parallelograme equall to the fourth part of the ſquare of halfe of the line A, and wanting in figure by a ſquare, after this maner.

How to deuide the line B C redely in ſuch ſort as is required in the propoſitiõ.

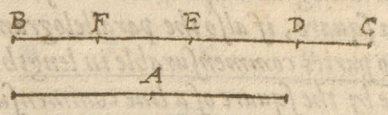
Deuide the line B C into two lines in ſuch ſort that halfe of the line A ſhalbe the meane proportionall betwene thoſe two lines, which is poſſible, when as the line B C is ſuppoſed to be greater then the line A, and may thus be done. Deuide the line B C into two equal partes in the point E and deſcribe vpon the line B C a ſemicircle B H C. And vnto the line B C, and from the point C erect a perpẽdicular line C K and put the line C K equall to halfe of the line A. And by the point K draw vnto the line E C a parallel line K H cutting the ſemicircle in the point H, (which it muſt needes cut, foraſmuch as the line B C is greater then the line A). And frõ the point H draw vnto the line B C a perpendicular line H D: which line H D, foraſmuch as by the 34. of the firſt it is equall vnto the line K C, ſhall alſo be equall to halfe of the line A: draw the lines B H and H C. Now then by the 31. of the third the angle B H C is a right angle. Wherefore by the corollary of the eight of the ſixt booke the line H D is the meane proportionall betwene the lines B D and D C. Wherefore the halfe of the line A which is equall vnto the line H D is the meane proportionall betwene the lines B D and D C. Wherefore that which is contained vnder the lines B D and D C is equall to the fourth part of the ſquare of the line A. And ſo if vpon the line B D be deſcribed a rectangle parallelograme hauing his other ſide equall to the line D C, there ſhalbe applied vpon the line B C a rectangle parallelograme equall vnto the ſquare of halfe of the line A, and wanting in figure by a ſquare: which was required to be done.



¶ The 15. Theoreme. The 18. Propoſition.

If there be two right lines vnequall, and if vpon the greater be applied a parallelograme equall vnto the fourth part of the ſquare of leſſe, and wanting in figure by a ſquare, if alſo the parallelograme thus applied deuide the line whereupon it is applied into partes incommenſurable in length: the greater line ſhalbe in power more then the leſſe line by the ſquare of a line incommenſurable in length vnto the greater line. And if the greater line be in power more then the leſſe line, by the ſquare of a line incommẽſurable in length vnto the greater, and if alſo vpon the greater be applied a parallelograme equall vnto the fourth part of the ſquare of the leſſe and wanting in figure by a ſquare: then ſhall it deuide the greater line into partes incommenſurable in length.

Suppose that these two right lines A and B C be vnequall the one to the other, of which let B C be the greater. And vpon the same B C apply a parallelograme equall vnto the fourth part of the square of the line A, and wanting in figure by a square: which how to doo was before taught in the end of the former proposition. And let the sayd parallelograme be that which is contained vnder the lines B D and D C. And let B D be incommensurable in length vnto D C. Then I say that the line B C is in power more then the line A, by the square of a right line incommensurable in length vnto the line B C.



First let the same order of construction, and demonstration be obserued in this which was in the former proposition. And we may in like sort proue that the line B C is in power more thē the line A by the square of the line D F. Now then must we proue that the lines B C and D F are incommensurable in length. Foraſmuch as by ſuppoſition the line B D is incommenſurable in lēgth vnto the line D C: therefore (by the 16. of the tenth) the line B C is incommenſurable in length vnto the line C D. But D C is commenſurable to these two lines B F and D C added together. For B F is equall vnto D C. Wherefore (by the 13. of the tenth) B C is incommenſurable vnto these two lines B F and D C compoſed. Wherefore by the ſecond part of the 16. of the tenth the line compoſed of the lines B F and D C taken as one line is incommenſurable in length vnto the line F D Wherefore by the firſt part of the ſame 16. propoſition the line B C is incommenſurable in length vnto the line F D. Wherefore the line B C is in power more then the line A by the ſquare of a line incommenſurable in length vnto the line B C.

Demonſtratiō of the firſt part.

But now ſuppoſe that the line B C be in power more then the line A by the ſquare of a line incommenſurable in length vnto B C. And vpon the line B C let there be applied a parallelogramme equall vnto the fourth part of the ſquare of the line A, and wanting in figure by a ſquare, and let the ſaid parallelogramme be that which is contained vnder the lines B D & D C. Then muſt we proue that the line B D is vnto the line D C incommenſurable in lēgth. The ſame order of conſtruction and demonſtration being kept, we may in like ſort proue that the line B C is in power more then the line A by the ſquare of the line F D. But now (by ſuppoſition) the line B C is in power more then the line A by the ſquare of a line incommenſurable in length vnto B C. Wherefore the line B C is vnto the line F D incommenſurable in length. Wherfore the line compoſed of B F and D C taken as one line, ſhall be incommenſurable in length to the line F D (by the ſecond part of the 16. of the tenth): wherefore alſo by the firſt part of the ſame, the line B C ſhall be incommenſurable in length to the line compoſed of the lines B F and D C. But the line compoſed of the lines B F and D C is commenſurable in length to the line D C (for that B F (as before hath bene proued) is equall to D C). Wherfore the line B C is incōmenſurable in length to the line D C (by the 13. of the tenth). Wherefore by the ſecond part of the 16. of the tenth, the line B D is incommēſurable in lēgth vnto the line D C. If therfore there be two right lines vnequall, and if vpon the greater be applied a parallelogramme equall vnto the fourth part of the ſquare of the leſſe line, & wanting in figure by a ſquare, if alſo the parallelogramme thus applied deuide the line wherupon it is applied into partes incommenſurable in length: the greater line ſhall be in power more then the leſſe line by the ſquare of a line incommenſurable in length vnto the greater. And if the greater line be in power more then the leſſe, by the ſquare of a line incommenſurable in length vnto the greater, and if alſo vpon the greater be applied a parallelogramme equall vnto the fourth part of the ſquare of the leſſe line, and wanting in figure by a ſquare, then ſhall it deuide the greater line into partes incommenſurable in length: which was required to be demonſtrated.

Demonſtration of the ſecond part which is the conuerſe of the former.

This Propoſition may alſo be demonſtrated by the former propoſition, namely, the firſt part of this by

by the ſecond part of the former,and the ſecond part of this by the firſt part of the former,by an argument leading to an abſurditie. For as touching the firſt part of this propoſition,the line B C contayning in power more then the line A by the ſquare of the line F D,if the line B C be not incommenſurable vnto the line F D,then is it commenſurable vnto it. Wherfore (by the ſecond part of the 17 propoſition) the lines B D and D C alſo are commenſurable,which is impoſſible,for they are ſuppoſed to be incommenſurable. So likewiſe as touching the ſecond parte of the ſame, the line B C contayning in power more then the line A by the ſquare of the line F D,if the line D B be not incommenſurable to the lyne D C,then is it commenſurable vnto it : wherfore(by the firſt part of the 17.propoſition) the lines B C and F D are alſo commenſurable,which were abſurde.For the lines B C and F D are ſuppoſed to be incommenſurable : which was required to be proued.

An other demonſtration by an argumēt leading to an abſurditie.

¶ An aſſumpt.

An Aſſumpt.

Foraſmuch as it hath bene proued that lines commenſurable in length, are alwayes alſo commenſurable in power,but lines commenſurable in power are not alwayes commenſurable in length,but may be in length both commenſurable and alſo incommenſurable:it is manifeſt,that if vnto the line propounded,which is called rationall of purpoſe, a certayne line be cōmenſurable in length,it ought to be called rationall and cōmenſurable vnto it,not only in length,but alſo in power : for lines commenſurable in length are alſo alwayes commenſurable in power.But if vnto the line propounded which is called rationall of purpoſe, a certayne line be commenſurable in power,then if it be alſo commenſurable vnto it in length, it is called rationall and commenſurable vnto it both in length and in power.But againe if vnto the ſaid line geuen which is called rationall,a certayne line be commenſurable in power,and incommenſurable in length,that alſo is called rationall,commenſurable in power onely.

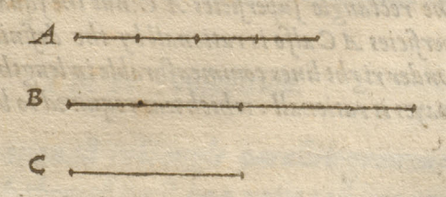
An annotacion of Proclus.

He calleth theſe lines rationall,which are vnto the rationall line firſt ſet cōmenſurable in length & in power,or in power only.And there are alſo other right lines, which are vnto the rationall line firſt ſet, incommenſurable in length, and are vnto it commenſurable in power only,and therfore they are called rationall, & commēſurable the one to the other: for which cauſe they are rationall.But euen theſe lines may be commenſurable the one to the other, either in length, and therefore in power, or els in power onely . Now if they be commenſurable in length,then are thoſe lines called rationall,commenſurable in length,but yet ſo that they be vnderſtand to be in power commenſurable:but if they be commenſurable the one to the other in power onely,they alſo are called rationall commenſurable in power onely.

¶A Corollary.

A Corollary added by Montaureus.

And that two lines or more being rationall and commenſurable in length to the rational line firſt ſet,are alſo commenſurable the one to the other in length,hereby it is manifeſt: for foraſmuch as they are rationall and commenſurable in length to the rationall line firſt ſet, but thoſe magnitudes whiche are commenſurable to one and the ſelfe ſame magnitude,are alſo commenſurable the one to the other (by the 12.of the tenth) wherfore the rationall lines,commenſurable in length to the rational lyne firſt ſet, are alſo commenſurable in length the one to the other.And as touching thoſe which are rationall commenſurable in power onely to the rationall line firſt ſet,they alſo muſt needes be at the leaſt commenſurable in power the one to the other. For foraſmuch as their ſquares are rationall they ſhall bee commenſurable to the ſquare of the rationall line firſt ſet.Wherfore by the 12. of this booke,they are alſo commenſurable the one to the other. Wherefore their lines are at the leaſt commenſurable in power the one to the other.And it is poſſible alſo that they may be commēſurable in lēgth the one to the other. For ſuppoſe that A be a rationall line firſt ſet,and let the line B be vnto the ſame rationall line A commenſurable in power onely,that is, incommenſurable in length vnto it. Let there be alſo an other line C commen-



ſurable in length to the lyne B (which is poſſible by the principles of this booke.) Now by the 13. of the tenth, it is manifeſt that the line C is incommenſurable in length vnto the line A. But the ſquare of the line A is cõmẽſurable to the ſquare of the line B by ſuppoſition, and the ſquare of the line C is alſo commenſurable to the ſquare of the line B by ſuppoſition. Wherefore by the 12. of this booke, the ſquare of the line C is commenſurable to the ſquare of the line A. Wherfore by the definition, the line C ſhall be rationall commenſurable in power onely to the line A, as alſo is the line B. Wherefore there are geuen two rationall lines commenſurable in power onely to the rationall line firſt ſet, and commẽſurable in length the one to the other.

Cauſe.

Cauſe of increaſing the difficulty of this booke.

Note.

Here is to be noted, which thing alſo we before noted in the definitions, that *Campane* and others which followed him, brought in theſe phraſes of ſpeaches, to call ſome lynes rationall in power onely, and other ſome rationall in length and in power, which we cannot finde that Euclide euer vſed. For theſe wordes in length and in power are neuer referred to rationalitie or irrationalitie, but alwayes to the commenſurabilitie or incommenſurablitie of lines. Which peruerting of wordes (as was there declared) hath much increaſed the difficulty and obſcurenes of this booke. And now I thinke it good agayne to put you in minde, that in theſe propoſitions which follow, we muſt euer haue before our eyes the rationall line firſt ſet, vnto which other lines compared are either rationall or irrationall, according to their commenſurability or incommenſurabilitie.

¶ The 16. Theoreme. The 19. Propoſition.

A rectangle figure comprehended vnder right lines commenſurable in lengthe, being rationall according to one of the foreſaide wayes: is rationall.

Conſtruction.

Demonſtration.

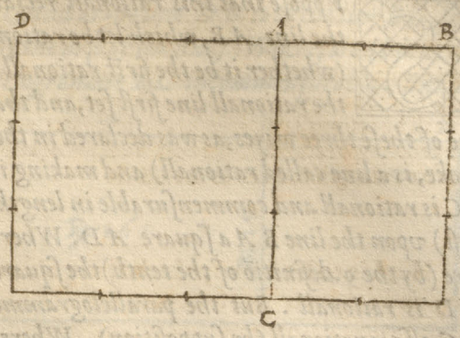
Suppoſe that this rectangle figure A C be comprehended vnder theſe right lines A B and B C being commenſurable in length, and rationall according to one of the foreſaid wayes. Then I ſay that the ſuperficies A C is rationall, deſcribe (by the 46. of the firſt) vpon the line A B a ſquare A D. Wherfore that ſquare A D is rationall by the definition. And foraſmuch as the line A B is commenſurable in length vnto the line B C, and the line A B is equall vnto the lyne B D, therefore the lyne B D is commenſurable in length vnto the line B C. And as the line B D is to the line B C, ſo is the ſquare D A to the ſuperficies A C (by the firſt of the ſixt): but it is proued that the line B D is commenſurable vnto the line B C, wherfore (by the 10. of the tenth) the ſquare D A is commenſurable vnto the rectangle ſuperficies A C. But the ſquare D A is rationall, wherfore the rectangle ſuperficies A C alſo is rationall by the definition. A rectangle figure therfore comprehended vnder right lines commenſurable in length, beyng rationall accordyng to one of the foreſayd wayes is rationall: which was required to be proued.

Diuers caſes in this propoſition.

The ſecond caſe.

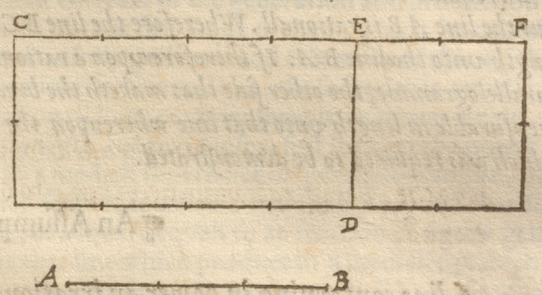
Where as in the former demonſtration the ſquare was deſcribed vpon the leſſe line, we may alſo demonſtrate the Propoſiton, if we deſcribe the ſquare vpon the greater line, and that after thys maner. Suppoſe that the rectangle ſuperficies B C be contayned of theſe vnequall lines A B and A C, which let be rationall commenſurable the one to the other in length. And let the line A C be the greater. And vpon

vpon the line A C describe the square D C. Then I say, that the parallelogramme B C is rationall. For the line A C is commensurable in length vnto the line A B by supposition, and the line D A is equall to the line A C. Wherefore the line D A is commensurable in length to the line A B. But what proportion the line D A hath to the line A B, the same hath the square D C to the parallelogramme C B (by the first of the sixt). Wherefore (by the 10. of this booke) the square D C is commensurable to the parallelogramme C B. But it is manifest, that the square D C is rationall, for that it is the square of a rationall line, namely, A C. Wherefore (by the definition) the parallelogrāme also C B is rationall.



The first kind of rationall lines commensurable in length.

Moreouer, forasmuch as those two former demonstrations seeme to speake of that parallelogrāme which is made of two lines, of which any one may be the line first set, which is called the first rationall line, from which (we sayd) ought to be taken the measures of the other lines compared vnto it, and the other is commēsurable in length to the same first rationall line, which is the first kinde of rationall lines cōmensurable in length: I thinke it good here to set an other case of the other kinde of rationall lines, of lines I say rationall cōmensurable in length compared to an other rationall line first set, to declare the generall truth of this Theoreme, and that we might see that this particle *according to any of the foresayd wayes* was not here in vaine put. Now then suppose first a rationall line A B. Let there be also a parallelogrāme C D contayned vnder the lines C E and E D, which lines let be rationall, that is commensurable in length to the first rationall line propounded A B. Howbeit, let those two lines C E and E D be diuers and vnequall lines vnto the first rationall line A B. Then I say, that the parallelogramme C D is rationall. Describe the square of the line D E, which let be D F. First it is manifest (by the 12. of this booke) that the lines C E & E D, are commensurable in lēgth the one to the other. For either of them is supposed to be commensurable in length vnto the line A B. But the line E D is equall to the line E F. Wherefore the line C E is commensurable in length to the line E F. But as the line C E is to the line E F, so is the parallelogramme C D to the square D F (by the first of the sixt). Wherefore (by the 10. of this booke) the parallelogramme C D shall be commensurable to the square D F. But the square D F is commensurable to the square of the line A B which is the first rationall line propounded. Wherfore (by the 12. of this booke) the parallelogramme C D is commensurable to the square of the line A B. But the square of the line A B is rationall (by the definition). Wherfore by the definition also of rationall figures, the parallelogramme C D shall be rationall.



This particle in the proposition (according to any of the foresayde wayes) was not in vayne put.

The second kinde of rationall lines cōmensurable in leng.h.

The third case.

Now resteth an other case of the thirde kinde of rationall lines commensurable in length the one to the other, which are to the rationall line A B first set commensurable in power onely, and yet are therfore rationall lines. And let the lines C E and E D be cōmensurable in length the one to the other. Now then let the selfe same construction remaine that was in the former: so that let the lines C E and E D be rationall commensurable in power onely vnto the line A B. But let them be commensurable in length the one to the other. Then I say, that in this case also the parallelogramme C D is rationall. First it may be proued as before, that the parallelogramme C D is commensurable to the square D F. Wherfore (by the 12. of this booke) the parallelogramme C D shall be commensurable to the square of the line A B. But the square of the line A B is rationall. Wherefore (by the definition) the parallelogrāme C D shall be also rationall. This case is well to be noted. For it serueth to the demonstration and vnderstanding of the 25. Proposition of this booke.

The third kinde of rationall lines commensurable in length.

The fourth case.

¶ The 17. Theoreme. The 20. Proposition.

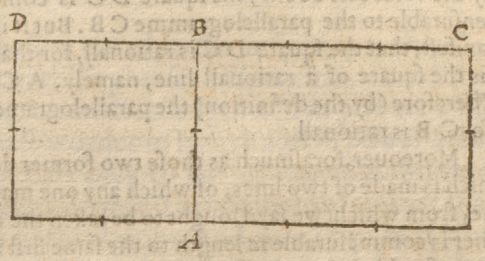
If vpon a rationall line be applied a rationall rectangle parallelogramme: the other side that maketh the breadth thereof shall be a rationall line and commensurable in length vnto that line wherupon the rationall parallelogramme is applied.

This proposition is the conuerse of the former propo-sition.

Construction.

Demonstration.

Supposse that this rationall rectangle parallelogramme AC, be applied vpon the line AB, which let be rationall according to any one of the foresaid wayes (whether it be the first rationall line set, or any other line commensurable to the rationall line first set, and that in length and in power, or in power onely: for one of these three wayes, as was declared in the Assumpt put before the 19. Proposition of this booke, is a line called rationall) and making in breadth the line BC. Then I say, that the line BC is rationall and commensurable in length vnto the line BA. Describe (by the 46. of the first) vpon the line BA a square AD. Wherefore (by the 9. definitiõ of the tenth) the square AD is rationall. But the parallelogramme AC also is rationall (by supposition). Wherefore (by the conuersion of the definition of rationall figures, or by the 12. of this booke) the square DA is commensurable vnto the parallelogramme AC. But as the square DA is to the parallelogramme AC, so is the line DB to the line BC (by the first of the sixt). Wherefore (by the 10. of the tenth) the line DB is commensurable vnto the line BC. But the line DB is equall vnto the line BA. Wherefore the line AB is cõmensurable vnto the line BC. But the line AB is rationall. Wherefore the line BC also is rationall and commensurable in length vnto the line BA. If therefore vpon a rationall line be applied a rationall rectangle parallelogramme, the other side that maketh the breadth therof shall be a rationall line commensurable in length vnto that line whereupon the rationall parallelogramme is applied: which was required to be demonstrated.



¶ An Assumpt.

A line containing in power an irrationall superficies, is irrationall.

An Assumpt.

Suppose that the line AB cõtaine in power an irrationall superficies, that is, let the square described vpon the line AB, be equal vnto an irrationall superficies. Then I say, that the line AB is irrationall. For if the line AB be rationall, thẽ shall the square of the line AB be also rationall. For so was it put in the definitions. But (by supposition) it is not. Wherefore the line AB is irrationall. A line therefore containing in power an irrationall superficies, is irrationall.

A B

¶ The 18. Theorene. The 21. Proposition.

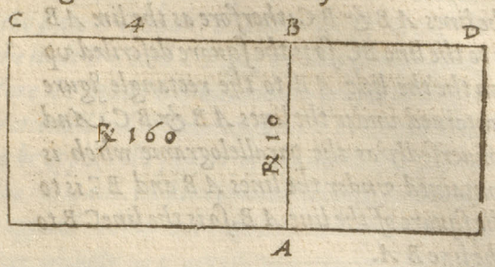
A rectangle figure comprehended vnder two rationall right lines commensurable in power onely, is irrationall. And the line which in power containeth that rectangle figure is irrationall, & is called a mediall line.

Construction. Demonstration.

Supposse that this rectangle figure AC be comprehended vnder these rationall right lines AB and BC commensurable in power onely. Then I say, that the superficies AC is irrationall: and the line which containeth it in power is irrationall, and is called a mediall line. Describe (by the 46. of the first) vpon the line AB a square AD. Wherefore the square AD is rationall. And forasmuch as the line AB is vnto the line BC incommensurable in length, for they are supposed to be commensurable in power onely, and the line AB is equall vnto the line BD, therefore also the line BD is

vnto

vnto the line B C incommenſurable in length. And as the line D B is to the line B C, ſo is the ſquare A D to the parallelogramme A C (by the firſt of the ſixt). Wherefore (by the 10. of the tenth) the ſquare D A is vnto the parallelogramme A C incommenſurable. But the ſquare D A is rationall. Wherefore the parallelogramme A C is irrationall. Wherefore alſo the line that contayneth the ſuperficies A C in power, that is whoſe ſquare is equall vnto the parallelogramme A C, is (by the Aſſumpt going before) irrationall. And it is called a mediall line, for that the ſquare which is made of it, is equall to that which is contayned vnder the lines A B and B C, and therefore it is (by the ſecond part of the 17. of the ſixt) a meane proportionall line betwene the lines A B and B C. A rectangle figure therefore comprehended vnder rationall right lines which are commenſurable in power onely, is irrationall. And the line which in power contayneth that rectangle figure is irrationall, and is called a mediall line.



At this Propoſition doth *Euclide* firſt entreateof the generation and production of irrationall lines. And here he ſearcheth out the firſt kinde of them, which he calleth a mediall line. And the definition therof is fully gathered and taken out of this 21. Propoſition, which is this. A mediall line is an irrationall line whoſe ſquare is equall to a rectangled figure contayned of two rationall linescommenſurable in power onely. It is called a mediall line, as *Theon* rightly ſayth, for two cauſes, firſt for that the power or ſquare which it produceth is equall to a mediall ſuperficies or parallelogramme. For as that line which produceth a rationall ſquare, iscalled a rationall line, and that line which produceth an irrationall ſquare, or a ſquareequall to an irrationall figure generally is called an irrationall line: ſo is that line which produceth a mediall ſquare, or a ſquare equall to a mediall ſuperficies, called by ſpeciall name a mediall line. Secondly it is called a mediall line, becauſe it is a meane proportionall betwene the two lines cõmenſurable in power onely which comprehend the mediall ſuperficies.

Diffinition of a mediall line.

¶ A Corollary added by *Flußates*.

A rectangle parallelogramme contayned vnder a rationall line and an irrationall line, is irrationall. For if the line A B be rationall, and if the line C B be irrationall, they ſhall be incommenſurable. But as the line B D (which is equall to the line B A) is to the line B C, ſo is the ſquare A D to the parallelogrãme A C. Wherefore the parallelogramme A C ſhall be incommenſurable to the ſquare A D which is rationall (for that the line A B wherupon it is deſcribed is ſuppoſed to be rationall). Wherefore the parallelogramme A C which is contayned vnder the rationall line A B, and the irrationall line B C, is irrationall.

A Corollary.

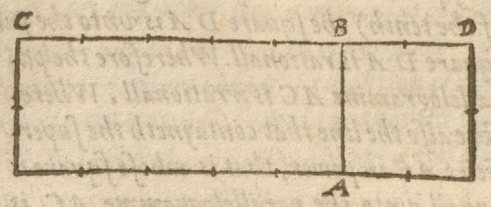
¶ An Aſſumpt.

If there be two right lines, as the firſt is to the ſecond, ſo is the ſquare which is deſcribed vpon the firſt, to the parallelograme which is contained vnder the two right lines.

Suppoſe that there be two right lines A B and B C. Then I ſay that as the line A B is to the line B C, ſo is the ſquare of the line A B, to that which is contained vnder the lines A B and B C. Deſcribe (by the 46. of the firſt) vpon the line A B a ſquare A D. And make perfect the parallelograme A C. Now for that as the line A E is to the line B C (for the line A B, is equall to the line B D), ſo is the ſquare A D to the parallelograme C A by the firſt of the ſixt and

This aſſumpt is nothing els but a part of the firſt propoſition of the ſixt booke.

and A D is the ſquare which is made of the line A B, and A C is that which is contained vnder the lines B D and B C, that is, vnder the lines A B & B C: therfore as the line A B is to the line B C, ſo is the ſquare deſcribed vpon the the line A B to the rectangle figure contained vnder the lines A B & B C. And conuerſedly as the parallelograme which is contained vnder the lines A B and B C is to the ſquare of the line A B, ſo is the line C B to the line B A.

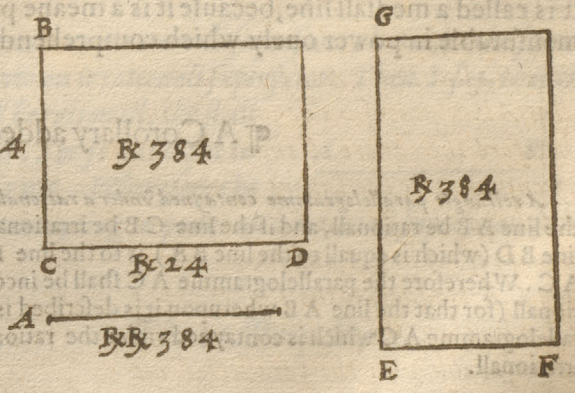


¶ The 19. Theoreme. The 22. Propoſition.

If vpon a rationall line be applied the ſquare of a mediall line: the other ſide that maketh the breadth thereof ſhalbe rationall, and incommenſurable in length to the line wherupon the parallelograme is applied.

Demonſtration.

Vppoſe that A be a mediall line, and let B C be a line rationall, and vpon the line B C deſcribe a rectangle parallelograme equall vnto the ſquare of the line A, and let the ſame be B D making in breadth the line C D. Then I ſay that the line C D is rationall and incõmenſurable in length vnto the line C B. For foraſmuch as A is a mediall line, it containeth in power (by the 21. of the tenth) a rectangle parallelograme comprehended vnder rationall right lines commenſurable in power onely. Suppoſe that it containe in power the parallelograme G F: and by ſuppoſition it alſo containeth in power the parallelograme B D. Wherefore the parallelograme B D is equall vnto the parallelograme G F: and it is alſo equiangle vnto it, for that they are ech rectãgle. But in parallelograms equall and equiangle the ſides which containe the equall angles, are reciprocall (by the 14. of the ſixt). Wherfore what proportiõ the line B C hath to the line E G, the ſame hath the line E F to the line C D. Therefore (by the 22. of the ſixt) as the ſquare of the line B C is to the ſquare of the line E G, ſo is the ſquare of the line E F to the ſquare of the line C D. But the ſquare of the line B C is commenſurable vnto the ſquare of the line E G (by ſuppoſition). For either of them is rationall. Wherefore (by the the 10. of the tenth) the ſquare of the line E F is commenſurable vnto the ſquare of the line C D. But the ſquare of the line E F is rationall. Wherefore the ſquare of the line C D is likewiſe rationall. Wherefore the line C D is rational. And foraſmuch as the line E F is incommenſurable in length vnto the line E G (for they are ſuppoſed to be commenſurable in power onely). But as the line E F is to the line E G, ſo (by the aſſumpt going before) is the ſquare of the line E F to the parallelograme which is contained vnder the lines E F and E G. Wherefore (by the 10. of the tenth) the ſquare of the line E F is incommenſurable vnto the parallelograme which is contained vnder the lines F E and E G. But vnto the ſquare of the line E F the ſquare of the line C D is commenſurable, for it is proued that either of them is



a rationall

a rationall line. And that which is contained vnder the lines D C and C B is commensurable vnto that which is contained vnder the lines F E and E G. For they are both equall to the square of the line A. Wherefore (by the 13. of the tenth) the square of the line C D is incommensurable to that which is contained vnder the lines D C and C B. But as the square of the line C D is to that which is contained vnder the lines D C and C B, so (by the assumpt going before) is the line D C to the line C B. Wherefore the line D C is incommensurable in length vnto the line C B. Wherefore the line C D is rationall and incommensurable in length vnto the line C B. If therefore vpon a rationall line be applied the square of a mediall line, the other side that maketh the breadth thereof shalbe rationall, and incommensurable in length to the line whereupon the parallelogramme is applied: which was required to be proued.

A square is sayd to be applied vpon a line, when it, or a parallelograme equall vnto it, is applied vpon the sayd line. If vpon a rationall line geuen we will apply a rectangle parallelograme equall to the square of a mediall line geuen, and so of any line geuen, we must, by the 11. of the sixt, finde out the third line proportionall with the rationall line and the mediall line geuen: so yet that the rationall line be the first, and the mediall line geuen, (which containeth in power the square to be applied) be the second. For then the superficies contained vnder the first and the third, shalbe equall to the square of the midle line, by the 17. of the sixt.

How a square is sayde to be applied vppon a line.

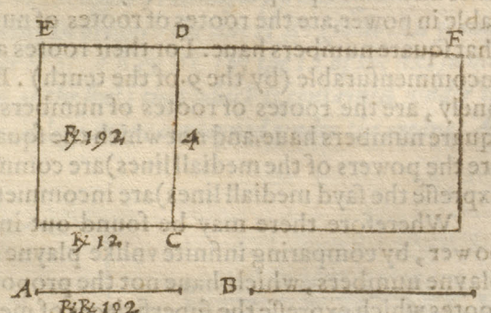
¶ The 20. Theoreme. The 23. Proposition.

A right line commensurable to a mediall line, is also a mediall line.

Vppose that A be a mediall line. And vnto the line A let the line B be commensurable, either in length, & in power, or in power only. Then I say that B also is a mediall line. Let there be put a rationall line C D. And vpon the line C D, apply a rectangle parallelograme C E, equall vnto the square of the line A, and making in breadth the line E D. Wherefore (by the proposition going before) the line E D is rationall and incommensurable in length vnto the line C D. And againe vpon the line C D apply a rectangle parallelograme C F equall vnto the square of the line B, and making in breadth the line D F. And forasmuch as the line A is commensurable vnto the line B, therefore the square of the line A is commensurable to the square of the line B. But the parallelograme E C is equall to the square of the line A, and the parallelograme C F is equall to the square of the line B: wherefore the parallelograme E C is cõmensurable vnto the parallelograme C F. But as the parallelograme E C, is to the parallelograme C F, so is the line E D to the line D F (by the first of the sixt). Wherefore (by the 10. of the tenth) the line E D is commensurable in length vnto the line D F. But the line E D is rationall and incõmensurable in length vnto the line D C, wherefore the line D F is rationall and incommensurable in length vnto the line D C (by the 13. of the tenth). Wherefore the lines C D and D F are rationall commensurable in power onely. But a rectangle figure comprehended vnder rationall right lines commensurable in power onely, is (by the 21. of the tenth) irrationall, and the line that containeth it in power is irrationall, and is called a mediall line. Wherefore the line that containeth in power that which is comprehended vnder the lines C D and D F is a mediall line. But the line B containeth in power the parallelograme which is comprehended vnder

Construction.

Demonstration.



der the lines C D and D F Wherefore the line B is a mediall line. Aright line therfore commensurable to a mediall line, is also a mediall line: which was required to be proued.

¶ *Corollary.*

Hereby it is manifest that a superficies commensurable vnto a mediall superficies, is also a mediall superficies.

For the lines which containe in power those superficieces are commensurable in power, of which the one is a mediall line (by the definitiō of a mediall line in the 21. of this tenth): wherefore the other also is a mediall line by this 23. propositiō. And as it was sayd of rationall lines so also is it to be sayd of mediall lines, namely, that a line commensurable to a mediall line, is also a mediall line, a line I say which is commensurable vnto a mediall line, whether it be commensurable in length, and also in power, or ells in power onely. For vniuersally it is true, that lines commensurable in length, are also commensurable in power. Now if vnto a mediall line there be a line commensurable in power, if it be commensurable in length, thē are those lines called mediall lines commensurable in length & in power. But if they be commensurable in power onely, they are called mediall lines commensurable in power onely.

There are also other right lines incommensurable in length to the mediall line, and commensurable in power onely to the same: and these lines are also called mediall, for that they are commensurable in power to the mediall line. And in as much as they are mediall lines, they are commensurable in power the one to the other. But being compared the one to the other, they may be commensurable either in length, and therefore in power, or ells in power onely. And then if they be commensurable in length, they are called also mediall lines commensurable in length, and so consequently they are vnderstanded to be commensurable in power. But if they be commensurable in power onely, yet notwithstanding they also are called mediall lines commensurable in power onely.

Flussates after this proposition teacheth how to come to the vnderstanding of mediall superficieces and lines, by surd numbers, after this maner. Namely to expresse the mediall superficieces by the rootes of numbers which are not square numbers: and the lines cōtaining in power such medial superficieces, by the rootes of rootes of numbers not square. Mediall lines also commensurable, are expressed by the rootes of rootes of like superficial numbers, but yet not square, but such as haue that proportion that the squares of square numbers haue. For the rootes of those numbers and the rootes of rootes are in proportion as numbers are, namely, if the squares be proportionall the sides also shalbe proportionall (by the 22. of the sixt). But mediall lines incommensurable in power, are the rootes of rootes of numbers, which haue not that proportion, that square numbers haue. For their rootes are the powers of mediall lines, which are incommensurable (by the 9. of the tenth). But mediall lines commensurable in power onely, are the rootes of rootes of numbers, which haue that proportion that simple square numbers haue, and not which the squares of squares haue. For the rootes (which are the powers of the mediall lines) are commēsurable, but the rootes of rootes (which expresse the sayd mediall lines) are incommensurable.

Wherefore there may be found out infinite mediall lines incommensurable in power, by comparing infinite vnlike playne numbers the one to the other. For vnlike playne numbers, which haue not the proportion of square numbers, doo make the rootes which expresse the superficieces of mediall lines incōmensurable (by the 9. of the tenth). And therefore the mediall lines containing in power those superficieces are incō mensurable in length. For lines incommensurable in power, are alwayes incommensurable in length (by the corrollary of the 9. of the tenth).

¶ *The 21. Theoreme.* *The 24. Proposition.*

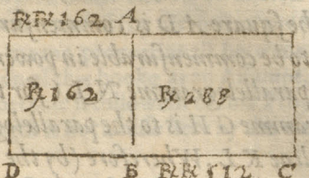
A rectangle parallelogramme comprehended vnder mediall lines cōmensurable in length, is a mediall rectangle parallelogramme.

Suppose

Suppose that the rectangle parallelogramme AC, be comprehended vnder these mediall right lines AB and BC, which let be commensurable in length. Then I say, that AC is a mediall rectangle parallelogramme. Describe (by the 46. of the first) vpon the line AB a square AD. Wherefore the square AD is a mediall superficies. And forasmuch as the line AB is commensurable in length vnto the line BC, and the line AB is equall vnto the line BD, therefore the line BD is commensurable in length vnto the line BC. But as the line DB is to the line BC, so is the square DA to the parallelogramme AC (by the first of the sixt). Wherfore (by the 10. of the tenth) the square DA is commensurable vnto the parallelogramme AC. But the square DA is mediall, for that it is described vpon a mediall line. Wherefore AC also is a mediall parallelogrãme (by the former Corollary). A rectangle &c: which was required to be proued.

Construction.

Demonstration.



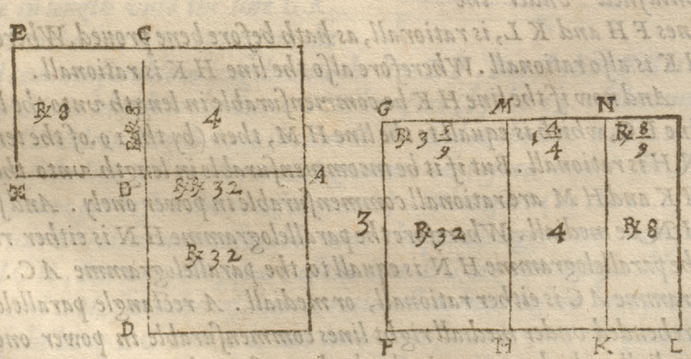
¶ The 22. Theoreme. The 25. Proposition.

A rectangle parallelogramme comprehended vnder mediall right lines commensurable in power onely, is either rationall, or mediall.

A Mediall line being geuen, there may be found an other line commensurable vnto it, in power onely (by the 11. of this booke) as was taught there touching rationall lines. Now then suppose that the rectangle parallelogramme AC be comprehended vnder these mediall right lines AB & BC. Then I say, that the parallelogramme AC is either rationall, or mediall. Describe (by the 46. of the first) vpon the lines AB and BC, their squares AD and BE. Wherefore either of these squares AD and BE is mediall (by the 21. of the tenth). Let there be put a rationall line FG. And vpon the line FG, let there be applied a rectangle parallelogramme GH equall to the square AD, and making the breadth the line FH. (How to do this was taught in the 22. of this booke). And vpon the line HM, apply a rectangle parallelogramme MK, equall to the parallelogramme AC, and making in breadth the line HK: (to do this ye must take a fourth line proportionall with the lines HM, AB & BC (by the 12. of the sixt) which fourth line let be HK: wherefore (by the 26. of the sixt) that which is contayned vnder the extremes HM and HK, is equall to the parallelogramme contayned vnder the meanes AB and BC). And moreouer vpon the line KN apply a rectangle parallelogramme NL equall to the square BE, and making in breadth the line KL. Wherfore the lines FH, HK & KL, are in one and the selfe same right line. (For those parallelogrammes so applied vpo the lines FG, HM, and KN, are rectangle, and the angles FHM and KHM are equall to two right angles, for they are right angles: wherefore the lines FH and HK are in one right line, by the 14. of the first. So also may be sayd of the angles HKN and LKN). And

Construction.

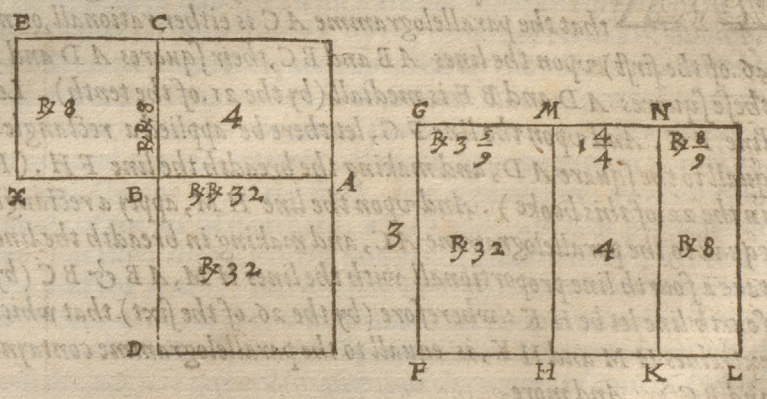
Demonstration.



foraſmuch as either of theſe ſquares A D and B E is mediall, and the ſquare A D is equall to the parallelogramme G H, and the ſquare B E to the parallelogramme N L: wherefore either of theſe parallelogrammes G H and N L is mediall. And they are applyed vpon a rationall line, namely, F G. Wherefore (by the 22. of the tenth) either of theſe right lines F H and K L is a rationall line, and incõmenſurable in length vnto the line F G. And foraſmuch as the ſquare A D is commenſurable to the ſquare B E (for the lines A B and B C are ſuppoſed to be commenſurable in power) therefore the parallelogramme G H is commenſurable to the parallelogramme N L (for they are equall vnto the ſayd ſquares). But as the parallelogramme G H is to the parallelogramme N L, ſo (by the firſt of the ſixt) is the line F H to the line K L. Wherefore (by the 10. of the tenth) the line F H is commenſurable in length to the line K L. Wherefore theſe right lines F H and K L, are rationall commenſurable in length the one to the other (commenſurable in length, I ſay, the one to the other, for vnto the line F G, by reaſon of which they are rationall, they are incõmenſurable in length as it hath bene proued). Wherefore the parallelogramme contayned vnder the lines F H and K L, is rationall (by the 19. of the tenth).

In theſe fewe words conſiſteth the full demonſtratiõ of the corollary following.

And foraſmuch as the line D B is equall to the line B A, and the line X B to the line B C: therefore as the line D B is to the line B C, ſo is the line A B to the line B X. But as the line D B is to the line B C, ſo (by the firſt of the ſixt) is the ſquare D A to the parallelogrãme A C. And as the line A B is to the line B X, ſo is the parallelogramme A C to the ſquare C X. Wherefore as the ſquare D A is to the parallelogramme A C, ſo is the parallelogramme A C to the ſquare C X. But the ſquare A D is equall to the parallelogramme G H, and to the parallelogramme A C is the parallelogramme M K alſo equall, and to the parallelogramme N L is equall the ſquare B E: Wherefore as the parallelogramme G H is to the parallelogramme M K, ſo is the parallelogramme M K to the parallelogramme N L. Wherefore (by the firſt of the ſixt) as the line F H is to the line H K, ſo is the line H K to the line K L. Wherefore (by the 17. of the ſixt) the parallelogrãme contayned vnder the lines F H and K L, is equall to the ſquare of the line H K. But the parallelogrãme contayned vnder the lines F H and K L, is rationall, as hath before bene proued. Wherefore the ſquare of the line H K is alſo rationall. Wherefore alſo the line H K is rationall.



Note.

And now if the line H K be commenſurable in length vnto the line H M, that is, vnto the line F G, which is equall to the line H M, then (by the 19. of the tenth) the parallelogramme N H is rationall. But if it be incommenſurable in length vnto the line F G, then the lines H K and H M are rationall commenſurable in power onely. And ſo ſhall the parallelogrãme H N be mediall. Wherefore the parallelogramme H N is either rationall, or mediall. But the parallelogramme H N is equall to the parallelogramme A C. Wherefore the parallelogramme A C is either rationall, or mediall. A rectangle parallelogramme therefore comprehended vnder mediall right lines commenſurable in power onely, is either rationall, or mediall: which was required to be demonſtrated.

How to finde mediall lines commenſurable in power onely contayning a rationall parallelogramme, and alſo other mediall lines commenſurable in power contayning a mediall

mediall parallelogramme, shall afterward be taught in the 27. and 28. Proposítions of thys booke.

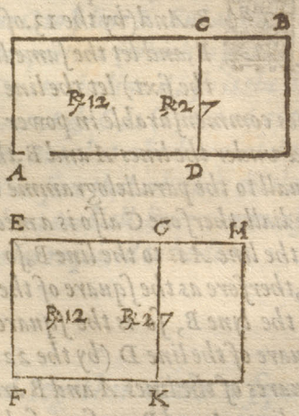
¶A Corollary.

Hereby it is manifest, that a rectangle parallelogramme contayned vnder two right lines, is the meane proportionall betwene the squares of the sayd lines. As it was manifest (by the first of the sixt) that that which is contayned vnder the lines A B and B C, is the meane proportionall betwene the squares A D and C X. This Corollary is put after the 33. Proposition of this booke as an Assumpt, and there demonstrated, which there in his place you shall finde. But becauſe it followeth of this Proposition so euidently and briefly without farther demonstration, I thought it not amisse here by the way to note it. *A Corollary.*

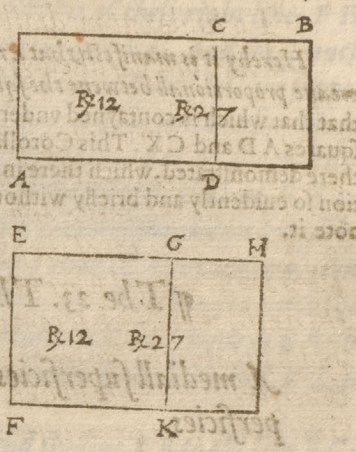
¶ The 23. Theoreme. The 26. Proposition.

A mediall superficies excedeth not a mediall superficies, by a rationall superficies.

FOr if it be possible, let A B being a mediall superficies, exceede A C being also a mediall superficies, by D B being a rationall superficies. And let there be put a rationall right line E F. And vpon the line E F apply a rectangle parallelogramme F H, equall vnto the mediall superficies A B, whose other side let be E H: and from the parallelogramme F H take away the parallelogramme F G, equall vnto the mediall superficies A C. Wherefore (by the third common sentence) the residue B D is equall to the residue K H. But (by suppoſition) the superficies D B is rationall. Wherfore the superficies K H is also rationall. And forasmuch as either of these superficieces A B and A C is mediall, and A B is equall vnto F H, & A C vnto F G: therefore either of these superficieces F H and F G is mediall: and they are applyed vpon the rationall line E F. Wherefore (by the 22. of the tenth) either of these lines H E and E G is rationall & incommensurable in length vnto the line E F. And forasmuch as the superficies D B is rationall, and the superficies K H is equall vnto it: therefore K H is also rationall: and it is applied vpō the rationall line E F (for it is applied vpon the line G K, which is equall to the line E F). Wherefore (by the 20. of the tenth) the line G H is rationall and commensurable in length vnto the line G K. But the line G K is equall to the line E F. Wherfore the line G H is rationall and commensurable in length vnto the line E F. But the line E G is rationall and incommensurable in length to the line E F. Wherefore (by the 13. of the tenth) the line E G is incommensurable in length vnto the line G H. And as the line E G is to the line G H, so is the square of the line E G to the parallelogramme contayned vnder the lines E G and G H (by the Assumpt put before the 21. of the tenth). Wherefore (by the 10. of the tenth) the square of the line E G is incommensurable vnto the parallelogramme contayned vnder the lines E G and G H. But vnto the square of the line E G are commensurable the squares of the lines E G and G H, for either of them is rationall, as hath before bene proued. Wherefore the squares of the lines E G and G H are incommensurable vnto the parallelogramme contayned vnder the lines E G and G H. But vnto the parallelogramme contayned vnder the lines E G and G H, is commensurable that which is contayned vnder the lines E G and G H twise (for they are in proportion the one to the other as number is to number, *Construction.* *Demonstration leading to an absurditie.*



namely, as vnitie is to the number 2, or as 2 is to 4: and therefore (by the 6. of this booke) they are commensurable). Wherefore (by the 13. of the tenth) the squares of the lines E G and G H are incommensurable vnto that which is contayned vnder the lines E G and G H twise. (This is more briefly concluded by the corollary of the 13. of the tenth). But the squares of the lines E G and G H together with that which is contayned vnder the lines E G and G H twise are equall to the square of the line E H (by the 4. of the second). Wherefore the square of the line E H is incōmensurable to the squares of the lines E G and G H (by the 16. of the tenth). But the squares of the lines E G & G H are rationall. Wherfore the square of the line E H is irrationall. Wherefore the line also E H is irrationall. But it hath before bene proued to be rationall: which is impossible. Wherefore a mediall superficies exceedeth not a mediall superficies by a rationall superficies: which was required to be proued.

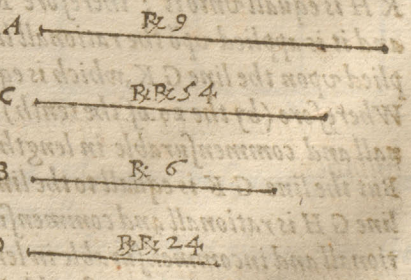


¶ The 4. Probleme. The 27. Proposition.

To finde out mediall lines commensurable in power onely, containing a rationall parallelogramme.

Constructiō. LEt there be put two rationall lines commensurable in power onely, namely, A and B. And (by the 13. of the six) take the meane proportionall betwene the lines A and B, and let the same line be C. And as the line A is to the line B, so (by the 12. of the sixt) let the line C be to the line D. And forasmuch as A and B are rationall lines commensurable in power onely, therfore (by the 21. of the tenth) that which is contayned vnder the lines A and B, that is, the square of the line C. For the square of the line C is equall to the parallelogramme contayned vnder the lines A anh B (by the 17. of the sixth) its mediall, therfore C also is a mediall line. And for that as the line A is to the line B, so is the line C to the line D, therfore as the square of the line A is to the square of the lyne B, so is the square of the line C to the square of the line D (by the 22. of the sixth). But the squares of the lines A and B are commensurable, for the lines A and B are supposed to be rationall commē-surable in power onely. Wherefore also the squares of the lines C and D are commensurable (by the 10. of the tenth) wherfore the lines C and D are commensu-rable in power onely. And C is a mediall line. Wherfore (by the 23. of the tenth) D also is a mediall line. Wherfore C and D are mediall lynes commensurable in power onely. Now also I say that they contayne a rationall parallelogramme. For for that as the line A is to the line B, so is the line C to the line D: therfore alternately also (by the 16. of the fift) as the line A is to the line C, so is the lyne B to the lyne D. But as the lyne A is to the lyne C, so is the line C to the lyne B: wherfore as the line C is to the line B, so is the line B to the lyne D. Wherfore the parallelogrāme cōtayned vnder the lines C and D is equal to the square of the line B. But the square of the lyne B is rationall. Wherfore the parallelograme which is contayned vnder the lynes C and D is also rationall. Wherfore there are found out mediall lines commensura-

Demonstra-tion.



ble

ble in power onely, contayning a rationall parallelogramme: which was required to be done.

The 5. Probleme. The 28. Proposition.

To finde out mediall right lynes commensurable in power onely, contayning a mediall parallelogramme.

LEt there be put three rationall right lines commensurable in power only, namely, A, B, and C, and (by the 13. of the sixt) take the meane proportional betwene the lines A and B, & let the same be D. And as the line B is to the line C, so (by the 12. of the sixt) let the line D be to the line E. And forasmuch as the lines A and B are rationall commensurable in power onely, therefore (by the 21. of the tenth) that which is contained vnder the lines A and B, that is the square of the line D, is mediall. Wherfore D is a mediall line. And forasmuch as the lines B and C are commensurable in power onely, and as the line B is to the line C, so is the line D to the line E: wherfore the lines D and E are commensurable in power onely (by the corollary of the tenth of this booke) but D is a mediall line. Wherefore E also is a mediall line (by the 23. of this booke.) Wherfore D & E are mediall lines commensurable in power onely. I say also that they containe a mediall parallelograme.

Construction.

Demonstration.

A Rt 16

D RtRt 128

B Rt 8

C Rt 2

E RtRt 72

For for that as the line B is to the line C, so is the line D to the line E: therfore alternately (by the 16 of the fift) as the line B is to the line D, so is the line C to ỹ line E. But as the lyne B is to the line D, so is the line D to the line A, by conuerse proportion (which is proued by the corollary of the fourth of the fifth) Wherfore as the line D is to the line A, so is the line C to the line E. Wherfore that which is contained vnder the lines A & C, is (by the 16. of the sixt) equall to that which is contayned vnder the lines D & E. But that which is contained vnder the lines A and C is medial (by the 21. of the tenth.) Wherfore that which is cõtained vnder the lines D and E is mediall. Wherfore there are found out mediall lines commensurable in power onely, containing a mediall superficies: which was required to be done.

An Assumpt.

To finde out two square numbers, which added together make a square number.

Let there be put two like superficiall numbers A B and B C (which how to finde out, hath bene taught after the 9. proposition of this booke) And let them both be either euen numbers or odde. And let the greater number be A B. And forasmuch as if from any euen number be taken away an euen number, or frõ an odde number be taken away an odde number, the residue shall be euen (by the 24. and 26 of the ninth). If therfore from A B being an euen number be taken away B C an euen number, or from A B being an odde number be taken away B C being also odde: the residue A C shall be euen. Deuide the number A C into two equall partes in D: wherefore the number which is produced of A B into B C together with the square number of C D, is (by the sixt of the second, as Barlaam demonstrateth it in numbers) equall to the square number of B D.

A ..... D ..... C ..... B

But that which is produced of A B into B C is a ſquare nũber. For it was proued (by the firſt of the ninth) that if two like plaine numbers multiplieng the one the other, produce any nũber, the number produced ſhalbe a ſquare number. Wherfore there are found out two ſquare numbers, the one being the ſquare number which is produced of A B into B C, and the other the ſquare number produced of C D, which added together make a ſquare number, namely, the ſquare number produced of B D multiplied into himſelfe, foraſmuch as they were demõſtrated equall to it.

A Corollary.

❊ A Corollary.

To finde out two ſquare numbers exceeding the one the other by a ſquare number.

And hereby it is manifeſt, that there are found out two ſquare numbers, namely, the one the ſquare number of B D, and the other the ſquare number of C D, ſo that that number wherin thone excedeth the other, the number (I ſay) which is produced of A B into B C, is alſo a ſquare number: namely, when A B & B C are like playne numbers. But when they are not like playne numbers, then are there found out two ſquare numbers, the ſquare number of B D, and the ſquare number of D C, whoſe exceſſe, that is, the number wherby the greater excedeth the leſſe, namely, that which is produced of A B into B C, is not a ſquare number.

A ..... D ..... C ..... B

¶ An Aſſumpt.

To finde out two ſquare numbers which added together make not a ſquare number.

An Aſſumpt.

Let A B and B C be like playne numbers, ſo that (by the firſt of the ninth) that which is produced of A B into B C is a ſquare number, and let A C be an euen number. And deuide C A into two equall partes in D. Now by that which hath before bene ſayd in the former aſſumpt, it is manifeſt that the ſquare number produced of A B into B C, together with the ſquare number of C D, is equall to the ſquare number of B D. Take away from C D vnitie D E. Wherfore that which is produced of A B into B C together with the ſquare of C E is leſſe then the ſquare number of B D. Now then I ſay that the ſquare number produced of A B into B C added to the ſquare number of C E, make not a ſquare number. For if they do make a ſquare number, then that ſquare number which they make, is either greater thẽ the ſquare number of B E, or equall vnto it, or leſſe then it. Firſt, greater it cannot be, for it is already proued that the ſquare number produced of A B into B C, together with the ſquare number of C E, is leſſe then the ſquare number of B D. But betwene the ſquare number of B D, and the ſquare number of B E, there is no meane ſquare number. For the number B D excedeth the number B E onely by vnitie: which vnitie can by no meanes be deuided into numbers. Or if the number produced of A B into B C together with the ſquare of the nũber C E, ſhould be greater then the ſquare of the number B E, then ſhould the ſelfe ſame number produced of A B into B C together with the ſquare of the number C E, be equall to the ſquare of the number B D, the contrary wherof is already proued. Wherfore if it be poſſible, let that which is produced of A B into B C together with the ſquare number of the number C E be equall to the ſquare number of B E. And let G A be double to vnitie D E, that is, let it be the number two. Now foraſmuch as the whole number A C is by ſuppoſition double to the whole number C D, of which the number A G is double to vnitie D E, therfore (by the 7. of the ſeventh

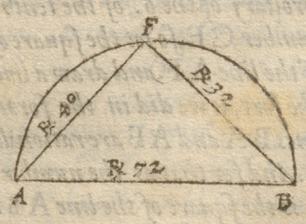
A .. G .. H . D . E . F ... C ..... B

uenth) the reſidue, namely, the number G C is double to the reſidue, namely, to the number B C. Wherfore the number G C is deuided into two equall partes in E. Wherefore that which is produced of G B into B C together with the ſquare number of C E is equall to the ſquare nũber of B E. But that which is produced of A B into B C, together with the ſquare number of C E, is ſuppoſed to be equal to the ſquare number of B E: wherfore that which is produced of G B into B C together with the ſquare number of C E is equall to that which is produced of A B into B C, together with the ſquare number of C E. Wherefore taking away the ſquare number of C E, which is common to them both, the number A B ſhall be equall to the number G B, namely, the greater to the leſſe, which is impoſſible. Wherfore that which is produced of A B into B C together with the ſquare number of C E, is not equall to the ſquare number of B E. I ſay alſo that that which is produced of A B into B C together with the ſquare number of C E is not leſſe then the ſquare number of B E. For if it be poſſible, thẽ ſhall it be equal to ſome ſquare number leſſe then the ſquare number of B E. Wherfore let the number produced of A B into B C together with the ſquare of the number C E be equal to the ſquare number of B F. And let the number H A be double to the number D F. Thẽ alſo it followeth that the number H C is double to the number C F, ſo that H C alſo is deuided into two equall partes in F, and therfore alſo the number which is produced of H B into B C, together with the ſquare number of F C, is equall to the ſquare number of the number B F. But by ſuppoſition, the number which is produced of A B into B C together with the ſquare number of C E is equall to the ſquare number of B F. Wherfore it followeth that the number produced of A B into B C together with the ſquare number of C E, is equall to that which is produced of H B into B C together with the ſquare number C F, which is impoſſible. For if it ſhould be equall, then foraſmuch as the ſquare of C F is leſſe then the ſquare of C E, the number produced of H B into B C ſhould be greater then the number produced of A B into B C. And ſo alſo ſhould the number H B be greater then the number A B, when yet it is leſſe then it. Wherfore the number produced of A B into B C together with the ſquare number of C E, is not leſſe then the ſquare nũber of B E. And it is alſo proued that it cannot be equall to the ſquare number of B E, neither greater then it. Wherfore that which is produced of A B into B C added to the ſquare number of C E, maketh not a ſquare number. And although it be poſſible to demonſtrate this many other wayes, yet this ſemeth to vs ſufficient, leaſt the matter beyng ouer long, ſhould ſeeme to much tedious.

¶ The 6. Probleme. The 29. Propoſition.

To finde out two ſuch rationall right lynes commenſurable in power only, that the greater ſhall be in power more then the leſſe, by the ſquare of a right line commenſurable in length vnto the greater.

LEt there be put a rational line A B, and take alſo two ſuch ſquare numbers C D and D E, that their exceſſe C E be not a ſquare number (by the corolary of the firſt aſſumpt of the 28. of the tenth) And vpon the line A B deſcribe a ſemicircle A F B. And by the corollary of the 6. of the tenth, as the number D C is to the number C E, ſo let the ſquare of the lyne B A be to the ſquare of the line A F. And draw a line from F to B. Now for that as the ſquare of the line B A is to the ſquare of the line A F, ſo is the number C D to the num-

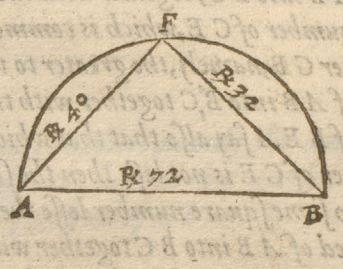


C ..... E .... D

Conſtruction.

Demonſtration.

ber C E, therfore the ſquare of the line B A hath to the ſquare of the line A F, that proportion that the nũber C D hath to the number C E. Wherfore the ſquare of the line B A is cõmẽſurable to the ſquare of the line A F (by the 6. of the tẽth). But the ſquare of the line A B is rational. Wherfore alſo the ſquare of the line A F is rational. Wherfore alſo the line AF is rationall. And foraſmuch as the number C D hath not vnto the number C E that proportion that a ſquare number hath to a ſquare number, therfore neither alſo hath the ſquare of the line A B to the ſquare of the line A F that proportion that a ſquare number hath to a ſquare number. Wherfore (by the 9. of the tẽth) the line A B is vnto the line A F incommenſurable in length. Wherfore the lines A F and A B are rationall commenſurable in power onely. And for that as the number D C is to the number C E, ſo is the ſquare of the line A B to the ſquare of the line A F: therfore by conuerſion or euerſe proportiõ which is demonſtrated (by the corollary of the 19. of the fifth) as the number C D is to the number D E, ſo is the ſquare of the line A B to the ſquare of the line B F, which is the exceſſe of the ſquare of the line A B aboue the ſquare of the line A F (by the aſsumpt put before the 14. of this booke). But the number C D hath to the number D E that proportion that a ſquare number hath to a ſquare number: wherfore the ſquare of the line A B hath to the ſquare of the line B F, that proportion that a ſquare number hath to a ſquare number. Wherefore (by the 9. of the tenth) the line A B is commenſurable in length vnto the line B F. And (by the 47. of the firſt) the ſquare of the line A B is equall to the ſquares of the lines A F and F B. Wherfore the line A B is in power more then the line A F by the ſquare of the line B F which is commenſurable in length vnto the line A B. Wherefore there are found out two ſuch rationall lines commenſurable in power onely, namely, A B and A F, ſo that the greater line A B is in power more then the leſſe line A F, by the ſquare of the line F B, which is commenſurable in length vnto the line A B: which was required to be done.



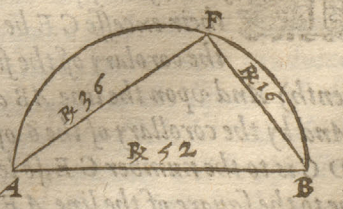
¶ The 1. Theoreme. The 30. Propoſition.

Montaureus maketh this an Aſsumpt: as the Greeke text ſeemeth to do likewiſe but without a cauſe.

To finde out two ſuch rationall lines commenſurable in power onely, that the greater ſhalbe in power more then the leſſe by the ſquare of a right line incommenſurable in length to the greater.

Conſtruction. LEt there be put a rationall line A B, and take alſo (by the 2. aſſumpt of the 28. of the tenth) two ſquare numbers C E and E D, which being added together make not a ſquare number, and let the numbers C E and E D added together make the number C D. And vpon the line A B deſcribe a ſencircle A F B. And (by the corollary of the 6. of the tenth) as the number D C is to the number C E, ſo let the ſquare of the line A B be to the ſquare of the line A F, and draw a line from F to B. And we may in like ſort, as we did in the former propoſition, proue that the lines B A and A F are rationall commenſurable in power onely. And for that as the number D C is to the number C E, ſo is the ſquare of the line A B to the ſquare of the line A F: therefore by conuerſion (by the corollary of the 19. of the fifte) as the number C D is to the number D E, ſo is the ſquare of the line A B to the ſquare to the line F B. But the number C D hath

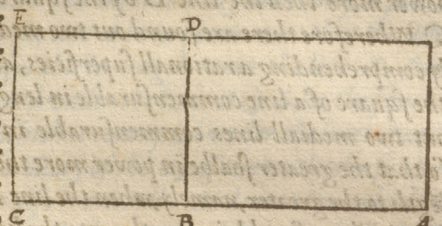
Demonſtration.



D hath not to the number D E that proportion that a ſquare number hath to a ſquare number. Wherefore neither alſo the ſquare of the line A B hath to the ſquare of the line B F that proportion that a ſquare number hath to a ſquare number. Wherefore the line A B is (by the 9 of the tenth) incommenſurable in length to the line B F. And the line A B is in power more then the line A F by the ſquare of the right line B F, which is incommenſurable in length vnto the line A B. Wherfore the lines A B and A F are rationall commenſurable in power onely. And the line A B is in power more then the line A F by the ſquare of the line F B which is commenſurable in length vnto the line A B. which was required to be done.

¶ An Aſſumpt.

If there be two right lines hauing betwene them ſelues any proportion: as the one right line is to the other, ſo is the parallelograme contained vnder both the right lines to the ſquare of the leſſe of thoſe two lines.



Suppoſe that theſe two right A B and B C be in ſome certaine proportion. Then I ſay that as the line A B is to the line B C, ſo is the parallelograme contained vnder A B and B C to the ſquare of B C. Deſcribe the ſquare of the line B C and let the ſame be C D, and make perfect the parallelograme A D now it is manifeſt that as the line A B is to the line B C, ſo is the parallelograme A D to the parallelograme or ſquare B E (by the firſt of the ſixt). But the parallelograme A D is that which is contained vnder the lines A B and B C, for the line B C is equall to the line B D and the parallelograme B E is the ſquare of the line B C. Wherefore as the line A B is to the line B C ſo is the parallelograme contained vnder the lines A B and B C to the ſquare of the line B C, which was required to be proued.

This Aſſumpe ſetteth forth nothing els but that which the firſt of the ſixt ſetteth forth, and therefore in ſome examplars it is not founde.

¶ The 8. Probleme. The 31. Propoſition.

To finde out two mediall lines commenſurable in power onely, comprehending a rationall ſuperficies, ſo that the greater ſhall be in power more then the leſſe by the ſquare of a line commenſurable in length vnto the greater.

LEt there be taken (by the 29. of the tenth) two rationall lines commenſurable in power onely A and B, ſo that let the line A being the greater be in power more then the line B, being the leſſe by the ſquare of a line commenſurable in length vnto the line A. And let the ſquare of the line C be equall to the parallelograme contained vnder the lines A and B which is done by finding out the meane proportionall line, namely the line C betwene the lines A and B (by the 13. of the ſixt). Now the parallelograme contained vnder the lines A and B is mediall (by the 21. of this booke). Wherefore (by the corollary of the 23. of the tenth) the ſquare alſo of the line C is mediall. Wherfore the line C alſo is mediall: Vnto the ſquare of the line B let the parallelograme contained vnder the lines C and D be equall (by finding out a third line proportionall) namely the line D to the two lines C and B (by the 11. of the ſixt). But the ſquare of the line B is rationall. Wherfore the parallelograme contained vnder the line C and D is rationall. And for that as the line A is to the line B,

Conſtruction.

Demonſtration.

ſo

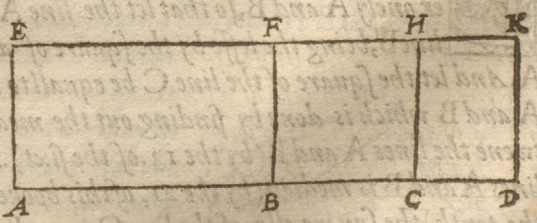
ſo is the parallelograme contained vnder the lines A *and* B *to the ſquare of the line* B *(by the aßumpt going before). But vnto the parallelograme contained vnder the lines* A *and* B *is equall the ſquare of the line* C, *and vnto the ſquare of the line* B *is equal the parallelograme contained vnder the lines* C *and* D, *as it hath now bene proued: therefore as the line* A *is to the line* B, *ſo is the ſquare of the line* C *to the parallelograme contained vnder the lines* C, D. *But as the ſquare of the line* C *is to that which is contayned vnder the lines* C *and* D, *ſo is the line* C *to the line* D. *Wherefore as the line* A *is to the line* B, *ſo is the line* C *to the line* D. *But (by ſuppoſition) the line* A *is commenſurable vnto the line* B *in power onely. Wherefore (by the* 11. *of the tenth) the line* C *alſo is vnto the line* D *commenſurable in power onely. But the line* C *is mediall. Wherefore by the* 23. *of the tenth) the line* D *alſo is mediall. And for that as the line* A *is to the line* B, *ſo is the line* C *to the line* D: *but the line* A *is in power more then the line* B, *by the ſquare of a line commenſurable in length vnto the line* A *(by ſuppoſition). Wherefore the line* C *alſo is in power more then the line* D *by the ſquare of a line commenſurable in length vnto the line* C. *Wherefore there are found out two mediall lines* C *and* D *commenſurable in power onely comprehending a rationall ſuperficies, and the line* C *is in power more then the line* D, *by the ſquare of a line commenſurable in length vnto the line* C. *And in like ſort may be found out two mediall lines commenſurable in power onely containing a rationall ſuperficies, ſo that the greater ſhalbe in power more thē the leſſe by the ſquare of a line incōmenſurable in length to the greater, namely, when the line* A *is in power more thē the line* B *by the ſquare of a line incōmenſurable in length vnto the line* A, *which to doo is taught by the* 30. *of this booke.*

A 64
B 1792
C 128
D 1748

The ſelfe ſame conſtruction remaining, that part of this propoſition frō theſe wordes. *And for that as the line A is to the line B*, to theſe wordes. *But (by ſuppoſition) the line A is commenſurable vnto the line B*, may more eaſely be demonſtrated after this maner. The lines C, B, D, are in continuall proportion by the ſecond part of the 17. of the ſixt. But the lines A, C, D are alſo in continuall proportion by the ſame. Wherefore by the 11. of the fifth, as the line A is to the line C, ſo is the line B to the line D. Wherfore alternately as the line A is to the line B ſo is the line C to the line D. &c. which was required to be doone.

¶ An aſſumpt.

If there be three right lines hauing betwene them ſelues any proportion: as the firſt is to the third, ſo is the parallelograme contained vnder the firſt and the ſecond, to the parallelograme contained vnder the ſecond and the third.



Conſtruction. *Suppoſe that theſe three lines* A B, B C, *and* C D *be in ſome certayne proportion. Then I ſay that as the line* A B *is to the line* C D, *ſo is the parallelograme contayned vnder the lines* A B *and* B C *to the parallelograme contayned vnder the lines* B C *and* C D. *From the point* A *raiſe vp vnto the line* A B *a perpendicular line* A E, *and let* A E *be equall to the line* B C: *and by the poynt* E *draw vnto the line* A D *a parallel line* E K: *and by euery one of the poyntes* B, C, *and* D *draw vnto the line* A E *parallel lines* B F, C H, *and* D K. *And for that as the line* A B *is to the line* B C *ſo is the parallelograme* A F *to the parallelograme* B H *(by the firſt of the ſixt): and as the line* B C *is to the line*

Demonſtration.

line C D, *so is the parallelograme* B H *to the parallelograme* C K. *Wherefore of equalitie as the line* A B *is to the line* C D, *so is the parallelograme* A F *to the parallelograme* C K. *But the parallelograme* A F *is that which is contayned vnder the lines* A B *and* B C, *for the line* A E *is put equall to the line* B C. *And the parallelograme* C K *is that which is contained vnder the lines* B C *and* C D, *for the line* B C *is equal to the line* C H, *for that the line* C H *is equall to the line* A E *(by the 34. of the first)*. *If therefore there be three right lines hauing betwene them selues any proportion: as the first is to the third, so is the parallelograme contained vnder the first and the second, to the parallelogramme cõtained vnder the second and the third: which was required to be demonstrated.*

¶ The 9. Probleme. The 32. Proposition.

To finde out two mediall lines commensurable in power onely, comprehending a mediall superficies, so that the greater shall be in power more then the lesse, by the square of a line commensurable in length vnto the greater.

Et there be taken three rationall lines cõmensurable in power onely, A, B, C, so, that (by the 29. of the tenth) let the line A be in power more then the line C, by the square of a line commensurable in length vnto the line A. And vnto the parallelogramme contayned vnder the lines A & B, let the square of the line D be equall. But that which is contayned vnder the lines A and B is mediall. Wherefore (by the Corollary of the 23. of the tenth) the square of the line D also is mediall. Wherefore the line D also is mediall. And vnto that which is contayned vnder the lines B and C, let be equall that which is contayned vnder the lines D and E (which is done by finding out a fourth line proportionall vnto the lines D, B, C, which let be the line E). And for that (by the Assumpt going before) as that which is contayned vnder the lines A and B is to that which is contayned vnder the lines B and C, so is the line A to the line C. But vnto that which is contayned vnder the lines A & B, is equall the square of the line D, and vnto that which is contayned vnder the lines B & C, is equall that which is contayned vnder the lines D and E. Wherefore as the line A is to the line C, so is the square of the line D, to that which is contayned vnder the lines D and E. But as the square of the line D is to that which is contayned vnder the lines D and E, so is the line D to the line E (by the Assumpt put before the 22. of the tenth). Wherefore as the line A is to the line C, so is the line D to the line E. But the line A is vnto the line C commensurable in power onely. Wherefore the line D is vnto the line E commensurable in power onely. But D is a mediall line. Wherefore (by the 23. of the tenth) E also is a mediall line. And for that as the line A is to the line C, so is the line D to the line E, and the line A is in power more then the line C, by the square of a line commensurable in length vnto the line A. Wherefore (by the 14. of the tenth) D is in power more then E, by the square of a line commensurable in length vnto the line D. I say moreouer that that which is contayned vnder the lines D and E is mediall. For forasmuch as that which is contayned vnder the lines B & C, is equall to that which is contayned vnder the lines D and E: but that which is contayned vnder the lines B and C is mediall. Wherefore that which is contayned vnder the lines D and E is also mediall. Wherefore there are found out two mediall lines D and E, cõmensurable in power onely, comprehending a mediall superficies, so that the greater is in power more then the lesse, by the square of a line commensurable in length to the greater: which was required to be

Construction.

Demonstration.

A — R 64

D — RR 3072

B — R 48

E — RR 2268.3/4

C — R 55

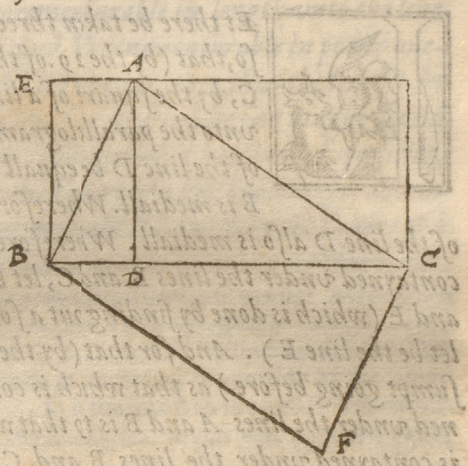
done

A Corollary. done. And thus is it euident, how in like sorte may be founde out two mediall lines commensurable in power onely, containing a mediall superficies, so that the greater shall be in power more then the lesse, by the square of a line incommensurable in length vnto the greater. When the line A is in power more then the line C, by the square of a line incomensurable in length vnto the line A: as the thirteth teacheth vs.

1. ¶ An Assumpt.

Suppose that there be a rectangle triangle A B C, hauing the angle B A C a right angle. And (by the 12. of the first) from the poynt A to the right line B C, a perpendicular line being drawen A D: then I say first, that the parallelogramme contayned vnder the lines
1. C B and B D, is equall to the square of the line B A. Secondly I say, that the parallelo-
2. gramme contayned vnder the lines B C and C D, is equall to the square of the line C A. Thirdly I say, that the parallelogramme contayned vnder the lines B D and D C, is equall to
3. the square of the line A D. And fourthly I say, that the parallelogramme contayned vnder
4. the lines B C & A D, is equall to the parallelogramme cõtayned vnder the lines B A & A C.

As touching the first, that the parallelogramme contayned vnder the lines C B and B D, is equall to the square of the line A B, is thus proued.



For forasmuch as in the rectangle triangle B A C, from the right angle vnto the base is drawen a perpendicular line A D, therfore (by the 8. of the sixt) the triangles A B D and A D C, are like to the whole triangle A B C, and are also like the one to the other. And for that the triangle A B C is like to the triangle A D B, therefore both the triangles are equiangle by the definitiõ of like figures. Wherfore (by the 4. of the sixt) as the line C B is to the line B A, so is the line A B to the line B D.

*A Corollary added by I. Dee.

* Therefore if you deuide the square of the side A B by the side B C: the quotient will be B D. Which maketh D C also knowen: by either of which (by the 47. of the first) the perpendicular A D is knowen: which consideration in practise, is very profitable, where the three sides A B, A C, and B C, are knowen or geuen.

I. Dee *The second Corollary.

* Therefore if you deuide the square of the side A C, by the side B C, the portion D C, will be the product. &c. as in the former Corollary.

* Wherefore (by the 17. of the sixt) the parallelogramme contained vnder the lines B C & B D, is equall to the, square of the line A B.

As touching the second, that the parellelogramme contained vnder the lines B C and C D, is equall to the square of the line A C is by the selfe same reason proued. For the triangle A B C is like to the triangle A D C. Wherefore as the line B C is to the line A C, so is the line, A C to the line D C. * Wherefore the parallelogramme contained vnder the lines B C and C D, is equall to the square of the line A C. As touching the third, that the parallelogramme contained vnder the lines B D and D C, is equall to the square of the line D A, is thus proued. For, forasmuch as if in a rectangle triangle be drawn frõ the right angle to the base a perpendicular line, the perpendicular so drawen is the meane proportional betwene the segmẽts of the base (by the corollary of the 8. of the sixt): therfore as the line B D is to the line D A, so is the line A D to the line D C. Wherefore (by the 17. of the sixt) the parallelogramme contayned vnder the lines B D and D C, is equall to the square of the line D A. As touching the fourth, that the parallelogramme contained vnder the lines B C and A D, is equall to the parallelogramme contained vnder the lines B A and A C, is thus proued. For forasmuch as (as we haue already declared) the triangle A B C is like, and therefore equiangle, to the triangle A B D, therefore as the line B C is to

the

the line A C, so is the line B A to the line A D (by the 4. of the sixt). But if there be foure right lines proportionall, that which is contained vnder the first and the last, is equall to that which is contained vnder the two meanes (by the 16. of the sixt). Wherefore that which is contained vnder the lines B C and A D, is equall to that which is contayned vnder the lines B A and A C.*

I. Dee **The third Corollary.*

* Therfore if the parallelogramme of B A, and A C, be deuided by B C, the product will geue the perpendicular D A. These three Corollaryes in practise Logisticall and Geometricall are profitable.

I say moreouer, that if there be made a parallelogramme complete, contained vnder the lines B C and A D, which let be E C: and if likewise be made complete the parallelogramme contained vnder the lines B A and A C, which let be A F, it may by an other way be proued that the parallelogramme E C is equall to the parallelogramme A F. For, forasmuch as either of them is double to the triangle A C B (by the 41. of the first): and thinges which are double to one and the selfe same thing, are equall the one to the other. Wherefore that which is contained vnder the lines B C and A D, is equall to that which is contained vnder the lines B A and A C.

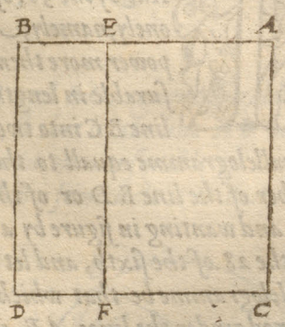
An other demonstration of this fourth part of the determinatiõ.

2. ¶ An Assumpt.

If a right line be deuided into two vnequall partes: as the greater part is to the lesse, so is the parallelogramme contayned vnder the whole line and the greater part, to the parallelogramme contayned vnder the whole line and the lesse part.

An Assumpt.

Deuide the right line A B into two vnequall partes in the point E: And let A E be the greater part. Then I say, that as the line A E is to the line E B, so is the parallelogramme contained vnder the lines B A and A E to the parallelogramme contained vnder the lines B A & B E. Describe the squari of the line A B, and let the same be A C D B. And from the point E draw vnto either of these lines A C and D B a pirallell line E F. Now it is manifest, that as the line A E is to the line E B, so is the parallelogramme A F to the parallelogramme B F (by the first of the sixt). But the parallelogramme A F is contayned vnder the lines B A and A E (for the line A C is equall to the line A B) and the parallelogramme B F is contained vnder the lines A B and B E (for the line D B is equall to the line A B). Wherefore as the line A E is to the line E B, so is the parallelogramme contained vnder the lines B A and A E, to the parallelogramme contained vnder the lines A B and B E: which was required to be demonstrated.

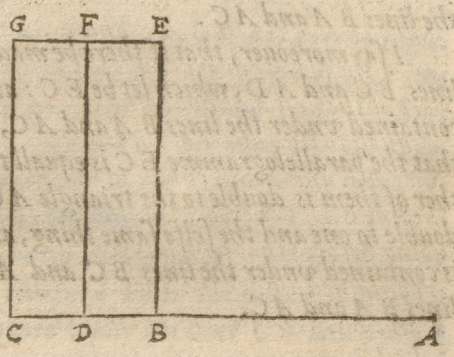


This Assumpt differeth litle from the first Proposition of the sixt booke.

3. ¶ An Assumpt.

If there be two vnequall right lines, and if the lesse be deuided into two equall partes: the parallelogramme contained vnder the two vnequall lines, is double to the parallelogramme contained vnder the greater line & halfe of the lesse line.

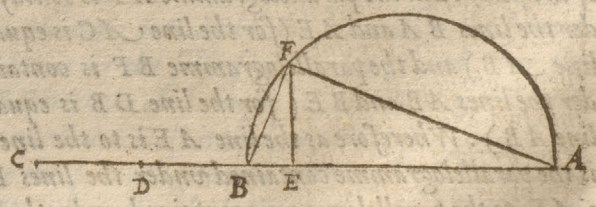
Suppose that there be two vnequall right lines A B and B C, of which let ⌒A B be the greater, and deuide the line B C into two equall partes in the point D. Then I say, that the parallelogramme contained vnder the lines A B & B C, is double to the parallelogramme contained vnder the lines A B and B D. From the point B raise vp vpon the right line B C, a perpendicular line B E, and let B E be equall to the line B A. And drawing from the point C and D, the lines C G and D F parallels and equall to B E: and then drawing the right line G F E, the figure is complete. Now for that as the line D B is to the line D C, so is the parallelogramme B F to the parallelogramme D G (by the 1. of the sixt): therfore by composition of proportion, as the whole line B C is to the line D C, so is the parallelogramme B G to the parallelogramme D G (by the 18. of the fift). But the line B C is double to the line D C. Wherefore the parallelogramme B G is double to the parallelogramme D G. But the parallelogramme B G is contained vnder the lines A B and B C) for the line A B is equall to the line B E (and the parallelogramme D G is contayned vnder the lines A B and B D) for the line B D is equall to the line D C, and the line ⌒A B to the line D F: which was required to be demonstrated.



¶ The 10. Probleme. The 33. Proposition.

To finde out two right lines incommensurable in power, whose squares added together make a rationall superficies, and the parallelogramme contained vnder them make a mediall superficies.

Construction. *TAke by the 30. of the tenth, two rationall right lines commensurable in power onely, namely, ⌒A B and B C, so that let the line A B, being the greater, be in power more then the line B C being the lesse, by the square of a line incommensurable in length vnto the line A B. And by the 10. of the first, deuide the line B C into two equall partes in the point D. And vpon the line A B apply a parallelogramme equall to the square either of the line B D or of the line D C, and wanting in figure by a square, by the 28. of the sixth, and let that parallelogramme be that which is contained vnder the lines A E and E B. And vpon the line A B describe a semicircle A F B. And by the 11. of the first, from the point E, raise vp vnto the line ⌒A B, a perpendiculer line E F, cutting the circumference in the point F. And draw lines from A to F, and from F to B. And forasmuch as there are two vnequall right lines A B and B C, and the line A B is in power more then the line B C, by the square of a line incommensurable in length vnto A B, and vpon the line A B is applied a parallelograme equall to the fourth part of the square of the line B C, that is, to the square of the halfe of the line B C, and wanting in figure by a square, and the said parallelogramme is that which is contained vnder the lines A E and E B. wherfore by the 2. part of the 18. of the tenth, the line A E is incommensurable in length vnto the line E B. But as the line A E is to the line E B, so is the parallelogramme contained vnder the lines B A and A E, to the parallelogramme contayned vnder the lines A B and B E by the second assumpt before put. And that which is contained vnder*



Demonstration.

the

the line B A and A E is equall to the square of the line A F, by the second part of the first assumpt before put. And that which is contained vnder the lines A B and B E is by the first part of the same assumpt, equall to the square of the line B F. Wherfore the square of the line A F is incommensurable to the square of the line B F. Wherfore the lines A F and B F are incommensurable in power. And forasmuch as A B is a rationall line (by supposition) therfore (by the 7. definition of the tenth) the square of the line A B is rationall. Wherefore also the squares of the lines A F and F B added together make a rationall superficies. For (by the 47. of the first) they are equal to the square of the line A B. Again forasmuch as (by the third part of the first assumpt going before) that which is contained vnder the lines A E and E B, is equall to the square of the line E F. But by supposition that which is contained vnder the lines A E and E B is equall to the square of the line B D. Wherfore the line F E is equall to the line B D. Wherfore the line B C is double to the line F E. Wherfore (by the third assumpt going before) that which is contained vnder the lines A B and B C, is double to that which is contained vnder the lines A B and E F. But that which is contained vnder the lines A B and B C, is by supposition mediall. Wherfore (by the corollary of the 23. of the tenth) that which is contained vnder the lines A B and E F is also mediall, but that whiche is contayned vnder the lines A B and E F, is (by the last parte of the first assumpt goyng before) equall to that which is contained vnder the lines A F and F B. Wherefore that which is contained vnder the lines A F & F B is a mediall superficies. And it is proued, that that which is composed of the squares of the lines A F and F B added together is rationall. Wherfore there are found out two right lines A F and F B incommensurable in power, whose squares added together, make a rationall superficies, and the parallelogramme contained vnder them, is a mediall superficies: which was required to be done.

The first part of the determination concluded.

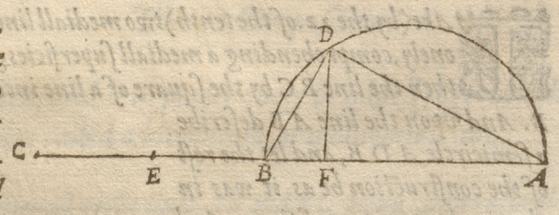
The second part concluded.

The totall conclusion.

¶ The 11. Probleme. The 34. Proposition.

To finde out two right lines incommensurable in power, whose squares added together make a mediall superficies, and the parallelogramme contayned vnder them, make a rationall superficies.

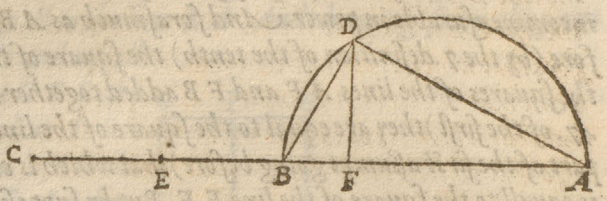
TAke (by the 31. of the tenth) two mediall lines A B and B C, commensurable in power onely, comprehending a rationall superficies, so that let the line A B be in power more then the line B C, by the square of a line incommensurable in length vnto the line A B. And describe vponthe line A B a semicircle A D B. And by the 10. of the first, deuide the line B C into two equall partes in the point E. And (by the 28. of the sixt) vpon the line A B apply a parallelogramme equall to the square of the line B E, and wantyng in figure by a square, and let that parallelogramme be that which is contayned vnder the lines A F and F B. Wherfore the line A F is incommensurable in length vnto the line F B (by the 2. part of the 18. of the tenth). And from the point F vnto the right line A B, raise vp (by the 11. of the first) a perpendiculer line F D, and draw lines from A to D, and from D to B. And forasmuch as the line A F is incommensurable vnto the line F B: but (by the second assumpt going before the 33. of the tenth) as the line A F is to the line F B, so is the parallelogramme contayned vnder the lines B A and A F, to the parallelogramme contained vnder the lines B A and B F, wherfore (by the tenth of the tenth) that which is contained vnder the lines B A and A F is incommensurable to that which is contayned vnder the lines A B and B F: but that which is



Construction.

Demonstration.

contained vnder the lines B A and A F is equall to the ſquare of the line A D, and that which is contained vnder the lines A B and B F is alſo equall to the ſquare of the line D B (by the ſecond part of the firſt aſsumpt going before the 33. of the tēth) wherfore the ſquare of the line A D is incōmenſurable to the ſquare of the line D B. Wherefore the lines A D and D B are incommenſurable in power. And foraſmuch as the ſquare of the line A B is mediall, therefore alſo the ſuperficies made of the ſquares of the lines A D and D B added together is mediall. For the ſquares of the lines A D and D B are (by the 47. of the firſt) equall to the ſquare of the line A B. And foraſmuch as the line B C is double to the line F D (as it was proued in the propoſition going before) therefore the parallelogramme contained vnder the lines A B and and B C is double to the parallelogramme contained vnder the lines A B and F D (by the third aſſumpt going before the 33. propoſition) wherefore it is alſo commenſurable vnto it (by the ſixt of the tenth) But that which is contained vnder the lines A B and B C is ſuppoſed to be rationall. Wherfore that which is contained vnder the lines A B and F D is alſo rationall. But that which is contained vnder the lines A B and F D, is equall to that which is contained vnder the lines A D and D B (by the laſt part of the firſt aſsumpt going before the 33. of the tenth) Wherfore that which is contayned vnder the lines A D and D B is alſo rationall. Wherefore there are found out two right lines A D and D B incommenſurable in power, whoſe ſquares added together, make a mediall ſuperficies, and the parallelogramme cōtayned vnder them, make a rationall ſuperficies: which was required to be done.



The firſt part of the determination concluded.

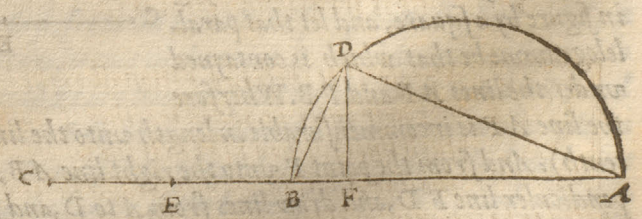
The ſecond part cōcluded.

The totall concluſion.

¶ The 12. Probleme. The 35. Propoſition.

To finde out two right lines incommenſurable in power, whoſe ſquares added together, make a mediall ſuperficies, and the parallelogramme contained vnder them, make alſo a mediall ſuperficies, which parallelogramme moreouer, ſhall be incommenſurable to the ſuperficies made of the ſquares of thoſe lines added together.

Conſtruction. *TAke (by the 32. of the tenth) two mediall lines A B and B C commenſurable in power onely, comprehending a mediall ſuperficies, ſo that let the line A B be in power more then the line B C by the ſquare of a line incommenſurable in length vnto the line A B. And vpon the line A B deſcribe a ſemicircle A D B, and let the reſt of the conſtruction be as it was in the two former propoſitions. And foraſmuch as (by the 2 part of the 18. of the tenth) the line A F is incommenſurable in length vnto the line F B, therfore the line A D is incommenſurable in power vnto the line D B (by that which was demonſtrated in the propoſitiō going before). And foraſmuch as the ſquare of the line A B is mediall, therefore that alſo which is compoſed of the ſquares of the lines A D and D B (which ſquares are equall to the ſquare of the line A B by the 47. of the firſt) is mediall. And foraſmuch as that which is con-*



Demonſtration.

The firſt part concluded.

contained vnder the lines A F and F B, is equall to either of the squares of the lines E B and F D, for by supposition the parallelogrammie contained vnder the lines A F and F B is equall to the square of the line E B, and the same parallelogramme is equall to the square of the line D F (by the third part of the first assumpt going before the 33. of the teth). Wherfore the line B E is equall to the line D F. Wherfore the line B C is double to the line F D. Wherefore that which is contained vnder the lines A B and B C is double to that which is contained vnder the lines A B and F D. Wherfore they are commensurable by the sixt of this boke: but that which is contained vnder the lines A B and B C is mediall by supposition. Wherfore also that which is contained vnder the lines A B and F D is mediall (by the corollary of the 23, of the tenth) but that which is contained vnder the lines A B and F D, is (by the fourth part of the first assumpt going before the 33. of the tenth) equall to that which is contained vnder the lines A D and D B: wherfore that which is contained vnder the lines A D and D B is also mediall. And forasmuch as the line A B is incommensurable in length vnto the line B C. But the line B C is commensurable in length vnto the line B E. Wherfore (by the 13 of the tenth) the line A B is incommensurable in length vnto the line B E. Wherefore the square of the line A B is incommensurable to that which is contained vnder the lines A B and B E (by the first of the sixt and 10. of this booke) But vnto the square of the line A B are equall the squares of the lines A D and D B added together (by the 47. of the first): and vnto that which is contayned vnder the lines A B and B E, is equall that which is contained vnder the lines A B and F D, that is, which is contained vnder A D and D B. For the parallelogramme contained vnder the lines A B and F D is equall to the parallelogramme contained vnder the lines A D and D B (by the last part of the first assumpt going before the 33. of this tenth booke). Wherfore that which is composed of the squares of the lines A D and D B is incommensurable to that which is contained vnder the lines A D and D B. Wherefore there are found out two right lines A B and D B incommensurable in power, whose squares added together, make a mediall superficies, and the parallelogramme contayned vnder them, make also a mediall superficies, which parallelogramme moreouer is incommensurable to the superficies composed of the squares of those lines added together, which was required to be done.

The second part cõcluded.

The third part cõcluded.

The totall conclusion.

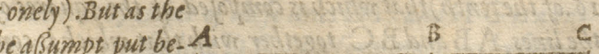
The beginning of the Senaries by Composition.

¶ The 25. Theoreme. The 36. Proposition.

If two rationall lines commensurable in power onely be added together: the whole line is irrationall, and is called a binomium, or a binomiall line.

The first Senary by composition.

SVppose that these two rationall right lines A B and B C being commensurable in power onely be added together (the 11. of the tenth teacheth to finde out two such lines). Then I say that the whole line A C is irrationall. For forasmuch as the line A B is incommensurable in length vnto the line B C, (for they are supposed to be commensurable in power onely). But as the line A B is to the line B C, so (by the assumpt put before the 22. of the tenth) is the parallelograme contained vnder the lines A B and B C to the square of the line B C, wherefore (by the 10. of this booke) the parallelograme contayned vnder the lines A



Demonstration.

B and B C is incommensurable to the square of the line B C. But vnto the parallelograme contained vnder the lines A B and B C is commensurable the parallelograme contained vnder A B and B C twise (by the 6. of the tenth): wherefore that which is contained vnder A B and B C twise is incommensurable to the square of the line B C (by the 13 of the tenth). But vnto the square of the line B C is commensurable that which is composed of the squares of the lines A B and B C (by the 15. of the tenth), for by supposition the lines A B and B C are commensurable in power onely. Wherefore (by the 13. of the tenth) that which is composed of the squares of the lines A B and B C added together is incommensurable to that which is contained vnder the lines A B and B C twise. Wherefore (by the 16. of the tenth) that which is contained vnder A B and B C twise together with the squares of the lines A B and B C, which (by the 4. of the second) is equall to the square of the whole line A C, is incommensurable to that which is composed of the squares of A B and B C added together. But that which is composed of the squares of A B and B C added together is rationall, for it is commensurable to either of the squares of the lines A B and B G of which either of them is rationall by supposition: wherfore the square of the line A C is (by the 10. definition of the tenth) irrationall. Wherefore the line A C also is irrationall, and is called a binomiall line.

Diffinition of a binomiall line.

This proposition sheweth the generation and production of the second kinde of irrational lines which is called a binomium, or a binomial line. The definition whereof is fully gathered out of this proposition, and that thus.

A binomium or a binomiall line, is an irrationall line composed of two rationall lines commensurable the one to the other in power onely. And it is called a binomium, that is, hauing two names, because it is made of two such lines as of his partes which are onely commensurable in power and not in length: and therefore ech part or line, or at the least the one of them, as touching length, is vncertaine and vnknowne. Wherefore being ioyned together their quantitie cannot be expressed by any one number or name, but ech part remayneth to be seuerally named in such sort as it may. And of these binomiall lines there are sixe seuerall kindes, the first binomiall, the second, the third, the fourth, the fifth, and the sixt, of what nature and condition ech of these is shalbe knowne by their definitious which are afterward set in their due place.

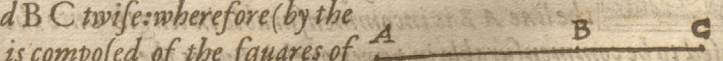
Sixe kindes of binomiall lines.

¶ The 25. Theoreme. The 37. Proposition.

If two mediall lines commensurable in power onely containing a rationall superficies, be added together: the whole line is irrationall, and is called a first bimediall line.

LEt these two mediall lines A B *and* B C *being commensurable in power onely, and containing a rationall superficies (the 27. of the tenth teacheth to finde out two such lines) be composed. Then I say that the whole line* A C *is irrationall. For as it was sayd in the proposition next going before that which is composed of the squares of the lines* A B *and* B C *is incommensurable to that which is contained vnder the lines* A B *and* B C *twise: wherefore (by the 16. of the tenth) that which is composed of the squares of the lines* A B *and* B C *together with that which is contained vnder the lines* A B *and* B C *twise, that is, the square of the line* A C *is incommensurable to that which is contayned vnder the lines* A B *and* B C *twise. But that which is contayned vnder the lines* A B *and* B C *twise is commensurable to that which is contayned vnder the lines* A B *and* B C *once (by the 6. of the tenth) wherfore*

Demonstration.



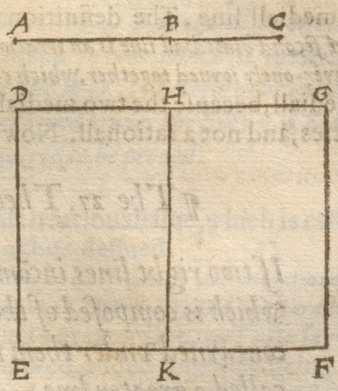
wherefore the ſquare of the whole line A C, *is (by the 13. of the tenth) incommenſurable to that which is contained vnder the lines* A B *and* B C *once. But by ſuppoſition the lines* A B *and* B C *comprehend a rationall ſuperficies. Wherefore the ſquare of the whole line* A C *is irrationall: wherefore alſo the line* A C *is irrationall. And it is called a firſt bimediall line.*

The third irrational line which is called a firſt bimediall line, is ſhewed by this propoſition, and the definition thereof is by it made manifeſt, which is this. *A firſt bimediall line, is an irrationall line, which is compoſed of two mediall lines commenſurable in power onely contayning a rationall parallelograme.* It is called a firſt bimediall line, bycauſe the two mediall lines or partes whereof it is compoſed contayne a rationall ſuperficies, which is preferred before an irrationall. *Diffinition of a firſt bimediall line.*

¶ The 26. Theoreme. The 38. Propoſition.

If two mediall lines commenſurable in power onely contayning a mediall ſuperficies, be added together: the whole line is irrationall, and is called a ſecond bimediall line.

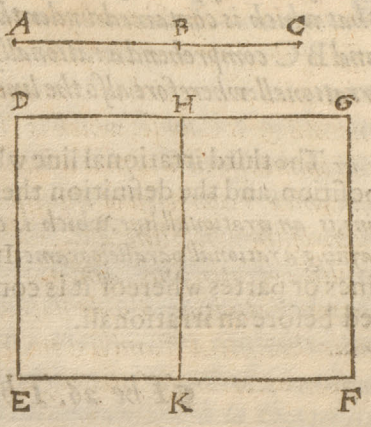
Et theſe two medial lines A B *and* B C *being commenſurable in power onely, and contayning a mediall ſuperficies (the 28. of the tenth teacheth to finde out two ſuch lines) be added together. Then I ſay that the whole line* A C *is irrationall. Take a rationall line* D E. *And (by the 44. of the firſt) vpon the line* D E *apply the parallelograme* D F, *equal to the ſquare of the line* A C, *whoſe other ſide let be the line* D G. *And foraſmuch as the ſquare of the line* A C *is (by the 4. of the ſecond) equall to that which is compoſed of the ſquares of the lines* A B *and* B C, *together with that which is contained vnder the lines* A B *and* B C *twiſe: but the ſquare of the line* A C *is equall to the parallelograme* D F. *Wherefore the parallelograme* D F *is equall to that which is compoſed of the ſquares of the lines* A B *and* B C *together with that which is contayned vnder the lines* A B *and* B C *twiſe. Now then agayne (by the 44. of the firſt) vpon the line* D E *apply the parallelograme* E H *equall to the ſquares of the lines* A B *and* B C. *Wherefore the parallelograme remayning, namely,* H F, *is equall to that which is contained vnder the lines* A B *and* B C *twiſe. And foraſmuch as either of theſe lines* A B *and* B C *is mediall, therefore the ſquares of the lines* A B *and* B C *are alſo mediall. And that which is contained vnder the lines* A B *and* B C *twiſe is (by the corollary of the 24. of the tenth) mediall. For by the 6. of this booke it is commẽſurable to that which is contained vnder the lines* A B *and* B C *once, which is by ſuppoſition medial. But vnto the ſquares of the lines* A B *and* B C *is equall the parallelograme* E H, *and vnto that which is contayned vnder the lines* A B *and* B C *twiſe is equall the parallelograme* H F: *wherefore either of theſe parallelogrames* H E *and* A F *is mediall: and they are applyed vpon the rationall line* E D. *Wherefore (by the 22. of the tenth) either of theſe lines* D H *and* H G *is a rationall line, and incommenſurable in length vnto the line* D E. *And foraſmuch as (by ſuppoſition) the line* A B *is incommenſurable in length vnto the line* B C. *But as the line* A B *is to the line* B C, *ſo is the ſquare of the line* A B *to the parallelograme which is contayned vnder the lines* A B *and* B C *(by the firſt of the ſixt). Wherefore (by the 10 of this booke) the ſquare of the line* A B *is incommenſurable to the parallelograme contayned vnder the lines*



Conſtruction.

Demonſtration.

A B and B C. *But to the ſquare of the line* A B *is commenſurable that which is compoſed of the ſquares of the lines* A B *and* B C *(by the* 15. *of the tenth). For the ſquares of the lines* A B *and* B C *are commenſurable (when as the lines* A B *and* B C *are put to be commenſurable in power onely). And to that which is contayned vnder the lines* A B *and* B C *is commenſurable that which is contained vnder the lines* A B *and* B C *twiſe (by the* 6 *of the tenth) wherefore that which is compoſed of the ſquares of the lines* A B *and* B C *is incommenſurable to that which is contayned vnder the lines* A B *and* B C *twiſe. But to the ſquares of the lines* A B *and* B C *is equall the parallelograme* E H. *And to that which is contayned vnder the lines* A B *and* B C *twiſe is equall the parallelograme* F H. *Wherfore the parallelograme* F H *is incommenſurable to the parallelograme* H E. *Wherfore the line* D H *is incommenſurable in length to the line* H G *(by the* 1 *of the ſixt and* 10 *of this booke). And it is proued that they are rationall lines. Wherefore the lines* D H *&* H G *are rationall commenſurable in power onely. Wherefore (by the* 36. *of the tenth) the whole line* D G *is irrationall. And the line* D E *is rationall. But a rectangle ſuperficies comprehended vnder a rationall line and an irrationall line is (by the corollary added after the* 21 *of the tenth) irrationall. Wherefore the ſuperficies* D F *is irrationall. And the line alſo which containeth it in power is irrational. But the line* A C *containeth in power the ſuperficies* D F. *Wherefore the line* A C *is irrationall. And it is called a ſecond bimediall line.*



This Propoſition ſheweth the generation of the fourth irrationall line, called a ſecond bimediall line. The definition wherof is euident by this Propoſition, which is thus.

Diffinition of a ſecond bimediall line.

A ſecond bimediall line is an irrationall line, which is made of two mediall lines commenſurable in power onely ioyned together, which comprehend a mediall ſuperficies. And it is called a ſecond bimediall, becauſe the two mediall lines of which it is compoſed, cõtaine a mediall ſuperficies, and not a rationall. Now a mediall is by nature & in knowledge after a rationall.

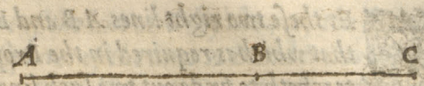
¶ The 27. Theoreme. The 39. Propoſition.

If two right lines incõmenſurable in power be added together, hauing that which is compoſed of the ſquares of them rationall, and the parallelogrãme contayned vnder them mediall: the whole right line is irrationall, and is called a greater line.

Demonſtration.

LEt theſe two right lines A B and B C being incommenſurable in power onely, and making that which is required in the Propoſition (The 33. *of the tenth teacheth to finde out two ſuch lines) be added together. Then I ſay, that the whole line A C is irrationall. For foraſmuch as (by ſuppoſition) the parallelogramme contained vnder the lines A B and B C is mediall, therefore the parallelogramme contained twiſe vnder the lines A B and B C is mediall. (For that which is contained vnder A B and B C twiſe, is commenſurable to that which is cõtained vnder A B and B C once (by the* 6. *of the tenth). Wherefore (by the Corollary of the* 23. *of the tenth) that which is contained vnder A B & B C twiſe, is mediall). But by ſuppoſition that which is compoſed of the ſquares of the lines A B and B C, is rationall. Wherefore that which is contained vnder the lines A B and B C twiſe is incommenſurable to that which is compoſed of the*

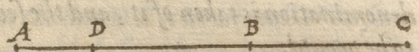
of the ſquares of the lines A B and B C. Wherfore (by the 16. of the tenth) that which is compoſed of the ſquares of the lines A B and B C together with that which is contayned vnder the lines A B & B C twiſe, which is (by the 4. of the ſecond) equall to the ſquare of the line A C, is incommenſurable to that which is compoſed of the ſquares of the lines A B and B C. But that which is compoſed of the ſquares of the lines A B and B C, is rationall. Wherefore the ſquare of the whole line A C, is irrationall. Wherefore the line A C alſo is irrationall. And is called a greater line. And it is called a greater line for that that which is compoſed of the ſquares of the lines A B & B C which are rationall, is greater then that which is contayned vnder the lines A B and B C twiſe, which are mediall. Now it is meete that the name ſhould be geuen according to the propertie of the rationall.



An Aſſumpt.

And that the ſuperficies compoſed of the ſquares of the lines A B and B C, is greater then that which is contained vnder the lines A B and B C twiſe, may thus be proued. Firſt it is manifeſt, that the lines A B and B C are vnequall: for if they were equall, then the ſquares of the lines A B and B C ſhould be equall to that which is contained vnder the lines A B and B C twiſe, ſo that that which is cõtained vnder A B and B C, ſhould alſo be rationall: which is contrary to the ſuppoſition. Wherefore the lines ⌣A B and B C are vnequall. Suppoſe then that the line ⌣A B be the greater, & let the line B D be equall to the line B C. Wherfore (by the 7. of the ſecond) the ſquares of the lines ⌣A B and B D, are equall to that which is contained vnder the lines A B and B C twiſe and to the ſquare of the line A D. But the line D B is equall to the line B C. Wherefore the ſquares of the lines A B and B C, are equall to that which is contained vnder the lines A B and B C twiſe and to the ſquare of the line A D. Wherefore the ſquares of the lines A B and B C are greater then that which is contayned vnder the lines ⌣A B and B C twiſe, by the ſquare of the line ⌣A D: which was required to be proued.

An Aſſumpt.



This Propoſition teacheth the production of the fift irrationall line, which is called a greater line: which is by the ſenſe of this Propoſition thus defined.

A greater line is an irrationall line, which is compoſed of two right lines which are incommenſurable in power, the ſquares of which added together, make a rationall ſuperficies, and the parallelogramme which they containe, is mediall. It is therefore called a greater line, as *Theon* ſayth, becauſe the ſquares of the two lines of which it is compoſed, added together being rationall, are greater then the mediall ſuperficies contained vnder them twiſe. And it is conuenient that the denomination be taken of the proprietie of the rationall part, rather then of the mediall part.

Diffinition of a greater line.

¶ The 28. Theoreme. The 40. Propoſition.

If two right lines incõmenſurable in power be added together, hauing that which is made of the ſquares of them added together mediall, and the parallelogramme contayned vnder them rationall: the whole right line is irrationall, and is called a line contayning in power a rationall and a mediall ſuperficies.

Let

LEt these two right lines A B and B C being incommensurable in power, & making that which is required in the Proposition be added together. (The 34. of the tenth teacheth to finde out two such lines). Then I say, that the whole line A C is irrationall. For forasmuch as that which is composed of the squares of the lines A B and B C is mediall, but that which is contained vnder the lines A B and B C twise is rationall, therefore that which is composed of the squares of the lines A B and B C, is incommensurable to that which is contained vnder the lines A B and B C twise. Wherefore (by the 16. of the tenth) that which is composed of the squares of the lines A B and B C together with that which is contayned vnder the lines A B and B C twise, which (by the 4. of the second) is the square of the whole line A C, is incommensurable to that which is contained vnder the lines A B and B C twise. But that which is contained vnder the lines A B and B C twise, is rationall, for that which is contained vnder the lines A B and B C once is put to be rationall. Wherefore the square of the whole line A C is irrationall. Wherefore the line A C also is irrationall. And it is therefore called a line containing in power a rationall and a mediall superficies, because the power thereof contayneth two superficieces, whereof the one is rationall, namely, that which is contained vnder the two partes, and the other mediall, namely, that which is made of the squares of the partes added together. And by reason that the rationall is in order of nature & of knowledge before the mediall, therfore the first part of the denomination is taken of it, and the second part is taken of the mediall: which was required to be proued.

Demonstration.

In this Proposition is taught the generation of the sixt irrationall line, which is called a line whose power is rationall and mediall. The definition of which is gathered of thys Proposition after this maner.

Diffinition of a line whose power is rationall and mediall.

A line whose power is rationall and mediall, is an irrationall line which is made of two right lines incommensurable in power added together, whose squares added together make a mediall superficies, but that superficies which they containe is rationall. The reason of the name is before set forth in the Proposition.

¶The 29. Theoreme. The 41. Proposition.

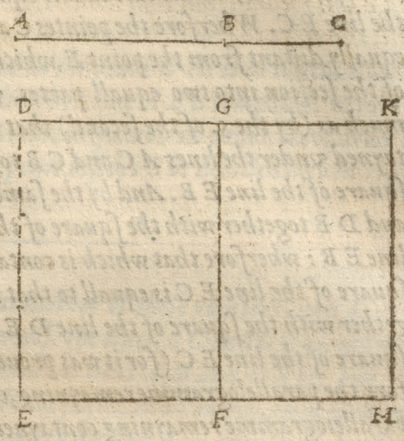
If two right lines incommensurable in power be added together, hauyng that which is composed of the squares of them added together mediall, and the parallelogramme contayned vnder them mediall, and also incommensurable to that which is composed of the squares of them added together: the whole right line is irrationall, and is called a line contayning in power two medials.

LEt these two right lines A B and B C being incommensurable in power, and hauing that which is composed of the squares of the lines A B and B C mediall, and the parallelogramme which is contayned vnder the lines A B and B C mediall, and also incommensurable to that which is composed of the squares of the lines A B and B C added together, be added together (the 35. of the tenth teacheth to finde out two such lines) Then I say that the whole line A C is irrationall. Take a rationall line D E, and (by the 44. of the first) vpon the line D E apply the parallelogramme D F equall to the squares of the lines A B and B C. And vpon the line G F, which is equall to the line D E, apply the parallelogrāme G H equall to that which is contained vnder the lines A B and B C twise. Wherefore the whole

Construction.

whole parallelogramme D H is equall to the ſquare of the line A C (by the 4. of the ſecond.) And foraſmuch as that which is compoſed of the ſquares of the lines A B and B C is mediall and is equall to the parrllelogramme D F: therfore D F alſo is mediall (by that which was ſaid in the 38. propoſition of this booke). And it is applied vpon the rationall line D E. Wherefore the line D G is rationall and incommenſurable in lẽgth vnto ẏ line D E (by the 22. of the teth). And by the ſame reaſon the lyne G K is rational & incõmẽſurable in length vnto the line G F, that is, vnto the line D E. And foraſmuch as that which is compoſed of ẏ ſquares of the lines A B & B C added together, is by ſuppoſition incõmenſurable to that which is cõtained vnder the lines A B and B C twiſe, therfore alſo the parallelogramme D F is incommenſurable vnto the parallelogramme G H. Wherfore alſo the line D G is incommenſurable vnto the line G K (by the firſt of the ſixt) and by the tenth of the tenth. But it is now proued that they are rationall. Wherfore the lines D G and G K are rationall commenſurable in power onely. Wherfore (by the 36. of the tenth) the whole line D K is rationall, and is called a binomiall line, but the line D E is irrationall. Wherfore the parallelogramme D H is irrationall (by the corollary added after the 22. propoſitiõ of the tenth). Wherfore alſo the line which containeth it in power is irrationall: but the line A C containeth it in power. Wherfore the line A C is irrationall, and is called a line contayning in power two medials. It is called a line containing in power two medials, for that it contayneth in power two mediall ſuperficieces, one of which is compoſed of the ſquares of the lynes A B and B C added together, and the other is that which is contained vnder the lines A B & B C twiſe: which was required to be demonſtrated.

Demonſtration.



In this propoſition is taught the nature of the 7. kinde of irrationall lines which is called a line whoſe power is two medials. The definition whereof is taken of this propoſition after this maner.

A line whoſe power is two medials, is an irrationall line which is compoſed of two right lines incommenſurable in power, the ſquares of which added together, make a mediall ſuperficies, and that which is contained vnder them is alſo mediall, and moreouer it is incommenſurable to that which is compoſed of the two ſquares added together.

Diffinition of a line containing in power two medials.

The reaſon why this line is called a line whoſe power is two medials, was before in the ende of the demonſtration declared.

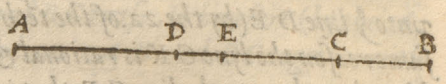
And that the ſaid irrationall lines are deuided one way onely, that is, in one point onely, into the right lines of which they are compoſed, and which make euery one of the kindes of thoſe irrationall lines, ſhall ſtraight way be demonſtrated: but firſt will we demonſtrate two aſſumptes here following.

¶ An Aſſumpt.

Take a right line and let the ſame be A B, and deuide it into two vnequall partes in the point C, and againe deuide the ſame line A B into two other vnequal partes, in an other point namely, in D, and let the line A C (by ſuppoſition) be greater then the line D B. Then I ſay that

An Aſſumpt.

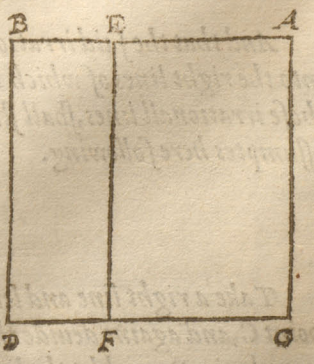
that the ſquares of the lines A C and B C added together, are greater then the ſquares of the lines A D and D B added together. Deuide the line A B (by the 10. of the firſt) into two equall partes in the point E. And foraſmuch as the line A C is greater then the line D B, take away the line D C which is common to them both: wherfore the reſidue A D is greater then the reſidue C B, but the line A E is equall to the line E B. Wherfore the line D E is leſſe then the line E C. Wherfore the pointes C and D are not equally diſtant from the point E, which is the point of the ſection into two equall partes. And foraſmuch as (by the 5. of the ſecond) that which is contayned vnder the lines A C and C B together with the ſquare of the line E C is equall to the ſquare of the line E B. And by the ſame reaſon that which is contayned vnder the lynes A D and D B together with the ſquare of the line D E, is alſo equall to the ſelf ſame ſquare of the line E B: wherfore that which is contained vnder the lines A C and C B together with the ſquare of the line E C is equall to that which is contained vnder the lines A D and D B together with the ſquare of the line D E: of which the ſquare of the line D E is leſſe then the ſquare of the line E C (for it was proued that the line D E is leſſe then the line E C). Wherfore the parallelogramme remayning, contayned vnder the lines A C and C B is leſſe thē the parallelogramme remayning contayned vnder the lines A D and D B. Wherfore alſo that which is contayned vnder the lines A C and C B twiſe is leſſe then that which is contayned vnder the lines A D and D B twiſe. But (by the fourth of the ſecond) the ſquare of the whole line A B is equall to that which is compoſed of the ſquares of the lines A C and C B together with that which is contained vnder the lynes A C and C B twiſe, and by the ſame reaſon the ſquare of the whole line A B is equall to that which is compoſed of the ſquares of the lines A D and D B together with that which is contayned vnder the lynes A D and D B twiſe: wherfore that which is compoſed of the ſquares of the lynes A C and C B together with that which is contayned vnder the lynes A C and C B twiſe, is equall to that which is compoſed of the ſquares of the lynes A D and D B, together with that which is contayned vnder the lynes A D and D B twiſe. But it is already proued that that which is contayned vnder the lynes A C and C B twiſe, is leſſe then that which is contayned vnder the lines A D & D B twiſe. Wherfore the reſidue, namely, that which is compoſed of the ſquares of the lines A C and C B is greater then the reſidue, namely, then that which is compoſed of the ſquares of the lines A D and D B: which was required to be demonſtrated.



¶ An Aſſumpt.

A rationall ſuperficies exceedeth a rationall ſuperficies, by a rationall ſuperficies.

Let A D be a rationall ſuperficies, and let it exceede A F being alſo a rationall ſuperficies by the ſuperficies E D. Then I ſay that the ſuperficies E D is alſo rationall. For the parallelogramme A D is commenſurable to the parallelogramme A F, for that either of them is rationall. Wherefore (by the ſecond part of the 15. of the tenth) the parallelogramme A F is commenſurable to the parallelogramme E D. But the the parallelogramme A F is rationall. Wherfore alſo the parallelogramme E D is rationall.



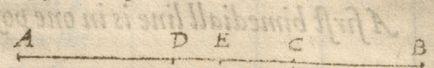
¶ The

¶ The 30. Theoreme. The 42. Proposition.

A binomiall line is in one point onely deuided into his names.

The second Senary by composition.

Vppose that A B be a binomiall line, and in the point C let it be deuided into his names, that is, into the lines wherof the whole line A B is composed. Wherefore these lines A C and C B are rationall commensurable in power onely. Now I say that the line A B cannot in any other point besides C be deuided into two rationall lines commensurable in power onely. For



Demonstration leading to an impossibilitie.

if it be possible, let it be deuided in the point D, so that let the lines A D and D B be rationall commensurable in power onely. First it is manifest, that neither of these pointes C and D deuideth the right line A B into two equall partes. Otherwise the lines A C and C B should be rationall commensurable in length, and so likewise should the lines A D and D B be. For euery line measureth it selfe, and any other line equall to it selfe. Moreouer the line D B is either one and the same with the line A C, that is, is equall to the line A C, or els it is greater then the line A C, either els it is lesse then it. If D B be equall to the line A C, then putting the line D B vpon the line A C eche endes of the one, shall agree with eche endes of the other. Wherfore putting the point B vpon the point A, the point D also shall fall vpon the point C, and the line A D which is the rest of the line A C, shall also be equall to the line C B, which is the rest of the line D B. Wherfore the line A B is deuided into his names in the point C. And so also shal the line A B being deuided in the point D be deuided in the self same point that the self same line A B was before deuided in the point C, which is cõtrary to the suppositiõ. For by suppositiõ it was deuided in sundry pointes, namely, in C & D. But if the line D B be greater thẽ the line A C, let the line A B be deuided into two equal partes in the point E. Wherfore the points C & D shal not equally be distant frõ the point E. Now (by the first aſsũpt going before this propositiõ) that which is cõposed of the squares of ỹ lines A D & D B is greater thẽ that which is composed of the squares of the lines A C & C B. But that which is composed of the squares of the lines A D & D B together with that which is cõtained vnder the lines A D & D B twise, is equall to that which is composed of the squares of the lines A C & C B together with that which is contained vnder the lines A C and C B twise, for either of them is equall to the square of the whole line A B (by the 4. of the second): wherefore how much that which is cõposed of the squares of the lines A D and D B added together is greater then that which is composed of the squares of the lines A C and C B added together, so much is that which is contained vnder the lines A C and C B twise greater then that which is contained vnder the lines A D and D B twise. But that which is composed of the squares of the lines A D and D B excedeth that which is composed of the squares of the lines A C and C B by a rationall superficies (by the 2. aſsumpt going before this proposition.) For that which is composed of the squares of the lines A D and D B is rationall, and so also is that which is composed of the squares of the lines A C and C B: for the lines A D and D B are put to be rationall commensurable in power onely, and so likewise are the lines A C and C B. Wherfore also that which is contained vnder the lines A C and C B twise, exceedeth that which is contained vnder the lines A D & D B twise by a rational superficies, whẽ yet notwithstãding they are both medial superficieces (by the 21. of the tenth): which (by the 26. of the same) is impoſsible. And if the line D B be lesse then the line A C, we may by the like demonstration proue the selfe same impoſsibilitie: Wherfore a binomiall line is in one point onely deuided into his names: Which was required to be demonstrated.

Corollary added by *Flussates.*

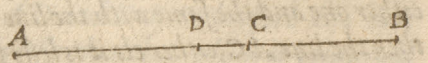
A Corollary. *Two rationall lines commensurable in power onely being added together cannot be equall to two other rationall lines commensurable in power onely added together.* For either of them should make a binomiall line, and so should a binomiall line be deuided into his names in moe poyntes then one: which by this proposition is proued to be impossible. The like shall follow in the fiue next irrationall lines as touching their two names.

¶ The 31. Probleme. The 43. Proposition.

A first bimediall line is in one poynt onely deuided into his names.

Demonstration leading to an impossibilitie.

Svppose that A B *be a first bimediall line, and let it be deuided into his partes in the point* C, *so that let the lines* A C *and* C B *be mediall cōmensurable in power onely, and containing a rationall superficies. Then I say that the line* A B *can not be deuided into his names in any other poynt then in* C. *For if it be possible let it be deuided into his names in the poynt* D, *so that let* A D *&* D B *be mediall lines commensurable in power onely, comprehending a rationall superficies. Now forasmuch as how much that which is contayned vnder the lines* A D *and* D B *twise differreth from that which is contayned vnder the lines* A C *and* C B *twise, so much differreth that which is composed of the squares of the lines* A D *and* D B *from that which is composed of the squares of the lines* A C *and* C B: *but that which is contayned vnder the lines* A D *and* D B *twise differreth from that which is contayned vnder the lines* A C *and* C B *twise, by a rationall superficies (by the second assumpt going before the 41. of the tenth). For either of those superficieces is rationall. Wherefore that which is composed of the squares of the lines* A C *and* C B *differeth from that which is composed of the squares of the lines* A D *and* D B *by a rationall superficies, when yet they are both mediall superficieces: which is impossible. Wherefore a first bimediall line is in one poynt onely deuided into his names: which was required to be proued.*



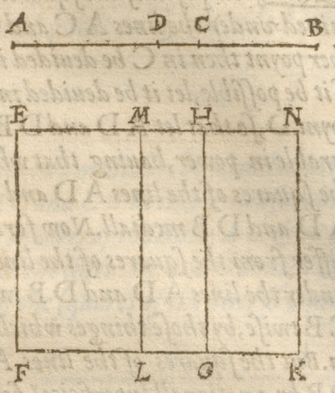
¶ The 32. Theoreme. The 44. Proposition.

A second bimediall line is in one poynt onely deuided into his names.

Demonstratiō leading to an impossibilitie.

Svppose that the line A B *being a second bimediall line, be deuided into hys names in the poynt* C: *so that let the lines* A C *and* C B *be mediall lines commensurable in power onely, comprehending a mediall superficies. It is manifest that the poynt* C *deuideth not the whole line* A B *into two equall partes. For the lines* A C *and* C B *are not commensurable in length the one to the other. Now I say that the line* A B *cannot be deuided into his names in any other poynt but onely in* C. *For if it be possible, let it be deuided into his names in the poynt* D, *so that let not the line* A C *be one and the same, that is, let it not be equall, with the line* D B. *But let it be greater then it. Now it is manifest (by the first assumpt going before the 42. proposition of this booke) that the squares of the lines* A C *and* C B *are greater then the squares of the lines* A D *and* D B. *And also that the lines* A D *and* D B *are mediall lines commensurable in power onely, comprehending a mediall superficies. Take a rationall line* E F. *And (by the 44. of the first) vpon the line* E F *apply a rectangle parallelograme* E K *equall to the square of the line* A B. *From which parallelograme take away the parallelograme* E G *equall to the squares of the lines* A C *and* C B. *Wherefore the residue, namely, the parallelograme* H K *is equall to that which is contained*

ned vnder the lines A C and C B twise. Agayne from the the parallelograme E K take away the parallelograme E L equall to the squares of the lines A D and D B which are lesse then the squares of the lines A C and C B. Wherefore the residue, namely, the parallelograme M K is equall to that which is contayned vnder the lines A D and D B twise. And forasmuch as the squares of the lines A C and C B are mediall, therefore the parallelograme E G also is mediall. And it is applyed vpon the rationall line E F: wherefore the line E H is rationall and incommensurable in length to the line E F. And by the same reason, the parallelograme H K is mediall (for that which is equall vnto it, namely, that which is contayned vnder the lines A C and C B twise is mediall) therefore the line H N is also rationall and incommensurable in length vnto the line E F. And forasmuch as the lines A C and C B are mediall lines commensurable in power onely, therefore the line A C is incommensurable in length vnto the line C B. But as the line A C is to the line C B, so is the square of the line A C to that which is contayned vnder the lines A C and C B (by the 1. of the sixt). Wherefore the square of the line A C is incommensurable to that which is contayned vnder the lines A C and C B. But (by the 16. of the tenth) vnto the square of the line A C are commensurable the squares of the lines A C and C B added together, for the lines A C and C B are commensurable in power onely. And vnto that which is contayned vnder the lines A C and C B is commensurable that which is contayned vnder the lines A C and C B twise. Wherefore that which is composed of the squares of the lines A C and C B, is incommensurable to that which is contained vnder the lines A C and C B twise. But to the squares of the lines A C and C B is equall the parallelograme E G, and to that which is contained vnder the lines A C and C B twise is equall the paralelograme H K. Wherfore the parallelograme E G is incommensurable to the parallelograme H K. Wherefore also the line E H is incommensurable in length to the line H N. And the lines E H and H N are rationall. Wherefore they are rationall commensurable in power onely: but if two rationall lines commensurable in power onely be added together, the whole line is irrationall, and is called a binomiall line (by the 36. of the tenth). Wherefore the binomiall line E N is in the poynt H deuided into his names. And by the same reason also may it be proued that the lines E M and M N are rationall lines commensurable in power onely. Wherfore E N being a binomiall line is deuided into his names in sundry poyntes, namely, in H and M, neither is the line E H one and the same, that is, equal with M N. For the squares of the lines A C and C B are greater then the squares of the lines B D and A D (by the 1. assumpt put after the 41. of the tenth). But the squares of the lines A D and D B are greater then that which is contayned vnder the lines A D and D B twise (by the assumpt put after the 39. of the tenth). Wherefore the squares of the lines A C and C B, that is, the parallelograme E G is much greater then that which is contained vnder the lines A D and D B twise, that is then the parallelograme M K. Wherfore (by the first of the sixt) the line E H is greater then the line M N. Wherefore E H is not one and the same with M N. Wherefore a binomiall line is in two sundry poyntes deuided into his names. Which is impossible. The selfe same absurditie also will follow if the line A C be supposed to be lesse then the line D B. A second binomiall line therefore is not deuided into his names in sundry poyntes. Wherefore it is deuided in one onely: which was required to be demonstrated.



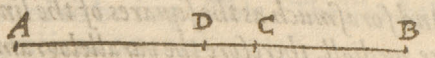
Demonstration leading to an impossibilitie.

¶ The 33. Theoreme. The 45. Proposition.

A greater line is in one poynt onely deuided into his names.

LEt A B being a greater line be deuided into his names in the poynt, C so that let the lines A C and C B be rationall incommensurable in power, hauing that which is composed of the squares of the lines A C and C B rationall, and that which is contained vnder the lines A C and C B mediall. Then I say that the line A B can not in any other poynt then in C be deuided into his names. For if it be possible, let it be deuided into his names in the poynt D, so that let A D and D B be lines incommensurable in power, hauing that which is composed of the squares of the lines A D and D B rationall, and that which is contayned vnder the lines A D and D B mediall. Now forasmuch as how much the squares of the lines A C and C B differ from the squares of the lines A D and D B, so much differeth that which is contained vnder the lines A D and D B twise from that which is contained vnder the lines A C and C B twise, by those thinges which haue bene sayd in the demonstration of the 42. proposition. But the squares of the lines A C and C B exceede the squares of the lines A D and D B by a rationall superficies (for they are either of them rationall). Wherfore that which is contained vnder the lines A D and D B twise exceedeth that which is contained vnder the lines A C and C B twise by a rationall superficies: when as either of them is a mediall superficies. Which is impossible (by the 26. of the tenth). Wherefore a greater line is in one poynt onely deuided into his names: which was required to be proued.

Demonstration leading to an impossibitie.

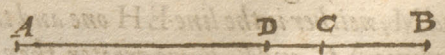


¶ The 34. Theoreme. The 46. Proposition.

A line contayning in power a rationall and a mediall, is in one point onely deuided into his names.

LEt A B being a line containing in power a rationall and a mediall, be deuided into his names in the point C, so that let the lines A C & C B be incommensurable in power, hauing that which is composed of the squares of the lines A C and C B mediall, and that which is contained vnder the lines A C and C B rationall. Then I say, that the line A B can not in any other point be deuided into his names but onely in the point C. For if it be possible, let it be deuided into his names in the point D, so that let the lines A D and D B be incommensurable in power, hauing that which is composed of the squares of the lines A D and D B mediall, and that which is contayned vnder the lines A D and D B rationall. Now forasmuch as how much that which is contained vnder the lines A D & D B twise differeth from that which is contained vnder the lines A C and C B twise, so much differ the squares of the lines A C & C B added together from the squares of the lines A D and D B added together. But that which is contayned vnder the lines A C and C B twise exceedeth that which is contained vnder the lines A D and D B twise by a rationall superficies (for either of them is rationall). Wherfore also the squares of the lines A C and C B added together, exceede the squares of the lines A D and D B added together by a rationall superficies, when yet ech of them is a mediall superficies: which is impossible. Wherefore a line containing in power a rationall and a mediall, is in one point onely deuided into his names: which was required to be demonstrated.

Demonstration leading to an impossibitie.



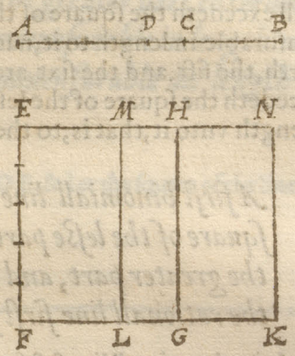
¶ The

¶ The 35. Theoreme. The 47. Proposition.

A line containing in power two medials, is in one point onely deuided into his names.

SVppose that A B being a line containing in power two medialls, be deuided into his names in the point C, so that let the lines A C and C B be incommensurable in power, hauing that which is composed of the squares of the lines A C & C B mediall, and that also which is contained vnder the lines A C and C B mediall, and moreouer incommensurable to that which is composed of the squares of the lines A C and C B. Then I say, that the line A B can in no other point be deuided into his names but onely in the point C. For if it be possible, let it be deuided into his names in the point D, so that let not the line A C be one and the same, that is, equall with the line D B: but by supposition let the line A C be the greater. And take a rationall line E F. And (by the 43. of the first) vpon the line E F apply a rectangle parallelogrãme E G equall to that which is cõposed of the squares of the lines A C and C B: and likewise vpon the line H G, which is equall to the line E F, apply the parallelogramme H K equall to that which is contained vnder the lines A C and C B twise. Wherefore the whole parallelogramme E K is equall to the square of the line A B. Againe vpon the same line E F describe the parallelogramme E L equall to the squares of the lines A D and D B. Wherefore the residue, namely, that which is contayned vnder the lines A D and D B twise, is equall to the parallelogramme remaining, namely, to M K. And forasmuch as that which is cõposed of the squares of the lines A C and C B, is (by supposition) mediall, therefore the parallelogrãme E G which is equall vnto it, is also mediall: and it is applied vpon the rationall line E F. Wherefore (by the 22. of the tenth) the line H E is rationall and incommensurable in length vnto the line E F. And by the same reason also the line H N is rationall and incommensurable in length to the same line E F. And forasmuch as that which is composed of the squares of the lines A C and C B is incommensurable to that which is contained vnder the lines A C and C B twise (for it is supposed to be incommensurable to that which is cõtained vnder the lines A C and C B once): therefore the parallelogramme E G is incommensurable to the parallelogramme H K. Wherefore the line E H also is incommensurable in length to the line H N, and they are rationall lines: wherfore the lines E H and H N are rationall commensurable in power onely. Wherefore the whole line E N is a binomiall line, and is deuided into his names in the point H. And in like sort may we proue, that the same binomiall line E N is deuided into his names in the point M, and that the line E H is not one and the same that is equall with the line M N, as it was proued in the end of the demonstration of the 44. of this booke. Wherefore a binomiall line is deuided into his names in two sundry pointes: which is impossible (by the 42. of the tenth). Wherefore a line containing in power two medials, is not in sundry pointes deuided into his names. Wherefore it is deuided in one point onely: which was required to be demonstrated.

Construction.



Demonstration leading to an absurditie.

¶Second Definitions.

Sixe kindes of binomiall lines.

IT was ſhewed before that of binomiall lines there were ſixe kindes, the definitions of all which are here now ſet, and are called ſecond definitiõs. All binomiall lines, as all other kindes of irrationall lines, are cõceaued, cõſidered, and perfectly vnderſtanded onely in reſpecte of a rationall line (whoſe partes as before is taught, are certayne, and knowen, and may be diſtinctly expreſſed by number) vnto which line they are compared. Thys rationall line muſt ye euer haue before your eyes, in all theſe definitions, ſo ſhall they all be eaſie inough.

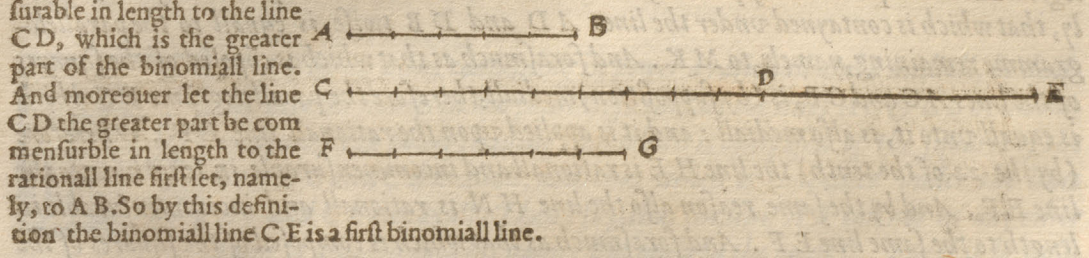
A binomiall line conſiſteth of two partes.

A binomiall line (ye know) is made of two partes or names, wherof the one is greater then the other. Wherfore the power or ſquare alſo of the one is greater then the power or ſquare of the other. The three firſt kindes of binomiall lines, namely, the firſt, the ſecond, & the third, are produced, when the ſquare of the greater name or part of a binomiall excedeth the ſquare of the leſſe name or part, by the ſquare of a line which is commenſurable in length to it, namely, to the greater. The three laſt kindes, namely, the fourth, the fift, and the ſixt, are produced, when the ſquare of the greater name or part exceedeth the ſquare of the leſſe name or part, by the ſquare of a line incommenſurable in length vnto it, that is, to the greater part.

Firſt diffinition.

A firſt binomiall line is, whoſe ſquare of the greater part exceedeth the ſquare of the leſſe part by the ſquare of a line commenſurable in length to the greater part, and the greater part is alſo commenſurable in length to the rationall line firſt ſet.

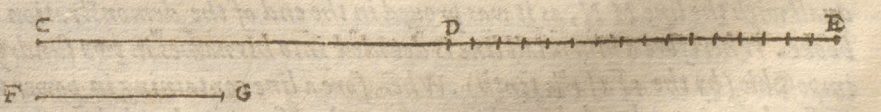
As let the rationall line firſt ſet be A B: whoſe partes are diſtinctly knowen: ſuppoſe alſo that the line C E be a binomiall line, whoſe names or partes let be C D and D E. And let the ſquare of the line C D the greater part excede the ſquare of the line D E the leſſe part by the ſquare of the line F G: which line F G, let bee commenſurable in length to the line C D, which is the greater part of the binomiall line. And moreouer let the line C D the greater part be commenſurble in length to the rationall line firſt ſet, namely, to A B. So by this definition the binomiall line C E is a firſt binomiall line.



Second diffinition.

A ſecond binomiall line is, when the ſquare of the greater part exceedeth the ſquare of the leſſe part by the ſquare of a line commenſurable in length vnto it, and the leſſe part is commenſurable in length to the rationall line firſt ſet.

As (ſuppoſing euer the rationall line) let C E be a binomiall line deuided in the poynt D. The ſquare of whoſe greater part C D let exceede the ſquare of the leſſe part D E by the ſquare of the line F G, which line F G let be cõmenſurable in length vnto the line C D the greater part of the binomiall line. And let alſo the line D E the leſſe part of the binomiall line be commenſurable in length to the rationall line firſt ſet A B. So by this definition the binomiall line C E is a ſecond binomiall line.



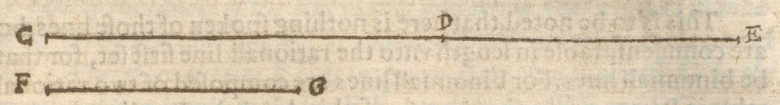
Third diffinition.

A third binomiall line is, when the ſquare of the greater part excedeth the

ſquare

ſquare of the leſſe part, by the ſquare of a line cõmenſurable in length vnto it. And neither part is commenſurable in length to the rationall line geuẽ.

As ſuppoſe the line C E to be a binomiall line: whoſe partes are ioyned together in the poynt D: and let the ſquare of the line C D the greater part exceede the ſquare of the leſſe part D E by the ſquare of the line F G, and let the line F G be commenſurable in length to the line C D the greater part of the binomiall.

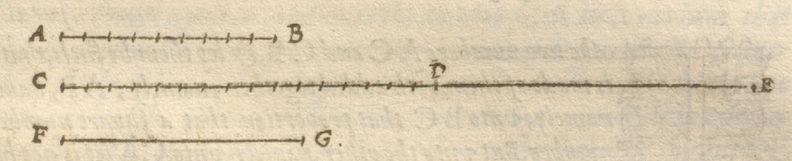


Moreouer, let neither the greater part C D, nor the leſſe part D E, be commenſurable in length to the rationall line A B, then is the line C E by this definition a third binomiall line.

A fourth binomiall line is, when the ſquare of the greater part exceedeth the ſquare of the leſſe by the ſquare of a line incommenſurable in length vnto the greater part. And the greater is alſo commenſurable in length to the rationall line.

Fourth diffinition.

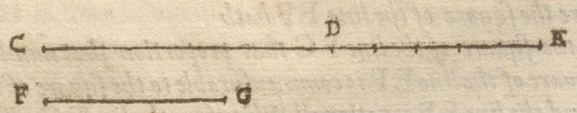
As let the line C E be a binomiall line, whoſe partes let be C D & D E, & let the ſquare of the line C D the greater part exceede the ſquare of the line D E the leſſe, by the ſquare of the line F G. And let the line F G be incommenſurable in length to the line C D the greater. Let alſo the line C D the greater part be commenſurable in length vnto the rationall line A B. Then by this definition the line C E is a fourth binomiall line.



A fift binomiall line is, when the ſquare of the greater part exceedeth the ſquare of the leſſe part, by the ſquare of a line incommenſurable vnto it in length. And the leſſe part alſo is commenſurable in length to the rationall line geuen.

Fifth diffinition.

As ſuppoſe that C E be a binomiall line, whoſe greater part let be C D, and let the leſſe part be D E. And let the ſquare of the line C D excede the ſquare of the line D E by the ſquare of the line F G, which let be incõmenſurable in length vnto the line C D the greater part of the binomiall line. And let the line D E the ſecond part of the binomiall line be commenſurable in length vnto the rationall line A B. So is the line C E by this definition a fift binomiall line.

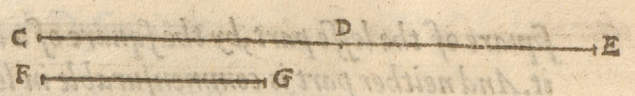


A ſixt binomiall line is, when the ſquare of the greater part exceedeth the ſquare of the leſſe, by the ſquare of a line incommenſurable in length vnto it. And neither part is commenſurable in length to the rationall line geuen.

Sixth diffinition.

As let the line C E be a binomiall line, deuided into his names in the poynt D. The ſquare of whoſe greater part C D let exceede the ſquare of the leſſe part D E by the ſquare of the line F G, and let the line F G be incommenſurable in length to the line C D the greater part of the binomiall line. Let alſo

neither C D the greater part, nor D E the leſſe part be commenſurable in length to the rationall line A B. And ſo by this definition the line C E is a ſixt binomiall line. So ye ſee that by theſe definitions, & their examples, and declarations, all the kindes of binomiall lines are made very playne.



This is to be noted that here is nothing ſpoken of thoſe lines, both whoſe portions are commenſurable in length vnto the rationall line firſt ſet, for that ſuch lines cannot be binomiall lines. For binomiall lines are compoſed of two rationall lines commenſurable in Power onely (by the 36. of this booke). But lines both whoſe portions are commenſurable in length to the rationall line firſt ſet are not binomiall lines. For that the partes of ſuch lines ſhould by the 12. of this booke be commenſurable in length the one to the other. And ſo ſhould they not be ſuch lines as are required to the compoſition of a binomiall line. Moreouer ſuch lines ſhould not be irrationall but rationall, for that they are commenſurable to ech of the parts whereof they are cõpoſed (by the 15. of this booke). And therefore they ſhould be rationall for that the lines which compoſe them are rationall.

¶ The 13. Probleme. The 48. propoſition.

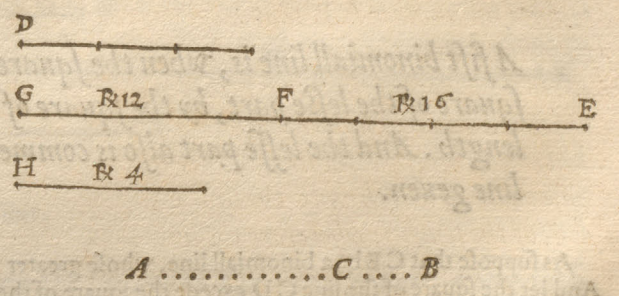
To finde out a firſt binomiall line.

The third Senary by compoſition.

Take two numbers A C *and* C B, *& let them be ſuch, that the number which is made of them both added together, namely,* A B, *haue vnto one of them, namely, vnto* B C *that proportion that a ſquare number hath to a ſquare number. But vnto the other, namely, vnto* C A *let it not haue that proportion that a ſquare number hath to a ſquare number (ſuch as is euery ſquare number which may be deuided into a ſquare number and into a number not ſquare).*

Conſtruction.

Take alſo a certayne rationall line, and let the ſame be D. *And vnto the line* D *let the line* E F *be commenſurable in length. Wherefore the line* E F *is rationall. And as the number* A B *is to the nũber* A C, *ſo let the ſquare of the line* E F *be to the ſquare of an other line, namely, of* F G *(by the corollary of the ſixt of ỹ tẽth). Wherfore the ſquare of the line* E F *hath to the ſquare of the line* F G *that proportion that number hath to number. Wherefore the ſquare of the line* E F *is commenſurable to the ſquare of the line* F G *(by the 6. of this booke) And the line* E F *is rationall. Wherefore the line* F G *alſo is rationall. And foraſmuch as the number* A B *hath not to the number* A C, *that proportion that a ſquare number hath to a ſquare number, neither ſhal the ſquare of the line* E F *haue to the ſquare of the line* F G *that proportion that a ſquare number hath to a ſquare number. Wherefore the line* E F *is incommenſurable in length to the line* F G *(by the 9. of this booke). Wherefore the lines* E F *and* F G *are rationall commenſurable in power onely. Wherefore the whole line* E G *is a binomiall line (by the 36. of the tenth). I ſay alſo that it is a firſt binomiall line. For for that as the number* B A *is to the number* A C, *ſo is the ſquare of the line* E F *to the ſquare of the line* F G: *but the number* B A *is greater then the number* A C: *wherefore the ſquare of the line* E F *is alſo greater then the ſquare of the line* F G. *Vnto the ſquare of the line* E F *let the ſquares of the lines* F G *and* H *be equall (which how to finde out is taught in the aſſumpt put after*



Demonſtration.

after the 13. of the tenth). And forthat as the number BA is to the number AC, so is the ſquare of the line EF to the ſquare of the line FG: therefore (by conuerſion or euerſion of proportion (by the corollary of the 19. of the fift) as the number AB is to the number BC, ſo is the ſquare of the line EF to the ſquare of the line H. But the number AB hath to the number BC that proportion that a ſquare number hath to a ſquare number. Wherefore alſo the ſquare of the line EF hath to the ſquare of the line H that proportion that a ſquare number hath to a ſquare number. Wherefore the line EF is commenſurable in length to the line H (by the 9. of this booke. Wherefore the line EF is in power more then the line FG by the ſquare of a line commenſurable in length to the line EF. And the lines EF and FG are rationall commenſurable in power onely. And the line EF is commenſurable in length to the rationall line D. Wherefore the line EG is a firſt binomiall line: which was required to be doone.

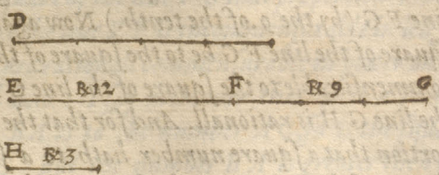
The 14. Probleme. The 49. Propoſition.

To finde out a ſecond binomiall line.

TAke two numbers AC and CB, and let them be ſuch that the number made of them both added together, namely, AB haue vnto BC that proportiõ that a ſquare number hath to a ſquare number, and vnto the number CA let it not haue that proportion that a ſquare number hath to a ſquare number, as it was declared in the former propoſition. Take alſo a rationall line, and let the ſame be D, and vnto the line D let the line FG be commenſurable in length. Wherefore FG is a rationall line. And as the number CA is to the number AB ſo let the ſquare of the line GF be to the ſquare of the line FE (by the 6. of the tenth), Wherefore the ſquare of the line GF is commenſurable to the ſquare of the line FE. Wherfore alſo FE is a rationall line. And foraſmuch as the nũber CA hath not vnto the number AB that proportiõ that a ſquare number hath to a ſquare nũber, therfore neither alſo the ſquare of the line GF hath to the ſquare of the line FE that proportion that a ſquare number hath to a ſquare number. Wherefore the line GF is incommenſurable in length vnto the line FE (by the 9. of the tenth): wherefore the lines FG and FE are rationall commenſurable in power onely. Wherefore the whole line EG is a binomiall line. I ſay moreouer that the line EG is a ſecond binomiall line. For for that by contrary proportion as the number BA is to the number AC, ſo is the ſquare of the line EF to the ſquare of the line FG. But the number BA is greater then the number AC, wherefore alſo the ſquare of the line EF is greater then the ſquare of the line FG. Vnto the ſquare of the line EF, let the ſquares of the lines GF and H be equall. Now by conuerſion (by the corollary of the 19. of of the fift) as the number AB is to the number BC, ſo is the ſquare of the line EF to the ſquare of the line H. But the number AB hath to the number BC that proportion that a ſquare number hath to a ſquare number. Wherefore the ſquare of the line EF hath to the ſquare of the line H, that proportion that a ſquare number hath to a ſquare number. Wherefore (by the 9. of the tenth) the line EF is commenſurable in length vnto the line H. Wherefore the line EF is in power more then the line FG by the ſquare of a line commenſurable in length vnto the line EF: and the lines

Conſtruction.

Demonſtration.



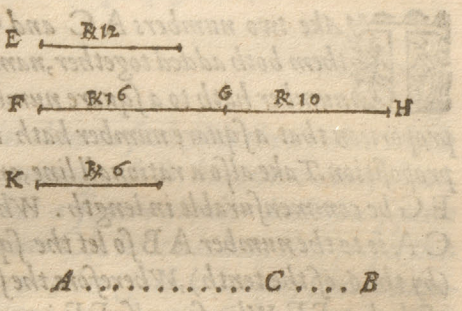
EF

EF and FG are rationall commensurable in power onely, and FG being the lesse name is commensurable in length vnto the rationall line geuen, namely, to D. Wherefore EG is a second binomiall line: which was required to be done.

¶ The 15. Probleme. The 50. Proposition.

To finde out a third binomiall line.

Construction. *TAke two numbers AC and CB, and let them be such that the number made of them both added together, namely, AB, haue to the number BC that proportion that a square number hath to a square number. But to the number AC let it not haue that proportion that a square number hath to a square number, as it was declared in the two former. And take also some other number that is not a square number, and let the same be D, and let not the number D haue either to the number BA, or to the number AC that proportion that a square number hath to a square number. And take a rational line and let the the same be E. And as the number D is to the number AB, so let the square of the line E be to the square of the line FG. Wherfore the square of the line E is cõmensurable to the square of the line FG, but the line E is rational, wherfore the line FG also is rational. And for that the nũber D hath not to the nũber AB that proportion that a square number hath to a square number, neither also shall the square of the line E haue to the square of the line FG that proportion that a square number hath to a square nũber. Wherfore the line E is incommensurable in length to the line FG (by the 9. of the tenth.) Now againe as the nũber AB is to the nũber AC, so let the square of the line FG be to the square of the line GH. Wherfore the square of the line FG is commensurable to the square of the line GH. And the line FG is rationall. Wherfore also the line GH is rationall. And for that the number BA hath not to the nũber AC, that proportion that a square number hath to a square number, therefore neither also hath the square of the line FG to the square of the line GH that proportion that a square number hath to a square number. Wherfore the line FG is incõmensurable in lẽgth to the line GH. Wherfore the lines FG & GH are rational cõmensurable in power only. Wherfore the whole line FH is a binomial line. I say moreouer that it is a third binomial line. For for that as the nũber D is to the nũber AB, so is the square of the line E to the square of the line FG: but as the nũber AB is to the number AC, so is the square of the line FG to the square of the line GH: therfore of equalitie (by the 22. of the fift) as the number D is to the number AC, so is the square of the line E to the square of the line GH. But the nũber D hath not to the nũber AC that proportiõ that a square nũber hath to a square nũber. Wherfore neither also hath the square of the line E to the square of the line GH that proportiõ that a square number hath to a square number. Wherfore the line E is incommensurable in lẽgth to the line GH. And for that as the number AB is to the number AC, so is the square of the line FG to the square of the line GH, therfore the square of the line FG is greater then the square of the line GH. Vnto the square of the line FG, let the squares of the lines GH and K be equal. Wherfore (by euerse proportiõ by the corollary of the 19. of the fift) as the nũber AB is to the number BC, so is the square of ỹ line FG to the square of the line K. But the nũber AB hath to the nũber*



Demonstration.

BC

B C that proportiõ that a ſquare number hath to a ſquare number. Wherfore alſo the ſquare of the line F G hath to the ſquare of the line K that proportion that a ſquare number hath to a ſquare number. Wherfore the line F G is commenſurable in length to the line K. Wherfore the line F G is in power more then the line G H by the ſquare of a line commenſurable in length vnto it. And the lines F G and G H are rationall commenſurable in power onely, and neither of them is commenſurable in length vnto the rationall line E: wherfore the lyne F H is a third binomiall line: which was required to be done.

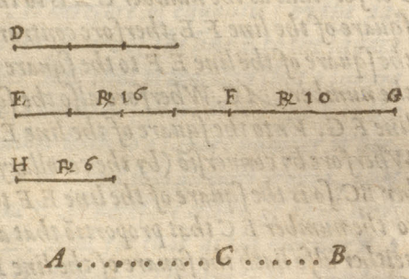
¶ The 16. Probleme. The 51. Propoſition.

To finde out a fourth binomiall line.

Take two numbers A C and C B, & let thẽ be ſuch that the nũber made of thẽ both added together, namely, A B, haue to neither of the nũbers A C and C B that proportiõ that a ſquare nmuber hath to a ſquare nũber (ſuch as is euery ſquare nũber to two nũbers not ſquare, which are leſſe thẽ it & make the ſaid ſquare nũber). And take a rationall line, and let the ſame be D. And vnto the line D let the line E F be cõmenſurable in length. Wherfore E F is a rationall line, and as the number B A is to the number A C, ſo let the ſquare of the line E F be to the ſquare of ỹ line F G. Wherefore the ſquare of the line E F is commenſurable to the ſquare of the line F G, and the line E F is a rationall line. Wherfore alſo the line F G is a rationall line. And for that the number B A hath not to the number A C that proportion that a ſquare number hath to a ſquare number, neither alſo ſhall the ſquare of the line E F haue to the ſquare of the line F G that proportion that a ſquare number hath to a ſquare number. Whrfore the line E F is incommenſurable in length to the line F G. Wherfore the lines E F and F G are rationall commenſurable in power onely. Wherefore the whole line E G is a binomiall line. I ſay moreouer that it ii a fourth binomiall lyne. For for that as the number B A is to the number A C, ſo is the ſquare of the line E F to the ſquare of the line F G. But the number B A is greater then eht number A C. Wherefore alſo the ſquare of the line E F is greater then the ſquare of the line F G. Vnto the ſquare of the line E F let the ſquares of the lines F G and H be equall. Wherfore by conuerſion (by the corollary of the 19. of the fift) as the number A B is to the number B C, ſo is the ſquare of the line E F to the ſquare of the line H. But the number A B hath not to the number B C that proportion that a ſquare number hath to a ſquare number: therfore neither alſo hath the ſquare of the line E F to the ſquare of the line H that proportiõ that a ſquare nũber hath to a ſquare nũber. Wherfore (by the 9. of the tẽth) the line E F is incõmenſurable in length vnto the line H. Wherfore the line E F is in power more thẽ the line F G by the ſquare of a line incommenſurable in length vnto it. And the lynes E F and F G are rationall commenſurable in power onely, and the line E F is commenſurable in length to the rationall line D. Wherfore the line E G is a fourth binomiall line: which was required to be found out.

Conſtruction.

Demonſtration.



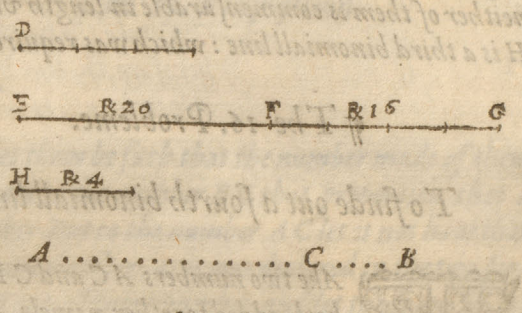
¶ The 17. Probleme. The 52. Propoſition.

To finde out a fift binomiall lyne.

Take

Construction. TAke two numbers A C and C B, and let them be such, that the number A B haue to neither of the numbers A C or C B that proportion that a square number hath to a square number, as in the former proposition. And take a rationall line and let the same be D. And vnto the line D let the line F G be commensurable in length. Wherfore the line F G is rationall. And as the number C A is to the number A B, so let the square of the line G F be to the square of the line E F. Wherefore the square of the line G F is commensurable to the square of the line F E. Wherefore also the line F E is rationall. And for that the number C A hath not to the number A B that proportiõ that a square number hath to a square number, therfore neither also hath the square of the line G F to the square of the line F E that proportion that a square number hath to a square number. Wherfore (by the 9. of the tenth) the line G F is incommensurable in length to the line F E. Wherfore the lines E F and F G are rationall commensurable in power only. Wherfore the whole line E G is a binomiall line. I say moreouer that it is a fift binomiall line. For for that as the number C A is to the number A B, so is the square of the line G F to the square of the line F E, therfore contrariwise, as the number B A is to the number A C, so is the square of the line E F to the square of the line F G. but the number B A is greater then the number A C. Wherfore also the square of the line E F is greater then the square of the line F G. Vnto the square of the line E F, let the squares of the lines F G and H be equall. Wherfore by conuersiõ (by the corollary of the 19. of the fift) as the nũber A B is to the number B C, so is the square of the line E F to the square of the line H. But the nũber A B hath not to the number B C that proportiõ that a square number hath to a square number. Wherefore neither also hath the square of the line E F to the square of the line H that proportion that a square number hath to a square number. Wherfore (by the 9. of the tenth) the line E F is incommensurable in length to the line H. Wherfore the line E F is in power more then the line F G by the square of a line incommensurable in length vnto it. And the lines E F and F G are rationall commensurable in power onely. And the line F G being the lesse name, is commensurable in length to the rationall line geuen, namely, to D. Wherfore the whole line E G is a fift binomiall line: which was required to be found out.

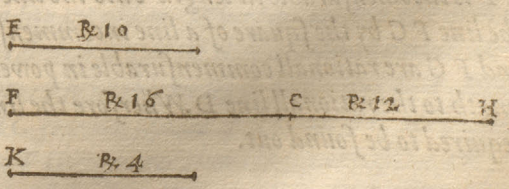
Demonstration.



¶ The 18. Probleme. The 53. Proposition.

To finde out a sixt binomiall line.

Construction. TAke two numbers A C & C B, and let thẽ be such that the number which is made of them both added together, namely, A B, haue to neither of the numbers A C or C B that proportion that a square nũber hath to a square number. Take also any other number which is not a square number, and let the same be D. And let not the number D haue to any one of these numbers A B and A C that proportion that a square number hath to a square nũber. Let there be put moreouer a rationall line,



and

and let the same be E. And as the number D is to the number A B, so let the square of the line E be to the square of F G. Wherefore (by the 6. of the tenth) the line E is commensurable in power to the line F G, & the line E is rationall. Wherfore also the line F G is rationall. And for that the number D hath not to the number A B that proportion that a square nũber hath to a square number, therefore neither also shall the square of the line E haue to the square of the line F G that proportion that a square number hath to a square number. Wherefore the line F G is incommensurable in length to the line E. Againe, as the number B A is to the number A C, so let the square of the line F G be to the square of the line G H. Wherefore (by the 6. of the tenth) the square of the line F G is commensurable to the square of the line G H. And the square of the line F G is rationall. Wherefore the square of the line G H is also rationall. Wherefore also the line G H is rationall. And for that the number A B hath not to the number A C, that proportion that a square number hath to a square number: therefore neither also hath the square of the line F G to the square of the line G H, that proportion that a square number hath to a square number. Wherefore the line F G is incommensurable in length to the line G H. Wherefore the lines F G and G H are rationall commensurable in power onely. Wherefore the whole line F H is a binomiall line. I say moreouer, that it is a sixt binomiall line. For for that as the number D is to the number A B, so is the square of the line E to the square of the line F G. And as the number B A is to the number A C, so is the square of the line F G to the square of the line G H. Wherefore, of equalitie (by the 22. of the fift) as the number D is to the number A C, so is the square of the line E to the square of the line G H. But the number D hath not to the nũber A C that proportion that a square number hath to a square number. Wherefore neither also hath the square of the line E to the square of the line G H that proportion that a square number hath to a square number. Wherfore the line E is incommensurable in length to the line G H. And it is already proued, that the line F G is also incommensurable in length to the line E. Wherefore either of these lines F G and G H is incommensurable in length to the line E. And for that as the number B A is to the number A C, so is the square of the line F G to the square of the line G H: therfore the square of the line F G is greater then the square of the line G H. Vnto the square of the line F G, let the squares of the lines G H and K be equall. Wherefore by euersion of proportion, as the number A B is to the number B C, so is the square of the line F G to the square of the line K. But the number A B hath not to the number B C that proportion that a square number hath to a square number. Wherefore neither also hath the square of the line F G to the square of the line K that proportion that a square number hath to a square number. Wherefore the line F G is incommensurable in length vnto the line K. Wherefore the line F G is in power more then the line G H, by the square of a line incommensurable in length to it. And the lines F G and G H are rationall commensurable in power onely. And neither of the lines F G & G H is commensurable in length to the rationall line geuen, namely, to E. Wherefore the line F H is a sixt binomiall line: which was required to be found out.

Demonstration.

¶ A Corollary added out of *Flussates*.

By the 6. former Propositions it is manifest, how to deuide any right line geuen into the names of euery one of the sixe foresayd binomiall lines. For if it be required to deuide a right line geuen into a first binomiall line, then by the 48. of this booke finde out a first binomiall line. And this right line being so found out deuided into his names, you may by the 10. of the sixt, deuide the right line geuen in like sort. And so in the other fiue following.

A Corollary added by Flussates.

Although I here note vnto you this Corollary out of *Flussas*, yet, in very conscience and of gratefull minde, I am enforced to certifie you, that, many yeares, before the trauailes of *Flussas* (vpõ *Euclides* Geometricall Elementes) were published, the order how to deuide, not onely the 6. Binomiall lines into their names, but also to adde to the 6. Residuals their due partes: and farthermore to deuide all the o-

M. Dee his booke called Tyrocinium Mathematicum.

ther irrationall lines (of this tenth booke) into the partes distinct, of which they are composed: with many other straunge conclusions Mathematicall, to the better vnderstanding of this tenth booke and other Mathematicall bookes, most necessary, were by *M. Iohn Dee* inuented and demonstrated: as in his booke, whose title is *Tyrocinium Mathematicum* (dedicated to *Petrus Nonnius*, An. 1559.) may at large appeare. Where also is one new arte, with sundry particular pointes, whereby the Mathematicall Sciences, greatly may be enriched. Which his booke, I hope, God will one day allowe him opportunitie to publishe: with diuers other his Mathematicall and Metaphysicall labours and inuentions.

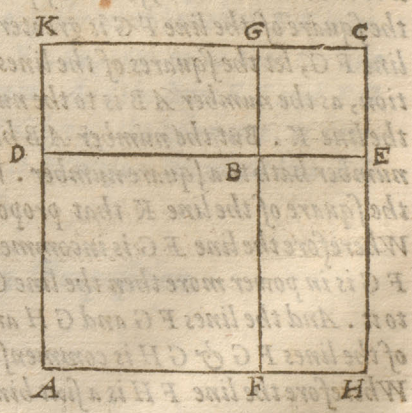
¶ An Assumpt.

If a right line be deuided into two partes how soeuer: the rectangle parallelogramme contayned vnder both the partes, is the meane proportionall betwene the squares of the same parts. And the rectangle parallelogramme contained vnder the whole line and one of the partes, is the meane proportionall betwene the square of the whole line and the square of the sayd part.

This Assumpt as was before noted, followeth most briefly without farther demonstration of the 25. of this booke.

Demonstration.

Suppose that there be two squares A B and B C, and let the lines D B and B E so be put that they both make one right line. Wherefore (by the 14. of the first) the lines F B and B G make also both one right line. And make perfect the parallelogramme A C. Then I say, that the rectangle parallelogramme D G is the meane proportionall betwene the squares A B and B C: and moreouer, that the parallelogramme D C is the meane proportionall betwene the squares A C and C B. First the parallelogramme A C is a square. For forasmuch as the line D B is equall to the line B F, and the line B E vnto the line B G, therfore the whole line D E is equall to the whole line F G. But the line D E is equall to either of these lines A H & K C, and the line F G is equall to either of these lines A K and H C (by the 34. of the first). Wherfore the parallelogrãme A C is equilater, it is also rectangle (by the 29. of the first). Wherefore (by the 46. of the first) the parallelogrãme A C is a square. Now for that as the line F B is to the line B G, so is the line D B to the line B E. But as ỹ line F B is to the line B G, so (by the 1. of the sixt) is the parallelogrãme A B, which is the square of the line D B, to the parallelogramme D G, and as the line D B is to the line B E, so is the same parallelogrãme D G to the parallelogramme B C, which is the square of the line B E. Wherefore as the square A B is to the parallelogramme D G, so is the same parallelogramme D G to the square B C. Wherefore the parallelogramme D G is the meane proportionall betwene the squares A B and B C. I say moreouer, that the parallelogramme D C is the meane proportionall betwene the squares A C and C B. For for that as the line A D is to the line D K, so is the line K G to the line G C (for they are ech equall to eche). Wherefore by composition (by the 18. of the fift) as the line A K is to the line K D, so is the line K C to the line C G. But as the line A K is to the line K D, so is the square of the line A K, which is the square A C, to the parallelogramme cõtayned vnder the lines A K and K D, which is the parallelogramme C D: and as the line K C is to the line C G, so also is the parallelogramme D C to the square of the line G C, which is the square B C. Wherefore as the square A C is to the parallelogramme D C, so is the parallelogrãme D C to the square B C. Wherefore the parallelogramme D C is the meane proportionall betwene the squares A C and B C: which was required to be demonstrated.



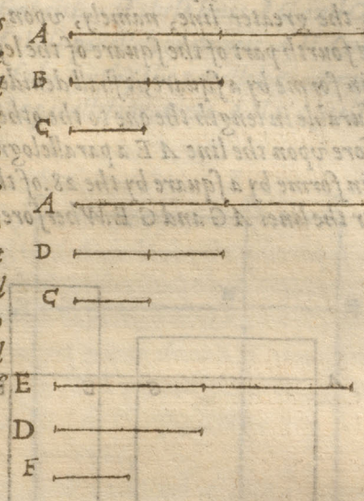
¶ An

¶ An Assumpt.

Magnitudes that are meane proportionalls betwene the selfe same or equall magnitudes, are also equall the one to the other.

An Assumpt.

Suppose that there be three magnitudes A, B, C. And as A is to B, so let B be to C. And likewise as the same magnitude A is to D, so let D be to the same magnitude C. Then I say that B and D are equall the one the other. For the proportion of A vnto C is double to that proportion which A hath to B (by the 10. definition of the fift) and likewise the selfe same proportion of A to C is (by the same definition) double to that proportion which A hath to D. But magnitudes whose equemultiplices are either equall or the selfe same, are also equall. Wherefore as A is to B, so is A to D. Wherefore (by the 9. of the fift) B and D are equall the one to the other. So shall it also be if there be other magnitudes equal to A and C, namely, E and F, betwene which let the magnitude D be the meane proportionall.



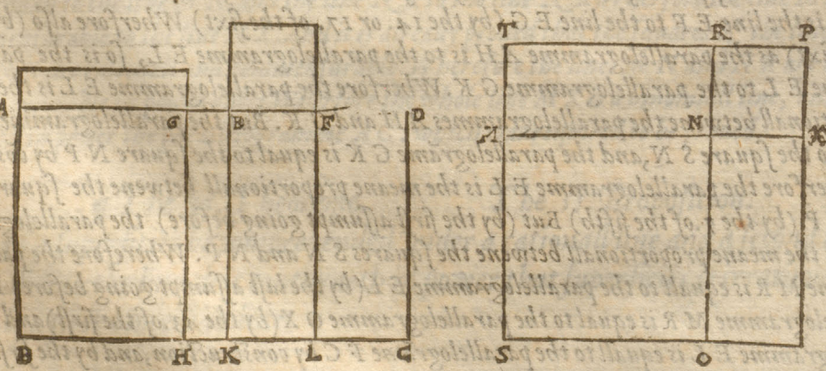
¶ *The 36. Theoreme.* *The 54. Proposition.*

If a superficies be contained vnder a rationall line & a first binomiall line: the line which containeth in power that superficies is an irrationall line, & a binomiall line.

The fourth Senary by composition.

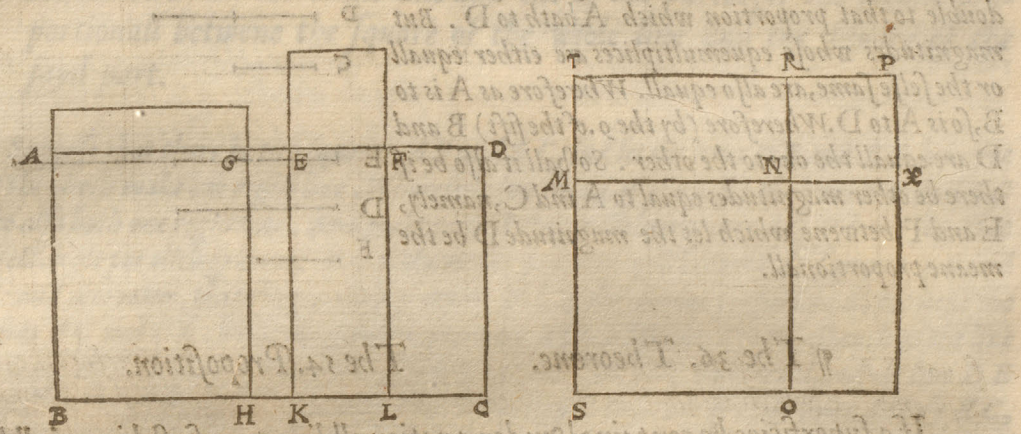
Suppose that the superficies A B C D, be contained vnder the rational line A B, and vnder a first binomial line A D. Then I say that the line which containeth in power the superficies AC, is an irrational line, and a binomial line. For forasmuch as the line A D is a first binomial line, it is in one only point deuided into

Construction.



his names (by the 42. of this tenth): let it be deuided into his names in the point E. And let A E be the greater name. Now it is manifest that the lines A E and E D are rationall cem-

mensurable in power onely, and that the line A E is in power more then the line E D, by the square of a line commensurable in length to the line A E, and moreouer that the line A E is commensurable in length to the rationall line geuen A B by the definition of a first Binomiall line set before the 48. proposition of this tenth. Deuide (by the 10. of the first) the line E D into two equall partes in the point F. And forasmuch as the line A E is in power more then the line E D by the square of a line commensurable in length vnto the line A E, therefore if vpon the greater line, namely, vpon the line A E be applied a parallelogramme equall to the fourth part of the square of the lesse line, that is, to the square of the line E F, & wanting in forme by a square, it shall deuide the greater line, namely, A E into two partes commensurable in length the one to the other (by the second part of the 17. of the tenth.) Apply therfore vpon the line A E a parallelogramme equall to the square of the line E F, and wanting in forme by a square by the 28. of the sixt, and let the same be that which is contained vnder the lines A G and G E. Wherfore the line A G is commensurable in length to the line



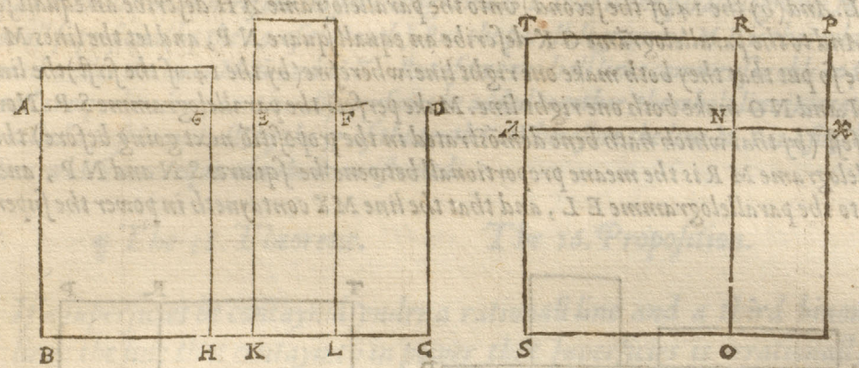
G E. Draw by the pointes G, E, and F, to either of these lines A B and D C these parallel lines G H, E K, and F L (by the 31. of the first) And (by the 14. of the second) vnto the parallelogramme A H describe an equall square S N. And vnto the parallelogramme G K describe (by the same) an equal square N P. And let these lines M N & N X be so put, that they both make one right line. Wherfore (by the 14. of the first) the lines R N and N O make also both one right line. Make perfect the parallelogramme S P. Wherfore the parallelogramme S P is a square by those thinges which were demonstrated after the determination in the first assumpt going before. And forasmuch as that which is contained vnder the lines A G and G E is equall to the square of the line E F (by construction) : therfore as the line A G is to the E F, so is the line E F to the line E G (by the 14. or 17. of the sixt) Wherfore also (by the 1. of the sixt) as the parallelogramme A H is to the parallelogramme E L, so is the parallelogramme E L to the parallelogramme G K. Wherfore the parallelogramme E L is the meane proportionall betwene the parallelogrammes A H and G K. But the parallelogramme A H is equal to the square S N, and the parallelogrãme G K is equal to the square N P by cõstruction. Wherfore the parallelogramme E L is the meane proportionall betwene the squares S N and N P (by the 7. of the fifth) But (by the first assumpt going before) the parallelogramme M R is the meane proportionall betwene the squares S N and N P. Wherefore the parallelogramme M R is equall to the parallelogramme E L (by the last assumpt going before) But the parallelogramme M R is equal to the parallelogramme O X (by the 43. of the first) and the parallelogrãme E L is equall to the parallelogrãme F C by construction, and by the first of the sixt. Wherfore the whole parallelogramme E C is equal to the two parallelogrammes M R & O X. And the parallelogrammes A H and G K are equall to the squares S N and N P by construction. Wherfore the whole parallelogramme A C is equal to the whole square S P, that

Demonstration.

is,

is, to the ſquare of the line M X. Wherefore the line M X containeth in power the parallelogramme A C. I ſay moreouer that the line MX is a binomiall line. For foraſmuch as (by the 17. of the tenth) the line A G is commenſurable in length to the line E G. Therefore (by the 15. of the tenth) the whole line A E is commenſurable in length to either of theſe lines A G and G E. But by ſuppoſition the line A E is commenſurable in length to the line A B. Wherfore (by the 12. of the tenth) either of the lines A G & G E are commenſurable in legth to the line A B. But the line A B is rationall. Wherefore either of theſe lines A G and G E is rationall. Wherfore (by the 19. of the tenth) either of theſe parallelogrammes A H and G K is rationall. Wherfore (by the firſt of the ſixt, and 10. of the tenth) the parallelogramme A H is commenſurable to the parallelogramme G K. But the parallelogramme A H is equall to the

The firſt part of this demonſtration concluded.



ſquare S N, and the parallelogramme G K is equall to the ſquare N P. Wherfore the ſquares S N and N P, which are the ſquares of the lines M N and N X are rationall and commenſurable. And foraſmuch as (by ſuppoſition) the line A E is incommenſurable in length to the line E D. But the line A E is commenſurable in length to the line A G. And the line D E is commenſurable in length to the line E F, for it is double to it by conſtruction. Wherfore (by the 13. of the tenth) the line A G is incommenſurable in length to the line E F. Wherfore the parallelogramme A H is incommenſurable to the parallelogramme E L. But the parallelogramme A H is equal to the ſquare S N, and the parallelogramme E L is equall to the parallelogramme M R. Wherfore the ſquare S N is incommenſurable to the parallelogramme M R. But as the ſquare S N is to the parallelogramme M R, ſo is the line O N to the line N R by the 1. of the ſixt. Wherfore the line O N is incommenſurable to the line N R. But the line O N is equall to the line M N, and the line N R to the line N X. Wherfore the line M N is incommenſurable to the line N X. And it is already proued that the ſquares of the lines M N and N X are rationall and commenſurable. Wherefore the lines M N and N X are rationall commenſurable in power onely. Wherfore the whole line M X is a binomiall line, and it containeth in power the parallelogramme A C: which was required to be proued.

The ſecõd part of the demonſtration concluded.

The third part cõcluded.

The totall concluſion.

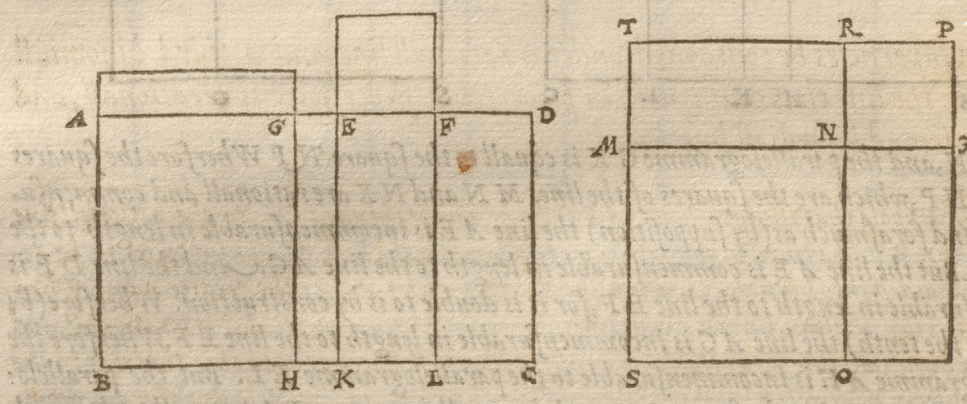
¶ The 37. Theorome. The 55. Propoſition.

If a ſuperficies be comprelænded vnder a rationall line and a ſecond binomiall line: the line that containeth in power that ſuperficies is irrationall, and is a firſt bimediall line.

SVppoſe that the ſuperſcies A B C D be contayned vnder a rationall line A B, and vnder a ſecond binomiall line A D. Then I ſay that the line that containeth in power the ſuperficies A C is a firſt bimediall line. For foraſmuch as A D is a ſecond binomiall line, it can in one onely point be deuided into his names, by the

43. of this tenth: let it therefore by ſuppoſition be deuided into his names in the poynt E, ſo that let A E be the greater name. Wherefore the lines A E and E D are rationall commenſurable in power onely, and the line A E is in power more then the line E D by the ſquare of a line commenſurable in length to A E, and the leſſe name, namely, E D is commenſurable in length to the line A B by the definition of a ſecond binomiall line, ſet before the 48. propoſitiõ of this tenth. Deuide the line E D (by the 10th of the firſt) into two equall partes in the poynt F. And (by the 28. of the ſixt) vpon the line A E apply a parallelogramme equal to the ſquare of the line E F, and wanting in figure by a ſquare. And let the ſame parallelogramme be that which is contayned vnder the lines A G and G E. Wherefore (by the ſecond part of the 17. of this tenth) the line A G is commenſurable in length to the line G E. And (by the 31. of the firſt) by the poyntes G, E, F, draw vnto the lines A B and C D theſe parallel lines, G H, E K, & F L. And (by the 14. of the ſecond) vnto the parallelogrãme A H deſcribe an equall ſquare S N. And to the parallelogrãme G K deſcribe an equall ſquare N P, and let the lines M N & N X be ſo put that they both make one right line: wherefore (by the 14. of the firſt) the lines alſo R N, and N O make both one right line. Make perfect the parallelogramme S P. Now it is manifeſt (by that which hath bene demõſtrated in the propoſitiõ next going before) that the parallelograme M R is the meane proportionall betwene the ſquares S N and N P, and is equall to the parallelogramme E L, and that the line M X contayneth in power the ſuperficies

Demonſtration.



AC. Now reſteth to proue that the line M X is a firſt bimediall line. Foraſmuch as the line A E is incommenſurable in length to the line E D, and the line E D is commenſurable in length to the line A B, therefore (by the 13. of the tenth) the line A E is incommenſurable in length to the line A B. And foraſmuch as the line A G is commenſurable in length to the line G E therefore the whole line A E is (by the 15. of the tenth) commenſurable in length to either of theſe lines A G and G E. But the line A E is rationall: wherefore either of theſe lines A G and G E is rationall. And foraſmuch as the line A E is incommenſurable in length to the line A B, but the line A E is commenſurable in lẽgth to either of theſe lines A G and G E: wherefore either of the lines A G and G E are incommenſurable in length to the line A B (by the 13. of the tenth). Wherefore the lines A B, A G, and G E are rationall commenſurable in power onely. Wherefore (by the 21. of the tenth) either of theſe parallelogrãmes A H and G K is a mediall ſuperficies. Wherefore alſo either of theſe ſquares S N and N P is a mediall ſuperficies by the corollary of the 23. of ỹ tenth. Wherfore the lines M N & N X are mediall lines by the 21. of this tẽth. And foraſmuch as the line A G is cõmenſurable in lẽgth to the line G E, therefore (by the 1. of the ſixt and 11. of the tenth) the parallelogrãme A H is cõmenſurable to the parallelogramme G K that is, the ſquare S N to the ſquare N P, that is, the ſquare of the line M N to the ſquare of the line N X. Wherefore the lines M N and N X are medialls commenſurable in power. And foraſmuch as the line A E is incommenſurable

The firſt part of this demonſtration concluded.

The third part cõcluded.

in

in length to the line E D, but the line A E is commenſurable in length to the line A G, and the line E D is commenſurable in length to the line E F, therefore (by the 13.of the tenth) the line A G is incommenſurable in length to the line E F. Wherefore (by the 1.of the ſixt and 11.of the tenth) the parallelograme A H is incõmenſurable to the parallelogramme E L, that is, the ſquare S N to the parallelogramme M R, that is, the line O N is incommenſurable to the line N R, that is the line M N to the line N X. And it is proued that the lines M N and N X are mediall lines commenſurable in power. Wherefore the lines M N and N X are mediall lines commenſurable in power onely. Now I ſay moreouer that they comprehend a rational ſuperficies. For foraſmuch as by ſuppoſition the line D E is commẽſurable in length to either of theſe lines A B and E F, therefore the line F E is commenſurable in length to the line E K which is equall to the line A B (by the 12.of the tenth). And either of theſe lines E F and E K is a rationall line. Wherefore the parallelogrãme E L, that is the parallelogrãme M R, is a rationall ſuperficies (by the 19.of the tenth). But the parallelogramme M R is that which is contayned vnder the lines M N and N X. But if two mediall lines commenſurable in power onely and comprehending a rationall ſuperficies be added together, the whole line is irrational and is called a firſt bimediall (by the 37.of the tenth). Wherefore the line M X is a firſt bimediall line: which was required to be demonſtrated.

The fourth part cõcluded.

The fift part concluded.

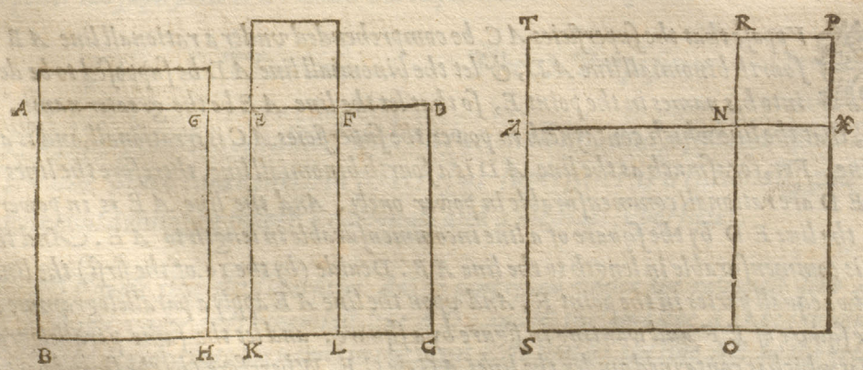
The totall concluſion.

¶ The 38. Theoreme. The 56. Propoſition.

If a ſuperficies be contayned vnder a rationall line and a third binomiall line: the line that contayneth in power that ſuperficies is irrationall, and is a ſecond bimediall line.

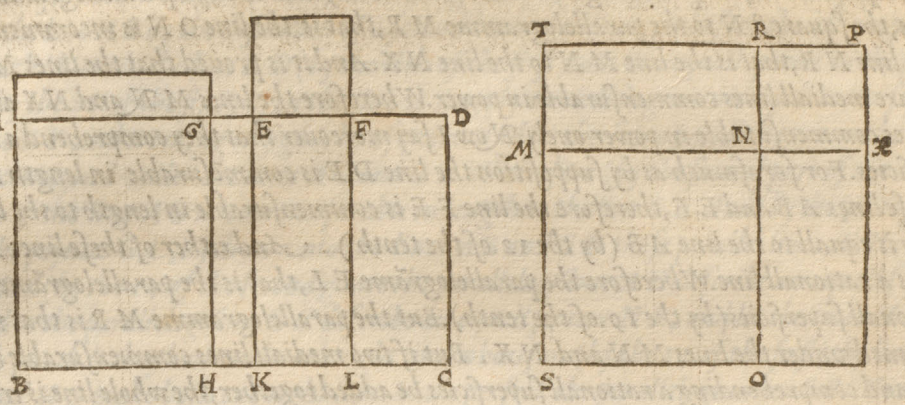
Vppoſe that the ſuperficies A B C D be comprehended vnder the rationall line A B and a third binomiall line A D, and let the line A D be ſuppoſed to be deuided into his names in the point E, of which let A E be the greater name. Then I ſay, that the line that containeth in power the ſuperficies A C is irrationall, and is a ſecond bimediall line. Let the ſame conſtruction of the figures be in this, that was in the two Propoſitions next going before. And now foraſmuch as the line A D is a third binomiall line, therefore theſe lines A E and E D are rationall commenſurable in power onely. And

Demonſtration.



the line A E is in power more thẽ the line E D by the ſquare of a line cõmenſurable in length to the line A E, and neither of the lines A E nor E D is commenſurable in length to the line A B by the definition of a third binomiall line ſet before the 48. Propoſition. As in the former Propoſitions it was demonſtrated, ſo alſo may it in this Propoſition be proued, that the

line M X containeth in power the ſuperficies A C, and that the lines M N and N X are mediall lines commenſurable in power onely. Wherefore the line M X is a bimediall line. Now reſteth to proue that it is a ſecond bimediall line. Foraſmuch as the line D E is (by ſuppoſition)



incommenſurable in length to the line A B, that is, to the line E K. But the line E D is commenſurable in length to the line E F. Wherefore (by the 13. of the tenth) the line E F is incommenſurable in length to the line E K. And the lines F E and E K are rationall. For by ſuppoſition the line E D is rationall, vnto which the line F E is commenſurable. Wherefore the lines F E and E K are rationall lines commenſurable in power onely. Wherefore (by the 21. of the tenth) the parallelogramme E L, that is, the parallelogramme M R which is contayned vnder the lines M N and N X is a mediall ſuperficies. Wherefore that which is contayned vnder the lines M N and N X is a mediall ſuperficies. Wherefore the line M X is a ſecond bimediall line (by the 38. Propoſition and definition annexed thereto): which was required to be proued.

¶ The 39. Theoreme. The 57. Propoſition.

If a ſuperficies be contained vnder a rationall line, and a fourth binomiall line: the line which contayneth in power that ſuperficies is irrationall, and is a greater line.

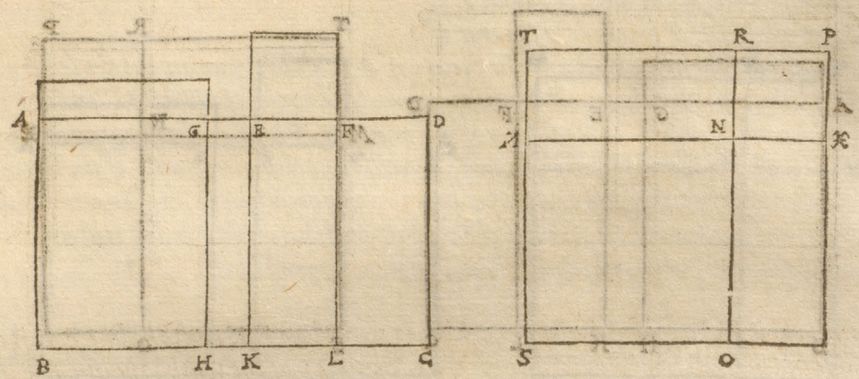
Conſtruction.

Vppoſe that the ſuperficies A C be comprehended vnder a rationall line A B and a fourth binomiall line A D, & let the binomiall line A D be ſuppoſed to be deuided into his names in the point E, ſo that let the line A E be the greater name. Then I ſay, that the line which contayneth in power the ſuperficies A C is irrationall, and is a greater line. For, foraſmuch as the line A D is a fourth binomiall line, therefore the lines A E and E D are rationall commenſurable in power onely. And the line A E is in power more then the line E D by the ſquare of a line incommenſurable in length to A E. And the line A E is commenſurable in length to the line A B. Deuide (by the 10. of the firſt) the line D E into two equall partes in the point F. And vpon the line A E apply a parallelogramme equall to the ſquare of E F and wanting in figure by a ſquare: and let the ſame parallelogramme be that which is contayned vnder the lines A G & G E. Wherefore (by the ſecond part of the 18. of the tenth) the line A G is incommenſurable in length to the line E G. Draw vnto the line A B, by the pointes G, E, F, parallell lines G H, E K, and F L, and let the reſt of the conſtruction be as it was in the three former Propoſitions. Now it is manifeſt, that the line M X contayneth in power the ſuperficies A C. Now reſteth to proue that the line M X is an

Demonſtration.

irrationall

irrationall line, and a greater line. For asmuch as the line AG is incommensurable in length to the line EG, therefore (by the 1. of the sixt, and 11. of the tenth) the parallelogramme AH is incommensurable to the parallelogramme GK, that is, the square SN to the square NP: Wherefore the lines MN and NX are incommensurable in power. And forasmuch as 2.
the line AE is commensurable in length to the rationall line AB, therefore the parallelo-



gramme AK is rationall. And it is equall to the squares of the lines MN and NX. Wherfore that which is composed of the squares of the lines MN and NX added together is rationall. And forasmuch as the line ED is incommensurable in length to the line AB, that is, to the line EK, but the line ED is commensurable in length to the line EF, therefore the 3.
line EF is incommensurable in length to the line EK. Wherefore the lines EK and EF are 5.
rationall commensurable in power onely. Wherefore (by the 21. of the tenth) the parallelogramme LE, that is, the parallelogramme MR is mediall. And the parallelogrāme MR 4.
is that which is contayned vnder the lines MN, and NX. Wherefore that which is contayned vnder the lines MN and NX is mediall. And that which is composed of the squares of the lines MN & NX is proued to be rationall, & the line MN is demonstrated to be incommensurable in power to the line NX. But if two lines incommensurable in power be added 6.
together, hauing that which is made of the squares of them added together rationall, & that which is vnder them mediall, the whole line is irrationall, and is called a greater line (by the 39. of the tenth). Wherefore the line MX is irrationall, and is a greater line, and it containeth in power the superficies AC: which was required to be demonstrated.

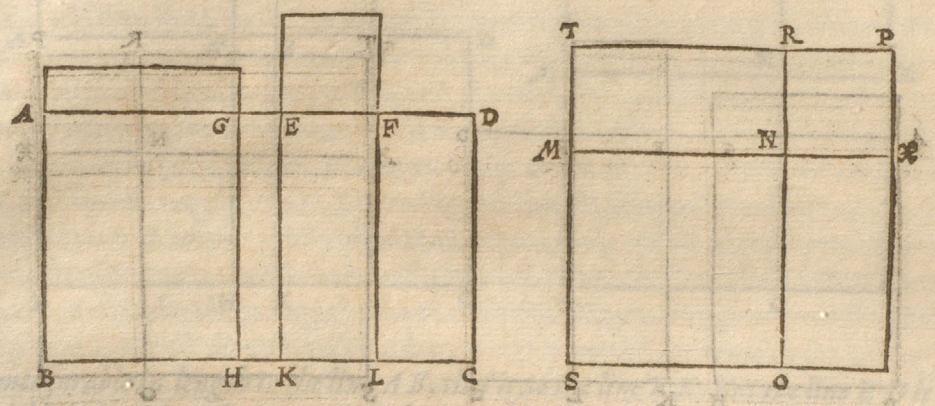
¶ The 40. Theoreme. The 58. Proposition.

If a superficies be contained vnder a rationall line and a fift binomiall line: the line which contayneth in power that superficies is irrationall, and is a line contayning in power a rationall and a mediall superficies.

SVppose that the superficies AC be contayned vnder the rationall line AB, and vnder a fift binomiall line AD: and let the same line AD be supposed to be deuided into his names in the poynt E, so that let the line AE be the greater name. Then I say, that the line which contayneth in power the superficies AC is irrationall, and is a line contayning in power a rationall and a mediall superficies. Let the selfe same constructions be in this, that were in the foure Proposition next going before. And it is manifest, that the line MX contayneth in power the superficies AC. Now resteth to proue that the line MX is a line contayning in power a rationall & a mediall superficies. For- Demonstration. 1.

asmuch

asmuch as the line A G is incommensurable in length to the line G E, therefore (by the 1. of the sixt, and 10. of the tenth) the parallelogramme A H is incommensurable to the parall-
2. logramme H E, that is, the square of the line M N to the square of the line N X. Wherefore the lines M N and N X are incommensurable in power. And forasmuch as the line A D is a fift binomiall line, and his lesse name or part is the line E D, therefore the line E D is com-



mensurable in length to the line A B. But the line A E is incommensurable in length to the line E D. Wherefore (by the 13. of the tenth) the line A B is incommensurable in length to the line A E. Wherefore the lines A B and A E are rationall commensurable in power onely. Wherefore (by the 21. of the tenth) the parallelogramme A K is mediall, that is, that
3. which is composed of the squares of the lines M N & N X added together. And forasmuch as the line D E is commensurable in length to the line A B, that is, to the line E K, but the line D E is commensurable in length to the line E F, wherefore (by the 12. of the tenth) the line E F is also commensurable in length to the line E K. And the line E K is rationall. Wherefore (by the 19. of the tenth) the parallelogramme E L, that is, the parallelogramme M R, which is contayned vnder the lines M N and N X is rationall. Wherefore the lines
4. M N and N X are incommensurable in power, hauing that which is composed of the squares of them added together, Mediall, and that which is contayned vnder them, Rationall. Wherefore (by the 40. of the tenth) the whole line M X is a line contayning in power a rationall and a mediall superficies, and it contayneth in power the superficies A C: which was required to be proued.

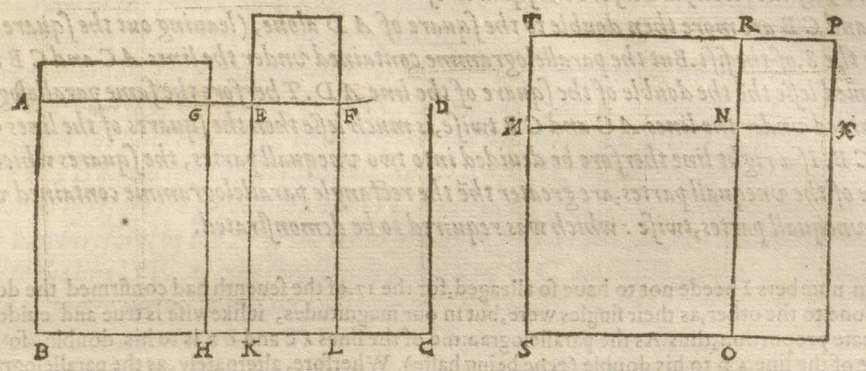
¶The 41. Theoreme. The 59. Proposition.

If a superficies be contayned vnder a rationall line, and a sixt binomiall line, the lyne which contayneth in power that superficies, is irrationall, & is called a line contayning in power two medials.

SVppose that the superficies A B C D be contained vnder the rationall line A B, and vnder a sixt binomiall line A D, and let the line A D be supposed to be deuided into his names in the point E, so that let the line A E be the greater name. Then I say that the line that containeth in power the superficies A C is irrationall, and is a line contay-
Demonstration. ning in power two medials. Let the selfe same constructiõs be in this, that were in the former propositions. Now it is manifest that the line M X containeth in power the superficies A C,
1. and that the line M N is incommensurable in power to the line N X. And forasmuch as the
2. line A E is incommensurable in length to the line A B, therfore the lines A E and A B are

rationall

rationall commensurable in power only. Wherfore (by the 21. of the tenth) the parallelogrãme A K that is, that which is composed of the squares of the lines M N and N X added together is mediall. Againe forasmuch as the line E D is incommensurable in length to the line A B, therefore also the line E F is incõmẽsurable in lẽgth to the line E K. Wherfore the lines E F and E K are rationall commensurable in power onely. Wherfore the parallelogramme E L, that is, the parallelogramme M R which is contained vnder the lines M N and N X is me- 3.

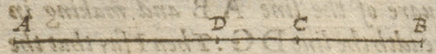


diall. And forasmuch as the line A E is incommensurable in length to the line E F, therfore the parallelogramme A K is also incommensurable to the parallelogramme E L (by the first of the sixt, and 10. of the tenth.) But the parallelogramme A K is equal to that which is composed of the squares of the lines M N and N X added together. And the parallelogramme E L is equall to that which is cõtained vnder the lines M N and N X. Wherfore that which is composed of the squares of the lines M N and N X added together, is incommensurable to that which is contained vnder the lines M N and N X: and either of them, namely, that which is composed of the squares of the lines M N and N X added together, and that which is contained vnder the lines M N and N X, is proued mediall, and the lines M N and N X are proued incommensurable in power. Wherfore (by the 41. of the tenth) the whole line M X is a line contayning in power two medials, and it containeth in power the superficies A C: which was required to be demonstrated. 4. 5.

❊ An Assumpt.

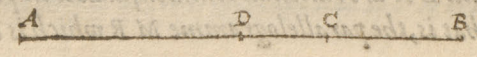
If a right line be deuided into two vnequall partes, the squares which are made of the vnequall partes, are greater then the rectangle parallelogramme cõtayned vnder the vnequall partes, twise.

Suppose that A B be a right line, and let it be deuided into two vnequall partes in the point C. And let the line A C be the greater part. Then I say, that the squares of the lines A C and C B, are greater thẽ that which is contained vnder the lines A C and C B, twise. Deuide (by the 10. of the first) the line A B into two equall partes, in the point D. Now forasmuch as the right line A B is deuided into two equall partes in the point D, and into two vnequall partes in the point C, therfore (by the 5. of the second) that which is contained vnder the lines A C and C B, together with the square of the line C D, is equall to the square of the line A D. Wherefore that which is contained vnder



the

*the lines A C and C B, (omitting the ſquare of the line C D) is leſſe then the ſquare of the A D (by the 9. common ſentence, and the ſeuenth of the fifth.) Wherefore that which is contained vnder the lines A C and C B, twiſe, is leſſe then the double of the ſquare of the line A D (that is, thē twiſe the ſquare of the line A D) by * alternate proportiō, and the 14. of the fift. But the ſquares of the lines A C and C B are double to the ſquares of the lines A D and D C (by the 9. of the ſecōd). Therfore the ſquares of A C and C B are more then double tō the ſquare of A D alone, (leauing out the ſquare of D C) by the 8. of the fift. But the parallelogramme contained vnder the lines A C and C B twiſe, is proued leſſe thē the double of the ſquare of the line A D. Therfore the ſame parallelograme contained vnder the lines A C and C B twiſe, is much leſſe then the ſquares of the lines A C and C B. If a right line therfore be deuided into two vnequall partes, the ſquares which are made of the vnequall partes, are greater thē the rectangle parallelogramme contained vnder the vnequall partes, twiſe: which was required to be demonſtrated.*



* *Looke after the Aſſumpt concluded at this marke: for plainer opening of this place.*

* In numbers I neede not to haue ſo alleaged, for the 17. of the ſeuenth had confirmed the doubles to be one to the other, as their ſingles were, but in our magnitudes, it likewiſe is true and euident by alternate proportion, thus. As the parallelogramme of the lines A C and C B is to his double, ſo is the ſquare of the line A D to his double (eche being halfe). Wherfore, alternately, as the parallelogramme is to the ſquare, ſo is the parallelograme his double to the double of the ſquare. But the parallelograme was proued leſſe then the ſquare: wherfore his double is leſſe then the ſquare his double, by the 14. of the fifth.

The vſe of this Aſſumpt is in the next propoſition, & other following.

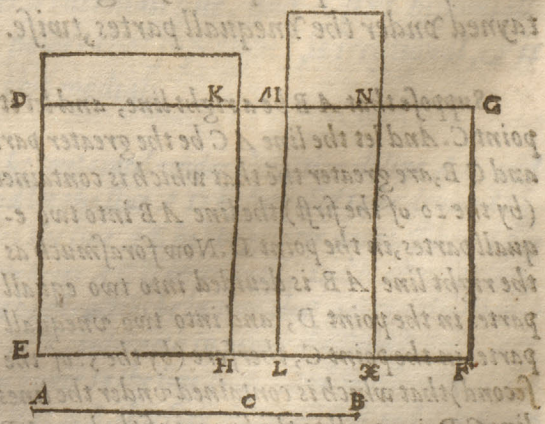
This Aſſumpt is in ſome bookes not read, for that in maner it ſemeth to be all one with that which was put after the 39. of this booke: but for the diuers maner of demonſtrating, it is neceſſary. For the feate of inuentiō is therby furthered. And though Zambert did in the demonſtration hereof, omitte that which P. Montaureus could not ſupply, but plainly doubted of the ſufficiencie of this proofe, yet M. Dee, by onely allegation of the due places of credite, whoſe pithe & force, Theon his wordes do containe, hath reſtored to the demonſtration ſufficiently, both light and authoritie, as you may perceiue, and chiefly ſuch may iudge, who can compare this demonſtration here (thus furniſhed) with the Greeke of Theon, or latine tranſlation of Zambert.

The fift Senary by compoſition.

¶ The 42. Theoreme. The 60. Propoſition.

The ſquare of a binomiall line applyed vnto a rationall line, maketh the breadth or other ſide a firſt binomiall line.

Conſtruction. *Vppoſe that the line A B be a binomiall line, and let it be ſuppoſed to be deuided into his names in the poynt C, ſo that let A C be the greater name. And take a rationall line D E. And (by the 44. of the firſt) vnto the line D E apply a rectangle parallelograme D E F G equall to the ſquare of the line A B and making in breadth the line D G. Then I ſay that the line D G is a firſt binomiall line. Vnto the line D E apply the parallelograme D H equall to the ſquare of the line A C, and vnto the line K H which is equall to the*



line

line DE apply the parallelogrãme KL equall to the ſquare of the line BC. Wherefore the reſidue namely, that which is contayned vnder the lines AC & CB twiſe is equall to the reſidue, namely, to the parallelograme MF, by the 4. of the ſecond. Deuide (by the 10. of the firſt) the line MG into two equall partes in the poynt N. And (by the 31. of the firſt) draw the line NX parallel to either of theſe lines ML and GF. Wherefore either of theſe parallelogrammes MX and NF is equall to that which is contayned vnder the lines AC and CB once, by the 15. of the fifth. And foraſmuch as the line AB is a binomiallline, and is deuided into his names in the poynt C therefore the lines AC and CB are rationall commenſurable in power onely. Wherefore the ſquares of the lines AC and CB are rationall, and therefore commenſurable the one to the other. Wherefore (by the 15. of the tenth) that which is made of the ſquares of the lines AC and CB added together is commenſurable to either of the ſquares of the lines AC or CB: wherefore that which is made of the ſquares of the lines AC and CB added together is rationall. And it is equall to the parallelograme DL by conſtruction. Wherefore the parallelograme DL is rationall. And it is applyed vnto the rationall line DE: wherefore (by the 20. of the tenth) the line DM is rationall, and commenſurable in length to the line DE. Agayne foraſmuch as the lines AC and CB are rationall commenſurable in power onely, therefore that which is contayned vnder the lines AC and CB twiſe, that is, the parallelograme MF is mediall by the 21 of the tenth, and it is applyed vnto the rationall line ML. Wherefore the line MG is rationall and incommenſurable in length to the line ML (by the 22. of this tenth) that is, to the line DE. But the line MD is proued rationall and commenſurable in length to the line DE. Wherefore (by the 13. of the tenth) the line DM is incommenſurable in length to the line MG. Wherefore the lines DM & MG are rationall commenſurable in power onely. Wherefore (by the 36. of the tenth) the whole line DG is a binomiall line. Now reſteth to proue that it is a firſt binomiall line. Foraſmuch as (by the the aſſumpt going before the 54. of the tenth) that which is contayned vnder the lines AC and CB is the meane proportionall betwene the ſquares of the lines AC and CB, therefore the parallelograme MX is the meane proportionall betwene the parallelogrammes DH and KL. Wherefore as the parallelogame DH is to the parallelograme MX, ſo is the parallelograme MX to the parallelograme KL, that is, as the line DK is to the line MN, ſo is the line MN to the line MK. Wherefore that which is contayned vnder the lines DK and KM is equall to the ſquare of the line MN. And foraſmuch as the ſquare of the line AC is commenſurable to the ſquare of the line CB, the parallelograme DH is commenſurable to the prarallelograme KL. Wherefore (by the 1. of the ſixt and 10. of the tenth) the line DK is cõmẽſurable in lẽgth to the line KM. And foraſmuch as the ſquares of the lines AC and CB are greater then that which is contayned vnder the lines AC and CB twiſe by the aſſumpt going before this propoſition, or by the aſſumpt after this 39. of the tenth, therefore the parallelograme DL is greater then the parallelograme MF. Wherefore (by the firſt of the ſixt) the line DM is greater then the line MG. And that which is contayned vnder the lines DK and KM is equall to the ſquare of the line MN, that is to the fourth part of the ſquare of the line MG. But (by the 17. of the tenth), if there be two vnequall right lines, and if vpon the greater be applyed a parallelograme equall to the fourth part of the ſquare made of the leſſe line and wanting in figure by a ſquare, if alſo the parallelograme thus applyed deuide the line whereupon it is applyed into parts commenſurable in length, then ſhall the greater line be in power more then the leſſe by the ſquare of a line commenſurable in length to the greater. Wherefore the line DM is in power more then the line MG by the ſquare of a line commenſurable in length vnto the line DM. And the lines DM and MG are proued rationall commenſurable in power onely. And the line DM is proued the greater name and commenſurable in length to the rationall line geuen DE. Wherefore by the definition of a firſt binomiall line ſet before the 48. propoſition of this booke, the line DG is a firſt binomiall

Demonſtration.

1.

2.

Concluded that DG is a binomiall line.

3.

4.

5.

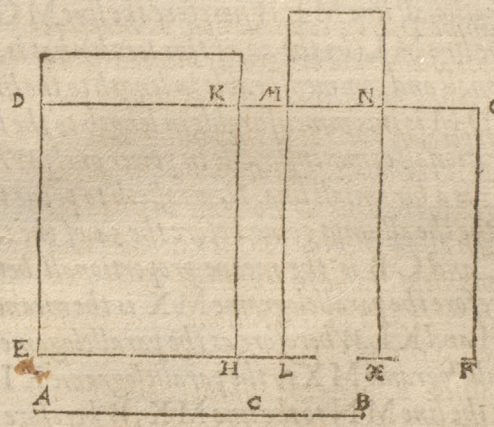
line: which was required to be proued.

This propoſition and the fiue following are the conuerſes of the ſixe former propoſitions.

¶ The 43. Theoreme. The 61. Propoſition.

The ſquare of a firſt bimediall line applied to a rationall line, maketh the breadth or other ſide a ſecond binomiall line.

Vppoſe that the line A B be a firſt bimediall line, and let it be ſuppoſed to be deuided into his partes in the point C, of which let A C be the greater part. Take alſo a rationall line D E, and (by the 44. of the firſt) apply to the line D E the parallelogrãme D F equall to the ſquare of the line A B, & making in breadth the line D G. Then I ſay, that the line D G is a ſecond binomiall line. Let the ſame conſtructions be in this, that were in the Propoſition going before. And foraſmuch as the line A B is a firſt bimediall line, and is deuided into his partes in the point C, therefore (by the 37. of the tenth) the lines A C and C B are mediall commenſurable in power onely, cõprehending a rationall ſuperficies. Wherfore alſo the ſquares of the lines A C and C B are mediall. Wherefore the parallelogramme D L is mediall (by the Corollary of the 23. of the tenth) and it is applied vpon the rationall line D E. Wherefore (by the 22. of the tenth) the line M D is rationall and incommenſurable in length to the line D E. Againe foraſmuch as that which is cõtayned vnder the lines A C and C B twiſe is rationall, therefore alſo the parallelogramme M F is rationall, and it is applied vnto the rationall line M L. Wherefore the line M G is rationall and commenſurable in length to the line M L, that is, to the line D E (by the 20. of the tenth). Wherefore the line D M is incommenſurable in length to the line M G, and they are both rationall. Wherefore the lines D M and M G are rationall commenſurable in power onely. Wherefore the whole line D G is a binomiall line. Now reſteth to proue that it is a ſecond binomiall line. Foraſmuch as the ſquares of the lines A C and C B are greater then that which is contayned vnder the lines A C and C B twiſe (by the Aßumpt before the 60. of this booke): therefore the parallelogramme D L is greater then the parallelogrrmme M F. Wherefore alſo (by the firſt of the ſixt) the line D M is greater then the line M G. And foraſmuch as the ſquare of the line A C is commenſurable to the ſquare of the line C B, therefore the parallelogramme D H is commenſurable to the parallelogramme K L. Wherefore alſo the line D K is commenſurable in length to the line K M. And that which is contayned vnder the lines D K and K M is equall to the ſquare of the line M N, that is, to the fourth part of the ſquare of the line M G. Wherefore (by the 17. of the tenth) the line D M is in power more then the line M G, by the ſquare of a line commenſurable in length vnto the line D M: and the line M G is commenſurable in length to the rationall line put, namely, to D E. Wherefore the line D G is a ſecond binomiall line: which was required to be proued.



Conſtruction.

Demonſtration.

1.

2.

3.

Concluded that D G is a binomiall line.

4.

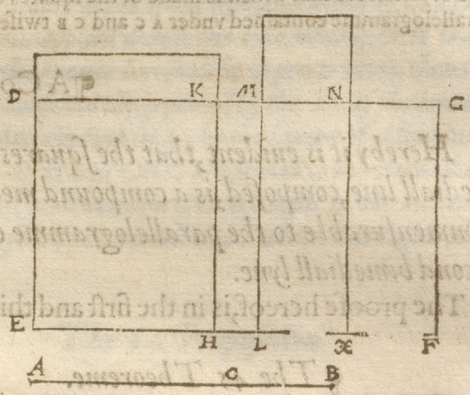
5.

¶ The

¶ The 44. Theoreme. The 62. Proposition.

The ſquare of a ſecond bimediall line, applied vnto a rationall line: maketh the breadth or other ſide therof, a third binomiall lyne.

SVppoſe that A B be a ſecond bimediall line, and let A B be ſuppoſed to be deuided into his partes in the point C, ſo that let A C be the greater part. And take a rationall line D E. And (by the 44. of the firſt) vnto the line D E apply the parallelogramme D F equall to the ſquare of the line A B, and making in breadth the line D G. Then I ſay that the line D G is a third binomiall line. Let the ſelfe ſame conſtructions be in this that were in the propoſitions next going before. And foraſmuch as the line A B is a ſecond bimediall line, and is deuided into his partes in the point C, therfore (by the 38. of the tenth) the lines A C and C B are medials commenſurable in power only, comprehẽding a mediall ſuperficies. Wherfore † that which is made of the ſquares of the lines A C and C B added together, is mediall, and it is equall to the parallelogramme D L by conſtruction. Wherefore the parallelogramme D L is mediall, and is applied vnto the rationall line D E, wherfore (by the 22. of the tenth) the line M D is rationall and incommenſurable in length to the line D E. And by the lyke reaſon alſo * the line M G is rationall and incommenſurable in length to the line M L, that is, to the line D E. Wherfore either of theſe lines D M and M G is rational, and incommenſurable in length to the line D E. And foraſmuch as the line A C is incommenſurable in length to the line C B, but as the line A C is to the line C B, ſo (by the aſſumpt going before the 22. of the tenth) is the ſquare of the line A C to that which is contained vnder the lines A C and C B. Wherfore the ſquare of the line A C is incommenſurable to that which is contayned vnder the lines A C and C B. Wherfore that †† that which is made of the ſquares of the lines A C and C B added together, is incommenſurable to that which is contained vnder the lines A C and C B twiſe, that is, the parallelogramme D L to the parallelogramme M F. Wherfore (by the firſt of the ſixt, and 10. of the tenth) the line D M is incommenſurable in length to the line M G. And they are proued both rationall, wherfore the whole line D G is a binomiall line by the definition in the 36. of the tenth. Now reſteth to proue that it is a third binomiall line. As in the former propoſitions, ſo alſo in this may we conclude that the line D M is greater then the line M G, and that the line D K is commenſurable in length to the line K M. And that that which is contained vnder the lines D K and K M is equall to the ſquare of the line M N. Wherfore the line D M is in power more then the line M G by the ſquare of a line commenſurable in length vnto the line D M, and neither of the lines D M nor M G is commenſurable in length to the rational line D E. Wherfore (by the definition of a third binomiall line) the line D G is a third binomiall line: which was required to be proued.



Conſtruction.

Demonſtration.

†

*

††

D G, concluded a binomiall line.

¶ Here follow certaine annotations by M. Dee, made vpon three places in the demonſtration, which were not very euident to yong beginners.

† (The ſquares of the lines A C and C B are medials (as is taught after the 21. of this tenth) and therfore foraſmuch as they are (by ſuppoſition) commẽſurable th'one to the other: (by the 15. of the tẽth)

the compound of them both is commenſurable to ech part. But the partes are medials, therfore (by the corollary of the 23. of the tenth) the compound ſhall be mediall.

* For that M X is equall (by conſtruction) to that which is contayned vnder the lines A C and C B, which is proued mediall: therfore (by the corollary of the 23. of this tenth) M X is mediall, and therfore (by the ſame corollary) his double M F is mediall. And it is applied to a rationall line, M L (beyng equall to D E) therfore by the 22. of the tenth, the line M G is rationall and incommenſurable in length to M L, that is, to D E.

†† Becauſe the compound of the two ſquares (of the lines A C and C B) beyng commenſurable one to the other, is alſo to eyther ſquare (by the 15.) commenſurable, therfore to the ſquare of A C: But the ſquare of A C is proued incommenſurable to that which is contained vnder A C & C B once. Wherfore (by the 13. of the tenth) the compound of the two ſquares (of the lines A C and C B) is incommenſurable, to that which is cõtained vnder the lines A C and C B once. But to that which is twiſe contained vnder the ſame lines A C and C B, the parallelogãrme once contayned, is commenſurable (for it is as 1. is to 2.) therfore that which is made of the ſquares of the lines A C and C B is incommenſurable to the parallelogramme contained vnder A C and C B twiſe, by the ſayd 13. of this tenth.

¶A Corollary.

A Corollary added by M. Dee.

Hereby it is euident, that the ſquares made of the two partes of a ſecond bimediall line, compoſed, is a compound mediall, and that the ſame compound is incommenſurable to the parallelogramme contayned vnder the two partes of the ſecond bimediall lyne.

The proofe hereof, is in the firſt and third annotations here before annexed.

¶ *The 45. Theoreme.* *The 63. Propoſition.*

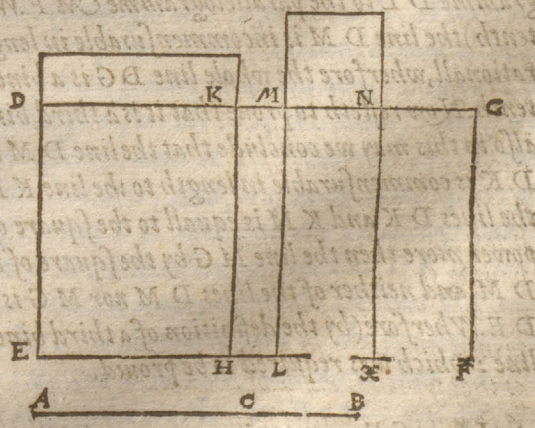
The ſquare of a greater line applied vnto a rationall line, maketh the breadth or other ſide a fourth binomiall line.

SVppoſe that the line A B be a greater line, and let it be ſuppoſed to be deuided into his partes in the point C, ſo that let A C be the greater part. And take a rationall line D E. And (by the 44. of the firſt) vnto the line D E, apply the parallelogramme D F equall to the ſquare of the line A B, and making in breadth the line D G. Then I ſay, that the line D G is a fourth binomiall line. Let the ſelfe ſame conſtruction be in this, that was in the former Propoſitions. And foraſmuch as the line A B is a greater line, & is deuided into his partes in the point C: therefore the lines A C and C B are incommenſurable in power, hauing that which is made of the ſquares of them added together rationall, and the parallelogramme which is contayned vnder them, mediall. Now foraſmuch as that which is made of the ſquares of the lines A C and C B added together is rationall, therefore the parallelogramme D L is rationall. Wherefore alſo the line M D is rationall and commenſurable in lẽgth to the line D E (by the 20. of this tenth). Againe foraſmuch as that which is cõtained vnder the lines A C and C B twiſe is mediall, that is, the parallelogrãme M F,

Conſtruction.

Demonſtration.

I.



M F, and it is applied vnto the rationall line M L, therefore (by the 22. of the tenth) the line M G is rationall and incommensurable in length to the line D E. Therefore (by the 13. of the tenth) the line D M is incommensurable in length to the line M G. Wherefore the lines D M and M G are rationall commensurable in power onely. Wherfore the whole line D G is a binomiall line. Now resteth to proue, that it is also a fourth binomiall line. Euen as in the former Propositions, so also in this may we conclude, that the line D M is greater then the line M G. And that that which is contayned vnder the lines D K and K M is equall to the square of the line M N. Now forasmuch as the square of the line A C is incommensurable to the square of the line C B, therefore the parallelogramme D H is incommensurable to the parallelogramme K L. Wherefore (by the 1. of the sixt, and 10. of the tenth) the line D K is incommensurable in length to the line K M. But if there be two vnequall right lines, and if vpon the greater be applied a parallelogramme equall to the fourth part of the square made of the lesse, and wanting in figure by a square, and if also the parallelogramme thus applied deuide the line wherupon it is applied into partes incommensurable in length, the greater line shall be in power more then the lesse, by the square of a line incōmensurable in length to the greater (by the 18. of the tenth). Wherefore the line D M is in power more then the line M G, by the square of a line incōmensurable in length to D M. And the lines D M and M G are proued to be rationall cōmensurable in power onely. And the line D M is commensurable in length to the rationall line geuen D E. Wherefore the line D G is a fourth binomiall line: which was required to be proued.

2.

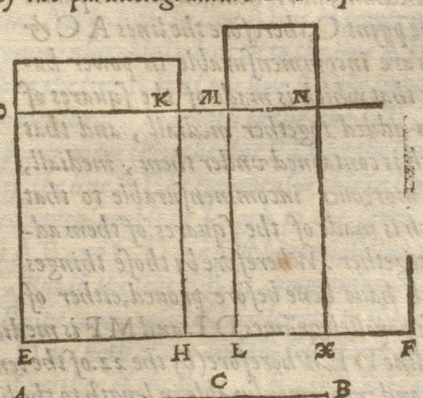
3.

4.

¶ The 46. Theoreme. The 64. Proposition.

The square of a line contayning in power a rationall and a mediall superficies applied to a rationall line, maketh the breadth or other side a fift binomiall line.

Svppose that the line A B be a line contayning in power a rationall and a mediall superficies, and let it be supposed to be deuided into his partes in the point C, so that let A C be the greater part, and take a rationall line D E. And (by the 44. of the first) vnto the line D E apply the parallelogramme D F equall to the square of the line A B, and making in breadth the line D G. Then I say, that the line D G is a fift binomiall line. Let the selfe same cōstruction be in this, that was in the former. And forasmuch as A B is a line contayning in power a rationall and a mediall superficies, and is deuided into his partes in the poynt C, therefore the lines A C & C B are incōmensurable in power, hauing that which is made of the squares of thē added together mediall, and that which is contayned vnder them rationall. Now forasmuch as that which is made of the squares of the lines A C and C B added together is mediall, therefore also the parallelogramme D L is mediall. Wherefore (by the 22. of the tenth) the line D M is rationall and incommensurable in length to the line D E. Againe forasmuch as that which is contayned vnder the lines A C and C B twise, that is, the parallelogramme M F, is rationall, therefore by the 20. the line M G is rationall & cōmensurable in length



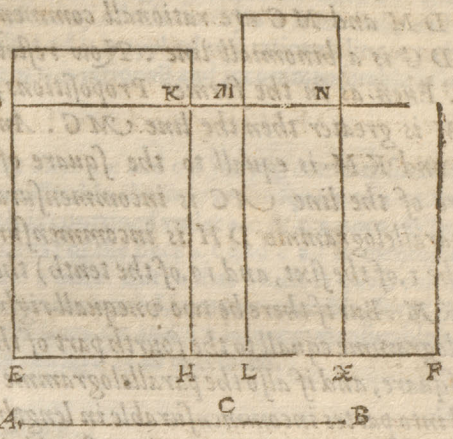
Construction.

Demonstration.

1.

2.

to the line D E. Wherefore (by the 13. of the tenth) the line D M is incommensurable in 3.
length to the line M G. Wherefore the lines D M and M G are rationall commensurable in power onely. Wherefore the whole line D G is a binomiall line. I say moreouer, that it is a fift binomiall. For, as in the former, so also in this may it be proued, that that which is contayned vnder the lines D K and K M is equall to the square of M N the halfe of the lesse: and that the line D K is incommensurable in length to the line K M. Wherefore (by the 18. of the tenth) the line D M is in power more thē the line M G by the square of a line incommensurable in length to the line D M. And the lines D M and M G are rationall commensurable in power onely, and the lesse line, namely, M G is commensurable in length to the rationall line geuen D E. Wherefore the line D G is a fift binomiall line: which was required to be demonstrated.



¶ The 47. Theoreme. The 65. Proposition.

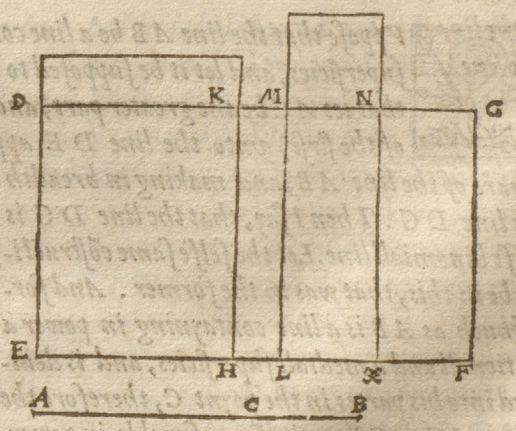
The square of a line contayning in power two medialls applyed vnto a rationall line, maketh the breadth or other side a sixt binomiall line.

Supposed that the line A B be a line contayning in power two medialls, and let it be supposed to be deuided into his partes in the poynt C. And take a rationall line D E. And (by the 44. of the first) vnto the rationall line D E apply the parallelogrāme D F equall to the square of the line A B and making in breadth the line D G. Then I say that the line D G is a sixt binomiall line.

Construction.

Let the selfe same construction be in thys that was in the former. And forasmuch as the line A B is a line contayning in power two medialls, and is deuided into his partes in the poynt C, therefore the lines A C & C B are incommensurable in power hauing that which is made of the squares of them added together mediall, and that which is contayned vnder them, mediall, and moreouer incommensurable to that which is made of the squares of them added together. Wherefore by those thinges which haue bene before proued, either of these parallelogrāmes D L and M F is mediall, and either of them is applyed vpon the rationall line D E. Wherefore (by the 22. of the tenth) either of these lines D M and M G is rationall and incommensurable in length to the line D E. And forasmuch as that which is made of the squares of the lines A C and C B, added together is incommensurable to that which is contayned vnder the lines A C and C B twise, therefore the parallelogrāme D L is incommensurable to the parallelogrāme M F. Wherefore (by the 1. of the sixt and 10. of the tenth) the line D M is incommensurable in length to the line M G. Wherefore the lines D M and M G are rationall commensurable in power onely. Wherefore the whole line D G

Demonstration.



1.

2.

is a

is a binomiall line. I say also that is a sixt binomiall line. For euen as in the other propositions it hath bene proued so also in this may it be proued, that that which is contayned vnder the lines D K *and* K M *is equall to the square of the line* M N, *and that the line* D K *is incommensurable in length to the line* K M, *and therfore (by the 18. of the tenth) the line* D M *is in power more then the line* M G *by the square of a line incommensurable in length to the line* D M. *And neither of these lines* D M *nor* M G *is commensurable in length to the rationall line geuen* D E. *Wherefore the line* D G *is a sixt binomiall line: which was required to be demonstrated.*

¶ *The 48. Theoreme.* *The 66. Proposition.*

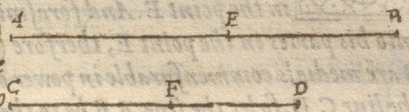
A line commensurable in length to a binomiall line, is also a binomiall line of the selfe same order.

The sixt Senary.

Construction.

Demonstration.

Suppose that the line A B *be a binomiall line, and vnto the line* A B *let the line* C D *be commensurable in length. Then I say that the line* C D *is a binomiall line and of the selfe same order that the line* A B *is. For forasmuch as* A B *is a binomiall line let it be deuided into his names in the poynt* E, *and let* A E *be the greater name. Wherefore the lines* A E *and* E B *are rationall commensurable in power onely. And as the line* A B *is to the line* C D, *(so by the 12. of the sixt) let the line* A E *be to the line* C F. *Wherefore (by the 19. of the fift) the residue, namely, the line* E B *is to the residue, namely, to the line* F D, *as the line* A B *is to the line* C D. *But (by supposition) the line* A B *is commensurable in length to the line* C D. *Wherefore (by the 10. of the tenth) the line* A E *is commensurable in length to the line* C F, *and the line* E B *to the line* F D. *And the lines* A E *and* E B *are rationall. Wherefore the lines* C F *and* F D *are also rationall. And for that as the line* A E *is to the line* C F, *so is the line* E B *to the line* F D, *therefore alternately (by the 16. of the fift) as the line* A E *is to the line* E B, *so is the line* C F *to the line* F D. *But the lines* A E *and* E B *are commensurable in power onely, wherefore the lines* C F *and* F D *are also commensurable in power onely, and they are rationall. Wherefore the whole line* C D *is a binomiall line. I say also that it is of the selfe same order of binomiall lines that the line* A B *is. For the line* A E *is in power more then the line* E B *either by the square of a line commensurable in length to the line* A E, *or by the square of a line incommensurable in length to the line* A E. *If the line* A E *be in power more then the line* E B *by the square of a line commensurable in length to the line* A E, *the line also* C F *(by the 14. of the tenth) shalbe in power more then the line* F D *by the square of a line commensurable in length to* C F. *And if the line* A E *be commensurable in length to a rationall line geuen, the line* C F *also shalbe commensurable in length to the same (by the 12. of the tenth). And so either of these lines* A B *and* C D *is a first binomiall line, that is, they are both of one and the selfe same order. But if the line* E B *be commensurable in length to the rationall line put, the line* F D *also shalbe commensurable in length to the same. And by that meanes agayne the lines* A B *and* C D *are both of one and the selfe same order, for either of them is a second binomiall line. But if neither of the lines* A E *nor* E B *be commensurable in length to the rationall line put, neither also of these lines* C F *nor* F D *shalbe commensurable in length to the same. And so either of the lines* A B *and* C D *is a third binomiall line. But if the line* A E *be in power more then the line* E B *by the square of a line incommensurable in length to the line* A E, *the line also* C F *shalbe in power more then the line* F D *by the square of a line incommensurable in length to the line* C F, *(by the 14. of*



1.

2.

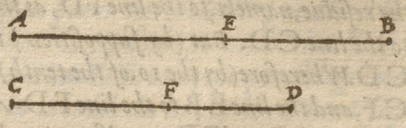
3.

the tenth). And then if the line AE be commensurable in length to the rationall line put, the line CF also shalbe commensurable in length to the same, and so either of the lines AB,
4. and CD shalbe a fourth binomiall line. And if the line EB be commensurable in length to the rationall line geuen, the line FD also shalbe commensurable in length to the same. And
5. so either of the lines AB and CD shalbe a fift binomiall line. But if neither of the lines AE nor EB be commensurable in length to the rationall line geuen, neither also of the lines CF nor FD shalbe commensurable in length to the same, and so either of the lines AB and
6. CD shalbe a sixt binomiall line. A line, therefore commensurable in length to a binomiall line, is also a binomiall line of the selfe same order: which was required to be proued.

¶ The 49. Theoreme. The 67. Proposition.

A line commensurable in length to a bimediall line, is also a bimediall lyne and of the selfe same order.

Construction. Vppose that the line AB be a bimediall line, And vnto the line AB, let the lyne CD be commensurable in length. Then I say that the line CD is a bimediall line, and of the self order that the line AB is. Deuide the line AB into his partes in the poynt E. And forasmuch as the line AB is a bimediall line, and is deuided into his partes in the poynt E, therfore (by the 37. and 38. of the tenth) the lines AE and EB are medials commensurable in power onely. And (by the 12. of the sixt) as the line AB is to the line CD, so let the line AE be to the line CF.



Demonstration. Wherfore (by the 19. of the fift) the residue, namely, the line EB is to the residue, namely, to the line FD, as the line AB is to the line CD. But the line AB is commensurable in length to the lyne CD. Wherfore the line AE is commensurable in length to the line CF, and the line EB to the line FD. Now the lines AE and EB are mediall, wherfore (by the 23. of the tenth) the lines CF and FD are also mediall. And for that as the line AE is to the line EB, so is the line CF to the line FD. But the lines AE and EB are commensurable in power onely, wherfore the lines CF and FD are also commensurable in power onely. And it is proued that they are mediall. Wherfore the lyne CD is a bimediall line. I say also that it is of the selfe same order that the line AB is. For, for that as the line
1. AE is to the line EB, so is the line CF to the line FD, but as the line CF is to FD, so is the square of the lyne CF to the parallelogramme contained vnder the lynes CF and FD, by the first of the sixt. Therfore as the line AE is to the line EB, so (by the 11. of the fift) is the square of the line CF to the parallelogramme contained vnder the lines CF and FD: but as AE is to EB, so by the 1. of the sixt, is the square of the line AE, to the parallelogramme
1. contained vnder the lines AE and EB, therfore (by the 11. of the fift) as the square of the line AE is to that which is contained vnder the lines AE and EB, so is the square of the line CF to that which is contained vnder the lines CF and FD. Wherfore alternately (by
2. the 16. of the fift) as the square of the line AE is to the square of the line CF, so is that which is contained vnder the lines AE and EB to that which is contained vnder the lines CF & FD. But the square of the line AE is commensurable to the square of the line CF, because AE and CF are commensurable in length. Wherfore that which is contained vnder the lines
3. AE and EB is commensurable to that which is contained vnder the lines CF and FD. If therfore that which is contained vnder the lines AE and EB be rationall, that is, if the line

AB

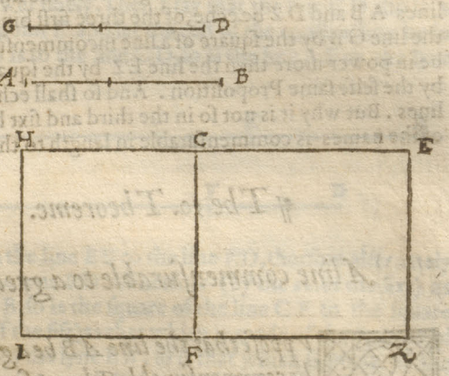
A B be a firſt bimediall line, that alſo which is contained vnder the lines C F and F D is rationall. Wherfore alſo the line C D is a firſt bimediall line. But if that which is contained vnder the lines A E and E B be mediall, that is, if the line A B be a ſecond bimediall line, that alſo which is contayned vnder the lines C F and F D is mediall: wherfore alſo the line C D is a ſecond bimediall line. Wherfore the lines A B and C D are both of one and the ſelfe ſame order: which was required to be proued. 3. 4.

¶ A Corollary added by *Fluſſates*: but firſt noted by *P. Montaureus.*

A line commenſurable in power onely to a bimediall line, is alſo a bimediall line, and of the ſelfe ſame order.

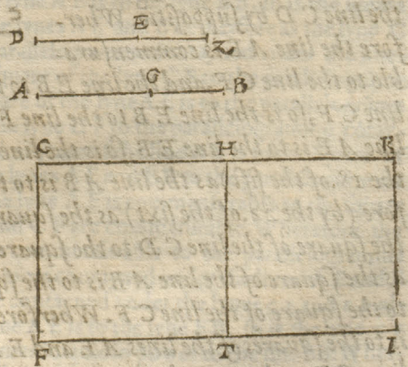
A Corollary added by Fluſſates.

Suppoſe that A B be a bimediall line, either a firſt or a ſecond, whervnto let the line G D be cõmenſurable in power onely. Take alſo a rationall line E Z, vpon which (by the 45. of the firſt) apply a rectangle parallelogramme equall to the ſquare of the line A B, which let be E Z F C, and let the rectangle parallelogramme C F I H be equall to the ſquare of the line G D. And foraſmuch as vpon the rationall line E Z is applyed a rectangle parallelogramme E F equall to the ſquare of a firſt bimediall line, therefore the other ſide therof, namely, E C, is a ſecond binomiall line, by the 61. of this booke. And foraſmuch as by ſuppoſition the ſquares of the lines A B & G D are commenſurable, therefore the parallelogrammes E F and C I (which are equall vnto them) are alſo commenſurable. And therefore by the 1. of the ſixt, the lines E C and C H are commenſurable in length. But the line E C is a ſecond binomiall line. Wherefore the line C H is alſo a ſecond binomiall line, by the 66. of this booke. And foraſmuch as the ſuperficies C I is contayned vnder a rationall line E Z or C F, and a ſecond binomiall line C H, therefore the line which contayneth it in power, namely, the line G D is a firſt bimediall line, by the 55. of this booke. And ſo is the line G D in the ſelfe ſame order of bimediall lines that the line A B is. The like demonſtration alſo will ſerue if the line A B be ſuppoſed to be a ſecond bimediall line. For ſo ſhall it make the breadth E C a third binomiall line whereunto the line C H ſhall be commenſurable in length, and therefore C H alſo ſhall be a third binomiall line, by meanes whereof the line which contayneth in power the ſuperficies C I, namely, the line G D ſhall alſo be a ſecond bimediall line. Wherefore a line commenſurable either in length, or in power onely to a bimediall line, is alſo a bimediall line of the ſelfe ſame order.



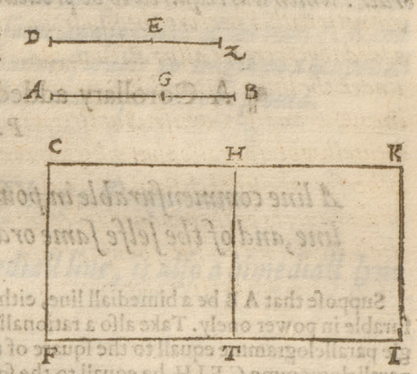
Note.

But ſo is it not of neceſſitie in binomiall lines, for if their powers onely be commenſurable, it followeth not of neceſſitie that they are binomialls of one and the ſelfe ſame order, but they are eche binomialls eyther of the three firſt kindes, or of the three laſt. As for example. Suppoſe that A B be a firſt binomiall line, whoſe greater name let be A G, and vnto A B let the line D Z be cõmẽſurable in power onely. Then I ſay, that the line D Z is not of the ſelfe ſame order that the line A B is. For if it be poſſible, let the line D Z be of the ſelfe ſame order that the line A B is. Wherefore the line D Z may in like ſort be deuided as the line A B is, by that which hath bene demonſtrated in the 66. Propoſition of this booke: let it be ſo deuided in the poynt E. Wherefore it can not be ſo deuided in any other poynt, by the 42. of this booke. And for that the line A B is to the line D Z, as the line A G is to the line D E, but the lines A G & D E, namely, the greater names, are commenſurable in length the one to the other (by the 10. of this booke) for that they are commenſurable in length to one and the ſelfe ſame rationall line, by the firſt definition of binomiall lines. Wherefore the lines A B and D Z are commenſurable in length, by the 13. of this booke. But by ſuppoſition they are commenſurable in power onely; which is impoſſible.



The

The selfe same demonstration also will serue, if we suppose the line A B to be a second binomiall line: for the lesse names G B and E Z being commensurable in length to one and the selfe same rationall line, shall also be commensurable in length the one to the other. And therefore the lines A B and D Z which are in the selfe same proportion with them, shall also be commensurable in length the one to the other: which is contrary to the supposition. Farther, if the squares of the lines A B and D Z be applyed vnto the rationall line C F, namely, the parallelogrammes C T and H L, they shall make the breadthes C H and H K first binomiall lines, of what order soeuer the lines A B & D Z (whose squares were applyed vnto the rational line) are, (by the 60. of this booke). Wherefore it is manifest, that vnder a rationall line and a first binomiall line, are confusedly contayned all the powers of binomiall lines (by the 54. of this booke). Wherfore the onely commensuration of the powers doth not of necessitie bryng forth one and the selfe same order of binomiall lines. The selfe same thyng also may be proued, if the lines A B and D Z be supposed to be a fourth or fifth binomiall line, whose powers onely are commensurable, namely, that they shall as the first bring forth binomiall lines of diuers ordes. Now forasmuch as the powers of the lines A G and G B, and D E and E Z are commensurable & proportionall, it is manifest, that if the line A G be in power more then the line G B by the square of a line commensurable in length vnto A G, the line D E also shall be in power more then the line E Z by the square of a line commensurable in length vnto the line D E (by the 16. of this booke). And so shall the two lines A B and D Z be eche, of the three first binomiall lines. But if the line A G be in power more then the line G B by the square of a line incommensurable in length vnto the line A G, the line D E shall also be in power more then the line E Z by the square of a line incomensurable in length vnto the line D E, by the selfe same Proposition. And so shall eche of the lines A B and D Z be of the three last binomiall lines. But why it is not so in the third and sixt binomiall lines, the reason is: For that in them neither of the names is commensurable in length to the rationall line put F C.

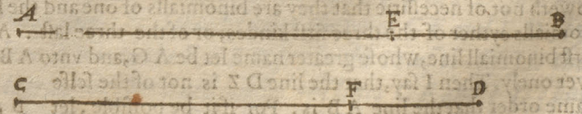


¶ The 10. Theoreme. The 68. Proposition.

A line commensurable to a greater line, is also a greater line.

Construction. Suppose that the line A B be a greater line. And vnto the line A B let the line C D be commensurable. Then I say that the line C D also is a greater line. Deuide the line A B into his partes in the point E. Wherfore (by the 39. of the tenth) the lines A E and E B are incommensurable in power, hauing that which is made of the squares of them added together rationall, and that which is contained vnder the mediall. And let the rest of the construction be in this, as it was in the former. And for that as the line A B is to the line C D, so is the line A E to the line C F, & the line E B to the line F D, but the line A B is commensurable to the line C D by supposition. Wherfore the line A E is commensurable to the line C F, and the line E B to the line F D. And for that as the line A E is to the line C F, so is the line E B to the line F D. Therfore alternately (by the 16. of the fift) as the line A E is to the line E B, so is the line C F to the line F D. Wherfore by composition also (by the 18. of the fift) as the line A B is to the line E B, so is the line C D to the line F D. Wherefore (by the 22. of the sixt) as the square of the line A B is to the square of the line E B, so is the square of the line C D to the square of the line F D. And in like sort may we proue that as the square of the line A B is to the square of the line A E, so is the square of the line C D, to the square of the line C F. Wherfore (by the 11. of the fift) as the square of the lyne A B is to the squares of the lines A E and E B, so is the square of the line C D to the squares of the lines C F and F D. Wherfore alternately (by the 16. of the fift) as the square of the line A B

Demonstration.

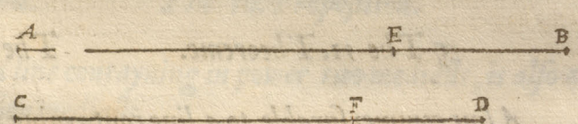


is to the ſquare of the line C D, ſo are the ſquares of the lines A E and E B to the ſquares of the lines C F and F D. But the ſquare of the line A B is commenſurable to the ſquare of the line C D (for the line A B is commenſurable to the line CD by ſuppoſitiō). Wherfore alſo the ſquares of the lines A E and E B are commenſurable to the ſquares of the lines C F and F D. But the ſquares of the lines A E and E B are incommenſurable, and being added together are rationall. Wherfore the ſquares of the lines C F and F D are incommenſurable, & being added together, are alſo rationall. And in like ſort may we proue that that which is contained vnder the lines A E and E B twiſe, is commenſurable to that which is contained vnder the lines C F and F D twiſe. But that which is contained vnder the lines A E and E B twiſe, is mediall, wherfore alſo that which is contained vnder the lines C F and F D twiſe is medial. Wherfore the lines C F and F D are incommenſurable in power, hauing that which is made of the ſquares of them added together rationall, and that which is contained vnder thē mediall. Wherfore (by the 39. of the tenth) the whole line C D is irrationall, & is called a greater line. A line therfore commenſurable to a greater line, is alſo a greater line.

An other demonſtration of *Peter Montaureus* to proue the ſame.

An other demonſtration after P. Montaureus.

Suppoſe that the line A B be a greater line, and vnto it let the line C D be commenſurable any way, that is, either both in length and in power, or els in power onely. Then I ſay that the line C D alſo is a greater line. Deuide the line A B into his partes in the point E. and let the reſt of the conſtruction be in this as it was in the former. And for that as the line A B is to the line C D, ſo is the line A E to the lyne C F, and the line E B to the line F D, therfore as the line A E is to the lyne C F, ſo is the line E B to the line F D, but the line A B is commenſurable to the line C D. Wherfore alſo the lyne A E is commenſurable to the lyne C F, and likewiſe the line E B to the line F D. And for that as the line A E is to the line C F, ſo is the line E B to the line F D, therfore alternately as the lyne A E is to the line E B, ſo is the line C F to the lyne F D. Wherfore (by the 22. of the ſixt) as the ſquare of the lyne A E is to the ſquare of the line E B, ſo is the ſquare of the line C F to the ſquare of the line F D. Wherfore by compoſition (by the 18. of the fift) as that which is made of the ſquares of the lynes A E and E B added together is to the ſquare of the lyne E B, ſo is that which is made of the ſquares of the lynes C F and F D added together to the ſquare of the lyne F D. Wherefore by contrary proportion as the ſquare of the line E B is to that which is made of the ſquares of the lines A E and E B added together, ſo is the ſquare of the lyne F D to that which is made of the ſquares of the lynes C F and F D added together. Wherfore alternately as the ſquare of the line E B is to the ſquare of the lyne F D, ſo is that which is made of the ſquares of the lynes A E and E B added together to that whiche is made of the ſquares of the lynes C F and F D added together. But the ſquare of the lyne E B is cōmenſurable to the ſquare of the lyne F D, for it hath already bene proued that the lines E B and F D are cōmēſurable. Wherfore that which is made of the ſquares of the lines A E & E B added together is commēſurable to that which is made of the ſquares of C F & F D added together. But that which is made of the ſquares of the lines A E and E B added together is rationall by ſuppoſitiō. Wherfore that which is made of the ſquares of the lynes C F and F D added together is alſo rationall. And as the lyne A E is to the lyne E B, ſo is the line C F to the lyne F D. But as the lyne A E is to the lyne E B, ſo is the ſquare of the line A E to the parallelogramme contayned vnder the lynes A E and E B: therfore as the lyne C F is to the lyne F D, ſo is the ſquare of the lyne A E to the parallelogramme contayned vnder the lines A E and E B: & as the lyne C F is to the lyne F D, ſo is the ſquare of the lyne C F to the parallelogrāme contayned vnder the lynes C F & F D. Wherfore as the ſquare of the lyne A E is to the parallelogrāme contained vnder the lines A E and E B, ſo is the ſquare of the lyne C F to the parallelogramme cōtayned vnder the lynes C F and F D. Wherfore alternately as the ſquare of the line A E is to the ſquare of the lyne C F, ſo is the parallelogramme contained vnder the lynes A E and E B to the parallelogramme contayned vnder the lines C F and F D. But the ſquare of the lyne A E is commenſurable to the ſquare of the lyne C F, for it is already proued that the lynes A E and C F are commēſurable. Wherefore the parallelogramme contayned vnder the lynes A E and E B is commenſurable to the parallelogramme contayned vnder the lynes C F and F D. But the parallelogramme contayned vnder the lines A E and E B is mediall by ſuppoſition. Wherfore the parallelogramme contayned vnder the lynes C F and F D alſo is mediall. And (as it hath already bene proued) as the line A E is to the lyne E B, ſo is the lyne C F to the lyne F D. But the lyne A E was by ſuppoſition incommenſurable in power to the line E B. Wherfore (by the 10. of the tenth) the lyne C F is incommenſurable in power to the lyne F D. Wherfore the



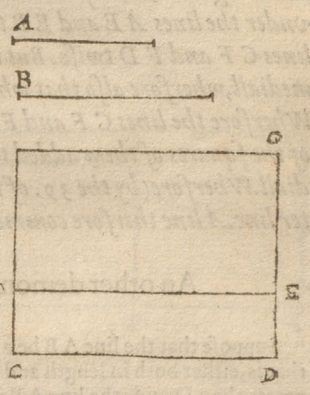
lynes

lynes C F and F D are incommensurable in power, hauing that which is made of the squares of them added together rationall, and that which is contayned vnder them mediall. Wherfore the whole lyne C D is (by the 39. of the tenth) a greater lyne. Wherfore a lyne commensurable to a greater lyne is also a greater lyne: which was required to be demonstrated.

An other more briefe demonstration of the same after Campane.

An other demonstration after Campane.

Suppose that A be a greater line, vnto which let the line B be commēsurable, either in length and power, or in power onely. And take a rational line C D. And vpon it apply the superficies C E equall to the square of the line A: and also vpō the line F E (which is equall to the rationall line C D) apply the parallelogramme F G equall to the square of the line B. And forasmuch as the squares of the two lines A and B are commensurable by supposition, the superficies C E, shalbe commensurable vnto the superficies F G: and therefore by the first of the sixt and tenth of this booke, the line D E is commensurable in length to the line G E. And forasmuch as (by the 63. of this booke) the line D E is a fourth binomiall line, therefore by the 66. of this booke the line G E is also a fourth binomiall line: wherefore by the 57. of this booke the line E which contayneth in power the superficies F G is a greater line.

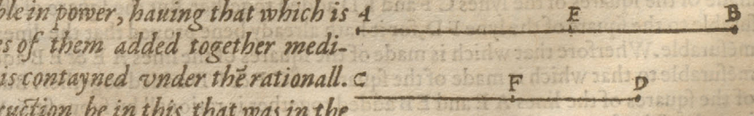


¶ The 51. Theoreme. The 69. Proposition.

A line commensurable to a line contayning in power a rationall and a mediall: is also a line contayning in power a rationall and a mediall.

Construction.

Vppose that A B *be a line contayning in power a rationall and a mediall. And vnto the line* A B *let the line* C D *be commensurable, whether in length and power, or in power onely. Thē I say that the line* C D *is a line cōtayning in power a rationall & a mediall. Duide the line* A B *into his parts in the poynt* E. *Wherfore (by the 40. of the tenth) the lines* A E *and* E B *are incommensurable in power, hauing that which is made of the squares of them added together mediall, and that which is contayned vnder thē rationall.*



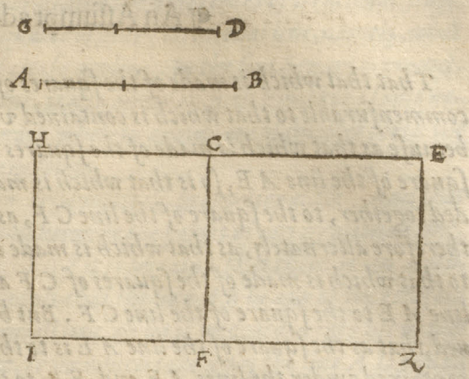
Demonstration.

Let the same construction be in this that was in the former. And in like sort we may proue that the lines C F *and* F D *are incommensurable in power, and that that which is made of the squares of the lines* A E *and* E B *is commensurable to that which is made of the squares of the lines* C F *and* F D, *and that that also which is contayned vnder the lines* A E *and* E B *is commēsurable to that which is contayned vnder the lines* C F *and* F D. *Wherefore that which is made of the squares of the lines* C F *and* F D *is mediall, and that which is contayned vnder the lines* C F *and* F D *is rationall. Wherefore the whole line* C D *is a line contayning in power a rationall and a mediall: which was required to be demonstrated.*

An other demonstration of the same after *Campane.*

Suppose

Supoſe that *A B* be a line contayning in power a rationall and a mediall: whereunto let the line *G D* be commenſurable either in length and power, or in power onely. Then I ſay that the line *G D* is a line contayning in power a rationall and a mediall. Take a rational line *E Z*, vpō which by the 45. of the firſt apply a rectangle parallelogrāme *E Z F C* equall to the ſquare of the line *A B*: and vpon the line *C F* (which is equall to the line *E Z*) applye the parallelogramme *F C H I* equall to the ſquare of the line *G D*: and let the breadths of the ſayd parallelogrammes be the lines *E C* and *C H*. And foraſmuch as the line *AB* is commenſurable to the line *G D* at the leaſt in power onely, therefore the parallelogrammes *E F* and *F H* (which are equall to their ſquares) ſhalbe commenſurable. Wherefore by the 1. of the ſixt the right lines *E C* and *C H* are cōmēſurable in lēgth. And foraſmuch as the parallelogramme *E F* (which is equall to the ſquare of the line *A B* which contayneth in power a rationall and a mediall) is applyed vpon the rationall *E Z*, making in breadth the line *E C*, therefore the line *E C* is a fifth binomiall line (by the 64. of this booke) vnto which line *E C* the line *C H* is cōmēſurable in length, wherefore by the 66. of this booke the line *C H* is alſo a fifth binomiall line. And foraſmuch as the ſuperficies *C I* is contayned vnder the rationall line *E Z* (that is *C F*) and a fifth binomall line *C H*, therefore the line which contayneth in power the ſuperficies *C I*, which by ſuppoſition is the line *G D* is a line contayning in power a rationall and a mediall by the 58. of this booke. A line therefore commenſurable to a line contayning in power a rationall and a mediall. &c.



An other demonſtratiō after Campane.

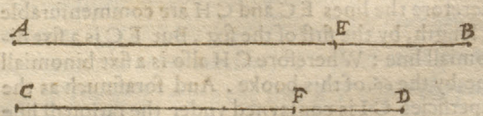
¶ The 52. Theoreme. The 70. Propoſition.

A line commenſurable to a line contayning in power two medialls, is alſo a line contayning in power two medialls.

Vppoſe that A B be a line contayning in power two medialls. And vnto the line A B let the line C D be commenſurable, whether in length & power, or in power onely. Then I ſay, that the line C D is a line contayning in power two medialls.

Conſtruction.

Foraſmuch as the line A B is a line contayning in power two medialls, let it be deuided into his partes in the point E. Wherefore (by the 41. of the tenth) the lines A E and E B are incommenſurable in power, hauing that which is made of the ſquares of them added together mediall, and that alſo which is contained vnder them mediall, and that which is made of the ſquares of the lines A E & E B is incommenſurable to that which is contained vnder the lines A E and E B. Let the ſelfe ſame conſtruction be in this, that was in the former. And in like ſort may we proue, that the lines C F & F D are incommenſurable in power, and that that which is made of the ſquares of the lines A E and E B added together, is commenſurable to that which is made of the ſquares of the lines C F and F D added together, and that that alſo which is contained vnder the lines A E and E B is commenſurable to that which is contained vnder the lines C F and F D. Wherefore that which is made of the ſquares of the lines C F and F D is mediall (by the Corollary of the 23. of the tenth): and that which is contayned vnder the lines C F and F D is mediall (by the ſame Corollary): and moreouer, that which is made of the ſquares of the lines C F & F D is incommenſurable to that which is contained vnder the lines C F and F D. Wherefore the line C D is a line containing in power two medialls: which was required to be proued.



Demonſtration.

¶ An Assumpt added by *Montaureus*.

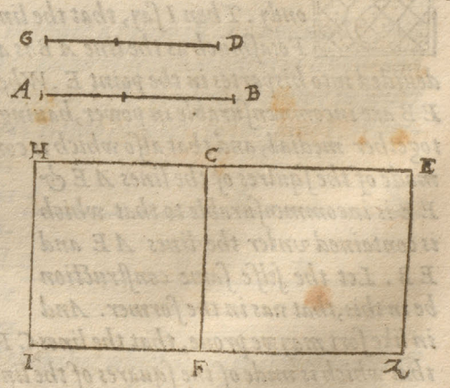
An Assumpt.

That that which is made of the squares of the lines C F and F D added together, is incommensurable to that which is contained vnder the lines C F and F D is thus proued. For, because as that which is made of the squares of the lines A E and E B added together is to the square of the line A E, so is that which is made of the squares of the lines C F and F D added together, to the square of the line C F, as it was proued in the Propositions going before: therefore alternately, as that which is made of the squares of A E and E B added together is to that which is made of the squares of C F and F D added together, so is the square of the line A E to the square of the line C F. But before, namely, in the 68. Proposition, it was proued, that as the square of the line A E is to the square of the line C F, so is the parallelogrãme contained vnder the lines A E and E B to the parallelogramme contained vnder the lines C F and F D. Wherefore as that which is made of the squares of the lines A E and E B is to that which is made of the squares of the lines C F and F D, so is the parallelogramme contained vnder the lines A E and E B to the parallelogramme contained vnder the lines C F and F D. Wherefore alternately, as that which is made of the squares of the lines A E and E B is to the parallelogramme contained vnder the lines A E and E B, so is that which is made of the squares of the lines C F and F D to the parallelogramme contained vnder the lines C F and F D. But by supposition that which is made of the squares of the lines A E and E B, is incommensurable to the parallelogramme contained vnder the lines A E & E B. Wherefore that which is made of the squares of the lines C F and F D added together, is incommensurable to the parallelogramme contained vnder the lines C F and F D: which was required to be proued.

✲ An other demonstration after *Campane*.

An other demonstration after Campane.

Suppose that A B be a line contayning in power two medialls: wherunto let the line G D be commensurable either in length, and in power, or in power onely. Then I say, that the line G D is a line cõtayning in power two medialls. Let the same construction be in this, that was in the former. And forasmuch as the parallelogramme E F is equall to the square of the line A B, and is applyed vpon a rationall line E Z, it maketh the breadth E C a sixt binomiall line, by the 65. of this booke. And forasmuch as the parallelogrammes E F & C I (which are equall vnto the squares of the lines A B and G D, which are supposed to be commensurable) are commensurable, therefore the lines E C and C H are commensurable in length, by the first of the sixt. But E C is a sixt binomiall line: Wherefore C H also is a sixt binomiall line, by the 66. of this booke. And forasmuch as the superficies C I is contayned vnder the rationall line C F and a sixt binomiall line C H, therefore the line which cõtayneth in power the superficies C I, namely, the line G D is a line contayning in power two medialls, by the 59. of this booke. Wherefore a line commensurable to a line contayning in power two medialls. &c.



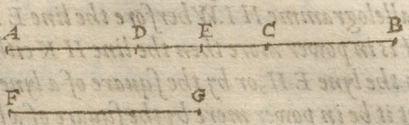
✲ An Annotation.

Hetherto hath bene spoken of sixe Senarys, *of which the first Senary contayneth the production of irrationall lines by composition: the second, the diuision of them, namely, that those lines are in one point onely deuided: the third, the finding out of binomiall lines, of the first, I say, the second, the third, the fourth, the fift, and the sixt: after that beginneth the fourth Senary, containing the difference of irrationall lines betwene them selues. For by the*

nature

nature of euery one of the binomiall lines are demonſtrated the differences of irrational lines. The ſixth entreateth of the applications of the ſquares of euery irrational line, namely, what irrationall lines are the breadthes of euery ſuperficies ſo applied. In the ſixt Senary is proued, that any line commenſurable to any irrationall line, is alſo an irrationall line of the ſame nature. And now ſhall be ſpoken of the ſeuenth Senary, wherein againe are plainly ſet forth the reſt of the differences of the ſaid lines betwene them ſelues.

Note. And there is euen in thoſe irrationall lines an arithmeticall proportionalitie. And that line which is the arithmeticall meane proportionall betwene the parties of any irrationall line, is alſo an irrationall line of the ſelfe ſame kinde. Firſt it is certaine that there is an arithmeticall proportion betwene thoſe partes. For ſuppoſe that the line A B be any of the foreſaid irrationall lines, as for example, let it be a binomiall line, & let it be deuided into his names in the point C. And let A C be the greater name, from which take away the line A D equall to the leſſe name, namely, to C B. And deuide the line C D into two equall partes in the point E. It is manifeſt that the line A E is equall to the line E B. Let the line F G be equall to either of them. It is plaine that how much the line A C differeth frō the line F G, ſo much the ſame line F G differeth from the line C B: for in eche is the difference of the line D E or E C, which is the propertie of arithmeticall proportionalitie. And it is manifeſt, that the line F G is commenſurable in length to the line A B, for it is the halfe thereof. Wherfore (by the 66. of the tenth) the line F G is a binomiall line. And after the ſelfe ſame maner may it be proued touching the reſt of the irrationall lines.



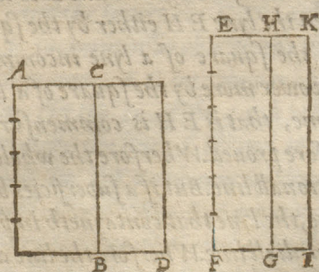
¶ The 53. Theoreme. The 71. Propoſition.

If two ſuperficieces, namely, a rationall and a mediall ſuperficies be cōpoſed together, the line which containeth in power the whole ſuperficies, is one of theſe foure irrationall lines, either a binomial line, or a firſt bimediall lyne, or a greater lyne, or a lyne containing in power a rationall and a mediall ſuperficies.

The ſeuenth Senary.

Conſtruction. Suppoſe that there be two ſuperficieces A B and C D, of which let the ſuperficies A B be rationall, & the ſuperficies C D mediall. Thē I ſay that the lyne containing in power ỹ whole ſuperficies A D, is either a binomial line, or a firſt bimediall line, or a greater line, or a line cōtayning in power a rationall & a mediall ſuperficies. For the ſuperficies A B is either greater or leſſe thē the ſuperficies C D (for they cā by no meanes be equall, whē as the one is rationall, and the other mediall). Firſt let it be greater, and take a rationall line E F. And (by the 44. of the firſt) vnto the line E F apply the parallelogramme E G equal to the ſuperficies A B, & making in breadth the line E H, and to the ſame line E F, that is, to the line H G apply the parallelogramme H I equall to the ſuperficies D C, and making in breadth the line H K. And foraſmuch as the ſuperficies A B is rationall, and is equall to the parallelogramme E G, therfore the parallelogramme E G is alſo rational, and it is applied vnto the rational line E F, making in breadth the line E H. Wherfore the line E H is rationall and commenſurable in length to the line E F (by the 20. of the tenth). Againe for aſmuch as the ſuperficies C D is mediall, and is e-

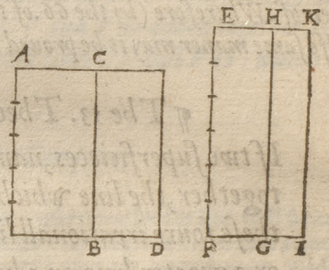
Two caſes in this propoſition, of which eche alſo hath his two caſes. Firſt caſe.



Demonſtration.

qual to the parallelogrãme H I, therfore the parallelogrãme H I is alſo mediall, and is applied vnto the rationall line E F, that is, vnto the lyne H G making in breadth the line H K. Wherfore the lyne H K is rationall and incommenſurable in length to the line E F (by the 22 of the tenth.) And foraſmuch as the ſuperficies C D is mediall, and the ſuperficies A B is rationall, therfore the ſuperficies A B is incommenſurable to the ſuperficies C D. Wherfore alſo the parallelogramme E G is incommenſurable to the parallelogramme H I. But as the parallelogramme G E is to the parallelogramme H I, ſo (by the 1. of the ſixt) is the line E H to the lyne H K. Wherfore (by the 10. of the tenth) the line E H is incommenſurable in length to the line H K, and they are both rationall. Wherfore the lines E H and H K are rationall commenſurable in power onely. Wherfore the whole line E K is a binomiall line, and is deuided into his names in the poynt H. And foraſmuch as the ſuperficies A B is greater then the ſuperficies C D, but the ſuperficies A B is equall to the parallelogramme E G, and the ſuperficies C D to the parallelogramme H I. Wherfore the parallelogramme E G is greater then the parallelogramme H I. Wherfore the line E H is greater then the line H K. Wherfore the line E H is in power more then the line H K either by the ſquare of a line commenſurable in length to the lyne E H, or by the ſquare of a lyne incommenſurable in length to the lyne E H. Firſt let it be in power more by the ſquare of a lyne cõmẽſurable in lẽgth vnto the line E H. Now the greater name, namely, E H is commenſurable in length to the rational line geuen E F, as it hath already bene proued. Wherfore the whole line E K is a firſt binomiall lyne. And the lyne E F is a rationall lyne. But if a ſuperficies be contayned vnder a rationall line, and a firſt binomiall lyne, the lyne that contayneth in power the ſame ſuperficies, is (by the 54. of the tenth) a binomiall line. Wherefore the lyne containing in power the parallelogramme E I is a binomiall line. Wherefore alſo the line containing in power the ſuperficies A D is a binomiall line.

First part of the firſt caſe.



Second part of the firſt caſe.

But now let the lyne E H be in power more then the line H K by the ſquare of a line incommenſurable in length to the line E H: now the greater name that is, E H is commenſurable in length to the rationall line geuen E F. Wherfore the line E K is a fourth binomiall line. And the line E F is rationall. But if a ſuperficies be contained vnder a rationall line and a fourth binomiall line, the line that containeth in power the ſame ſuperficies is (by the 57. of the tenth) irrational, and is a greater line. Wherfore the line which containeth in power the parallelogramme E I is a greater line. Wherefore alſo the line containing in power the ſuperficies A D is a greater lyne.

The ſecond caſe.

But now ſuppoſe that the ſuperficies A B which is rationall, be leſſe then the ſuperficies C D which is mediall. Wherfore alſo the parallelogramme E G is leſſe then the parallelogrãme H I. Wherfore alſo the line E H is leſſe then the line H K. Now the line H K is in power more then the lyne E H either by the ſquare of a line cõmenſurable in length to the line H K, or by the ſquare of a lyne incommenſurable in length vnto the lyne H K. Firſt let it be in power more by the ſquare of a line commenſurable in length vnto H K: now the leſſe name, that is E H is commenſurable in length to the rationall line geuen E F, as it was before proued. Wherfore the whole line E K is a ſecond binomiall line. And the line E F is a rationall line. But if a ſuperficies be contained vnder a rationall line and a ſecond binomiall lyne, the lyne that contayneth in power the ſame ſuperficies, is (by the 55. of the tenth) a firſt bimediall line. Wherfore the line which contayneth in power the parallelograme E I is a firſt bimediall line. Wherfore alſo the line that containeth in power the ſuperficies A D is a firſt bimediall lyne.

First part of the ſecõd caſe.

Second part of the ſecõd caſe.

But now let the line H K be in power more then the line E H, by the ſquare of a line incommen-

cõmenſurable in length to the lyne H K, now the leſſe name, that is, E H is cõmenſurable in length to the rationall lyne geuen E F. Wherfore the whole line E K is a fift binomiall lyne. And the lyne E F is rationall. But if a ſuperficies be contayned vnder a rationall lyne, and a fift binomiall lyne, the line that contayneth in power the ſame ſuperficies, is (by the 58. of the tenth) a line containing in power a rationall and a mediall. Wherefore the lyne that contayneth in power the parallelogramme E I is a line contayning in power a rationall and a mediall. Wherfore alſo the lyne that containeth in power the ſuperficies A D is a lyne contayning in power a rationall and a mediall. If therfore a rationall and a mediall ſuperficies be added together, the lyne which contayneth in power the whole ſuperficies, is one of theſe foure irrationall lines, namely, either a binomiall line, or a firſt bimediall line, or a greater lyne, or a lyne contayning in power a rationall and a mediall: which was required to be demonſtrated.

¶ The 54. Theoreme. The 72. Propoſition.

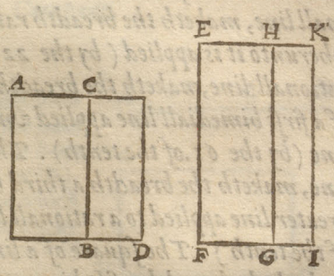
If two mediall ſuperficieces incommenſurable the one to the other be compoſed together: the line contayning in power the whole ſuperficies is one of the two irrationall lines remayning, namely, either a ſecond bimediall line, or a line contayning in power two medialls.

LEt theſe two mediall ſuperficieces A B and C D being incommenſurable the one to the other be added together. Then I ſay, that the line which contayneth in power the ſuperficies A D is either a ſecond bimediall line, or a line contayning in power two medialls. For the ſuperficies A B is either greater or leſſe then the ſuperficies C D (for they can by no meanes be equall, when as they are incommenſurable). Firſt let the ſuperficies A B be greater then the ſuperficies C D. And take a rationall line E F. And (by the 44. of the firſt) vnto the line E F apply the parallelogramme E G equall to the ſuperficies A B, and making in breadth the line E H: and vnto the ſame line E F, that is, to the line H G, apply the parallelogramme H I equall to the ſuperficies C D, & making in breadth the line H K. And foraſmuch as either of theſe ſuperficieces A B & C D is mediall, therefore alſo either of theſe parallelogrammes E G and H I is mediall. And they are eche applied to the rationall line E F, making in breadth the lines E H and H K. Wherefore (by the 22. of the tenth) either of theſe lines E H and H K is rationall and incommenſurable in length to the line E F. And foraſmuch as the ſuperficies A B is incommenſurable to the ſuperficies C D, and the ſuperficies A B is equall to the parallelogramme E G, and the ſuperficies C D to the parallelogramme H I: therefore the parallelogramme E G is incommenſurable to the parallelogramme H I. But (by the 1. of the ſixt) as the parallelogramme E G is to the parallelogramme H I, ſo is the line E H to the line H K. Wherefore (by the 10. of the tenth) the line E H is incommenſurable in length to the line H K. Wherefore the lines E H and H K are rationall commenſurable in power onely. Wherfore the whole line E K is a binomiall line. And as in the former Propoſition ſo alſo in this may it be proued, that the line E H is greater then the line H K. Wherefore the line E H is in power more then the line H K, either by the ſquare of a line commenſurable in length to the line E H, or by the ſquare of a line incommenſurable in length to the line E H. Firſt let it be greater by the ſquare of a line commenſurable in length vnto the line E H. Now neither of theſe lines E H and H K is commenſurable in length to the rationall line geuen E F. Wherefore the whole line E K is a third bi-

Construction.

Two caſes in this Propoſition.

The firſt caſe.



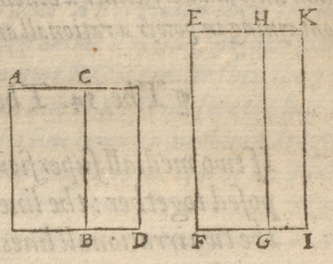
The firſt part of the firſt caſe.

nomiall line. And the line E F is a rationall line. But if a ſuperficies be contayned vnder a rationall line & a third binomiall line, the line that cõtaineth in power the ſame ſuperficies, is (by the 56. of the tenth) a ſecond bimediall line. Wherefore the line that containeth in power the ſuperficies E I, that is, the ſuperficies A D, is a ſecond bimediall line.

The ſecond part of the firſt caſe.

But now ſuppoſe that the line E H be in power more then the line H K by the ſquare of a line incommenſurable in length to the line E H. And foraſmuch as either of theſe lines E H and H K is incommenſurable in length to the rationall line geuen E F, therfore the line E K is a ſixt binomiall line. But if a ſuperficies be contained vnder a rationall line and a ſixt binomiall line, the line that containeth in power the ſame ſuperficies, is (by the 59. of the tenth) a line containing in power two medialls. Wherefore the line that containeth in power the ſuperficies A D, is a line contayning in power two medialls. And after the ſelfe ſame maner, if the ſuperficies A B be leſſe then the ſuperficies C D, may we proue, that the line that contayneth in power the ſuperficies A D, is either a ſecond bimediall line, or a line containing in power two medialls. If therefore two mediall ſuperficieces incommenſurable the one to the other be added together, the line contayning in power the whole ſuperficies is one of the two irrationall lines remayning, namely, either a ſecond bimediall line, or a line cõtaining in power two medialls: which was required to be proued.

The ſecond caſe.



¶ A Corollary following of the former Propoſitions.

A Corollary.

A binomiall line and the other irrationall lines following it, are neither mediall lines, nor one and the ſame betwene them ſelues. For the ſquare of a mediall line applied to a rationall line, maketh the breadth rationall and incommenſurale in length to the rationall line, wherunto it is applied (by the 22. of the tenth). The ſquare of a binomiall line applyed to a rationall line, maketh the breadth a firſt binomiall line (by the 60. of the tenth). The ſquare of a firſt bimediall line applied vnto a rationall line, maketh the breadth a ſecond binomiall line (by the 61. of the tenth). The ſquare of a ſecond bimediall line applied vnto a rationall line, maketh the breadth a third binomiall line (by the 62. of the tenth). The ſquare of a greater line applied to a rationall line, maketh the breadth a fourth binomiall line (by the 63. of the tenth). The ſquare of a line containing in power a rationall & a mediall ſuperficies, maketh the breadth a fift binomiall line (by the 64. of the tenth). And the ſquare of a line containing in power two medialls, applied vnto a rationall line, maketh the breadth a ſixt binomiall line (by the 65. of the tenth). Seing therefore that theſe foreſaid breadthes differ both from the firſt breadth, for that it is rationall, and differ alſo the one from the other, for that they are binomials of diuers orders: it is manifeſt that thoſe irrationall lines differ alſo the one from the other.

Here beginneth the Senaries by ſubſtraction.

¶ The 55. Theoreme. The 73. Propoſition.

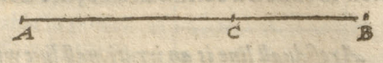
If from a rationall line be taken away a rationall line commenſurable in power onely to the whole line: the reſidue is an irrationall line, and is called a reſiduall line.

Suppoſe

The first Senary by substraction.

Demonstration.

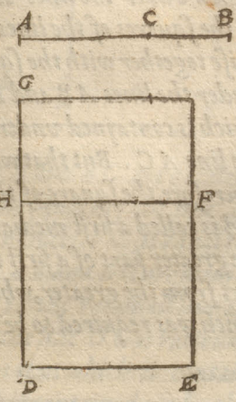
Vppose that A B be a rationall line, and from A B take away a rationall line B C commensurable in power onely to the whole line A B. Then I say that the line remayning, namely A C is irrationall and is called a residuall line. For forasmuch as the line A B is incommensurable in length vnto the line B C, and (by the assumpt going before the 22. of the tenth) as the line A B is to the line B C, so is the square of the line A B to that which is contayned vnder the lines A B and B C: wherefore (by the 10. of the tenth) the square of the line A B is incommensurable to that which is contayned vnder the lines A B and B C. But vnto the square of the line A B are commensurable the squares of the lines A B and B C (by the 15. of the tenth). Wherefore the squares of the lines A B and B C are incommensurable to that which is contayned vnder the lines A B and B C. But vnto that which is contayned vnder the lines A B and B C is commensurable that which is contayned vnder the lines A B and B C twise. Wherefore the squares of the lines A B and B C are incõmẽsurable to that which is contayned vnder the lines A B and B C twise. But the squares of the lines A B and B C are equall to that which is contayned vnder the lines A B and B C twise, and to the square of the line A C (by the 7. of the second). Wherefore that which is contayned vnder the lines A B and B C twise together with the square of the line A C is incommensurable to that which is cõtayned vnder the lines A B and B C twise. Wherefore (by the 2 part of the 16. of the tẽth) that which is cõtayned vnder the lines A B and B C twise, is incõmẽsurable to the square of the line A C. Wherefore (by the first part of the same) that which is contayned vnder the lines A B and B C twise together with the square of the line A C, that is, the squares of the lines A B and B C are incommensurable to the square of the line A C. But the squares of the lines A B and B C are rationall, for the lines A B and B C are put to be rationall: wherfore the line A C is irrationall and is called a residuall line: which was required to be proued.



❊ An other demonstration after *Campane*.

An other demonstration after Campane.

Campane demonstrateth this Proposition by a figure more briefly after this maner. Let the superficies E G be equall to the squares of the lines A B and B C added together: which shall be rationall (for that the lines A B and B C are supposed to be rationall cõmensurable in power onely). Frõ which superficies take away the superficies D F equall to that which is contayned vnder the lines A B & B C twise, which shall be mediall (by the 21. of this booke). Now by the 7. of the second, the superficies F G is equall to the square of the line A C. And forasmuch as the superficies E G is incommensurable to the superficies D F (for that the one is rationall and the other mediall): therefore (by the 16. of this booke) the same superficies E G is incommensurable to the superficies F G. Wherfore the superficies F G is irrationall. And therefore the line A C which contayneth it in power is irrationall: which was required to be proued.



An annotation of *P. Montaureus*.

This Theoreme teacheth nothing els but that that portion of the greater name of a binomiall line which remayneth after the taking away of the lesse name from the greater name is irrationall, which is called a residuall line, that is to say, if from the greater name of a binomiall line, which greater name is a rationall line cõmensurable in power onely to the lesse name, be taken away the lesse name, which selfe lesse name is also commensurable in power onely to the greater name (which greater name this Theoreme calleth the whole line) the rest of the line which remaineth is irrational, which he calleth

a residuall line. Wherfore all the lines which are intreated in this Theoreme, and in the fiue other which follow are the portions remayning of the greater partes of the whole lines which were intreated of in the 36.37.38.39.40.41. propositiõs, after the taking away the lesse part from the greater.

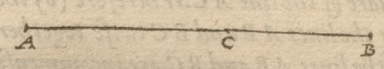
In this proposition is set forth the nature of the eight kinde of irrationall lines which is called a residuall line, the definition whereof by this proposition is thus.

Diffinition of the eight irrationall line.

A residuall line is an irrationall line which remayneth, when from a rationall line geuen, is taken away a rationall line commensurable to the whole line in power onely.

¶ The 56. Theoreme. The 74. Proposition.

If from a mediall line be taken away a mediall line commensurable in power onely to the whole line, and comprehending together with the whole line a rationall superficies: the residue is an irrationall line, and is called a first mediall residuall line.



Suppose that A B be a mediall line. And from the line A B take away a mediall line B C commensurable in power onely to the whole line A B and comprehending together with the line A B a rationall superficies, that is, let that which is comprehended vnder the lines A B and B C be rationall. Then I say that the line remayning, namely, A C is irrationall and is called a first mediall residuall line. For forasmuch as the lines A B and B C are mediall, therefore also the squares of the lines A B and B C are mediall. But that which is contayned vnder the lines A B and B C twise is rationall. Wherefore that which is composed of the squares of the lines A B and B C, that is, that which is contayned vnder A B and B C twise together with the square of the line A C is incommensurable to that which is contained vnder the lines A B and B C twise. Wherefore (by the second part the 16. of the tenth) that which is contayned vnder the lines A B and B C twise is incommensurable to the square of the line A C. But that which is contayned vnder the lines A B and B C twise is rationall, wherefore the square of the line A C is irrationall. Wherefore also the line A C is irrationall: and is called a first mediall residuall line. This first mediall residuall line is also that part of the greater part of a first bimediall line, which remayneth after the taking away of the lesse part from the greater, wherof it hath also his name, and is called a first mediall residuall line: which was required to be proued.

Demonstration.

Out of this proposition is taken the definition of the ninth kinde of irrationall lines, which is called a first residuall mediall line the difinition whereof is thus.

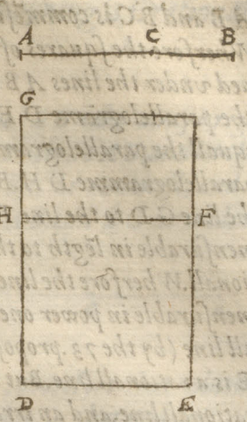
Diffinition of the ninth irrationall line.

A first residuall mediall line is an irrationall line which remayneth, when from a mediall line is taken away a mediall line commensurable to the whole in power onely, and the part taken away and the whole line contayne a mediall superficies.

An other demonstration after Campane.

Let the line D E be rationall, vpon which apply the superficies D F equall to that which is contained

ned vnder the lynes A B and B C twiſe, and let the ſuperficies G E be equal to that which is compoſed of the ſquares of the lynes A B and B C: wherfore by the 7. of the ſecond, the ſuperficies F G is equal to the ſquare of the lyne A C. And foraſmuch as (by ſuppoſition) the ſuperficies E G is mediall, therfore (by the 22. of the tenth) the lyne D G is rationall cõmenſurable in power onely to the rational lyne DE. And foraſmuch as by ſuppoſition the ſuperficies EH is rational, therfore by the 20. of the tenth, the line D H is rational commenſurable in length vnto the rationall line DE. Wherfore the lynes D G and D H are rationall commenſurable in power only (by the aſſumpt put before the 13. of this boke). Wherfore by the 73 of this boke, the lyne G H is a reſiduall lyne, and is therefore irrationall. Wherfore (by the corollary of the 21. of this boke) the ſuperficies F G is irrational. And therfore the line A C which cõtayneth it in power is irrationall, and is called a firſt medial reſiduall lyne.



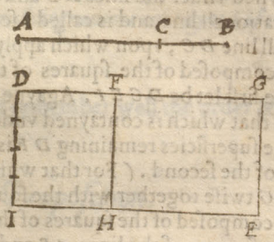
An other demonſtratiõ after Campane.

¶ The 57. Theoreme. The 75. Propoſition.

If from a mediall lyne be taken away a mediall lyne commenſurable in power only to the whole lyne, and comprehending together with the whole lyne a mediall ſuperficies, the reſidue is an irrationall lyne, and is called a ſecond mediall reſiduall lyne.

Vppoſe that A B be a mediall line, and from A B take away a mediall line C B commenſurable in power onely to the whole line A B, and comprehending together with the whole line A B a mediall ſuperficies, namely, the parallelogramme contained vnder the lines A B and B C. Then I ſay that the reſidue, namely, the line A C is irrationall, and is called a ſecond mediall reſiduall line. Take a rationall line D I, and (by the 44. of the firſt) vnto the line D I apply the parallelogramme D E equall to the ſquares of the lines A B & B C, and making in bredth the line D G. And vnto the ſame line D I apply the parallelogramme D H equall to that which is cõtained vnder the lines A B & B C twiſe, and makyng in breadth the line D F. Now the parallelogramme D H is leſſe then the parallelogramme D E, for that alſo the ſquare of the lines A B and B C are greater then that which is contained vnder the lines A B and B C twiſe, by the ſquare of the line A C by the 7. of the ſecond. Wherfore the parallelogramme remayning, namely, F E, is equal to the ſquare of the line A C. And foraſmuch as the ſquares of the lines A B and B C are mediall, therfore alſo the parallelogramme D E is mediall, and is applied to the rationall line D I, making in bredth the line D G. Wherfore (by the 22. of the tenth) the line D G is rational and incommenſurable in length to the line D I. Againe foraſmuch as that which is contained vnder the lines A B and B C is mediall, therfore alſo that which is contained vnder the lines A B and B C twiſe is mediall, but that which is contained vnder the lines A B and B C twiſe is equall to the parallelogramme D H. Wherfore the parallelogramme D H is mediall and is applied to the rationall line D I making in breadth the line D F. Wherfore the line D F is rationall and incommenſurable in length to the line D I. And foraſmuch as the lines A B and B C are cõmenſurable in power onely, therfore the line A B is incommẽſurable in lẽgth to the line B C. Wherfore (by the aſſumpt going before the 22. of the tenth, and by the 10. of the tenth) the ſquare of the line A B is incommenſurable to that which is contained vnder the lines A B and B C. But vnto the ſquare of the line A B are commenſurable the ſquares of A B and B C (by the 15. of the tenth). And vnto that which is contained vnder the lines

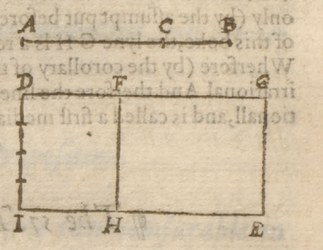
Conſtruction.



Demonſtration.

A B

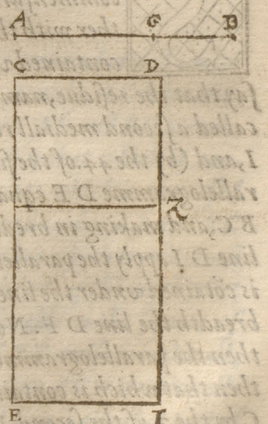
A B and B C is commẽsurable to that which is contained vnder the lines A B and B C twise. Wherfore the ſquares of the lines A B and B C are incommẽſurable to that which is contained vnder the lines A B and B C twiſe. But vnto the ſquares of the lines A B and B C is equal the parallelogrãme D E, and to that which is cõtained vnder the lines A B and B C twiſe, is equall the parallelogramme D H. Wherefore the parallelogrãme D E is incommẽſurable to the parallelogramme D H. But as the parallelogramme D E is to the parallelogramme D H, ſo is the line G D to the line D F. Wherfore the line G D is incommenſurable in lẽgth to the line D F. And either of thẽ is rationall. Wherfore the lines G D and D F are rationall commenſurable in power onely. Wherfore the line F G is a reſiduall line (by the 73. propoſition of the tenth) And the line D E is a rationall line. But a ſuperficies comprehended vnder a rationall line, and an irrationall line is irrationall (by the 21 of the tenth) and the line which containeth in power the ſame ſuperficies is irrationall (by the aſſumpt going before the ſame). Wherfore the parallelogrãme F E is irrationall. But the line A C containeth in power the parallelogramme F E. Wherfore the line A C is an irrationall line and is called a ſecond mediall reſiduall line. And this ſecond mediall reſiduall line is that part of the greater part of a bimediall line which remayneth after the taking away of the leſſe part from the greater: which was required to be proued.



An other demonſtrtion more briefe after *Campane.*

An other demonſtration after Campane.

Suppoſe that *A B* be a mediall line, from which take away the mediall line *G B* commenſurable vnto the whole line *A B* in power onely and contayning with it a mediall ſuperficies, namely, that which is contayned vnder the lines *A B* and *B G*. Then I ſay that the reſidue *A G* is an irrationall line, and is called a ſecond mediall reſiduall line. Take a rationall line *D C*, vpon which apply a parallelogramme equall to that which is compoſed of the ſquares of the lines *A B* and *B G*, which by the 45. of the firſt let be *D C E I*. Agayne let the parallelogramme *Z F E Y* be equall to that which is contayned vnder the lines *A B* and *B G* twiſe. Wherfore the ſuperficies remaining *D F* is equal to the ſquare of the line *A G* by the 7. of the ſecond. (For that which is contayned vnder the lines *A B* and *B G* twiſe together with the ſquare of the line *A G* is equall to that which is compoſed of the ſquares of the lines *A B* and *B G*). And foraſmuch as the ſquares of the lines *A B* and *B G* are mediall, for that they are deſcribed of medial lines: the parllelogrãme *D E* which is equal vnto thẽ, ſhall alſo be mediall. And foraſmuch as that which is cõtained vnder the lines *A B* and *B G* is by ſuppoſition mediall, therfore the ſuperficies *Z E* which is double vnto it is alſo mediall. But the mediall ſuperficies *D E* excedeth



not the mediall ſuperficies *Z E* by a rational ſuperficies (by the 26. of this booke). Wherfore the exceſſe, namely, the ſuperficies *D F* is irrationall, vnto which the ſquare of the line *A G* is equall: wherefore the ſquare of the line *A G* is irrationall, and therefore the line *A G* which contayneth it in power is irrationall by the aſſumpt put before the 21. of this booke, and is called a ſecond mediall reſiduall line.

This propoſition ſetteth forth the nature of the tenth kinde of irrational lines, which is called a ſecond reſiduall mediall line, which is thus defined.

Diffinition of the tenth irrationall line.

A ſecond reſiduall mediall lyne is an irrationall lyne which remayneth, when from a medial line is taken away a mediall lyne commenſurable to the whole in power onely, and the part taken away & the whole lyne contayne a mediall ſuperficies.

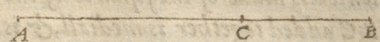
¶ The 58. Theoreme. The 76. Propoſition.

If from a right line be taken away a right line incommenſurable in power to the whole, and if that which is made of the ſquares of the whole line and of the line taken away added together be rationall, and the parallelogrãme

con-

contained vnder the same lines mediall: the line remayning is irrationall, and is called a lesse line.

SVppose that A B be a right line, and from the right line A B take away a right line B C incommensurable in power to the whole line, namely, to A B, and let that which is composed of the squares of the lines A B and B C be rationall, and let the parallelogramme contayned vnder the same lines A B and B C be mediall. Then I say, that the line remayning, namely, the line A C is irrationall, & is called a lesse line. For forasmuch as that which is cõposed of the squares of the lines A B and B C is rationall, and that which is contayned vnder the lines A B and B C twise is mediall, therefore that which is composed of the squares of the lines A B and B C is incommensurable to that which is contained vnder the lines A B and B C twise. Wherefore the squares of the lines A B and B C, are incõmensurable to the square of A C, as it was said in the 73. Proposition. But that which is made of the squares of the lines A B and B C is rationall. Wherefore the square of the line A C is irrationall: wherefore also the line A C is irrationall: and is called a lesse line. And is therefore so called, for that it is that portiõ of the greater part of a greater line, which remaineth after the taking away of the lesse part frõ the greater: which was required to be proued.



Demonstration.

In thys Proposition is contayned the definition of the eleuenth kinde of irrationall lines, which is called a lesse line, whose definition is thus.

A lesse line is an irrationall line which remayneth, whẽ from a right line is taken away a right line incommensurable in power to the whole, and the square of the whole line, & the square of the part taken away added together, make a rationall superficies, and the parallelogramme contayned of them is mediall.

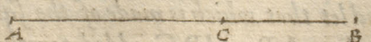
Diffinition of the eleuẽth irrationall line.

This Proposition may after *Campanes* way be demonstrated, if you remember well the order & positions which he in the three former Propositions vsed.

¶ The 59. Theoreme. The 77. Proposition.

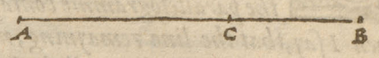
If from a right line be taken away a right line incommensurable in power to the whole line, and if that which is made of the squares of the whole line and of the line taken away added together be mediall, and the parallelogramme contained vnder the same lines rationall: the line remaining is irrationall, and is called a line making with a rationall superficies the whole superficies mediall.

SVppose that A B be a right line, and from the right line A B take away a right line B C incommensurable in power to the whole line A B, and let that which is made of the squares of the lines A B and B C added together, be mediall, and the parallelogramme contained vnder the same lines rationall. Then I say, that the line remayning, namely, the line A C, is irrationall, and is called a line making with a rationall superficies the whole superficies mediall. For forasmuch as that which is made of the squares of the lines A B and B C added together is mediall, and that which is contained vnder the lines A B and



Demonstration.

A B and B C twiſe is rationall, therefore that which is made of the ſquares of the lines A B and B C added together, is incommenſurable to that which is contained vnder the lines A B and B C twiſe. Wherefore (by the 16. of the tenth) the reſidue, namely, the ſquare of the line A C is incommenſurable to that which is contained vnder the lines A B and B C twiſe. But that which is contained vnder the lines A B and B C twiſe is rationall. Wherfore the ſquare of the line A C is irrationall. Wherefore alſo the line A C is irrationall: and is called a line making with a rationall ſuperficies the whole ſuperficies mediall: and is therfore ſo called for that that which is made of the ſquares of the lines A B and B C added together is mediall, & is a certaine whole ſuperficies, part whereof is that which is contained vnder the lines A B & B C, which is a rationall ſuperficies. For the ſquares of the lines A B and B C, are equall to that which is contained vnder the lines A B and B C twiſe, and to the ſquare of the line A C (by the 7. of the ſecond). Or it is therefore ſo called for that the ſquare thereof added to a rationall ſuperficies, maketh the whole ſuperficies mediall, as ſhall be proued by the 109. Propoſition of this booke: which was required to be proued.



In this Propoſition is declared the nature of the twelueth kind of irrationall lines, which is called a line making with a rationall ſuperficies the whole ſuperficies mediall, whoſe definition is thus.

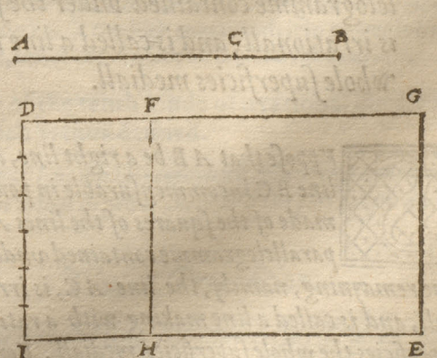
Diffinition of the twelueth irrationall liue.

A line making with a rationall ſuperficies the whole ſuperficies mediall, is an irrationall line which remaineth, whē frō a right line is taken away a right line incōmenſurable in power to the whole line, and the ſquare of the whole line & the ſquare of the part taken away added together make a mediall ſuperficies, and the parallelogramme contained of them is rationall.

This Propoſition alſo may after *Campanes* way be demonſtrated, obſeruing the former caution.

¶ The 60. Theoreme. The 78. Propoſition.

If from a right line be taken away a right line incommenſurable in power to the whole line, and if that which is made of the ſquares of the whole line and of the line taken away added together be medial, and the parallelogramme contayned vnder the ſame lines be alſo mediall, and incommenſurable to that which is made of the ſquares of the ſayd lines added together: the line remayning is irrationall, and is called a line making with a mediall ſuperficies the whole ſuperficies mediall.



Vppoſe that A B *be a right line, and from* A B *take away a right line* B C *incommenſurable in power to the whole line* A B. *And let that which is made of the ſquares of the lines* A B *and* B C *added together be mediall, and let the parallelograme contayned vnder the lines* A B *and* B C *be alſo medial, and let that which is made of the ſquares of the lines* A B *and* B C *added together be incommenſurable to that which is contayned vnder the lines* A B *and* B C. *Thē I ſay that the line remayning, namely, the line* A C *is*

irrationall

irrationall and is called a line making with a mediall superficies the whole superficies medial. Take a rationall line D I. *And (by the 44. of the first) vnto the line* D I *apply the parallelogramme* D E *equall to that which is made of the squares of the lines* A B *and* B C *added together, and making in breadth the line* D G. *And vnto the same line* D I *apply the parallelogramme* D H *equall to that which is contayned vnder the lines* A B *and* B C *twise and making in breadth the line* D F. *Wherefore the parallelogramme remayning, namely the parallelogramme* F E, *is equall to the square of the line* A C. *Wherefore the line* A C *contayneth in power the parallelograme* F E. *And forasmuch as that which is made of the squares of the lines* A B *and* B C *added together is mediall, and is equall to the parallelogramme* D E, *therefore also the parallelogramme* D E *is mediall. And the parallelogramme* D E *is applyed to the rationall line* D I *making in breadth the line* D G. *Wherfore (by the 22. of the tenth) the line* D G *is rationall and incommensurable in length to the line* D I. *Agayne forasmuch as that which is contayned vnder the lines* A B *and* B C *twise is mediall and is equall to the parallelogramme* D H, *therefore the parallelogramme* D H *is mediall. And the parallelograme* D H *is applyed vnto the rationall line* D I *making in breadth the line* D F, *wherefore the line* D F *is rationall and incommensurable in length to the line* D I. *And forasmuch as that which is made of the squares of the lines* A B *and* B C *added together is incommensurable to that which contayned vnder the lines* A B *&* B C *twise, therefore the parallelogramme* D E *is incommensurable to the parallelogramme* D H. *But as the parallelogramme* D E *is to the parallelogramme* D H, *so (by the first of the sixt) is the line* D G *to the line* D F: *wherfore the line* D G *is incommensurable in length to the line* D F. *And they are both rationall lines. Wherefore the lines* D G *and* D F *are rationall commensurable in power onely, wherefore the line* F G *is a residuall line by the 73. of this booke. But the line* F H *is rationall for that it is equall vnto the line* D I. *But a rectangle parallelogramme contayned vnder a rationall line and an irrationall line is irrationall, and the line also that contayneth in power the same parallelogramme is irrationall (by the 21. of the tenth). But the line* C A *contayneth in power the parallelogramme* F E. *Wherefore the line* A C *is irrationall and is called a line making with a mediall superficies the whole superficies mediall. And is therfore so called, for that that which is made of the squares of the lines* A B *and* B C *added together is mediall, & is a certayne whole superficies, part whereof is that which is cõtayned vnder the lines* A B *and* B C, *which is also mediall: you shall also by the 110. proposition of this booke vnderstand an other cause why it is so called.*

Construction.

Demonstration.

This proposition may thus more briefely be demonstrated: forasmuch as that which is composed of the squares of the lines A B and B C is mediall, and that also which is contayned vnder them is mediall, therefore the parallelogrammes D E and D H which are equall vnto them are mediall: but a mediall superficies exceedeth not a mediall superficies by a rationall superficies. Wherefore the superficies F E which is the excesse of the mediall superficies D E aboue the mediall superficies D H is irrational. And therefore the line A C which contayneth it in power is irrationall. &c.

In this proposition is shewed the conditiõ and nature of the thirteenth and last kinde of irrationall lines, which is called a line making with a mediall superficies the whole superficies mediall, whose definition is thus.

A line making with a mediall superficies the whole superficies mediall is an irrationall line which remayneth, when from a right line is taken away a right line incommensurable in power to the whole line, and the squares of the whole line and of the line taken away added together make a mediall superficies, and the parallelogramme contayned of thẽ is also a mediall superficies, moreouer the squares of them are incommensurable to the parallelogramme contayned of them.

Diffinition of the thirteenth and last irrationall line.

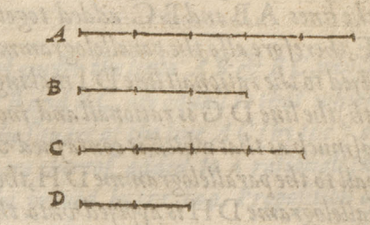
An assumpt of *Campane.*

If there be fower quãtities, & if the difference of the first to the second, be as the difference of the third to the fourth, then alternately, as the difference of the first is to the third, so is the difference of the second to the fourth.

An Assumpt of Campane.

I. Dee. Though Campanes lemma be true, yet the maner of demonstrating it, (narrowly considered) is not artificiall.

This is to be vnderstand of quãtities in like sort referred the one to the other, that is if the first be greater then the second, the third ought to be greater then the fourth and if the first be lesse then the second, the third ought to be lesse then the fourth: and is also to be vnderstand in arithmeticiall proportionality. As for example let the difference of A be vnto B as the difference of C is to D. Then I say that as the difference of A is to C, so is the difference of B to D. For (by this common sẽtence, the difference of the extreames is composed of the differences of the extreames to the meanes), the difference of A to C is composed of the difference of A to B and of the difference of B to C. And (by the same common sentence) the difference of B to D is composed of the difference of B to C, and of the difference of C to D. And forasmuch as (by supposition) the difference of A to B is as the difference of C to D, and the difference of B to C is common to them both. Wherefore it followeth, that as the difference of A is to C, so is the difference of B to D: which was required to be proued.



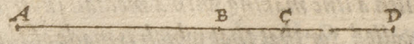
¶ The 61. Theoreme. The 79. Proposition.

Vnto a residual line can be ioyned one onely right lyne rational, and commensurable in power onely to the whole lyne.

Second Senary.

Demonstration leading to an impossibilitie.

LEt A B be a residuall line, and vnto it let the line B C be supposed to be ioyned, so that let the lines A C and B C be rationall commensurable in power onely. Then I say that vnto the line A B cannot be ioyned any other rationall line commensurable in power onely to the whole line. For if it be possible, let B D be such a line added vnto it. Wherfore the lines A D and D B are rationall commensurable in power onely. And forasmuch as how much the squares of the lines A D and D B do exceede that which is contained vnder the lines A D and D B twise, so much also do the squares of the lines A C and C B exceede that which is contained vnder the lines AC and C B twise, for the excesse of eche is one and the same, namely, the square of the line A B (by the 7. of the second.) Wherfore alternately (by the former assumpt of Campanus) how much the squares of the lines A D and D B do exceede the squares of the lines A C & C B, so much also excedeth that which is contayned vnder the lines A D and D B twise, that which is contained vnder the lines A C and C B twise. But that which is made of the squares of the lines A D and D B added together, exceedeth that which is made of the squares of the lynes A C and C B added together by a rationall superficies: (for they are either of them rational). Wherefore that which is contained vnder the lines A D and D B twise, exceedeth that which is contained vnder the lines A C and C B twise by a rationall superficies. But that which is contained vnder the lines A D and D B twise, is mediall, for it is commensurable to that which is contained vnder the lines A D and D B once, which superficies is mediall (by the 21. of the tenth) and by the same reason also that which is contained vnder the lynes A C and C B twise is mediall. Wherfore a mediall superficies differeth from a mediall superficies by a rationall superficies, which (by the 26. of the tenth) is impossible. Wherfore vnto the lyne A B cannot be ioyned any other rationall line besides B C commensurable in power onely, to the whole line. Wherfore vnto a residuall line can be ioyned one onely right line rationall and commensurable in power onely to the whole lyne: which was required to be demonstrated.

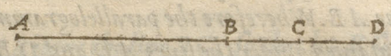


¶ *The*

¶ The 62. Theoreme. The 80. Proposition.

Vnto a first medial residuall line can be ioyned one onely mediall right lyne, commensurable in power onely to the whole lyne, and comprehendyng wyth the whole lyne a rationall superficies.

Suppose that A B be a first mediall residuall line, & vnto A B ioyne the lyne B C, so that let the lynes A C and B C be mediall commensurable in power onely, & let that which is contained vnder the lines A C and B C be rationall. Then I say that vnto the lyne A B cannot be ioyned any other mediall line commensurable in power onely to the whole lyne, and comprehending together with the whole lyne a rationall superficies. For if it be possible let the line B D be such a line. Wherfore the lynes A D and D B are mediall commensurable in power onely, and that which is contayned vnder the lynes A D and D B is rational. Demonstration leading to an absurditie.

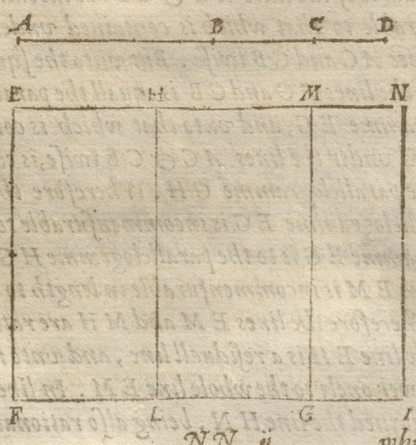


And forasmuch as how much the squares of the lynes A D and D B exceede that which is contayned vnder the lynes A D and D B twise, so much also exceede the squares of the lynes A C & B C, that which is contayned vnder the lynes A C and C B twise (for the excesse of eche is one and the same, namely, the square of the lyne A B). Wherfore alternately (as it was sayd in the former proposition) how much the squares of the lynes A D and D B exceede the squares of the lines A C and C B, so much also that which is contained vnder the lines A D and D B twise, excedeth that which is contained vnder the lines A C and C B twise. But that which is contained vnder the lines A D and D B twise, excedeth that which is contained vnder the lines A C and C B twise by a rationall superficies, for they are either of them a rationall superficies. Wherfore that which is made of the squares of the lines A D & D B excedeth that which is made of the squares of the lines A C & C B by a rationall superficies, which (by the 26. of the tenth) is impossible. For they are either of them mediall (for those foure lines were put to be mediall.) Wherfore vnto a first mediall residuall line can be ioyned onely one right mediall line commensurable in power onely to the whole line, and comprehending with the whole line a rationall superficies: which was required to be proued.

¶ The 63. Theoreme. The 81. Proposition.

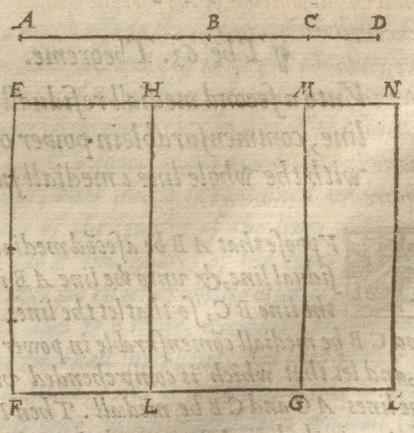
Vnto a second mediall residuall line can be ioyned onely one mediall right line, commensurable in power onely to the whole line, and comprehending with the whole line a mediall superficies.

Suppose that A B be a secõd mediall residual line, & vnto the line A B ioyne the line B C, so that let the lines A C and C B be mediall cõmensurable in power onely, and let that which is comprehended vnder the lines A C and C B be mediall. Then I say, that vnto the line A B can not be ioyned any other mediall right line cõmensurable in power onely to the whole line, and comprehending together with the whole line a mediall superficies. For if it be possible, let the line B D be such a line. Wherefore the lines A D & D B are mediall commensurable in power onely, and that



Construction.

which is contained vnder the lines AD and DB is also mediall. Take a rationall line EF. And (by the 44. of the first) vnto the line EF apply the parallelogramme EG equall to the squares of the lines AC and CB, and making in breadth the line EM: and from that parallelogramme EG take away the parallelogramme HG equall to that which is contained vnder AC and CB twise, and making in breadth the line HM. Wherefore the parallelogramme remayning, namely, EL, is (by the 7. of the second) equall to the square of the line AB. Wherefre the line AB containeth in power the parallelograme EL. Againe, vnto the line EF apply (by the 44. of the first) the parallelogramme EI equall to the squares of the lines AD and DB, and making in breadth the line EN. But the squares of the lines AD and DB are equall to that which is contained vnder the lines AD and DB twise, and to the square of the line AB. Wherefore the parallelogramme EI is equall to that which is contained vnder the lines AD & DB twise, and to the square of the line AB. But the parallelogramme EL is equall to the square of the line AB. Wherefore the parallelogramme remaining, namely, HI, is equall to that which is contained vnder the lines AD and DB twise. And forasmuch as the lines AC and CB are mediall, therefore the squares also of the lines AC and CB are mediall: and they are equall to the parallelogramme EG: wherefore the parallelogramme EG is (by that which was spoken in the 25. Proposition) mediall: and it is applied vnto the rationall line EF, making in breadth the line EM. Wherefore (by the 22. of the tenth) the line EM is rationall, and incommensurable in length to the line EF. Againe, forasmuch as that which is contayned vnder the lines AC and CB is mediall, therefore (by the Corollary of the 23. of the tenth) that which is contained vnder the lines AC and CB twise is also mediall: and it is equall to the parallelogramme HG: wherefore also the parallelogramme HG is mediall, and is applyed to the rationall line EF, making in breadth the line HM. Wherefore (by the 22 of the tenth) the line HM is rationall, and incommensurable in length to the line EF. And forasmuch as the lines AC and CB are commensurable in power onely, therefore the line AC is incommensurable in length to the line CB. But as the line AC is to the line CB, so (by the Assumpt going before the 22. of the tenth) is the square of the line AC to that which is contayned vnder the lines AC & CB. Wherefore (by the 10. of the tenth) the square of the line AC is incommensurable to that which is contained vnder the lines AC and CB. But vnto the square of the line AC are commensurable the squares of AC & CB, and vnto that which is contained vnder the lines AC and CB, is commensurable that which is contained vnder the lines AC and CB twise. Wherefore the squares of the lines AC & CB are incommensurable to that which is contained vnder the lines AC and CB twise. But vnto the squares of the lines AC and CB is equall the parallelogramme EG, and vnto that which is contained vnder the lines AC & CB twise, is equall the parallelogramme GH. Wherefore the parallelogramme EG is incommensurable to the parallelogramme HG. But as the parallelogramme EG is to the parallelogrmme HG, so is the line EM to the line HM. Wherefore the line EM is incommensurable in length to the line HM. And they are both rationall lines. Wherefore the lines EM and MH are rationall commensurable in power onely. Wherefore the line EH is a residuall line, and vnto it is ioyned a rationall line HM commensurable in power onely to the whole line EM. In like sort also may it be proued, that vnto the line EH is ioyned the line HN, being also rationall, and commensurable in power onely to the whole



Demonstration leading to an absurditie.

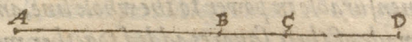
line

line E N. Wherefore vnto a reſiduall line is ioyned more then one onely line commenſurable in power onely to the whole line: which (by the 79. of the tenth) is impoſsible. Wherefore vnto a ſecond mediall reſiduall line can be ioyned onely one mediall right line commenſurable in power onely to the whole line, and comprehending with the whole line a mediall ſuperficies: which was required to be demonſtrated.

¶ The 64. Theoreme. The 82. Propoſition.

Vnto a leſſe line can be ioyned onely one right line incommenſurable in power to the wholelyne, and making together with the whole lyne that which is made of their ſquares added together rationall, and that which is contayned vnder them mediall.

Vppoſe that A B be a leſſe line, and to A B ioyne the line B C, ſo that let B C be ſuch a line as is required in the Theoreme. Wherfore the lines A C and C B are incõmenſurable in power, hauing that which is made of the ſquares of them added together rationall, and that which is contained vnder them mediall. Then I ſay that vnto A B cannot be ioyned any other ſuch right line. For if it be poſsible, let the lyne B D be ſuch a line. Wherfore the lines A D & D B are incommenſurable in power, hauing that which is made of the ſquares of them added together, rationall, and that which is contained vnder them mediall. And for that how much the ſquares of the lines A D and D B excede the ſquares of the lines A C and C B, ſo much that which is contained vnder the lines A D and D B twiſe, excedeth that which is contained vnder the lines A C and C B twiſe (by thoſe things which were ſpoken in the 79. propoſition) But that which is made of the ſquares of the lines A D and D B alded together excedeth that which is made of the ſquares of the lines A C and C B added together by a rationall ſuperficies, for they are either of them rationall by ſuppoſition. Wherfore that which is contained vnder the lines A D and D B twiſe, excedeth that which is contained vnder the lines A C and C B twiſe by a rationall ſuperficies: which (by the 26. of the tenth) is impoſsible, for either of them is mediall by ſuppoſition. Wherfore vnto a leſſe line can be ioyned onely one right line incommenſurable in power to the whole line, and making together with the whole line that which is made of their ſquares added together rationall, and that which is contained vnder them mediall: which was required to be demonſtrated.



Demonſtration leading to an abſurditie.

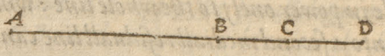
¶ The 65. Theoreme. The 83. Propoſition.

Vnto a line making with a rationall ſuperficies the whole ſuperficies mediall, can be ioyned onely one right lyne incommenſurable in power to the whole lyne, and making together with the whole line that which is made of their ſquares added together mediall, and that which is contained vnder them rationall.

Vppoſe that A B be a line making with a rationall ſuperficies the whole ſuperficies mediall, and vnto it let the line B C be ioyned, ſo that let B C be ſuch a line as is required in the Theoreme. Wherfore the lines A C and C B are incommenſurable in

power, hauing that which is made of the squares of the lines A C and C B added together mediall, and that which is contained vnder the lynes A C and C B rationall. Then I say that vnto the lyne A B cannot be ioyned any other such line. For if it be possible, let the line B D be such a line. Wherfore the lines A D and D B are incommensurible in power, hauing that which is made of the squares of the lines A D and D B added together mediall, and that which is contained vnder the lines A D and D B rationall. Now for that how much the squares of the lines A D and D B exceede the squares of the lines A C and C B, so much that which is contained vnder the lines A D and D B twise exceedeth that which is contayned vnder the lynes A C and C B twise, by that which was spoken in the 79. proposition. But that which is contained vnder the lines A D and D B twise, exceedeth that which is contained vnder the lines A C and C B twise by a rationall superficies, for they are either of them rationall by suppositiõ. Wherfore that which is made of the squares of the lines A D and D B added together, excedeth that which is made of the squres of the lines A C and C B added together by a rationall superficies, which by the 26. of the tenth, is impossible, for they are either of them mediall by supposition. Wherfore vnto the line A B cannot be ioyned any other lyne besides B C, making that which is required in the proposition. Wherfore vnto a line making with a rationall superficies the whole superficies mediall can be ioyned onely one right line incõmensurable in power to the whole line, and making together with the whole line that which is made of their squares added together mediall, and that which is cõtained vnder them rationall: which was required to be proued.

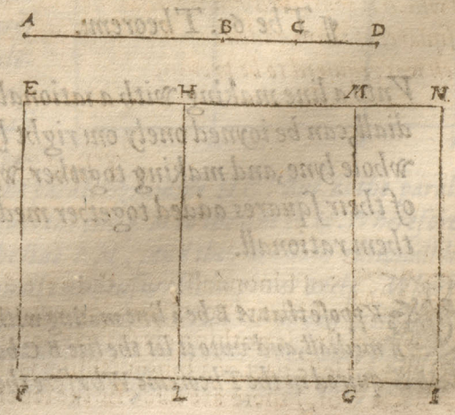
Demonstratiõ leading to an impossibili. ie.



¶ The 66. Theoreme. The 84. Proposition.

Vnto a line making with a medial superficies the whole superficies medial, can be ioyned onely one right line incommensurable in power to the whole line, and making together with the whole line that which is made of their squares added together mediall, and that which is contained vnder them mediall, and moreouer making that which is made of the squares of them added together incommensurable to that which is contayned vnder them.

Suppose that A B be a line making with a mediall superficies the whole superficies mediall, and vnto it let the line B C be ioyned, so that let B C be such a line as is required in the Theoreme. Wherefore the lines A C and C B are incommensurable in power, hauing that which is made of the squares of the lines A C and C B added together mediall, & that which is contained vnder the lines A C and C B mediall, and moreouer that which is made of the squares of the lines A C and C B is incommensurable to that which is contained vnder the lines A C and C B. Then I say, that vnto the line A B can be ioyned no other such line. For if it be possible, let B D be such a line. Wherefore the lines A D and D B are incõmensurable in power, hauing that which is made of the squares of the lines A D and D B



DB added together mediall, and that which is contained vnder the lines AD and DB mediall, and moreouer that which is made of the ſquares of the lines AD and and DB added together, is incommenſurable to that which is contained vnder the lines AD and DB. Take a rationall line EF. And (by the 44. of the firſt) vnto the line EF apply the parallelogramme EG equall to the ſquares of the lines AC and CB, and making in breadth the line EM: and from the parallelogramme EG take away the parallelogramme HG equall to that which is contained vnder the lines AC & CB twiſe, and making in breadth the line HM. Wherefore the reſidue, namely, the ſquare of the line AB is equall to the parallelogramme EL (by the 7. of the ſecond). Wherefore the line AB contayneth in power the parallelogramme EL. Againe (by the 44. of the firſt) vnto the line EF apply the parallelogramme EI equall to the ſquares of the lines AD and DB, and making in breadth the line EN. But the ſquare of the line AB is equall to the parallelogrãme EL. Wherefore the reſidue, namely, the parallelogramme HI is equall to that which is contained vnder the lines AD and DB wiſe. And foraſmuch as that which is made of the ſquares of the lines AC and CB is medull, and is equall to the parallelogramme EG, therefore alſo the parallelogramme EG is mediall. And it is applied vnto the rationall line EF, making in breadth the line EM. Wherefore (by the 22. of the tenth) the line EM is rationall and incommenſurable in length to the line EF. Againe, foraſmuch as that which is contained vnder the lines AC and CB wiſe is mediall, and is equall to the parallelogramme HG. Wherefore the parallelogramme HG is mediall, which parallelogramme HG is applied to the rationall line EF, making in breadth the line HM. Wherefore the line HM is rationall and incommenſurable in length to the line EF. And foraſmuch as the ſquares of the lines AC and CB are incommenſurable to that which is contained vnder the lines AC and CB twiſe, therefore the parallelogramme EG is incommenſurable to the parallelogramme HG. Wherefore the line EM is incommenſurable in length to the line MH, and they are both rationall. Wherefore the lines EM and MH are rationall commenſurable in power onely. Wherefore EH is a reſiduall line. And the line ioyned vnto it is HM. And in like ſort may we proue, that the line EH is a reſiduall line, and that the line HN is ioyned vnto it. Wherefore vnto a reſiduall line is ioyned two ſundry lines, being eche commenſurable in power onely to the whole line: which (by the 79. of the tenth) is impoſsible. Wherefore vnto the line AB can not be ioyned any other right line beſides the line BC, which ſhall be incommenſurable in power to the whole line, & haue together with the whole line that which is made of their ſquares added together mediall, and that which is contained vnder them mediall, and moreouer incommenſurable to that which is made of their ſquares added together. Wherefore vnto a line making with a mediall ſuperficies the whole ſuperficies mediall, can be ioyned onely one right line incommenſurable in power to the whole line, and making together with the whole line that which is made of their ſquares added together mediall, and that which is contained vnder them mediall, and moreouer making that which is made of the ſquares of them added together incommenſurable to that which is contained vnder thē: which was required to be proued.

Conſtruction.

Demonſtration leading to an abſurdite.

¶ Third Definitions.

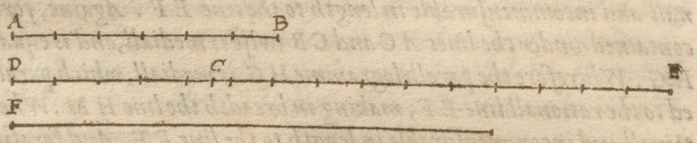
AS of binomiall lynes, there are 6. diuers kindes, ſo alſo of reſiduall lynes which are correſpondett vnto them and depend of them (for a reſiduall line is nothing els (as was before ſaid) but that which remayneth whē the leſſe part of a binomiallline is taken from the greater part or name therof,) there are likewiſe ſxe ſeuerall kindes. All which are knowne and con-

Sixe kindes of residuall lines.

sidered in comparison to a rationall line set forth & appointed, and these residual line haue the selfe same order of production that the binomials had. For as the three first kindes of binomiall lines, namely, the first, second, and third, were produced when the square of the greater part of the binomiall excedeth the square of the lesse part thereof by the square of a line commensurable vnto it in length: so in likewise, the first three kindes of residuall lines, namely, the first, second, and third, are produced, when the square of the whole, namely, of that which is made of the residuall line, and the line ioyned vnto it added together, excedeth the square of the line ioyned to the residuall, by the square of a lyne which is commensurable vnto it in length. And as the three last kindes of binomials, namely, the fourth, fifth, and sixth were produced when the square of the greater part excedeth the square of the lesse, by the square of a line incommensurable in length vnto it, euen so the three last kyndes of residuall lynes are produced whē the square of the whole excedeth the square of the lyne adioined, by the square of a line incommensurable vnto it in length. As ye may perceiue by their definitions following.

First diffinition.

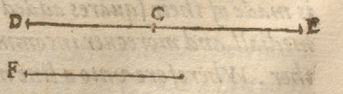
A first residuall line is, when the square of the whole excedeth the square of the lyne adioyned, by the square of a lyne commensurable vnto it in lēgth, and also the whole is commensurable in length to the rationall line first set.



As let A B be a rationall line, whose partes are certaine, distinct, & to be expressed by number. And let the residual lyne be C D, and let the line ioyned vnto it be E C, and let the whole being composed of the residuall C D, and the line adioyned E C, be the line E D: let moreouer the square of the whole line E D, excede the square of the line adioyned E C by the square of the line F, which line F let be commensurable in length to the whole lyne E D, and let the whole line E D be also commensurable in length to the rationall line A B: then is the residuall lyne C D by this definition a first residuall line.

Second diffinition.

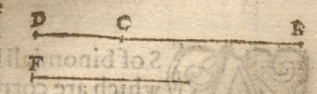
A second residual line is, when the square of the whole excedeth the square of the line adioyned, by the square of a line commensurable vnto it in lēgth, and also the line adioyned is commensurable in length to the rationall lyne.



As suppose the line C D to be a residuall, and let the line adioyned vnto it be E C, and the whole made of them both, let be the line E D: & let the square of E D the whole line excede the square of the lyne adioyned E C, by the square of the lyne F, and let the lyne F be commensurable in length to the whole lyne E D, moreouer let the line adioyned E C be commensurable in length to the rationall line A B: then by this definition, the residuall line C D is a second residuall line.

Third diffinition.

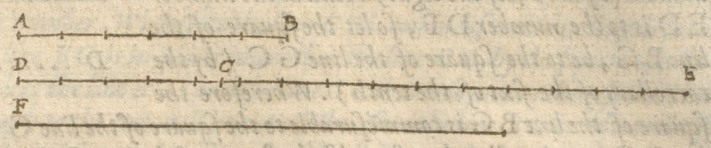
A third residuall line is, when the square of the whole excedeth the square of the lyne adioyned, by the square of a line commensurable vnto it in lēgth and neither the whole line, nor the line adioyned is cōmensurable in length to the rationall lyne.



As (the former supposition standing) suppose that the square of the whole lyne E D exceede the square of the lyne adioyned E C by the square of the lyne F, and let the lyne F be commensurable in length to the whole lyne E C, and let neither the whole lyne E D, nor the line adioyned E C be commensurable in length to the rationall lyne A B, then by this definition the residuall lyne C D is a third residuall lyne.

A fourth

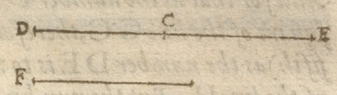
A fourth reſiduall line is, when the ſquare of the whole lyne excedeth the ſquare of the lyne adioyned, by the ſquare of a lyne incommenſurable vnto it in length, and the whole lyne is alſo commenſurable in length to the rationall lyne. *Fourth diffinition.*



As the reſiduall lyne beyng as before C D, & the lyne adioyned E C, and the whole E D, let the ſquare of the whole lyne E D exceede the ſquare of the line adioyned E C by the ſquare of the lyne F, and let the lyne F be incommenſurable in length to the whole line E D, and let E D the whole line be commenſurable in length to the rationall lyne A B, then is the reſiduall line C D by this declaration a fourth reſiduall lyne.

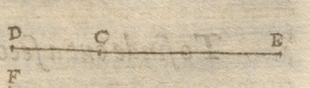
A fiueth reſiduall line is, when the ſquare of the whole lyne exceedeth the ſquare of the lyne adioyned, by the ſquare of a lyne incommenſurable vnto it in length, and the lyne adioyned is commenſurable in length to the rationall lyne. *Fifth diffinition.*

As the reſiduall line beyng C D, the lyne adioyned E C, and the whole lyne E D, let the ſquare of the whole lyne E D exceede the ſquare of the line adioyned E C by the ſquare of the lyne F, and let the line F be incommenſurable in length to the whole lyne E D, and let alſo E C the lyne adioyned be commenſurable in length to the rationall line A B, then ſhall the reſiduall C D be by this diffinition a fifth reſiduall lyne.



A ſixth reſiduall line is when the ſquare of the whole line, exceedeth the ſquare of the line adioyned, by the ſquare of a line incommenſurable vnto it in length, and neither the whole line nor the line adioyned is commenſurable in length to the rationall line. *Sixth diffinition.*

As ſuppoſe the reſiduall line to be C D, and the lyne adioyned to be E C, and the whole lyne compoſed of them let be E D, and let the ſquare of the whole lyne E D exceede the ſquare of the line adioyned by the ſquare of the lyne F, which line F let be incommenſurable in length to the whole lyne E D: moreouer let neither the whole lyne C D nor the line adioyned E C, be commenſurable in length to the rationall line A B, then ſhall the reſiduall lyne C D be by this explication a ſixt reſiduall lyne, and the laſt.



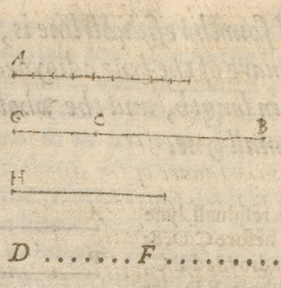
¶ *The 19. Probleme.* *The 85. Propoſition.*

To finde out a firſt reſiduall line.

TAke a rationall line and let the ſame be A, and vnto it let the line B G be commenſurable in length. Wherefore the line B G alſo is rationall. And take two ſquare numbers D E and E F which let be ſuch, that the exceſſe of the greater, namely, of *Third Senary. Conſtruction.*

D E,

DE, *aboue the leſſe* EF (*which exceſſe let be the number* DF) *be no ſquare number (by the corollary of the firſt aſſumpt of the 28. of the tenth). Wherfore the number* ED *hath not to the number* DF *that proportion that a ſquare number hath to a ſquare number (by the 24. of the eight). And as the number* ED *is to the number* DF, *ſo let the ſquare of the line* BG, *be to the ſquare of the line* GC (*by the corrollary of the ſixt of the tenth*). *Wherefore the*



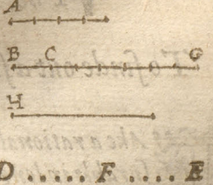
Demonſtration. *ſquare of the line* BG *is commẽſurable to the ſquare of the line* GC. *But the ſquare of the line* BG *is rationall, wherefore alſo the ſquare of the line* GC *is rationall. Wherefore the line* GC *is alſo rationall. And foraſmuch as the number* ED *hath not to the number* DF, *that proportiõ that a ſquare nũber hath to a ſquare nũber, therfore neither alſo hath ỹ ſquare of the line* BG *to the ſquare of the line* GC *that proportion that a ſquare number hath to a ſquare number. Wherfore (by the 9. of the tenth) the line* BG *is incommenſurable in length to the line* GC. *And they are both rationall. Wherefore the lines* BG *and* GC *are rational commenſurable in power onely. Wherefore the line* BC *is a reſiduall line. I ſay moreouer that it is a firſt reſiduall line. For foraſmuch as the ſquare of the line* BG *is greater then the ſquare of the line* GC (*that it is greater it is manifeſt, for by ſuppoſition the ſquare of the line* BG *is to the ſquare of the line* GC, *as the greater number, namely,* ED *is to the number* DF) *vnto the ſquare of the line* BG *let the ſquares of the lines* GC *and* H *be equall. And for that as the number* DE *is to the number* DF, *ſo is the ſquare of the line* BG *to the ſquare of the line* GC, *therefore by conuerſion of proportion (by the corrollary of the 9. of the fifth) as the number* DE *is to the number* EF, *ſo is the ſquare of the line* BG *to the ſquare of the line* H. *But the number* DE *hath to the number* EF *that proportion that a ſquare numher hath to a ſquare number, for either of them is a ſquare number, wherefore alſo the ſquare of the line* BG *hath to the ſquare of the line* H *that proportion that a ſquare number hath to a ſquare number. Wherefore the line* GB *is commenſurable in length to the line* H. *Wherefore the line* GB *is in power more then the line* GC *by the ſquare of a line commenſurable in length to the line* GB: *and the whole line, namely,* GB *is commenſurable in length to the rationall line* A. *Wherefore the line* BC *is a firſt reſiduall line. Wherefore there is founde out a firſt reſiduall line: which was required to be done.*

¶ The 20. Probleme. The 86. Propoſition.

To finde out a ſecond reſiduall line.

Conſtruction. TAke a rationall line, and let the ſame be A, and vnto it let the line GC be commenſurable in length. And take two ſquare numbers DE and EF, and let them be ſuch that the exceſſe of the greater, namely, DF, be no ſquare number. And as the number DF is to the number DE, ſo let the ſquare of the line GC be to the ſquare of the line GB. Wherefore both the ſquares are commenſu-

Demonſtration. rable. And foraſmuch as the ſquare of the line GC is rationall, therefore the ſquare of the line BG is alſo rationall: Wherefore alſo the line BG is rationall. And foraſmuch as the ſquares of the lines BG & GC haue not that proportion the one to the other that a ſquare number hath to a ſquare number, therefore the lines BG and GC are incommenſurable in length, and they are both rationall. Wherefore the lines BG and GC are rationall commenſura-



ble

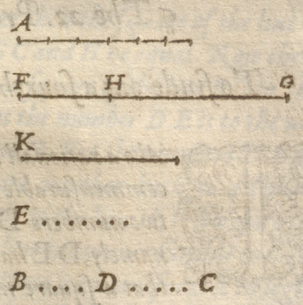
ble in power onely. Wherefore the line B C is a reſiduall line. I ſay moreouer, that it is a ſecond reſidual line. For foraſmuch as the ſquare of the line B G is greater thē the ſquare of the line G C, vnto the ſquare of the line B G let the ſquares of the lines G C & H be equall. And for that as the number D E is to the nūber D F, ſo is the ſquare of the line G B to the ſquare of the line G C, therefore (by conuerſion of proportion) as the number D E is to the number E F, ſo is the ſquare of the line B G to the ſquare of the line H. But either of theſe numbers D E and E F is a ſquare number. Wherefore the line G B is commenſurable in length to the line H. Wherefore the line B G is in power more then the line G C, by the ſquare of a line commenſurable in length to the line B G: and the line G C that is ioyned to the reſiduall line is commenſurable in length to the rationall line A. Wherefore the line B C is a ſecond reſiduall line. Wherefore there is found out a ſecond reſiduall line: which was required to be done.

¶ The 21. Probleme. The 87. Propoſition.

To finde out a third reſiduall line.

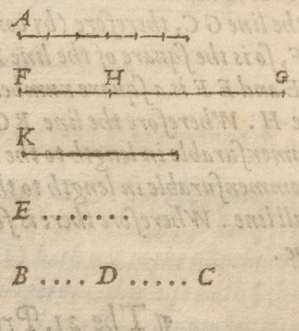
TAke a rationall line, & let the ſame be A: and take three numbers E, B C, and C D, not hauing the one to the other that proportion that a ſquare nūber hath to a ſquare number: and let the number B C haue to the number B D that proportion that a ſquare number hath to a ſquare number. And let the number B C be greater then the number C D. And as the number E is to the number B C, ſo let the ſquare of the line A be to the ſquare of the line F G: and as the number B C is to the number C D, ſo let the ſquare of the line F G be to the ſquare of the line H G. Wherefore the ſquare of the line A is commenſurable to the ſquare of the line F G. But the ſquare of the line A is rationall. Wherefore alſo the ſquare of the line F G is rationall: wherefore the line F G is alſo rationall. And foraſmuch as the number E hath not to the number B C that proportion that a ſquare number hath to a ſquare number, therefore neither alſo hath the ſquare of the line A to the ſquare of the line F G that proportion that a ſquare number hath to a ſquare number. Wherefore the line A is incommenſurable in length to the line F G. Againe for that as the number B C is to the number C D, ſo is the ſquare of the line F G to the ſquare of the line H G, therefore the ſquare of the line F G is commenſurable to the ſquare of the line H G. But the ſquare of the line F G is rationall. Wherefore alſo the ſquare of the line H G is rationall. Wherefore alſo the line H G is rationall. And for that the number B C hath not to the number C D, that proportion that a ſquare number hath to a ſquare number, therefore neither alſo hath the ſquare of the line F G to the ſquare of the line H G, that proportion that a ſquare nūber hath to a ſquare number. Wherefore the line F G is incommenſurable in length to the line H G: and they are both rationall. Wherefore the lines F G & H G are rationall cōmenſurable in power onely. Wherfore the line F H is a reſiduall line. I ſay moreouer, that it is a third reſiduall line. For for that as the number E is to the number B C, ſo is the ſquare of the line A to the ſquare of the line F G: and as the number B C is to the number C D, ſo is the ſquare of the line F G to the ſquare of the line H G: therefore by equalitie of proportion, as the number E is to the number C D, ſo is the ſquare of the line A to the ſquare of the line H G: but the number E hath not to the number C D that proportion that a ſquare number hath to a ſquare number, therefore

Conſtruction.



Demonſtration.

fore neither also hath the square of the line A to the square of the line H G that proportion that a square number hath to a square number, therefore the line A is incommensurable in length to the line H G. Wherefore neither of the lines F G and H G is commensurable in length to the rationall line A. And forasmuch as the square of the line F G is greater then the square of the line H G (that the line F G is greater then the line H G it is manifest, for by supposition the number B C is greater then the number C D) vnto the square of the line F G let the squares of the lines H G & K be equall. And for that as the nũber B C is to the number C D, so is the square of the line F G to the square of the line H G, therfore (by conuersion of proportion) as the number B C is to the number B D, so is the square of the line F G to the square of the line K. But the nũber B C hath to the number B D that proportion that a square number hath to a square number. Wherefore the square of the line F G hath to the square of the line K that proportion that a square number hath to a square number. Wherefore the line F G is commensurable in length to the line K. Wherefore the line F G is in power more then the line H G, by the square of a line commensurable in length to the line F G, and neither of the lines F G and G H is commensurable in length to the rationall line A: when yet notwithstanding either of the lines F G and G H is rationall. Wherefore the line F H is a third residuall line. Wherefore there is found out a third residuall line: which was required to be done.

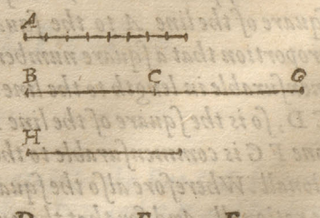


¶ The 22. Probleme. The 88. Proposition.

To finde out a fourth residuall line.

Construction.

TAke a rationall line and let the same be A: and vnto it let the line B G be commensurable in length. Wherefore the line B G is rationall. And take two numbers D F and F E, and let them be such that the whole number, namely, D E haue to neither of the numbers D F and F E that proportion that a square number hath to a square number. And as the number D E is to the number E F, so let the square of the line B G be to the square of the line G C: wherefore the square of the line B G is commensurable to the square of the line G C, wherefore also the square of the line G C is rationall, and the line G C is also rationall. And for that the number D E hath not the number E F that proportion that a square number hath to a square number, therefore the line B G is incommensurable in length to the line G C. And they are both rationall: wherefore the line B C is a residuall line. I say moreouer that it is a fourth residuall line. For forasmuch as the square of the line B G is greater then the square of the line G C, vnto the square of the line B G let the squares of the lines C G and H be equall. And for that as the number D E is to the number E F, so is the square of the line B G to the square of the line G C, therefore by conuersion of proportion as the number D E is to the number D F, so is the square of the line B G to the square of the line H. But the numbers D E and D F haue not the one to the other that porportion that a square number hath to a square number. Wherefore the line B G is incommensurable in length to the line H. Wherefore the line B G is in power more then



Demonstration.

the

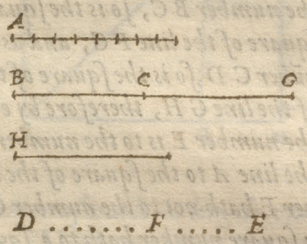
the line G C by the ſquare of a line incommenſurable in length to the line B G: and the whole line B G is commenſurable in length to the rationall line A. Wherefore the line B C is a fourth reſiduall line. Wherefore there is founde out a fourth reſiduall line: which was required to be doone.

¶ The 23. Probleme. The 89. Propoſition.

To finde out a fift reſiduall lyne.

TAke a rational line and let the ſame be A, and vnto it let the line C G be commenſurable in length. Wherefore the line C G is rationall. And take two numbers D F and F E, which let be ſuch, that the number D E haue to neither of theſe numbers D F nor F E that proportion that a ſquare number hath to a ſquare number. And as the number F E is to the number D E, ſo let the ſquare of the line C G be to the ſquare of the line B G. Wherfore the ſquare of the line C G is commenſurable to the ſquare of the line B G. Wherefore the ſquare of the line B G is rationall, and the line B G is alſo rational. But the numbers D E and E F haue not that proportion the one to the other that a ſquare number hath to a ſquare nũber. Wherfore the lines B G and G C are rationall commenſurable in power onely. Wherefore the line B C is a reſiduall line. I ſay moreouer that it is a fift reſiduall line. For foraſmuch as the ſquare of the line B G is greater then the ſquare of the line G C, vnto the ſquare of the line B G let the ſquares of the lines G C and H be equal. Now therfore for that as the number D E is to the number E F, ſo is the ſquare of the line B G to the ſquare of the line G C, therfore by conuerſion of proportion, as the number D E is to the nũber D F, ſo is the ſquare of the line B G to the ſquare of the line H. But the numbers D E & D F haue not that proportion the one to the other that a ſquare number hath to a ſquare number. Wherefore the line B G is incommenſurable in length to the line H. Wherefore the line B G is in power more then the line C G by the ſquare of a line incommenſurable in lẽgth to the line B G, and the line C G which is ioyned to the reſidual line is commẽſurable in lẽgth to the rationall line A. Wherfore the line B C is a fift reſiduall line. Wherfore there is found out a fift reſiduall line: which was required to be done.

Conſtruction.



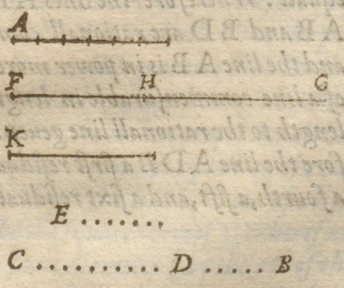
Demonſtration.

¶ The 24. Probleme. The 90. Propoſition.

To finde out a ſixth reſiduall line.

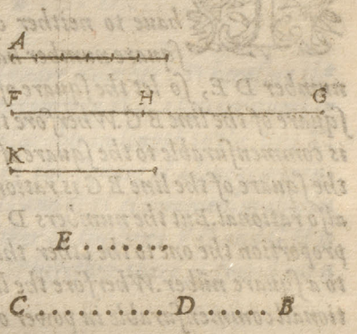
TAke a rational line and let the ſame be A. And take three numbers E, B C, and C D, not hauing the one to the other that proportion that a ſquare number hath to a ſquare number. And let not the number B C haue to the number B D that proportion that a ſquare number hath to a ſquare number. And let the number B C be greater then the number C D, & as the number E is to the number B C, ſo let the ſquare of the line A be to the ſquare of the lyne F G. And as the number B C is to the number C D, ſo let the ſquare

Conſtruction.



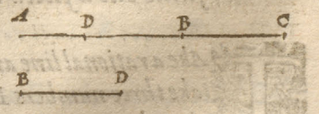
Demonstration.

of the line F G be to the ſquare of the line G H. Now therfore for that as the number E is to the nũber B C, ſo is the ſquare of the line A to the ſquare of the line F G, therfore the ſquare of the line A is commenſurable to the ſquare of the line F G. Wherfore the ſquare of the line F G is rationall, and the line F G is alſo rationall. And for that the number E hath not to the number B C that proportion that a ſquare number hath to a ſquare number, therfore the line A is incommenſurable in length to the line F G. Againe for that as the number B C is to the number C D, ſo is the ſquare of the line F G to the ſquare of the line G H, therefore the ſquare of the line F G is commenſurable to the ſquare of the line G H. But the ſquare of the line F G is rationall, wherfore the ſquare alſo of the line G H is rationall, wherfore the line G H is alſo rationall. And for that the number B C hath not to the number C D that proportion that a ſquare number hath to a ſquare number, therfore the line F G is incommenſurable in length to the line G H, and they are both rationall. Wherefore the lines F G and G H are rationall commenſurable in power onely. Wherfore the lyne F H is a reſiduall line. I ſay moreouer that it is a ſixt reſiduall line. For for that as the number E is to the number B C, ſo is the ſquare of the line A to the ſquare of the line F G, and as the number B C is to the nũber C D, ſo is the ſquare of the line F G to the ſquare of the line G H, therefore by equalitie of proportion as the number E is to the number C D, ſo is the ſquare of the line A to the ſquare of the line G H. But the number E hath not to the number C D that proportion that a ſquare number hath to a ſquare number. Wherefore the line A is incommenſurable in length to the line G H, and neither of theſe lines F G nor G H is commenſurable in length to the rationall line A. And foraſmuch as the ſquare of the line F G is greater then the ſquare of the line G H, vnto the ſquare of the line F G let the the ſquares of the lines G H and K be equall. Now therfore for that as the number B C is to the number C D, ſo is the ſquare of the line F G to the ſquare of the line G H, therefore by conuerſion of proportion as the number B C is to the number B D, ſo is the ſquare of the line F G to the ſquare of the line K. But the number B C hath not to the number B D that proportion that a ſquare number hath to a ſquare number, therfore the line F G is incommenſurable in length to the line K. Wherfore the line F G is in power more then the lyne G H by the ſquare of a line incommenſurable in length to the line F G, and neither of the lines F G nor G H is commenſurable in length to the rationall line A. Wherfore the line F H is a ſixt reſidual line. Wherfore there is found out a ſixt reſiduall line: which was required to be done.



An other more redie way to finde out the ſixe reſiduall lines.

There is alſo a certayne other redier way to finde out euery one of the forſayd ſixe reſiduall lines which is after this maner. Suppoſe that it were required to finde out a firſt reſiduall line. Take a firſt binomiall line A C, & let the greater name thereof be A B. And vnto the line B C let the line B D be equall. Wherefore the lines A B and B C, that is the lines A B and B D are rationall commenſurable in power onely, and the line A B is in power more then the line B C, that is, then the line B D by the ſquare of a line commenſurable in length to the line A B. And the line A B is commenſurable in length to the rationall line geuen. For the line A C is put to be a firſt binomiall line. Wherefore the line A D is a firſt reſidual line. And in like maner may ye finde out a ſecond, a third, a fourth, a fift, and a ſixt reſiduall line, if ye take for eche a binomiall line of the ſame order.



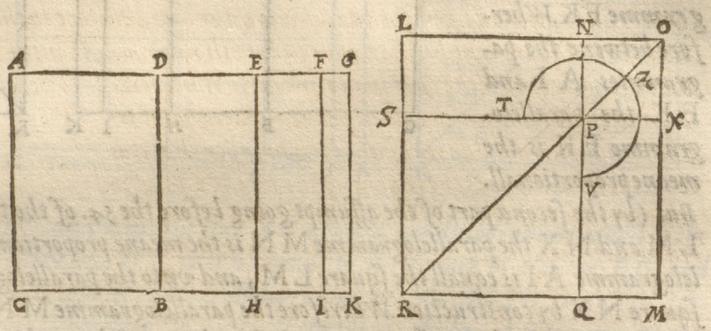
¶ The 67. Theoreme. The 91. Proposition.

If a superficies be contayned vnder a rationall line & a first residuall line: the line which contayneth in power that superficies, is a residuall line.

Fourth Senary.

SVppose that there be a rectangle superficies A B contayned vnder a rationall line A C and a first residuall line A D. Then I say that the line which contayneth in power the superficies A B is a residuall line. For forasmuch as A D is a first residuall line, let the line ioyned vnto it be D G (by the line ioyned vnto it vnderstand such a line as was spoken of in the end of the 79. proposition). Wherefore the lines A G and G D are rationall cõmẽsurable in power only, & the whole line A G is cõmensurable in length to the rationall line A C, and the line A G is in power more then the line G D by the square of a line commensurable in length vnto A G, by the definition of a first residuall line.

The first part of the Construction.



Deuide the line G D into two equall partes in the poynt E. And vpon the line A G apply a parallelogramme equall to the square of the line E G and wanting in figure by a square, and let the sayd parallelogramme be that which is cõtayned vnder the lines A F and F G. Wherefore the line A F is commensurable in length to the line F G (by the 17. of the tenth). And by the poyntes E, F and G, draw vnto the line A C these parallel lines E H, F I, and G K. And make perfect the parallelogrãme A K.

The first part of the demonstration.

And forasmuch as the line A F is commensurable in length to the line F G, therefore also the whole line A G is commensurable in length to either of the lines A F and F G (by the 15. of the tenth). But the line A G is commensurable in length to the line A C. Wherefore either of the lines A F and F G is commensurable in length to the line A C. But the line A C is rationall, wherefore either of the lines A F and F G is also rationall. Wherefore (by the 19. of the tenth) either of the parallelogrammes A I and F K is also rationall. And forasmuch as the line D E is commẽsurable in length to the line E G, therfore also (by the 15. of the tenth) the line D G is commensurable in length to either of the lines D E and E G. But the line D G is rationall, wherefore either of the lines D E and E G is rationall, and the selfe same line D G is incommensurable in length to the line A C (by the definition of a first residuall line, or by the 13. of the tenth). For the line D G is incommensurable in length to the line A G, which line A G is cõmensurable in length to the line A C: wherfore either of the lines D E and E G is rationall and incommensurable in length to the line A C. Wherefore (by the 21. of the tenth) either of these parallelogrammes D H and E K is mediall.

Note. A I and F K concluded rational parallelogramme.

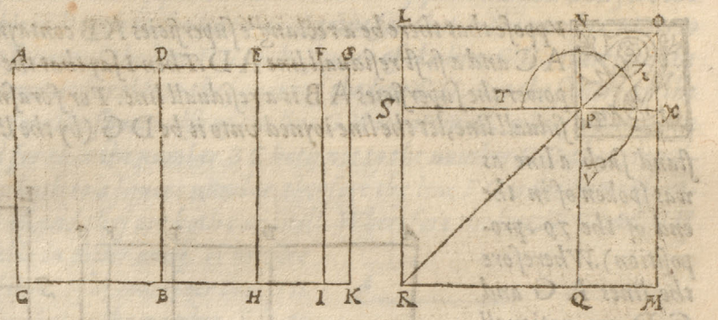
Note. D H and F K parallelogrammes mediall.

Vnto the parallelogramme A I let the square L M be equall, and vnto the parallelogramme F K let the square N X be equall, being taken away from the square L M, and hauing the angle L O M common to them both. (And to doo this, there must be founde out the meane proportionall betwene the lines F I and F G. For the square of the meane proportionall is equall to the parallelogramme contayned vnder the lines F I and F G. And from the line L O cut of a line equall to the meane proportionall so founde out, and describe the square thereof). Wherefore both the squares L M and N X are about one and the selfe same diameter (by the 26. of the

Second part of the construction.

Second part of the demonstration.

sixth) let their diameter be O R and describe the figure as it is here set forth. Now then forasmuch as the parallelogramme contayned vnder the lines A F & F G is equal to the square of the line E G, therefore (by the 17. of the sixth) as the line A F is to the line E G, so is the line E G to the line F G. But as the line A F is to the line E G, so is the parallelogramme A I to the parallelogramme E K. And as the line E G is to the line F G, so is the parallelogramme E K to the parallelogramme F K. Wherfore betwene the pagrammes A I and F K the parallelogramme E K is the meane proportionall.



But (by the second part of the assumpt going before the 54. of the tenth) betwene the squares L M and N X the parallelogramme M N is the meane proportionall. And vnto the parallelogramme A I is equall the square L M, and vnto the parallelogramme F K is equall the square N X by construction. Wherefore the parallelogramme M N is equall to the parallelogramme E K (by the 2. assumpt going before the 54. of the tenth). But the parallelogramme E K is (by the first of the sixth) equall to the parallelogramme D H, and the parallelogramme M N is (by the 43. of the first) equall to the parallelogramme L X. Wherefore the whole parallelogramme D K is equall to the gnomon V T Z (which gnomon consisteth of those parallelogrammes by which ye see in the figure paßeth a portion of a circle greater then a semicircle) and moreouer to the square N X: and the parallelogramme A K is equall to the squares L M and N X by construction: and it is now proued, that the parallelogramme D K is equall to the gnomō V T Z, and moreouer to the square N X. Wherfore the residue namely the parallelogramme A B is equall to the square S Q which is the square of the line L N. Wherefore the square of the line L N is equall to the parallelogramme A B. Wherefore the line L N contayneth in power the parallelogramme A B. I say moreouer that the line L N is a residuall line. For forasmuch as either of these parallelogrammes A I and F K is rationall, as it is before sayd, therefore the squares L M and N X which are equall vnto them, that is, the squares of the lines L O and O N are rationall. Wherefore the lines L O and O N are also rationall. Agayne forasmuch as the parallelogramme D H that is L X is mediall, therefore the parallelogramme L X is incommensurable to the square N X. Wherefore (by the 1. of the sixth, and 10. of the tenth) the line L O is incōmensurable in length to the line O N: and they are both rationall. Wherefore they are lines rationall commensurable in power onely. Wherefore L N is a residuall line by the definition, and it contayneth in power the parallelogramme A B. If therefore a superficies be contayned vnder a rationall line and a first residuall line, the line which contayneth in power that superficies, is a residuall line: which was required to be demonstrated.

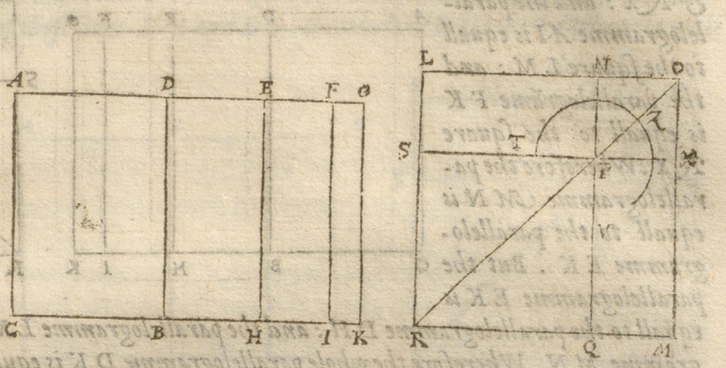
L N, is the onely line that we sought & considered.

¶ The 68. Theoreme. The 92. Proposition.

If a superficies be contained vnder a rationall line and a second residuall line: the line which containeth in power that superficies, is a first mediall residuall line.

Suppose

SVppose that A B be a superficies contained vnder a rationall line A C, and a second residuall line A D. Then I say, that the line that containeth in power the superficies A B is a first mediall residuall line. For let the line ioyned to the line A D be D G. Wherefore the lines A G and G D are rationall commensurable in power onely, and the line that is ioyned to the residual line, namely, the line D G is cõmensurable in length to the rationall line A C: and the line A G is in power more then the line D G, by the square of a line cõmensurable in length to the line A G. Deuide the line D G into two equall partes in the point E. And vnto the line A G apply a parallelogramme equall to the fourth part of the square of the line D G, that is, equall to the square of the line E G, and wanting in figure by a square, and let that parallelogramme be that which is contained vnder the lines A F and F G. Wherefore (by the 17. of the tenth) the line A F is commensurable in length to the line F G. And by the pointes E, F, and G, draw vnto the line A C these parallell lines E H, F I, and G K: and forasmuch as the line A F is cõmensurable in length to the line F G, therefore the whole line A G is commensurable in length to either of these lines A F and F G. But the line A G is rationall and incommensurable in length to the line A C. Wherefore either of these lines A F and F G are rationall and incommensurable in length to the line A C. Wherefore either of these parallelogrammes A I and F K is (by the 21. of the tenth) mediall. Againe, forasmuch as the line D E is commensurable in length to the line E G, therefore the line D G is commensurable in length to either of these lines D E and E G. But the line D G is commensurable in length to the rationall line A C. Wherefore either of these lines D E and E G is rationall and commensurable in length to the line A C. Wherefore (by the 19. of the tenth) either of these parallelogrammes D H and E K is rationall. Vnto the parallelogramme A I describe an equall square L M, and vnto the parallelogramme F K let the square N X be equall, as in the Proposition going before. Wherefore the squares L M and N X are both about one and the same diameter. Let the diameter be O R, and describe the figure as is in the former Proposition expressed. Now therefore forasmuch as the parallelogrammes A I and F K are mediall and * commensurable the one to the other, and the squares of the lines L O & O N which are equall to those parallelogrãmes, are mediall, therefore the lines L O and O N are also mediall commensurable in power. (And it is manifest, that the lines L O and O N are cõmensurable in power, for their squares are commensurable, and those squares, namely, the squares of the lines L O & O N are commensurable, for they are equall to the parallelogrammes A I and F K, which are commensurable the one to the other: and that those parallelogrammes A I and F K are commensurable the one to the other, hereby it is manifest, for that it was before proued that the lines A F and F G are commensurable in length. Wherefore (by the 1. of the sixt, and 10. of the tenth) the parallelogrammes A I and F K are commensurable the one to the other. Wherefore it is now manifest by the way of resolution, that the lines L O & O N are cõmensurable in power). And forasmuch as the parallelogramme contained vnder the lines A F and F G is equall to the square of the line E G, therefore as the line A F is to the line E G, so is the line E G to the line F G. But as the line A F is to the line E G, so is the parallelogramme A I to the parallelogramme E K: and as the line E G is to the line F G, so is the parallelogramme E K to



First part of the construction.

The first part of the demonstration.

A I and F K concluded parallelogrãmes mediall.

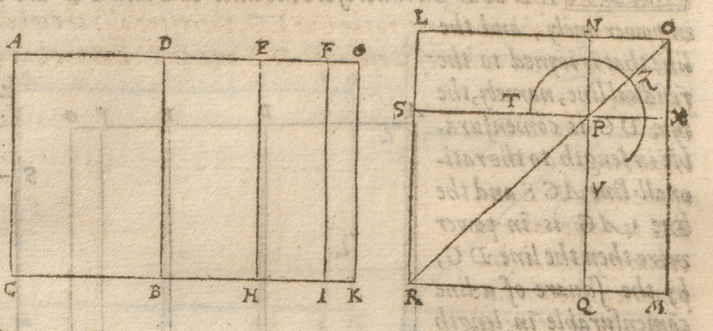
D H & E K, rationall.

The second part of the construction.

The second part of the demonstration.

* Analytically the prose hereof followeth amõge other thinges.

the parallelogramme F K. Wherefore the parallelogramme E K is the meane proportionall betwene the parallelogrammes A I and F K: and the parallelogramme M N is also the meane proportionall betwene the squares L M & N X: and the parallelogramme A I is equall to the square L M: and the parallelogrãme F K is equall to the square N X. Wherefore the parallelogramme M N is equall to the parallelogramme E K. But the parallelogramme E K is



equall to the parallelogramme D H: and the parallelogramme L X is equall to the parallelogramme M N. Wherefore the whole parallelogramme D K is equall to the Gnomon V T Z, and to the square N X. Wherefore the residue, namely, the parallelogramme A B is equall to the square S Q, that is, to the square of the line L N. Wherefore the line L N containeth in power the superficies A B. I say moreouer, that the line L N is a first mediall residuall line. For forasmuch as the parallelogramme E K is rationall, and is equall to the parallelogramme M N, that is, L X, therefore L X, that is, the parallelogramme contained vnder the lines L O and O N is rationall. But the square N X is mediall, for it is already proued that the parallelogramme F K which is equall to the square N X, is mediall. Wherefore the parallelogramme L X is incommensurable to the square N X. But as the parallelogramme L X is to the square N X, so is the line L O to the line O N (by the 1. of the sixt). Wherefore (by the 10. of the tenth) the lines L O and O N are incommensurable in length. And it is already proued, that they are mediall commensurable in power. Wherefore the lines L O & O N are mediall commensurable in power onely containing a rationall superficies. Wherefore the line L N is a first mediall residuall line, and containeth in power the superficies A B, which is contained vnder a rationall line and a second residuall line. If therefore a superficies be contained vnder a rationall line and a second residuall line: the line which containeth in power that superficies, is a first mediall residuall line: which was required to be demonstrated.

The line L N found, which is the principall drift of all the former discourse.

¶ The 69. Theoreme. The 93. Proposition.

If a superficies be contained vnder a rationall line and a third residuall line: the line that containeth in power that superficies is a second mediall residuall line.

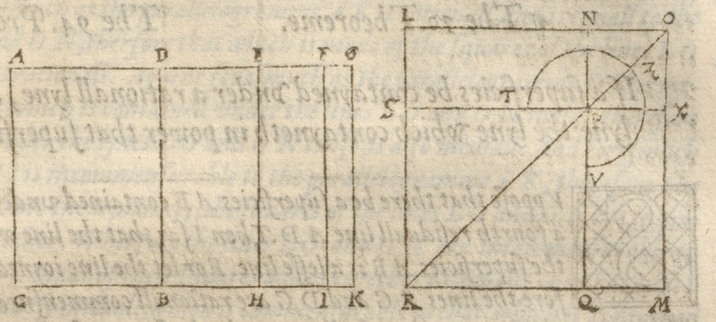
SVppose that A B be a superficies contained vnder a rationall line A C, & a third residuall line A D. Then I say, that the line which containeth in power the superficies A B is a second mediall residuall line. Let the line ioyned vnto A D, be D G. Wherefore the lines A G and G D are rationall commensurable in power onely, and neither of the lines A G nor G D is commensurable in length to the rationall line A C, and the whole line A G is in power more then the line G D, by the square of a line commensurable in length to the line A G. Let the rest of the construction be as it was in the former Propositions. Wherefore the lines A F and F G are commensurable in length: and the parallelogramme A I is commensurable to the parallelogramme F K. And forasmuch as the lines A F and F G are commensurable in length, therefore the whole line A G is commensu-

The first part of the Construction.

The first part of the demonstration.

rable

rable in length to either of these lines A F and F G. But the line A G is rationall and incommensurable in length to the line A C. Wherefore either of these lines A F and F G is rationall and incommensurable in length to the line A C. Wherefore (by the 21. of the tenth) either of these parallelogrammes A I and F K is mediall. Againe, forasmuch as the line D E is commensurable in length to the line E G, therefore also the whole line D G is commensurable in length to either of these lines D E and E G. But the line D G is rationall commensurable in power only to the line A C. Wherfore also either of the lines D E and E G is rationall and commensurable in power only to the line A C. Wherefore either of these parallelogrammes D H and E K is mediall. Againe forasmuch as the lines A G and D G are commensurable in power onely, therefore they are incommensurable in length. But the line A G is commensurable in length to the line A F: and the line D G is commensurable in length to the line G E. Wherefore the line A F is incommensurable in length to the line E G. But as the line A F is to the line E G, so is the parallelogramme A I to the parallelogramme E K. Wherefore the parallelogramme A I is incommensurable to the parallelogramme E K. Vnto the parallelogramme A I describe an equall square L M: and vnto the parallelogramme F K describe an equall square N X: and describe the figure as you did in the former Proposition. Now forasmuch as the parallelogramme contained vnder the lines A F and F G is equall to the square of the line E G, therefore as the line A F is to the line E G, so is the line E G to the line F G. But as the line A F is to the line E G, so is the parallelogramme A I to the parallelogramme E K: and as the line E G is to the line F G, so is the parallelogramme E K to the parallelogramme F K. Wherefore as the parallelogrāme A I is to the parallelogramme E K, so is the parallelogramme E K to the parallelogramme F K. Wherefore the parallelogramme E K is the meane proportionall betwene the parallelogrāmes A I and F K. But the parallelogramme M N is the meane proportionall betwene the squares L M and N X. Wherefore the parallelogramme E K is equall to the parallelogramme M N. Wherefore the whole parallelogramme D K is equall to the Gnomon V T Z, & to the square N X. And the parallelogramme A K is equall to the squares L M and N X. Wherefore the residue, namely, the parallelogramme A B is equall to the square Q S, that is, to the square of the line L N. Wherefore the line L N containeth in power the superficies A B. I say moreouer, that the line L N is a second mediall residuall line. For for that as it is proued, the parallelogrammes A I and F K are mediall, therefore the squares that are equall vnto them, namely, the squares of the lines L O and O N are also mediall. Wherefore either of these lines L O and O N is mediall. And forasmuch as the parallelogramme A I is * commensurable to the parallelogramme F K, therefore the squares that are equall to them, namely, the squares of the lines L O and O N are also commensurable. Againe, forasmuch as it is proued, that the parallelogramme A I is incommensurable to the parallelogramme E K, therfore the square L M is incommensurable to the parallelogramme M N, that is, the square of the line L O to the parallelogramme contained vnder the lines L O & O N. Wherfore also the * line L O is incommensurable in length to the line O N. Wherefore the lines L O and O N are mediall commensurable in power onely. I say moreouer, that they containe a mediall superficies. For forasmuch as it is proued, that the parallelogramme E K is mediall, therefore the



Note. A I and F K mediall.

Note D H and E K mediall.

Note. A I incommensurable to E K. Second part of the Construction.

The principall line, L N, founde.

* Because the lines A F and F G are proued commensurable in length.

* By the first of the sixth and tenth of the tenth.

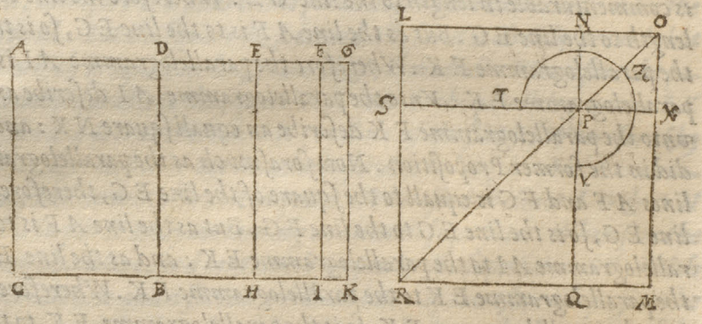
parallelogramme which is equall vnto it, namely, the parallelogramme contained vnder the lines L O and O N is also mediall. Wherefore the line L N is a second mediall residuall line, and containeth in power the superficies A B. Wherefore the line that containeth in power the superficies A B is a second mediall residuall line. If therefore a superficies be contained vnder a rationall line and a third residuall line, the line that containeth in power that superficies, is a second mediall residuall line: which was required to be demonstrated.

¶ The 70. Theoreme. The 94. Proposition.

If a superficies be contayned vnder a rationall lyne, and a fourth residuall lyne: the lyne which contayneth in power that superficies, is a lesse lyne.

SVppose that there be a superficies A B contained vnder a rationall line A C, and a fourth residuall line A D. Then I say that the line which containeth in power the superficies A B is a lesse line. For let the line ioyned vnto it be D G. Wherefore the lines A G and D G are rationall commensurable in power only, and the line A G is in power more then the line D G, by the square of a line incōmensurable in length to the line A G, and the line A G is commensurable in length to the line A C. Deuide the line D G into two equall partes in the point E. And vnto the line A G apply a parallelogramme equal to the

The first part of the construction.



square of the line E G, and wanting in figure by a square, and let that parallelograme be that which is contayned vnder the lines A F and F G. Wherfore (by the 18. of the tenth) the line A F is incōmensurable in length to the line F G. Draw by the pointes E, F, & G, vnto the lines A C and D B these parallel lines E H, F I, and G K. Now forasmuch as the line A G is rational, and commensurable in length to the line A C, therfore the whole parallelogramme A K is (by the 19. of the tenth) rational. Againe forasmuch as the line D G is incommensurable in length to the line A C (for if the line D G were commensurable in length to the line A C, then forasmuch as the line A G is commensurable in length to the same line A C, the lines A G and D G should be commensurable in length the one to the other, when yet they are put to be commensurable in power onely) and both these lines A C and D G are rationall. Wherfore the parallelograme D K is mediall. Againe forasmuch as the line A F is incommensurable in length to the line F G, therfore the parallelogramme A I is incommensurable to the parallelogramme F K. Vnto the parallelogramme A I describe an equal square L M, and vnto the parallelograme F K describe an equall square N X, and let the angle L O M be common to both those squares. Wherefore the squares L M and N X are about one and the selfe same diameter. Let their diameter be O R, and describe the figure. And forasmuch as the parallelograme contained vnder the lines A F and F G is equall to the square of the line E G, therfore proportionally as the line A F is to the line E G, so is the line E G to the line F G, but as the line A F is to the line E G, so is the parallelogramme A I to the parallelogramme E K (by the 1. of the sixt.) And as the line E G is to the line F G, so is the parallelogramme E K to the parallelogramme F K. Wherfore

The first part of the demonstration.

Note. A K rational.

Note. D K mediall. A I and F K incommensurable.

The second part of the construction.

The second part of the demonstration.

the

the parallelogramme E K is the meane proportionall betwene the parallelogrammes A I and F K: wherfore as it was said in the former proposition, the parallelogramme M N is equal to the parallelogramme E K: but the parallelogramme D H is equall to the parallelograme E K, and the parallelogramme M N to the parallelogramme L X. Wherfore the whole parallelogramme D K is equall to the gnomon V T Z, and to the square N X. Wherfore the residue, namely, the parallelogramme A B is equall to the residue, namely, to the square S Q, that is to the square of the line L N. I say moreouer that L N is that irrationall line which is called a lesse line. For forasmuch as the parallelogramme A K is rationall, and is equall to the squares of the lines L O and O N, therfore that which is made of the squares of the lines L O and O N added together is rationall. Againe forasmuch as the parallelogramme D K is mediall, and is equall to that which is contained vnder the lines L O and O N twise, therefore that which is contained vnder the lynes L O and O N twise, is also mediall. And forasmuch as the parallelogramme A I is incommensurable to the parallelogramme F K, therefore the squares which are equall vnto them, namely, the squares of the lines L O and O N are incommensurable the one to the other. Wherfore the lines L O and O N are incommensurable in power, hauyng that which is made of their squares added together rationall, and that which is contained vnder them twise mediall, which is commensurable to that which is contayned vnder them once. Wherfore that which is contained vnder them once is also mediall. Wherfore L N is that irrationall line which is called a lesse line, and it containeth in power the superficies A B. If therfore a superficies be contained vnder a rationall line and a fourth residuall line, the line which containeth in power that superficies is a lesse line: which was required to be demonstrated.

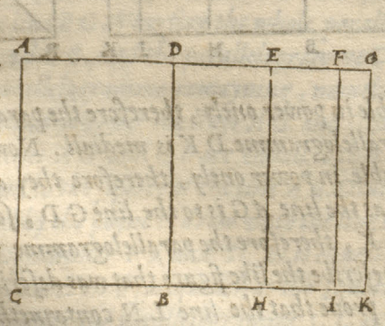
L N (the chiefe line of this theoreme) founde.

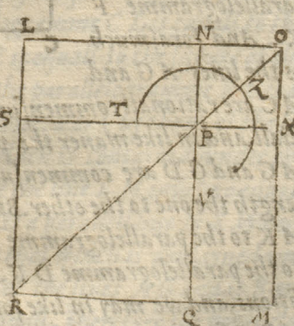
¶ The 71. Theoreme. The 95. Proposition.

If a superficies be contained vnder a rationall line and a fift residual line: the line that cõtayneth in power the same superficies, is a line making with a rationall superficies, the whole superficies mediall.

Vppose that there be a superficies A B contained vnder a rationall line A C and a fift residuall line A D. Thẽ I say that the line that cõtaineth in power ỹ superficies A B, is a line making with a rationall superficies the whole superficies mediall. For vnto the line A D let the line D G be ioyned, which shal be cõmẽsurable in lẽgth to the rational line A C. And let the rest of the constructiõ be as in the proposition next going before. And forasmuch as the line A G is incõmensurable in lẽgth to the line A C and they are both rationall, therfore the parallelogrãme A K is medial. Againe forasmuch as the line D G is rationall and commensurable in length to the lyne A C, therefore the parallelogramme D K is rationall. Vnto the parallelogramme A I describe an equall square L M, and vnto the parallelograme F K describe an equall square N X, and as in the proposition next going

Demonstration.





ing

The line LN.

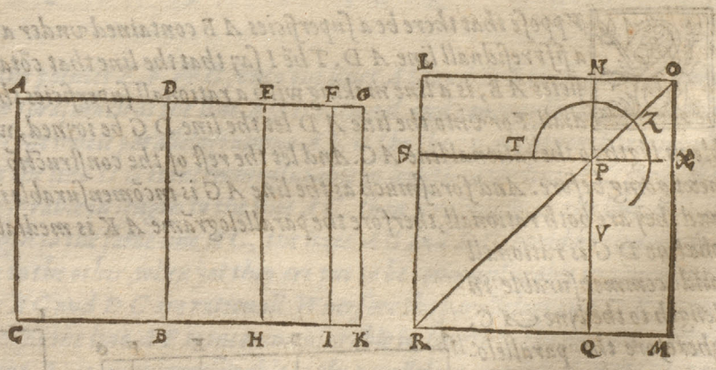
ing before, so also in this may we proue, that the line L N containeth in power the superficies A B. I say moreouer that that line L N is a line making with a rationall superficies the whole superficies mediall. For forasmuch as the parallelogramme A K is mediall, therefore that which is equall vnto it, namely, that which is made of the squares of the lines L O and O N added together is also mediall. Againe forasmuch as the parallelogramme D K is rationall, therfore that which is equall vnto it, namely, that which is contained vnder the lines L O and O N twise, is also rationall. And forasmuch as the line A F is incommensurable in length to the line F G, therfore (by the 1. of the sixt, & 10. of the tenth) the parallelogräme A I is incommensurable to the parallelogramme F K, wherfore also the square of the lyne L O is incommensurable to the square of the line O N. Wherfore the lines L O and O N are incommensurable in power hauing that which is made of their squares added together mediall, and that which is contayned vnder them twise rationall. Wherfore the line L N is that irrationall line which is called a lyne making with a rationall superficies the whole superficies mediall, and it contayneth in power the superficies A B. Wherfore the line contayning in power the superficies A B, is a line making with a rationall superficies the whole superficies mediall. If therfore a superficies be contayned vnder a rationall lyne & a fift residuall line, the line that contayneth in power the same superficies, is a line makyng with a rationall superficies the whole superficies mediall: which was required to be proued.

¶ The 72. Theoreme. The 96. Proposition.

If a superficies be contayned vnder a rationall line and a sixth residuall line, the line which contayneth in power the same superficies is a line making with a mediall superficies the whole superficies mediall.

Demonstration.

Vppose that A B be a superficies contayned vnder a rationall line A C, & a sixt residuall line A D. Then I say that the line which contayneth in power the superficies A B, is a line making with a medial superficies the whole superficies mediall. For vnto the line A D let the line D G be ioyned. And let the rest be as in the propositions going before. And forasmuch as the line A F is incommensurable in length to the line F G, therefore the parallelogramme A I is incōmensurable to the parallelogramme F K. And forasmuch as the lines A G and A C are rationall commensurable in power onely, therefore the parallelogramme A K is mediall, and in like maner the parallelogramme D K is mediall. Now forasmuch as the lines A G and G D are commensurable in power onely, therefore they are incommensurable in length the one to the other. But as the line A G is to the line G D, so is the parallelogramme A K to the parallelogramme D K, therefore the parallelogramme A K is incommensurable to the parallelogramme D K. Describe the like figure that was described in the former propositions, and we may in like sort proue that the line L N contayneth in power the superficies A B. I say moreouer that it is a line making with a mediall superficies the whole superficies



mediall

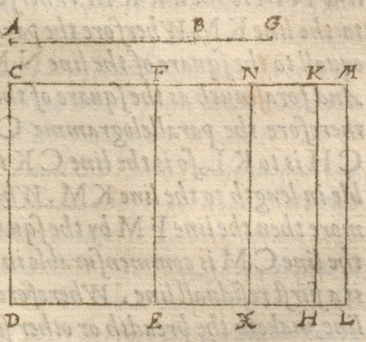
mediall. For the parallelograme A K is mediall, wherefore that which is equal vnto it, namely, that which is made of the ſquares of the lines L O and O N added together is alſo mediall. And foraſmuch as the parallelogramme D K is mediall, therefore that which is equall vnto it, namely, that which is contayned vnder the lines L O and O N twiſe is alſo mediall. And foraſmuch as the parallogramme A K is incommenſurable to the parallelogramme D K, therefore the ſquares of the lines L O and O N are incommenſurable to that which is contained vnder the lines L O and O N twiſe. And foraſmuch as the parallelogramme A I is incommenſurable to the parallelogramme F K, therefore alſo the ſquare of the line L O is incommenſurable to the ſquare of the line O N. Wherefore the lines L O and O N are incommenſurable in power, hauing that which is made of the ſquares of the lines L O and O N mediall, and that which is contayned vnder them twiſe mediall, and moreouer that which is made of the ſquares of them is incommenſurable to that which is contayned vnder them twiſe. Wherfore the line L N is that irrationall line which is called a line making with a mediall ſuperficies the whole ſuperficies mediall, and it contayneth in power the ſuperficies A B. Wherefore the line which contayneth in power the ſuperficies A B is a line making with a mediall ſuperficies the whole ſuperficies mediall. If therefore a ſuperficies be contayned vnder a rationall line and a ſixth reſiduall line, the line which contayneth in power the ſame ſuperficies is a line making which a mediall ſuperficies the whole ſuperficies mediall. which was required to be demonſtrated.

¶ The 73. Theoreme. The 97. Propoſition.

The ſquare of a reſiduall line applyed vnto a rationall line, maketh the breadth or other ſide a firſt reſiduall line. — The fiueth Senary.

These ſixe propoſitions following are the conuerſes of the ſixe former propoſitions.

Construction.

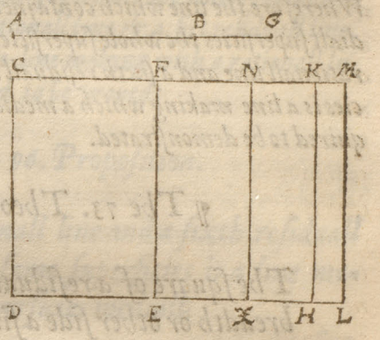


SVppoſe that A B be a reſiduall line, and let C D be a rationall line. And vnto the line C D apply the parallelogramme C E equall to the ſquare of the line A B, and making the breadth the line C F. Then I ſay that the line C F is a firſt reſiduall line. For vnto the line A B let the line cõueniently ioyned be ſuppoſed to be B G (which ſelfe line is alſo called a line ioyned, as we declared in the end of the 79. propoſition). Wherefore the lines A G and G B are rationall commenſurable in power onely. And vnto the line C D apply the parallelogramme C H equall to the ſquare of the line A G, and vnto the line K H (which is equall to the line C D) apply the parallelogramme K L equall to the ſquare of the line B G. Wherfore the whole parallelogramme C L is equall to the ſquares of the lines A G and G B. And the parallelogramme C E is equall to the ſquare of the line A B, wherefore the parallelogramme remayning, namely, the parallelogramme F L is equall to that which is contayned vnder the lines A G and G B twiſe. For (by the 7. of the ſecond) the ſquares of the lines A G and G B are equall to that which is contayned vnder the lines A G & G B twiſe, and to the ſquare of the line A B. Deuide the line F M into two equall partes in the point N. And by the poynt N draw vnto the line C D a parallell line N X. Wherfore either of the parallelogrammes F X and N L is equall to that which is contayned vnder the lines A G and G B once. And foraſmuch as the ſquares of the lines A G and G B are rationall, vnto which ſquares the parallelogramme C L is equall, therefore the parallelogramme C L alſo is rationall: wherefore the line G M is rationall and* commenſurable in length

Demonſtration.

*By the 20. of the tenth.

length to the line C D. Agayne forasmuch as that which is contayned vnder the lines A G and G B twise, is ** mediall, therefore the parallelogramme equall vnto it, namely the parallelogramme F L is also mediall. Wherefore the line F M is rationall and * incommensurable in length to the line C D. And forasmuch as the squares of the lines A G and G B are rationall, and that which is contayned vnder the lines A G and G B twise, is mediall, therefore the squares of the lines A G and G B are incōmensurable to that which is contayned vnder the lines A G and G B twise. But vnto the squares of the lines A G and G B is equall the parallelogramme C L, and to that which is contayned vnder the lines A G and G B twise is equall the parallelogramme F L, wherefore the parallelogramme C L is incommensurable to the parallelogramme F L. Wherefore also the line C M is incommensurable in length to the line F M: and they are both rationall. Wherefore the lines C M and F M are rationall commensurable in power onely: and therefore the line C F is a residuall line (by the 73. of the tenth). I say moreouer that it is a first residuall line. For forasmuch as that which is contayned vnder the lines A G and G B is the meane proportionall betwene the squares of the lines A G and G B (by the assumpt going before the 54. of the tenth). And vnto the square of the line A G is equall the parallelogramme C H, and vnto that which is contayned vnder the lines A G and G B is equall the parallelogramme N L, and vnto the square of the line G B is equall the parallelograme K L. Wherefore the parallelogramme N L is the meane proportionall betwene the parallelogrammes C H and K L. Wherefore as C H is to N L, so is N L to K L. But as C H is to N L, so is the line C K to the line N M & as N L is to K L, so is the line N M to the line K M. Wherfore as the line C K is to the line N M, so is the line N M to the line K M. Wherfore the parallelogramme contayned vnder the lines C K and K M is equall to the square of the line N M, that is to the fourth part of the square of the line F M. And forasmuch as the square of the line A G is commensurable to the square of the line G B, therefore the parallelogramme C H is cōmensurable to the parallelogramme K L. But as C H is to K L, so is the line C K to the line K M: wherefore the line C K is commensurable in length to the line K M. Wherefore (by the 17. of the tenth) the line C M is in power more then the line F M by the square of a line commensurable in length to the line C M. But the line C M is commensurable in length to the rationall line C D. Wherefore the line C F is a fiirst residuall line. Wherefore the square of a residuall line applyed vnto a rationall line, maketh the breadth or other side a first residuall line: which was required to be demonstrated.



** By the 21. of the tenth.

* By the 22. of the tenth.

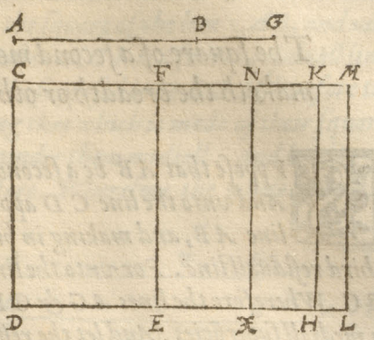
C F cōcluded a residual line.

¶ The 14. Theoreme. The 98. Proposition.

The square of a first mediall residuall line applied to a rationall line, maketh the breadth or other side a second residuall line.

Construction.

SVppose that A B be a first mediall residual line, and let C D be a rationall line. And vnto the line C D apply the parallelogramme C E equall to the square of the lyne A B, and making in breadth the line C F. Then I say that the line C F is a second residuall



reſiduall line. For vnto the line A B, let the lyne cõueniently ioyned be ſuppoſed to be B G. Wherefore the lines A G and B G are mediall commenſurable in power onely, comprehending a rationall ſuperficies. And vnto the line C D apply the parallelogramme C H equal to the ſquare of the line A G, and making in bredth the line C K, and vnto the line K H (which is equall to the line C D) apply the parallelogramme K L equall to the ſquare of the line G B and making in breadth the line K M. Wherfore the whole parallelogramme C L is equall to both the ſquares of the lines A G and G B which are mediall & commenſurable the one to the other. Wherfore the parallelogrammes C H and K L are mediall and commenſurable the one to the other. Wherfore (by the 15. of the tenth) the whole parallelogramme C L is commenſurable to either of theſe parallelogrammes C H and K L. Wherfore (by the corollary of the 23. of the tenth) the whole parallelogramme C L is alſo mediall. Wherefore (by the 22. of the tenth) the line C M is rationall and incommenſurable in length to the line C D. And foraſmuch as the parallelogramme C L is equal to the ſquares of the lines A G and G B: and the ſquares of the lines A G and G B are equall to that which is cõtained vnder the lines A G and G B twiſe, together with the ſquare of the line A B (by the 7. of the ſecond): and vnto the ſquare of the line A B is equall the parallelogramme C E. Wherfore the reſidue, namely, that which is contained vnder the lines A G and G B twiſe, is equal to the reſidue, namely, to the parallelogrãme F L. But that which is contained vnder the lines A G & G B twiſe is rational. Wherfore the parallelogramme F L is alſo rationall. Wherfore the line F M is rationall and cõmenſurable in length to the line C D (by the 20. of the tenth.) Now foraſmuch as the parallelogramme C L is mediall, and the parallelogramme F L is rational, therfore they are incõmenſurable the one to the other. Wherfore alſo the lyne C M is incommenſurable in length to the lyne F M, and they are both rationall. Wherfore the line C F is a reſiduall line. I ſay morcouer that it is a ſecond reſiduall lyne. For deuide the line F M into two equall partes in the point N, from which point draw vnto the line C D a parallel line N X. Wherfore either of theſe parallelogrammes F X and N L is equall to the parallelogramme contained vnder the lines A G and G B. And foraſmuch as the parallelogramme contained vnder the lines A G and G B is the meane proportionall betwene the ſquares of the lines A G and B G. Therefore the parallelogramme N L is the meane proportionall betwene the parallelogrammes C H and K L. But as C H is to N L, ſo is the line C K to the line N M, and as N L is to K L, ſo is the line N M to the line K M. Wherfore as the line C K is to the lyne N M, ſo is the lyne N M to the line K M. Wherfore the parallelogrãme contayned vnder the lines C K and K M is equall to the ſquare of the line N M, that is, to the fourth part of the ſquare of the lyne F M. But the parallelogrãmme C H is commenſurable to the parallelogramme K L. Wherfore alſo the lyne C K is commenſurable in length to the lyne K M. Wherfore (by the 17. of the tenth) the line C M is in power more then the line F M, by the ſquare of a line commenſurable in length to the line C M. And the line F M which is the line conueniently ioyned, is commenſurable in length to the rationall lyne C D. Wherfore the lint C F is a ſecond reſiduall lyne. Wherfore the ſquare of a firſt mediall reſidual line applied to a rationall line, maketh the breadth or other ſide a ſecond reſiduall lyne: which was required to be proued.

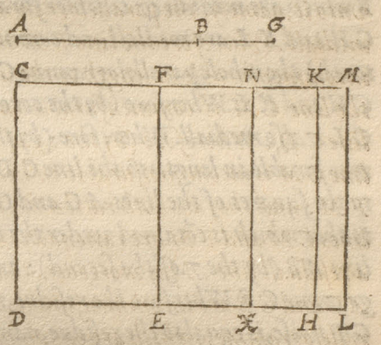
Demonſtration.

C F concluded a reſiduall line.

¶ The 75. Theoreme. The 99. Proposition.

The square of a second mediall residuall line applied vnto a rationall line, maketh the breadth or other side a third residuall line.

Construction. *Vppose that A B be a second mediall residuall line, and let C D be a rationall line. And vnto the line C D apply the parallelogramme C E equall to the square of the line A B, and making in breadth the line C F. Then I say, that the line C F is a third residuall line. For vnto the line A B let the line conueniently ioyned be supposed to be B G. Wherefore the lines A G & G B are mediall commensurable in power onely containing a mediall superficies. And let the rest of the construction be as in the Proposition next going before. Wherefore the line C M is rationall and incommensurable in length to the rationall line C D. And either of the parallelogrammes F X and N L is equall to that which is contained vnder the lines A G and G B. But that which is contayned vnder the lines A G & G B is mediall. Wherfore that which is contained vnder the lines A G and G B twise is also mediall. Wherfore the whole parallelogramme F L is also mediall. Wherefore the line F M is rationall and incommensurable in length to the line C D. And forasmuch as the lines A G and G B are incōmensurable in length, therefore also the square of the line A G is incommensurable to the parallelogramme contayned vnder the lines A G and G B. But vnto the square of the line A G are commensurable the squares of the lines A G and G B: and vnto the parallelogramme contained vnder the lines A G and G B, is commensurable that which is contained vnder the lines A G and G B twise. Wherefore the squares of the lines A G and G B are incommensurable to that which is contained vnder the lines A G and G B twise. Wherefore the parallelogrammes which are equall vnto them, namely, the parallelogrammes C L and F L are incommensurable the one to the other. Wherefore also the line C M is incommensurable in length to the line F M: and they are both rationall. Wherefore the line C F is a residuall line. I say moreouer, that it is a third residuall line. For forasmuch as the square of the line A G, that is, the parallelogramme C H is commensurable to the square of the line B G, that is, to the parallelogramme K L, therefore the line C K is commensurable in length to the line K M. And in like sort as in the former Proposition, so also in this may we proue, that the parallelogramme contayned vnder the lines C K and K M, is equall to the square of the line N M, that is, to the fourth part of the square of the line F M. Wherefore the line C M is in power more then the line F M, by the square of a line commensurable in length to the line C M: and neither of the lines C M nor F M is commensurable in length to the rationall line C D. Wherefore the line C F is a third residual line. Wherfore the square of a second mediall residual line applied vnto a rationall line, maketh the breadth or other side a third residuall line: which was required to be demonstrated.*



Demonstration.

C F concluded a residual line.

¶ The 76. Theoreme. The 100. Proposition.

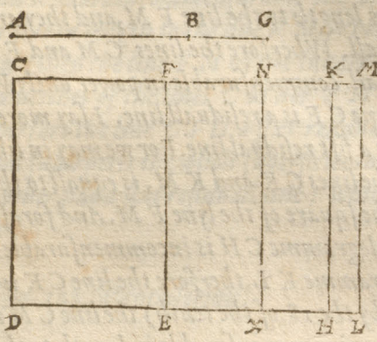
The square of a lesse line applied vnto a rationall line, maketh the breadth or other side a fourth residuall line.

Suppose

SVppoſe that A B be a leſſe line, and let C D be a rationall line. And vnto the line C D apply the parallelogramme C E equall to the ſquare of the line A B, and making in breadth the line C F. Then I ſay, that the line C F is a fourth reſiduall line. For vnto the line A B let the line conueniently ioyned be ſuppoſed to be B G. Wherefore the lines A G & G B are incommenſurable in power, hauing that which is made of their ſquares added together rationall, and that which is contained vnder them mediall. And let the reſt of the conſtruction be as in the Propoſitions going before. Wherefore the whole parallelogramme C L is rationall. Wherefore the line C M is alſo rationall, and commenſurable in length to the line C D. And foraſmuch as that which is contained vnder the lines A G and G B twiſe is mediall, therefore the parallelogramme which is equall vnto it, namely, the parallelogramme F L, is alſo mediall. Wherefore the line F M is rationall and incommenſurable in length to the line C D. But the line C M is commenſurable in length to the line C D. Wherefore (by the 13. of the tenth) the line C M is incommenſurable in length to the line F M: and they are both rationall. Wherefore the lines C M and F M are rationall commenſurable in power onely. Wherefore the line C F is a reſiduall line. I ſay moreouer, that it is a fourth reſiduall line. For foraſmuch as the lines A G and G B are incommenſurable in power, therefore the ſquares of them, that is, the parallelogrammes, which are equall vnto them, namely, the parallelogrammes C H and K L, are incommenſurable the one to the other. Wherefore alſo the line C K is incommenſurable in length to the line K M. And in like ſort may we proue, that the parallelogramme contayned vnder the lines C K and K M, is equall to the ſquare of the line N M, that is, to the fourth part of the ſquare of the line F M. Wherefore (by the 18. of the tenth) the line C M is in power more then the line F M, by the ſquare of a line incommenſurable in length to the line C M. And the whole line C M is commenſurable in length to the rationall line C D. Wherefore the line C F is a fourth reſiduall line. Wherefore the ſquare of a leſſe line applied vnto a rationall line, maketh the breadth or other ſide a fourth reſiduall line: which was required to be proued.

Conſtruction.

Demonſtration.



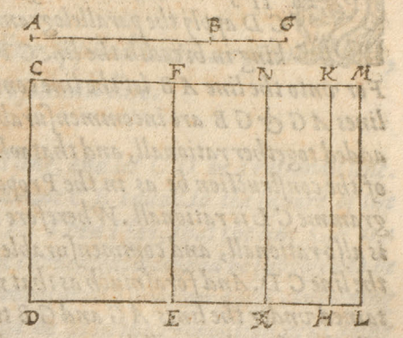
C F proued a reſiduall line.

¶ The 77. Theoreme. The 101. Propoſition.

The ſquare of a lyne making with a rationall ſuperficies the whole ſuperficies mediall applied vnto a rational line, maketh the breadth or other ſide a fift reſiduall lyne.

SVppoſe that A B be a line making with a rationall ſuperficies the whole ſuperficies mediall, and let C D be a rationall line. And vnto the line C D apply the parallelogramme C E equall to the ſquare of the line A B, and making in breadth the line C F. Then I ſay that the line C F is a fift reſiduall line. For vnto the line A B let the line conueniently ioyned be ſuppoſed to be B G. Wherfore the lines A G and G B are incommenſurable in power, hauing that which is made of their ſquares added together mediall, and that which is contained vnder them rationall. Let the reſt of the conſtruction be in this as it was in the former propoſitions. Wherfore the whole parallelogramme C L is mediall. Wherefore the line C M is rationall and incommenſurable in length to the line C D. And either of the

parallelogramme F X & N L is rationall. Wherfore the whole parallelogramme F L is also rationall. Wherfore also the line F M is rationall and commensurable in legth to the line C D. And forasmuch as the parallelogramme C L is mediall, and the parallelogramme F L is rationall, therefore C L and F L are incommensurable the one to the other, and the line C M is incommensurable in length to the line F M, and they are both rationall. Wherfore the lines C M and F M are rationall commensurable in power onely. Wherfore the lyne C F is a resíduall line. I say moreouer that it is a fift resídual line. For we may in like sort proue, that the parallelograme contained vnder the lines C K and K M, is equall to the square of the line N M, that is, to the fourth part of the square of the lyne F M. And forasmuch as the square of the line A G, that is, the parallelogramme C H is incommensurable to the square of the line B G, that is to the parallelogramme K L, therfore the line C K is incommensurable in length to the line K M. Wherfore (by the 18. of the tenth) the line C M is in power more then the line F M, by the square of a line incommensurable in length to the line C M. And the line conueniently ioyned, namely, the line F M is commensurable in length to the rationall line C D. Wherfore the line C F is a fift resíduall line. Wherfore the line C F is a fift resíduall line. Wherfore the square of a line making with a rationall superficies the whole superficies medial, applied vnto a rational line maketh the breadth or other side a fift resíduall lyne: which was required to be demonstrated.



C F proued a resíduall line.

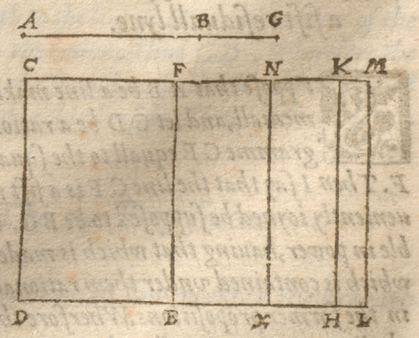
¶ The 78. Theoreme. The 102. Proposition.

The square of a lyne making with a mediall superficies, the whole superficies mediall applied to a rationall line, maketh the breadth or other side, a sixt resíduall line.

Construction.

Vppose that A B be a line making with a mediall superficies, the whole superficies mediall, and let C D be a rationall line. And vnto the line C D apply the parallelogramme C E equall to the square of the line A B and making in breadth the line C F. Then I say that the line C F is a sixt resídual line. For vnto the line A B let the line conueniently ioyned be B G. Wherfore the lines A G and B G are incommensurable in power hauing that which is made of their squares added together mediall, & that which is contained vnder them mediall, and moreouer that which is made of their squares added together is incommensurable to that which is contained vnder them. Let the rest of the construction be in this, as it was in the propositiõs going before. Wherfore the whole parallelogramme G L is mediall, (for it is equall to that which is made of the squares of the lines A G & G B added together, which is supposed to be mediall). Wherefore the line C M is rationall and incommensurable in length to the line C D: and in like manner the parallelogramme F L is mediall. Wherfore also the line F M is rationall and incommensurable in length to the line C D. And forasmuch as that

Demonstration.



which

which is made of the ſquares of the lines A G and G B added together, is incommenſurable to that which is contained vnder the lines A G and G B twiſe, therefore the parallelogrãmes equall to them, namely, the parallelogrammes C L and F L are incommenſurable the one to the other. Wherfore alſo the lines C M and F M are incommenſurable in length, and they are both rational. Wherfore they are rationall cõmenſurable in power only. Wherfore the line C F is a reſiduall line. I ſay moreouer that it is a ſixt reſiduall line. Let the reſt of the demonſtration be as it was in the former propoſitions. And foraſmuch as the lines A G and B G are incommenſurable in power, therfore their ſquares, that is, the parallelogrammes which are equall vnto them, namely, the parallelogrammes C H and K L are incommenſurable the one to the other. Wherfore alſo the line C K is incommenſurable in length to the line K M. Wherfore (by the 18. of the tenth) the line C M is in power more then the line F M by the ſquare of a line incommenſurable in length to the line C M. And neither of the lines C M nor F M is commenſurable in length to the rationall line C D. Wherfore the line C F is a ſixt reſiduall line. Wherfore the ſquare of a line making with a medial ſuperficies the whole ſuperficies mediall applied to a rationall line, maketh the breadth or other ſide a ſixt reſiduall line: which was required to be demonſtrated.

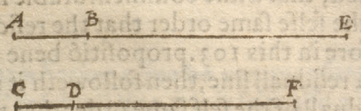
C F bruedd a reſiduall.

¶ The 79. Theoreme. The 103. Propoſition.

A line commenſurable in length to a reſiduall line: is it ſelfe alſo a reſiduall line of the ſelfe ſame order.

The ſixt Senary.

SVppoſe that A B be a reſiduall line, vnto which let the line C D be commenſurable in length. Then I ſay, that the line C D is alſo a reſiduall line, and of the ſelfe ſame order of reſiduall lines that the line A B is. For foraſmuch as the line A B is a reſiduall line, let the line conueniently ioyned vnto it be ſuppoſed to be B E. Wherefore the lines A E and B E are rationall commenſurable in power onely. As the line A B is to the line C D, ſo (by the 12. of the ſixt) let the line B E be to the line D F. Wherefore (by the 12. of the fift) as one of the antecedentes is to one of the conſequentes, ſo are all the antecedentes to all the conſequentes. Wherefore as the line A B is to the line C D, ſo is the whole line A E to the whole line C F, and the line B E to the line D F. Wherefore (by the 10. of the tenth) the line A E is commenſurable in length to the line C F, and the line B E to the line D F. But the line A E is rationall. Wherefore the line C F is alſo rationall. And in like ſort the line D F is rationall, for that the line B E, to whom it is commenſurable, is alſo rationall. And for that as the line B E is to the line A E, ſo is the line D F to the line C F. But the lines B E and A E are commenſurable in power onely: Wherefore the lines C D and D F are commenſurable in power onely. Wherefore the line C D is a reſiduall line. I ſay moreouer, that it is a reſiduall line of the ſelfe ſame order that the line A B is. For for that as we haue before ſaid, as the line A E is to the line C F, ſo is the line B E to the line D F, therefore alternately, as the line A E is to the line B E, ſo is the line C F to the line D F. But the line A E is in power more then the line E B, either by the ſquare of a line commenſurable in length to the line A E, or by the ſquare of a line incommenſurable in length to the line A E. If A E be in power more then B E, by the ſquare of a line commenſurable in length to A E, then the line C F ſhall alſo (by the 14. of the tenth) be in power more then the line D F, by the ſquare of a line commenſurable in length to the line C F, and ſo if the line A E be commenſurable in length to the rationall line put, foraſmuch as the line

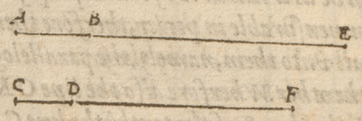


Conſtruction.

Demonſtration.

C D cõcluded a reſiduall line.

AE is commensurable in length to the line CF, therefore (by the 12. of the tenth) the line CF shall also be commensurable in length to the same rationall line. Wherefore either of the
1. *lines AB and CD is a first residuall line. And if the line BE be commensurable in length to the rationall line put, forasmuch as the line BE is commensurable in length to the line DF, therefore the line DF shall also be commensurable in length to the rationall line put:*
2. *and then either of the lines AB and CD is a second residuall line. And if neither of the lines AE nor BE be commensurable in length to the rationall line put, then neither of the lines CF nor DF shall be commensurable in length to the same rationall line (by the 13. of the teth). And so either*



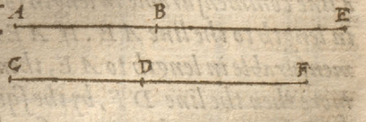
3. *of the lines AB & CD is a third residuall line. But if the line AE be in power more the the line BE, by the square of a line incomensurable in length to the line AE, the line CF shall in like sort (by the 14. of the tenth) be in power more then the line DF, by the square of a line incommensurable in length to the line CF: and then if the line AE be commensurable in length to the rationall line, the line CF shall also in like sort be*
4. *commensurable in length to the same rationall line: and so either of the lines AB and CD is a fourth residuall line. And if the line BE be cõmẽsurable in lẽgth to the rationall line, the line DF shall also be cõmensurable in lẽgth to the same line: and so either of the lines AB &*
5. *CD is a fift residuall line. And if neither of the lines AE nor BE be commensurable in length to the rationall line, in like sort neither of the lines CF nor DF shall be cõmensurable*
6. *in lẽgth to the same rational line. And so either of the lines AB & CD is a sixt residual line. Wherefore the line CD is a residuall line of the selfe same order that the line AB is. A line therfore commensurable in length to a residuall line, is it selfe also a residuall line of the selfe same order: which was required to be proued.*

Note. As before touching binomiall lines, so also touching residuall lines, this is to be noted, that a line commensurable in length to a residuall line, is alwayes a residuall line of the selfe same order that the residuall line is, vnto whom it is cõmensurable, as hath before in this 103. propositiõ bene proued. But if a line be cõmensurable in power only to a residuall line, then followeth it not, yea it is impossible, that that line should be a residuall of the self same order that the residual line, is vnto whom it is commensurable in power onely. Howbeit those two lines shall of necessitie be both either of the three first orders of residuall lines, or of the three last orders: which is not hard to proue, if ye marke diligently the former demonstration, and that which was spoken of binomiall lines as touching this matter.

¶ The 80. Theoreme. The 104. Proposition.

A line commensurable to a mediall residuall line, is it selfe also a medial residuall line, and of the selfe same order.

Construction. *Demonstration.* *Vppose that* AB *be a mediall residuall line, vnto whome let the line* CD *be commensurable in length and in power, or in power onely. Then I say that* CD *is also a mediall residuall line, and of the selfe same order. For forasmuch as the line* AB *is a mediall residuall line, let the line conueniently ioyned vnto it be* BE: *wherefore the lines* AE *and* BE *are mediall commensurable in power onely. As* AB *is to* CD, *so by the 22. of the sixth) let* BE *be to* DF. *And in like sort as in the former so also in this may we proue, that the line* AE *is commensurable in length and in power, or in power one-*



ly vnto

ly vnto the line C F, *& the line* B E *to the line* D F. *Wherefore (by the 23. of the tenth) the line* C F *is a mediall line, and the line* D F *is also a mediall line, for that it is commensurable to the mediall line* B E. *And in like sort the lines* C F *and* D F *are commensurable in power onely: for that they haue the selfe same proportiõ the one to the other, that the lines* A E *and* E B *haue, which are commensurable in power onely. Wherefore the line* C D *is a mediall residuall line. I say moreouer that it is of the selfe same order that the line* A B *is. For for that as the line* A E *is to the line* B E, *so is the line* C F *to the line* D F. *But as the line* A E *is to the line* B E, *so is the square of the line* A E *to the parallelogramme contayned vnder the lines* A E *and* B E *(by the first of the sixth): and as the line* C F *is to the line* D F, *so is the square of the line* C F *to the parallelogramme contayned vnder the lines* C F *and* D F. *Wherefore as the square of the line* A E *is to the parallelogramme contayned vnder the lines* A E *and* B E, *so is the square of the line* C F *to the parallelogramme contayned vnder the lines* C F *and* D F. *Wherefore alternately as the square of the line* A E *is to the square of the line* C F, *so is the parallelogramme contayned vnder the lines* A E *and* B E, *to the parallelogramme contained vnder the lines* C F *and* D F. *But the square of the line* A E *is commensurable to the square of the line* C F *(for the line* A E *is commensurable to the line* C F). *Wherefore also the parallelogramme contayned vnder the lines* A E *and* B E, *is commensurable to the parallelogramme contayned vnder the lines* C F *and* D F. *Wherefore if the parallelogramme contayned vnder the lines* A E *and* E B *be rationall, the parallelogramme also contayned vnder the lines* C F *and* F D *shall be rationall. And then either of the lines* A B *and* C D *is a first mediall residuall line. But if the parallelogramme contayned vnder the lines* A E *and* B E *be mediall, the parallelogramme also contayned vnder the lines* C F *and* F D *shall be also mediall (by the corollary of the 23. of the tẽth): and so either of the lines* A B *and* C D *is a second mediall residuall line. Wherefore the line* C D *is a mediall residuall line of the selfe same order that the line* A B *is. A line therefore commensurable to a mediall residuall line, is it selfe also a mediall residuall line of the selfe same order: which was required to be demonstrated.*

C D proued a mediall.

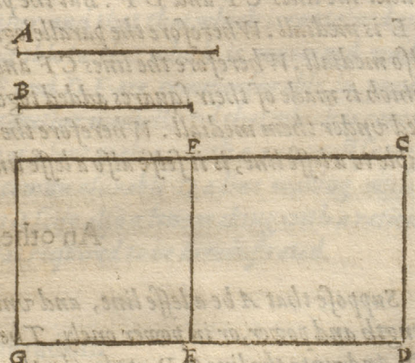
1.

2.

This Theoreme is vnderstanded generally, that whether a line be commensurable in length & in power, or in power onely to a mediall residuall line, it is it selfe also a mediall residuall line, and of the selfe same order, which thing also is to be vnderstanded of the three Theoremes which follow.

An other demonstration after *Campane*.

Suppose that *A* be a mediall residuall line, vnto whome let the line *B* be commensurable in length, or in power onely. And take a rationall line *C D*, vnto which apply the parallelogramme *C E* equall to the square of the line *A*, and vnto the line *F E* (which is equall to the line *C D*) apply the parallelogramme *F G* eqnall to the square of the line *B*. Now then the parallelogrammes *C E* and *F G* shall be commensurable; for that the lines *A*, *B* are commensurable in power: wherefore by the 1. of the sixth and 10. of this booke, the lines *D E* and *F G* are commensurable in length. Now then if *A* be a first mediall residuall line, then is the line *D E* a second residuall line by the 98. of this booke: and if the line *A* be a second mediall residuall line, then is the line *D E* a third residuall line by the 99. of this booke. But if *D E* be a second residuall line, *G E* also shall be a second residuall line (by the 103. of this boke). And if *D E* be a third re-



Construction.

Demonstration.

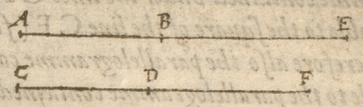
siduall line, *G E* also shall (by the same) be also a third residuall line. Wherefore it followeth by the 92. and 93. of this booke, that *B* is either a first medial residuall line or a second mediall residuall line, according as the line *A* is supposed to be: which was required to be proued.

¶ The 81. Theoreme. The 105. Proposition.

A line commensurable to a lesse line: is it selfe also a lesse line.

Construction. **Demonstration.**

SVppose that A B be a lesse line, vnto whom let the line C D be commensurable. Then I say, that the line C D is also a lesse line. For let the same construction be in this, that was in the former Propositions. And forasmuch as the lines A E and E B are incommensurable in power, therefore (by the 22. of the sixt, and 10. of the tenth) the lines C F & F D are incommensurable in power. Againe (by the 22. of the sixt) as the square of the line A E is to the square of the line B E, so is the square of the line C F to the square of the line D F. Wherfore by cõposition as the squares of the lines A E and B E are to the square of the line B E, so are the squares of the lines C F and D F, to the square of the line D F: and alternately, as the squares of the lines A E and B E are to the squares of the lines C F and D F, so is the square of the line B E to the square of the line D F. But the square of the line B E is commensurable to the square of the line D F (for the lines B E and D F are commensurable). Wherefore that which is made of the squares of the lines A E and B E added together, is commensurable to that which is made of the squares of the lines C F and D F added together. But that which is made of the squares of the lines A E and B E added together, is rationall. Wherefore that which is made of the squares of the lines C F and D F added together, is also rationall. Againe, for that as the square of the line A E is to the parallelogramme contained vnder the lines A E and B E, so is the square of the line C F to the parallelogramme contained vnder the lines C F and D F (as we declared in the Proposition next going before): therefore alternately, as the square of the line A E is to the square of the line C F, so is the parallelogramme contained vnder the lines A E and B E, to the parallelogramme contained vnder the lines C F and D F. But the square of the line A E is commensurable to the square of the line C F, for the lines A E & C F are commensurable. Wherefore the parallelogramme contained vnder the lines A E and B E, is commensurable to the parallelogramme contained vnder the lines C F and D F. But the parallelogramme contained vnder the lines A E and B E is mediall. Wherefore the parallelogramme contained vnder the lines C F and D F is also mediall. Wherefore the lines C F and D F are incommensurable in power, hauing that which is made of their squares added together rationall, and the parallelogramme contained vnder them mediall. Wherefore the line C D is a lesse line. A line therefore commensurable to a lesse line, is it selfe also a lesse line: which was required to be proued.



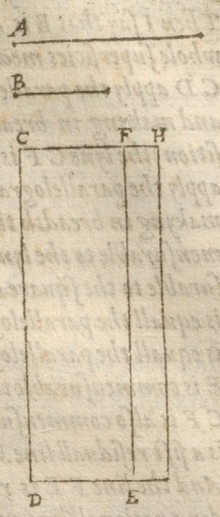
An other demonstration.

Construction.

Suppose that A be a lesse line, and vnto A let the line B be commensurable whether in length and power, or in power onely. Then I say that B is a lesse line. Take a rationall line C D. And vnto the line C D apply (by the 44. of the first) the parallelogramme C E equall to the square of the line A, and making in bredth the line C F. Wherefore (by the 100. propo-

sition

ſition) the line C F is a fourth reſiduall line. Vnto the line F E apply (by the ſame) the parallelogramme E H equall to the ſquare of the line B, and making in breadth the line F H. Now foraſmuch as the line A is commenſurable to the line B, therefore alſo the ſquare of the line A is cõmenſurable to the ſquare of the line B. But vnto the ſquare of the line A is equall the parallelogramme C E, & vnto the ſquare of the line B is equal the parallelogramme E H. Wherfore the parallelogramme C E is commenſurable to the parallelogramme E H. But as the parallelogramme C E is to the parallelogramme E H, ſo is the line C F to the line F H. Wherfore the line C F is commenſurable in length to the line F H. But the line C F is a fourth reſiduall line. Wherfore the line F H is alſo a fourth reſiduall line (by the 103. of the tenth): and the line F E is rationall. But if a ſuperficies be contained vnder a rationall line, and a fourth reſiduall lyne, the line that containeth in power that ſuperficies is (by the 94. of the tenth) a leſſe lyne. But the line B containeth in power the ſuperficies E H. Wherfore the line B is a leſſe line: which was required to be proued.

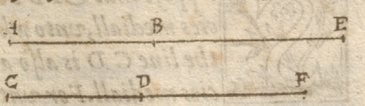


Demonſtration.

¶ The 82. Theoreme. The 106. Propoſition.

A line commenſurable to a lyne making with a rationall ſuperficies the whole ſuperficies mediall, is it ſelfe alſo a lyne making with a rationall ſuperficies the whole ſuperficies mediall.

Vppoſe that A B be a line making with a rationall ſuperficies the whole ſuperficies mediall, vnto whom let the line C D be commenſurable. Then I ſay that the line C D is a line making with a rationall ſuperficies the whole ſuperficies mediall. Vnto the line A B let the line conueniently ioyned be B E. Wherefore the lines A E and E B are incommenſurable in power, hauing that which is made of their ſquares added together mediall, and the parallelogramme contained vnder them rationall. Let the conſtruction be in this as it was in the former propoſitions. And in like ſort may we proue that as the line A E is to the line B E, ſo is the line C F to the line D F, and that that which is made of the ſquares of the lines A E and B E added together is commenſurable to that which is made of the ſquares of the lines C F and D F added together, and that that which is contained vnder the lynes A E and E B, is in like ſort commenſurable to that which is contained vnder the lines C F and D F. Wherfore alſo the lines C F and D F are commenſurable in power, hauing that which is made of their ſquares added together mediall, and that which is contained vnder them rationall. Wherfore the line C D is a lyne making with a rationall ſuperficies the whole ſuperficies mediall. Wherfore a line commenſurable to a line making with a rationall ſuperficies the whole ſuperficies mediall, is it ſelfe alſo a lyne making with a rational ſuperficies the whole ſuperficies mediall: which was required to be demonſtrated.



Conſtruction.

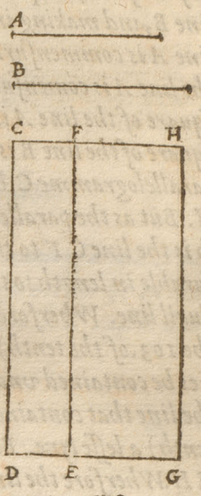
Demonſtration.

An other demonſtration.

Suppoſe that A be a line making with a rationall ſuperficies the whole ſuperficies mediall, and

and vnto it let the lyne B be commensurable either in length and in power, or in power onely.

Construction. Then I say that B is a lyne making with a rationall superficies the whole superficies mediall. Take a rational line C D, and vnto the line C D apply the parallelogramme C E equal to the square of the line A and makyng in breadth the lyne C F. Wherfore (by the 101. proposition) the lyne C F is a fift residuall lyne. Againe vnto the line F E apply the parallelogramme F G equall to the square of the line B, and makyng in breadth the lyne F H. Now forasmuch as the line A is cõmensurable to the lyne B, therfore the square of the lyne A is commẽsurable to the square of the line B. But vnto the square of the lyne A is equall the parallelogramme C E, and vnto the square of the line B is equall the parallelogramme F G. Wherfore the parallelogramme C E is commensurable to the parallelogramme F G. Wherefore the line C F is also commensurable in length to the line F H. But the line C F is a fift residuall line. Wherfore also the line F H is a fift residual line. And the line F E is rationall. But if a superficies be contayned vnder a rationall line and a fift residuall lyne, the lyne that contayneth in power that superficies, is (by the 95. of the tenth) a lyne making with a rationall superficies the whole superficies mediall. But the lyne B containeth in power the parallelogramme F G. Wherfore the lyne B is a lyne making with a rationall superficies, the whole superficies mediall: which was required to be demonstrated.

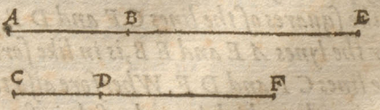


Demonstration.

¶ The 83. Theoreme. The 107. Proposition.

A line cõmensurable to a line, making with a mediall superficies, the whole superficies mediall, is it selfe also a line making with a mediall superficies the whole superficies mediall.

Vppose that A B be a line making with a mediall superficies the whole superficies mediall, vnto whome let the line C D be commensurable. Then I say that the line C D is also a line making with a mediall superficies the whole superficies mediall. For vnto the line A B let the line conueniently ioyned be B E. And let the rest of the construction be in this as it was in the former propositions. Wherefore the lines A E and B E are incõmensurable in power, hauing that which is made of their squares added together mediall, and that which is contained vnder them also mediall, and moreouer that which is made of their squares added together is incommensurable to that which is contained vnder them. But the lines A E and B E (as we haue before proued) are commensurable to the lines C F & D F, and that which is made of the squares of the lines A E and B E added together, is commensurable to that which is made of the squares of the lines C F and F D added together, and the parallelogramme contained vnder the lines A E and B E is commensurable to the parallelogramme contained vnder the lines C F and D F. Wherfore the lines C F and D F are incommensurable in power hauing that which is made of their squares added together mediall, and that which is contained vnder them also mediall, and moreouer that which is made of their squares added together, is incommensurable to that which is contained vnder them. Wherfore the line C D is a line making with a mediall superficies the whole superficies mediall. A line therefore commensurable to a lyne



Construction.

Demonstration.

making

making with a mediall ſuperficies the whole ſuperficies mediall, is it ſelfe alſo a line making with a mediall ſuperficies the whole ſuperficies mediall: which was required to be proued.

This propoſition may alſo be an other way demonſtrated, as the three former propoſitions were. If vpon a rationall line you apply parallelogrammes equall to the ſquares of the lines A B and C D, the breadthes of which parallelogrames ſhall be eche a ſixth reſidual line, and therfore the lines which cõtayne them in power, namely, the lines A B and C D ſhall be both ſuch lines as is required in the propoſition, which is eaſy to conclude marking the order of the demonſtration in the three former propoſitions.

¶ The 84. Theoreme. The 108. Propoſition.

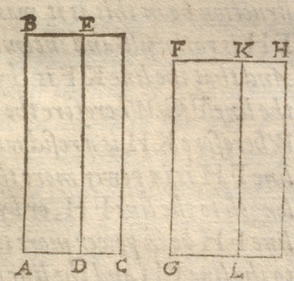
If from a rationall ſuperficies be taken away a mediall ſuperficies, the line which containeth in power the ſuperficies remayning, is one of theſe two irrationall lines, namely, either a reſiduall line, or a leſſe line.

Seuenth Senary.

Vppoſe that B C be a rationall ſuperficies, and from it take away a mediall ſuperficies, namely, B D. Then I ſay, that the line which containeth in power the ſuperficies remayning, namely, the ſuperficies E C, is one of theſe two irrationall lines, namely, either a reſiduall line, or a leſſe line. Take a rationall line F G.

Conſtruction.

And vpon F G deſcribe (by the 44. of the firſt) a rectangle parallelogramme G H equall to the ſuperficies B C. And from the parallelogramme G H take away the parallelogramme G K equall to the ſuperficies B D. Wherefore (by the third common ſentence) the ſuperficies remayning, namely, E C, is equall to the parallelogramme remayning, namely, to L H. And foraſmuch as B C is rationall, and B D is mediall, and B C is equall to the parallelogramme G H, and B D to the parallelogramme G K: therefore G H is rationall, and G K is mediall: and the parallelogramme G H is applied vnto the rationall line F G. Wherefore (by the 20. of the tenth) the line F H is rationall and commenſurable in length to the line F G. And the parallelogramme G K is alſo applied vnto the rationall line F G.



Demonſtration.

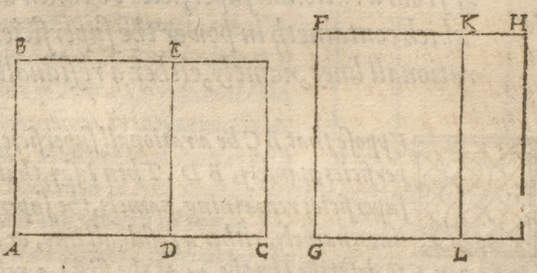
Wherefore (by the 22. of the tenth) the line F K is rationall and incommenſurable in length to the line F G. Wherefore (by the Aſſumpt of the 12. of the tenth) the line F H is incommenſurable in length to the line F K. And they are both rationall. Wherefore the lines F H and F K are rationall commenſurable in power onely. Wherefore the line K H is a reſiduall line: and the line conueniently ioyned vnto it is K F. Now the line F H is in power more then the line K F, either by the ſquare of a line commenſurable in length to the line F H, or by the ſquare of a line incommenſurable in length to the line F H. Firſt let it be in power more then the line F K, by the ſquare of a line commenſurable in length to the line F H, and the whole line F H is commenſurable in length to the rationall line put, namely, to F G. Wherefore the line K H is a firſt reſiduall line. But if a ſuperficies be contained vnder a rationall line, and a firſt reſiduall line, the line that containeth in power that ſuperficies, is (by the 91. of the tenth) a reſiduall line. Wherefore the line which containeth in power L H, that is, the ſuperficies E C, is a reſiduall line. But if the line H F be in power more then the line F K, by the ſquare of a line incommenſurable in length to the line F H, and the whole line F H is commenſurable in length to the rationall line geuen F G. Wherefore the line K H is a fourth reſiduall line. But a line containing in power a ſuperficies contained vnder a rationall line and a fourth reſiduall line, is a leſſe line (by the 94. of the tenth). Wherefore the line that containeth in power the ſuperficies L H, that is, the ſuperficies E C, is a leſſe line. If therefore from a rationall ſuperficies be taken away a mediall ſuperficies, the line which containeth

in

in power the superficies remayning, is one of these two irrationall lines, namely, either a residuall line, or a lesse line: which was required to be proued.

¶ The 85. Theoreme. The 109. Proposition.

If from a mediall superficies be taken away a rationall superficies, the line which contayneth in power the superficies remayning is one of these two irrationall lines, namely either a first mediall residuall line, or a line making with a rationall superficies the whole superficies mediall.



Vppose that B C *be a mediall superficies and from it take away a rationall superficies, namely,* B D. *Then I say that the line which contayneth in power the superficies remayning, namely, the superficies* E C *is one of these two irrationall lines, either a first mediall residuall line, or a line making with a rationall superficies the whole superficies mediall.*

Construction. *Take a rationall line* F G, *and let the rest of the construction be in this as it was in the former proposition. Wherefore it followeth that the line* F H *is rationall and incommensurable in length to the line* F G (*by the 22. of the tenth*).

Demonstration. *And that the line* K F *is (by the 20. of the tenth) rationall and commensurable in length to the line* F G. *Wherefore the lines* F H *and* F K *are rationall commensurable in power onely. Wherefore* K H *is a residuall line. And the line conueniently ioyned vnto it is* F K. *Now the line* F H *is in power more then the line* F K, *either by the square of a line commensurable in*

1. *length to the line* F H, *or by the square of a line incommensurable in length vnto it. If the line* F H *be in power more then the line* F K *by the square of a line commensurable in length to the line* F H *and the line cõueniently ioyned vnto it, namely,* F K, *is cõmensurable in legth*

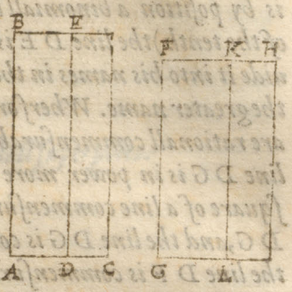
2. *to the rationall line* F G. *Wherefore the line* K H *is a second residuall line. And the line* F G *is a rationall line. But a line contayning in power a superficies comprehended vnder a rationall line and a second residuall line is (by the 92. of the tenth) a first mediall residuall line. Wherefore the line that contayneth in power the superficies* L H, *that is, the superficies* C E *is a first mediall residuall lne. But if the line* H F *be in power more then the line* F K *by the square of a line incommensurable in length to the line* F H, *and the line conueniently ioyned, namely, the line* F K *is commensurable in length to the rationall line put, namely, to* F G: *wherefore the line* K H *is a fift residuall line. Wherefore (by the 95. of the tenth) the line that contayneth in power the superficies* L H, *that is, the superficies* E C, *is a line making with a rationall superficies, the whole superficies mediall: which was required to be proued.*

¶ The 86. Theoreme. The 110. Proposition.

If from a mediall superficies be taken away a mediall superficies incommensurable to the whole superficies, the line which containeth in power the superficies which remaineth, is one of these two irrationall lines, namely, either a second mediall residuall line, or a line making with a mediall superficies the whole superficies mediall.

AS in the former descriptions, so here also take away frō the mediall superficies B C a mediall superficies B D, and let B D be incommensurable to the whole superficies B C. Then I say, that the line which containeth in power the superficies E C, is one of these two irrationall lines, namely, either a second mediall residuall line, or a line making with a mediall superficies the whole superficies mediall. For forasmuch as either of these superficiecs B C and B D is mediall, and B C is incommensurable to B D, it followeth (by the 22. of the tenth) that either of these lines F H and F K is rationall and incommensurable in length to the line F G. And forasmuch as the superficies B C is incommensurable to the superficies B D, that is, the superficies G H, to the superficies G K, therefore (by the first of the sixt, & 10. of the tenth) the line F H is incommensurable in length to the line F K. Wherfore the lines H F and F K are rational cōmensurable in power onely. Wherefore (by the 73. of the tenth) the line K H is a residuall line, and the line cōueniently ioyned vnto it is F K. Now the line H F is in power more then the line F K, either by the square of a line cōmensurable in length to the line H F, or by the square of a line incommensurable in length vnto it. If the line H F be in power more then the line F K, by the square of a line cōmensurable in length to the line F H, and neither of the lines H F nor F K is commensurable to the rationall line put F G. Wherfore the line K H is a third residuall. But the line G F, that is, the line K L, is rationall. And a rectangle superficies contained vnder a rationall line and a third residuall line, is irrationall, and the line which containeth in power that superficies, is (by the 93. of the tenth) a second mediall residuall line. Wherefore the line that containeth in power the superficies L H, that is, the superficies E C is a second mediall residuall line. But if the line H F be in power more then the line F K, by the square of a line incommensurable in length to the line F H, and neither of the lines H F nor F K is commensurable in length to the line F G. Wherefore the line H K is a sixt residuall line. But a line containing in power a superficies contained vnder a rationall line and a sixt residuall line, is (by the 96. of the tenth) a line making with a mediall superficies the whole superficies mediall. Wherefore the line that containeth in power the superficies L H, that is, the superficies E C, is a line making with a mediall superficies the whole superficies mediall. If therefore from a mediall superficies be taken away a mediall superficies, incommensurable to the whole superficies, the line that containeth in power the superficies which remaineth, is one of the two irrationall lines remaining, namely, either a second mediall residuall line, or a line making with a mediall superficies the whole superficies mediall: which was required to be proued.

Demonstration.



1.

2.

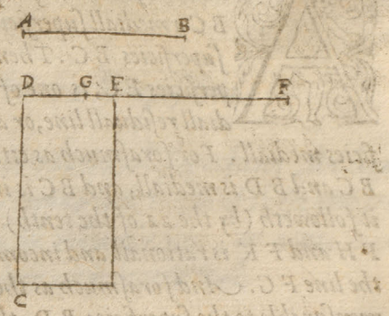
¶ The 87. Theoreme. The 111. Proposition.

A residuall line, is not one and the same with a binomiall lyne.

SVppose that A B be a residuall line. Then I say that A B is not one and the same with a binomiall line. For if it be possible let it be a binomiall line. And take a rationall line D C. And (by the 44. of the first) vnto the line C D apply a rectangle parallelogramme C E equall to the square of the line A B, and making in bredth the line D E. And forasmuch as A B is a residuall line, therfore (by the 97. of the tenth) the line D E is a first residuall line. Let the line conueniently ioyned vnto it be E F. Wherfore the lines D F and

Construction.

Demonstration leading to an impossibilitie.



F and F E are rationall commensurable in power only, and the line D F is in power more then the line F E by the square of a line commensurable in length to the line D E & the line D F, is cõmensurable in lẽgth to the rational line put D C. Again forasmuch as A B is by position a binomiall line, therefore (by the 60. of the tenth) the line D E is a first binomiall line. Deuide it into his names in the point G. And let D G be the greater name. Wherfore the lines D G and G E are rationall commensurable in power onely. And the line D G is in power more then the line G E by the square of a line commensurable in length to the lyne D G, and the line D G is commensurable in length to the rationall line put D C. Wherefore the line D F is commensurable in length to the line D G. Wherfore (by the 15. of the tenth) the whole line D F is commensurable in lẽgth to the line remaining, namely, to the line G F. And forasmuch as the line D F is cõmẽsurable to the line F G, but the line F D is rationall. Wherfore the line F G is also rationall. And forasmuch as the line F D is commensurable in length to the line F G, but the line D F is incommensurable in length to the line F E. Wherfore the line F G is incommensurable in length to the line F E (by the 13. of the tenth) and they are both rationall lines. Wherfore the lines G F and F E are rationall commensurable in power onely. Wherfore (by the 73. of the tenth) the line E G is a residuall line, but it is also rationall (as before hath bene proued): which is impossible, namely, that one & the same line should be both rationall and irrationall. Wherfore a residuall line is not one and the same with a binomiall line, that is, is not a binomiall line: which was required to be demonstrated.

A Corollary.

¶ *A Corollary.*

A residuall lyne and the other fiue irrationall lynes following it, are neither mediall lines, nor one and the same betwene themselues. that is, one is vtterly of a diuers kinde frõ an other. For the square of a mediall line applied to a rationall line, maketh the breadth rationall and incommensurable in length to the rationall lyne, whereunto it is applied (by the 22. of the tenth) The square of a residuall line applied to a rationall line, maketh the breadth a first residuall line (by the 97. of the tenth). The square of a first mediall residuall line applied to a rationall line, maketh the breadth a second residuall lyne (by the 98. of the tenth) The square of a second mediall residuall line applied vnto a rationall line, maketh the breadth a third residuall line (by the 99. of the tenth) The square of a lesse line applied to a rationall line, maketh the breadth a fourth residuall line (by the 100. of the tenth) The square of a line making with a rationall superficies the whole superficies mediall applied to a rationall line, maketh the breadth a fift residuall line (by the 101. of the tenth) And the square of a line making with a mediall superficies the whole superficies mediall applied to a rationall line, maketh the breadth a sixt residuall line (by the 102. of the tẽth) Now forasmuch as these foresaid sides which are the breadthes differ both from the first breadth, for that it is rational, and differ also the one frõ the other, for that they are residuals of diuers orders and kindes, it is manifest that those irrationall lines differ also the one from the other. And forasmuch as it hath bene proued in the 111. proposition, that a residual line is not one and the same with a binomiall line, and it hath also bene proued that the squares of a residuall line and of the fiue irrationall lines that follow it being applied to a rational line do make their breadthes one of the residuals of that order of which they were, whose squares were applied to the rationall line, likewise also the squares of a binomiall line, and of the fiue irrationall lines which follow it, being applied to a rationall line, do make the breadthes one

of the binomials of that order of which they were, whose ſquares were applied to the rationall line. Wherfore the irrationall lines which follow the binomiall line, and the irrationall lines which follow the reſiduall line, differ the one from the other, ſo that all the irrationall lynes are 13. in number, namely, theſe.

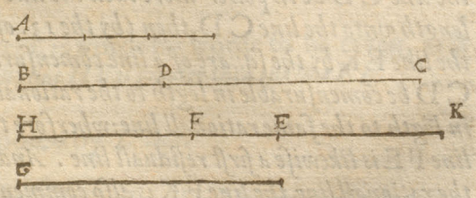
1 A mediall line.
2 A binomiall line.
3 A firſt bimediall line.
4 A ſecond bimediall line.
5 A greater line.
6 A line containing in power a rationall ſuperficies and a mediall ſuperficies.
7 A line contayning in power two mediall ſuperficieces.
8 A reſiduall line.
9 A firſt mediall reſiduall line.
10 A ſecond mediall reſiduall line.
11 A leſſe line.
12 A line making with a rationall ſuperficies the whole ſuperficies mediall.
13 A line making with a mediall ſuperficies the whole ſuperficies mediall.

¶ The 88. Theoreme. The 112. Propoſition.

The ſquare of a rationall line applyed vnto a binomiall line, maketh the breadth or other ſide a reſiduall line, whoſe names are commenſurable to the names of the binomiall line, & in the ſelfe ſame proportiõ: & moreouer that reſiduall line is in the ſelfe ſame order of reſiduall lines, that the binomiall line is of binomiall lines.

The determination hath ſundry partes orderly to be proued.

Vppoſe that A be a rationall line, and B C a binomiall line whoſe greater name let be C D. And vnto the ſquare of the line A let the parallelogramme contayned vnder the lines B C and E F (ſo that E F be the breadth) be equall. Then I ſay that E F is a reſiduall line, whoſe names are commenſurable to the names of the binomiall line B C, which names let be C D and D B, and are in the ſame proportion with them: and moreouer the line E F is in the ſelfe ſame order of reſidual lines, that the line B C is of binomiall lines. Vnto the ſquare of the line A let the parallelogramme contayned vnder the lines B D and G be equal.



Conſtruction.

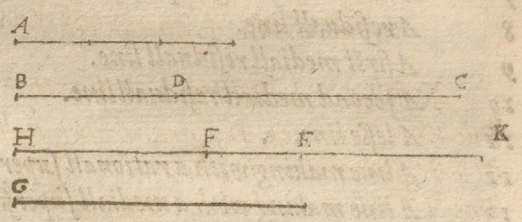
Now foraſmuch as that which is cõtayned vnder the lines B C & E F is equal to that which is contayned vnder the lines B D and G, therfore reciprocally (by the 14. of the ſixth) as the line C B is to the B D, ſo is the line G to the line E F. But the line B C is greater then the line B D, wherefore the line G is greater then the line E F. Vnto the line G let the line E H be equall. Wherefore (by the 11. of the fift) as the line C B is to the line B D, ſo is the line H E to the line F E. Wherefore by deuiſion (by the 17. of the fifth) as the line C D is to the line B D, ſo is the line H F to the line F E. *As the line H F is to the F E ſo let the line F K be to the line K E* (how this is to be done we will declare at the end of this demonſtration). Wherefore (by the 12. of the fift) the whole line H K* is to the whole line K F as the line F K is to the line K E. For as one of the antecedentes is to one of the conſequentes, ſo are all the antecedentes to all the conſequentes. But as the line F K is the line K E, ſo is the line C D to the line D B (for F K is to E K as H F is to F E, and H F is to F E as C D is D B). Wherfore (by the 11.

Demonſtration.

This is an Aſſumpt, problematicall, artificially vſed and demonſtrated.

**Therfore thoſe three lines are in continuall proportion.*

of the fift) as the line HK is to the line KF, so is the line CD to the line DB. But the square of the line CD is commensurable to the square of the line DB: wherefore (by the 10. of the tenth) the square of the line HK is cõmensurable to the square of the line FK. But these three lines HK, FK, and EK are proportionall in cõtinuall proportiõ (as it hath already bene proued). Wherefore (by the second corrollary of the 20. of the sixth) the square of the line HK is to the square of the line FK as the line HK is to the line EK: wherfore the line HK is commensurable in length to the line EK. Wherefore (by the 15. of the tenth) the line HE is commensurable in length to the line FK. And forasmuch as the square of the line A is equall to that which is contayned vnder the lines EH and BD, but the square of the line A is rationall, wherefore that which is contayned vnder the lines EH and BD is rationall. And it is applyed vnto the rationall line BD. Wherefore (by the 20. of the tenth) the line EH is rationall and commensurable in length to the line BD. Wherefore also the line EK which is commensurable in length to the line HE is



1. rational and commensurable in length to the line BD. Now for that as the line CD is to the line DB, so is the line FK to the line EK (for it was before proued, that as CD is to DB, so is HF to FE, and as HF is to FE, so is FK to EK) but the lines CD and DB are commensurable in power onely, wherefore (by the 10. of the tenth) the lines FK and KE are also commensurable in power onely. And for that as the line CD is to the line DB, so is the line FK to the line EK, therefore by contrary proportion as DB is to CD, so is EK
2. to FK, and alternately as DB is to EK, so is CD to FK: but the lines BD and EK are commensurable in length (as it hath already bene proued). Wherfore also the lines CD and FK are commensurable in length. But the line CD is rationall: wherefore also the line FK is rationall. Wherefore the lines FK and EK are rationall commensurable in power onely.
3. Wherefore the line FE is a residuall line: whose names FK and KE are commensurable to the names CD and BD of the binomiall line BC and in the same proportion as is proued.

FE concluded a residuall line, which is somwhat preposterously, in respect of the order propounded, both in the propositiõ, and also in the determinatiõ.

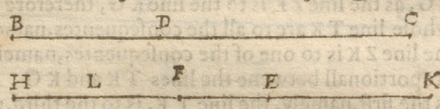
4. I say moreouer that it is a residuall line of the selfe same order that the binomiall line is. For the line CD is in power more then the line BD either by the square of a line commensurable in length to the line CD, or by the square of a line incommensurable in length. Now if the line CD be in power more then the line BD by the square of a line commensurable in length vnto the line CD, then (by the 13. of the tenth) the line FK is in power more then the line EK by the square of a line cõmensurable in length to the line FK. And so if the line CD be cõmensurable in lẽgth to the rationall line put, the line FK also shalbe cõmensurable in lẽgth to the same rationall line: wherfore then the line BC is a first binomiall line, & the line FE is likewise a first residuall line. And if the line BD be commensurable in length to
2. the rationall line, the line EK is also commensurable in length to the same, and then the line BC is a second binomiall line, and the line FE a second residuall line. And if neither of the
3. lines CD nor DB be commensurable in length vnto the rationall line, neither of the lines FK nor EK are commensurable in length vnto the same, and then the line BC is a third binomiall line, & the line FE is a third residuall line. And if the line CD be in power more then the line BD, by the square of a line incommensurable in length to the line CD, the line FK is also (by the 14. of the tenth) in power more then the line EK by the square of a line incommensurable in length to the line FK. And so if the line CD be commensurable in
4. lengh to a rationall line put, the line FK also is commensurable in length to the same, wherefore the line BC is a fourth binomiall line, and the line FE is a fourth residuall line. And if
5. the line BD be commensurable in length to the rationall line, the line EK is likewise commensurable in length to the same, and then the line BC is a fifth binomiall line and the line

EF

E F *a fifth reſiduall line. And if neither of the lines* C D *nor* D B *be commenſurable in length to the rationall line, neither alſo of the lines* F K *nor* E K *is commenſurable in length to the ſame, and then the line* B C *is a ſixth binomiall line, and the line* F E *a ſixth reſiduall line. Wherfore the line* F E *is a reſiduall line, whoſe names, namely,* F K *and* E K *are commẽſurable to the names of the binomiall line, namely, to the names* C D *and* D B, *and are in the ſelfe ſame proportion, and the reſiduall line* E F *is in the ſelfe ſame order of reſiduall lines, that the binomall line* B C *is of binomall lines. Wherefore the ſquare of a rationall line applyed vnto a binomiall line maketh the breadth (or other ſide) a reſiduall line, whoſe names are commenſurable to the names of the binomiall line, and in the ſelfe ſame proportion, and moreouer that reſiduall line is in the ſelfe ſame order of reſiduall lines, that the binomiall line is of binomiall lines: which was required to be demonſtrated.* 6.

✻ Here is the Aſſumpt (of the foregoing Propoſition) confirmed.

Now let vs declare how as the line H F is to the line F E, ſo to make the line F K to the line E K. The line C D is greater then the line B D by ſuppoſition. Wherefore alſo the line H F is greater then the line F E (by alternate proportion, and the 14. of the fifth). From the line H F take away the line F L equall to the line F E. Wherefore the line remayning, namely, H L, is leſſe then the line H F, for the line H F is equall to the lines H L & L F. As H L is to H F, ſo (by the 12. of the ſixt) let F E be to F K. Conſtruction.



Wherfore by contrary proportion (by the Corollary of the 4. of the fifth) as H F is to H L, ſo is F K to F E. Wherefore by conuerſion of proportion (by the Corollary of the 19. of the fifth) as H F is to L F, that is, to the line equall vnto it, namely, to F E, ſo is the line F K to the line E K. Demonſtration.

M. Dee, of this Aſſumpt, maketh (ποριςμαλιῶς, that is, *Acquiſitiuely,*) a Probleme vniuerſall, thus:

Two vnequall right lines being propounded, to adioyne vnto the leſſe, a right line, which take̅ with the leſſe (as one right line) ſhall haue the ſame proportion, to the line adioyned, which, the greater of the two propounded, hath to the leſſe. 1.

The conſtruction and demonſtration hereof, is worde for worde to be taken, as it ſtandeth here before: after theſe wordes: *The line H F is greater then the line F E.*

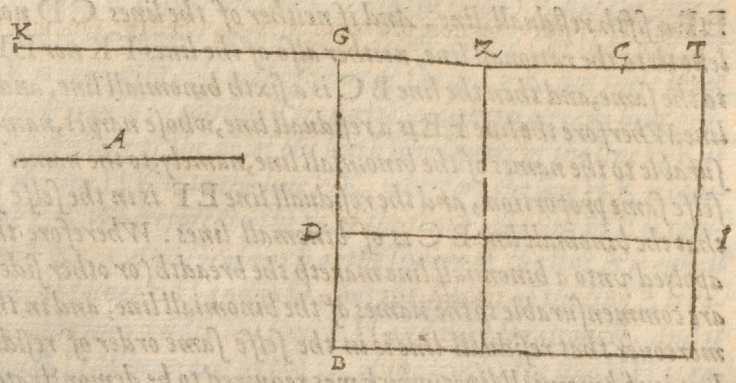
¶ A Corollary alſo noted by *I. Dee.*

It is therefore euident, that thus are three right lines (in our handling) in continuall proportion: it is to weete, the greater, the leſſe and the adioyned, make the firſt, the leſſe with the adioyned, make the ſecond: and the adioyned line is the third. 2.

This is proued in the beginning of the demonſtration, after the Aſſumpt vſed.

✻ An other demonſtration after *Fluſſas.*

Take a rationall line A, and let G B be a binomiall line, whoſe greater line let be G D: and vpon the line G B apply (by the 45. of the firſt) the parallelograme B Z equall to the ſquare of the line A, and making in breadth the line G Z. Likewiſe vpon the line D B (by the ſame) apply the parallelogramme B I equall alſo to the ſquare of the line A, and making in breadth the line D I: and put the line G Z T equall to the line D I. Then I ſay, that G Z is ſuch a reſiduall line as is required in the Propoſition. Conſtruction.



Foraſmuch as the parallelogrammes B Z & B I are equall, therefore (by the 14. of the ſixt) reciprocally as the line G B is to the line B D, ſo is the line D I or the line G T, (which is equall vnto it) vnto the line G Z. Wherefore by diuiſion, as the line G D is to the line D B, ſo is the line T Z to the line Z G (by the 17. of the fifth). Wherefore the line T Z is greater then the line Z G. (For the line G D is the greater name of the binomiall line G B). Vnto the line Z G put the line Z C equall. And as the line T C is to the line T Z, ſo (by the 11. of the ſixth) let the line Z G be to the line Z K. Wherefore contrarywiſe (by the Corollary of the 4. of the fifth) the line T Z is to the line T C, as the line Z K is to the line Z G. Wherefore by conuerſion of proportion (by the 19. of the fifth) as the line T Z is to the line Z C (that is, to Z G, which is equall vnto it) ſo is the line Z K to the line K G. But the line T Z is to the line Z G, as the line G D is to the line D B. Wherefore (by the 11. of the fifth) the line Z K is to the line K G, as the line G D is to the line D B. But the lines G D and D B are commenſurable in power onely. Wherefore alſo the lines Z K and K G are commenſurable in power onely, by the 10. of this booke. Farther, foraſmuch as the line T Z is to the line Z G, as the line Z K is to the line K G, therefore by the 12. of the fifth, all the antecedentes, namely, the whole line T K are to all the conſequentes, namely, to the line K Z, as one of the antecedentes, namely, the line Z K is to one of the conſequentes, namely, to the line K G. Wherefore the line Z K is the meane proportionall betwene the lines T K and K G. And therefore (by the Corollary of the 20. of the ſixth) as the firſt, namely, the line T K, is to the third, namely, to the line K G: ſo is the ſquare of the line T K to the ſquare of the ſecond, namely, of the line K Z. And foraſmuch as the parallelogrãme B I (which is equall to the ſquare of the rationall line A) is applied vpon the rationall line D B, it maketh the breadth D I rationall and commenſurable in length vnto the line D B, by the 20. of the tenth. And therfore the line G T (which is equall vnto the line D I) is commenſurable in length to the ſame line D B. And for that as the line G D is to the line D B, ſo is the line K Z to the line K G, but as the line K Z is to the line K G, ſo is the line T K to the line K Z, therefore (by the 11. of the fifth) as the line G D is to the line D B, ſo is the line T K to the line K Z. Wherefore (by the 22. of the ſixth) as the ſquare of the line G D is to the ſquare of the line D B, ſo is the ſquare of the line T K to the ſquare of the line K Z. But the ſquare of the line G D is commenſurable to the ſquare of the line D B (for the names G D and D B of the binomiall line G B are commenſurable in power). Wherefore the ſquare of the line T K ſhall be commenſurable to the ſquare of the line K Z, by the 10. of this booke. But as the ſquare of the line T K is to the ſquare of the line K Z, ſo is it proued, that the right line T K is to the right line K G. Wherefore the right line T K is commenſurable in length to the right line K G. Wherefore it is alſo commenſurable in length to the line T G (by the 15. of the tenth). Which line T G is (as it hath bene proued) a rationall line, and equall to the line D I. Wherefore the lines T K and K G are rationall commenſurable in length. And foraſmuch as it hath bene proued, that the line Z K is commenſurable in power onely vnto the rationall line K G, therefore the lines Z K and K G are rationall commenſurable in power onely. Wherefore the line G Z is a reſiduall line. And foraſmuch as the rationall line T G is commenſurable in length to either of theſe lines D B and K G. Wherefore the lines D B & K G ſhall be commenſurable in length, by the 12. of the tenth. But the line Z K is to the line K G, as the line G D is to the line D B. Wherefore alternately, by the 16. of the fifth, the line K Z is to the line G D, as the line K G is to the line D B. Wherefore the line Z K is commenſurable in length vnto the line G D. Wherefore the lines Z K and K G (the names of the reſiduall line G Z) are commenſurable in length to the lines G D and D B, which are the names of the binomiall line G B: and the line Z K is to the line K G in the ſame proportion, that the line G D is to the line D B. Wherefore if the whole line Z K be in power more then the line conueniently ioyned K G, by the ſquare of a line commenſurable in length to the line Z K, then the greater name G D ſhall be in power more then the leſſe name D B, by the ſquare of a line commenſurable in length to the line G D, by the 14. of the tenth. And if the line Z K be in power more then the line K G, by the ſquare of a line incommenſurable in length to the line Z K, the line alſo G D ſhall be in power more then the line D B, by the ſquare of a line incommenſurable in length vnto the line G D (by the ſame Propoſition). And if the greater or leſſe name of the one be commenſurable in length to the rationall line put, the greater or leſſe name alſo of the other ſhall be commenſurable in length to the ſame rationall line, (by the 12. of this booke). But if neither name of the one be commenſurable in length to the rationall line put, neither name of the other alſo ſhall be commenſurable in length to the ſame rationall line put (by the 13. of the ſame). Wherefore the reſiduall line G Z ſhall be in the ſelfe ſame order of reſiduall lines, that the binomiall line G B is of binomiall lines (by the definitions of reſiduall and binomiall

Demonſtration.

1.

2. *Here are the fower partes of the propoſitiõ more orderly hãdled then in the former demõſtration.*

3.

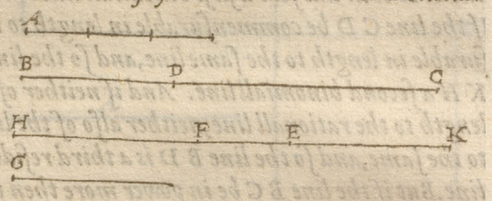
4.

lines.

lines. The ſquare therefore of a rationall line applied to a binomiall line. &c: which was required to be proued.

¶ The 89. Theoreme. The 113. Propoſition.

The ſquare of a rational line applied vnto a reſiduall, maketh the breadth or other ſide a binomial line, whoſe names are commenſurable to the names of the reſiduall line, and in the ſelfe ſame proportion: and moreouer that binomiall line is in the ſelfe ſame order of binomiall lynes, that the reſiduall line is of reſiduall lynes.



SVppoſe that A be a rationall line, and B D a reſiduall line. And vnto the ſquare of the line A let that which is contained vnder the lines B D and K H be equall. Wherfore the ſquare of the rationall line A applied vnto the reſiduall line B D maketh the breadth or other ſide K H. Then I ſay that the line K H is a binomiall line, whoſe names are commenſurable to the names of the reſiduall line B D, and in the ſelfe ſame proportion, and that the line K H is in the ſelfe ſame order of binomiall lines, that the line B D is of reſiduall lines. Vnto the line B D let the line conueniently ioyned be D C. Wherfore the lines B C and D C are rationall commenſurable in power onely. And vnto the ſquare of the line A let the parallelogramme contained vnder the lines B C and G be equall. But the ſquare of the line A is rationall. Wherfore the parallelogramme contained vnder the lines B C and G is alſo rationall. Wherfore alſo the line G is rationall and commenſurable in length to the line B C (by the 20. of the tenth). Now foraſmuch as the parallelogramme contained vnder the lines B C and G is equall to that which is contained vnder the lines B D and K H, therfore (by the 16. of the ſixt) as the line B C is to the line B D, ſo is the line K H to the line G. But the line B C is greater then the line B D. Wherfore alſo the line K H is greater then the line G. Vnto the line G let the line K E be equall. Wherfore the line K E is rationall and cõmenſurable in length to the line B C, as alſo the line G was (by the 12. of the tenth). And for that as B C is to B D, ſo is K H to K E. Wherfore by cõuerſion of proportiõ (by the corollary of the 19. of the fift), as B C is to D C, ſo is K H, to E H. As K H is to E H, ſo let the line F H be to the line E F (how this is to be done, we will decare at the ende of this demonſtration). Wherfore thereſidue K F is to the reſidue F H, as the whole K H is to the whole H E (by the 19. of the fift) that is, as the line B C is to the line C D. But the lines B C and C D are commenſurable in power onely. Wherefore alſo the lines K F and F H are commenſurable in power only. And for that as K H is to H E, ſo is K F to F H, but as K H is to H E, ſo is alſo H F to F E, therfore as K F is to F H, ſo is F H to F E. Wherfore (by the corollary of the 19. of the ſixt) as the firſt is to the third, ſo is the ſquare of the firſt, to the ſquare of the ſecond. Wherefore as K F is to F E, ſo is the ſquare of the line K F to the ſquare of the line F H, but theſe ſquares are commenſurable, for the lines K F and F H are commenſurable in power. Wherfore the lines K F and F E are commenſurable in length. Wherfore (by the ſecond part of the 15. of the tenth) the lines K E and E F are commenſurable in length. Wherfore (by the ſame) the lines K F and F E are commenſurable in length. But the line K E is rationall and commenſurable in length to the line B C: wherefore the line K F is alſo rationall and commenſurable in legth to the line B C. And for that as the line B C is to the line C D, ſo is K F to F H, therfore alternately (by the 16. of the fift) as B C is to K F, ſo is C D to F H. But the line B C is commenſurable in length

Conſtruction.

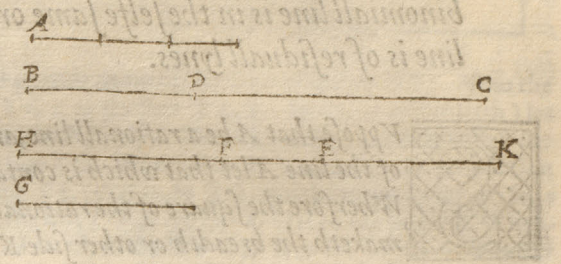
Demonſtration.

An Aſſumpt.

to the line K F. Wherfore the line C D is commenſurable in length to the line F H. But the line C D is rationall. Wherfore alſo the line F H is rationall. And the lines B C and C D are rationall commenſurable in power onely. Wherfore the lines K F and F H are rationall commenſurable in power onely. Wherfore the line K H is a binomiall line, whoſe names are commenſurable to the names of the reſiduall line, and in the ſame proportion. I ſay moreouer that it is a binomiall of the ſelfe ſame order of binomial lines, that the line B D is of reſidual lines.

1. 2. 3. 4.

For if the line B C be in power more then the line C D by the ſquare of a line commenſurable in length to the line B C, the line K F is alſo in power more then the line F H by the ſquare of a line commenſurable in length to the line K F (by the 14. of the tenth). And if the line B C be commenſurable in length to the rationall line put, the line K F is alſo (by the 12. of the tenth) commenſurable in length to the rationall line, and ſo the lyne B D is a firſt reſiduall lyne, and the line K H is in like ſort a firſt binomiall line.



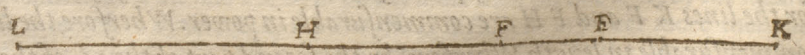
1.

If the line C D be commenſurable in length to the rational line, the line F H is alſo commenſurable in length to the ſame line, and ſo the line B D is a ſecond reſiduall line, and the line K H a ſecond binomiall line. And if neither of the lines B C nor C D be commenſurable in length to the rationall line, neither alſo of the lines K F nor F H is commenſurable in length to the ſame, and ſo the line B D is a third reſiduall line, and the line K H a third binomiall line. But if the line B C be in power more then the line C D by the ſquare of a line incommenſurable in length to the line B C, the line K F is in power more thē the line F H by the ſquare of a line incommenſurable in length to the line K F (by the 14. of the tenth) And if the line B C be commenſurable in length to the rationall line put, the line K F is alſo commenſurable in length to the ſame line, and ſo the line B D is a fourth reſiduall line, and the line K H a fourth binomiall line. And if the line C D be cōmēſurable in lēgth to the rational line, the line F H is alſo cōmēſurable in lēgth to the ſame, & ſo the line B D is a fift reſiduall line, & the line K H a fift binomiall line. And if neither of the lines B C nor C D be commenſurable in length to the rationall line, neither alſo of the lines K F nor F H is commenſurable in length to the ſame, and ſo the line B D is a ſixt reſiduall line, and the line K H is a ſixt binomiall line. Wherfore K H is a binomiall line, whoſe names K F and F H are commenſurable to the names of the reſiduall line B D, namely, to B C and C D, and in the ſelfe ſame proportion, and the binomiall line K H is in the ſelfe ſame order of binomiall lines, that the reſiduall B D, is of reſiduall lines. Wherefore the ſquare of a rationall line applied vnto a reſiduall line, maketh the breadth or other ſide a binomiall line, whoſe names are commenſurable to the names of the reſiduall line, and in the ſelfe ſame proportion, and moreouer the binomiall line is in the ſelfe ſame order of binomiall lincs, that the reſiduall line is of reſiduall lines: which was required to be demonſtrated.

2. 3. 4. 5. 6.

✻ The Aſſumpt confirmed.

Now let vs declare how, as the line K H is to the line E H, ſo



to make the line H F to the line F E. Adde vnto the line K H directly a line equall to H E, and let the whole line be K L, and (by the tenth of the ſixt) let the line H E be deuided as the whole line K L is deuided in the point H: let the line H E be ſo deuided in the point F. Wherfore

fore as the line K H is to the line H L, that is, to the line H E, so is the line H F to the line F E.

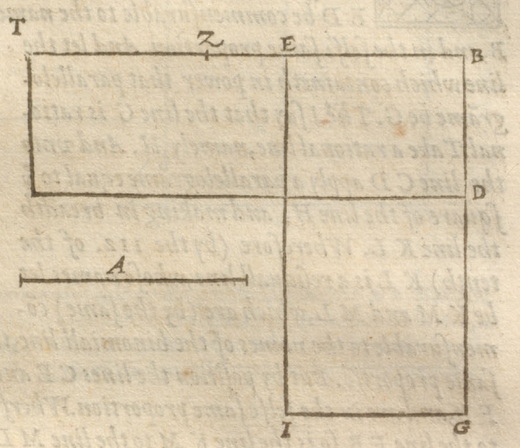
An other demonstration after *Flussas*.

An other demonstratiõ after Flussas.

Construction.

Suuppose that *A* be a rationall line, and let *B D* be a residuall line. And vpon the line *B D* apply the parallelogramme *D T* equall to the square of the line *A* (by the 45. of the first) making in breadth the line *B T*. Then I say that *B T* is a binominall line such a one as is required in the proposition. Forasmuch as *B D* is a residuall line, let the line cõueniently ioyned vnto it be *G D*. Wherfore the lines *B G* and *G D* are rationall commensurable in power onely. Vpon the rationall line *B G* apply the parallelogramme *B I* equall to the square of the line *A* and making in breadth the line *B E*. Wherefore the line *B E* is rationall and commensurable in length to the line *B G* (by the 20. of the tenth). Now forasmuch as the parallelogrammes *B I* and *T D* are equall (for that they are eche equall to the square of the line *A*): therfore reciprokally (by the 14. of the sixth) as the line *B T* is to the line *B E*, so is the line *B G* to the line *B D*. Wherefore by conuersion of proportion (by the corrollary of the 19. of the fifth) as the line *BT* is to the line *T E*, so is the line *B G* to the line *G D*. As the line *B G* is to the line *G D*, so let the line *T Z* be to the line *Z E* by the corrollary of the 10. of the sixth. Wherefore by the 11. of the fifth the line *B T* is to the line *T E*, as the line *T Z* is to the line *Z E*. For either of them are as the line *B G* is to the line *G D*. Wherefore the residue B Z is to the residue *Z T*, as the whole *B T* is to the whole *T E* by the 19. of the fifth. Wherefore by the 11. of the fifth the line *B Z* is to the line *Z T* as the line *Z T* is to the line *Z E*. Wherfore the line *T Z* is the meane proportionall betwene the lines *BZ* and *Z E*. Wherefore the square of the first, namely, of the line *BZ*, is to the square of the second, namely, of the line *Z T*, as the first, namely, the line *B Z*, is to the third, namely, to the line *Z E* (by the corollary of the 20. of the sixth). And for that as the line *B G* is to the line *G D*, so is the line *T Z* to the line *Z E*: but as the line *T Z* is to the line *Z E*, so is the line *B Z* to the line *Z T*. Wherefore as the line *B G* is to the line *G D*, so is the line *B Z* to the line *Z T* (by the 11. of the fifth). Wherfore the lines *B Z* and *Z T* are commensurable in power onely, as also are the lines *B G* and *G D* (which are the names of the residuall line *B D*) by the 10. of this booke. Wherfore the right lines *B Z* and *Z E* are cõmensurable in length, for we haue proued that they are in the same proportion that the squares of the lines *B Z* and *Z T* are. And therefore (by the corollary of the 15. of this booke) the residue *B E* (which is a rationall line) is commensurable in length vnto the same line *B Z*. Wherefore also the line *B G* (which is commensurable in length vnto the line *B E*) shall also be commensurable in length vnto the same line *B Z* (by the 12. of the tenth). And it is proued that the line *BZ* is to the line *Z T* commensurable in power onely. Wherefore the right lines *B Z* and *Z T* are rationall commensurable in power onely. Wherefore the whole line *B T* is a binomiall line (by the 36. of this booke). And for that as the line *B G* is to the line *G D*, so is the line *B Z* to the line *Z T*: therefore alternately (by the 16. of the fifth) the line *B G* is to the line *B Z*, as the line *G D* is to the line *Z T*. But the line *B G* is commensurable in length vnto the line *B Z*. Wherefore (by the 10. of this booke) the line *G D* is commensurable in length vnto the line *Z T*. Wherefore the names *B G* and *G D* of the residuall line *B D* are commensurable in length vnto the names *B Z* and *Z T* of the binomiall line *BT*: and the line *B Z* is to the line *Z T* in the same proportion that the line *B G* is to the line *G D* as before it was more manifest. And that they are of one and the selfe same order is thus proued. If the greater or lesse name of the residuall line, namely, the right lines *B G* or *G D* be cõmensurable in length to any rationall line put: the greater name also or lesse, namely, *B Z* or *Z T* shalbe commensurable in length to the same rationall line put by the 12. of this booke. And if neither of the names of the residuall line be commensurable in length vnto the rationall line put, neither of the names of the binomiall line shalbe commensurable in length vnto the same rationall line put (by the 13. of the tenth). And if the greater name *B G* be in power more then the lesse name by the square of a line commensurable in length vnto the line *B G*, the greater name also *B Z* shalbe in power more then the lesse by the square of a line commensurable in length vnto the line *B Z*. And if the one be in power more by the square of a line incommensurable in length, the other also shalbe in power more by the square of a line incommensurable in length by the 14. of this booke. The square therefore of a rationall line. &c. which was required to be proued.

Demonstration.



1.

2.

3.

4.

The

¶ The 90. Theoreme. The 114. Proposition.

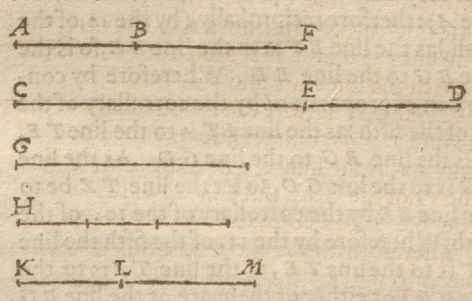
This is in a maner the conuerse of both the former propositions, ioyntly.

If a parallelogrãme be cõtained vnder a residuall line & a binomiall lyne, whose names are commensurable to the names of the residuall line, and in the selfe same proportion: the lyne which contayneth in power that superficies is rationall.

Suppose that a parallelogramme be contained vnder a residuall line A B and a binomiall line C D, and let the greater name of the binomiall line be C E, and the lesse name be E D, and let the names of the binomiall line, namely, C E and E D be commensurable to the names of the residuall line, namely, to A F and F B, and in the selfe same proportion. And let the line which containeth in power that parallelogrãme be G. Thẽ I say that the line G is rational. Take a rational line, namely, H. And vnto the line C D apply a parallelogrãme equal to ỹ square of the line H, and making in breadth the line K L. Wherefore (by the 112. of the tenth) K L is a residuall line, whose names let be K M and M L, which are (by the same) cõmensurable to the names of the binomiall line, that is to C E and E D, and are in the selfe same proportiõ. But by position the lines C E and E D are cõmensurable to the lines A F and F B, and are in the selfe same proportion. Wherfore (by the 12. of the tenth) as the line A F is to the line F B, so is the line K M to the line M L. Wherfore alternately (by the 16. of the fift) as the line A F is to the line K M, so is the line B F to the line L M. Wherfore the residue A B is to the residue K L, as the whole A F is to the whole K M. But the line A F is commensurable to the line K M, for either of the lines A F and K M is commensurable to the line C E. Wherfore also the line A B is commensurable to the line K L. And as the line A B is to the line K L, so (by the first of the sixt) is the parallelogramme contained vnder the lines C D and A B to the parallelogramme contained vnder the lines C D and K L. Wherfore the parallelogramme contained vnder the lines C D and A B is commensurable to the parallelogrimme contained vnder the lines C D and K L. But the parallelogramme contained vnder the lines C D and K L is equall to the square of the line H. Wherfore the parallelogrãme cõtained vnder the lines C D & A B is cõmensurable to the square of the line H. But the parallelogrãme contained vnder the lines C D and A B is equall to the square of the line G. Wherfore the square of the line H is commensurable to the square of the line G. But the square of the line H is rationall. Wherfore the square of the line G is also rationall. Wherfore also the line G is rational, and it containeth in power the parallelogramme contained vnder the lines A B and C D. If therfore a parallelogramme be contained vnder a residuall line and a binomiall line, whose names are commensurable to the names of the residuall line, and in the selfe same proportion, the line which containeth in power that superficies, is rationall: which was required to be proued.

Construction.

Demonstration.



¶ Corollary.

Hereby it is manifest, that a rationall parallelogramme may be contained vnder irrationall lines.

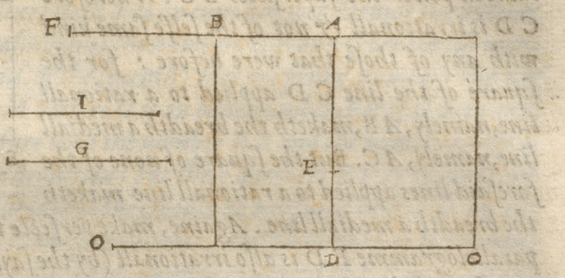
An

¶ An other demonſtration after *Fluſſas*.

Suppoſe that the ſuperficies D B be contayned vnder a reſiduall line A B, and a binomiall line A D: whoſe names A E and E D let be commenſurable in length vnto the names of the reſiduall line A B, which let be A F and F B. And let the line A E be to the line E D, in the ſame proportion that the line A F is to the line F B. And let the right line I contayne in power the ſuperficies D B. Then I ſay, that the line I is a rationall line. Take a rationall line, which let be G. And vpon the line A D deſcribe (by the 45. of the firſt) a parallelogramme equall to the ſquare of the line G, and making in breadth the line D C. Wherefore (by the 112. of this booke) C D is a reſiduall line, whoſe names (which let be C O and O D) ſhall be cõmenſurable in lẽgth vnto the names A E and E D, and the line C O ſhall be vnto the line O D, in the ſame proportion that the line A E is to the line E D. But as the line A E is to the line E D, ſo by ſuppoſition, is the line A F to the line F B. Wherfore as the line C O is to the line O D, ſo is the line A F to the line F B. Wherefore the lines C O and O D are commenſurable with the lines A F and F B (by the 12. of this boke). Wherfore the reſidue, namely, the line C D is to the reſidue, namely, to the line A B, as the line C O is to the line A F (by the 19. of the fifth). But it is proued, that the line C O is cõmenſurable vnto the line A F. Wherefore the line C D is commenſurable vnto the line A B. Wherefore (by the firſt of the ſixth) the parallelogramme C A is commenſurable to the parallelogramme D B. But the parallelogramme C A is (by conſtruction) rationall (for it is equall to the ſquare of the rationall line G). Wherefore the parallelogramme B D is alſo rationall. Wherefore the line I which by ſuppoſition cõtayneth in power the ſuperficies B D, is alſo rationall. If therfore a parallelogrãme be contayned. &c: which was required to be proued.

Conſtruction.

Demonſtration.

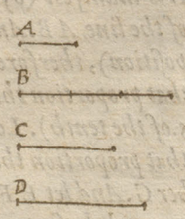


¶ *The 91. Theoreme.* *The 115. Propoſition.*

Of a mediall line are produced infinite irrationall lines, of which none is of the ſelfe ſame kinde with any of thoſe that were before.

SVppoſe that A be a mediall line. Then I ſay, that of the line A may be produced infinite irrationall lines, of which none ſhall be of the ſelfe ſame kinde with any of thoſe that were before. Take a rationall line B. And vnto that which is contained vnder the lines A and B let the ſquare of the line C be equall (by the 14. of the ſecond). Wherefore the line C is irrationall. For a ſuperficies contained vnder a rationall line and an irrationall line, is (by the Aſſumpt following the 38. of the tenth) irrationall: and the line which containeth in power an irrationall ſuperficies, is (by the Aſſumpt going before the 21. of the tenth) irrationall. And it is not one and the ſelfe ſame with any of thoſe thirtene that were before. For none of the lines that were before applied to a rationall line maketh the breadth mediall. Againe vnto that which is contained vnder the lines B and C, let the ſquare of D be equall. Wherefore the ſquare of D is irrationall. Wherefore alſo the line D is irrationall and not of the ſelf ſame kinde with any of thoſe that were before. For the ſquare of none of the lines which were before, applied to a rationall line, maketh the breadth the line C. In like ſort alſo ſhall it ſo followe, if a man proceede infinitely. Wherefore it is manifeſt, that of a mediall line are produced infinite irrationall lines, of which none is of the ſelfe ſame kinde with any of thoſe that were before: which was required to be proued.

Demonſtration.

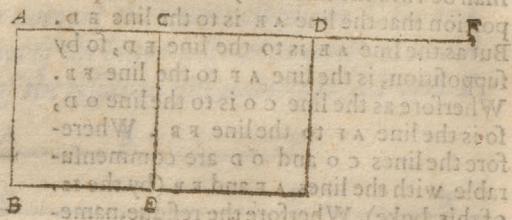


An

An other demonstration.

An other demonstration.

Suppose that A C be a mediall line. Then I say, that of the line A C may be produced infinite irrationall lines, of which none shall be of the selfe same kinde with any of those irrationall lines before named. Vnto the line A C and from the point A, draw (by the 11. of the first) a perpẽdicular line A B, and let A B be a rationall line, and make perfecte the parallelogramme B C. Wherefore B C is irrationall, by that which was declared and proued (in maner of an Assumpt) in the end of the demonstration of the 38: and the line that containeth it in power is also irrationall. Let the line C D containe in power the superficies B C. Wherefore C D is irrationall & not of the selfe same kind with any of those that were before: for the square of the line C D applied to a rationall line, namely, A B, maketh the breadth a mediall line, namely, A C. But the square of none of the foresaid lines applied to a rationall line maketh the breadth a mediall line. Againe, make perfecte the parallelogramme E D. Wherefore the parallelogramme E D is also irrationall (by the sayd Assumpt in the end of the 38. his demonstration briefly proued) and the line which containeth it in power is irrationall: let the line which containeth it in power be D F. Wherefore D F is irrationall and not of the selfe same kinde with any of the foresaid irrationall lines. For the square of none of the foresayd irrationall lines applied vnto a rationall line, maketh the breadth the line C D. Wherefore of a mediall line are produced infinite irrationall lines, of which none is of the selfe same kinde with any of those that were before: which was required to be demonstrated.

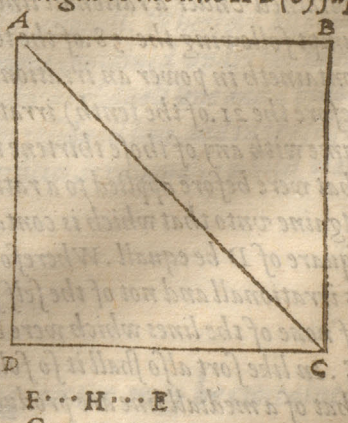


¶ The 92. Theoreme. The 116. Proposition.

Now let vs proue that in square figures, the diameter is incommensurable in length to the side.

Demonstratiõ leading to an impossibilitie.

Vppose that A B C D be a square, and let the diameter therof be A C. Then I say that the diameter A C is incommensurable in length to the side A B. For if it be possible, let it be cõmensurable in lẽgth. I say that thẽ this will follow, that one and the selfe same nũber shall be both an euen number & an odde number. It is manifest (by the 47. of the first) that the square of the line A C is double to the square of the line A B. And for that the line A C is commensurable in length to the line A B (by supposition), therfore the lyne A C hath vnto the line A B that proportion that a number hath to a number (by the 5. of the tenth). Let the lyne A C haue vnto the line A B that proportion that the number E F hath to the number G. And let E F and G be the least numbers that haue one and the same proportion with them. Wherfore E F is not vnitie. For if E F be vnitie, and it hath to the number G that proportion that the line A C hath to the lyne A B, and the line A C is greater then the lyne A B. Wherfore vnitie E F is greater then the number G, which is impossible. Wherfore F E is not vnitie, wherfore it is a number. And for that as the square of the line A C is to the square of the lyne A B, so is the square number of the number E F, to the square number of the

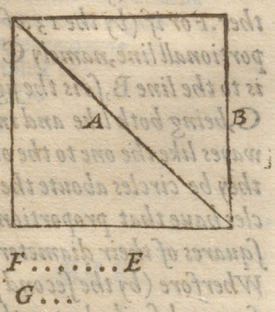


of the number G, for in eche is the proportion of their ſides doubled (by the corollary of the 20 of the ſixt and 11 of the eight): and the proportion of the line A C to the line A B doubled, is equal to the proportiõ of the nũber E F to the number G, doubled, for as the line A C is to the line A B, ſo is the nũber E F to the number G. But the ſquare of the line A C is double to the ſquare of the line A B. Wherfore the ſquare number produced of the number E F is double to the ſquare number produced of the number G. Wherefore the ſquare number produced of E F is an euen number. Wherfore E F is alſo an euen number. For if E F were an odde number, the ſquare number alſo produced of it, ſhould (by the 23. and 29. of the ninth) be an odde number. For if odde numbers how many ſoeuer be added together, and if the multitude of thẽ be odde, the whole alſo ſhal be odde. Wherfore E F is an euen number. Deuide the number E F into two equall partes in H. And foraſmuch as the numbers E F and G are the leſt numbers in that proportion, therfore (by the 24. of the ſeuenth) they are prime numbers the one to the other. And E F is an euen number. Wherfore G is an odde number. For if G were an euen number, the number two ſhould meaſure both the number E F and the number G (for euery euen nũber hath an halfe part by the definition) but theſe numbers E F & G are prime the one to the other. Wherfore it is impoſſible that they ſhould be meaſured by two or by any other number beſides vnitie. Wherfore G is an odde number. And foraſmuch as the number E F is double to the number E H, therfore the ſquare number produced of E F is quadruple to the ſquare number produced of E H. And the ſquare number produced of E F is double to the ſquare number produced of G. Wherfore the ſquare number produced of G is double to the ſquare number produced of E H. Wherfore the ſquare number produced of G is an euen number. Wherfore alſo by thoſe thinges which haue bene before ſpoken, the number G is an euen number, but it is proued that it is an odde number, which is impoſsible. Wherefore the line A C is not commenſurable in length to the line A B, wherfore it is incommenſurable.

An other demonſtration.

An other demonſtration leading to an impoſſibilitie.

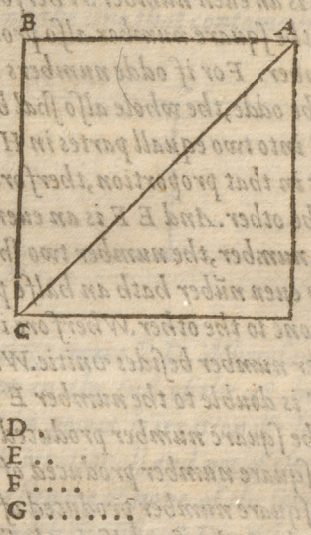
We may by an other demonſtration proue, that the diameter of a ſquare is incommenſurable to the ſide thereof. Suppoſe that there be a ſquare, whoſe diameter let be A and let the ſide thereof be B. Then I ſay that the line A is incommenſurable in length to the line B. For if it be poſſible let it be commenſurable in length. And agayne as the line A is to the line B ſo let the number E F be to the number G: and let them be the leaſt that haue one and the ſame proportion with them: wherefore the numbers E F and G, are prime the one to the other. Firſt I ſay that G is not vnitie. For if it be poſſible let it be vnitie. And for that the ſquare of the line A is to the ſquare of the line B, as the ſquare number produced of E F is to the ſquare number produced of G (as it was proued in the former demonſtration) but the ſquare of the line A is double to the ſquare of the line B. Wherfore the ſquare nũber produced of E F is double to the ſquare number produced of G. And by your ſuppoſition G is vnitie. Wherefore the ſquare number produced of E F is the number two which is impoſſible. Wherefore G is not vnitie. Wherefore it is a number. And for that as the ſquare of the line A is to the ſquare of the line B, ſo is the ſquare number produced of E F to the ſquare number produced of G. Wherefore the ſquare number produced of E F is double to the ſquare number produced of G. Wherefore the ſquare number produced of G meaſureth the ſquare number produced of E F. Wherefore alſo (by the 14. of the eight) the number G meaſureth the number E F: and the number G alſo meaſureth it ſelfe. Wherefore the number G meaſureth theſe numbers E F and G, when yet they are prime the one to the



other: which is impoſſible. Wherefore the diameter A *is not commenſurable in length to the ſide* B. *Wherefore it is incommenſurable: which was required to be demonſtrated.*

An other demonſtration after *Fluſſas.*

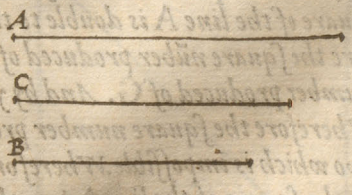
Suppoſe that vppon the line A B be deſcribed a ſquare whoſe diameter let be the line A C. Then I ſay that the ſide A B is incommenſurable in length vnto the diameter A C. For-aſmuch as the lines A B and B C are equall, therefore the ſquare of the line A C is double to the ſquare of the line A B by the 47. of the firſt. Take by the 2. of the eight nũbers how many ſoeuer in continuall proportion frõ vnitie, and in the proportion of the ſquares of the lines A B and A C. Which let be the numbers D, E, F, G. And foraſmuch as the firſt from vnitie namely E is no ſquare number, for that it is a prime number, neither is alſo any other of the ſayd numbers a ſquare number except the third from vnitie and ſo all the reſt leuing one betwene, by the 10. of the ninth. Wherefore D is to E, or E to F, or F to G, in that proportion that a ſquare number is to a number not ſquare. Wherefore by the corrollary of the 25. of the eight, they are not in that proportion the one to the other that a ſquare number is to a ſquare number. Wherefore neither alſo haue the ſquares of the lines A B and A C (which are in the ſame proportion) that porportion that a ſquare number hath to a ſquare number. Wherefore by the 9. of this booke their ſides, namely, the ſide A B and the diameter A C are incommenſurable in length the one to the other which was required to be proued.



This demonſtration I thought good to adde, forthat the former demonſtrations ſeme not ſo full, and they are thought of ſome to be none of *Theons*, as alſo the propoſition to be none of *Euclides*.

Here followeth an inſtruction by ſome ſtudious and ſkilfull Grecian (perchance *Theon*) which teacheth vs of farther vſe and fruite of theſe irrationall lines.

Seing that there are founde out right lines incommenſurable in length the one to the other, as the lines A *and* B, *there may alſo be founde out many other magnitudes hauing lẽgth and breadth (ſuch as are playne ſuperficieces) which ſhalbe incommẽſurable the one to the other. For if (by the 13. of the ſixth) betwene the lines* A *and* B *there be taken the meane proportionall line, namely* C, *then (by the ſecond corrollary of the 20. of the ſixth) as the line* A *is to the line* B, *ſois the figure deſcribed vpon the line* A *to the figure deſcribed vpon the line* C, *being both like and in like ſort deſcribed, that is, whether they be ſquares (which are alwayes like the one to the other), or whether they be any other like rectiline figures, or whether they be circles aboute the diameters* A *and* C. *For circles haue that proportion the one to the other, that the ſquares of their diameters haue (by the 2. of the twelfth). Wherfore (by the ſecond part of the 10. of the tenth) the figures deſcribed vpon the lines* A *and* C *being like and in like ſort deſcribed are incommenſurable the one to the other. Wherfore by this meanes there are founde out ſuperficieces incommenſurable the one to the other. In like ſort there may be founde out figures cõmẽſurable the one to the other, if ye put the lines* A *and* B *to be cõmenſurable in lẽgth the one to the other. And ſeing that it is ſo, now let vs alſo proue that euen in ſolides alſo or bodyes there are ſome commenſurable the one to the other, and other ſome incommenſurable the one to the other. For if from eche of the ſquares of the lines* A *and* B, *or from any other rectiline figures equal to theſe ſquares*



be

be erected solides of equall altitude, whether those solides be composed of equidistant superficieces, or whether they be piramids, or prismes, those solides so erected shalbe in that proportiõ the one to the other that theyr bases are (by the 32. of the eleuenth and 5. and 6. of the twelfth) Howbeit there is no such proposition concerning prismes. And so if the bases of the solides be commensurable the one to the other, the solides also shall be commensurable the one to the other, and if the bases be incommensurable the one to the other, the solides also shall be incommensurable the one to the other (by the 10. of the tenth). And if there be two circles A and B: and vpon ech of the circles be erected Cones or Cilinders of equal altitude, those Cones & Cilinders shall be in that proportion the one to the other that the circles are, which are their bases (by the 11. of the twelfth): and so if the circles be commensurable the one to the other, the Cones and Cilinders also shall be commensurable the one to the other. But if the circles be incõmensurable the one to the other, the Cones also and Cilinders shalbe incõmensurable the one to the other, (by the 10. of the tenth). Wherefore it is manifest that not onely in lines and superficieces, but also in solides or bodyes is found commensurabilitie or incommensurability.

An aduertisement by *Iohn Dee.*

Although this proposition were by *Euclide* to this booke alotted, (as by the auncient grecian published vnder the name of *Aristoteles* Περὶ ἀτόμων γραμμῶν, it would seme to be, and also the property of the same, agreable to the matter of this booke, and the proposition it selfe, so famous in Philosophy and Logicke, as it was, would in maner craue his elemẽtal place, in this tẽth boke) yet the dignitie & perfection, of Mathematicall Method can not allow it here: as in due order following: But most aptly after the 9. propositiõ of this booke, as a Corrollary of the last part thereof. And vndoubtedly the propositiõ hath for this 2000. yeares bene notably regarded among the greke Philosophers: and before *Aristotles* time was concluded with the very same inconuenience to the gaynesayer, that the first demonstration here induceth, namely, *Odde number to be equall to euen*: as may appeare in *Aristotles* worke, named *Analitica prima*, the first booke and 40. chapter. But els in very many places of his workes he maketh mention of the proposition. Euident also it is that *Euclide* was about *Aristotles* time, and in that age the most excellent Geometrician among the Grekes. Wherefore, seing it was so publike in his time, so famous, and so appertayning to the property of this booke: it is most likely, both to be knowne to *Euclide*, and also to haue bene by him in apt order placed. But of the disordring of it, can remayne no doubt, if ye consider in *Zamberts* translation, two other propositions going next before it, so farre misplaced, that where they are, word for word, before duely placed, being the 105. and 106. yet here (after the booke ended), they are repeated with the numbers of 116. and 117. proposition. *Zambert* therein was more faythfull to follow as he found in his greke example, than he was skilfull or carefull to doe what was necessary. Yea and some greke written auncient copyes haue them not so: Though in deede they be well demonstrated, yet truth disorded, is halfe disgraced: especially where the patterne of good order, by profession is auouched to be. But through ignoraunce, arrogancy and temeritie of vnskilfull Methode Masters, many thinges remayne yet, in these Geometricall Elementes, vnduely tumbled in: though true, yet with disgrace: which by helpe of so many wittes and habilitie of such, as now may haue good cause to be skilfull herein, will (I hope) ere long be taken away: and thinges of importance (wanting) supplied.

Amos Hacon dwelle ?? this Booke

The end of the tenth booke of Euclides Elementes

¶ The eleuenth booke of Euclides Elementes.

The argument of the eleuenth booke.

HITHERTO HATH EVCLIDE IN THESE former bookes with a wonderfull Methode and order entreated of ſuch kindes of figures ſuperficial, which are or may be deſcribed in a ſuperficies or plaine. And hath taught and ſet forth their properties, natures, generations, and productions euen from the firſt roote, ground, and beginning of them: namely, from a point, which although it be indiuiſible, yet is it the beginning of all quantitie, and of it and of the motion and flowing therof is produced a line, and conſequently all quantitie cõtinuall, as all figures playne and ſolide what ſo euer. *Euclide* therefore in his firſt booke began with it, and from thence went he to a line, as to a thing moſt ſimple next vnto a point, then to a ſuperficies, and to angles, and ſo through the whole firſt booke, he intreated of theſe moſt ſimple and plaine groundes. In the ſecond booke he entreated further, and went vnto more harder matter, and taught of diuiſions of lines, and of the multiplication of lines, and of their partes, and of their paſſions and properties. And for that rightlined figures are far diſtant in nature and propertie from round and circular figures, in the third booke he inſtructeth the reader of the nature and conditiõ of circles. In the fourth booke he compareth figures of right lines and circles together, and teacheth how to deſcribe a figure of right lines with in or about a circle: and contrariwiſe a circle with in or about a rectiline figure. In the fifth booke he ſearcheth out the nature of proportion (a matter of wonderfull vſe and deepe conſideration), for that otherwiſe he could not compare figure with figure, or the ſides of figures together. For whatſoeuer is compared to any other thing, is compared vnto it vndoubtedly vnder ſome kinde of proportion. Wherefore in the ſixth booke he compareth figures together, one to an other, likewiſe their ſides. And for that the nature of proportion, can not be fully and clearely ſene without the knowledge of number, wherein it is firſt and chiefely found: in the ſeuenth, eight, and ninth bookes, he entreateth of number, & of the kindes and properties thereof. And becauſe that the ſides of ſolide bodyes, for the moſt part are of ſuch ſort, that compared together, they haue ſuch proportion the one to the other, which can not be expreſſed by any number certayne, and therefore are called irrational lines, he in the tẽth boke hath writtẽ & taught which lines are cõmẽſurable or incõmẽſurable the one to the other, and of the diuerſitie of kindes of irrationall lines, with all the conditions & proprieties of them. And thus hath *Euclide* in theſe ten foreſayd bokes, fully & moſt plẽteouſly in a meruelous order taught, whatſoeuer ſemed neceſſary, and requiſite to the knowledge of all ſuperficiall figures, of what ſort & forme ſo euer they be. Now in theſe bookes following he entreateth of figures of an other kinde, namely, of bodely figures: as of Cubes, Piramids, Cones, Columnes, Cilinders, Parallelipipedons, Spheres and ſuch others: and ſheweth the diuerſitie of thẽ, the generation, and production of them, and demonſtrateth with great and wonderfull art, their proprieties and paſſions, with all their natures and conditions. He alſo compareth one of them to an other, whereby to know the reaſon and proportion of the one to the other, chiefely of the fiue bodyes which are called regular bodyes. And theſe are the thinges of all other entreated of in Geometrie, moſt worthy and of greateſt dignitie, and as it were the end and finall entent of the whole are of Geometrie, and for whoſe cauſe hath bene written, and ſpoken whatſoeuer hath hitherto in the former bookes bene ſayd or written. As the firſt booke was a ground, and a neceſſary entrye to all the reſt following, ſo is this eleuenth booke a neceſſary entrie and ground to the reſt which follow. And as that contayned the declaration of wordes, and definitions of thinges requiſite

A point the beginning of all quantitie continuall.

The methode vſed by Euclide in the ten former bookes.

First booke.

Second booke.

Third booke.

Fourth booke.

Fiueth booke.

Sixth booke.

Seuenth booke

Eight booke.

Ninth booke.

Tenth booke.

What is entreated of in the fiue bookes following.

Fiue regular bodies, the finall ende & ſcope of Euclides Geometricall Elementes.

Compariſon of the firſt booke and this eleuẽth booke together.

requisite to the knowledge of superficiall figures, and entreated of lines (and of their diuisions and sections) which are the termes and limites of superficiall figures: so in this booke is set forth the declaration of wordes and definitions of thinges pertayning to solide and corporall figures: and also of superficieces which are the termes & limites of solides: moreouer of the diuision and intersection of them, and diuers other thinges, without which the knowledge of bodely and solide formes can not be attayned vnto. And first is set the definitions as followeth.

Definitions.

A solide or body is that which hath length, breadth, and thicknes, and the terme or limite of a solide is a superficies. — *First diffinition*

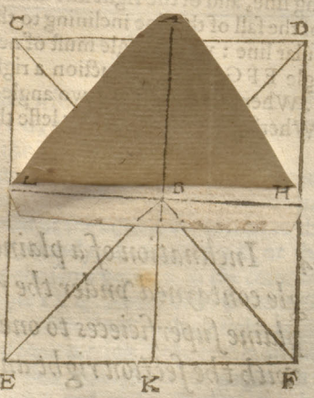
There are three kindes of continuall quantitie, a line, a superficies, and a solide or body: the beginning of all which (as before hath bene sayd) is a poynt, which is indiuisible. Two of these quantities, namely, a line and a superficies, were defined of *Euclide* before in his first booke. But the third kinde, namely, a solide or body he there defined not, as a thing which pertayned not then to his purpose: but here in this place he setteth the definitiō therof, as that which chiefely now pertayneth to his purpose, and without which nothing in these thinges can profitably be taught. *A solide* (sayth he) *is that which hath lēgth, breadth, and thicknes, or depth.* There are (as before hath bene taught) three reasons or meanes of measuring, which are called cōmonly dimensions, namely, length, breadth, and thicknes. These dimensions are ascribed vnto quantities onely. By these are all kindes of quantitie defined, & are counted perfect or imperfect, according as they are pertaker of fewer or more of them. As *Euclide* defined a line, ascribing vnto it onely one of these dimensions, namely, length: Wherefore a line is the imperfectest kinde of quantitie. In defining of a superficies, he ascribed vnto it two dimensions, namely, length, and breadth: whereby a superficies is a quantitie of greater perfection then is a line, but here in the definitiō of a solide or body. *Euclide* attributeth vnto it all the three dimensiōs, lēgth, breadth, and thicknes. Wherfore a solide is the most perfectest quantitie, which wanteth no dimension at all, passing a lyne by two dimensions, and passing a superficies by one. This definition of a solide is without any designation of forme or figure easily vnderstanded, onely conceiuing in minde, or beholding with the eye a piece of timber or stone, or what matter so euer els, whose dimensions let be equall or vnequall. For example let the length therof be 5. inches, the breadth 4. and the thicknes 2. if the dimensions were equall, the reason is like, and all one, as it is in a Sphere and in a cube. For in that respect and consideration onely, that it is long, broade, and thicke, it beareth the name of a solide or body, and hath the nature and properties therof. There is added to the ende of the definition of a solide, that the terme and limite of a solide is a superficies. Of thinges infinitie there is no Arte or Science. All quantities therfore in this Arte entreated of, are imagined to be finite, and to haue their endes and borders as hath bene shewed in the first booke, that the limites and endes of a line are pointes, and the limites or borders of a superficies are lines, so now he saith that the endes, limites, or borders of a solide are superficieces. As the side of any square piece of timber, or of a table, or die, or any other like, are the termes and limites of them.

A solide the most perfectest quantitie.

No science of thinges infinite.

2 *A right line is then erected perpendicularly to a plaine superficies, whē the right line maketh right angles with all the lines which touch it, and are drawen vpon the ground plaine superficies.* — *Second diffinition.*

Suppose that vpon the grounde playne superficies C D E F from the pointe B be erected a right line, namely, B A, so that let the point A be aloft in the ayre. Drawe also from the poynte B in the playne superficies C-D E F, as many right lines as ye list, as the lines B C, B D, B E, B F, B G, B K, B H, and B L. If the erected line B A with all these lines drawen in the superficies C D E F make a right angle, so that all these angles A B C, A B D, A B E, A B F, A B G, A B K, A B H, A B L, and so of others, be right angles, then by this definition, the line A B is a line erected vpon the superficies C D E F: it is also called commonly a perpendicular line or a plumb line, vnto or vpon a superficies.

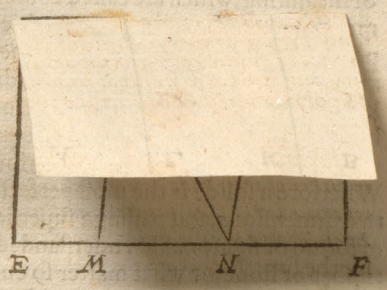


Third diffinition.

3 *A plaine ſuperficies is then vpright or erected perpendicularly to a plaine ſuperficies, when all the right lines drawen in one of the plaine ſuperficieces vnto the common ſection of thoſe two plaine ſuperficieces, making therwith right angles, do alſo make right angles to the other plaine ſuperficies. Inclination or leaning of a right line, to a plaine ſuperficies, is an acute angle, contained vnder a right line falling from a point aboue to the plaine ſuperficies, and vnder an other right line, from the lower end of the ſayd line (let downe) drawen in the ſame plaine ſuperficies, by a certaine point aſsigned, where a right line from the firſt point aboue, to the ſame plaine ſuperficies falling perpendicularly, toucheth.*

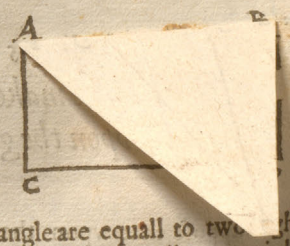
Two diffinitions included in this diffinition. Declaratiõ of the firſt part.

In this third definition are included two definitions: the firſt is of a plaine ſuperficies erected perpendicularly vpon a plaine ſuperficies. The ſecond is of the inclination or leaning of a right line vnto a ſuperficies: of the firſt take this example. Suppoſe ye haue two ſuperficieces A B C D and C D E F. Of which let the ſuperficies C D E F be a ground plaine ſuperficies, and let the ſuperficies A B C D be erected vnto it, and let the line C D be a common terme or interſection to them both, that is, let it be the end or bound of either of them, & be drawen in either of them: in which line note at pleaſure certaine pointes, as the point G, H. From which pointes vnto the line C D, draw perpendicular lines in the ſuperficies A B C D, which let be G L and H K, which falling vpon the ſuperficies C D E F, if they cauſe right angles with it, that is, with lines drawen in it from the ſame pointes G and H, as if the angle L G M or the angle L G N contayned vnder the line L G drawen in the ſuperficies erected, and vnder the G M or G N drawen in the ground ſuperficies C D E F lying flat, be a right angle, then by this definition, the ſuperficies A B C D is vpright or erected vpon the ſuperficies C D E F. It is alſo commonly called a ſuperficies perpendicular vpon or vnto a ſuperficies.



Declaration of the ſecond part.

For the ſecond part of this definition, which is of the inclination of a right line vnto a plaine ſuperficies, take this example. Let A B C D be a ground plaine ſuperficies, vpon which from a point being aloft, namely, the point E, ſuppoſe a right line to fall, which let be the line E G, touching the plaine ſuperficies A B C D at the poynt G. Againe, from the point E, being the toppe or higher limite and end of the inclining line E G, let a perpendicular line fall vnto the plaine ſuperficies A B C D, which let be the line E F, and let F be the point where E F toucheth the plaine ſuperficies A B C D. Then from the point of the fall of the line inclining vpon the ſuperficies vnto the point of the falling of the perpendicular line vpon the ſame ſuperficies, that is, from the point G to the point F, draw a right line G F. Now by this definition, the acute angle E G F is the inclination of the line E G vnto the ſuperficies A B C D. Becauſe it is contayned of the inclining line, and of the right line drawen in the ſuperficies, from the point of the fall of the line inclining to the point of the fall of the perpendicular line: which angle muſt of neceſſitie be an acute angle. For the angle E F G is by conſtruction a right angle, and three angles in a triangle are equall to two right angles. Wherefore the other two angles, namely, the angles E G F, and G E F, are equall to one right angle. Wherfore either of them is leſſe then a right angle. Wherfore the angle E G F is an acute angle.

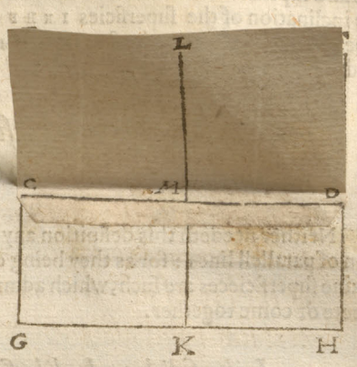


Fourth diffinition.

4 *Inclination of a plaine ſuperficies to a plaine ſuperficies, is an acute angle contayned vnder the right lines, which being drawen in either of the plaine ſuperficieces to one & the ſelf ſame point of the cõmon ſection, make with the ſection right angles.*

Suppoſe

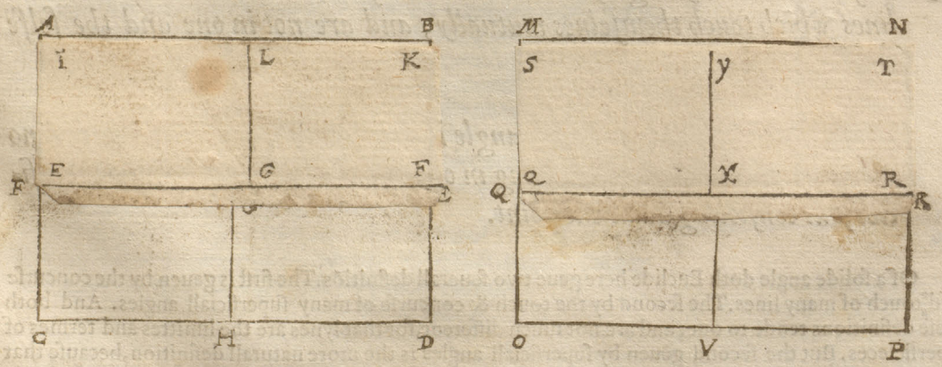
Suppoſe that there be two ſuperficieces A B C D & E F G H, and let the ſuperficies A B C D be ſuppoſed to be erected not perpendicularly, but ſomewhat leaning and inclining vnto the plaine ſuperficies E F G H, as much or as litle as ye will : the cõmon terme or ſection of which two ſuperficieces let be the line C D. From ſome one point, as from the point M aſſigned in the common ſection of the two ſuperficieces, namely, in the line C D, draw a perpendicular line in either ſuperficies. In the ground ſuperficies E F G H draw the line M K, and in the ſuperficies A B C D draw the line M L. Now if the angle L M K be an acute angle, then is that angle the inclination of the ſuperficies A B C D vnto the ſuperficies E F G H, by this definition, becauſe it is contained of perpendicular lines drawen in either of the ſuperficieces to one and the ſelf ſame point being the common ſection of them both.



5 Plaine ſuperficieces are in like ſort inclined the one to the other, when the ſayd angles of inclination are equall the one to the other. *Fifth diffinition.*

This definition needeth no declaration at all, but is moſt manifeſt by the definition laſt going before. For in conſidering the inclinations of diuers ſuperficieces to others, if the acute angles contayned vnder the perpendicular lines drawen in them from one point aſſigned in ech of their common ſections be equall, as if to the angle L M K in the former example be geuen an other angle in the inclination of two other ſuperficieces equall, then is the inclination of theſe ſuperficieces like, and are by this definition ſayd in like ſort to incline the one to the other.

Theodoſius geueth an other definition of like inclination of plaine ſuperficieces the one to the other, after this maner. *One plaine ſuperficies is like inclined to an other, as an other ſuperficies is to an other, when in either of the plaine ſuperficieces right lines being drawen, and making right angles with their common ſection, containe in the ſame pointes equall angles.* This definition is in ſubſtance the ſame with that geuen of *Euclide*, and is an elucidation of it. For example let A B C D be a ground plaine ſuperficies, vnto which let the ſuperficies E F I K incline and leane. And let the common ſection of theſe two ſuperficieces be the line E F. Then drawe in eche of theſe ſuperficieces right lines to ſome one point of the common ſection E F, which let be the point G : with which ſection let them make right angles. As in the ſuper-



ficies A B C D draw the line H G, which in the point G let it make with the common ſectiõ a right angle H G F or H G E. Alſo in the ſuperficies E F I K draw the line L G, which in the point G together with the common ſection E F let make alſo a right angle L G F, or the right angle L G E.

Now alſo let there be an other ground plaine ſuperficies, namely, the ſuperficies M N O P, vnto whom alſo let leane and incline the ſuperficies Q R S T, and let the common ſection or ſegment of them be the line Q R. And draw in the ſuperficies M N O P to ſome one point of the cõmon ſection as to the point X the line V X, making with the common ſection right angles, namely, the angle V X R, or the angle V X Q : alſo in the ſuperficies S T Q R draw the right line Y X to the ſame point X in the common ſection, making therwith right angles, as the angle Y X R, or the angle Y X Q. Now (as ſayth the definition) if the angles contayned vnder the right lines drawen in theſe ſuperficieces & making right angles with the common ſection, be in the pointes, that is, in the pointes of their meting in the common ſection, equall : then is the inclination of the ſuperficieces equall. As in this example, if the angle L G H contayned vnder the line L G being in the inclining ſuperficies I K E F and vnder the line H G being in the ground ſuperficies A B C D, be equall, to the angle Y X V contayned vnder the line V X being in the

ground superficies M N O P and vnder the line Y X being in the inclining superficies S T Q R: then is the inclination of the superficies I K E F vnto the superficies A B C D, like vnto the inclination of the superficies S T Q R vnto the superficies M N O P. And so by this definition these two superficieces are sayd to be in like sort inclined.

Sixth diffinition.

6 *Parallell plaine superficieces are those, which being produced or extended any way neuer touch or concurre together.*

Neither needeth this definition any declaration, but is very easie to be vnderstanded by the definition of parallell lines: for as they being drawen on any part, neuer touch or come together: so parallel plaine superficieces are such, which admitte no touch, that is, being produced any way infinitely neuer meete or come together.

Seuenth diffinition.

7 *Like solide or bodily figures are such, which are contained vnder like plaine superficieces, and equall in multitude.*

What plaine superficieces are called like, hath in the beginning of the sixth booke, bene sufficiently declared. Now when solide figures or bodies be contained vnder such like plaine superficieces as there are defined, and equall in number, that is, that the one solide haue as many in number as the other, in their sides and limites: they are called like solide figures, or like bodies.

Eighth diffinition.

8 *Equall and like solide (or bodely) figures are those which are contained vnder like superficieces, and equall both in multitude and in magnitude.*

In like solide figures it is sufficient, that the superficieces which containe them be like and equall in number onely, but in like solide figures and equall, it is necessary that the like superficieces contaynyng them, be also equal in magnitude. So that besides the likenes betwene them, they be (eche being compared to his correspondent superficies) of one greatnes, and that their areas or fieldes be equal. When such superficieces contayne bodies or solides, then are such bodies equall and like solides or bodies.

Ninth diffinition.

9 *A solide or bodily angle, is an inclination of moe then two lines to all the lines which touch themselues mutually, and are not in one and the selfe same superficies.*

Or els thus: *A solide or bodily angle is that which is contayned vnder mo then two playne angles, not being in one and the selfe same plaine superficies, but consisting all at one point.*

Of a solide angle doth Euclide here geue two seuerall definitiõs. The first is geuen by the concurse and touch of many lines. The second by the touch & concurse of many superficiall angles. And both these definitions tende to one, and are not much different, for that lynes are the limittes and termes of superficieces. But the second geuen by superficiall angles is the more naturall definition, because that superficieces are the next and immediate limites of bodies, and so are not lines. An example of a solide angle cannot wel and at fully be geuẽ or described in a plaine superficies. But touchyng this first definitiõ, lay before you a cube or a die, and cõsider any of the corners or angles therof, so shal ye see that at euery angle there concurre thre lines (for two lines cõcurring cannot make a solide angle) namely, the line or edge of his breadth, of his legth, and of his thicknes, which their so inclining & cõcurring touether, make a solide angle, and so of others. And now cõcerning the second definitiõ, what superficial or plaine angles be, hath bene taught before in the first boke, namely, that it is the touch of two right lines. And as a superficiall or playne angle is causẽd & cõtained of right lines, so si a solide angle causẽd & cõtayned of plaine superficiall angles. Two right lines touching together, make a plaine angle, but two plaine angles ioyned together can not make a solide angle, but according to the definitiõ, they must be moe thẽ two, as three, foure, fiue, or moe: which also must not be in one & the selfe same superficies, but must be in diuers superficieces, meeting at one point. This definition is not hard, but may easily be cõceiued in a cube or a die, where ye see three angles of any three superficieces or sides of the die concurre and meete together in one point, which three playne angles so ioyned together, make a solide angle. Likewise in a Pyramis or a spire of a steple or any other such thing, all the sides therof tẽding vp-

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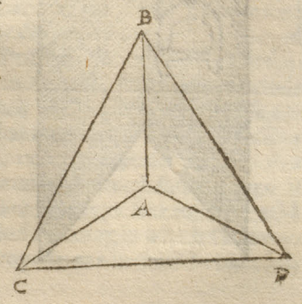
ward narower and narower, at length ende their angles (at the heigth or toppe therof) in one point. So all their angles there ioyned together, make a ſolide angle. And for the better ſight thereof, I haue ſet here a figure wherby ye ſhall more eaſily conceiue it, the baſe of the figure is a triangle, namely, A B C, if on euery ſide of the triangle A B C, ye rayſe vp a triangle, as vpon the ſide A B, ye raiſe vp the triangle A F B, and vpon the ſide A C the triangle A F C, and vpon the ſide B C, the triangle B F C, and ſo bowing the triangles raiſed vp, that their toppes, namely, the pointes F meete and ioyne together in one point, ye ſhal eaſily and plainly ſee how theſe three ſuperficiall angles A F B B F C, C F A, ioyne and cloſe together, touching the one the other in the point F, and ſo make a ſolide angle.

10 *A Pyramis is a ſolide figure contained vnder many playne ſuperficieces ſet vpon one playne ſuperficies, and gathered together to one point.* *Tenth diffinition.*

Two ſuperficieces rayſed vpon any ground can not make a Pyramis, forthat two ſuperficiall angles ioyned together in the toppe, cannot (as before is ſayd) make a ſolide angle. Wherfore whē thre, foure, fiue, or moe (how many ſoeuer) ſuperficieces are raiſed vp frō one ſuperficies being the ground, or baſe, and euer aſcēding diminiſh their breadth, till at the lēgth all their angles cōcurre in one point, making there a ſolide angle: the ſolide incloſed, bounded, and terminated by theſe ſuperficieces is called a Pyramis, as ye ſee in a taper of foure ſides, and in a ſpire of a towre which containeth many ſides, either of which is a Pyramis.

And becauſe that all the ſuperficieces of euery Pyramis aſcend from one playne ſuperficies as from the baſe, and tende to one poynt, it muſt of neceſſitie come to paſſe, that all the ſuperficieces of a Pyramis are trianguler, except the baſe, which may be of any forme or figure except a circle. For if the baſe be a circle, then it aſcendeth not with ſides, or diuers ſuperficieces, but with one round ſuperficies, and hath not the name of a Pyramis, but is called (as hereafter ſhall appeare) a Cone.

Of Pyramids there are diuers kindes. For according to the varietie of the baſe is brought forth the varietie and diuerſitie of kindes of Pyramids. If the baſe of a Pyramis be a triangle, then is it called a triangled Pyramis. If the baſe be a figure of fower angles, it is called a quadrangled Pyramis. If the baſe be a Pentagon, then is it a Pentagonall or fiueangled Pyramis. And ſo forth according to the increaſe of the angles of the baſe infinitely. Although the figure of a Pyramis can not be well expreſſed in a playne ſuperficies, yet may ye ſufficiently conceaue of it both by the figure before ſet in the definition of a ſolide angle, and by the figure here ſet, if ye imagine the point A together with the lines A B, A C, and A D, to be eleuated on high. And yet that the reader may more clerely ſee the forme of a Pyramis, I haue here ſet two ſundry Pyramids which will appeare bodilike, if ye erecte the papers wherin are drawen the triangular ſides of eche Pyramis, in ſuch ſort that the pointes of the angles F of ech triangle may in euery Pyramis concurre in one point, and make a ſolide angle: one of which hath to his baſe a fower ſided figure, and the other a fiue ſided figure. The forme of a triangled Pyramis ye may before beholde in the example of a ſolide angle. And by theſe may ye conceaue of all other kindes of Pyramids.



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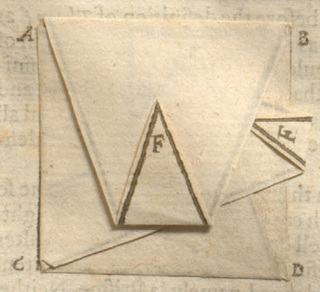

ground ſuperficies M N O P and vnder the line Y X being in the inclining ſuperficies S T Q R: then is the inclination of the ſuperficies I K E F vnto the ſuperficies A B C D, like vnto the inclination of the ſuperficies S T Q R vnto the ſuperficies M N O P. And ſo by this definition theſe two ſuperficieces are ſayd to be in like ſort inclined.

Sixth diffinition.

6 *Parallell plaine ſuperficieces are thoſe, which being produced or extended any way neuer touch or concurre together.*

Neither needeth this definition any declaration, but is very eaſie to be vnderſtanded by the definition of parallell lines: for as they being drawen on any part, neuer touch or come together: ſo parallel plaine ſuperficieces are ſuch, which admitte no touch, that is, being produced any way infinitely neuer meete or come together.

Seuenth diffinition.

7 *Like ſolide or bodily figures are ſuch, which are contained vnder like plaine ſuperficieces, and equall in multitude.*

What plaine ſuperficieces are called like, hath in the beginning of the ſixth booke, bene ſufficiently declared. Now when ſolide figures or bodies be contained vnder ſuch like plaine ſuperficieces as there are defined, and equall in number, that is, that the one ſolide haue as many in number as the other, in their ſides and limites: they are called like ſolide figures, or like bodies.

Eighth diffinition.

8 *Equall and like ſolide (or bodely) figures are thoſe which are contained vnder like ſuperficieces, and equall both in multitude and in magnitude.*

In like ſolide figures it is ſufficient, that the ſuperficieces which containe them be like and equall in number onely, but in like ſolide figures and equall, it is neceſſary that the like ſuperficieces contaynyng them, be alſo equal in magnitude. So that beſides the likenes betwene them, they be (eche being compared to his correſpondent ſuperficies) of one greatnes, and that their areas or fieldes be equal. When ſuch ſuperficieces contayne bodies or ſolides, then are ſuch bodies equall and like ſolides or bodies.

Ninth diffinition.

9 *A ſolide or bodily angle, is an inclination of moe then two lines to all the lines which touch themſelues mutually, and are not in one and the ſelfe ſame ſuperficies.*

Or els thus: *A ſolide or bodily angle is that which is contayned vnder mo then two playne angles, not being in one and the ſelfe ſame plaine ſuperficies, but conſiſting all at one point.*

Of a ſolide angle doth Euclide here geue two ſeuerall definitiõs. The firſt is geuen by the concurſe and touch of many lines. The ſecond by the touch & concurſe of many ſuperficiall angles. And both theſe definitions tende to one, and are not much different, for that lynes are the limittes and termes of ſuperficieces. But the ſecond geuen by ſuperficiall angles is the more naturall definition, becauſe that ſuperficieces are the next and immediate limites of bodies, and ſo are not lines. An example of a ſolide angle cannot wel and at fully be geuẽ or deſcribed in a plaine ſuperficies. But touchyng this firſt definitiõ, lay before you a cube or a die, and cõſider any of the corners or angles therof, ſo ſhal ye ſee that at euery angle there concurre thre lines (for two lines cõcurring cannot make a ſolide angle) namely, the line or edge of his breadth, of his legth, and of his thicknes, which their ſo inclining & cõcurring touether, make a ſolide angle, and ſo of others. And now cõcerning the ſecond definitiõ, what ſuperficial or plaine angles be, hath bene taught before in the firſt boke, namely, that it is the touch of two right lines. And as a ſuperficiall or playne angle is cauſed & cõtained of right lines, ſo si a ſolide angle cauſed & cõtayned of plaine ſuperficiall angles. Two right lines touching together, make a plaine angle, but two plaine angles ioyned together can not make a ſolide angle, but according to the definitiõ, they muſt be moe thẽ two, as three, foure, fiue, or moe: which alſo muſt not be in one & the ſelfe ſame ſuperficies, but muſt be in diuers ſuperficieces, meeting at one point. This definition is not hard, but may eaſily be cõceiued in a cube or a die, where ye ſee three angles of any three ſuperficieces or ſides of the die concurre and meete together in one point, which three playne angles ſo ioyned together, make a ſolide angle. Likewiſe in a Pyramis or a ſpire of a ſteple or any other ſuch thing, all the ſides therof tẽding vp-

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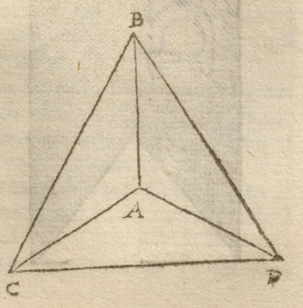
ward narower and narower, at length ende their angles (at the heigth or toppe therof) in one point. So all their angles there ioyned together, make a ſolide angle. And for the better ſight thereof, I haue ſet here a figure wherby ye ſhall more eaſily conceiue it, the baſe of the figure is a triangle, namely, A B C, if on euery ſide of the triangle A B C, ye rayſe vp a triangle, as vpon the ſide A B, ye raiſe vp the triangle A F B, and vpon the ſide A C the triangle A F C, and vpon the ſide B C, the triangle B F C, and ſo bowing the triangles raiſed vp, that their toppes, namely, the pointes F meete and ioyne together in one point, ye ſhal eaſily and plainly ſee how theſe three ſuperficiall angles A F B B F C, C F A, ioyne and cloſe together, touching the one the other in the point F, and ſo make a ſolide angle.

10 *A Pyramis is a ſolide figure contained vnder many playne ſuperficieces ſet vpon one playne ſuperficies, and gathered together to one point.* *Tenth diffinition.*

Two ſuperficieces rayſed vpon any ground can not make a Pyramis, forthat two ſuperficiall angles ioyned together in the toppe, cannot (as before is ſayd) make a ſolide angle. Wherfore whē thre, foure, fiue, or moe (how many ſoeuer) ſuperficieces are raiſed vp frō one ſuperficies being the ground, or baſe, and euer aſcēding diminiſh their breadth, till at the lēgth all their angles cōcurre in one point, making there a ſolide angle: the ſolide incloſed, bounded, and terminated by theſe ſuperficieces is called a Pyramis, as ye ſee in a taper of foure ſides, and in a ſpire of a towre which containeth many ſides, either of which is a Pyramis.

And becauſe that all the ſuperficieces of euery Pyramis aſcend from one playne ſuperficies as from the baſe, and tende to one poynt, it muſt of neceſſitie come to paſſe, that all the ſuperficieces of a Pyramis are trianguler, except the baſe, which may be of any forme or figure except a circle. For if the baſe be a circle, then it aſcendeth not with ſides, or diuers ſuperficieces, but with one round ſuperficies, and hath not the name of a Pyramis, but is called (as hereafter ſhall appeare) a Cone.

Of Pyramids there are diuers kindes. For according to the varietie of the baſe is brought forth the varietie and diuerſitie of kindes of Pyramids. If the baſe of a Pyramis be a triangle, then is it called a triangled Pyramis. If the baſe be a figure of fower angles, it is called a quadrangled Pyramis. If the baſe be a Pentagon, then is it a Pentagonall or fiueangled Pyramis. And ſo forth according to the increaſe of the angles of the baſe infinitely. Although the figure of a Pyramis can not be well expreſſed in a playne ſuperficies, yet may ye ſufficiently conceaue of it both by the figure before ſet in the definition of a ſolide angle, and by the figure here ſet, if ye imagine the point A together with the lines A B, A C, and A D, to be eleuated on high. And yet that the reader may more clerely ſee the forme of a Pyramis, I haue here ſet two ſundry Pyramids which will appeare bodilike, if ye erecte the papers wherin are drawen the triangular ſides of eche Pyramis, in ſuch ſort that the pointes of the angles of ech triangle may in euery Pyramis concurre in one point, and make a ſolide angle: one of which hath to his baſe a fower ſided figure, and the other a fiue ſided figure. The forme of a triangled Pyramis ye may before beholde in the example of a ſolide angle. And by theſe may ye conceaue of all other kindes of Pyramids.



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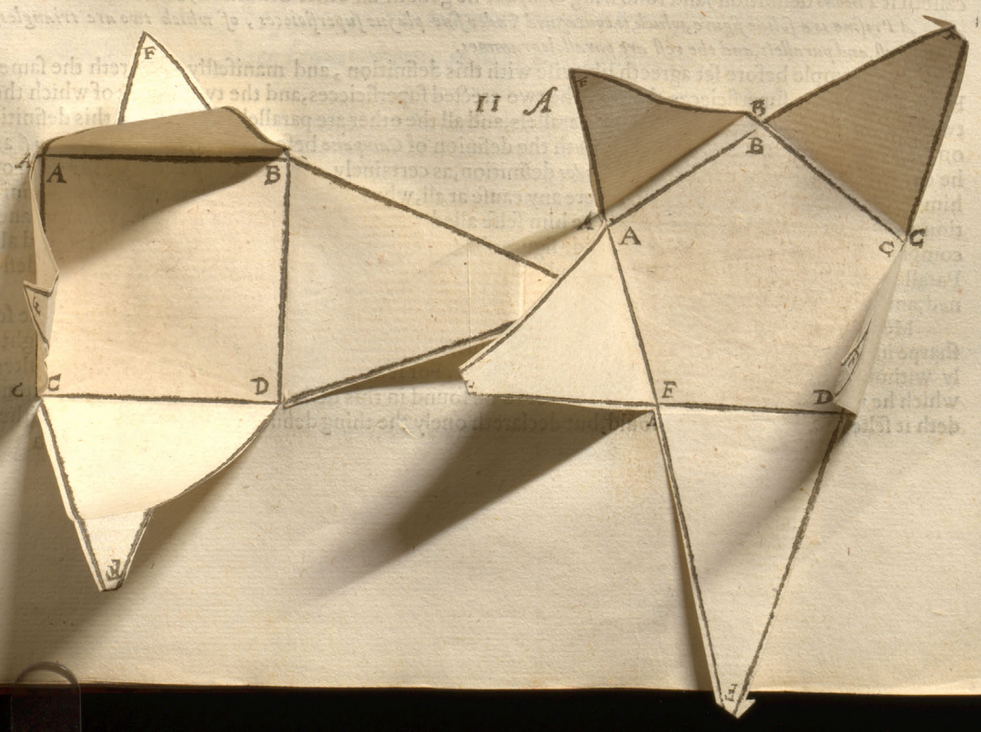
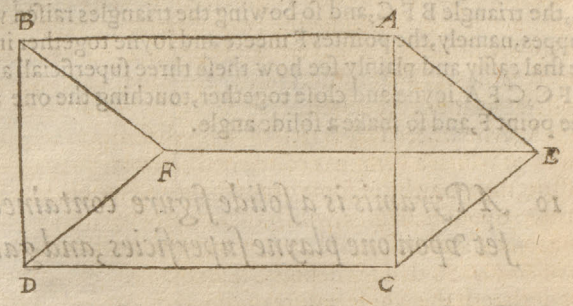


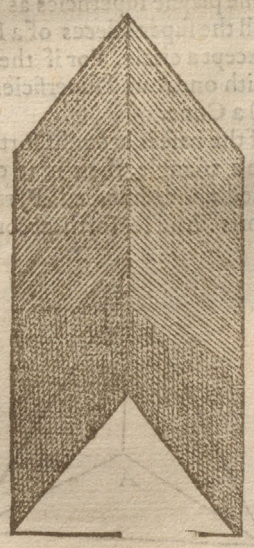
Eleuenth diffinition.

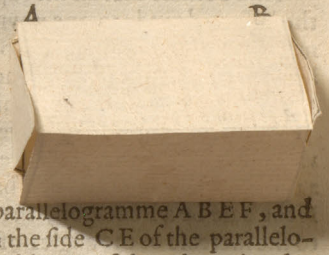
11 A prisme is a solide or a bodily figure contained vnder many plaine superficieces, of which the two superficieces which are opposite, are equall and like, and parallells, & all the other superficieces are parallelogrämes.

This diffinition agreeth to the generall name of a prisme, which also may be applied to Parallelepedons (as may appeare in the 12. booke following & 10. propositiõ, his demonstratiõ) and to some other bodie likewise.

Although you may in a plaine superficies by this figure here set, without any hardnes conceaue what a prisme is, namely, if ye imagine the superficies A B D C to be the ground & base of the solide, and the two superficieces, namely, the superficies A E F B, and the superficies C E F D to be erected vp on the sides of the base, the one on the one side, namely, on the line A B, and the other on the other side, namely, on the line D C, not perpendicularly, but inclining and bending the one to the other, till they meete in the toppe, namely, on the line E F. For so ye see that this solide figure is contained vnder many plaine superficieces, of which two, namely, the superficies A E C, and the superficies B F D, which are the endes of the solide, and opposite the one to the other, are equall like and parallels, and all the other superficieces, namely, the base A B C D, & the two erected superficieces, that is, the superficies A E F B, and the superficies C E F D are parallelogrammes. Yet notwithstanding, to make the thing more clere vnto the reader, I haue here set a Prisme which will appeare bodilike, if you erecte bending wise the papers wherein are drawen the parallelogrämes A B E F, & C D E F, that they may concurre in the line E F in the toppe, and so erect the papers wherein are drawen the triangles A C E and B D F, that the side A E of the one triangle may exactly agree with the side A E of the one parallelogramme A B E F, and the side C E of the same triangle, with the side C E of the parallelogramme C D E F: and moreouer, the side B F of the other triangle, with the side D F of the parallelogramme C D E F: and finally, the side B F of the same triangle, with the side B F of the parallelogramme A B E F. And so shall you most easilie see the forme of a Prisme: that it consisteth of two equall, like, and parallell triangular superficieces, and of three parallelogrammes: wherof the one is the base, and the other two are erected bending wise. Here also beholde the forme thereof as it is by arte described in a plaine to appeare bodilike.







Fluſſas here noteth that *Theon* and *Campane* disagree in defining a Prisme, and he preferreth the definition geuen of *Campane* before the definition geuen of *Euclide* (which becaus he may seme with out lesse offence to reiect, he calleth it *Theons* definition) and following *Campane* he geueth an other definition, which is this.

An other diffinition of a prisme, which is a speciall diffinition of a prisme: as it is commonly called and vsed.

A Prisme is a solide figure, which is contaynced vnder fiue playne superficieces, of which two are triangles, like, equall, and parallels, and the rest are parallelogrammes.

The example before set agreeth likewise with this definition, and manifestly declareth the same. For in it were fiue superficieces, the base, the two erected superficieces, and the two endes: of which the two endes are triangles like, equall and parallels, and all the other are parallelogrammes as this definition requireth. The cause why he preferreth the definion of *Campane* before the definition of *Theon* (as he calleth it, but in very deede it is *Euclides* definition, as certainely, as are all those which are geuen of him in the former bookes, neither is there any cause at all, why it should be doubted in this one definition more then in any of the other) as he him selfe alledgeth, is, for that it is (as he sayth) to large, and comprehendeth many mo kindes of solide figures besides Prismes, as Columnes hauing sides, and all Parallelipipedons, which a definition should not doo: but should be conuertible with the thing defined, and declare the nature of it onely, and stretch no farther.

Me thinketh *Fluſſas* ought not to haue made so much a doo in this matter, nor to haue bene so sharpe in sight and so quicke as to see and espy out such faultes, which can of no man that will see rightly without affection be espyed for such great faultes. For it may well be aunswered that these faultes which he noteth (if yet they be faultes) are not to be found in this definion. It may be sayd that it extendeth it selfe not farther then it should, but declareth onely the thing defined, namely, a Prisme. Neither

doth

doth it agree (as *Flussas* cauilleth) with all Parallelipipedons and Columnes hauing sides. All Parallelipipedons what so euer right angled, or not right angled which are described of equidistant sides or superficieces, haue their sides opposit. So that in any of them there is no one side, but it hath a side opposit vnto it. So likewise is it of eué sided Columnes, eche hath his opposite side directly agaynst it, which agreeth not with this definition of *Euclide*. Here it is euidently sayd, that of all the superficieces, the two which are opposite are equall, like, and parallels, meaning vndoubtedly onely two & no moe. Which is manifest by that which followeth. The other (sayth he) are parallelogrammes, signifiing most euidently that none of the rest besides the two aforesayd, which are equall, like, and parallels, are opposite: but two of necessitie are raysed vp, and concurre in one common line, and the other is the base. So that it contayneth not vnder it the figures aforesayd, that is sided Columnes, & al Parallelipipedons, as *Flussas* hath not so aduisedly noted.

Agayne where *Flussas* setteth in his definition, as an essentiall part thereof, that of the fiue superficieces, of which a Prisme is contayned, two of them must be triangles, that vndoubtedly is not of necessitie, they may be of some other figure. Suppose that in the figure before geuen that in the place of the two opposite figures, which there were two triangles, were placed two pentagōs: yet should the figure remayne a Prisme still, and agree with the definition of *Euclide*, and falleth not vnder the definition of *Flussas*. So that his definitiō seemeth to be to narrow and stretcheth not so farre as it ought to do, nor declareth the whole nature of the thing defined. Wherefore it is not to be preferrd before *Euclides* definition, as he woulde haue it. This figure of *Euclide* called a Prisme, is called of *Campane* and certayne others *Figura Serratilis*, for that it represēteth in some maner the forme of a Sawe. And of some others it is called *Cuneus*, that is, a Wedge, because it beareth the figure of a wedge.

This bodie called Figura Serratilis.

Moreouer although it were so, that the definitiō of a Prisme should be so large, that it should cōtaine all these figures noted of *Flussas* as sided Columnes, & all Parallelipipedons: yet should not *Flussas* haue so great a cause to finde so notably a fault, so vtterly to reiect it. It is no rare thing in all learninges, chiefely in the Mathematicalls, to haue one thing more generall then an other. Is it not true that euery Isosceles is a triangle, but not euery triangle is an Isosceles? And why may not likewise a Prisme be more generall, then a Parallelepipedon, or a Columne hauing sides (and contayne them vnder it as a triangle cōtayneth vnder it an Isosceles and other kinds of triangles). So that euery Prallelipipedon, or euery sided Columne be a Prisme, but not euery Prisme a Parallelipipedō or a sided Columne. This ought not to be so much offensiue. And indeede it semeth manifestly of many, yea & of the learned so to be také, as clearely appeareth by the wordes of *Pſellus* in his Epitome of Geometrie, where he entreateth of the production and constitution of these bodyes. His wordes are these. *All rectilīne figures being erected vpon their playnes or bases by right angles, make Prismes.* Who perceaueth not but that a Pentagon erected vpō his base of fiue sides maketh by his motion a sided Columne of fiue sides? Likewise an Hexagon erected at right angles produceth a Columne hauing sixe sides: and so of all other rectiline figures. All which solides or bodyes so produced, whether they be sided Columnes or Parallelipipedons, be here in most plaine words (of this excellēt and auncient Greke author *Pſellus*) called Prismes. Wherfore if the definitiō of a Prisme geué of *Euclide* should extend it selfe so largely as *Flussas* imagineth, and should enclude such figures or bodyes, as he noted: he ought not yet for all that so much to be offended, and so narowly to haue sought faultes. For *Euclide* in so defining mought haue that meaning & sense of a Prisme which *Pſellus* had. So ye see that *Euclide* may be defended either of these two wayes, either by that that the definition extendeth not to these figures, and so not to be ouer generall nor stretch farther then it ought: or ells by that that if it should stretch so far it is not so haynous. For that as ye se many haue také it in that sense. In deede cōmonly a Prisme is taken in that significatiō and meaning in which *Campanus Flussas* and others take it. In which sense it semeth also that in diuers propositions in these bookes following it ought of necessitie to be taken.

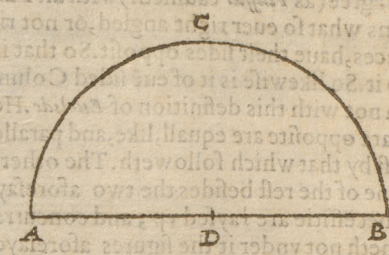
Pſellus.

12 *A Sphere is a figure which is made, when the diameter of a semicircle abiding fixed, the semicircle is turned round about, vntill it returne vnto the selfe same place from whence it began to be moued.*

Twelueth diffinition.

To the end we may fully and perfectly vnderstand this definition, how a Sphere is produced of the motion of a semicircle, it shall be expedient to cōsider how quantities Mathematically are by imagination conceaued to be produced, by flowing and motion, as was somewhat touched in the beginning of the first booke. Euer the lesse quantitie by his motion bringeth forth the quātitie next aboue it. As a point mouing, flowing, or gliding, bringeth forth a line, which is the first quantitie, and next to a point. A line mouing produceth a superficies, which is the second quantitie, and next vnto a line. And last of all, a superficies mouing bringeth forth a solide or body, which is the third & last quantitie. These thinges well marked, it shall not be very hard to attaine to the right vnderstanding of this definition. Vpon the line A B being the diameter, describe a

semicircle



ſemicircle A C B, whoſe centre let be D: the diameter A B being fixed on his endes or pointes, imagine the whole ſuperficies of the ſemicircle to moue round from ſome one point aſſigned, till it returne to the ſame point againe. So ſhall it produce a perfect Sphere or Globe, the forme whereof you ſee in a ball or bowle. And it is fully round and ſolide, for that it is deſcribed of a ſemicircle which is perfectly round, as our countrey man *Iohannes de Sacro Buſco* in his booke of the Sphere, of this definition which he taketh out of *Euclide*, doth well collecte. But it is to be noted and taken heede of, that none be deceaued by the definition of a Sphere geuen by *Iohannes de Sacro Buſco*: *A Sphere* (ſayth he) *is the paſſage or mouing of the circumference of a ſemicircle, till it returne vnto the place where it beganne*, which agreeth not with *Euclide*. *Euclide* plainly ſayth, that a Sphere is the paſſage or motion of a ſemicicle, and not the paſſage or motion of the circumference of a ſemicircle: neither can it be true that the circumference of a ſemicircle, which is a line, ſhould deſcribe a body. It was before noted that euery quantitie moued, deſcribeth and produceth the quantitie next vnto it. Wherefore a line moued can not bring forth a body, but a ſuperficies onely. As if ye imagine a right line faſtened at one of his endes to moue about from ſome one point till it returne to the ſame againe, it ſhall deſcribe a plaine ſuperficies, namely, a circle. So alſo if ye likewiſe conceaue of a crooked line, ſuch as is the circumference of a ſemicicle, that his diameter faſtened on both the endes it ſhould moue from a point aſſigned till it returne to the ſame againe, it ſhould deſcribe & produce a round ſuperficies onely, which is the ſuperficies and limite of the Sphere, and ſhould not produce the body and ſoliditie of the Sphere. But the whole ſemicircle, which is a ſuperficies, by his motion, as is before ſaid, produceth a body, that is, a perfect Sphere. So ſee you the errour of this deſmition of the author of the Sphere: which whether it happened by the author him ſelfe, which I thinke not: or that that particle was thruſt in by ſome one after him, which is more likely, it it not certaine. But it is certaine, that it is vnaptly put in, and maketh an vntrue definition: which thing is not here ſpoken, any thing to derogate the author of the booke, which aſſuredly was a man of excellent knowledge: neither to the hindrance or diminiſhing of the worthines of the booke, which vndoubtedly is a very neceſſary booke, then which I know none more meete to be taught and red in ſcholes touching the groundes and principles of Aſtronomie and Geographie: but onely to admoniſhe the young and vnſkilfull reader of not falling into errour. *Theodoſius* in his booke *De Sphericis* (a booke very neceſſary for all thoſe which will ſee the groundes and principles of Geometrie and Aſtronomie, which alſo I haue tranſlated into our vulgare tounge, ready to the preſſe) defineth a Sphere after thys maner: *A Sphere is a ſolide or body contained vnder one ſuperficies, in the midle wherof there is a point, frō which all lines drawen to the circumference are equall*. This definition of *Theodoſius* is more eſſentiall and naturall, then is the other geuen by *Euclide*. The other did not ſo much declare the inward nature and ſubſtance of a Sphere, as it ſhewed the induſtry and knowledge of the producing of a Sphere, and therfore is a cauſall definition geuen by the cauſe efficient, or rather a deſcription then a definition. But this definition is very eſſentiall, declaring the nature and ſubſtance of a Sphere. As if a circle ſhould be thus defined, as it well may: *A circle is the paſſage or mouing of a line from a point till it returne to the ſame point againe*: it is a cauſall definition, ſhewing the efficient cauſe wherof a circle is produced, namely, of the motion of a line. And it is a very good deſcription fully ſhewing what a circle is. Such like deſcription is the definition of a Sphere geuen of *Euclide* by the motion of a ſemicircle. But when a circle is defined to be a plaine ſuperficies, in the middeſt wherof is a point, from which all lines drawen to the circumference therof, are equall: this definition is eſſentiall and formall, and declareth the very nature of a circle. And vnto this definition of a circle, is correſpondent the definition of a Sphere geuē by *Theodoſius*, ſaying: that it is a ſolide or body, in the middeſt whereof there is a point, from which all the lines drawen to the circumference are equall. So ſee you the affinitie betwene a circle and a Sphere. For what a circle is in a plaine, that is a Sphere in a Solide. The fulnes and content of a circle is deſcribed by the motion of a line moued about: but the circumference therof, which is the limite and border thereof, is deſcribed of the end and point of the ſame line moued about. So the fulnes, content, and body of a Sphere or Globe is deſcribed of a ſemicircle moued about. But the Sphericall ſuperficies, which is the limite and border of a Sphere, is deſcribed of the circumference of the ſame ſemicircle moued about. And this is the ſuperficies ment in the definition, when it is ſayd, that it is contained vnder one ſuperficies, which ſuperficies is called of *Iohannes de Sacro Buſco* & others, the circumference of the Sphere.

Galene in his booke *de diffinitionibus medicis*, geueth ye an other definitiō of a Sphere, by his propertie or cōmon accidēce of mouing, which is thus. *A Sphere is a figure moſt apt to all motion, as hauing no baſe whereon to ſtay*. This is a very plaine and witty definition, declaring the dignitie thereof aboue all figures generally. All other bodyes or ſolides, as Cubes, Pyramids, and others haue ſides, baſes, and angles, all which are ſtayes to reſt vpon, or impedimentes and lets to motion. But the Sphere hauing no ſide or

baſe

What is to be taken heede of in the diffinition of a ſphere geuen by Iohannes de Sacrō Buſco.

Theodoſius diſſinition of a ſphere.

The circumference of a ſphere.

Galens diffinition of a ſphere.

The dignitie of a ſphere.

baſe to ſtay one, nor angle to let the courſe thereof, but onely in a poynt touching the playne wherein it ſtandeth, moueth freely and fully with out let. And for the dignity and worthines thereof, this circular and Spherical motion is attributed to the heauens, which are the moſt worthy bodyes. Wherefore there is aſcribed vnto them this chiefe kinde of motion. This ſolide or bodely figure is alſo commonly called a Globe.

A ſphere called a Globe.

13 The axe of a Sphere is that right line which abideth fixed, about which the ſemicircle was moued.

Thirteenth diffinition.

As in the example before geuen in the definition of a Sphere, the line A B, about which his endes being fixed, the ſemicircle was moued (which line alſo yet remayneth after the motion ended) is the axe of the Sphere deſcribed of that ſemicircle. *Theodoſius* defineth the axe of a Sphere after this maner. *The axe of a Sphere is a certayne right line drawen by the centre, ending on either ſide in the ſuperficies of the Sphere, about which being fixed the Sphere is turned.* As the line A B in the former example. There nedeth to this definition no other declaration, but onely to conſider, that the whole Sphere turneth vpon that line A B, which paſſeth by the centre D, and is extended one either ſide to the ſuperficies of the Sphere, wherefore by this definition of *Theodoſius* it is the axe of the Sphere.

Theodoſius diffinition of the axe of a ſphere.

14 The centre of a Sphere is that poynt which is alſo the centre of the ſemicircle.

Fourtenth diffinition.

This definition of the centre of a Sphere is geuen as was the other definition of the axe, namely, hauing a relation to the definition of a Sphere here geuen of *Euclide*: where it was ſayd that a Sphere is made by the reuolution of a ſemicircle, whoſe diameter abideth fixed. The diameter of a circle and of a ſemicrcle is all one. And in the diameter either of a circle or of a ſemicircle is contayned the center of either of them, for that the diameter of eche euer paſſeth by the centre. Now (ſayth *Euclide*) the poynt which is the center of the ſemicircle, by whoſe motion the Sphere was deſcribed, is alſo the centre of the Sphere. As in the example there geuen, the poynt D is the centre both of the ſemicircle & alſo of the Sphere. *Theodoſius* geueth an other definition of the centre of a Sphere which is thus. *The centre of a Sphere is a poynt with in the Sphere, from which all lines drawen to the ſuperficies of the Sphere are equall.* As in a circle being a playne figure there is a poynt in the middeſt, from which all lines drawen to the circumfrence are equall, which is the centre of the circle: ſo in like maner with in a Sphere which is a ſolide and bodely figure, there muſt be conceaued a poynt in the middeſt thereof, from which all lines drawen to the ſuperficies thereof are equall. And this poynt is the centre of the Sphere by this definition of *Theodoſius*. *Fluſſas* in defining the centre of a Sphere comprehendeth both thoſe definitions in one, after this ſort. *The centre of a Sphere is a poynt aſſigned in a Sphere, from which all the lines drawen to the ſuperficies are equall, and it is the ſame which was alſo the centre of the ſemicircle which deſcribed the Sphere.* This definition is ſuperfluous and contayneth more then nedeth. For either part thereof is a full and ſufficient diffinition, as before hath bene ſhewed. Or ells had *Euclide* bene inſufficient for leauing out the one part, or *Theodoſius* for leauing out the other. Paradienture *Fluſſas* did it for the more explication of either, that the one part might open the other.

Theodoſius diffinition of the center of a ſphere.

Fluſſas diffinition of the center of a ſphere.

15 The diameter of a Sphere is a certayne right line drawen by the cẽtre, and one eche ſide ending at the ſuperficies of the ſame Sphere.

Finetenth diffinition

This definitiõ alſo is not hard, but may eaſdy be couceaued by the definitiõ of the diameter of a circle. For as the diameter of a circle is a right lire drawne frõ one ſide of the circũfrence of a cirtle to the other, paſſing by the centre of the circle: ſo imagine you a right line to be drawen from one ſide of the ſuperficies of a Sphere to the other, paſſing by the center of the Sphere, and that line is the diameter of the Sphere. So it is not all one to ſay, the axe of a Sphere, and the diameter of a Sphere. Any line in a Sphere drawen from ſide to ſide by the centre is a diameter. But not euery line ſo drawen by the centre is the axe of the Sphere, but onely one right lne about which the Sphere is imagnined to be moued. So that the name of a diameter of a Sphere is more general, then is the name of an axe. For euery axe in a Sphere is a diameter of the ſame: but not euery diameter of a Sphere is an axe of the ſame. And therefore *Fluſſas* ſetteth a diameter in the definitior of an axe as a more generall word in this maner. *The axe*

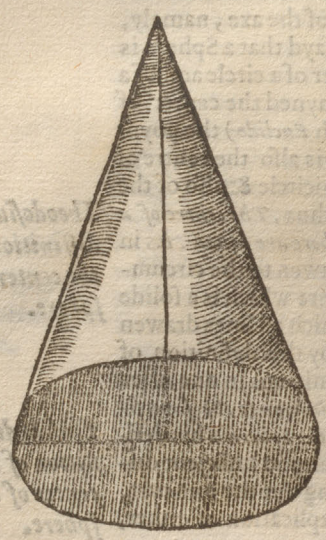
Difference betweene the diameter & axe of a ſphere.

of a Sphere, is that fixed diameter about which the Sphere is moued. A Sphere (as alſo a circle) may haue infinite diameters, but it can haue but onely one axe.

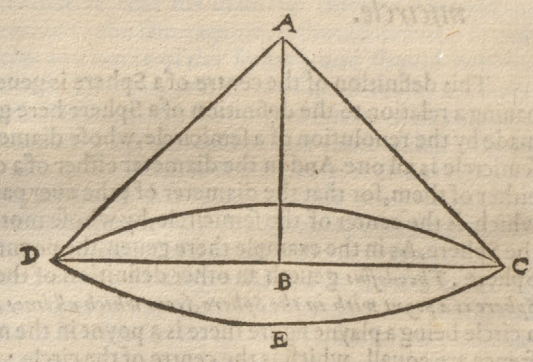
Seuenteenth diffinition.

16 A cone is a ſolide or bodely figure which is made, when one of the ſides of a rectangle triangle, namely, one of the ſides which contayne the right angle, abiding fixed, the triangle is moued about, vntill it returne vnto the ſelfe ſame place from whence it began firſt to be moued. Now if the right line which abideth fixed be equall to the other ſide which is moued about and containeth the right angle: then the cone is a rectangle cone. But if it be leſſe, then is it an obtuſe angle cone. And if it be greater, thē is it an acute angle cone,

This definition of a Cone is of the nature and condition that the definition of a Sphere was, for either is geuen by the motion of a ſuperficies. There, as to the production of a Sphere was imagined a ſemicircle to moue round, from ſome one point till it returned to the ſame point againe: ſo here muſt ye imagine a rectangle triangle to moue about till it come againe to the place where it beganne. Let A B C be a rectangle triangle, hauing the angle A B C a right angle, which let be contained vnder the lines A B and B C.



Now ſuppoſe the ſide A B, namely, one of the lines which cōtaine the right angle A B C to be faſtened, and about it ſuppoſe the triangle A B C to be moued from ſome one poynt aſſigned till it returne to the ſame agayne (as vppon the diameter in the definition of a Sphere ye imagined a ſemicircle to moue about): ſo ſhall the ſolide or body thus deſcribed be a perfect Cone. As you may imagine by this figure here ſet. And the forme of a Cone you may ſufficiently conceaue by the figure ſet in the margent. There are of Cones three kindes, namely, a rectangle Cone, an obtuſeangle Cone, and an acute angle Cone, all which were before in the former definitiō defined: Namely, the firſt kinde after this maner.

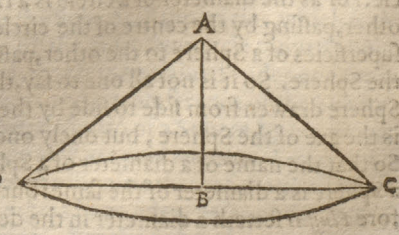


If the right line which abideth fixed, be equall to the other ſide which moueth round about, and containeth the right angle, then the Cone is a rectangle Cone.

Firſt kinde of Cones.

As ſuppoſe in the former example, that the line A B which is fixed, and about which the triangle was moued, and after the motion yet remayneth, be equall to the line B C, which is the other line contayning the right angle, which alſo is moued about together with the whole triangle: then is the Cone deſcribed, as the Cone A D C in this example, a right angled Cone: ſo called for that the angle at the toppe of the Cone is a right angle. For foraſmuch as the lines A B and B C of the triangle A B C are equall, the angle B A C is equall to the angle B C A (by the 5. of the firſt). And eche of them is the halfe of the right angle A B C (by the 32. of the firſt). In like ſort may it be ſhewed in the triangle A B D, that the angle B D A is equall to the angle B A D, and that eche of them is the halfe of a right angle. Wherefore the whole angle C A D, which is compoſed of the two halfe right angles, namely, D A B and C A B is a right angle. And ſo haue ye what is a right angled Cone.

But if it be leſſe, then is it an obtuſeangle Cone. As in this example, the line A B fixed is leſſe then the line B C moued about. Wherefore the Cone deſcribed of the circumuolution of the triangle A B C about the line A B, is an obtuſeangle Cone, for that the angle at the toppe D A C is greater then a right angle. Wherefore it is an obtuſeangle. And therefore the Cone is called an obtuſe angle Cone.



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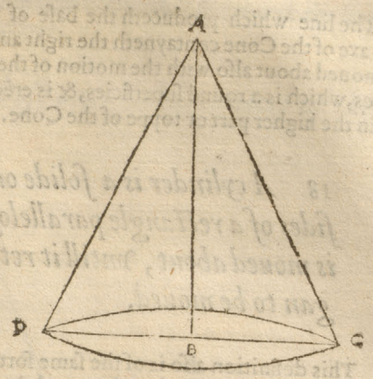
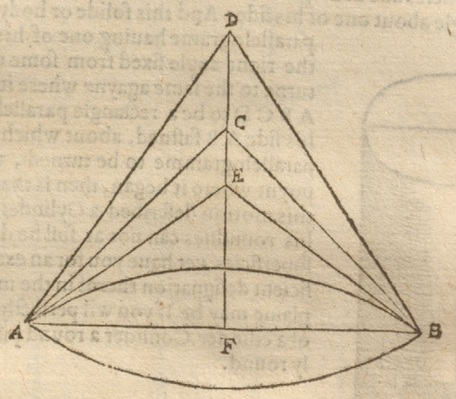


And if it be greater, then is it an acuteangle Cone. As in this figure, the line A B faſtened, is greater then the line B C moued about. Wherefore the Cone deſcribed by the motion and turning of the triangle A B C about A B is an acuteangle Cone, hauing the angle at the toppe B A C an acute angle. Of whome the Cone is called an acuteangle Cone. For the eaſier ſight & cõſideration of all theſe kindes of Cones, and alſo for the plainer demonſtration of the varieties of their angles in their toppes, I haue deſcribed them all three in one playne figure, of which the Cone A C B is a right angled Cone, hauyng his fixed ſide C F equall to the line F B, and hys angle A C B a right angle: the Cone A E B is an obtuſe angle Cone, and A D B an acuteangle Cone. By which figure ye may eaſily demonſtrate (by the 21. of the firſt) that the angle A D B of the Cone A D B, whoſe fixed line D F is greater then the ſide F B, is leſſe then the right angle A C B, and ſo is an acute angle. And alſo (by the ſame 21. of the firſt) ye ſhall with like facilitie perceaue how the angle A E B of the Cone A E B whoſe fixed line E F is leſſe then the ſide F B, is greater then the right angle A C B: and therefore is an obtuſe angle.

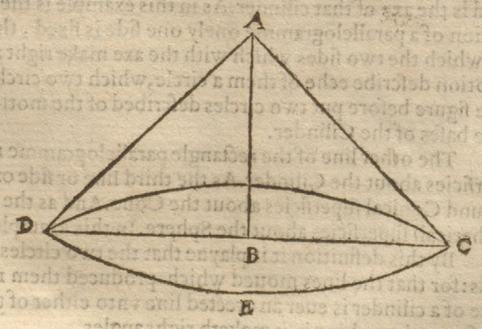


A Cone called of Campane a rounde Piramis.

This figure of a Cone is of *Campane*, of *Vitellio*, and of others which haue written in theſe latter times, called a round Pyramis, which is not ſo aptly. For a Pyramis, and a Cone, are farre diſtant, & of ſundry natures. A Cone is a regular body produced of one circumuolution of a rectangle triangle, and limited and bordered with one onely round ſuperficies. But a Pyramis is terminated and bordered with diuers ſuperficieces. Therefore can not a Cone by any iuſt reaſon beare the name of a Pyramis. This ſolide of many is called *Turbo*, which to our purpoſe may be Engliſhed a *Top* or *Ghyg*: and moreouer, peculiarly *Campane* calleth a Cone the Pyramis of a round Columne, namely, of that Columne which is produced of the motion of a parallelogramme (contained of the lines A B and B C) moued about, the line A B being fixed. Of which Columnes ſhall be ſhewed hereafter.

17 *The axe of a Cone is that line, which abideth fixed, about which the triangle is moued. And the baſe of the Cone is the circle which is deſcribed by the right line which is moued about.*

Seuenteenth diffinition.



As in the example the line *A B* is ſuppoſed to be the line about which the right angled triangle *A B C* (to the production of the Cone) was moued: and that line is here of *Euclide* called the axe of the Cone deſcribed. *The baſe of the Cone is the circle which is deſcribed by the right line which is moued about.* As the line *A B* was fixed and ſtayed, ſo was the line *B C* (together with the whole triangle *A B C*) moued and turned about. A line moued, as hath bene ſayd before, produceth a ſuperficies: and becauſe the line *B C* is moued about a point, namely, the point *B*, being the end of the axe of the Cone *A B*, it produceth by his motion, and reuolution a circle, which circle is the baſe of the Cone: as in this example, the circle *C D E*.

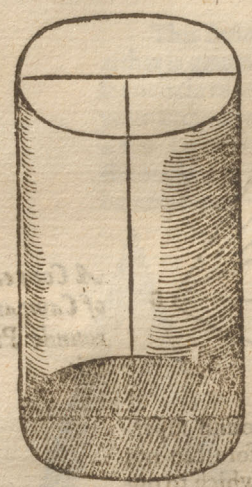
The line which produceth the baſe of the Cone, is the line of the triangle which together with the axe of the Cone contayneth the right angle. The other ſide alſo of the triangle, namely, the line *A C*, is moued about alſo with the motion of the triangle, which with his reuolution deſcribeth alſo a ſuperficies, which is a round ſuperficies, & is erected vpon the baſe of the Cone, & endeth in a point, namely, in the higher part or toppe of the Cone. And it is commonly called a Conicall ſuperficies.

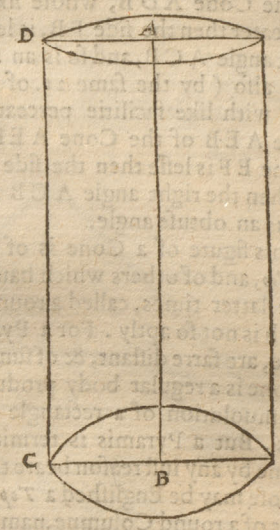
A conicall ſuperficies.

Eighteenth diffinition.

18 A cylinder is a ſolide or bodely figure which is made, when one of the ſides of a rectangle parallelogramme, abiding fixed, the parallelogramme is moued about, vntill it returne to the ſelfe ſame place from whence it began to be moued.

This definition alſo is of the ſame ſort and condition, that the two definitions before geuē were, namely, the definition of a Sphere and the definition of a Cone. For all are geuen by mouing of a ſuperficies about a right line fixed, the one of a ſemicircle about his diameter, the other of a rectangle triangle about one of his ſides. And this ſolide or body here defined is cauſed of the motion of a rectangle parallelogrāme hauing one of his ſides contayning the right angle fixed from ſome one poynt till it returne to the ſame agayne where it began. As ſuppoſe A B C D to be a rectangle parallelogramme, hauing his ſide A B faſtned, about which imagine the whole parallelogramme to be turned, till it returne to the poynt where it began, then is that ſolide or body, by this motion deſcribed, a Cylinder: which becauſe of his roundnes can not at full be deſcribed in a playne ſuperficies, yet haue you for an example thereof a ſufficient deſignation therof in the margent ſuch as in a plaine may be. If you wil perfectly behold the forme of a cilinder. Conſider a round piller that is perfectly round.





Ninetenth diffinition.

19 The axe of a cilinder is that right line which abydeth fixed, about which the parallelogramme is moued. And the baſes of the cilinder are the circles deſcribed of the two oppoſite ſides which are moued about.

Euen as in the deſcription of a Sphere the line faſtened was the axe of the Sphere produced: and in the deſcription of a cone, the line faſtened was the axe of the cone brought forth: ſo in this deſcription of a cilinder the line abiding, which was fixed, about which the rectangle parallelogramme was moued is the axe of that cilinder. As in this example is the line *A B. The baſes of the cilinder &c.* In the reuolution of a parallelogramme onely one ſide is fixed, therefore the three other ſides are moued about: of which the two ſides which with the axe make right angles, and which alſo are oppoſite ſides, in their motion deſcribe eche of them a circle, which two circles are called the baſes of the cilinder. As ye ſee in the figure before put two circles deſcribed of the motiō of the two oppoſit lines A D and BC, which are the baſes of the Cilinder.

A cillindricall ſuperficies.

The other line of the rectangle parallelogramme moued, by his motion deſcribeth the round ſuperficies about the Cilinder. As the third line or ſide of a rectangle triangle by his motion deſcribed the round Conical ſuperficies about the Cone. And as the circūferēce of the ſemicircle deſcribed the round ſphericall ſuperficies about the Sphere. In this example it is the ſuperficies deſcribed of the line D C.

Corollary.

By this definition it is playne that the two circles, or baſes of a cilinder are euer equall and parallels: for that the lines moued which produced them remayned alwayes equall and parallels. Alſo the axe of a cilinder is euer an erected line vnto either of the baſes. For with all the lines deſcribed in the baſes, and touching it, it maketh right angles,

A rounde Columne or ſphera.

Campane, Vitellio, with other later writers, call this ſolide or body a round Columne or piller. And *Campane* addeth vnto this definition this, as a corrollary. That of a round Columne, of a Sphere, and

of

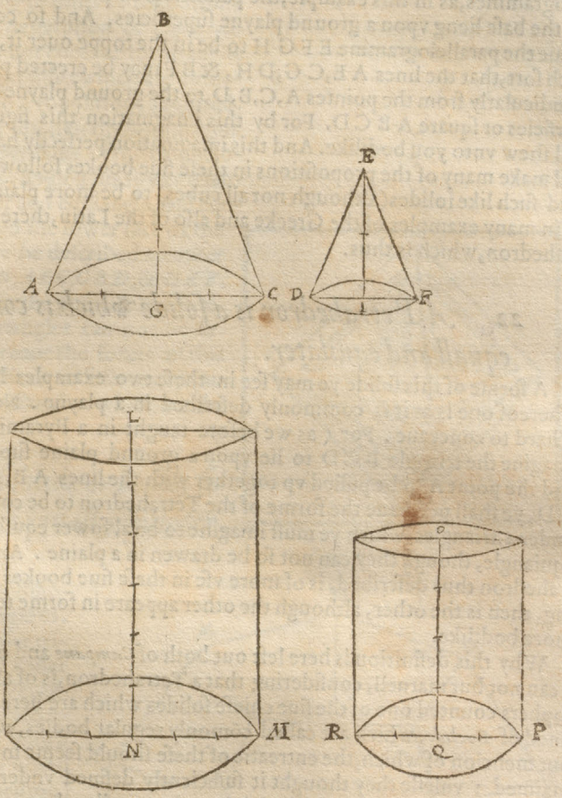
of a circle the cẽtre is one and the ſelfe ſame. That is (as he him ſelfe declareth it & proueth the ſame) where the Columne, the Sphere, and the circle haue one diameter.

A Corollary added by Campane.

20 Like cones and cilinders are thoſe, whoſe axes and diameters of their baſes are proportionall

Twenty diffinition.

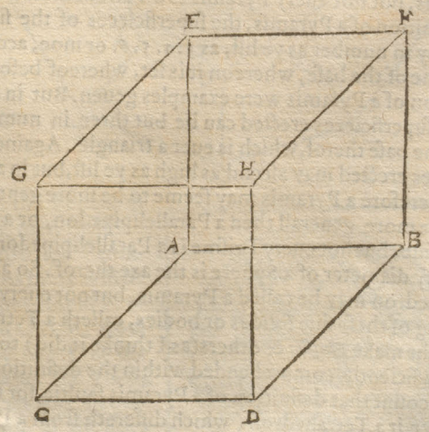
The ſimilitude of cones and cilinders ſtandeth in the proportion of thoſe right lines, of which they haue their originall and ſpring. For by the diameters of their baſes is had their length and breadth, and by their axe is had their heigth or deepenes. Wherefore to ſee whether they be like or vnlike, ye muſt compare their axes together, which is their depth, and alſo their diameters together, which is thier length & breadth. As if the axe *B G* of the cone *A B C* be to to the axe *E I* of the cone *D E F*, as the diameter *A C* of the cone *A B C* is to the diameter *D F* of the cone *D E F*, then are the cones *A B C* and *D E F* like cones. Likewiſe in the cilinders. If the axe *L N* of the cilinder *L H M N* haue that proportion to the axe *O Q* of the cilinder *R O P Q*, which the diameter *H M* hath to the diameter *R P*: then are the cilinders *H L M N* and *R O P Q* like cilinders, and ſo of all others.



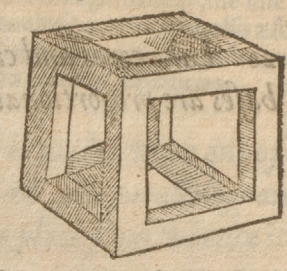
21 A Cube is a ſolide or bodely figure contayned vnder ſixe equall ſquares.

Twenty one diffinition.

As is a dye which hath ſixe ſides, and eche of them is a full and perfect ſquare, as limites or borders vnder which it is contayned. And as ye may conceiue in a piece of timber contayning a foote ſquare euery way, or in any ſuch like. So that a Cube is ſuch a ſolide whoſe three dimenſions are equall, the length is equall to the breadth thereof, and eche of them equall to the depth. Here is as it may be in a playne ſuperficies ſet an image therof, in theſe two figures wherof the firſt is as it is commonly deſcribed in a playne, the ſecond (which is in the beginning of the other ſide of this leafe) is drawn as it is deſcribed by arte vpõ a playne ſuperficies to ſhew ſomwhat bodilike. And in deede the latter deſcriptiõ is for the ſight better thẽ the firſt. But the firſt for the demõſtrations of Euclides propoſitions in the fiue bookes following is of more vſe, for that in it may be conſidered and ſene



all the fixe sides of the Cube. And so any lines or sections drawen in any one of the fixe sides. Which can not be so wel sene in the other figure described vpon a playne. And as touching the first figure (which is set at the ende of the other side of this leafe) ye see that there are fixe parallelogrammes which ye must conceyue to be both equilater and rectangle, although in dede there can be in this description onely two of them rectangle, they may in dede be described al equilater. Now if ye imagine one of the fixe parallelogrammes, as in this example, the parallelogramme A B C D to be the base lieng vpon a ground playne superficies. And so conceiue the parallelogramme E F G H to be in the toppe ouer it, in such sort, that the lines A E, C G, D H, & B F may be erected perpendicularly from the pointes A, C, B, D, to the ground playne superficies or square A B C D. For by this imagination this figure wil shew vnto you bodilike. And this imagination perfectly had, wil make many of the propositions in these fiue bookes following, in which are required to be described such like solides (although not all cubes) to be more plainly and easily conceiued.

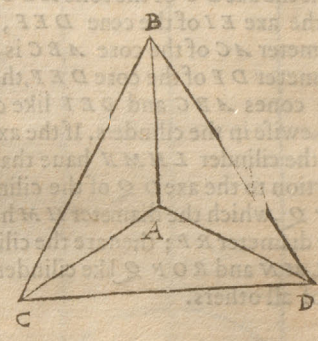


In many examples of the Greeke and also of the Latin, there is in this place set the diffinition of a Tetrahedron, which is thus.

Twenty two diffinition.

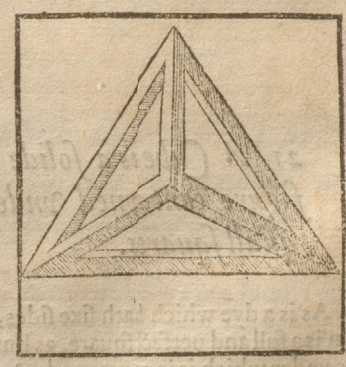
22 *A Tetrahedron is a solide which is contained vnder fower triangles equall and equilater.*

A forme of this solide ye may see in these two examples here set, whereof one is as it is commonly described in a playne. Neither is it hard to conceaue. For (as we before taught in a Pyramis) if ye imagine the triangle B C D to lie vpon a ground plaine superficies, and the point A to be pulled vp together with the lines A B, A C, and A D, ye shall perceaue the forme of the Tetrahedron to be contayned vnder 4. triangles, which ye must imagine to be al fower equilater and equiangle, though they can not so be drawen in a plaine. And a Tetrahedron thus described, is of more vse in these fiue bookes following, then is the other, although the other appeare in forme to the eye more bodilike.



A Tetrahedron one of the fiue regular bodyes.

Why this definition is here left out both of *Campane* and of *Flussas*, I can not but maruell, considering that a Tetrahedron, is of all Philosophers counted one of the fiue chiefe solides which are here defined of *Euclide*, which are called cõmonly regular bodies, without mencion of which, the entreatie of these should seeme much maimed: vnlesse they thought it sufficiently defined vnder the definition of a Pyramis, which plainly and generally taken, includeth in deede a Tetrahedron, although a Tetrahedron properly much differeth from a Pyramis, as a thing speciall or a particular, from a more generall. For so taking it, euery Tetrahedron is a Pyramis, but not euery Pyramis is a Tetrahedron. By the generall definition of a Pyramis, the superficieces of the sides may be as many in number as ye list, as 3. 4. 5. 6. or moe, according to the forme of the base, whereon it is set, whereof before in the definition of a Pyramis were examples geuen. But in a Tetrahedron the superficieces erected can be but three in number according to the base therof, which is euer a triangle. Againe, by the generall definition of a Pyramis, the superficieces erected may ascend as high as ye list, but in a Tetrahedron they must all be equall to the base. Wherefore a Pyramis may seeme to be more generall then a Tetrahedron, as before a Prisme seemed to be more generall then a Parallelipipedon, or a sided Columne: so that euery Parallelipipedon is a Prisme, but not euery Prisme is a Parallelipipedon. And euery axe in a Sphere is a diameter: but not euery diameter of a Sphere is the axe therof. So also noting well the definition of a Pyramis, euery Tetrahedron may be called a Pyramis, but not euery Pyramis a Tetrahedron. And in dede *Psellus* in numbring of these fiue solides or bodies, calleth a Tetrahedron a Pyramis in manifest wordes. This I say might make *Flussas* & others (as I thinke it did) to omitte the definition of a Tetrahedron in this place, as sufficiently comprehended within the definition of a Pyramis geuen before. But why then did he not count that definition of a Pyramis faultie, for that it extendeth it selfe to large, and comprehendeth vnder it a Tetrahedron (which differeth from a Pyramis by that it is contayned of equall triangles) as he not so aduisedly did before the definition of a Prisme.



Difference betwene a Tetrahedron and a Piramis.

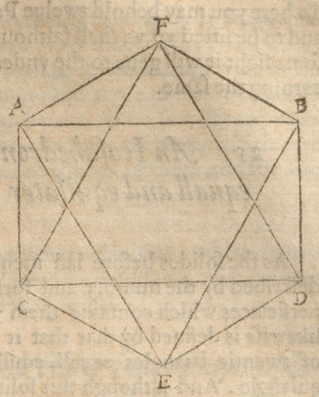
Psellus calleth a Tetrahedron a Piramis.

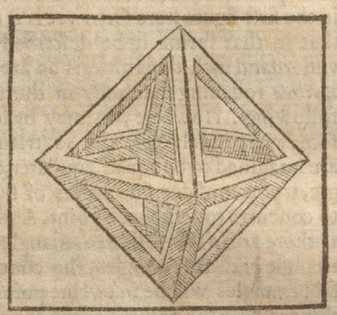
Twenty three definition.

23 *An Octohedron is a solide or bodily figure cõtained vnder eight equall and equilater triangles.*

As

As a Cube is a ſolide figure contayned vnder ſixe ſuperficiall figures of foure ſides or ſquares which are equilater, equiangle, and equall the one to the other: ſo is an Octohedron a ſolide figure contained vnder eight triangles which are equilater and equall the one to the other. As ye may in theſe two figures here ſet beholde. Whereof the firſt is drawen according as this ſolide is commonly deſcribed vpon a plaine ſuperficies. The ſecond is drawen as it is deſcribed by arte vpona plaine, to ſhewe bodilike. And in deede although the ſecond appeare to the eye more bodilike, yet as I before noted in a Cube, for the vnderſtanding of diuers Propoſitions in theſe fiue bookes following, is the firſt deſcription of more vſe yea & of neceſſitie. For without it, ye can not cõceaue the draught of lines and ſections in any one of the eight ſides which are ſometimes in the deſcriptions of ſome of thoſe Propoſitions required. Wherefore to the conſideration of this firſt deſcription, imagine firſt that vppon the vpper face of the ſuperficies of the parallelogramme *A B C D*, be deſcribed a Pyramis, hauing his fower triangles *A F B*, *A F C*, *C F D*, and *D F B*, equilater and equiangle, and concurring in the point *F*. Thẽ cõceaue that on the lower face of the ſuperficies of the former parallelogramme be deſcribed an other Pyramis, hauing his fower triangles *A E B*, *A E C*, *C E D*, & *D E B*, equilater and equiangle, and concurring in the point *E*. For ſo although ſomewhat groſly by reaſon the triangles can not be deſcribed equilater, you may in a plaine perceaue the forme of this ſolide, and by that meanes conceaue any lines or ſections required to be drawen in any of the ſayd eight triangles which are the ſides of that body.

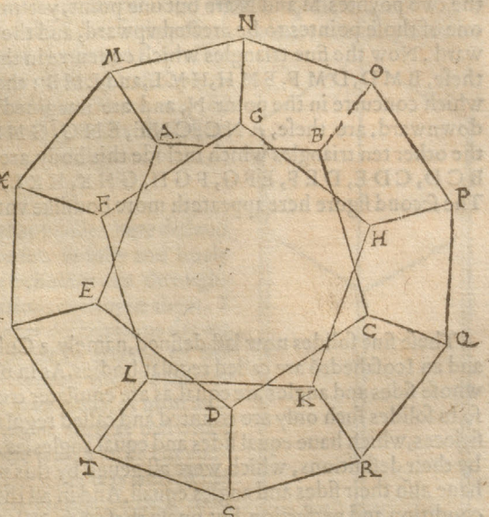


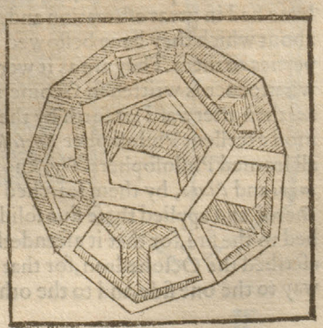


Twẽty fower definition.

24 *A Dodecahedron is a ſolide or bodily figure cõtained vnder twelue equall, equilater, and equiangle Pentagons.*

As a Cube, a Tetrahedron, and an Octohedron, are contayned vnder equall plaine figures, a Cube vnder ſquares, the other two vnder triangles: ſo is this ſolide figure contained vnder twelue equilater, equiangle, and equall Pentagons, or figures of fiue ſides. As in theſe two figures here ſet you may perceaue. Of which the firſt (which thinge alſo was before noted of a Cube, a Tetrahedron, and an Octohedron) is the common deſcription of it in a plaine, the other is the deſcription of it by arte vppon a plaine to make it to appeare ſomwhat bodilike. The firſt deſcription in deede is very obſcure to conceaue, but yet of neceſſitie it muſt ſo, neyther can it otherwiſe, be in a plaine deſcribed to vnderſtãd thoſe Propoſitions of *Euclide* in theſe fiue bokes following which concerne the ſame. For in it although rudely, may you ſee all the twelue Pentagons, which ſhould in deede be all equall, equilater, and equiangle. And now how you may ſomewhat conceaue the firſt figure deſcribed in the plaine to be a body. Imagine firſt the Pentagon A B C D E to be vpon a ground plaine ſuperficies, then imagine the Pentagon F G H K L to be on high oppoſite vnto the Pentagon A B C D E. And betwene thoſe two Pentagons there will be ten Pentagons pulled vp, fiue frõ the fiue ſides of the ground Pentagon, namely, from the ſide A B the Pentagon A B O N M, from the ſide B C the Pentagon B C Q P O, from the ſide C D the Pentagon C D S R Q, from the ſide D E, the Pentagon D E V T S, from the ſide E A the Pentagon E A M X V, the other fiue Pentagons haue eche one of their ſides common with one of the ſides of the Pentagon F G H K L, which is oppoſite vnto the Pentagon in the ground ſuperficies: namely, theſe are the other fiue Pentagons F G N M X, G H P O N, H K R Q P, K L R S T, L F X V T.



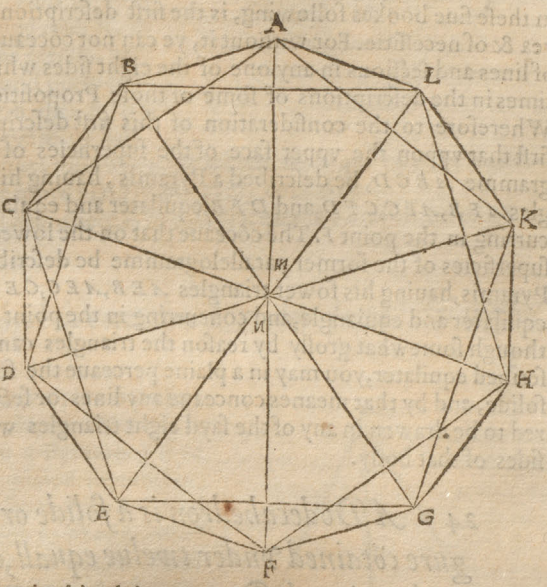


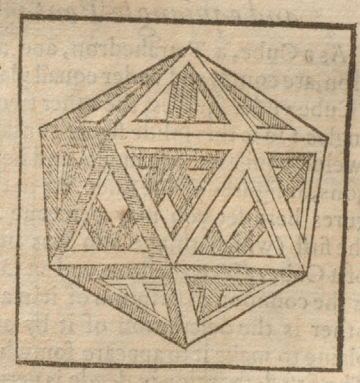
So here you may behold twelue Pentagons, which if you imagine to be equall, equilater, & equiangle, and to be lifted vp, ye ſhall (although ſomewhat rudely) conceaue the bodily forme of a Pentagon. And ſome light it will geue to the vnderſtanding of certaine Propoſitions of the fiue bookes following concerning the ſame.

Twenty fiue diffinition.

25 *An Icoſahedron is a ſolide or bodily figure contained vnder twentie equall and equilater triangles.*

As the ſolides before laſt mentioned are all deſcribed by the number and forme of the ſuperficieces which containe them: ſo this body likewiſe is defined by that that it is contayned of twentie triangles equall, equilater, and equiangle. And although this ſolide alſo be very hard to conceaue, as it is commonly deſcribed vpon a plaine (an example wherof you haue in the firſt figure here ſet): yet is it of neceſſitie that in that forme it be deſcribed, if we will vnderſtand ſuch deſcriptions as are ſet forth of *Euclide* touching that body in the fiue bookes following. Howbeit you may by it (although ſomewhat rudely) ſee the 20. triangles, which are imagined to be equall, equilater, and equiangle, if you conſider fiue angles of fiue triangles to concurre together at a point. And foraſmuch as there are in this ſolide 20. triangles, and euery triangle hath three angles, the concurſe of the ſaid triangles will be in twelue pointes. As in this example the pointes of the concurſe are A, B, C, D, E, F, G, H, K, L, M, & N. Where note that in this plaine the two poyntes M and N are but one point, yet muſt ye imagine one of thoſe pointes to be erected vpward, and the other downward. Now the fiue triangles which concurre in the point M, are theſe, B M D, D M F, F M H, H M L, and L M B: the fiue triangles which concurre in the point N, and are imagined to be erected downward, are theſe, A N C, C N E, E N G, G N K, and K N A: the other ten triangles which include this body, are theſe, A B C, B C D, C D E, D E F, E F G, F G H, G H K, H K L, K L A, L A B. The ſecond figure here appeareth more bodilike vnto the eye.





Fiue regular bodies.

Theſe fiue ſolides now laſt defined, namely, a Cube, a Tetrahedrõ, an Octohedron, a Dodecahedron and an Icoſahedrõ are called regular bodies. As in plaine ſuperficieces, thoſe are called regular figures, whoſe ſides and angles are equal, as are equilater triangles, equilater pentagons, hexagons, & ſuch lyke, ſo in ſolides ſuch only are counted and called regular, which are cõprehẽded vnder equal playne ſuperficieces, which haue equal ſides and equal angles, as all theſe fiue foreſayd haue, as manifeſtly appeareth by their definitions, which were all geuen by this proprietie of equalitie of their ſuperficieces, which haue alſo their ſides and angles equall. And in all the courſe of nature there are no other bodies of this condition and perfection, but onely theſe fiue. Wherfore they haue euer of the auncient Philoſophers bene had in great eſtimation and admiration, and haue bene thought worthy of much contemplacion, about which they haue beſtowed moſt diligent ſtudy and endeuour to ſearche out the natures & properties of them. They are as it were the ende and perfection of all Geometry, for whoſe ſake is written whatſoeuer is written in Geometry. They were (as men ſay) firſt inuented by the moſt witty *Pithagoras* then afterward ſet forth by the diuine *Plato*, and laſt of all meruelouſly taught and declared by the moſt excellent Philoſopher *Euclide* in theſe bookes following, and euer ſince wonderfully embraced of all learned Philoſophers. The knowledge of them containeth infinite ſecretes of nature. *Pithagoras*, *Timeus* and *Plato*, by them ſearched out the cõpoſition of the world, with the harmony and preſeruation therof, and applied theſe fiue ſolides to the ſimple partes therof, the Pyramis, or Tetrahedrõ they aſcribed to the fire, for that it aſcendeth vpward according to the figure of the Pyramis. To the ayre they aſcribed the Octohedron, for that through the ſubtle moiſture which it hath, it extendeth it ſelfe euery way to the one ſide, and to the other, accordyng as that figure doth. Vnto the water they aſſigned the

The dignity of theſe bodies.

A Tetrahedron aſcribed vnto the fire.

An octohedron aſcribed vnto the ayre.

Ikoſahedron

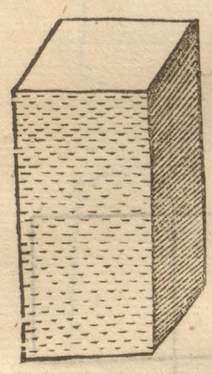
Ikoſahedron, for that it is continually flowing and mouing, and as it were makyng angles on euery ſide according to that figure. And to the earth they attributed a Cube, as to a thing ſtable, firme and ſure as the figure ſignifieth. Laſt of all a Dodecahedron, for that it is made of Pentagõs, whoſe angles are more ample and large then the angles of the other bodies, and by that meanes draw more to roundnes, & to the forme and nature of a ſphere, they aſſigned to a ſphere, namely, to heauen. Who ſo will read *Plato* in his *Tineus*, ſhall read of theſe figures, and of their mutuall proportion, ſtraunge matters, which here are not to be entreated of, this which is ſayd, ſhall be ſufficient for the knowledge of them, and for the declaration of their diffinitions.

An Ikoſahedron aſſigned vnto the water.
A cube aſſigned vnto the earth.
A dodecahedron aſſigned to heauen.

After all theſe diffinitions here ſet of *Euclide*, *Fluſſas* hath added an other diffinition, which is of a Parallelipipedon, which bicauſe it hath not hitherto of *Euclide* in any place bene defined, and becauſe it is very good and neceſſary to be had, I thought good not to omitte it, thus it is.

A parallelipipedon is a ſolide figure comprehended vnder foure playne quadrangle figures, of which thoſe which are oppoſite are parallels.

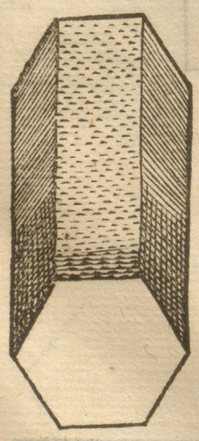
Diffinition of a parallelipipedon.

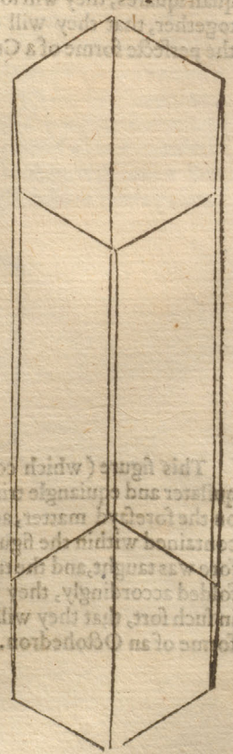


As in playne ſuperficieces a parallelogramme is that which is contained vnder foure ſides beyng lines, and whoſe oppoſite ſides are equidiſtant and parallel lynes, ſo in ſolide figures a Parallelipipedon is that ſolide which is contayned vnder foure quadrangle ſuperficieces, whoſe oppoſite ſides are alſo parallels, as it is eaſily to be ſene and conceaued in a cube or die, all whoſe oppoſite ſides are parallel ſuperficieces, & ſo of others like, ye may alſo ſomewhat conceiue therof by the example in the margent.

There is alſo in theſe bookes following, mencion made of ſolides, whoſe two baſes are Poligonon figures, lyke, equall, equilater, and parallels, and the ſides ſet vpon the baſes are parallelogrammes: which kynde of ſolides Campane calleth ſided Columnes (and which as was before noted, may be cõpreded vnder the definition of a Priſme) a forme wherof although groſely, behold in this example, whoſe baſes are two like equall, equilater, equiangle, and parallel hexagons, and the ſides ſet vppon thoſe baſes are ſixe parallelogrãmes: ye may better cõceiue the forme therof by the figure put vnder the figure of the parallelipipedon, which apeareth more bodilike. There may of theſe be infinite formes according to the diuerſitie of their baſes.

A ſided Columne.

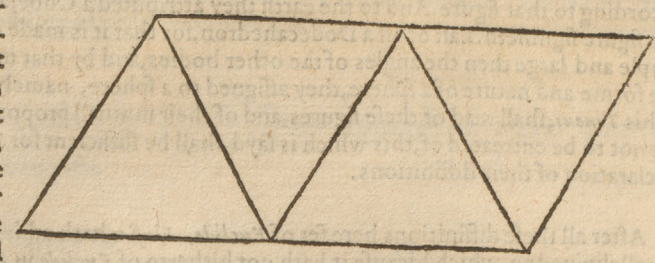




Becauſe theſe fiue regular bodies here defined are not by theſe figures here ſet, ſo fully and liuely expreſſed, that the ſtudious beholder can throughly according to their definitions conceyue them. I haue here geuen of them other deſcriptions drawn in a playne, by which ye may eaſily attayne to the knowledge of them. For if ye draw the like formes in matter that wil bow and geue place, as moſt aptly ye may do in fine paſted paper, ſuch as paſtwiues make womẽs paſtes of, & thẽ with a knife cut euery line finely, not through, but halfe way only, if thẽ ye bow and bende them accordingly, ye ſhall moſt plainly and manifeſtly ſee the formes and ſhapes of theſe bodies, euen as their definitions ſhew. And it ſhall be very neceſſary for you to haue ſtore of that paſted paper by you, for ſo ſhal you vpon it deſcribe the formes of other bodies, as Priſmes and Parallelipopedons, and ſuch like ſet forth in theſe fiue bookes following, and ſee the very formes of thoſe bodies there mẽcioned: which will make theſe bokes concerning bodies, as eaſy vnto you as were the other bookes, whoſe figures you might plainly ſee vpon a playne ſuperficies.

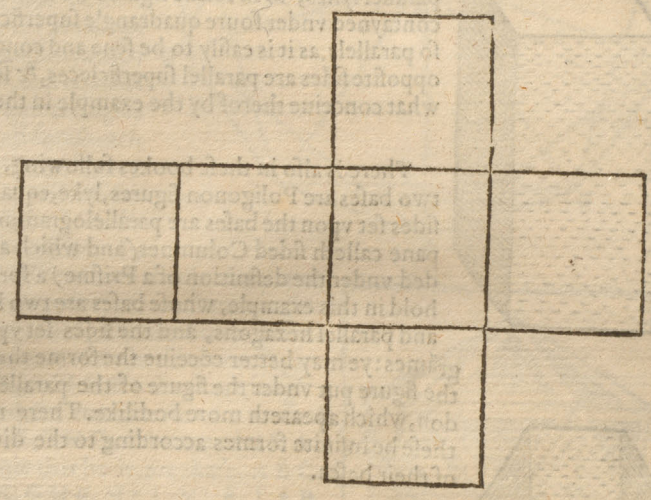
A Tetrahedron.

If ye draw this figure consisting as ye see of fower equilater and equiangle triangles vpō pasted paper, or vpon any other such like matter that will bowe and geue place, and then cut not through the paper, but as it were halfe the thicknes of the paper, the three lines contained within the figure, and bowe & folde in the fower triangles accordingly: they will close together in such sort, that they will make the perfecte forme of a Tetrahedron.



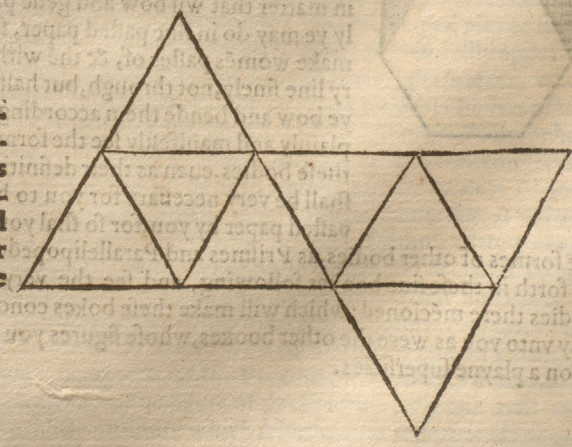
A cube.

This figure (consisting of sixe equall squares) drawen vpon pasted paper, and the siue lines contained within the figure being cut finely halfe the thicknes of the paper, or not through, if then ye bowe and folde accordingly the sixe equall squares, they will so close together, that they will cause the perfecte forme of a Cube.



An octohedron.

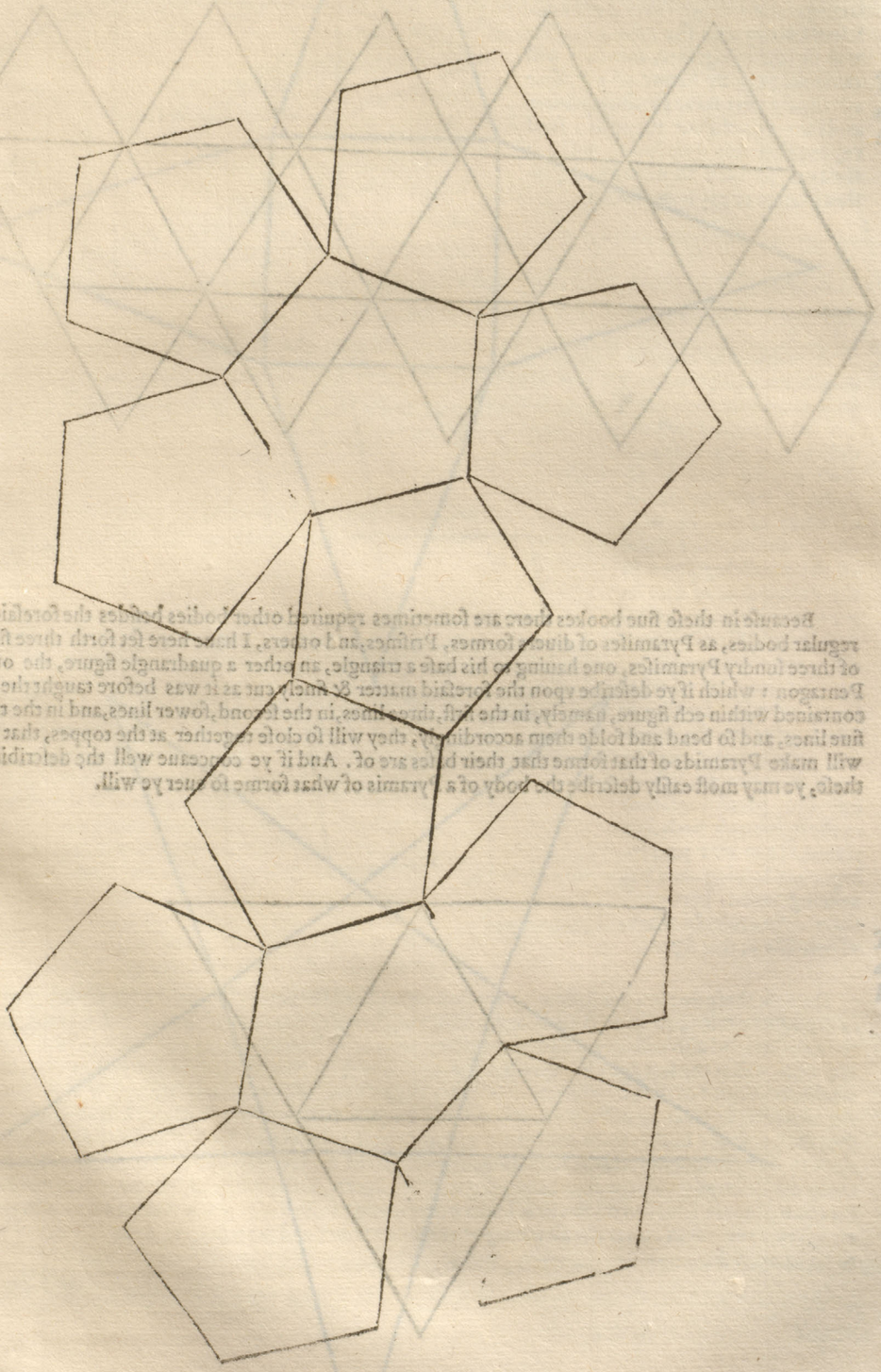
This figure (which consisteth of eight equilater and equiangle triangles) drawen vpon the foresayd matter, and the seuen lines contained within the figure being cut as before was taught, and the triangles bowed and folded accordingly, they will close together in such sort, that they will make the perfecte forme of an Octohedron.



Describe

Deſcribe this figure which conſiſteth of twelue equilater and equiangle Pentagons, vpon the foreſaid matter, and finely cut as before was taught the eleuen lines contained within the figure, and bow and folde the Pentagons accordingly. And they will ſo cloſe together, that they will make the very forme of a Dodecahedron.

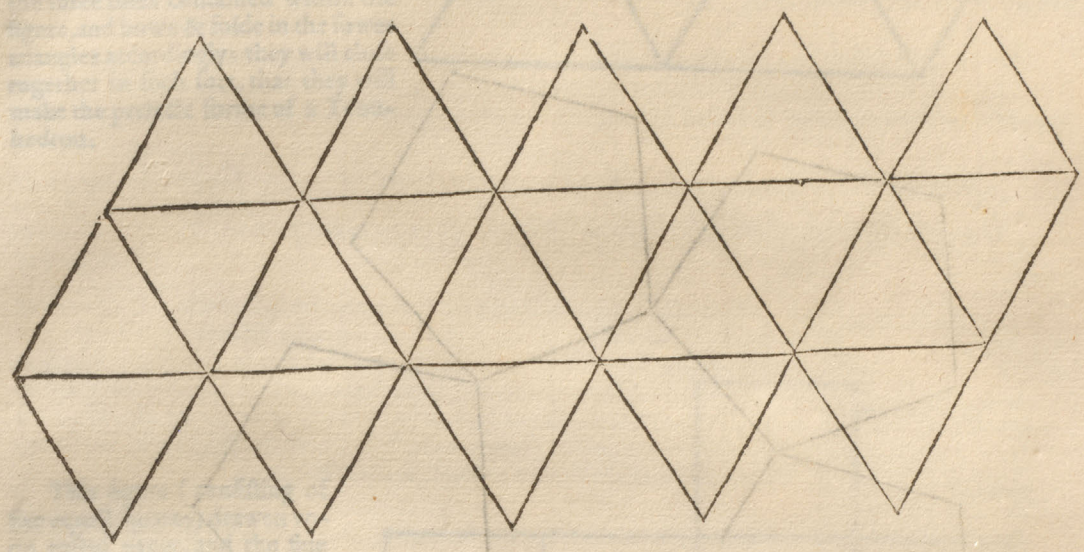
A Dodecahedron.



If ye

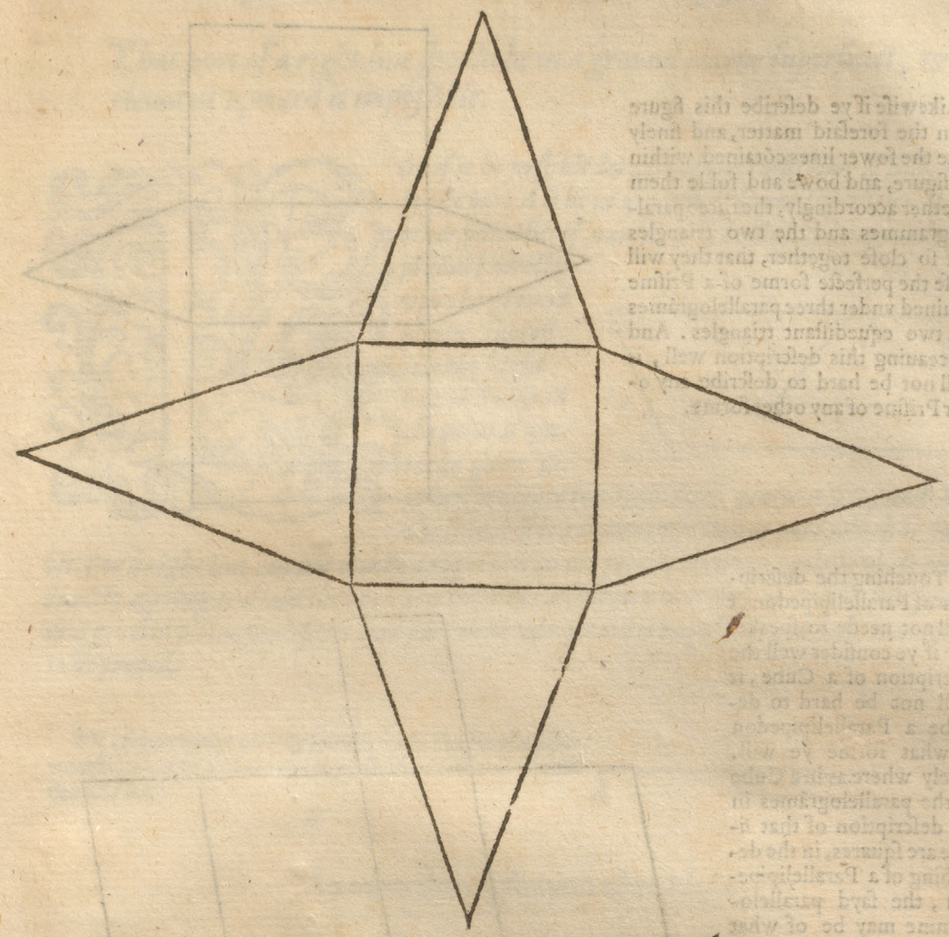
An Icoſahedron.

If ye deſcribe this figure which conſiſteth of twentie equilater and equiangle triangles vpon the foreſaid matter, and finely cut as before was ſhewed the ninetene lines which are contayned within the figure, and then bowe and folde them accordingly, they will in ſuch ſort cloſe together, that there will be made a perfecte forme of an Icoſahedron.

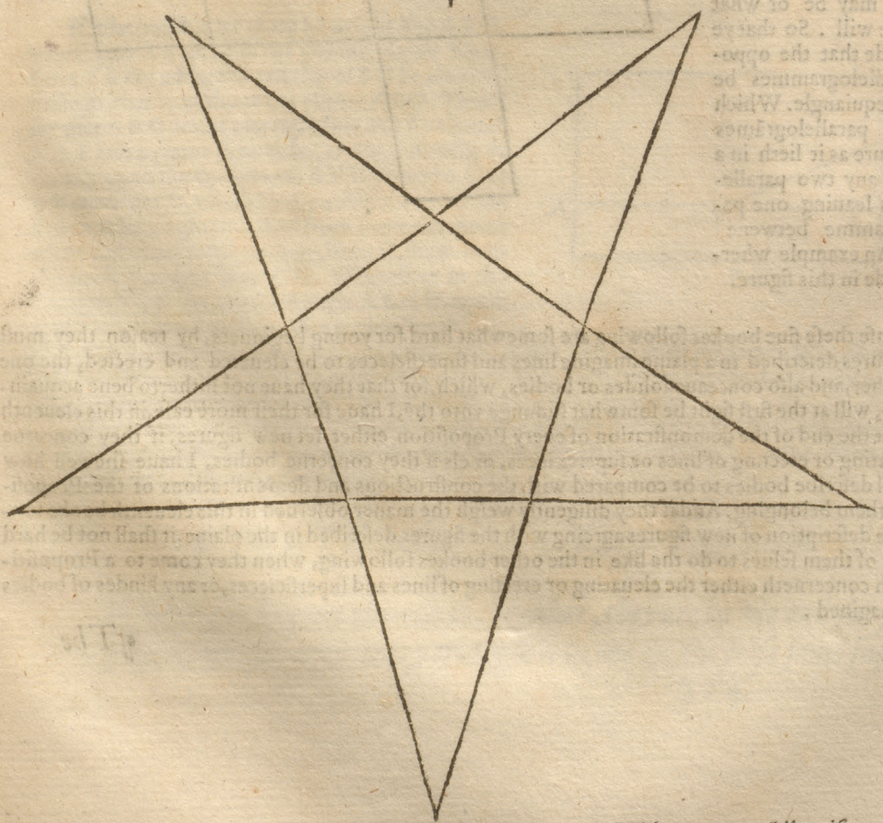


Becauſe in theſe fiue bookes there are ſometimes required other bodies beſides the foreſaid fiue regular bodies, as Pyramiſes of diuers formes, Priſmes, and others, I haue here ſet forth three figures of three ſundry Pyramiſes, one hauing to his baſe a triangle, an other a quadrangle figure, the other a Pentagon: which if ye deſcribe vpon the foreſaid matter & finely cut as it was before taught the lines contained within ech figure, namely, in the firſt, three lines, in the ſecond, fower lines, and in the third, fiue lines, and ſo bend and folde them accordingly, they will ſo cloſe together at the toppes, that they will make Pyramids of that forme that their baſes are of. And if ye conceaue well the deſcribing of theſe, ye may moſt eaſily deſcribe the body of a Pyramis of what forme ſo euer ye will.

A triangled Pyramis.



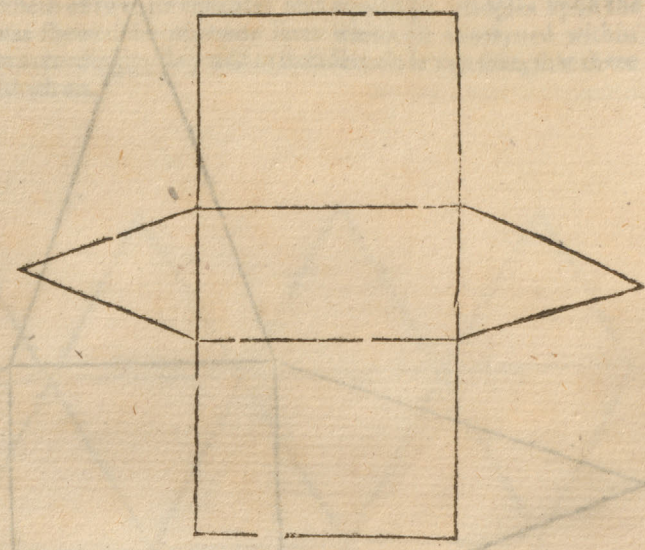
The forme of a quadrangled Pyramis.



The forme of a fiue angled Pyramis.

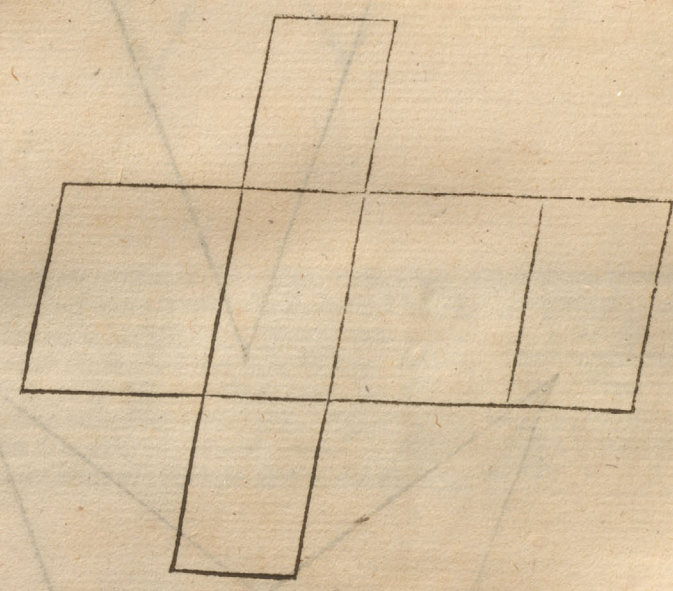
The forme of a prisme.

Likewise if ye describe this figure vpon the foresaid matter, and finely cutte the fower lines cõtained within the figure, and bowe and folde them together accordingly, the three parallelogrammes and the two triangles will so close together, that they will cause the perfecte forme of a Prisme cõtained vnder three parallelogrãmes and two equedistant triangles. And conceauing this description well, it shall not be hard to describe any other Prisme of any other forme.



The forme of a parallelipipedon.

Touching the description of Parallelipipedons I shall not neede to speake. For if ye consider well the description of a Cube, it shall not be hard to describe a Parallelipipedon of what forme ye will. Onely where as in a Cube all the parallelogrãmes in the description of that figure are squares, in the describing of a Parallelipipedon, the sayd parallelogramme may be of what forme ye will. So that ye take heede that the opposite parallelogrammes be equal & equiangle. Which opposite parallelogrãmes in the figure as it lieth in a plaine, is any two parallelogrãmes leauing one parallelogramme betwene them. An example wherof beholde in this figure.



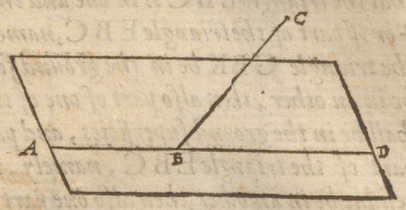
Because these fiue bookes following are somewhat hard for young beginners, by reason they must in the figures described in a plaine imagine lines and superficieces to be eleuated and erected, the one to the other, and also conceaue solides or bodies, which, for that they haue not hitherto bene acquainted with, will a: the first sight be somwhat straunge vnto thẽ, I haue for their more ease, in this eleuenth booke, at the end of the demonstration of euery Proposition either set new figures, if they concerne the eleuating or erecting of lines or superficieces, or els if they concerne bodies, I haue shewed how they shall describe bodies to be compared with the constructions and demonstrations of the Propositions to them belonging. And if they diligently weigh the maner obserued in this eleuenth booke touching the description of new figures agreing with the figures described in the plaine, it shall not be hard for them of them selues to do the like in the other bookes following, when they come to a Proposition which concerneth either the eleuating or erecting of lines and superficieces, or any kindes of bodies to be imagined.

¶ The

¶ The 1. Theoreme. The 1. Propoſition.

That part of a right line ſhould be in a ground playne ſuperficies, & part eleuated vpward is impoſsible.

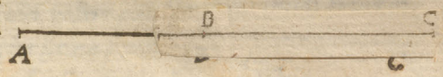
Or if it be poſſible, let part of the right line A B C, namely, the part A B be in a ground playne ſuperficies, and the other part therof, namely, B C be eleuated vpwarde. And produce directly vpō the ground playne ſuperficies the right line A B beyond the point B vnto the point D.



Wherfore vnto two right lines geuen A B C, and A B D, the line A B is a common ſection or part, which is impoſsible. For a right line can not touche a right line in mo pointes then one, vnleſſe thoſe right be exactly agreing and laid the one vpon the other. Wherfore that part of a right line ſhould be in a ground plaine ſuperficies, and part eleuated vpward is impoſſible: which was required to be proued.

Demonſtration leading to an impoſsibilitie.

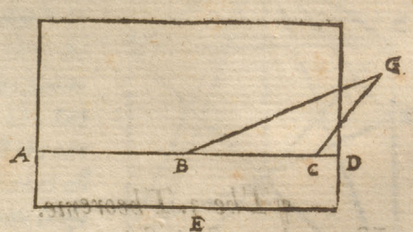
This figure more plainly ſetteth forth the foreſaid demonſtratiō, if ye eleuate the ſuperficies wherin is drawn the line B C.



An other demonſtration after *Fluſsas.*

An other demonſtration after Fluſſas.

If it be poſſible let there be a right line A B G, whoſe part A B let be in the ground playne ſuperficies A E D, and let the reſt therof B G be eleuated on high, that is, without the playne A E D. Then I ſay that A B G is not one right line. For foraſmuch as A E D is a plaine ſuperficies, produce directly & equally vpon the ſayd playne A E D the right lyne A B towardes D, which by the 4. definition of the firſt ſhall be a right line. And from ſome one point of the right line A BD, namely, from C, draw vnto the point G a right lyne C G. Wherefore in the triangle B C G the outward angle A B G is equall to the two inward and oppoſite angles (by the 32. of the firſt) and therfore it is leſſe then two right angles (by the 17. of the ſame) Wherfore the lyne A B G foraſmuch as it maketh an angle, is not a right line. Wherefore that part of a right line ſhould be in a ground playne ſuperficies, and part eleuated vpward is impoſſible.



If ye marke well the figure before added for the playner declaration of Euclides demonſtration, it will not be hard for you to conceiue this figure which Fluſſas putteth for his demonſtration: wherein is no difference but onely the draught of the lyne G C.

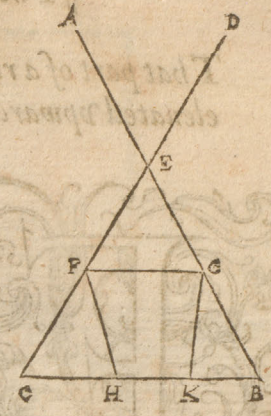
¶ The 2. Theoreme. The 2. Propoſition.

If two right line cut the one to the other, they are in one and the ſelfe ſame playne ſuperficies: & euery triangle is in one & the ſelfe ſame ſuperficies.

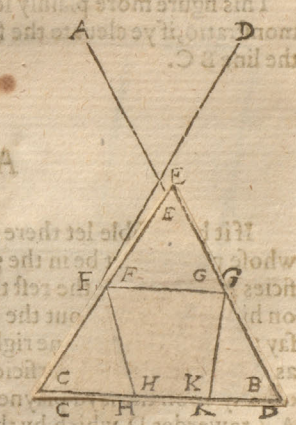
SVppose that these two right lines AB and CD doo cutte the one the other in the poynt E. Then I say that these lines AB and CD are in one and the selfe same superficies, and that euery triangle is in one & selfe same playne superficies. Take in the lines EC and EB points at all auentures, and let the same be F and G, and draw a right line from the poynt B to the point C, and an other from the point F to the point G. And draw the lines FH and GK. First I say that the triangle EBC is in one and the same ground superficies. For if part of the triangle EBC, namely the triangle FCH, or the triangle GBK be in the ground superficies, and the residue be in an other, then also part of one of the right lines EC or EB shall be in the ground superficies, and part in an other. So also if part of the triangle EBC, namely, the part EFG be in the ground superficies and the residue be in an other, then also one part of eche of the right lines EC and EB shall be in the ground superficies, & an other part in an other superficies, which (by the first of the eleuenth) is proued to be impossible. Wherfore the triangle EBC is in one and the selfe same playne superficies. For in what superficies the triangle BCE is, in the same also is either of the lines EC and EB, and in what superficies either of the lines EC and EB is, in the selfe same also are the lines AB and CD. Wherfore the right lines lines AB and CD are in one & the selfe same playne superficies, and euery triangle is in one & the selfe same playne superficies: which was required to be proued.

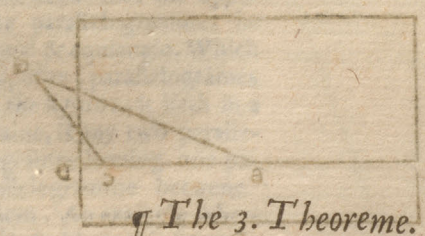
Construction.

Demonstration leading to an impossibilitie.



In this figure here set may ye more playnely conceaue the demonstration of the former proposition where ye may eleuate what part of the triangle ECB ye will, namely the part FCH or the part GBK, or finally the part FCGB as is required in the demonstration.



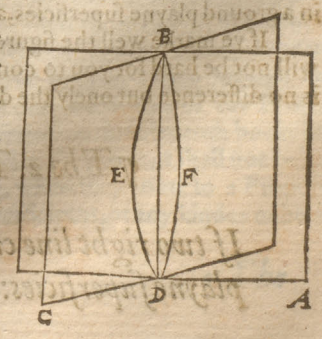


¶ The 3. Theoreme. The 3. Proposition.

If two playne superficieces cutte the one the other: their common section is a right line.

SVppose that these two superficiecs AB & BC do cutte the one the other, and let their common section be the line DB. Then I say that DB is a right line. For if not, draw from the poynt D to the point B a right line DFB in the playne superficies AB, and likewise from the same poyntes draw an other right line DEB in the playne superficies BC. Now therfore two right lines DEB and DFB shall haue the selfe same endes, and therefore doo include a superficies which (by the last common sentence) is impossible. Wherefore the lines DEB and DFB are not right lines. In

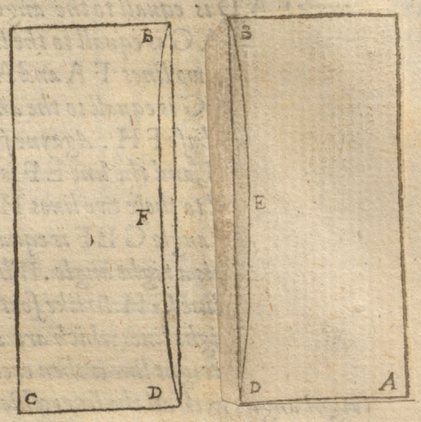
Demonstration leading to an impossibilitie.



like

like ſort alſo may we proue that no other right line can be drawne from the poynt D *to the point* B *beſides the line* D B *which is the common ſection of the two ſuperficieces* A B *and* B C. *If therefore two playne ſuperficieces cutte the one the other, their common ſection is a right line: which was required to be demonſtrated.*

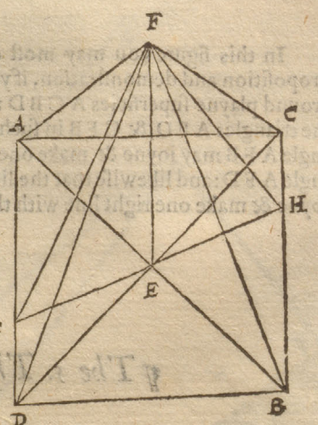
This figure here ſet, ſheweth moſt playnely not onely this third propoſition, but alſo the demonſtration thereof, if ye eleuate the ſuperficies A B, and ſo compare it with the demonſtration.



¶ The 4. Theoreme. The 4. Propoſition.

If from two right lines, cutting the one the other, at their common ſection, a right line be perpendicularly erected: the ſame ſhall alſo be perpendicularly erected from the playne ſuperficies by the ſayd two lines paſſing.

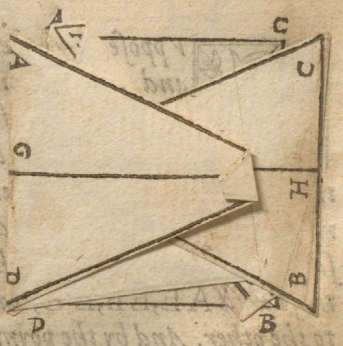
VPpoſe that there be two right lines A B *and* C D *cutting the one the other in the poynt* E. *And from the poynt* E *let there be erected a right line* E F *perpendicularly to the ſayd two right lines* A B *and* C D: *then I ſay that the right line* E F, *is alſo erected perpendicular to the plaine ſuperficies which paſſeth by the lines* A B *and* C D. *Let theſe lines* A E, E B, E C, *and* E D *be put equall the one to the other. And by the poynt* E *extend a right line at all auentures, and let the ſame be* G E H. *And drawe theſe right lines* A D, C B, F A, F G, F D, F C, F H, *and* F B. *And foraſmuch as theſe two right lines* A E *&* E D *are equall to theſe two lines* C E *and* E B, *and they comprehend equall angles (by the 15. of the firſt): therefore (by the 4. of the firſt) the baſe* A D *is equall to the baſe* C B, *and the triangle* A E D *is equall to the triangle* C E B. *Wherefore alſo the angle* D A E, *is equall to the angle* E B C. *But the angle* A E G *is equall to the angle* B E H *(by the 15 of the firſt). Wherefore there are two triangles* A G E, *and* B E H *hauing two angles of the one equall to two angles of the other, eche to his correſpondent angle, and one ſide of the one equall to one ſide of the other, namely one of the ſides which lye betwene the equall angles, namely, the ſide* A E *is equall to the ſide* E B. *Wherefore (by the 26. of the firſt) the ſides remayning are equall to the ſides remayning. Wherefore the ſide* G E *is equall to the ſide* E H, *and the ſide* A G *to the ſide* B H. *And foraſmuch as the line* A E *is equall to the line* E B, *and the line* F E *is common to them both,*



Conſtruction.

Demonſtration.

and maketh with them right angles, wherefore (by the fourth of the first) the base F A *is equall to the base* F B. *And (by the same reason) the base* F C *is equall to the base* F D. *And forasmuch as the line* A D *is equall to the line* B C, *and the line* F A *is equal to the line* F B *as it hath bene proued. Therefore these two lines* F A *and* A D *are equall to these two lines* F B *&* B C, *the one to the other, & the base* F D *is equall to the base* F C. *Wherfore also the angle* F A D *is equall to the angle* F B C. *And againe forasmuch as it hath bene proued, that the line* A G *is equall to the line* B H, *but the line* F A *is equall to the line* F B. *Wherefore there are two lines* F A *and* A G *equall to two lines* F B *and* B H *and it is proued that the angle* F A G *is equall to the angle* F B H: *wherefore (by the 4. of the first) the base* F G *is equal to the base* F H. *Agayne forasmuch as it hath bene proued that the line* G E *is equal to the line* E H, *and the line* E F *is common to them both: wherefore these two lines* G E *and* E F *are equall to these two lines* H E *and* E F, *and the base* F H *is equall to the base* F G: *wherefore the angle* G E F *is equall to the angle* H E F. *Wherefore either of the angles* G E F, *and* H E F *is a right angle. Wherefore the line* E F *is erected, from the point* E, *perpendicularly to the line* G H. *In like sort may we proue, that the same line* F E *maketh right angles with all the right lines which are drawne vpon the ground playne superficies and touch the point* B. *But a right line is then erected perpendicularly to a plaine superficies, when it maketh right angles with all the lines which touch it, and are drawne vpon the ground playne superficies (by the 2. definition of the eleuenth). Wherefore the right line* F E *is erected perpendicularly to the ground playne superficies. And the ground plaine superficies is that which passeth by these right lines* A B *and* C D. *Wherefore the right line* F E *is erected perpendicularly to the playne superficies which passeth by the right lines* A B *and* C D. *If therefore from two right lines cutting the one the other and at their common section a right line be perpendicularly erected: it shall also be erected perpendicularly to the plaine superficies by the sayd two lines passing: which was required to be proued.*



In this figure you may most euidently conceaue the former proposition and demonstration, if ye erect perpendicularly vnto the ground playne superficies A C B D the triangle A F B: and eleuate the triangles A F D, & C F B in such sort, that the line A F of the triangle A F B may ioyne & make one line with the line A F of the triangle A F D: and likewise that the line B F of the triangle A F B may ioyne & make one right line with the line B F of the triangle B F C.

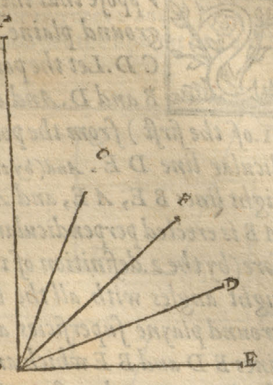
¶ The 5. Theoreme. The 5. Proposition.

If vnto three right lines which touch the one the other, be erected a perpendicular line from the common point where those three lines touch: those three right lines are in one and the selfe same plaine superficies.

Suppose that vnto these three right lines B C, B D, *and* B E, *touching the one the other in the poynt* B, *be erected perpendicularly from the poynt* B, *the line* A B. *Then I say, that those thre right lines* B C, B D *and* B E, *are in one & the selfe same plaine superficies. For if not, then if it be possible, let the lines* B D *&*

B E

BE *be in the ground ſuperficies, and let the line* BC *be erected vpward (now the lines* AB *and* BC *are in one and the ſame playne ſuperficies (by the 2. of the eleuenth) for they touch the one the other in the point* B*). Extend the plaine ſuperficies wherein the lines* AB *and* BC *are, and it ſhall make at the length a common ſection with the ground ſuperficies, which common ſection ſhall be a right line (by the 3. of the eleuenth): let that common ſection be the line* BF. *Wherefore the three right lines* AB, BC, *and* BF *are in one and the ſelfe ſame ſuperficies, namely, in the ſuperficies wherein the lines* AB *and* BC *are. And foraſmuch as the right line* AB *is erected perpendicularly to either of theſe lines* BD *and* BE, *therefore the line* AB *is alſo (by the 4. of the eleuenth) erected perpendicularly to the plaine ſuperficies, wherein the lines* BD *and* BE *are. But the ſuperficies wherein the lines* BD *and* BE *are is the ground ſuperficies. Wherefore the line* AB *is erected perpendicularly to the ground plaine ſuperficies. Wherefore (by the 2. definition of the eleuenth) the line* AB *maketh right angles with all the lines which are drawne vpon the ground ſuperficies and touch it. But the line* BF *which is in the ground ſuperficies doth touch it. Wherfore the angle* ABF *is a right angle. And it is ſuppoſed that the angle* ABC *is a right angle. Wherefore the angle* ABF *is equall to the angle* ABC, *and they are in one and the ſelfe ſame plaine ſuperficies which is impoſſible. Wherefore the right line* BC *is not in an higher ſuperficies. Wherefore the right lines* BC, BD, BE *are in one and the ſelfe ſame plaine ſuperficies. If therefore vnto three right lines touching the one the one the other, be erected a perpendicular line from the common point where thoſe three lines touch: thoſe three right lines are in one and the ſelfe ſame plaine ſuperficies: which was required to be demonſtrated.*



Demonſtration leading to an impoſſibilitie.

This figure here ſet more playnely declareth the demonſtration of the former propoſition, if ye erect perpendicularly vnto the ground ſuperficies, the ſuperficies wherein is drawne the line AB and ſo compare it with the ſayd demonſtration.

The 6. Theoreme. The 6. Propoſition.

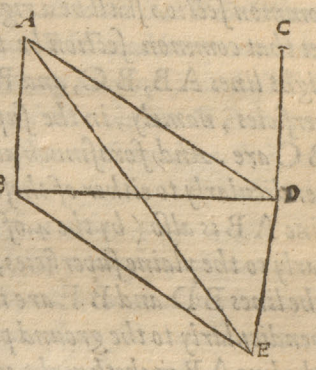
If two right lines be erected perpendicularly to one & the ſelfe ſame plaine ſuperficies: thoſe right lines are parallels the one to the other.

Construction.

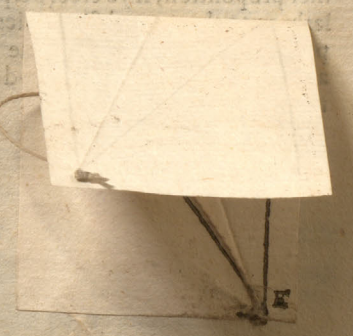
SVppoſe that theſe two right lines A B and C D be erected perpendicularly to a ground plaine ſuperficies. Then I ſay that the line A B is a parallel to the line C D. Let the pointes which thoſe two right lines touch in the plaine ſuperficies be B and D. And draw a right line from the point B to the point D. And (by the 11. of the firſt) from the point D, draw vnto the line B D in the ground ſuperficies a perpendicular line D E. And (by the 2. of the firſt)* put the line D E equall to the line A B. And draw theſe right lines B E, A E, and A D. And foraſmuch as the line A B is erected perpendicularly to the ground ſuperficies, therfore (by the 2. definition of the eleuenth) the line A B maketh right angles with all the lines which are drawne vpon the ground playne ſuperficies and touch it. But either of theſe lines B D and B E which are in the ground ſuperficies, touch the line A B, wherefore either of theſe angles A B D and A B E is a right angle: and by the ſame reaſon alſo either of the angles C D B, & C D E is a right angle. And foraſmuch as the line A B is equall to the line D E, and the line B D is common to them both, therfore theſe two lines A B and B D, are equall to theſe two lines E D and D B, and they contayne right angles: wherefore (by the 4. of the firſt) the baſe A D is equall to the baſe B E. And foraſmuch as the line A B is equall to the line D E, and the line A D to the line B E, therefore theſe two lines A B and B E are equall to theſe two lines E D and D A, and the line A E is a common baſe to them both. Wherefore the angle A B E is (by the 8. of the firſt) equal to the angle E D A. But the angle A B E is a right angle: whefore alſo the angle E D A is a right angle: wherfore the line E D is erected perpẽdicularly to the line D A: and it is alſo erected perpẽdicularly to either of theſe lines B D and D C, wherefore the line E D is vnto theſe three right lines B D, D A, and D C erected perpendicularly from the poynt where theſe three right lines touch the one the other: wherefore (by the 5. of the eleuenth) theſe three right lines B D, D A, and D C are in one and the ſelfe ſame ſuperficies. And in what ſuperficies the lines B D and D A are, in the ſelfe ſame alſo is the line B A: for euery triangle is (by the 2. of the eleuenth) in one and the ſelfe ſame ſuperficies. Wherefore theſe right lines A B, B D, and D C are in one and the ſelfe ſame ſuperficies, and either of theſe angles A B D and B D C is a right angle (by ſuppoſition). Wherefore (by the 28. of the firſt) the line A B is a parallel to the line C D. If therefore two right lines be erected perpendicularly to one and the ſelfe ſame playne ſuperficies, thoſe right lines are parallels the one to the other: which was required to be proued.

*An Aſſumpt as M. Dee proueth it.

Demonſtration.



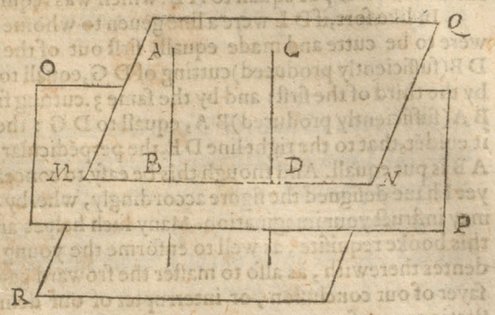
Here for the better vnderſtanding of this 6. propoſition I haue deſcribed an other figure: as touching which if ye erect the ſuperficies A B D perpendicularly to the ſuperficies B D E, and imagine only a line to be drawne from the poynt A to the poynt E (if ye will ye may extend a thred from the ſaide poynt A to the poynt E) and ſo compare it with the demonſtration, it will make both the propoſition, and alſo the demonſtration moſt cleare vnto you.



¶ An other demonſtration of the ſixth propoſition by M. Dee.

Suppoſe that the two right lines A B & C D be perpendicularly erected to one & the ſame playne ſuperficies

perficies, namely the playne superficies O P. Then I say that A B and C D are parallels. Let the end points of the right lines A B and C D, which touch the plaine superficies O P, be the poyntes B and D, frō B to D let a straight line be drawne (by the first petition) and (by the second petition) let the straight line B D be extēded, as to the poynts M & N. Now forasmuch as the right line A B, from the poynt B produced, doth cutte the line M N (by construction). Therefore (by the second proposition of this eleuenth booke) the right lines A B & M N are in one plaine superficies. Which let be Q R, cutting the superficies O P in the right line M N. By the same meanes may we conclude the right line C D to be in one playne superficies with the right line M N. But the right line M N (by supposition) is in the plaine superficies Q R: wherefore C D is in the plaine superficies Q R. And A B the right line was proued to be in the same plaine superficies Q R. Therfore A B and C D are in one playne superficies, namely Q R. And forasmuch as the lines A B and C D (by



suppositiō) are perpendicular vpon the playne superficies O P, therefore (by the second definition of this booke) with all the right lines drawne in the superficies O P and touching A B and C D, the same perpēdiculars A B and C D, do make right angles. But (by construction) M N, being drawne in the plaine superficies O P toucheth the perpendiculars A B and C D at the poyntes B and D. Therefore the perpendiculars A B and C D, make with the right line M N two right angles namely A B N, and C D M: and M N the right line is proued to be in the one and the same playne superficies, with the right lines A B & C D: namely in the playne superficies Q R. Wherefore by the second part of the 28. proposition of the first booke, the right lines A B and C D are parallels. If therefore two right lines be erected perpendicularly to one and the selfe same playne superficies those right lines are parallels the one to the other: which was required to be demonstrated.

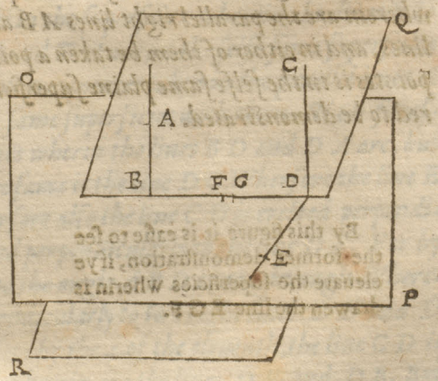
A Corollary added by *M. Dee.*

Hereby it is euident that any two right lines perpendicularly erected to one and the selfe same playne superficies, are also themselues in one and the same playne superficies, which is likewise perpendicularly erected to the same playne superficies, vnto which the two right lines are perpendicular. 1. 2.

The first part hereof is proued by the former construction and demonstration, that the right lines A B and C D are in one and the same playne superficies Q R. The second part is also manifest (that is, that the playne superficies Q R is perpendicularly erected vpon the playne superficies O P) for that A B and C D being in the playne superficies Q R, are by supposition perpendicular to the playne superficies O P: wherefore by the third definition of this booke Q R is perpendicularly erected to, or vpon O P: which was required to be proued. 1. 2.

Io. Dee his aduise vpon the Assumpt of the 6.

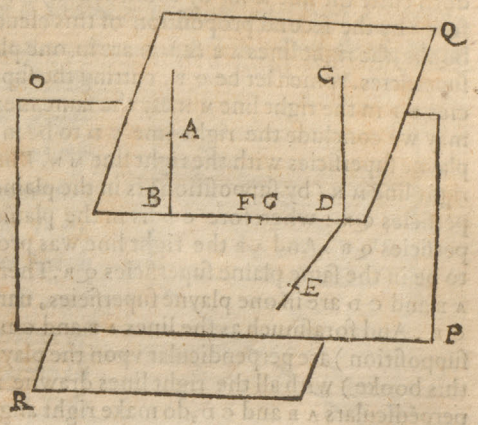
* As concerning the making of the line D E, equall to the right line A B, verely the second of the first, without some farther consideration, is not properly enough alledged. And no wonder it is, for that in the former bookes, whatsoeuer hath of lines bene spoken, the same hath alwayes bene imagined to be in one onely playne superficies considered or executed. But here the perpendicular line A B, is not in the same playne superficies, that the right line D E is. Therfore some other helpe must be put into the handes of young beginners, how to bring this probleme to execution: which is this, most playne and briefe. Vnderstand that B D the right line, is the common section of the playne superficies, wherein the perpendiculars A B and C D are, & of the other playne superficies, to which they are perpendiculars. The first of these (in my former demonstration of the 6), I noted by the playne superficies Q R: and the other, I noted by the plaine superficies O P. Wherfore B D being a right line common to both the playne superficieces Q R & O P, therby the ponits B and D are cōmon to the playnes Q R and O P. Now



from

from B D(ſufficiently extended)cutte a right line equall to A B, (which ſuppoſe to be B F) by the third of the firſt,and orderly to B F make D E equall,by the 3.of the firſt,if D E be greater then B F. (Which alwayes you may cauſe ſo to be,by producing of D E ſufficiently).Now foraſmuch as B F by conſtruction is cutte equall to A B,and D E alſo , by conſtruction, put equall to B F , therefore by the 1. common ſentence,D E is put equall to A B:which was required to be done.

In like ſort,if D E were a line geuen to whome A B were to be cutte and made equall, firſt out of the line D B(ſufficiently produced)cutting of D G,equall to DE by the third of the firſt: and by the ſame 3.cutting from B A(ſufficiently produced)B A, equall to D G : then is it euidēt,that to the right line D E,the perpēdicular line A B is put equall. And though this be eaſy to conceaue, yet I haue deſigned the figure accordingly, wherby you may inſtruct your imagination. Many ſuch helpes are in this booke requiſite , as well to enforme the young ſtudentes therewith , as alſo to maſter the froward gayneſayer of our concluſion , or interrupter of our demonſtrations courſe.

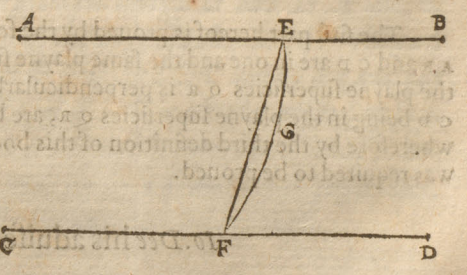


¶ The 7. Theoreme. The 7. Propoſition.

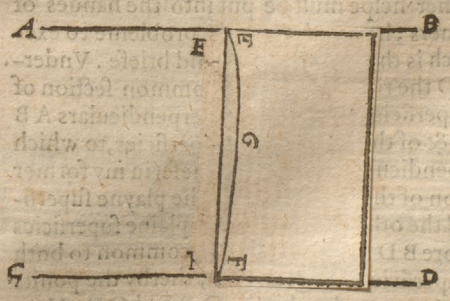
If there be two parallel right lines, and in either of them be taken a point at all aduentures : a right line drawen by the ſaid pointes is in the ſelf ſame ſuperficies with the parallel right lines.

Demonſtration leading to an impoſsibilitie.

Vppoſe that theſe two right lines A B and C D be parallels, and in either of thē take a point at all aduentures,namely, E and F . Then I ſay, that a right line drawen from the point E to the point F, is in the ſelfe ſame plaine ſuperficies that the parallel lines are . For if not, then if it be poſsible, let it be in an higher ſuperficies,as the line E G F is,and draw the ſuperficies wherin the line E G F is,& extend it, and it ſhall make a common ſection with the ground ſuperficies,which ſection ſhall (by the 3.of the eleuenth) be a right line : let that ſection be the right line E F . Wherefore two right lines E G F and E F include a ſuperficies: which (by the laſt common ſentence) is impoſsible. Wherfore a right line drawen from the point E to the point F,is not in an higher ſuperficies. Wherfore a right line drawen from the point E to the point F, is in the ſelfe ſame ſuperficies wherein are the parallel right lines A B and C D . If therefore there be two parallel right lines, and in either of them be taken a point at all aduentures, a right line drawen by thoſe pointes is in the ſelfe ſame plaine ſuperficies with the parallel right lines : which was required to be demonſtrated.



By this figure it is eaſie to ſee the former demonſtration, if ye eleuate the ſuperficies wherin is drawen the line E G F.



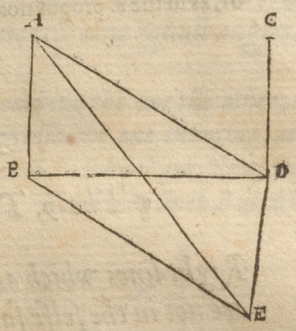
¶ The

The 8. Theoreme. The 8. Proposition.

If there be two parallel right lines, of which one is erected perpendicularly to a round playne superficies: the other also is erected perpendicularly to the selfe same ground playne superficies.

This proposition is as it were the conuerse of the sixth Construction.

SVppose that there be two parallel right lines A B and C D, and let one of them, namely, A B be erected perpendicularly to a ground superficies. Then I say that the line C D is also erected perpendicularly, to the selfe same ground superficies. Let the lines A B and C D fall vpon the ground superficies in the pointes B and D, and (by the first peticion) draw a right line from the point B to the point D. And drawe (by the 11. of the first) in the ground superficies from the point D vnto the line B D a perpendiculer line D E, and (by the 2. of the first) put the line D E equall to the line A B, and draw a right line from the point B to the point E, and an other from the point A to the point E, and an other from the point A to the point D. And forasmuch as the line A B is erected perpendicularly to the ground superficieces, therfore (by the 2. definition of the eleuenth) the line A B is erected perpendicularly to all the right lines that are in the ground superficies and touche it. Wherfore either of these angles A B D & A B E is a right angle. And forasmuch as vpon these parallel lines A B and C D falleth a certaine right line B D, therefore (by the 29. of the first) the angles A B D and C D B are equal to two right angles. But the angle A B D is a right angle, wherfore also the angle C D B is a right angle. Wherfore the line C D is erected perpendicularly to the line B D. And forasmuch as the line A B is equall to the line D E, and the line B D is common to them both, therfore these two lines A B and B D are equal to these two lines E D and D B, and the angle A B D is equall to the angle E D B, for either of them is a right angle. Wherfore (by the 4. of the first) the base A D is equall to the base B E. And forasmuch as the line A B is equall to the line D E, and the line B E to the line A D, therfore these two lines A B and B E are equall to these two lines A D & D E, the one to the other, and the line A E is a common base to them both. Wherfore (by the 8. of the first) the angle A B E is equall to the angle A D E: but the angle A B E is a right angle, wherfore the angle E D A, is also a right angle. Wherefore the line E D is erected perpendicularly to the line A D, and it is also erected perpendicularly to the line D B. Wherfore the line E D is erected perpendicularly to the plaine superficies wherin the lines B D and B A are (by the 4. of this booke) Wherfore (by the 2. definition of the eleuenth) the line E D is erected perpendicularly to all the right lines that touche it and are in the superficies wherein the lines B D and A D are. But in what superficies the lines B D and D A are, in the selfe same superficies is the line D C. For the line A D being drawen from two pointes taken in the parallel lines A B and C D is by the former proposition in the selfe same superficies with them. Now forasmuch as the lines A B and B D are in the superficies wherin the lines B D and D A are, but in what superficies the lines A B & B D are, in the same is the line D C. Wherfore the line E D is erected perpendicularly to the line D C. Wherfore also the line C D is erected perpendicularly to the line D E. And the line C D is erected perpendicularly to the line D B. For by the 29. of the first, the angle C D B being equall to the angle A B D is a right angle. Wherefore the line C D is from the point D erected perpendicularly to two right lines D E and D B cutting the one the other in the point D. Wherfore by the 4. of the eleuenth, the line C D is erected perpendiculaaly to the plaine superficies, wherein are the lines D E and D B. But



Demonstration.

the

the ground plaine ſuperficies is that wherin are the lines D E and D B, to which ſuperficies alſo the line A B is ſuppoſed to be erected perpendiculerly. Wherefore the line C D is erected perpendicularly to the ground plaine ſuperficies, wherunto the line A B is erected perpendicularly. If therfore there be two parallel right lines, of which one is erected perpendicularly to a ground plaine ſuperficies, the other alſo is erected perpendicularly to the ſelfe ſame ground plaine ſuperficies: which was required to be demonſtrated.

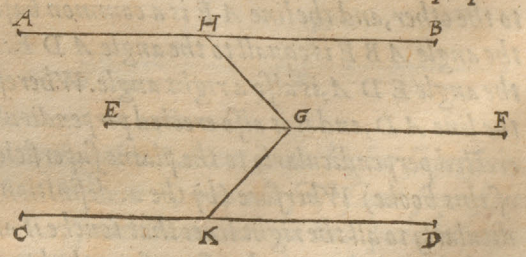
This figure will more clearely ſet forth the former demonſtration, if ye erect perpendicularly the ſuperficies A B D to the ſuperficies B D E, and imagine a lyne to be drawen from the point A to the point D, in ſtede wherof, as in the 6. propoſition ye may extende a threede.

¶ The 9. Theoreme. The 9. Propoſition.

Right lines which are parallels to one and the ſelfe ſame right line, and are not in the ſelfe ſame ſuperficies that it is in: are alſo parallels the one to the other.

Conſtruction. *Vppoſe that either of theſe right lines A B and C D be a parallel to the line E F not being in the ſelfe ſame ſuperficies with it. Then I ſay that the line A B is a parallel to the line C D. Take in the line E F a point at all aduentures, and let the ſame be G. And from the point G raiſe vp in the ſuperficies wherin are the lines E F and A B, vnto the line E F a perpendiculer line G H, and againe in the ſuperficies wherin are the lines E F and C D, raiſe vp from the ſame point G to the line E F a perpendiculer line G K.*

Demonſtration. *And foraſmuch as the line E F is erected perpendiculerly to either of the lines G H and G K, therfore (by the 4. of the eleuenth) the line E F is erected perpendicularly to the ſuperficies wherein the lines G H and G K are, but the line E F is a parallel line to the line A B. Wherfore (by the 8. of the eleuenth) the line A B is erected perpendicularly to the plaine ſuperficies, wherin are the lines G H and G K. And by the ſame reaſon alſo the line C D is erected perpendicularly to the plaine ſuperficies wherin are the lines G H & G K. Wherefore either of theſe lines A B and C D is erected perpendicularly to the plaine ſuperficies, wherin the lines G H and G K are. But if two right lines be erected perpendicularly to one and the ſelfe ſame plaine ſuperficies, thoſe right lines are parallels the one to the other (by the 6. of the eleuenth) Wherfore the line A B is a parallel to the line C D. Wherfore right lines which are parallels to one & the ſelfe ſame right line, and are not in the ſelf ſame ſuperficies with it are alſo parallels the one to the other: which was required to be proued.*



This

This figure more clearely manifesteth the former propoſition and demonſtration, if ye eleuate the ſuperficieces A B E F and C D E F that they may incline and concurre in the lyne E F.

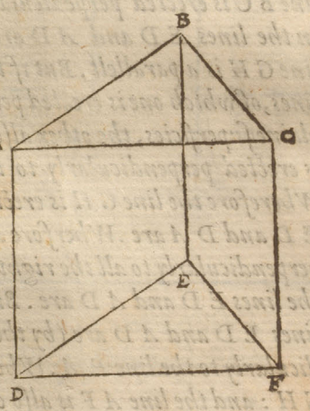
¶ The 10. Theoreme. The 10. Propoſition.

If two right lines touching the one the other be parallells to two other right lines touching the one the other, and not being in one and the ſelfe ſame ſuperficies with the two firſt: thoſe right lines cõtaine equall angles.

SVppoſe that theſe two right lines ∠A B and B C touching the one the other, be parallells to theſe two lines D E and E F touching alſo the one the other, and not being in the ſelfe ſame ſuperficies that the lines A B and B C are. The I ſay, that the angle ∠A B C is equall to the angle D E F. For let the lines B A, B C, E D, E F, be put equall the one to the other: and draw theſe right lines A D, C F, B E, A C, and D F. And foraſmuch as the line B A is equall to the line E D, and alſo parallell vnto it, therefore (by the 33. of the firſt) the line A D is equall and parallell to the line B E: and by the ſame reaſon alſo the line C F is equall & parallell to the line B E. Wherfore either of theſe lines A D and C F is equall & parallell to the line E B. But right lines which are parallells to one and the ſelfe ſame right line, and are not in the ſelfe ſame ſuperficies with it, are alſo (by the 9. of the eleuenth) parallells the one to the other. Wherefore the line A D is a parallell line to the line C F. And the lines A C and D F ioyne them together. Wherefore (by the 33. of the firſt) the line ∠A C is equall and parallell to the line D F. And foraſmuch as theſe two right lines A B & B C are equall to theſe two right lines D E and E F, and the baſe A C alſo is equall to the baſe D F: therefore (by the 8. of the firſt) the angle A B C is equall to the angle D E F. If therfore two right lines touching the one the other be parallells to two other right lines touching the one the other, and not being in one and the ſelfe ſame ſuperficies with the two firſt: thoſe right lines containe equall angles: which was required to be demonſtrated.

Conſtruction.

Demonſtration.



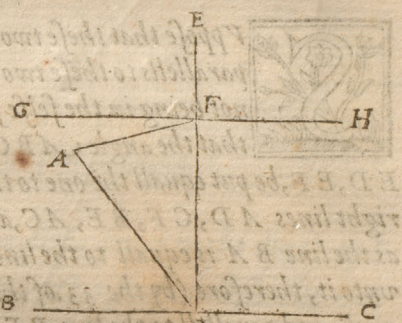
This figure here ſet more plainly declareth the former Propoſition and demonſtration, if ye eleuate the ſuperficieces D A B E, and F C B E, till they concurre in the line F E.

¶ The 1. Probleme. The 11. Proposition.

From a point geuen on high, to drawe vnto a ground plaine superficies a perpendicular right line.

Construction. Two cases in this proposition.

SVppose that the point geuen on high be A, and suppose a ground plaine superficies, namely, B C G H. It is required from the point A to draw vnto the ground superficies a perpendicular line. Drawe in the ground superficies a right line at aduentures, and let the same be B C. And (by the 12. of the first) from the point A draw vnto the line B C a perpendicular line A D. * Now if A D be a perpendicular line to the ground superficies, then is that done which was sought for. But if not, then (by the 11. of the first) from the point D raise vp in the ground superficies vnto the line B C a perpendicular line D E. And (by the 12. of the first) from the point A draw vnto the line D E a perpendicular line A F. And by the point F draw (by the 31. of the first) vnto the line B C a parallell line F H. And extend the line F H from the point F to the point G. And forasmuch as the line B C is erected perpendicularly to either of these lines D E and D A, therefore (by the 4. of the eleuenth) the line B C is erected perpēdicularly to the superficies wherin the lines E D and A D are: and to the line B C the line G H is a parallell. But if there be two parallell right lines, of which one is erected perpendicularly to a certaine plaine superficies, the other also (by the 8. of the eleuenth) is erected perpendicularly to the selfe same superficies. Wherefore the line G H is erected perpendicularly to the plaine superficies wherein the lines E D and D A are. Wherfore also (by the 2. definition of the eleuenth) the line G H is erected perpendicularly to all the right lines which touch it, and are in the plaine superficies wherein the lines E D and A D are. But the line A F toucheth it being in the superficies wherein the lines E D and A D are (by the 2. of this booke). Wherefore the line G H is erected perpendicularly to the line F A. Wherefore also the line F A is erected perpendicularly to the line G H: and the line A F is also erected perpendicularly to the line D E. Wherefore A F is erected perpendicularly to either of these lines H G and D E. But if a right line be erected perpendicularly from the common section of two right lines cutting the one the other, it shall also be erected perpendicularly to the plaine superficies of the said two lines (by the 4. of the eleuenth). Wherefore the line A F is erected perpendicularly to that superficies wherin the lines E D and G H are. But the superficies wherein the lines E D and G H are, is the ground superficies. Wherefore the line A F is erected perpendicularly to the ground superficies. Wherfore from a point geuen on high, namely, frō the point A, is drawen to the ground superficies a perpendicular line: which was required to be done.



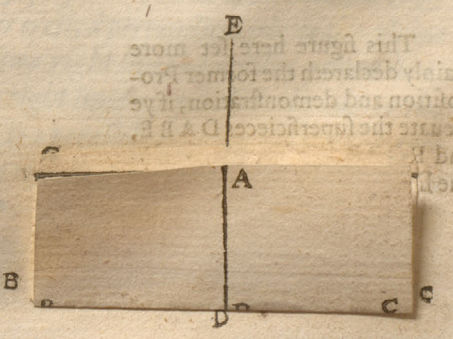
Iohn Dee.

This requireth the imagination of a plaine superficies passing by the pointe A, and the straight line B C. And so helpe your selfe in the lyke cases either Mathematically imagining, or Mechanically practising.

Second case.

Demonstration.

In this figure shall ye much more plainely see both the cases of this former demonstratiō. For as touching the first case, ye must erecte perpendicularly to the ground superficies, the superficies wherein is drawen the line A D, and compare it with the demonstration, and it will be clere vnto you. For the second case ye must erecte perpendicularly vnto the ground superficies the superficies wherein is drawen the line A F, and vnto it let the other superficies wherein is drawen the line A D, incline, so that the point A of the one may concurre with the point A of the other: and so with your figure thus ordered, compare it with the demonstration, and there will be in it no hardnes at all.

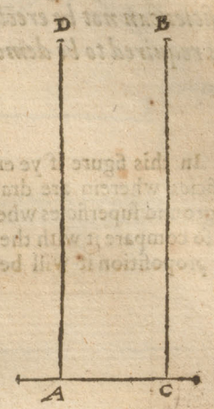


¶ The

¶ *The 2. Probleme.* *The 12. Proposition.*

Vnto a playne superficies geuen, and from a poynt in it geuen, to rayse vp a perpendicular line.

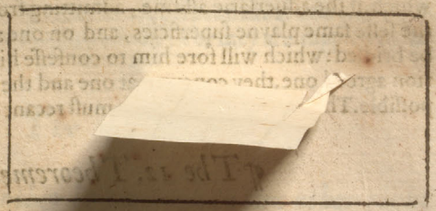
SVppose that there be a ground playne superficies geuen, and let the poynt in it geuen be A. *It is required from the point* A *to raise vp vnto the ground plaine superficies a perpendicular line. Vnderstand some certayne poynt on high, and let the same be* B. *And from the poynt* B *draw (by the 11. of the eleuenth) a perpendicular line to the ground superficies, and let the same be* B C. *And (by the 31. of the first) by the poynt* A *drawe vnto the line* B C *a parallel line* D A. *Now forasmuch as there are two parallel right lines* A D *and* C B, *and the one of them, namely,* C B *is erected perpendicularly to the ground superficies: wherefore the other line also, namely,* A D, *is erected perpendicularly to the same ground superficies (by the eight of the eleuenth). Wherefore vnto a playne superficies geuen, and from a poynt in it geuen, namely,* A, *is raysed vp a perpendicular lyne* A D: *which was required to be doone.*



Construction.

Demonstration.

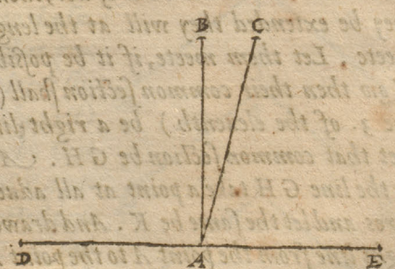
In this second figure ye may consider playnely the demonstration of the former proposition if ye erect perpendicularly the superficies wherein are drawne the lines A D and C B.



¶ *The 11. Theoreme.* *The 13. Proposition.*

From one and the selfe poynt, and to one and the selfe same playne superficies, can not be erected two perpendicular right lines on one and the selfe same side.

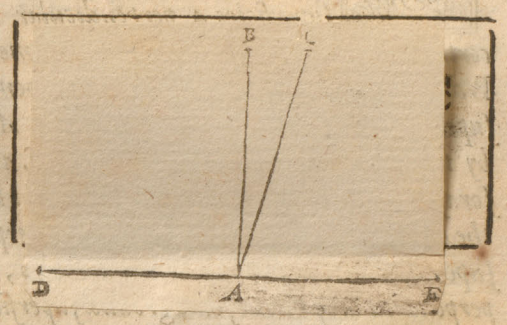
FOr if it be possible from the poynt A *let there be erected perpendicularly to one and the selfe same playne superficies two right lines* A B *and* A C *on one and the selfe same side. And extende the superficies wherein are the lines* A B *and* A C: *and it shall make at length a common section in the ground superficies which common section shall be a right line, and shall passe by the poynt* A: *let that common section be the line* D A E. *Wherefore (by the 3. of the eleuenth) the lines* A B, A C, *and* D A E *are in one and the selfe same playne superficies. And forasmuch as the line* C A *is erected perpendicularly to the ground superficies, therfore (by the*



Demonstration leading to an impossibilitie. Note this maner of imagination Mathematicall.

2. definition of the eleuenth) it maketh right angles with all the right lines that touch it, and are in the ground ſuperficies. But the line D A E *toucheth it, being in the ground ſuperficies. Wherefore the angle* C A E *is a right angle, and by the ſame reaſon alſo the angle* B A E *is a right angle. Wherefore (by the 4 petition) the angle* C A E *is equall to the angle* B A E *the leſſe to the more, both angles being in one & the ſelfe ſame playne ſuperficies: which is impoſſible. Wherefore from one and the ſelfe ſame poynt, and to one and the ſelfe ſame playne ſuperficies can not be erected two perpendicular right lines on one & the ſelfe ſame ſide: which was required to be demonſtrated.*

In this figure if ye erect perpendicularly the ſuperficies wherein are drawne the lines B A and C A to the ground ſuperficies wherein is drawn the line D A E, and ſo compare it with the the demonſtratiō of the former propoſition it will be cleare vnto you.



M. Dee his annotation.

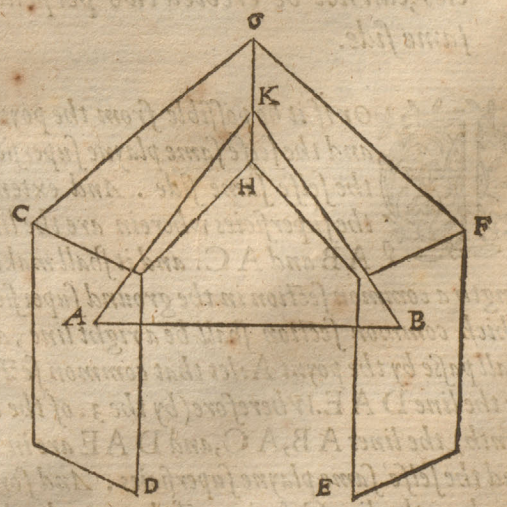
Euclides wordes in this 13. propoſition admit two caſes: one, if the poynt aſſigned be in the playne ſuperficies, (as cōmonly the demonſtrations ſuppoſe) the other, if the poynt aſſigned be any where without the ſayd playne ſuperficies, to which, the perpendiculars fall, is conſidered. Contrary to either of which, if the aduerſarie affirme, admitting from one poynt two right lines, perpendiculars to one and the ſelfe ſame playne ſuperficies, and on one and the ſame ſide thereof, by the 6. of the eleuenth he may be bridled: which will fore him to confeſſe his two perpendiculars to be alſo parallels. But by ſuppoſition agreed one, they concurre at one and the ſame poynt, which (by the definition of parallels) is impoſſible. Therefore our aduerſary muſt recant aud yelde to our propoſition.

¶ *The 12. Theoreme.* *The 14. Propoſition.*

To whatſoeuer plaine ſuperficieces one and the ſelfe ſame right line is erected perpendicularly: thoſe ſuperficieces are parallels the one to the other.

Demonſtration leading to an impoſſibilitie.

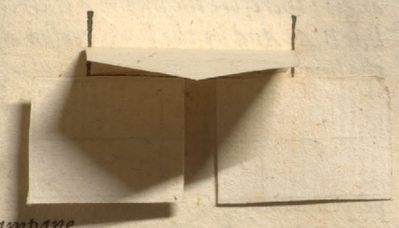
Vppoſe that a right line A B be erected perpēdicularly to either of theſe plaine ſuperficieces C D and E F. Then I ſay, that theſe ſuperficieces C D and E F are parallels the one to the other. For if not, then if they be extended they will at the length meete. Let them meete, if it be poſſible. Now then their common ſection ſhall (by the 3. of the eleuenth) be a right line. Let that common ſection be G H. And in the line G H take a point at all aduentures and let the ſame be K. And drawe a right line from the point A to the point K, and an other from the point B to the point K. And foraſmuch as the line A B is e-



rected

erected perpendicularly to the plaine ſuperficies E F, therefore the line A B is alſo erected perpendicularly to the line B K which is in the extended ſuperficies E F. Wherfore the angle A B K is a right angle. And by the ſame reaſon alſo the angle B A K is a right angle. Wherfore in the triangle A B K, theſe two angles A B K & B A K, are equall to two right angles: which (by the 17. of the firſt) is impoſsible. Wherefore theſe ſuperficieces C D and E F being extended meete not together. Wherefore the ſuperficieces C D and E F are parallells. Wherfore to what ſoeuer plaine ſuperficieces one and the ſelfe ſame right line is erected perpendicularly: thoſe ſuperficies are parallells the one to the other: which was required to be proued.

In this figure may ye plainly ſee the former demonſtration if ye erecte the three ſuperficieces, G D, G E, and K L M perpẽdiculary to the ground plaine ſuperficies: but yet in ſuch ſort that the two ſuperficieces G D and G E may concurre in the common line G K H, as is required in the demonſtration.



A corollary added by *Campane.*

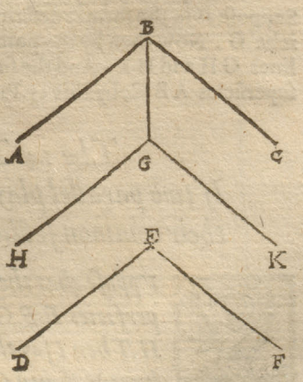
If a right line be erected perpendicularly to one of thoſe ſuperficies, it ſhall alſo be erected perpendicularly to the other.

For if it ſhould not be erected perpendicularly to the other, then it falling vpon that other ſhall make with ſome one line thereof an angle leſſe then a right angle: which line ſhould (by the 5. petition of the firſt) concurre with ſome one line of that ſuperficies whereunto it is perpendicular. So that thoſe ſuperficieces ſhould not be parallels: which is contrary to the ſuppoſition. For they are ſuppſed to be parallels.

¶ *The 13. Theoreme.* *The 15. Propoſition.*

If two right lines touching the one the other be parallels to two other right lines touching alſo the one the other and not being in the ſelfe ſame plaine ſuperficies with the two firſt: the plaine ſuperficieces extended by thoſe right lines, are alſo parallells the one to the other.

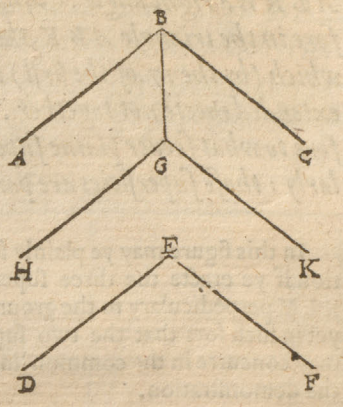
SVppoſe that theſe two right lines A B and B C touching the one the other be parallells to theſe two right lines D E & E F touching alſo the one the other, and not being in the ſelfe ſame plaine ſuperficies with the right lines A B and B C. Then I ſay, that the plaine ſuperficieces by the lines A B and B C, and the lines D E and E F being extended, ſhall not meete together, that is, they are equediſtant and parallels. From the point B draw (by the 11. of the eleuenth) a perpendicular line to the ſuperficies wherein are the lines D E and E F, and let that perpendicular line be B G. And by the point G in the plaine ſuperficies paſsing by D E, and E F, draw (by the 31. of the firſt) vnto the line E D a parallell line G H: and likewiſe by that point G drawe in the ſame ſuperficies vnto the line E F a parallell line G K. And foraſmuch as the line B G is erected perpendicularly to the ſuperficies wherein are the lines D E and E F, therefore (by the 2. definition of the eleuenth) it is alſo erected perpendicularly to all the right lines which touch it, and are in the ſelfe ſame ſuperficies wherein are the lines D E



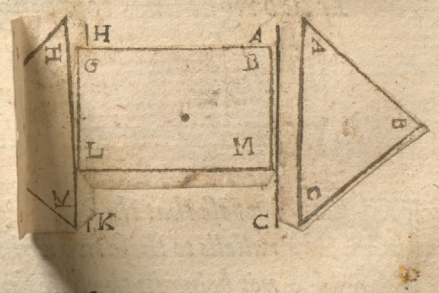
Conſtruction.

Demonſtration.

and E F. But either of these lines G H and G K touch it, and are also in the superficies wherein are the lines D E and E F, therefore either of these angles B G H, and B G K, is a right angle. And forasmuch as the line B A is a parallell to the line G H (that the lines G H and G K are parallells vnto the lines A B and B C it is manifest by the 9. of this booke): therefore (by the 29. of the first) the angles G B A and B G H are equall to two right angles. But the angle B G H is (by constructiō) a right angle, therfore also the angle G B A is a right angle: therefore the line G B is erected perpendicularly to the line B A. And by the same reason also may it be proued, that the line B G is erected perpendicularly to the line B C. Now forasmuch as the right line B G is erected perpendicularly to these two right lines B A and B C touching the one the other, therefore (by the 4. of the eleuenth) the line B G is erected perpendicularly to the superficies wherein are the lines B A and B C. And it is also erected perpendicularly to the superficies wherein are the lines G H and G K. But the superficies wherein are the lines G H and G K, is that superficies wherein are the lines D E and E F: wherefore the line B G is erected perpendicularly to the superficies wherein are the lines D E and E F. Wherefore the line B G is erected perpendicularly to the superficies wherein are the lines D E and E F, and to the superficies wherein are the lines A B and B C. But if one and the selfe same right line be erected perpendicularly to plaine superficieces, those superficieces are (by the 14. of the eleuenth) parallels the one to the other. Wherefore the superficies wherin are the lines A B and B C is a parallel to the superficies wherin are the lines D E and E F. If therefore two right lines touching the one the other be parallels to two other right lines touching also the one the other, and not being in the selfe same plaine superficies with the two first, the plaine superficieces extended by those right lines are also parallels the one to the other which was required to be demonstrated.



By this figure here put, ye may more clerely see both the former 15. Proposition and also the demonstration therof: if ye erecte perpendicularly vnto the ground superficies, the three superficieces A B C, K H E, and L H B M, and so compare it with the demonstration.



¶ A Corollary added by *Flussas*.

Vnto a plaine superficies being geuen, to drawe by a point geuen without it, a parallel plaine superficies.

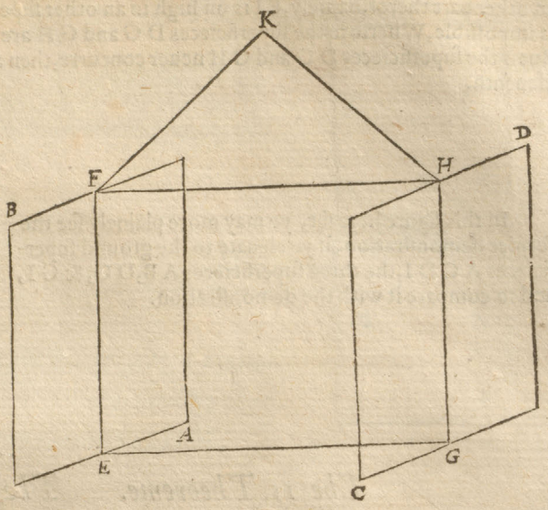
Suppose as in the former description that the superficies geuē be A B C, & let the point geuē without it be G. Now then by the point G drawe (by the 31. of the first) vnto the lines A B and B C parallel lines G H and H K. And the superficies extended by the lines G H and G K shall be parallel vnto the superficies A B C, by this 15. Proposition.

The 14. Theoreme. *The 16. Proposition.*

If two parallel playne superficieces be cut by some one playne superficies: their common sections are parallel lines.

Vppose that these two plaine superficieces A B and C D be cut by this plaine superficies E F G H, and let their common sections be the right lines E F and G H. Then I say that the line E F is a parallel to the line G H. For if not, then the lines E F and G H being produced, shall at the length meete together either on the

the ſide that the pointes F,H are, or on the ſide that the pointes E,G are. Firſt let them be produced on that ſide that the pointes F,H are,and let them mete in the point K. And foraſmuch as the line E F K is in the ſuperficies A B, therfore all the points which are in the line E F are in the ſuperficies A B (by the firſt of this booke) But one of the pointes which are in the right line E F K is the point K, therfore the point K is in the ſuperficies A B. And by the ſame reaſon alſo the point K is in the ſuperficies C D. Wherfore the two ſuperficieces A B and C D being produced do mete together, but by ſuppoſitiõ they mete not together,for they are ſuppoſed to be parallels.Wherfore the right lines E F and G H produced ſhall not meete together on that ſide that the pointes F,H are.In like ſort alſo may we proue that the right lines E F and G H produced meete not together on that ſide that the pointes E,G are. But right lines which being produced on no ſide mete together, are parallels (by the laſt definition of the firſt.) Wherfore the line E F is a parallel to the line G H.If therfore two parallel plaine ſuperficieces be cut by ſome one plaine ſuperficies their common ſections are parallel lines: which was required to be proued.



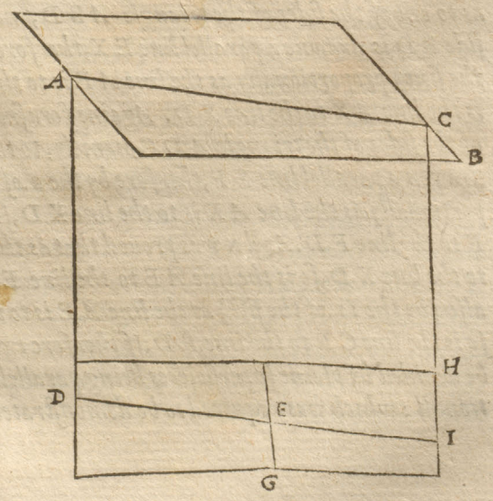
Demonſtration leading to an abſurditie.

This figure here ſet more plainly declares the demonſtration,if ye erect perpendicularly vpon the ſuperficies the three ſuperficieces A B,C D, and ſo compare it with the demonſtratiõ.

A Corollary added by *Fluſas.*

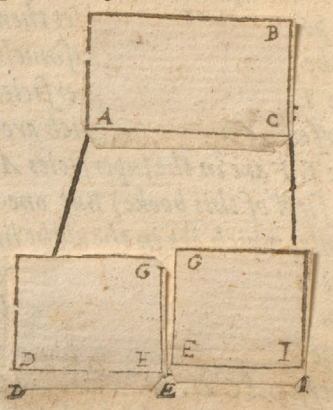
If two plaine ſuperficieces be parallels to one and the ſelfe ſame playne ſuperficies: they ſhall alſo be parallels the one to the other,or they ſhall make one and the ſelfe ſame plaine ſuperficies.

For if the plaine ſuperficieces D G and G H being parallels to one and the ſelfe ſame ſuperficies,namely, to A B be not alſo parallels the one to the other,then being produced they ſhall concurre (by the conuerſe of the ſixt definition of the eleuenth) Let them concurre in the right line G E. Then I ſay that the ſuperficieces G D and G H are in one and the ſelfe ſame playne ſuperficies. Draw in the playne ſuperficies A B a right line at all aduentures A C. And by that right lyne & the point E extende a playne ſuperficies, cutting the two ſuperficieces D G and G H by the right lines E D and E I. Wherfore the right lines A C and D E,alſo A C and E I are parallels by this propoſition. But the lines D E and E I foraſmuch as they concurre in the point E are not parallels the one to the other. Wherefore the right lines D E and E I make directly one right line (by that which is added after the 30. propoſiton of the firſt.) And therfore the plaine ſuperficieces D G and G H are in one and the ſelfe ſame playne ſuperficies. For



if they be not, then part of the right line D I, namely, the part D E is in the playne ſuperficies D G, and an other part therof, namely, E I is on high in an other ſuperficies G H, which by the firſt of the eleuẽth is impoſſible. Wherfore the ſuperficieces D G and G H are in one and the ſelfe ſame playne ſuperficies. But if the ſuperficieces D G and G H neuer concurre, then are they parallels by the 6. definition of the eleuenth.

In this figure here ſet, ye may more plainely ſee the former demonſtration, if ye eleuate to the ground ſuperficieces A C D I, the three ſuperficieces A B, D G, & G I, and ſo compare it with the demonſtration.



The 15. Theoreme. The 17. Propoſition.

If two right lines be cut by playne ſuperficieces being parallels: the partes of the lines deuided ſhall be proportionall.

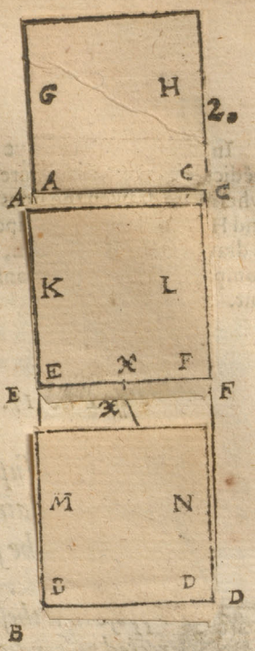
In this propoſition ye muſt vnderſtand the proportionall partes or ſections to be thoſe which are contained betwene the parallel ſuperficies.

Conſtruction.

Demonſtration.

Vppoſe that theſe two right lines A B and C D be deuided by theſe plaine ſuperficieces being parallels, namely, GH, KL, MN in the points A, E, B, C, F, D. Thẽ I ſay that as the right line A E is to the right line E B, ſo is the right line C F to the right line F D. Draw theſe right lines A C, B D and A D. And let the line A D and the ſuperficies K L concurre in the point X. And draw a right line from the point E to the point X and an other from the point X to the point F. And foraſmuch as theſe two parallel ſuperficieces K L and M N are cut by the ſuperficies E B D X, therfore their common ſections which are the lines E X and B D, are (by the 16. of the eleuenth) parallels the one to the other. And by the ſame reaſon alſo foraſmuch as the two parallel ſuperficies G H and K L be cut by the ſuperficies A X F C, their common ſections A C and X F are (by the 16. of the eleuenth) parallels. And foraſmuch as to one of the ſides of the triangle A B D, namely, to the ſide B D is drawne a parallel line E X, therfore (by the 2. of the ſixt) proportionally as the line A E is to the line E B, ſo is the line A X to the line X D. Againe foraſmuch as to one of the ſides of the triangle A D C, namely, to the ſide A C is drawen a parallel line X F, therfore by the 2. of the ſixt, proportionally as the line A X is to the line X D, ſo is the line C F to the line F D. And it was proued that as the line A X is to the line X D, ſo is the line A E to the line E B, therefore alſo (by the 11. of the fift) as the line A E is to the line E B, ſo is the line C F to the line F D. If therfore two right lines be deuided by plaine ſuperficieces being parallels, the parts of the lines deuided ſhal be proportionall: which was required to be demonſtrated.

In

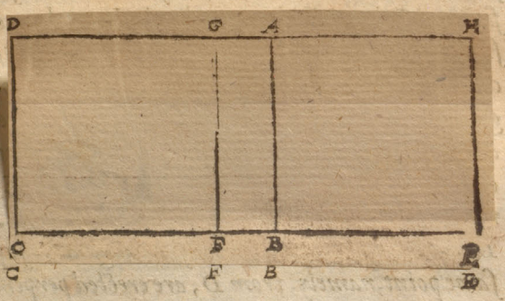


In this figure it is more easy to see the former demonstration, if ye erect perpendicularly vnto the ground superficies A C B D, the thre superficieces, G H, K L, and M N, or if ye so erect them that they be equedistant one to the other.

¶ The 16. Theoreme. The 18. Proposition.

If a right line be erected perpẽdicularly to a plaine superficies: all the superficieces extended by that right line, are erected perpendicularly to the selfe same plaine superficies.

SVppose that a right line A B be erected perpendicularly to a ground superficies. Thẽ I say, that all the superficieces passing by the line A B, are erected perpendicularly to the ground superficies. Extend a superficies by the line A B, and let the same be E D, & let the cõmon section of the plaine superficies and of the ground superficies be the right line C E. And take in the line C E a point at all aduentures, and let the same be F: and (by the 11. of the first) from the point F drawe vnto the line C E a perpendicular line in the superficies D E, and let the same be F G. And forasmuch as the line A B is erected perpendicularly to the ground superficies, therefore (by the 2. definition of the eleuenth) the line A B is erected perpendicularly to all the right lines that are in the ground plaine superficies, and which touch it. Wherefore it is erected perpendicularly to the line C E. Wherefore the angle A B E is a right angle. And the angle G F B is also a right angle (by construction). Wherefore (by the 28. of the first) the line A B is a parallel to the line F G. But the line A B is erected perpendicularly to the ground superficies: wherefore (by the 8. of the eleuenth) the line F G is also erected perpendicularly to the ground superficies. And forasmuch as (by the 3. definition of the eleuenth) a plaine superficies is then erected perpendicularly to a plaine superficies, when all the right lines drawen in one of the plaine superficieces vnto the common section of those two plaine superficieces making therwith right angles, do also make right angles with the other plaine superficies: and it is proued that the line F G drawen in one of the plaine superficieces, namely, in D E, perpendicularly to the common section of the plaine superficieces, namely, to the line C E, is erected perpendicularly to the ground superficies: wherefore the plaine superficies D E is erected perpendicularly to the ground superficies. In like sort also may we proue, that all the plaine superficieces which passe by the line A B, are erected perpendicularly to the ground superficies. If therefore a right line be erected perpendicularly to a plaine superficies all the superficieces passing by the right line, are erected perpendicularly to the selfe same plaine superficies: which was required to be demonstrated.

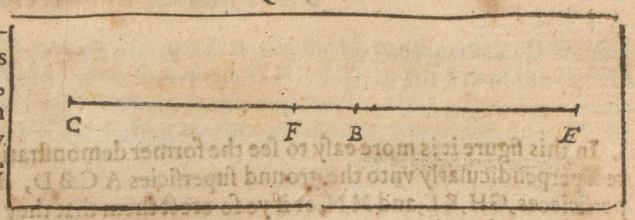


Construction.

Demonstration.

In

In this figure here set ye may erect perpendicularly at your pleasure the superficies wherin are drawen the lines D C, G F, A B, and H E, to the ground superficies wherin is drawen the line C F B E, and so plainly compare it with the demonstration before put.

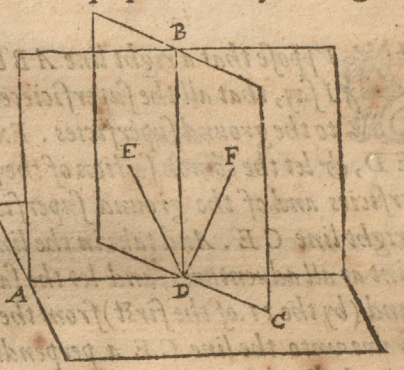


¶ The 17. Theoreme. The 19. Proposition.

If two plaine superficieces cutting the one the other be erected perpendicularly to any plaine superficies: their common section is also erected perpendicularly to the selfe same plaine superficies.

Suppose that these two plaine superficieces A B & B C cutting the one the other be erected perpendicularly to a ground superficies, and let their common section be the line B D. Then I say, that the line B D is erected perpendicularly to the ground superficies. For if not, then (by the 11. of the first) from the point D draw in the superficies A B vnto the right line D A a perpendicular line D E. And in the superficies C B draw vnto the line D C a perpendicular line D F. And forasmuch as the superficies A B is erected perpendicularly to the ground superficies, and in the plaine superficies A B vnto the common section of the plaine superficies and of the ground superficies, namely, to the line D A is erected a perpendicular line D E, therefore (by the conuerse of the 3. definition of this booke) the line D E is erected perpendicularly to the ground superficies. And in like sort may we proue, that the line D F is erected perpendicularly to the ground superficies. Wherefore from one and the selfe same point, namely, from D, are erected perpendicularly to the ground superficies two right lines both on one and the self same side: which is (by the 15. of the eleuenth) impossible. Wherefore from the point D can not be erected perpendicularly to the ground superficies any other right lines besides B D, which is the common section of the two superficieces A B and B C. If therefore two plaine superficieces cutting the one the other be erected perpendicularly to any plaine superficies, their common section is also erected perpendicularly to the selfe same plaine superficies: which was required to be proued.

Demonstration leading to an impossibilitie.



Here haue I set an other figure which will more plainly shewe vnto you the former demonstration, if ye erecte perpendicularly to the ground superficies A C the two superficieces A B and B C which cut the one the other in the line B D.

The 18. Theoreme. The 20. Proposition.

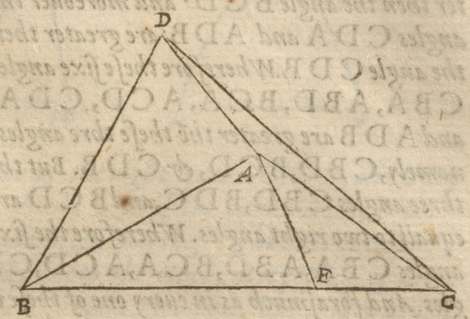
If a solide angle be contayned vnder three playne superficiall angles: euery

two

two of those three angles, which two so euer be taken, are greater then the third.

SVppose that the solide angle A *be contayned vnder three playne superficiall angles, that is, vnder* B A C, C A D, *and* D A B. *Then I say that two of these superficiall angles how so euer they be taken, are greater then the third. If the angles* B A C, C A D, *&* D A B *be equall the one to the other, then is it manifest that two of them which two so euer be taken are greater then the third. But if not, let the angle* B A C *be the greater of the three angles. And vnto the right line* A B *and from the poynt* A *make in the playne superficies* B A C *vnto the angle* D A B *an equall angle* B A E. *And (by the 2. of the first) make the line* A E *equall to the line* A D. *Now a right line* B E C *drawne by the poynt* E, *shall cut the right lines* A B *and* A C *in the poyntes* B *and* C: *draw a right line from* D *to* B, *and an other from* D *to* C. *And forasmuch as the line* D A *is equall to the line* A E, *and the line* A B *is common to the both, therefore these two lines* D A *and* A B *are equall to these two lines* A B *and* A E *and the angle* D A B *is equall to the angle* B A E. *Wherefore (by the 4. of the first) the base* D B *is equall to the base* B E. *And forasmuch as these two lines* D B *and* D C *are greater then the line* B C, *of which the line* D B *is proued to be equall to the line* B E. *Wherefore the residue, namely, the line* D C *is greater then the residue, namely, then the line* E C. *And forasmuch as the line* D A *is equall to the line* A E, *and the line* A C *is common to them both, and the base* D C *is greater then the base* E C, *therefore the angle* D A C *is greater then the angle* E A C. *And it is proued that the angle* D A B *is equall to the angle* B A E: *wherfore the angles* D A B *and* D A C *are greater then the angle* B A C. *If therefore a solide angle be contayned vnder three playne superficiall angles euery two of those three angles, which two so euer be taken are greater then the third: which was required to be proued.*

1.



2. Construction.

Demonstration.

In this figure ye may playnely behold the former demonstration, if ye eleuate the three triangles A B D, A B C and A C D in such sort that they may all meete together in the poynt A.

The 19. Theorcme. *The 21. Proposition.*

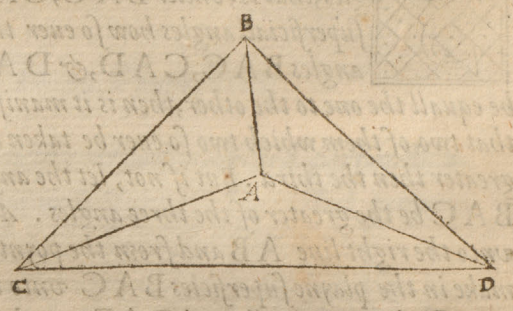
Euery solide angle is comprehended vnder playne angles lesse then fower right angles.

SVppose that A *be a solide angle contayned vnder these superficiall angles* B A C, D A C *and* D A B. *Then I say that the angles* B A C, D A C *and* D A B *are lesse then fower right angles. Take in euery one of these right lines* A C A B *and* A D

Construction

Demonſtration.

AD *a poynt at all aduentures and let the ſame be* B,C,D. *And draw theſe right lines* BC, CD *and* DB. *And foraſmuch as the angle* B *is a ſolide angle, for it is contayned vnder three ſuperficiall angles, that is, vnder* CB A, ABD *and* CBD, *therefore (by the 20. of the eleuenth) two of them which two ſo euer be taken are greater then the third. Wherefore the angles* CBA *and* ABD *are greater then the angle* CBD: *and by the ſame reaſon the angles* BCA *and* ACD *are greater then the angle* BCD: *and moreouer the angles* CDA *and* ADB *are greater then the angle* CDB. *Wherefore theſe ſixe angles* CBA, ABD, BCA, ACD, CDA, *and* ADB *are greater thē theſe thre angles, namely,* CBD, BCD, *&* CDB. *But the three angles* CBD, BDC, *and* BCD *are equall to two right angles. Wherefore the ſixe angles* CBA, ABD, BCA, ACD, CDA, *and* ADB *are greater thē two right angles. And foraſmuch as in euery one of theſe triangles* ABC, *and* ABD *and* ACD *three angles are equall two right angles (by the 32. of the firſt). Wherefore the nine angles of the thre triangles, that is, the angles* CBA, ACB, BAC, ACD, DAC, CDA, ADB, DBA *and* BAD *are equall to ſixe right angles. Of which angles the ſixe angles* ABC, BCA, ACD, CDA, ADB *and* DBA *are greater then two right angles. Wherefore the angles remayning, namely, the angles* BAC, CAD *and* DAB *which contayne the ſolide angle are leſſe then fower right angles. Wherefore euery ſolide angle is comprehended vnder playne angles leſſe then fower right angles: which was required to be proued.*



If ye will more fully ſee this demonſtration compare it with the figure which I put for the better ſight of the demonſtration of the propoſition next going before. Onely here is not required the draught of the line A E.

Although this demonſtration of *Euclide* be here put for ſolide angles contayned vnder three ſuperficiall angles, yet after the like maner may you proceede if the ſolide angle be contayned vnder ſuperficiall angles how many ſo euer. As for example if it be contayned vnder fower ſuperficiall angles, if ye follow the former conſtruction, the baſe will be a quadrangled figure, whoſe fower angles are equall to fower right angles: but the 8. angles at the baſes of the 4. triangles ſet vpon this quadrangled figure may by the 20. propoſition of this booke be proued to be greater then thoſe 4. angles of the quadrangled figure: As we ſawe by the diſcourſe of the former demonſtration. Wherefore thoſe 8. angles are greater then fower right angles: but the 12. angles of thoſe fower triangles are equall to 8. right angles. Wherefore the fower angles remayning at the toppe which make the ſolide angle are leſſe then fower right angles. And obſeruing this courſe ye may proceede infinitely.

¶ *The 20. Theoreme.* *The 22. Propoſition.*

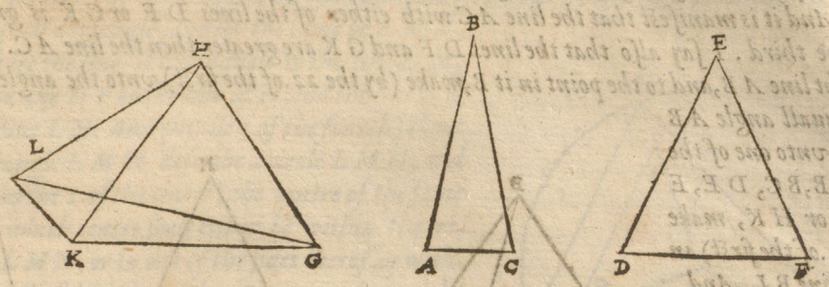
If there be three ſuperficiall plaine angles of which two how ſoeuer they be taken, be greater then the third, and if the right lines alſo which contayne thoſe angles be equall: then of the lines coupling thoſe equall right lines together, it is poſſible to make a triangle.

Vppoſe that there be thre ſuperficial angles A B C, D E F, and G H K, of which let two, which two ſoeuer be taken, be greater then the third, that is, let the angles A B C, and D E F be greater then the angle G H K, and let the angles D E F and G H K be greater then the angle A B C.: and moreouer let the angles G H K and A B C be greater then the angle D E F. And let the right lines A B, B C, D E, E F G H, and H K be equall the one to the other, and draw a right line from the point A to the

poynt

point C, and an other from the point D to the point F, and moreouer an other from the point G to the point K. Then I say that it is possible of three right lines equall to the lines A C, D F

Two cases in this propositiou.



and G K, to make a triangle, that is, that two of the right lynes A C, D F, and G K, which two soeuer be taken, are greater then the third. Now if the angles A B C, D E F, and G H K be equall the one to the other, it is manifest that these right lines A C, D F, and G K being also (by the 4. of the first) equall the one to the other, it is possible of three right lines equall to the lines A C, D F, and G K to make a triangle. But if they be not equall, let them be vnequall. And (by the 23. of the first) vnto the right line H K, and at the point in it H, make vnto the angle A B C an equall angle K H L. And by the 2. of the first) to one of the lines A B, B C, D E, E F, G H, or H K make the line H L equal, & draw these right lines K L and G L. And forasmuch as these two lines A B and B C, are equall to these two lines K H and H L, and the angle B is equall to the angle K H L, therfore (by the 4. of the first) the base A C is equall to the base K L. And forasmuch as the angles A B C, and G H K are greater then the angle D E F, but the angle G H L is equall to the angles A B C, & G H K: therfore the angle G H L is greater then the angle D E F. And forasmuch as these two lines G H and H L are equall to these two lines D E and E F, and the angle G H L is greater then the angle D E F, therfore (by the 25. of the first) the base G L is greater thē the base D F. But the lines G K and K L are greater then the line G L. Wherfore the lines G K & K L are much greater then the line D F. But the line K L is equal to the line A C. Wherfore the lines A C and G K are greater then the line D F. In like sort also may we proue that the lines A C and D F are greater then the line G K, and that the lines G K and D F are greater then the lyne A C. Wherfore it is possible to make a triangle of three lynes equall to the lines A C, D F, and G K: which was required to be demonstrated.

The first case.

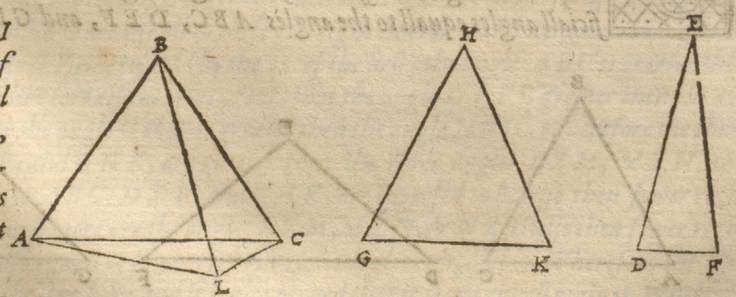
Second case. Constructiou.

Demonstration.

An other demonstration.

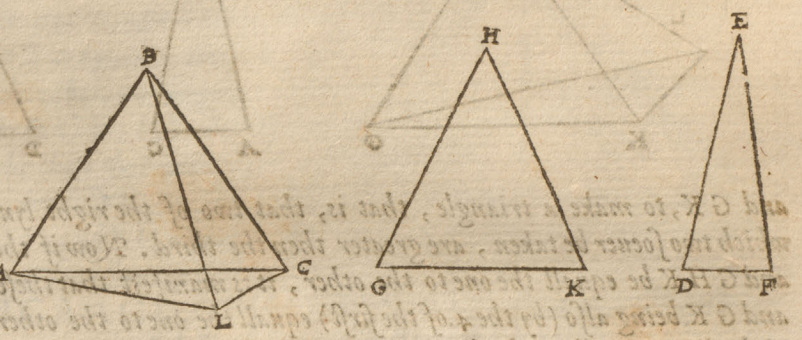
Suppose that the three superficiall angles be A B C, D E F, and G H K, of which angles, two howsoeuer they be taken, are greater then the third. And let them be contained vnder these equall right lines A B, B C, D E, E F, G H, H K, which equall right lines let these lines A C, D F, and G K ioyne together. Then I say that it is possible of three right lines equal to the lines A C, D F, and G K to make a triangle, which againe is as much to say, as that two of those lines which two soeuer be taken, are greater then the third. Now againe if the angles B, E, H, be equall, the lines also A C D F,

An other demonstration.



D F, and G K are equall, and ſo two of them ſhall be greater then the third. But if not, let the angles B, E, H, be vnequall, and let the angle B be greater then either of the angles E and H. Therfore (by the 24. of the firſt) the right line A C is greater then either of the lines D F & G K. And it is manifeſt that the line A C with either of the lines D F or G K is greater then the third. I ſay alſo that the lines D F and G K are greater then the line A C. Vnto the right line A B, and to the point in it B, make (by the 22. of the firſt) vnto the angle G H K an equall angle A B L, and vnto one of the lines A B, B C, D E, E F, G H or H K, make by the 2. of the firſt) an equall line B L. And draw a right line from the point A to ỹ point L, and an other frō the point L to the point C.

Conſtruction.



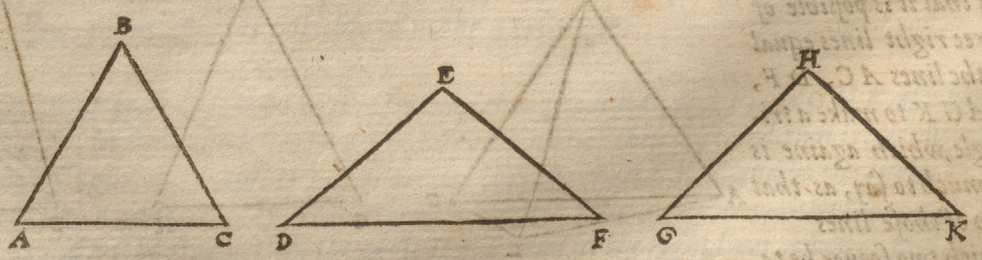
And foraſmuch as theſe two lines A B & B L are equal to theſe two lines G H & H K the one to the other, and they containe equal angles: therfore (by the 4. of the firſt) the baſe A L is equall to the baſe G K. And foraſmuch as the angles E and H be greater then the angle A B C, of which the angle G H K is equal to the angle A B L, therfore the angle remayning, namely, the angle E is greater then the angle L B C. And foraſmuch as theſe two lines L B and B C, are equall to theſe two lines D E and E F the one to the other, and the angle D E F is greater then the angle L B C, therfore (by the 25. of the firſt) the baſe D F is greater then the baſe L C: and it is proued that the line G K is equall to the line A L. Wherfore the lines D F & G K are greater then the lines A L & L C. But the lines A L and L C are greater then the line A C. Wherfore the lines D F & G K are much greater thē the line A C. Wherfore two of theſe right lines A C, D F & G K which two ſoeuer be taken are greater then the third. Wherfore it is poſſible of three right lines equall to the lines A C, D F and G K to make a triangle: which was required to be demonſtrated.

Demonſtration.

¶ The 3. Probleme. The 23. Propoſition.

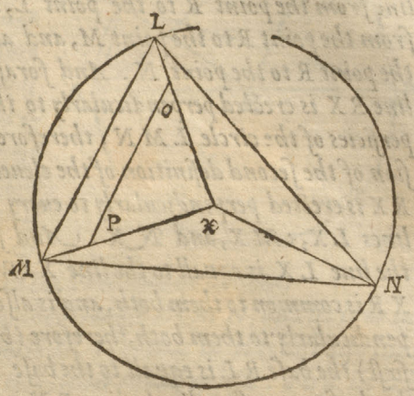
Of three plaine ſuperficiall angles, two of which how ſoeuer they be taken, are greater then the third, to make a ſolide angle: Now it is neceſſary that thoſe three ſuperficiall angles be leſſe then fower right angles.

Vppoſe that the ſuperficiall angles geuen be A B C, D E F, G H K: of which let two how ſoeuer they be taken, be greater then the third: and moreouer, let thoſe three angles be leſſe then fower right angles. It is required of three ſuperficiall angles equall to the angles A B C, D E F, and G H K, to make a ſolide



or

or bodily angle. Let the lines A B, B C, D E, E F, G H, and H K, be made equall: and drawe a right line from the point A to the point C, & an other from the point D to the point F, and an other from the point G to the point K. Now (by the 22. of the eleuenth) it is possible of three right lines equall to the right lines A C, D F, and G K, to make a triangle. Make such a triangle, and let the same be L M N, so that let the line A C be equall to the line L M, and the line D F to the line M N, and the line G K to the line L N. And (by the 5. of the fourth) about the triangle L M N describe a circle L M N, and take (by the 1. of the third) the centre of the same circle, which centre shall either be within the triangle L M N, or in one of the sides therof, or without it. First let it be within the triangle, and let the same be the point X, & drawe these right lines L X, M X, and N X. Now I say, that the line A B is greater then the line L X. For if not, then the line A B is either equall to the line L X, or els it is lesse then it. First let it be equall. And forasmuch as the line A B is equall to the line L X, but the line A B is equall to the line B C, therefore the line L X is equall to the line B C: and vnto the line L X the line X M is (by the 15. definition of the first) equall: wherefore these two lines A B and B C, are equall to these two lines L X and X M the one to the other: and the base A C is supposed to be equall to the base L M. Wherefore (by the 8. of the first) the angle A B C is equall to the angle L X M. And by the same reason also the angle D E F is equall to the angle M X N, and moreouer the angle G H K to the angle N X L. Wherefore these three angles A B C, D E F, and G H K, are equall to these three angles L X M, M X N, & N X L. But the three angles L X M, M X N, and N X L, are equall to fower right angles (as it is manifest to see by the 13. of the first, if any one of these lines M X, L X, or N X, be extended on the side that the point X is). Wherfore the three angles A B C, D E F, and G H K, are also equall to fower right angles. But they are supposed to be lesse then fower right angles: which is impossible. Wherefore the line A B is not equall to the line L X. I say also that the line A B is not lesse then the line L X. For if it be possible, let it be lesse: and (by the 2. of the first) vnto the line A B put an equall line X O: and to the line B C put an equall line X P, and draw a right line from the point O to the point P. And forasmuch as the line A B is equall to the line B C, therefore also the line X O is equall to the line X P. Wherefore the residue O L is equall to the residue M P. Wherfore (by the 2. of the sixt) the line L M is a parallel to the line O P: and the triangle L M X is equiangle to the triangle O P X. Wherefore as the line X L is to the line L M, so is the line X O to the line O P. Wherefore alternately (by the 16. of the fift) as the line L X is to the line X O, so is the line L M to the line O P. But the line L X is greater then the line X O. Wherfore also the line L M is greater then the line O P. But the line L M is put to be equall to the line A C: wherefore also the line A C is greater then the line O P. Now forasmuch as these two right lines A B and B C are equall to these two right lines O X and X P, and the base A C is greater then the base O P, therefore (by the 25. of the first) the angle A B C is greater then the angle O X P. In like sort also may we proue, that the angle D E F is greater then the angle M X N, and that the angle G H K is greater then the angle N X L. Wherefore the three angles A B C, D E F, and G H K, are greater then the three angles L X M, M X N, and N X L. But the angles A B C, D E F, and G H K, are supposed to be lesse then fower right angles: wherefore much more are the angles L X M, M X N, & N X L lesse then fower right angles. But they are also equall to fower right angles: which is impossible. Wherefore the line A B is not lesse then the line L X: and it is also proued that it is not equall vnto it. Wherfore



Construction.

Three cases in this proposition.

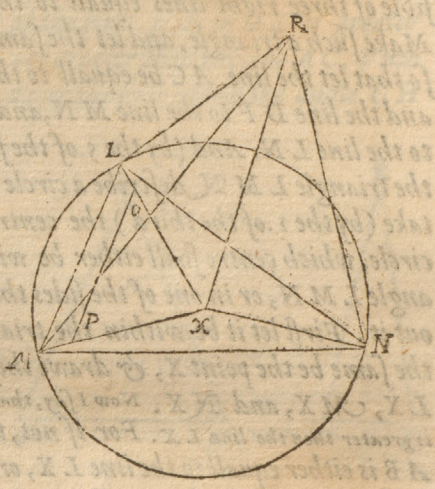
The first case.

A necessary thing to be proued before he proceede any farther in the construction of the Probleme.

the line A B is greater then the line L X.

Now from the point X raiſe vp vnto the plaine ſuperficies of the circle L M N a perpendicular line X R (by the 12. of the eleuenth). And vnto that which the ſquare of the line A B excedeth the ſquare of the line X L * let the ſquare of the line X R be equall. And draw a right line from the point R to the point L, and an other from the point R to the point M, and an other from the point R to the point N. And foraſmuch as the line R X is erected perpendicularly to the plaine ſuperficies of the circle L M N, therefore (by conuerſion of the ſecond definition of the eleuenth) the line R X is erected perpendicularly to euery one of theſe lines L X, M X, and N X. And foraſmuch as the line L X is equall to the line X M, & the line X R is common to them both, and is alſo erected perpendicularly to them both, therefore (by the 4. of the firſt) the baſe R L is equall to the baſe R M. And by the ſame reaſon alſo the line R N is equall to either of theſe lines R L and R M. Wherefore theſe three lines R L, R M, and R N, are equall the one to the other. And foraſmuch as vnto that which the ſquare of the line A B excedeth the ſquare of the line L X, the ſquare of the line R X is ſuppoſed to be equall, therefore the ſquare of the line A B is equall to the ſquares of the lines L X and R X. But vnto the ſquares of the lines L X and X R, the ſquare of the line L R is (by the 47. of the firſt) equall, for the angle L X R is a right angle. Wherefore the ſquare of the line A B is equall to the ſquare of the line R L. Wherefore alſo the line A B is equall to the line R L. But vnto the line A B is equall euery one of theſe lines B C, D E, E F, G H, and H K, and vnto the line R L is equall either of theſe lines R M and R N. Wherfore euery one of theſe lines A B, B C, D E, E F, G H, and H K, is equall to euery one of theſe lines R L, R M, and R N. And foraſmuch as theſe two lines R L and R M are equall to theſe two lines A B and B C, and the baſe L M is ſuppoſed to be equall to the baſe A C, therfore (by the 8. of the firſt) the angle L R M is equall to the angle A B C. And by the ſame reaſon alſo the M R N is equall to the angle D E F, and the angle L R N to the angle G H K. Wherefore of three ſuperficiall angles L R M, M R N, and L R N, which are equall to three ſuperficiall angles geuẽ, namely, to the angles A B C, D E F, & G H K, is made a ſolide angle R, comprehended vnder the ſuperficiall angles L R M, M R N, and L R N: which was required to be done.

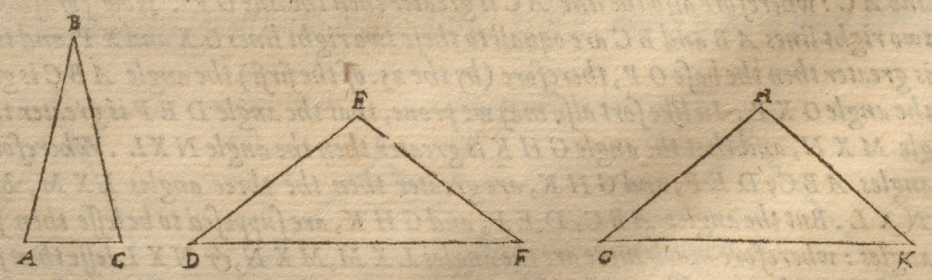
* Which how to finde out is taught at the end of this demonſtration, and alſo was taught in the aſſumpt put before the 14. propoſition of the téth boke.



Demonſtration of the firſt caſe.

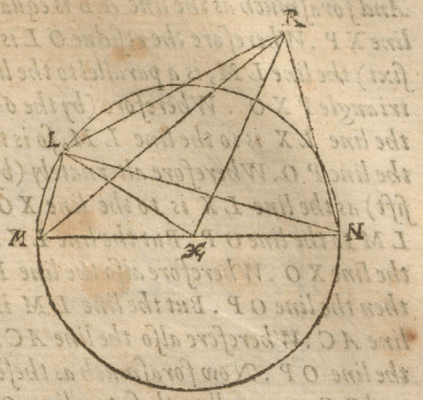
Second caſe.

But now let the centre of the circle be in one of the ſides of the triangle, let it be in the ſide M N, and let the centre be X. And draw a right line from the point L to the point X. I ſay againe, that the line A B is greater then the line L X. For if not, then A B is either equall to L X, or els it is leſſe then it. Firſt let it be equall. Now thẽ theſe two lines A B and

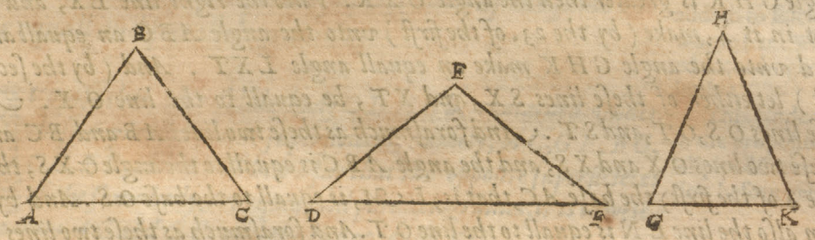


B C,

BC, that is, DE & EF are equall to these two lines MX and XL, that is, to the line MN. But the line MN is supposed to be equall to the line DF. Wherefore also the lines DE and EF are equall to the line DF: which (by the 20. of the first) is impossible. Wherfore the line AB is not equall to the line LX. In like sort also may we proue, that it is not lesse. Wherefore the line AB is greater then the line LX. And now if as before vnto the plaine superficies of the circle be erected frō the point X a perpendicular line RX whose square let be equall vnto that which the square of the line AB exceedeth the square of the line LX, and if the rest of the construction and demonstration be obserued in this that was in the former case, then shall the Probleme be finished.



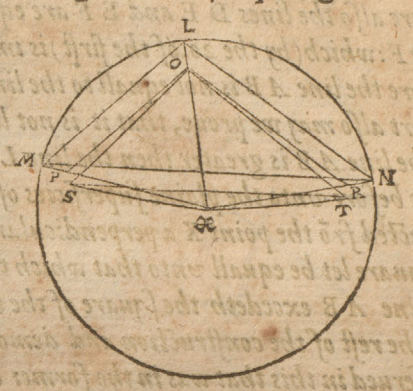
But now let the centre of the circle be without the triangle LMN, and let it be in the point X. And draw these right lines LX, MX, and NX. I say that in this case also the line AB is greater then the line LX. For if not, then is it either equall or lesse. First let it be e- Third case.



quall. Wherefore these two lines AB and BC are equall to these two lines MX and XL the one to the other, and the base AC is equall to the base ML. Wherefore (by the 8. of the first) the angle ABC is equall to the angle MXL: and by the same reason also the angle GHK is equall to the angle LXN. Wherefore the whole angle MXN is equall to these two angles ABC and GHK. But the angles ABC and GHK are greater then the angle DEF. Wherefore the angle MXN is greater then the angle DEF. And forasmuch as these two lines DE and EF are equall to these two lines MX and XN, and the base DF is equall to the base MN, therefore (by the 8. of the first) the angle MXN is equall to the angle DEF. And it is proued that it is also greater: which is impossible. Wherefore the line AB is not equall to the line LX. In like sort also may we proue, that it is not lesse. Wherfore the line AB is greater then the line LX. And againe if vnto the plaine superficies of the circle be erected perpendicularly from the point X a line XR, whose square is equall to that which the square of the line AB exceedeth the square of the line LX, and the rest of the construction be done in this that was in the former cases, then shall the Probleme be finished.

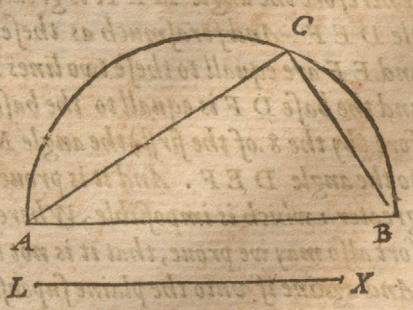
I say moreouer, that the line AB is not lesse then the line LX. For if it be possible, let it be lesse. And vnto the line AB, put (by the 2. of the first) the line XO equall: and vnto the line BC put the line XP equall: And draw a right line from the point O to the point P. An other demōstratiō to proue that the line AB is not lesse thē the line LX.

And forasmuch as the line A B is equall to the line B C, therefore the line X O is equall to the line X P. Wherefore the residue O L is equall to the residue M P. Wherefore (by the 2. of the sixt) the line L M, is a parallel to the line P O. And the triangle L X M, is equiangle to the triangle P X O. Wherefore (by the 6. of the sixt) as the line L X is to the line L M, so is the line X O to the line P O. Wherefore alternately (by the 16. of the fift) as the line L X is to the line X O, so is the line L M to the line O P. But the line L X is greater then the line X O. Wherefore also the line L M is greater then the line O P. But the line L M is equall to the line A C. Wherefore also the line A C is greater then the line O P. Now forasmuch as these two lines A B and B C are equall to these two lines O X & X P the one to the other, and the base A C is greater then the base O P, therefore (by the 25. of the first) the angle A B C is greater then the angle O X P. And in like sort if we put the line X R equall to either of these lines X O or X P, and draw a right line frō the point O to the point R, we may proue that the angle G H K is greater then the angle O X R. Vnto the right line L X, and vnto the point in it X, make (by the 23. of the first) vnto the angle A B C an equall angle L X S: and vnto the angle G H K make an equall angle L X T. And (by the second of the first) let either of these lines S X, and X T, be equall to the line O X. And drawe these lines O S, O T, and S T. And forasmuch as these two lines A B and B C are equall to these two lines O X and X S, and the angle A B C is equall to the angle O X S, therefore (by the 4. of the first) the base A C, that is, L M, is equall to the base O S. And by the same reason also the line L N is equall to the line O T. And forasmuch as these two lines L M and L N are equall to these two lines S O and O T, and the angle M L N is greater then the angle S O T, therefore (by the 25. of the first) the base M N is greater then the base S T. But the line M N is equall to the line D F. Wherefore the line D F is greater then the line S T. Now forasmuch as these two lines D E and E F are equall to these two lines S X and X T, and the base D F is greater then the base S T, therefore (by the 25. of the first) the angle D E F is greater then the angle S X T. But the angle S X T is equall to the angles A B C and G H K. Wherefore also the angle D E F is greater then the angles A B C and G H K. But it is also lesse: which is impossible.

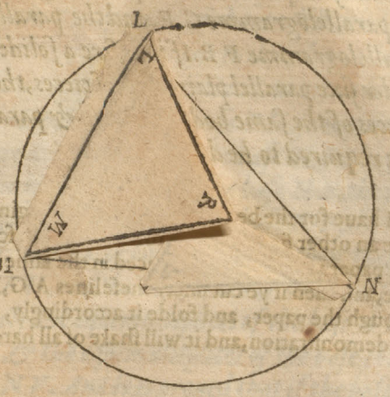


This was before taught in the tenth booke in the assumpt put before the 14. proposition.

But now let vs declare how to finde out the line X R, *whose square shall be equall to that which the square of the line* A B *exceedeth the square of the line* L X. *Take the two right lines* A B *and* L X, *and let* A B *be the greater, and vpon* A B *describe a semicircle* A C B, *and fron the poynt* A *apply into the semicircle a right line* A C *equall to the right line* L X: *and draw a right line from the poynt* C *to the poynt* B. *And forasmuch as in the semicircle* A C B *is an angle* A C B, *therefore (by the 31. of the third) the angle* A C B *is a right angle. Wherefore (by the 47. of the first) the square of the line* A B *is equall to the squares of the lines* A C *and* C B: *wherefore the square of the line* A B *is in power more then the square of the line* A C *by the square of the line* C B: *but the line* A C *is equall to the line* L X, *wherefore the square of the line* A B *is in power more then the square of the line* L X *by the square of the line* C B. *If therefore vnto the line* C B *we make the line* X R *equall, then is the square of the line* A B *greater then the square of the line* L X *by the square of the line* X R: *which was required to be doone.*



In this figure may ye more fully see the construction and demonstration of the first case of the former 23. Propositiō, if ye erect perpendicularly the triangle x R N, and vnto it bend the triangle L M R, that the angles R of eche may ioyne together in the point R. And so fully vnderstanding this case, the other cases will not be hard to conceaue.



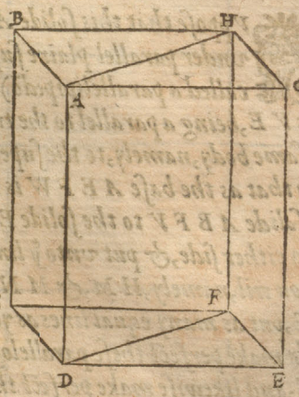
¶ The 21. Theoreme. The 24. Proposition.

*If a solide or body be contayned vnder * sixe parallel playne superficieces, the opposite plaine superficieces of the same body are equall and parallelogrammes.*

* M. Dee (to auoide cauillation) addeth to Euclides proposition this worde sixe: whome I haue followed accordingly, and not Zamberts, in this.

This kinde of body mencioned in the proposition is called a Parallelipipedō according to the diffinition before geuen thereof.

*Supposse that this solide body C D H G be contained vnder these 6. parallel plaine superficieces, namely, A C, G F, B G, C E, F B, and A E. Then I say that the opposite superficieces of the same body, are equal and parallelogrāmes, it is to wete, the two opposites A C and G F, and the two opposites B G and C E, and the two opposites F B and A E to be equall, and al to be parallelogrammes. For forasmuch as two parallel plaine superficieces, that is, B G, and C E are deuided by the plaine superficies A C, their common sections are (by the 16. of the eleuenth) parallels. Wherfore the line A B is a parallel to the line C D. Againe forasmuch as two parallel plaine superficieces F B and A E are deuided by the plaine superficies A C their common sections are by the same proposition, parallels. Wherfore the lyne A D is a parallel to the line B C. And it is also proued, that the line A B is a parallel to the line D C. Wherfore the superficies A C is a parallelogramme. In like sort also may we proue, that euery one of these superficices C E, G F, B G, F B, and A E are parallelogrammes. Draw a right line from the point A, to the point H, and an other from the point D to the point F. Aud forasmuch as the line A B is proued a parallel to the line C D, and the lyne B H to the line C F, therfore these two right lines A B and B H touching the one the other, are parallels to these two right lines D C and C F touching also the one the other, and not being in one and the selfe same plaine superficies. Wherfore (by the 10. of the eleuenth) they comprehend equall angles. Wherfore the angle A B H is equall to the angle D C F. And forasmuch as these two lines A B and B H are * equall to these two lines D C and C F, and the angle A B H is proued equall to the angle D C F, therfore (by the 4. of the first) the base A H is equall to the base D F, and the triangle A B H is equall to the triangle D C F. And forasmuch as (by the 41. of the first) the parallelogramme B G is double to the triangle A B H, and the parallelogramme C E is also double to the triangle D C F, therfore the parallelogramme*

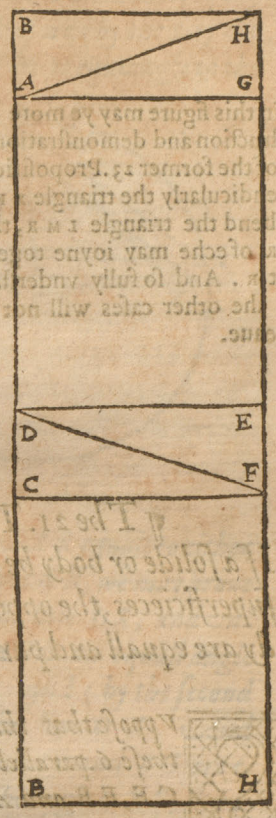


Demonstration that the opposite sides are parallelogrammes.

Demonstratiō that the opposite superficies are equall.

* A B is equall to D C, because the superficies A C is proued a parallelogrāme, and by the same reason, is B H equall to C F, because the superficies F B is proued a parallelogramme: therefore the 34. of the first is our proofe.

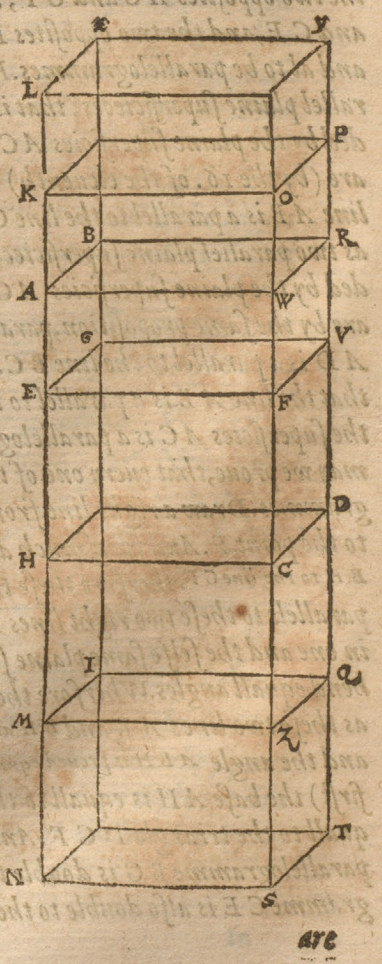
gramme B G is equall to the parallelogramme C E. In like ſort alſo may we proue that the parallelogramme A C is equall to the parallelogramme G F, and the parallelograme A E to the parallelogramme F B. If therfore a ſolide or body be contained vnder ſixe parallel plaine ſuperficieces, the oppoſite plaine ſuperficieces of the ſame body are equal & parallelogrammes: which was required to be demonſtrated.



I haue for the better helpe of young beginners, deſcribed here an other figure whoſe forme if it be deſcribed vpon paſted paper with the letters placed in the ſame order that it is here, and then if ye cut finely theſe lines A G, D E and C F not through the paper, and folde it accordingly, compare it with the demonſtration, and it will ſhake of all hardenes from it.

The 22. Theoreme. The 25. Propoſition.

If a Parallelipipedō be cutte of a playne ſuperficies beyng a parallel to the two oppoſite playne ſuperficieces of the ſame body: then, as the baſe is to the baſe, ſo is the one ſolide to the other ſolide.



Suppoſe that this ſolide A B C D being contained vnder parallel plaine ſuperficieces (and therfore called a parallelipipedō) be cut of the plaine ſuperficies V E, being a parallel to the two oppoſite ſuperficieces of the ſame body, namely, to the ſuperficieces A R & D H. Thē I ſay that as the baſe A E F W is to the baſe E H C F, ſo is the ſolide A B F V to the ſolide E G C D. Extēd the line A H on either ſide, & put vnto ey line E H as many equal lines as you wil, namely, H M, & M N: & likewiſe vnto the line A E, put as many equal lines as you will, namely, A K & K L, & make perfect theſe parallelogrāmes L O, K W, H Z, & M S, and likewiſe make perfect theſe ſolides or bodies L P, K R, D M, and M T. And foraſmuch as theſe right lines, L K, K A, and A E are equall the one to the other, therfore theſe parallelogrammes L O, K W, and A F, are alſo (by the firſt of the ſixt) equall the one to the other: and ſo alſo (by the ſame) are theſe parallelogrammes K X, K B, and A G equall the one to the other. And likewiſe (by the 24. of the eleuenth) are the parallelogrammes L Y, K P, and A R, equal, for they are oppoſite the one to the other. And by the ſame reaſon alſo the parallelogrammes E C, H Z, and M S are equall the one to the other, and the parallelogrammes H G, H I, and I N are equal the one to the other. And moreuer the parallelogrammes D H, M Q, and N T are (by the 24. of the eleuenth) equall the one to the other, for they

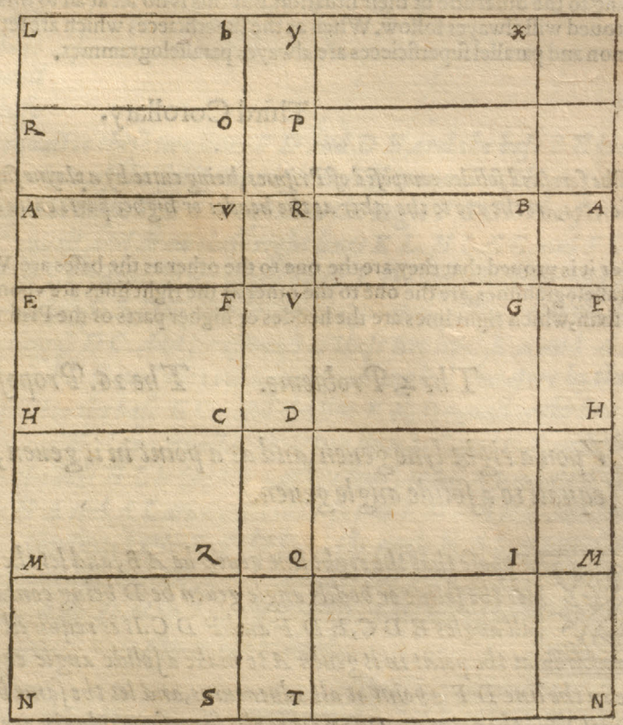
Conſtruction.

Demonſtration.

are

are oppoſite: wherefore three plaine ſuperficies of the ſolides L P, R K, and A V are equall to three plaine ſuperficies: but vnto eche of theſe three ſuperficieces are equall the three oppoſite ſuperficieces (by the 24. of the eleuẽth) Wherfore theſe three ſolides or bodies L P, K R, and A V, are equal the one to the other, by the 8. definition of the eleuẽth. And by the ſame reaſon alſo the three ſolides, E D, D M & M T are equal the one to the other. Wherfore how multiple x the baſe L F is, to the baſe A F, ſo multiplex is the ſolide L V to the ſolide A V. And by the ſame reaſon alſo how multiplex the baſe N F is to the baſe F H, ſo multiplex is the ſolide N V to the ſolide H V: ſo that if the baſe L F be equall to the baſe N F, the ſolide alſo L V ſhall be equall to the ſolide V N, and if the baſe L F exceede the baſe N F, the ſolide alſo L V ſhall exceede the ſolide V N, and if the baſe L F be leſſe then the baſe N F, the ſolide alſo L V ſhall be leſſe then the ſolide V N (by the 1. and 14. of the fift.) Now then there are foure magnitudes, namely, the two baſes A F and F H, and the two ſolides or bodies A V and V H, of which are take their equemultiplices, namely, the equemultiplices of the baſe A F, & of the ſolide A V, or the baſe L F, & the ſolide L V, & the equemultiplices of the baſe H F, & of the ſolide H V are the baſe N F, and the ſolide N V. And it is proued that if the baſe L F excede the baſe N F, the ſolide alſo L V excedeth the ſolide N V, & if it be equal, it is equal, and if it be leſſe, it is leſſe. Wherfore (by the 6. definitiõ of the fift) as the baſe A F is to the baſe F H, ſo is the ſolide A V to the ſolide H V. If therfore a parallelipipedon be cut of a playne ſuperficies being a parallel to the two oppoſite playne ſuperficieces of the ſame body, then, as the baſe is to the baſe, ſo is the one ſolide to the other ſolide: which was required to be proued.

I haue for the better ſight of the cõſtructiõ & demõſtration of the former 25. propoſitiõ, here ſet another figure, whoſe forme if ye deſcribe vpon paſted paper, and finely cut the three lines X I, B S, and T Y, not through the paper but halfe way, and then fold it accordingly, and compare it with the conſtruction and demõſtration, you ſhall ſee that it will geue great light therunto.



Here *Flußas* addeth three Corollaries.

First Corollary.

If a Priſme be cutte of a playne ſuperficies parallel to the oppoſite ſuperficieces, the ſections of the Priſme ſhall be the one to the other in that proportion, that the ſections of the baſe are the one to the other.

First Corollary.

For

For the ſections of the baſes, which baſes (by the 11. definitiō of this booke) are parallelogrammes, ſhall likewiſe be parallelogrammes, by the 16. of this booke (when as the ſuperficies which cutteth is parallelel to the oppoſite ſuperficieces) and ſhal alſo be equiangle. Wherfore if vnto the baſes (by producing the right lines) be added like and equall baſes, as was before ſhewed in a parallelipipedon, of thoſe ſections ſhalbe made as many like Priſmes as ye will. And ſo by the ſame reaſon, namely, by the commō exceſſe, equalitie, or want of the multiplices of the baſes & of the ſections by the 5. definitiō of the fifth may be proued, that euery ſection of the Priſme multiplyed by any multiplycation whatſoeuer, ſhall haue to any other ſection that proportion that the ſections of the baſes haue.

Second Corollary.

Second Corollary. These ſolides which he ſpeaketh of in this Corollary are of ſome called ſolid columnes.

Solides whoſe two oppoſite ſuperficieces are poligonon figures like equall and parallels, the other ſuperficies, which of neceſſitie are parallelogrammes, being cutte of a playne ſuperficies parallel to the two oppoſite ſuperficies: the ſections of the baſe are the one to the other, as the ſections of the ſolide are the one to the other.

Which thing is manifeſt, for ſuch ſolides are deuided into Priſmes, which haue one cōmon ſide, namely, the axe or right line, which is drawne by the centers of the oppoſite baſes. Wherefore how many parallelogrāmes or baſes are ſet vpon the oppoſite poligonon figures, of ſo many Priſmes ſhal the whole ſolide be cōpoſed. For thoſe poligonon figures are deuided into ſo many like triangles by the 20. of the ſixth, which deſcribe Priſmes. Which Priſmes being cut by a ſuperficies parallel to the oppoſite ſuperficieces, the ſectiōs of the baſes ſhal (by the former Corollary) be proportional with the ſectiōs of the Priſmes. Wherefore by the 12. of the fifth, as the ſections of the one are the one to the other, ſo are the ſections of the whole the one to the other.

Of theſe ſolides there are infinite kindes, according to the varietie of the oppoſite and parallel poligonon figures, which poligonon figures doo alter the angles of the parallelogrammes ſet vpon them according to the diuerſitie of their ſituation. But this is no let at all to this corollary, for that which we haue proued will alwayes follow. When as the ſuperficieces which are ſet vpō the oppoſite like & equal poligonon and parallel ſuperficieces are alwayes parallelogrammes.

Third Corollary.

Third Corollary.

The foreſayd ſolides compoſed of Priſmes, being cutte by a playne ſuperficies parallel to the oppoſite ſuperficieces, are the one to the other as the heades or higher parts cutte are.

For it is proued that they are the one to the other as the baſes are. Which baſes foraſmuch as they are parallelogrammes, are the one to the other as the right lines are vpon which they are ſet by, the firſt of the ſixth, which right lines are the heddes or higher parts of the Priſmes.

The 4. Probleme. The 26. Propoſition.

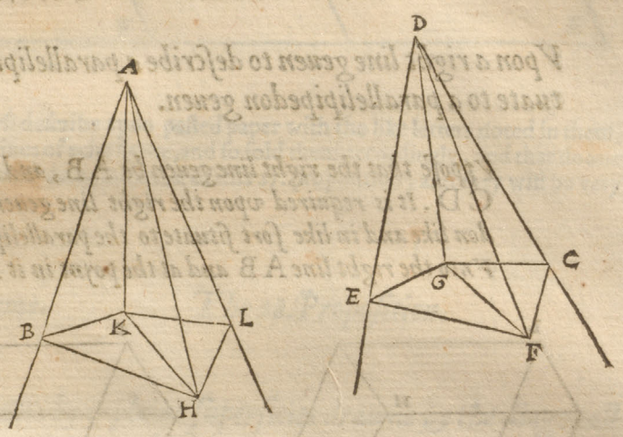
Vpon a right lyne geuen, and at a point in it geuen, to make a ſolide angle equall to a ſolide angle geuen.

Conſtruction.

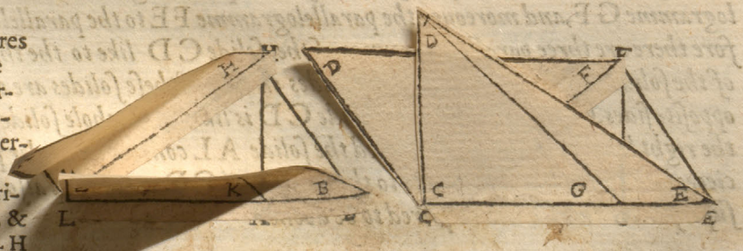
Vppoſe that the right line geuen be A B, and let the point in it geuen be A, and let the ſolide or bodily angle geuen be D being contained vnder theſe ſuperficiall angles E D C, E D F and F D C. It is required vpon the right line A B, & at the point in it geuen A to make a ſolide angle equall to the ſolide angle D. Take in the line D F a point at all aduentures, and let the ſame be F. And (by the 11. of the eleuenth) frō the point F. Draw vnto the ſuperficies wherin are the lines E D & D C a perpendicular line F G, and let it fall vpon the plaine ſuperficies in the point G, & draw a right line from the point D to the point G. And (by the 23. of the firſt) vnto the line A B, and at the point A make vnto the angle E D C an equall angle B A L, and vnto the angle E D G put the angle B A K equall: and by the 2. of the firſt, put the line A K equall to the line D G, and (by the 12. of the eleuenth) from the point K raiſe vp vnto the plaine ſuperficies B A L a perpendicular line K H, and put the line K H equall to the line G F, and draw a right lyne from

from the point H to the point A. Now I say that the solide angle A contained vnder the superficiall angles B A L, B A H, and H A L is equall to the solide angle D contained vnder the superficiall angles E D C, E D F, and F D C. Let the the lines A B and D E be put equall, and draw these right lines H B, B K, F E, and E G. And forasmuch as the line F G is erected perpendicularly to the ground superficies, therfore by the 2. definition of the eleuenth, the line F G is also erected perpendicularly to all the right lines that are in the ground superficies and touche it. Wherfore either of these angles F G D and F G E is a right angle, and by the same reason also either of the angles H K A and H K B is a right angle. And forasmuch as these two lines K A & A B are equall to these two lines G D & D E, the one to the other, and they containe equall angles (by construction). Wherfore (by the 4. of the first) the base K B is equall to the base E G, and the line K H is equall to the line G F, and they cõpreh̃ed right angles. Wherfore the line B H is equall to the line F E. Agayne, forasmuch as these two lines A K and K H are equal to these two lines D G and G F, and they containe right angles. Wherfore ỹ base A H is (by the 4. of the first) equall to the base D F. And the line AB is equall to the line D E. Wherfore these two lines A B and A H are equall to these two lines F D and D E, and the base B H is equall to the base F E. Wherfore (by the 8. of the first) the angle B A H is equall to the angle EDF. And by the same reason also the angle H K L is equall to the angle F G C. Wherfore if we put these lines A L and D C equall, and draw these right lines K L, H L, G C, and F C: forasmuch as the whole angle B A L is equall to the whole angle E D C, of which the angle B A K is supposed to be equall to the angle E D G, therfore the angle remayning, namely, K A L is equall to the angle remayning G D C. And forasmuch as these two lines K A and A L are equall to these two lines G D and D C, and they containe equall angles, therefore by the 4. of the first, the base K L is equall to the base G C, and the line K H is equall to the line G F, wherfore these two lines L K and K H are equall to these two lines C G and G F, and they cõtaine right angles. Wherfore the base H L is (by the 4. of the first) equal to the base F C. And forasmuch as these two lines H A and A L are equall to these two F D and D C, and the base H L is equall to the base F C, therfore (by the 8. of the first) the angle H A L is equall to the angle F D C, and by construction, the angle B A L is equall to the angle E D C. Wherefore vnto the right line geuen, and at the point in it geuen, namely, A, is made a solide angle equal to the solide angle geuen D: which was required to be done.

Demonstration.



In these two figures here put, you may more clearely conceiue the former construction and demonstratiõ, if ye erect perpendicularly vnto the ground superficies the triangles A L B and D C E, & eleuate the triangles A L H and D C F that the lynes



L A and C D of them may exactly agree with the lines L A and C D of the triangles erected. For so ordering them, if ye compare the former construction and demonstration with them, they will be playne vnto you.

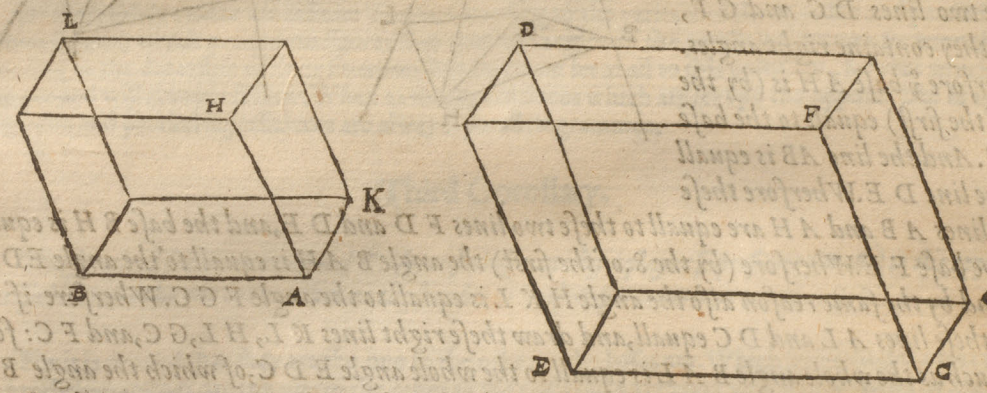
Although Euclides demõstration be touching solide angles which are contained only vnder three superficiall angles, that is, whose bases are triangles: yet by it may ye performe the Probleme touching solide angles contained vnder superficiall angles how many soeuer, that is, hauing to their bases any kinde of Poligonon figures. For euery Poligonon figure may by the 20. of the sixt, be resolued into like tringles. And so also shall the solide angle be deuided into so many solide angles as there be triangles in the base. Vnto euery one of which solide angles you may by this proposition describe an equall solide angle vpon a line geuen, and at a point in it geuen. And so at the length the whole solide angle after this maner described shall be equall to the solide angle geuen.

The 5. Theoreme. The 27. Proposition.

Vpon a right line geuen to describe a parallelipipedon like and in like sort situate to a parallelipipedon geuen.

Vppose that the right line geuen be A B, *and let the parallelipipedon geuen be* C D. *It is required vpon the right line geuen A B to describe a parallelipipedon like and in like sort situate to the parallelipipedon geuen, namely, to* C D.

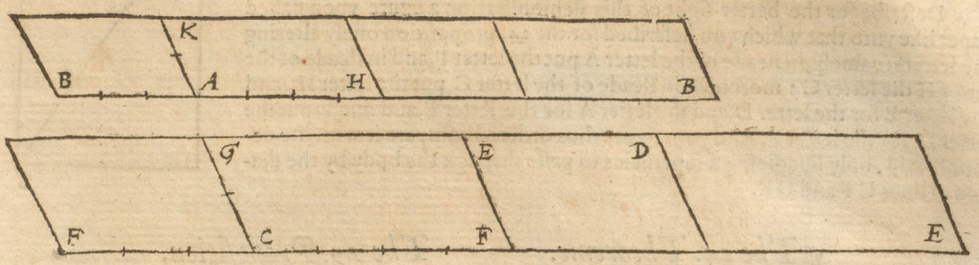
Construction. *Vnto the right line* A B *and at the poynt in it* A *describe (by the 26. of the eleuenth (a solide angle equall to the solide angle* C, *and let it be contayned vnder these superficiall angles* B A H, H A K, *and* K A B, *so that let the angle* B A H *be equall to the angle* E C F, *and the angle* B A K *to the angle* E C G, *and moreouer the angle* K A H *to the angle* G C F. *And as the line* E C *is to the line* C G, *so let the* A B *be to the line* A K *(by the 12. of the sixth) and as the line* G C *is to the line* C F, *so let the* K A *be to the line* A H.



Demonstration. *Wherefore of equalitie (by the 22. of the fift) as the line* E C *is to the line* C F, *so is the line* B A *to the line* A H. *Make perfect the parallelogramme* B H, *and also the solide* A L. *Now for that as the line* E C *is to the line* C G, *so is the line* B A *to the line* A K, *therefore the sides which contayne the equall angles, namely, the angles* E C G *and* B A K *are proportionall: wherefore (by the first definition of the sixth) the parallelogramme* G E *is like to the parallelogramme* K B. *And by the same reason the parallelogramme* K H *is like to the parallelogramme* G F, *and moreouer the parallelogramme* F E *to the parallelogramme* H B. *Wherfore there are three parallelogrammes of the solide* C D *like to the three parallelogrammes of the solide* A L. *But the three other sides in eche of these solides are equall and like to their opposite sides. Wherefore the whole solide* C D *is like to the whole solide* A L. *Wherfore vpon the right line geuen* A B *is described the solide* A L *contayned vnder parallel playne superficieces like and in like sort situate to the solide geuẽ* C D *contayned also vnder parallel playne superficieces: which was required to be doone.*

This demonstration is not hard to conceaue by the former figure as it is described in a playne, if ye haue

that imagination of parallelipipedons described in a playne which we before taught in the diffinition of a cube. Howbeit I haue here for the more ease of such as are not yet acquainted with solides, des-

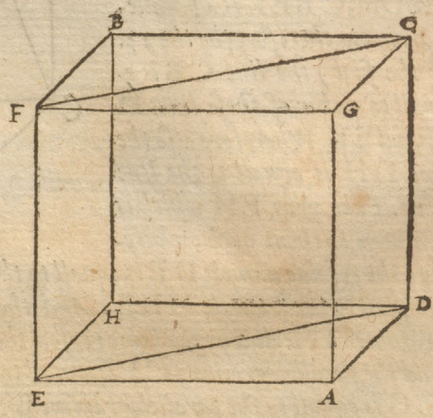


cribed two figures, whose formes first describe vpon pasted paper with the like letters noted in them, and then finely cutte the three midle lines of eche figure, and so fold them accordingly, and that doone compare them with the construction and demonstration of this 27. proposition, and they will be very easy to conceaue.

The 23. Theoreme. *The 28. Proposition.*

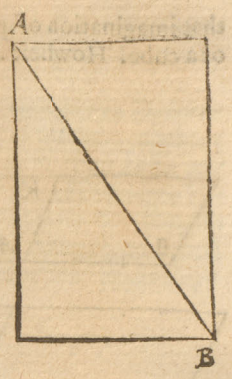
If a parallelipipedō be cutte by a plaine superficies drawne by the diagonall lines of those playne superficieces which are opposite: that solide is by this playne superficies cutte into two equall partes.

Suppose that the parallelipipedon AB *be cutte by the playne superficies* CDEF *drawne by the diagonall lines of ỹ plaine superficieces which are opposite, namely, of the superficieces* CF *and* DE. *Then I say that the parallelipipedon* AB *is cutte into two equall partes by the superficies* CDEF. *For forasmuch as (by the 34. of the first) the triangle* CGF *is equall to the triangle* CBF, *and the triangle* ADE *to the triangle* DEH, *and the parallelograme* CA *is equall to the parallelogramme* BE, *for they are opposite, and the parallelogramme* GE *is also equall to the parallelogramme* CH, *and the parallelogramme* CE, *is the common section by supposition: Wherfore the prisme contained vnder the two triangles* CGF, *and* DAE, *and vnder the three parallelogrammes* GE, AC, *and* CE *is (by the 8. definition of the eleuenth) equall to the prisme contayned vnder the two triangles* CFB *and* DEH *and vnder the three parallelogrammes* CH, BE, *and* CE. *For they are cōtayned vnder playne superficieces equall both in multitude and in magnitude. Wherefore the whole parallelipipedon* AB *is cutte into two equall partes by the playne superficies* CDEF: *which was required to be demonstrated.*



Demonstration.

A diagonall line is a right line which in angular figures is drawne from one angle and extended to his contrary angle as you see in the figure AB.



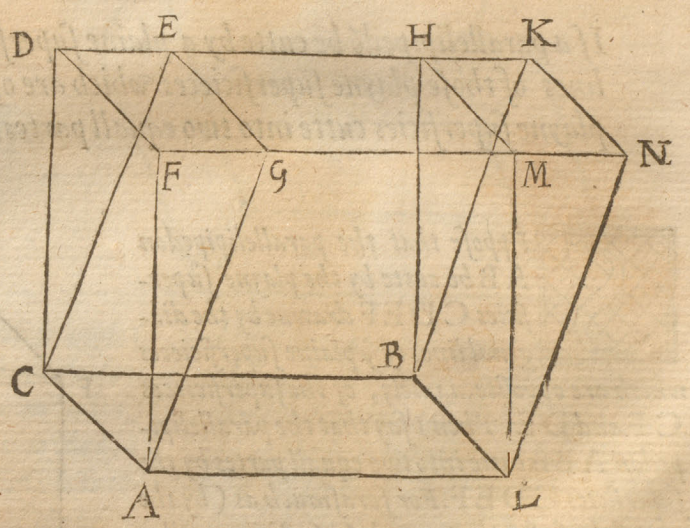
Describe for the better sight of this demonstration a figure vpon pasted paper like vnto that which you described for the 24. proposition onely altering the letters: namely, in steade of the letter A put the letter F, and in steade of the letter H the letter C: moreouer in steade of the letter C put the letter H, and the letter E for the letter D, and the letter A for the letter E, and finally put the letter D for the letter F. And your figure thus ordered compare it with the demonstratiõ, only imagining a superficies to passe through the body by the diagonall lines C F and D E.

¶ The 24. Theoreme. The 29. Proposition.

Parallelipipedons consisting vpon one and the selfe same base, and vnder one and the selfe same altitude, whose standing lines are in the selfe same right lines, are equall the one to the other.*

* *Looke at the end of the demonstration what is vnderstanded by stãding lines.*

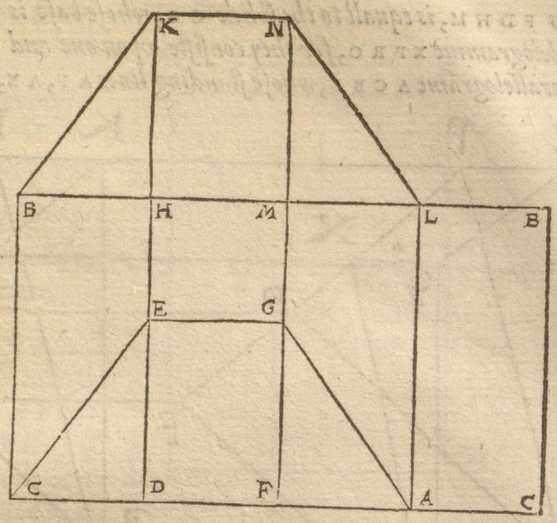
Vppose that that these parallelipipedons C M *and* C N *doo consist vpon one and the selfe same base, namely* A B, *and let them be vnder one and the selfe same altitude, whose standing lines, that is, the fower sides of eche solide which fall vpon the base, as the lines* A F, C D, B H, L M *of the solide* C M, *and the lines* C E, B K, A G, *and* L N *of the solide* C N, *let be in the selfe same right lines or parallel lines* F N, D K. *Then I say that the solide* C M *is equall to the solide* C N. *For forasmuch as either of these superficieces* C B D H, C B E K *is a parallelogramme, therefore (by the 34. of the first) the line* C B *is equall to either of these lines* D H *and* E K. *Wherefore also the line* D H *is equall to the line* E K. *Take away* E H *which is common to them both, wherefore the residue namely* D E *is equall to the residue* H K. *Wherfore also the triangle* D C E *is equall to the triangle* H K B. *And the parallelogramme* D G *is equall to the parallelogramme* H N. *And by the same reason the triangle* A G F *is equall to the triangle* M L N, *and the parallelogramme* C F *is equall to the parallelogramme* B M. *But the parallelogrãme* C G *is equall to the parallelogramme* B N, *by the 24. of the tenth for they are opposite the one to the other. Wherefore the prisme contayned vnder the two triangles* F A G *and* D C E *and vnder the three parrallelogrãmes* A D, D G, *and* C G *is equall to the prisme cõtayned vnder the two triangles* M L N *and* H B K, *and vnder the three parallelogrãmes, that is,* B M, N H, *and* B N. *Put that solide common to them both, whose base is the parallelogrãme* A B, *and the opposite side vnto the base is the superficies* G E H M. *Wherefore the whole parallelipipedon* C M *is equall to the whole parallelipipedon* C N. *Wherfore parallelipipedons consisting vpon one and the selfe same base, and vnder one and the selfe same altitude, whose standing lines are in the selfe same right lines, are equall the one to the other: which was required to be demonstrated.*



Iohn Dee his figure. By this figure it appeareth why such Prismes were called wedges: of the very shape of a wedge, as is the solide D E F G A C, &c.

Although

Although this demonſtration be not hard to a good imaginati-on to conceaue by the former figure (which yet by *M. Dees* reforming is much better then the figure of this propoſition commonly deſcribed in other copyes both greake and lattin): yet for the eaſe of thoſe which are young beginners in thys matter of ſolides, I haue here ſet an other figure whoſe forme if it be deſcribed vpon paſted paper, with the like letters to euery line as they be here put, and then if ye finely cut not thorough but as it were halfe way the three lines L A, N M G F, and K H E D, & ſo folde it accordingly, & compare it with the demonſtratiō, it will geue great light thereunto.



Stāding lines are called thoſe fower right lines of euery parallelipipedon which ioyne together the angles of the vpper and nether baſes of the ſame body. Which according to the diuerſitie of the angles of the ſolides, may either be perpendicular vpon the baſe, or fall obliquely. And foraſmuch as in thys propoſition and in the next propoſition following, the ſolides compared together are ſuppoſed to haue one and the ſelfe ſame baſe, it is manifeſt that the ſtanding lines are in reſpect of the lower baſe in the ſelfe ſame parallel lines, namely, in the two ſides of the lower baſe. But becauſe there are put two ſolides vpon one and the ſelfe ſame baſe, and vnder one and the ſelfe ſame altitude, the two vpper baſes of the ſolides may be diuerſly placed. For foraſmuch as they are equall and like (by the 24. of this booke) either they may be placed betwene the ſelfe ſame parallel lines: and thē the ſtanding lines are in either ſolide ſayd to be in the ſelfe ſame parallel lines, or right lines: namely, when the two ſides of eche of the vpper baſes are contayned in the ſelfe ſame parallel lines: but contrariwiſe if thoſe two ſides of the vpper baſes be not contayned in the ſelfe ſame parallel or right lines, neither ſhal the ſtanding lines which are ioyned to thoſe ſides be ſayd to be in the ſelfe ſame parallel or right lines. And therefore the ſtanding lines are ſayd to be in the ſelfe ſame right lines, when the ſides of the vpper baſes, at the leaſt two of the ſides are contayned in the ſelfe ſame right lines: which thing is required in the ſuppoſition of this 29. propoſition. But the ſtanding lines are ſayd not to be in the ſelfe ſame right lines, when none of the two ſides of the vpper baſes are contayned in the ſelfe ſame right lines, which thing the next propoſition following ſuppoſeth. *Stāding lines.*

¶ *The 25. Theoreme.* *The 30. Propoſition.*

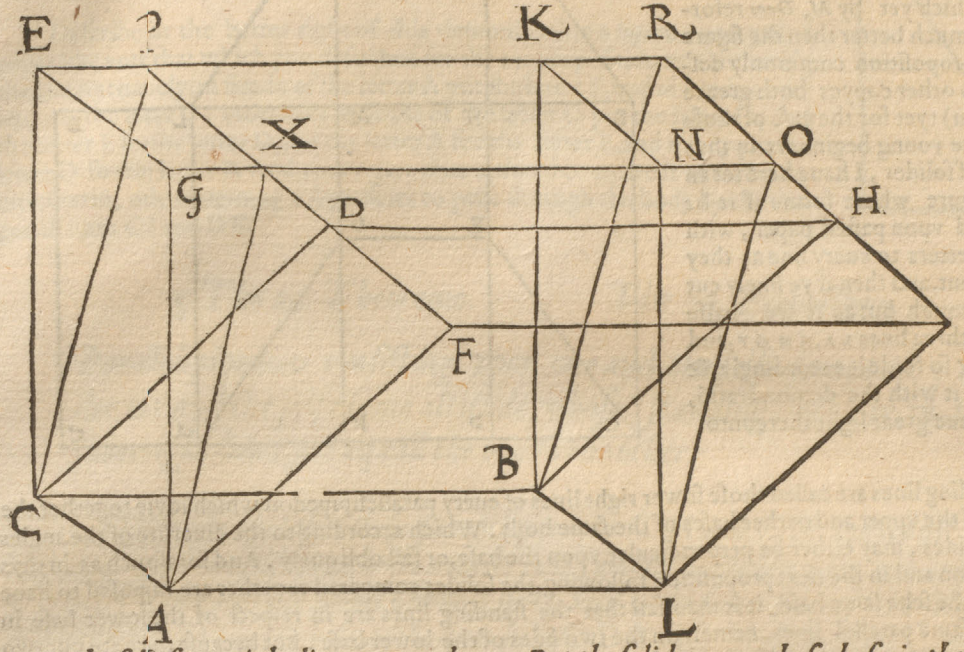
Parallelipipedons conſiſting vpon one and the ſelfe ſame baſe, and vnder the ſelfe ſame altitude, whoſe ſtanding lines are not in the ſelfe ſame right lines, are equall the one to the other.

Vppoſe that theſe Parallelipipedons C M *and* C N, *do conſiſt vpon one and the ſelfe ſame baſe, namely,* A B, *and vnder one and the ſelfe ſame altitude, whoſe ſtanding lines, namely, the lines* A F, C D, B H, *and* L M, *of the Parallelipipedon* C M, *and the ſtanding lines* A G, C E, B K, *and* L N, *of the Parallelipipedon* C N, *let not be in the ſelfe ſame right lines. Then I ſay, that the Parallelipipedon* C M, *is equall to the Parallelipipedon* C N. *Foraſmuch as the vpper ſuperficieces* F H *&* G K, *of the former Parallelipipedons, are in one and the ſelfe ſame ſuperficies (by reaſon they are ſuppoſed to be of one and the ſelfe ſame altitude): Extend the lines* F D *and* M H, *till they concurre with the lines* N G *and* K E *(ſufficiently both wais extended: for in diuers caſes their concurſe is diuers). Let* F D *extended, meete with* N G, *and cut it in the point* X: *and with* K E *in the point* P. *Let likewiſe* M H *extended, meete with* N G *(ſufficiently produced) in the point* O, *and with* K E *in the point* R. *And drawe theſe right lines* A X, L O, C P, *and* B R. *Now (by the 29. of the eleuenth) the ſolide* C M, *whoſe baſe is the parallelogramme* A C B L, *and the parallelogramme oppoſite vn-* *Conſtruction.*

Demonſtration.

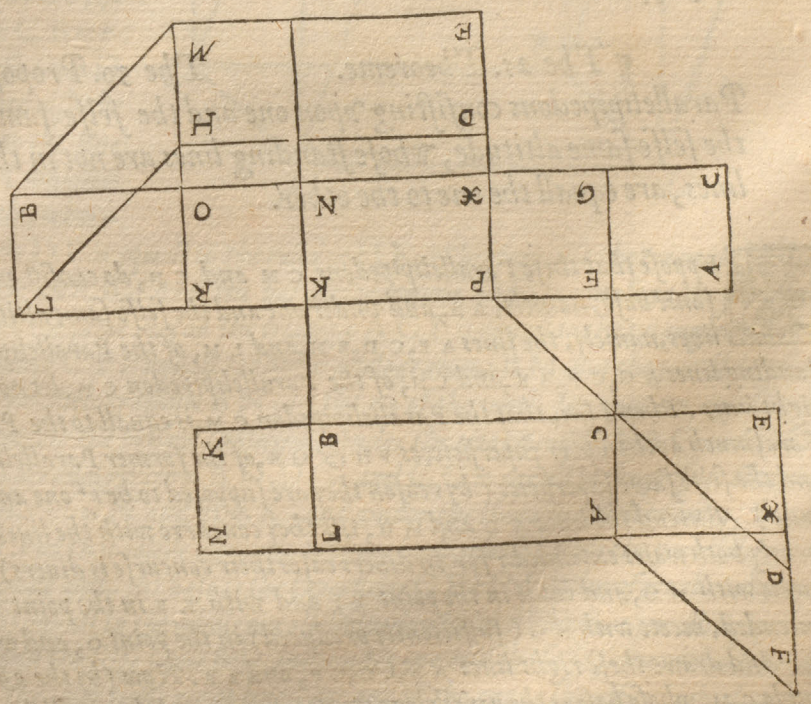
to it is FDHM, *is equall to the ſolide* CO, *whoſe baſe is* ACBL *and the oppoſite ſide the parallelogramme* XPRO, *for they conſiſte vpon one and the ſelfe ſame baſe, namely, vpon the parallelogrãme* ACBL, *whoſe ſtanding lines* AF, AX, LM, LO, CD, CP, BH, *and* BR,

I. Dees figure.



are in the ſelfe ſame right lines FP *and* MR. *But the ſolide* CO, *whoſe baſe is the parallelogramme* ACBL, *and the oppoſite ſuperficies vnto it is* XPRO, *is equall to the ſolide* CN, *whoſe baſe is he parallelogramme* ACBL, *and the oppoſite ſuperficies vnto it is the ſuperficies* GEKN. *for they are vpon one and the ſelfe ſame baſe, namely,* ACBL, *and their ſtanding lines* AG, AX, CE, CP, LN, LO, BK, *and* BR, *are in the ſelfe ſame right lines* NX, *and* PK. *Wherefore alſo the ſolide* CM, *is equall to the ſolide* CN. *Wherefore Parallelipipedons conſiſting vpon one and the ſelfe ſame baſe, and vnder the ſelfe ſame altitude, whoſe ſtanding lins are not in the ſelfe ſame right lines, are equall the one to the other: which was required to be proued.*

This demonſtration is ſomwhat harder then the former to conceaue by the figure deſcribed in the plaine (and yet before *M. Dee* inuented that figure which is placed for it, it was much harder) by reaon one ſolide is contained in an other. And therefore for the clerer light therof, deſcribe vpõ paſted paper this figure here put with the like letters and finely cut the lines AC, CB, EG, BL, EPK, ROHM, and folde it accordingly that euery line may exactly agree with his correſponden lyne (which obſeruing the letters of eury line



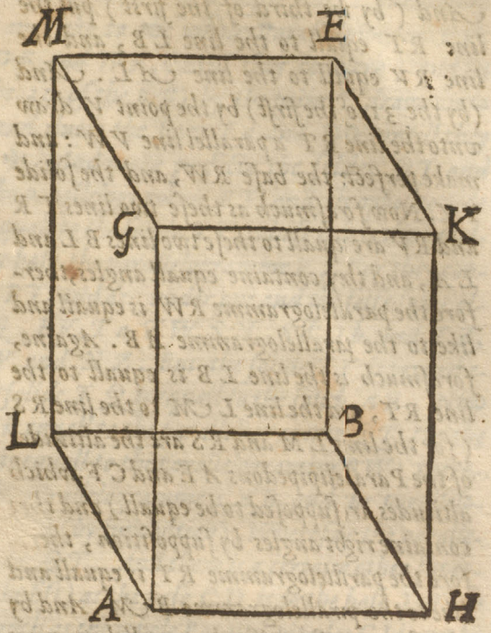
ye

ye ſhall eaſily do) and ſo cõpare your figure with the demonſtration, and it will make it very plaine vnto you.

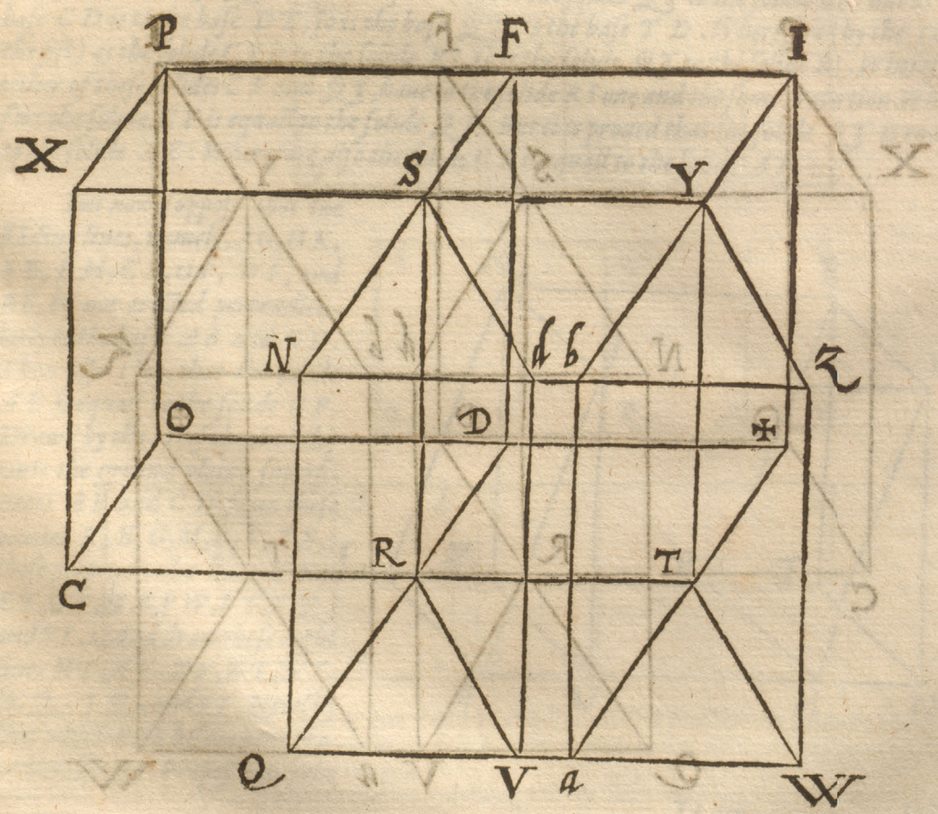
The 26. Theoreme. The 31. Propoſit.

Parallelipipedons conſiſting vpon equall baſes, and being vnder one and the ſelfe ſame altitude, are equall the one to the other.

VPpoſe that vppon theſe equall baſes A B and C D do conſiſte theſe parallelipipedons A E and C F, being vnder one & the ſelf ſame altitude. The I ſay, that the ſolide A E is equall to the ſolide C F. Firſt let the ſtanding lines, namely, H K, B E, A G, L M, O P, D F, C X, and R S, be erected perpendicularly to the baſes A B and C D, and let the angle ∠ A L B not be equall to the angle C R D. Extend the line C R to the point T. And (by the 23. of the firſt) vpon the right line R T, and at the point in it R, deſcribe vn-



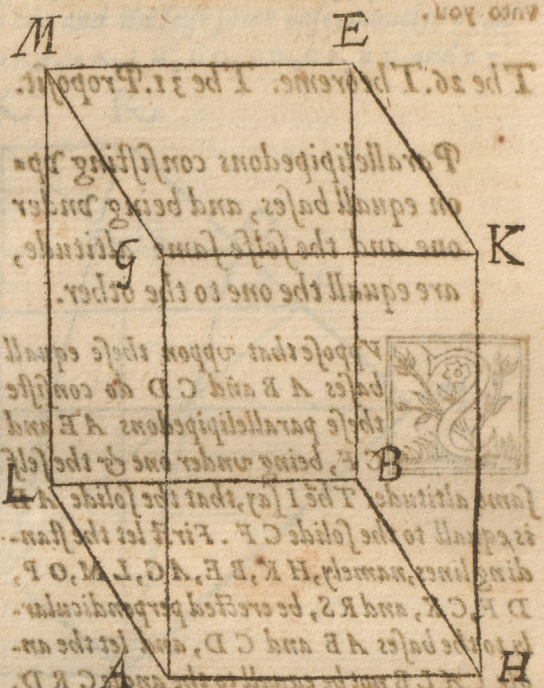
Two caſes in this propoſition.
The firſt caſe. Conſtruction.

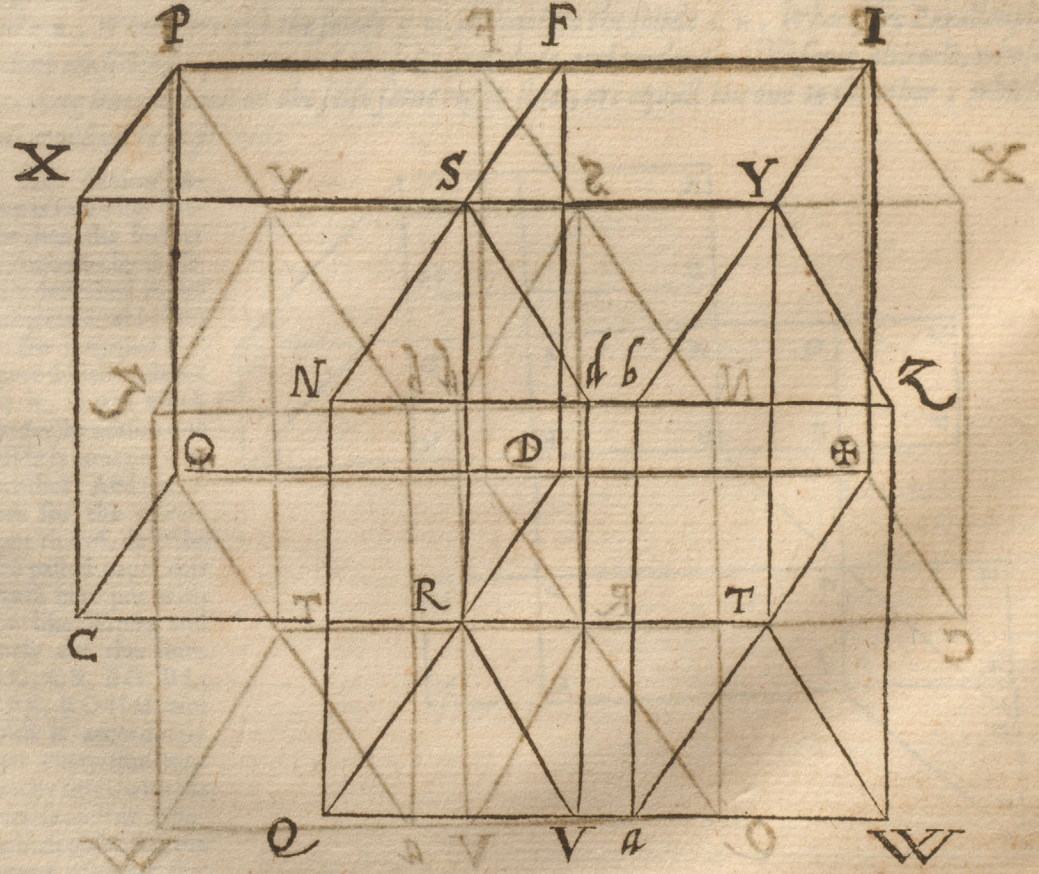


We are beholding to M. Dee for inuenting this figure, with other, which till his reforming were as much miſhappen as this was, and ſo both in the Greeke and Latine copies remaine.

to the angle ALB an equall angle TRV. And (by the third of the first) put the line RT equall to the line LB, and the line RV equall to the line AL. And (by the 31. of the first) by the point V draw vnto the line RT a parallel line VW: and make perfecte the base RW, and the solide TV. Now forasmuch as these two lines TR and RV are equall to these two lines BL and LA, and they containe equall angles, therfore the parallelogramme RW is equall and like to the parallelogramme AB. Againe, forasmuch as the line LB is equall to the line RT, and the line LM to the line RS (for the lines LM and RS are the altitudes of the Parallelipipedons AE and CF, which altitudes are supposed to be equall) and they containe right angles by supposition, therefore the parallelogramme RT is equall and like to the parallelogramme BM. And by the same reason also the parallelogramme LG is equall and like to the parallelogrãme SV. Wherefore three parallelogrammes of

Demonstration.



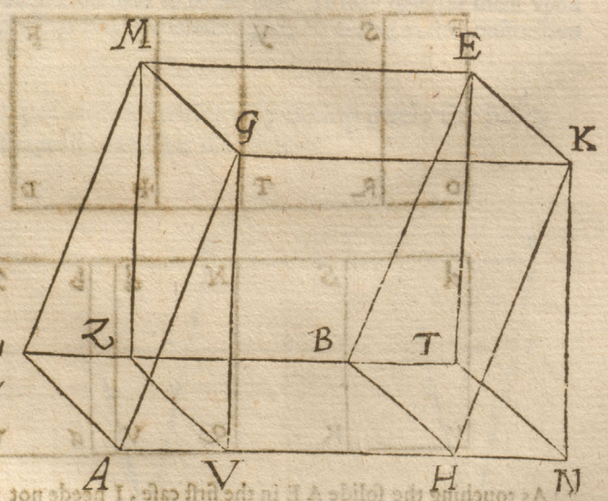


the solide AE are equall and like to the three parallelogrammes of the solide ϒV. But these three parallelogrammes are equall and like to the three opposite sides. Wherefore the whole Parallelipipedon AE is equall and like to the whole Parallelipipedon ϒV. Extend (by the second petition) the lines DR and WV, vntill they concurre, and let them concurre in the point Q. And (by the 31. of the first) by the point T drawe vnto the line RQ a parallel line Ta, and extend duely the lines Ta and DO vntill they concurre, and let them concurre in the point ✣. And make perfecte the solides QY and RI. Now the solide QY, whose * base is the parallelogramme RY, and the opposite side vnto the base the parallelogramme Qb, is equall to the solide ϒV, whose base is the parallelogramme RY, and the opposite side vnto the base the parallelogramme VZ. For they consiste vpon one and the selfe same base, namely, RY, and are vnder one and the selfe same altitude, and their standing lines, namely, RQ, RV, Ta, TW, SN, Sd, Yb, and YZ, are in the selfe same right lines, namely, QW, and NZ. But the solide ϒV is proued equall to the solide AE. Wherefore also the solide TQ is equall to the solide AE. Now forasmuch as the parallelogramme RVWT is equall to the parallelogramme QT (by the 35. of the first) and the parallelogramme AB is equall to the parallelogramme RW: therefore the parallelogramme QT is equall to the parallelogramme AB, and the parallelogramme CD is equall to the parallelogramme AB (by supposition). Wherefore the parallelogramme CD is equall to the parallelogramme QT. And there is a certaine other superficies, namely, DT. Wherefore (by the 7. of the fift) as the base CD is to the base DT, so is the base QT to the base DT. And forasmuch as the whole Parallelipipedon CI is cut by the plaine superficies RF, which is a parallel to either of the opposite plaine superficieces, therfore as the base CD is to the base DT, so is the solide CF to the solide RI (by the 25. of the eleuenth). And by the same reason also, forasmuch as the whole Parallelipipedon QI is cut by the plaine superficies RY, which is a parallel to either of the opposite plaine superficieces, therefore as the base QT is to the base DT, so is the solide QY to the solide RI. But as the base CD is to the base DT, so is the base QT to the base TD. Wherefore (by the 11. of the fift) as the solide CF is to the solide RI, so is the solide QY to the solide RI. Wherefore either of these solides CF and QY, haue to the solide RI one and the same proportion. Wherfore the solide CF is equall to the solide QY. But it is proued that the solide QY is equall to the solide AE. Wherefore also the solide CF is equall to the solide AE.

* Note now, how the base respectiuely is taken: for so may alteratiõ of respects alter the name of the bowndes eyther of solides or playnes.

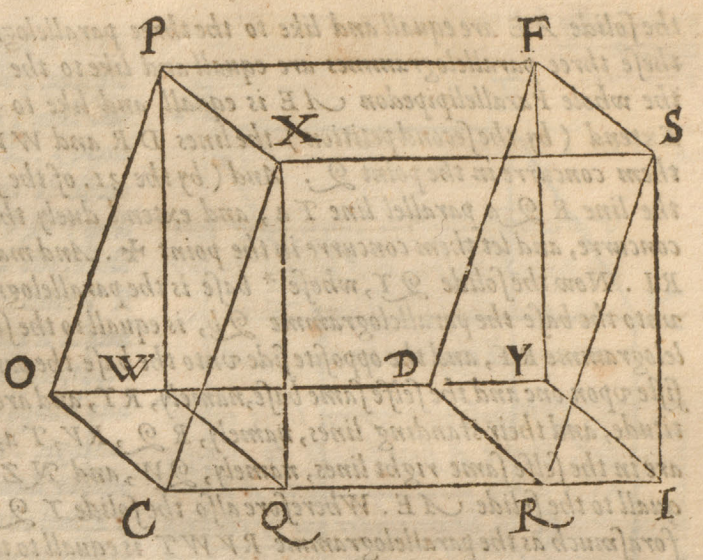
Second case.

Bnt now suppose that the stading lines, namely, AG, HK, BE, LM, CX, OP, DF, and RS, be not erected perpendicularly to the bases AB and CD. Then also I say, that the solide AE is equall to the solide CF. Draw (by the 11. of the eleuẽth) vnto the ground plaine superficieces AB and CD from these pointes K, E, G, M, P, F, X, S, these perpendicular lines KN, ET, GV, MZ, PW, FY, XQ, and SI. And draw these right lines NT, NV, ZV, ZT, WY, WQ, IQ, and IY. Now (by that which hath before bene proued in this 31. Proposition) the

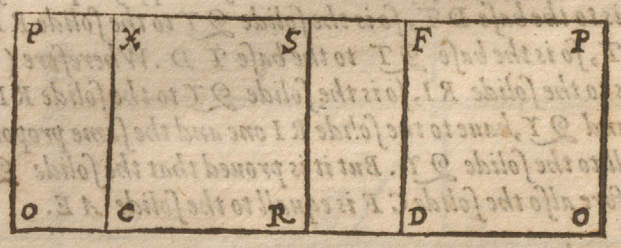


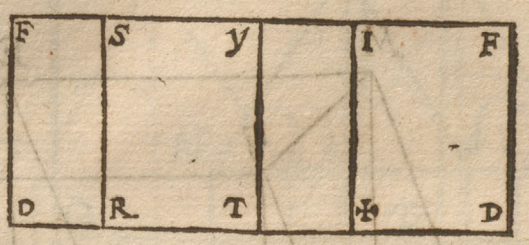
* There you perceaue how the base is diuersly considered & chosen: as before weaduertised you.

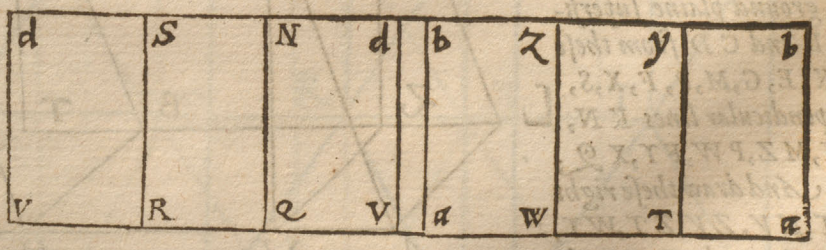
*ſolide K Z is equall to the ſolide P I, for they conſiſt vpon equall * baſes, namely, K M, and P S, and are vnder one and the ſelfe ſame altitude, whoſe ſtanding lines alſo are erected perpendicularly to the baſes. But the ſolide K Z is equall to the ſolide A E (by the 29. of the eleuenth): and the ſolide P I is (by the ſame) equall to the ſolide C F, for they conſiſt vppon one and the ſelfe ſame baſe, and are vnder one & the ſelfe ſame altitude, whoſe ſtanding lines are vpon the ſelfe ſame right lines. Wherefore alſo the ſolide A E is equall to the ſolide C F. Wherefore Parallelipipedons conſiſting vpon equall baſes: and being vnder one and the ſelfe ſame altitude, are equall the one to the other: which was required to be demonſtrated.*



The demonſtration of the firſt caſe of this propoſition is very hard to conceaue by the figure deſcribed for it in a playne. And yet before *M. Dee* inuented that figure which we haue there placed for it, it was much harder. For both in the Greke and Lattin *Euclide*, it is very ill made, and it is in a maner impoſſible to conceaue by it the conſtruction and demonſtration thereto appertayning. Wherefore I haue here deſcribed other figures, which firſt deſcribe vpon paſted paper, or ſuch like matter and then order them in maner following.





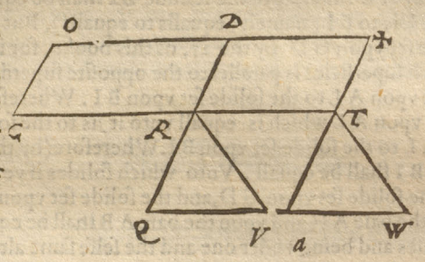


As touching the ſolide A E in the firſt caſe, I neede not to make any new deſcription. For it is playne inough to conceaue as it is there drawne. Although you may for your more eaſe of imagination deſcribe of paſted paper a parallelipipedon hauing his ſides equall with the ſides of the parallelipipedon A E before deſcribed, and hauing alſo the ſixe parallelogrammes thereof (contayned vnder thoſe ſides) equiangle

equiangle with the ſixe parallelogrammes of that figure,ech ſide and eche angle equall to his correſpon dent ſide,and to his correſpondent angle.

But concerning the other ſolide. When ye haue deſcribed theſe three figures vpon paſted paper: Where note for the proportion of eche line, to make your figure of paſted paper eqnall with the figure before deſcribed vpon the playne,let your lines O P,C X, R S,D F, &c. namely,the reſt of the ſtanding lines,of theſe figures,be equall to the ſtanding lines O P,C X,R S,D F, &c. of that figure. Likewiſe let the lines O C,C R,R D,D O,&c.namely,the ſides which cõtayne the baſes of theſe figures be equal to the lines O C,C R,R D,D O,&c. namely,to the ſids which cõtayne the baſes of that figure. Moreouer let the lines P X,X S,S F,F P,&c.namely,the reſt of the lines which cõtaine the vpper ſuperficieces of theſe figures,be equal to the lines P X,X S,S F,F P,&c.namely,to the reſt of the lines which cõtaine the vpper ſuperficieces of that figure(to haue deſcribed all thoſe foreſaid lines of theſe figures equal to all the lines of that figure,would haue required much more ſpace then here can be ſpared: I haue made them equall onely to the halues of thoſe lines,but by the example of theſe ye may,if ye will deſcribe the like figures hauing their lines equall to the whole lines of the figure in the playne,eche line to his correſpondent line). When I ſay ye haue as before is taught deſcribed theſe three figures, cut finely the lines X C, S R, F D of the firſt figure,and the lines S R,Y T,and I✠ of the ſecond figure: likewiſe the lines S R,N Q,Z V, and Y T,of the third figure, and fold theſe figures accordingly, which ye can not chuſe but doo if ye marke well the letters of euery line.

The three former figures being after this ſort deſcribed,ſet them vpon this figure here deſcribed vpon a playne,as vpõ their baſes,namely,the lines O C,C R,R D,D O:R T,T ✠, ✠ D, D R:V R,R T, T W,W a, V Q, and Q R of theſe three figures vpon the lines correſpondent vnto them in this figure. And they ſo ſtanding compare them with the conſtruction and demonſtration of the firſt caſe, and they will geue great light vnto it. This alſo ye muſt note,that if ye make the lines of the fore ſayd three figures equall to the lines of the figure of the plaine deſcribed before in the demõſtration of the firſt caſe: then muſt ye make a new baſe for them like vnto this, which is eaſy to doo, if ye draw a pallelogramme equall and like to the parallelogramme O C T ✠, and thẽ cut off from the ſame a parallelogramme D R T ✠ equall and in like ſort ſituate to the parallelogramme D R T ✠ of that figure: & vpon the line R T deſcribe two parallelogrammes, the one equall like, and in like ſort ſituate to the parallelogramme R T Q a of that figure, and the other equall, like and in like ſort ſituate to the parallelogramme R T V W of the ſame figure. The lines of this baſe which I haue here put are equall onely to the halues of the lines of that figure in the demonſtration.



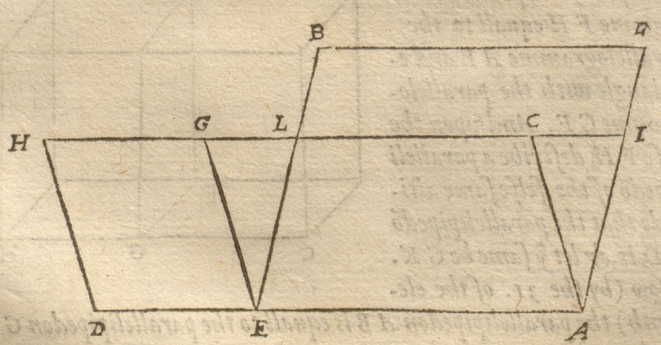
Conſtruction.

Demonſtration.

As touching the ſecond caſe ye neede no new figures, for it is playne to ſee by the figures (now reformed by *M. Dee*) deſcribed for it in the playne, eſpecially if ye remember the forme of the figure of the 29. propoſition of this booke, Only that which there ye conceaue to be the baſe, imagine here in both the figures of this ſecond caſe to be the vpper ſuperficies oppoſite to the baſe, and that which was there ſuppoſed to be the vpper ſuperficies conceaue here to be the baſe. Ye may deſcribe them vpon paſted paper for your better ſight, taking hede ye note the letters rightly according as the conſtruction requireth.

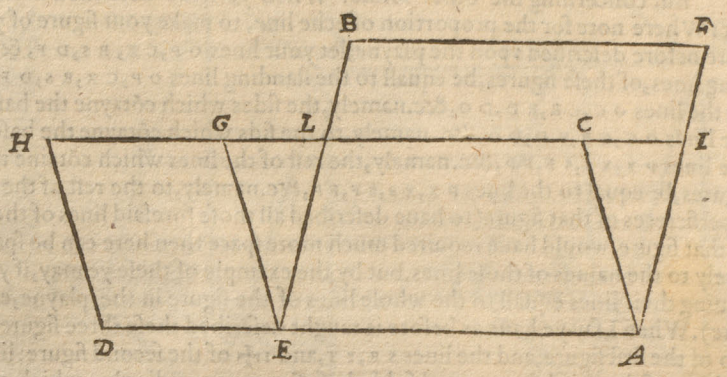
Fluſſas demonſtrateth this propoſition an other way taking onely the baſes of the ſolides, and that after this maner.

Take equall baſes(which yet for the ſurer vnderſtanding let be vtterly vnlike) namely, A E B F and A D C H, and let one of the ſides of eche concurre in one & the ſame right line A E D, & the baſes being vpon one and the ſelfe ſame playne let there be ſuppoſed to be ſet vpon thẽ parallelipipedons vnder one & the ſelfe ſame altitude. Then I ſay that the ſolide ſet vpõ the baſe A B is equal to the ſolide ſet vpon the baſe A H. By the poynt E draw vnto the line A C a



parallel

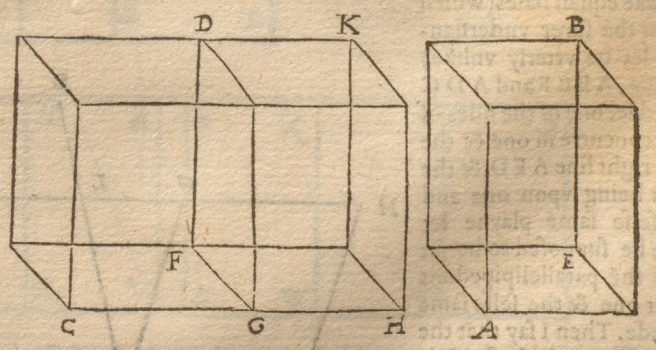
parallel line E G, which if it fall without the baſe A B, produce the right line H C to the poynt I. Now foraſmuch as A B and A H are parallelogrmães, therefore by the 24. of this booke, the triangles A C I and E G L ſhall be equaliter the one to the other: and by the 4. of the firſt, they ſhal be equiangle and equall: and by the firſt definition of the ſixth, and fourth Propoſition of the ſame, they ſhall be like. Wherfore Priſmes erected vppon thoſe triangles and vnder the ſame altitude that the ſolides A B and A H are, ſhall be equall and like, by the 8. definition of this booke. For they are contayned vnder like playne ſuperficieces equall both in multitude and magnitude. Adde the ſolide ſet vpon the baſe A C L E common to them both. Wherefore the ſolide ſet vppon the baſe A E G C, is equall to the ſolide ſet vpon the baſe A E L I. And foraſmuch as the ſuperficieces A E B F, and A D H C are equall (by ſuppoſition): and the part taken away A G is equall to the part taken away A L: therefore the reſidue B I ſhall be equall to the reſidue G D. Wherefore as A G is to G D as A L is to B I (namely, equalls to equalls). But as A G is to G D, ſo is the ſolide ſet vpon A G to the ſolide ſet vpon G D by the 25. of this booke, for it is cut by a playne ſuperficies ſet vpon the line G E, which ſuperficies is parallel to the oppoſite ſuperficieces. Wherefore as A L is to B I, ſo is the ſolide ſet vpon A L to the ſolide ſet vpon B I. Wherefore (by the 11. of the fifth) as the ſolide ſet vpon A G (or vpon A L which is equall vnto it) is to the ſolide ſet vpon G D, ſo is the ſame ſolide ſet vpon A G or A L to the ſolide ſet vpon B I. Wherefore (by the 2. part of the 9. of the fifth) the ſolides ſet vpon G D and B I ſhall be equall. Vnto which ſolides if ye adde equall ſolides, namely, the ſolide ſet vpon A G to the ſolide ſet vpon G D, and the ſolide ſet vpon A L to the ſolide ſet vpon B I: the whole ſolides ſet vpon the baſe A H and vpon the baſe A B ſhall be equall. Wherefore Parallelipedons conſiſting vpon equall baſes and being vnder one and the ſelfe ſame altitude are equall the one to the other: which was required to be proued.



¶ The 27. Theoreme. The 32. Propoſition.

Parallelipipedons being vnder one and the ſelfe ſame altitude, are in that proportion the one to the other that their baſes are.

Conſtruction. *Vppoſe that theſe parallelipipedons A B and C D be vnder one & the ſelfe ſame altitude. Then I ſay that thoſe parallelipipedons A B and C D are in that proportion the one to the other, that their baſes are, that is, that as the baſe A E is to the baſe C F, ſo is the parallelipipedon A B to the parallelipipedon C D. Vpon the line F G deſcribe (by the 45. of the firſt) the parallelogramme F H equall to the parallelogramme A E and equiangle with the parallelogramme C F. And vpon the baſe F H deſcribe a parallelipipedō of the ſelfe ſame altitude that the parallelipipedō C D is, & let ỹ ſame be G K.*



Demonſtration. *Now (by the 31. of the eleuenth) the parallelipipedon A B is equall to the parallelipipedon G K, for they conſiſt vpon equall*

quall baſes, namely, A E and F H, and are vnder one and the ſelfe ſame altitude. And foraſmuch as the parallelipipedon C K is cut by a plaine ſuperficies D G, being parallel to either of the oppoſite plaine ſuperficieces, therfore (by the 25. of the eleuenth) as the baſe H F is to the baſe F C, ſo is the parallelipipedon G K to parallelipipedon C D: but the baſe H F is equal to the baſe A E, and the parallelipipedon G K is proued equall to the parallelipipedon A B. Wherfore as the baſe A E is to the baſe C F, ſo is the parallelipedon A B, to the parallelipipedon C D. Wherfore parallelipipedons being vnder one and the ſelfe ſame altitude, are in that proportion the one to the other that their baſes are: which was required to be demonſtrated.

I neede not to put any other figure for the declaration of this demonſtration, for it is eaſie to ſee by the figure there deſcribed. Howbeit ye may for the more full ſight therof, deſcribe ſolides of paſted paper according to the conſtruction there ſet forth, which will not be hard for you to do, if ye remember the deſcriptions of ſuch bodies before taught.

A Corollary added by *Fluſas.*

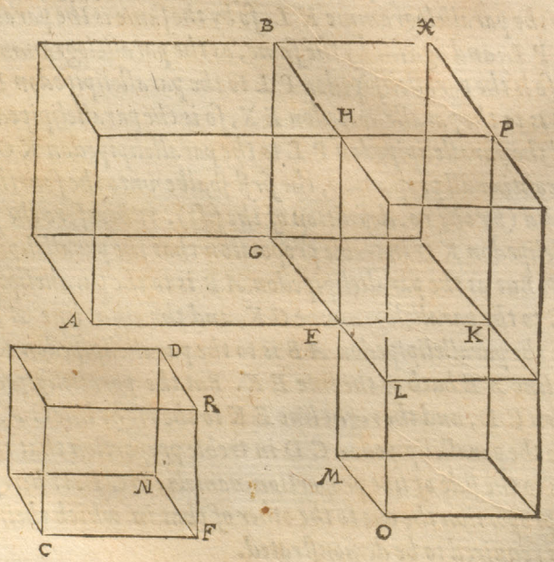
Equall parallelipipedons cõtained vnder one and the ſelfe ſame altitude, haue alſo their baſes equal. For if the baſes ſhould be vnequall, the parallelipipedons alſo ſhould be vnequal by this 32 propoſitiõ. *And equall parallelipipedons hauing equall baſes, haue alſo one and the ſelfe ſame altitude.* For if they ſhould haue a greater altitude, they ſhould exceede the equall parallelipipedons which haue the ſelfe ſame altitude: But if they ſhould haue a leſſe they ſhould want ſo much of thoſe ſelfe ſame equal parallelipipedons.

The 28. Theoreme. *The 33. Propoſition.*

Like parallelipipedons are in treble proportion the one to the other of that in which their ſides of like proportion are.

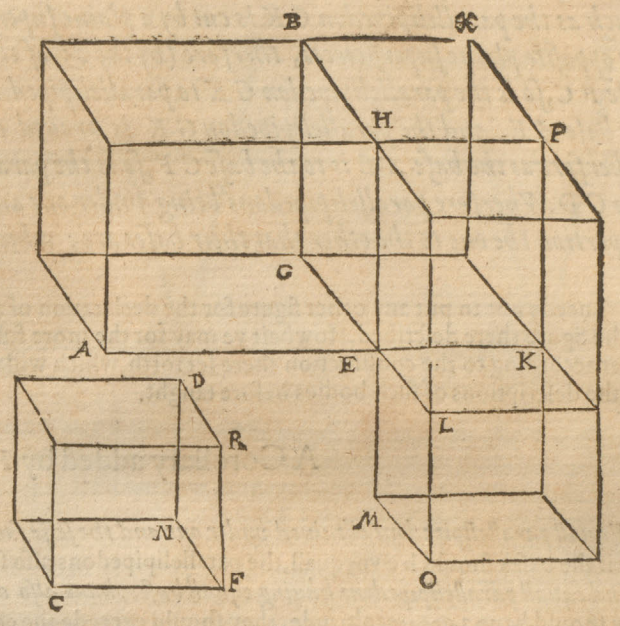
Vppoſe that theſe parallelipipedons A B and C D be like, & let the ſides A E and C F be ſides of like proportion. Then I ſay, the parallelipipedon A B is vnto the parallelipipedon C D in treble proportion of that in which the ſide A E is to the ſide C F. Extend the right lines A E, G E and H E to the pointes K, L, M. And

Conſtruction.



(by the 2. of the firſt) vnto the line C F put the line E K equal, and vnto the line F N put the line E L equall, and moreouer vnto the line F R put the line E M equall, and make perfect the parallelogramme K L, and the parallelipipedon K O. Now foraſmuch as theſe two lines E K and E L are equall to theſe two lines C F and F N, but the angle K E L is equall to the angle C F N (for the angle A E G is equall to the angle C F M by reaſon that the ſolides A B and C D are like). Wherfore the parallelogramme K L is equall and like to the parallelogramme C N, and by the ſame reaſon alſo the parallelogramme K M is equall and like

Demonſtration.



like to the parallelogramme C-R, and moreouer the parallelogramme O E to the parallelogramme F D. Wherefore three parallelogrammes of the parallelipipedon KO are like and equall to three parallelogrammes of the parallelipipedon C D: but thoſe three ſides are equall and like to the three oppoſite ſides: wherfore the whole parallelipipedon K O is equal and like to the whole parallelipipedon C D. Make perfect the parallelogramme G K. And vpon the baſes G K and K L make perfect to the altitude of the parallelipipedon A B, the parallelipipedons E X & L P. And foraſmuch as by reaſon that the parallelipipedons A B & C D are like, as the line A E is to the line C F, ſo is the line E G, to the line F N, and the line E H to the line F R. But the line C F is equall to the line E K, and the line F N to the line E L, and the line F R to the line E M, therefore as the line A E is to the line E K, ſo is the line G E to the line E L, and the line H E to the line E M (by conſtruction). But as the line A E is to the line E K, ſo is the parallelogramme A G to the parallelogramme G K (by the firſt of the ſixt). And as the line G E is to the line E L, ſo is the parallelogramme G K to the parallelogrãme K L. And moreouer as the line H E is to the line E M, ſo is the parallelogramme P E to the parallelogramme K M. Wherefore (by the 11. of the fift) as the parallelogramme A G is to the parallelogramme G K, ſo is the parallelogramme G K to the parallelogramme K L, and the parallelograme P E to the parallelogramme K M. But as the parallelogramme A G is to the parallelogramme G K, ſo is the parallelipipedon A B to the parallellpipedon E X, by the former propoſition, and as the parallelogramme G K is to the parallelogramme K L, ſo by the ſame is the parallelipipedon X E to the parallelipipedon P L: and agayne by the ſame, as the parallelogramme P E is to the parallelogramme K-M, ſo is the parallelipipedon P L to the parallelipipedon K O. Wherfore as the parallelipipedõ A B is to the parallelipipedon E X, ſo is the parallelipipedon E X to the parallelipipedon P L, and the parallelipipedon P L to the parallelipipedon K O. But if there be fower magnitudes in* continuall proportion, the firſt ſhalbe vnto the fourth in treble proportion that it is to the ſecond (by the 10. definition of the fift). Wherefore the parallelipipedon A B is vnto the parallelipedon K O in treble proportion that the parallelipipedon A B is to the parallelipipedon E X. But as the parallelipipedon A B is to the parallelipipedõ E X, ſo is the parallelogramme A G to the parallelogramme G K, and the right line A E to the right line E K. Wherefore alſo the parallelipipedon A B is to the parallelipipedon K O in treble proportiõ of that which the line A E hath to the line E K. But the parallelipipedon K O is equall to the parallelipipedon C D, and the right line E K to the right line C F. Wherefore the parallelipipedon A B is to the parallelipipedon C D in treble proportion that the ſide of like proportion, namely, A-E is to the ſide of like proportion, namely, to C F. Wherefore like parallelipipedons are in treble proportion the one to the other of that in which their ſides of like proportion are: which was requircd to be demonſtrated.

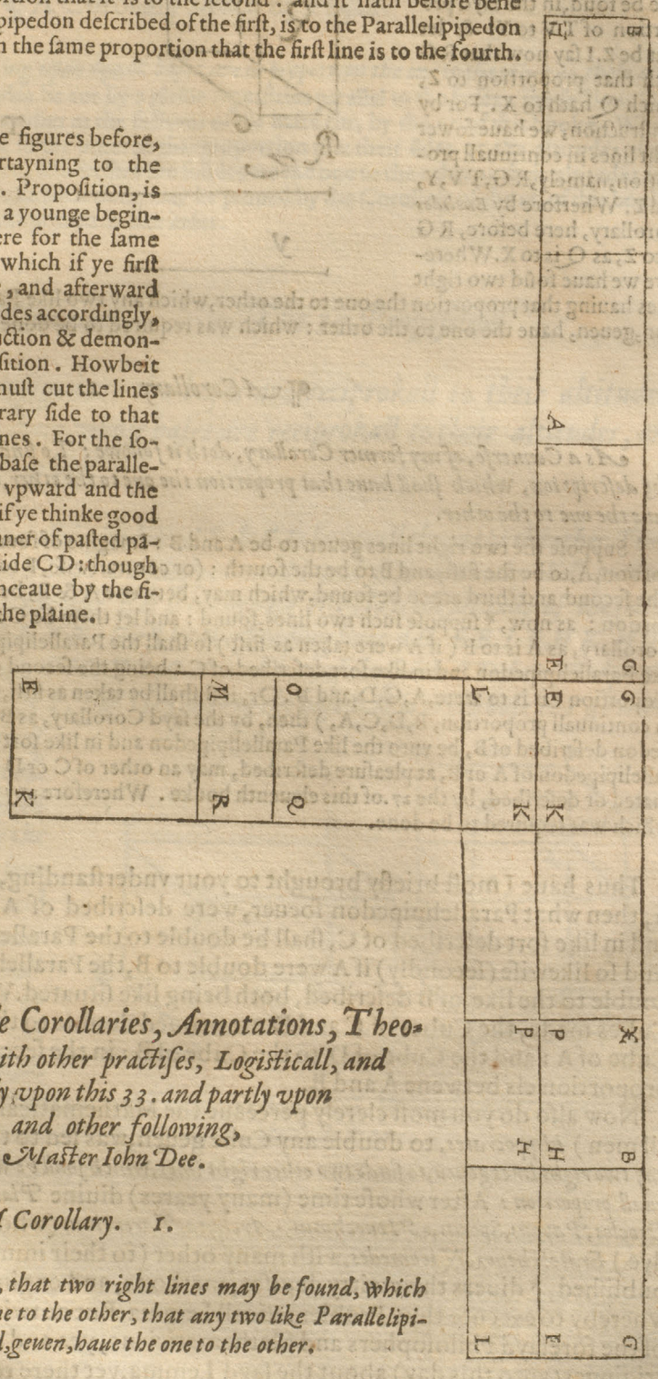
*

Becauſe

¶ *Corellary.*

*Hereby it is manifeſt, that if there be fower right lines in * continuall proportion, as the firſt is to the fourth, ſo ſhall the Parallelipipedon deſcribed of the firſt line, be to the Parallelipipedon deſcribed of the ſecond, both the Parallelipipedons being like and in like ſort deſcribed.* For the firſt line is to the fourth in treble proportion that it is to the ſecond: and it hath before bene proued that the Parallelipipedon deſcribed of the firſt, is to the Parallelipipedon deſcribed of the ſecond, in the ſame proportion that the firſt line is to the fourth.

Becauſe the one of the figures before, deſcribed in a plaine, pertayning to the demonſtration of this 33. Propoſition, is not altogether ſo eaſie to a younge beginner to conceaue, I haue here for the ſame deſcribed an other figure, which if ye firſt drawe vpon paſted paper, and afterward cut the lines & folde the ſides accordingly, will agree with the conſtruction & demonſtration of the ſayd Propoſition. Howbeit this ye muſt note that ye muſt cut the lines O Q & M R on the contrary ſide to that which ye cut the other lines. For the ſolides which haue to their baſe the parallelogramme L K are ſet on vpward and the other downward: Ye may if ye thinke good deſcribe after the ſame maner of paſted paper a body equall to the ſolide C D: though that be eaſie inough to conceaue by the figure thereof deſcribed in the plaine.



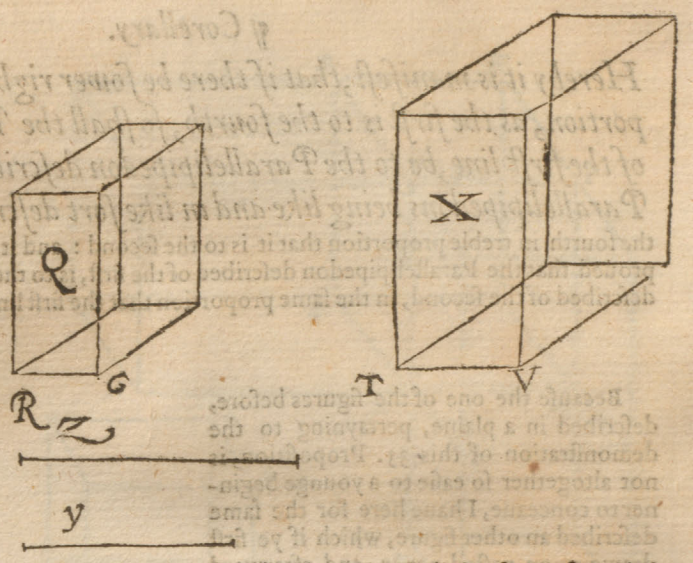
¶ *Certaine moſt profitable Corollaries, Annotations, Theoremes, and Problemes, with other practiſes, Logiſticall, and Mechanicall, partly vpon this 33. and partly vpon the 34. 36. and other following, added by Maſter Iohn Dee.*

¶ *A Corollary.* 1.

1. *Hereby it is manifeſt, that two right lines may be found, which ſhall haue that proportion, the one to the other, that any two like Parallelipipedons, and in like ſort deſcribed, geuen, haue the one to the other.*

Suppoſe Q and X to be two like Parallelipipedons, and in like ſort deſcribed. Of Q take any of the three lines, of which it is produced: as namely, R G. Of X, take that right line of his production, which line is aunſwerable to R G in proportion (which moſt aptly, after the Greke name, may be

called Omologall to R G) & let that be T V. By the 11. of the ſixth, to R G and T V, let the third line in proportion with them be founde, and let that be Y. By the ſame 11. of the ſixth, to T V and Y, let the thirde right line be foũd, in the ſayd proportion of T V to Y: & let that be Z. I ſay now that R G hath that proportion to Z, which Q hath to X. For by conſtruction, we haue fower right lines in continuall propiotion, namely, R G, T V, Y, and Z. Wherfore by *Euclides* Corollary, here before, R G is to Z, as Q is to X. Wherefore we haue foũd two right lines hauing that proportion the one to the other, which any two like Parallelipipedons of like deſcription, geuen, haue the one to the other: which was required to be done.



¶ *A Corollary.* 2.

As a Conuerſe, of my former Corollary, doth it followe: To finde two like Parallelipipedons of like deſcription, which ſhall haue that proportion the one to the other, that any two right lines geuen, haue the one to the other.

Suppoſe the two right lines geuen to be A and B: Imagine of foure right lines in continuall proportion, A, to be the firſt, and B to be the fourth: (or contrariwiſe, B to be firſt, & A to be the fourth). The ſecond and third are to be found, which may, betwene A & B, be two meanes in continuall proportion: as now, * ſuppoſe ſuch two lines, found: and let them be C and D. Wherefore by *Euclides* Corollary, as A is to B (if A were taken as firſt) ſo ſhall the Parallelipipedon deſcribed of A, be to the like Parallelipipedon and in like ſort deſcribed of C: being the ſecond of the fower lines in continuall proportion: it is to wete, A, C, D, and B. Or, if B ſhall be taken as firſt, (and that thus they are orderly in continuall proportion, B, D, C, A,) then, by the ſayd Corollary, as B is to A, ſo ſhall the Parallelipipedon deſcribed of B, be vnto the like Parallelipipedon and in like ſort deſcribed of D. And vnto a Parallelipipedon of A or B, at pleaſure deſcribed, may an other of C or D be made like, and in like ſort ſituated or deſcribed, by the 27. of this eleuenth booke. Wherefore any two right lines being geuẽ, &c: which was required to be done.

** Note this famous Lemma.*

Thus haue I moſt briefly brought to your vnderſtanding, if (firſt) B were double to A, then what Parallelipipedon ſoeuer, were deſcribed of A, the like Parallelipipedon and in like ſort deſcribed of C, ſhall be double to the Parallelipipedon deſcribed of A. And ſo likewiſe (ſecondly) if A were double to B, the Parallelipipedon of D, ſhoulde be double to the like, of B deſcribed, both being like ſituated. Wherefore if of A or B, were Cubes made, the Cubes of C and D are proued double to them: as that of C, to the Cube of A: and the Cube of D to the Cube of B: in the ſecond caſe. * And ſo of any proportion els betwene A and B.

The doubling of the Cube.
** Note.*

Now alſo do you moſt clerely perceaue the Mathematicall occaſion, whereby (firſt of all men) *Hippocrates*, to double any Cube geuen, was led to the former Lemma: *Betwene any two right lines geuen, to finde two other right lines, which ſhall be with the two firſt lines, in continuall proportion:* After whoſe time (many yeares) diuine *Plato, Heron, Philo, Appollonius, Diocles, Pappus, Sporus, Menechmus, Archytas Tarentinus* (who made the wodden doue to flye) *Eratoſthenes, Nicomedes*, with many other (to their immortall fame and renowme) publiſhed, * diuers their witty deuiſes, methods, and engines (which yet are extant) whereby to execute thys Problematicall Lemma. But notwithſtanding all the trauailes of the foreſayd Philoſophers and Mathematiciens, yea and all others doinges and contriuinges (vnto this day) about the ſayd Lemma, yet there remaineth ſufficient matter, *Mathematically ſo to demonſtrate the ſame, that moſt exactly & readily, it may alſo be Mechanically practiſed:* that who ſoeuer ſhall achieue that feate, ſhall not be counted a ſecond *Archi-*

** All their wayes of executing the Lemma are in Eutotius commentaries vpõ Archimedes booke de Sphera & Cylindro: where Archimedes vſeth the ſame Lemma.*

medes,

medes, but rather a pereles Mathematicien, and *Mathematicorum Princeps*. I will ſundry wayes (in my briefe additions and annotations vpon *Euclide*) excite you thereto, yea and bring before your eyes ſundry new wayes, by me inuented: and in this booke ſo placed, as matter thereof, to my inuentions appertayning, may geue occaſion: Leauing the farther, full, & abſolute my concluding of the Lemma, to an other place and time: which will, now, more cõpendiouſly be done: ſo great a part therof, being before hand in thys booke publiſhed.

Note what is yet lacking requiſite to the doubling of the Cube.

¶ A Corollary added by *Fluſas*.

Parallelipipedous conſiſting vpon equall baſes, are in proportion the one to the other as their altitudes are. For if thoſe altitudes be cut by a plaine ſuperficies parallel to the baſes: the ſections ſhall be in proportion the one to the other as the ſections of the baſes cut, by the 25. of this booke. Which ſections of the baſes are the one to the other in that proportion that their ſides or the altitudes of the ſolides are, by the firſt of the ſixt. Wherefore the ſolides are the one to the other as their altitudes are. But if the baſes be vnlike, the ſelfe ſame thing may be proued by the Corollary of the 25. of this booke, which by the 25. Propoſition was proued in like baſes.

¶ The 29. Theoreme. The 34. Propoſition.

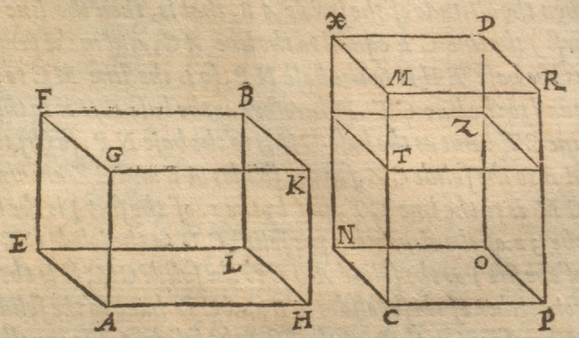
In equall Parallelipipedons the baſes are reciprokall to their altitudes. And Parallelipipedons whoſe baſes are reciprokall to their altitudes, are equall the one to the other.

Two caſes in the firſt part of this propoſition.

First caſe, which alſo may be two wayes.

First way.

Suppoſe that theſe Parallelipipedons A B & C D be equall the one to the other. Then I ſay, that the baſes of the Parallelipipedons A B and C D are reciprokall to their altitudes, that is, as the baſe E H is to the baſe N P, ſo is the altitude of the ſolide C D to the altitude of the ſolide A B. Firſt let the ſtanding lines A G, E F, L B, H K, of the ſolide A B, & the ſtãding lines C M, N X, O D, and P R, of the ſolide C D, be erected perpẽdicularly to the baſes E H & N P. Thẽ I ſay, that as the baſe E H is to the baſe N P, ſo is the line C M to the line A G. Now if the baſe E H be equal to the baſe N P, and the ſolide A B is equall to the ſolide C D, wherefore the line C M is equall to the line A G. For if the baſes E H and N P being equall, the altitudes A G and C M be not equall, neither alſo ſhall the ſolide A B be equall to the ſolide C D, but they are ſuppoſed to be equall. Wherefore the altitude C M is not vnequall to the altitude A G. Wherefore it is equall. And therefore as the baſe E H is to the baſe P N, ſo is the altitude C M to the altitude A G. Wherfore it is manifeſt, that the baſes of the Parallelipipedons A B and C D are reciprokall to their altitudes.*



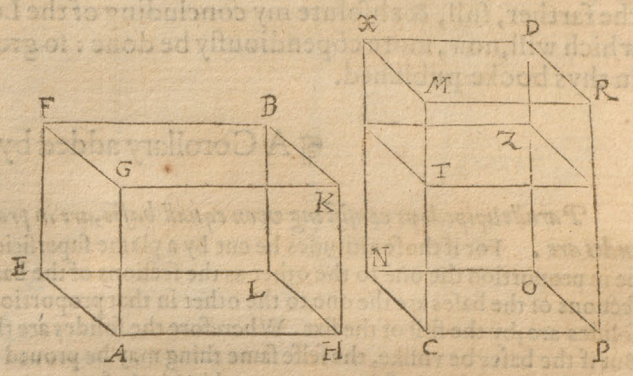
* *This followeth alſo of the Corollary added of Fluſas after the 33. propoſition of this booke.*

Second way.

But now ſuppoſe that the baſe E H be not equall to the baſe N P. But let the baſe E H be the greater. Now the ſolide A B is equall to the ſolide C D. Wherefore alſo the altitude C M is greater then the altitude A G. For if not, then againe are not the ſolides A B and C D equall: but they are by ſuppoſition equall. Wherefore (by the 2. of the firſt) put vnto the line A G an equall line C T. And vpon the baſe N P and the altitude being C T, make perfecte a ſolide cõtained vnder parallel plaine ſuperficieces, and let the ſame be C Z. And foraſmuch*

* *This alſo followeth of the former Corollary.*

as the ſolide A B is equall to the ſolide C D, and there is a certaine other ſolide, namely, C Z, but vnto one and the ſelfe ſame magnitude equall magnitudes haue one and the ſelfe ſame proportion (by the 7. of the fift). Wherefore as the ſolide A B is to the ſolide C Z, ſo is the ſolide C D to the ſolide C Z. But as the ſolide A B is to the ſolide C Z, ſo is the baſe E H to the baſe N P (by the 32. of the eleuenth) · for the ſolides A B and C Z are vnder equall altitudes. And as the ſolide C D is to the ſolide C Z, ſo is the baſe M P to the baſe P T, and the line M C to the line C T. Wherefore (by the 11. of the fift) as the baſe E H is to the baſe N P, ſo is the line C M to the line C T. But the line C T is equall to the line A G. Wherefore (by the 7. of the fift) as the baſe E H is to the baſe N P, ſo is the altitude C M to the altitude A G. Wherfore in theſe Parallelipipedons A B and C D the baſes are reciprokall to their altitudes.



The conuerſe of both the partes of the firſt caſe.

But now againe ſuppoſe that the baſes of the Parallelipipedons A B and C D be reciprokall to their altitudes, that is, as the baſe E H is to the baſe N P, ſo let the altitude of the ſolide C D be to the altitude of the ſolide A B. Then I ſay, that the ſolide A B is equall to the ſolide C D. For againe let the ſtanding lines be erected perpendicularly to their baſes.

1. And now if the baſe E H be equall to the baſe N P: but as the baſe E H is to the baſe N P, ſo is the altitude of the ſolide C D to the altitude of the ſolide A B. Wherefore the altitude of the ſolide C D is equall to the altitude of the ſolide A B. But Parallelipipedons conſiſting vpon equall baſes and vnder one and the ſelfe ſame altitude, are (by the 31. of the eleuenth) equall the one to the other. Wherefore the ſolide A B is equall to the ſolide C D.

2. But now ſuppoſe that the baſe E H be not equall to the baſe N P: but let the baſe E H be the greater. Wherefore alſo the altitude of the ſolide C D, that is, the line C M is greater then the altitude of the ſolide A B, that is, then the line A G. Put againe (by the 3. of the firſt) the line C T equall to the line A G, and make perfecte the ſolide C Z. Now for that as the baſe E H is to the baſe N P, ſo is the line M C to the line A G. But the line A G is equall to the line C T. Wherefore as the baſe E H is to the baſe N P, ſo is the line C M to the line C T. But as the baſe E H is to the baſe N P, ſo (by the 32. of the eleuenth) is the ſolide A B to the ſolide C Z, for the ſolides A B and C Z are vnder equall altitudes. And as the line C M is to the line C T, ſo (by the 1. of the ſixt) is the baſe M P to the baſe P T, and (by the 32. of the eleuenth) the ſolide C D to the ſolide C Z. Wherefore alſo (by the 11. and 9. of the fift) as the ſolide A B is to the ſolide C Z, ſo is the ſolide C D to the ſolide C Z. Wherfore either of theſe ſolides A B and C D haue to the ſolide C Z one and the ſame proportion. Wherefore (by the 7. of the fift) the ſolide A B is equall to the ſolide C D: which was required to be demonſtrated.

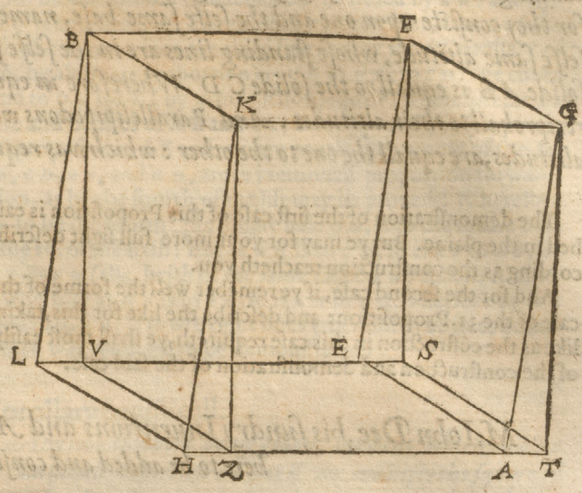
Second caſe.

Conſtruction.

But now ſuppoſe that the ſtanding lines, namely, F E, B L, G A, K H: X N, D O, M C, and R P, be not erected perpendicularly to their baſes. And (by the 11. of the eleuenth) from the pointes F, G, B, K: X, M, D, R, draw vnto the plaine ſuperficies of the baſes E H and N P perpendicular lines, and let thoſe perpendicular lines light vpō the pointes S, T, V, Z: W, Y, d, and Q, and make perfecte the Parallelipipedons F Z, and X Q. I ſay that euen in this caſe alſo, if the ſolides A B and C D be equall, their baſes are reciprokall to their altitudes, that is, as the baſe E H is to the baſe N P, ſo is the altitude of the ſolide C D to the altitude of the ſolide A B. For foraſmuch as the ſolide A B is equall to the ſolide

C D

CD, but the ſolide AB is equall to the ſolide BT (by the 20.of the eleuenth) for they are vpon one and the ſelfe ſame baſe, namely, the parallelogramme KF, and vnder one and the ſelfe ſame altitude, whoſe ſtanding lines are in the ſelfe ſame right lines, namely, HZAT, and LVES: and the ſolide CD is by the ſame reaſon equall to the ſolide DY, for they both conſiſt vpon one and theſelfe ſame baſe, namely, the parallelogramme XR, and are vnder one and the ſelfe ſame altitude, whoſe ſtanding lines are in the ſelfe ſame right lines, namely, PQCY, and OhNW. Wherefore the ſolide BT is equall to the ſolide DY. But in equall Parallelipipedons, whoſe altitudes are erected perpendicularly to their baſes, their baſes are reciprokall to their altitudes (by the firſt part of this Propoſition). Wherefore as the baſe FK is to the baſe XR, ſo is the altitude of the ſolide DY to the altitude of the ſolide BT. But the baſe FK is equall to the baſe EH, and the baſe XR to the baſe NP. Wherefore as the baſe EH is to the baſe NP, ſo is the altitude of the ſolide DY to the altitude of the ſolide BT. But the altitudes of the ſolides DY & BT, and of the ſolides DC & BA are one and the ſelfe ſame. Wherefore as the baſe EH is to the baſe NP, ſo is the altitude of the ſolide CD to the altitude of the ſolide AB. Wherfore the baſes of the Parallelipipedons AB and CD are reciprokall to their altitudes.



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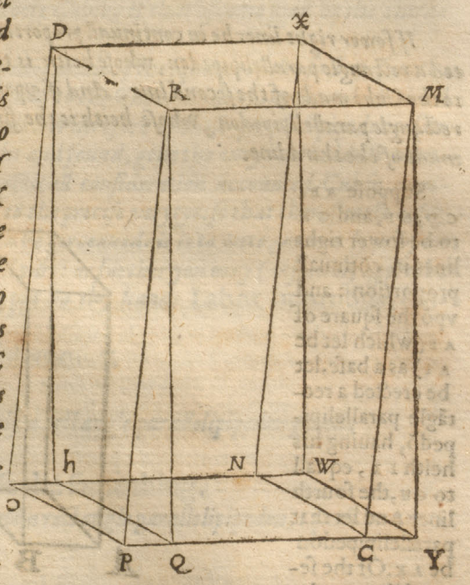


Againe ſuppoſe that the baſes of the Parallelipipedons AB and CD be reciprokall to their altitudes, that is, as the baſe EH is to the baſe NP, ſo let the altitude of the ſolide CD be to the altitude of the ſolide AB. Then I ſay, that the ſolide AB is equall to the ſolide CD. For the ſame order of conſtruction remayning, for that as the baſe EH is to the baſe NP, ſo is the altitude of the ſolide CD to the altitude of the ſolide AB: but the baſe EH is equall to the parallelogramme FK, and the baſe NP to the parallelogramme XR: wherefore as the baſe FK is to the baſe XR, ſo is the altitude of the ſolide CD to the altitude of the ſolide AB But the altitudes of the ſolides AB and BT are equall, and ſo alſo are the altitudes of the ſolides DC and DY. Wherefore as the baſe FK is to the baſe XR, ſo is the altitude of the ſolide DY to the altitude of the ſolide BT. Wherefore the baſes of the Parallelipipedons BT and DY are reciprokall to their altitudes. But Parallelipipedons whoſe altitudes are erected perpendicularly to their baſes, and whoſe baſes are reciprokall to their altitudes, are equall the one to the other (by this Propoſition). Wherefore the ſolide BT is equall to the ſolide DY. But the ſolide BT is equall to the ſolide BA (by the 29.of the eleuenth) for they conſiſt vpon one and the ſelfe ſame baſe, namely, FK, and are vnder one and the ſelf ſame altitude, whoſe

The conuerſe of the ſecond caſe.

ſtanding lines are in the ſelfe ſame right lines. And the ſolide D Y is equall to the ſolide D C, for they conſiſte vpon one and the ſelfe ſame baſe, namely, X R, and are vnder one and the ſelfe ſame altitude, whoſe ſtanding lines are in the ſelfe ſame right lines. Wherefore alſo the ſolide A B is equall to the ſolide C D. Wherefore in equall Parallelipipedons the baſes are reciprokall to their altitudes. And Parallelipipedons whoſe baſes are reciprokall with their altitudes, are equall the one to the other: which was required to be proued.

The generall concluſion.

The demonſtration of the firſt caſe of this Propoſition is eaſie to conceaue by the figure as it is deſcribed in the plaine. But ye may or your more full ſight deſcribe Parallelipipedons of paſted paper, according as the conſtruction teacheth you.

And for the ſecond caſe, if ye remēber well the forme of the figures which you made for the ſecond caſe of the 31. Propoſition: and deſcribe the like for this, taking heede to the letters that ye place them like as the cōſtruction in this caſe requireth, ye ſhall moſt eaſily by them come to the full vnderſtanding of the conſtruction and demonſtration of the ſaid caſe.

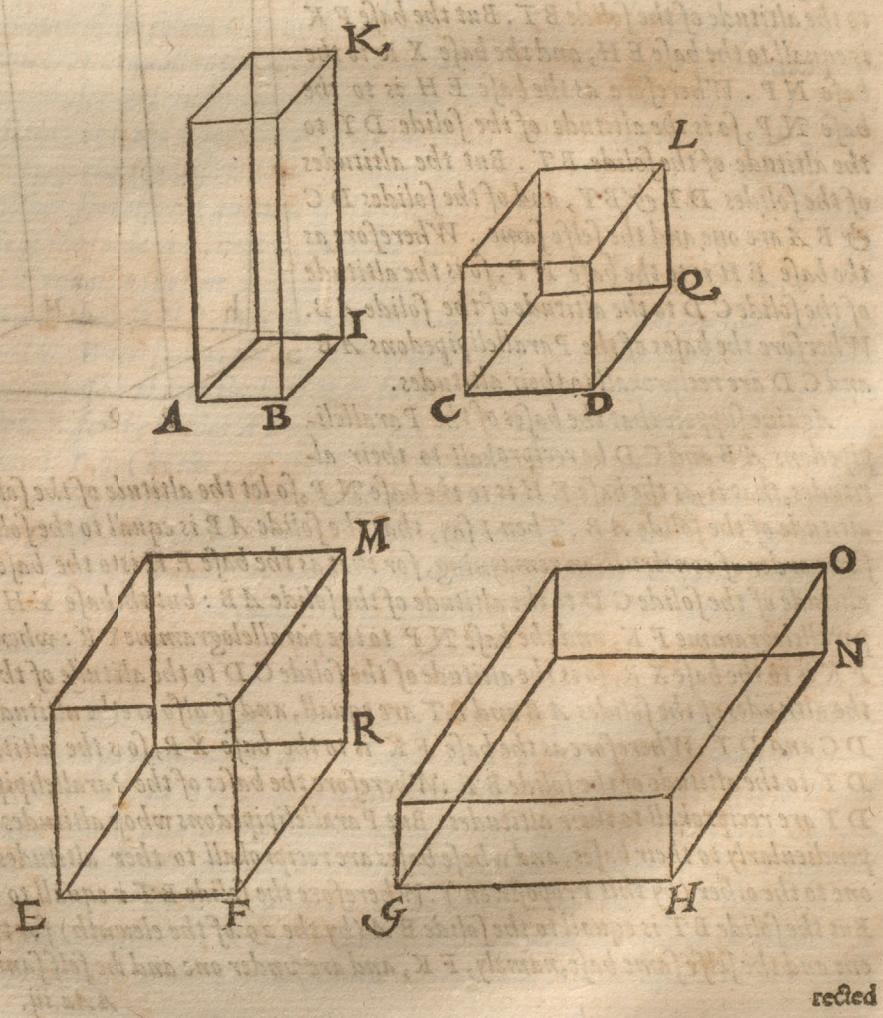
M. Iohn Dee, his ſundry Inuentions and Annotacions, very neceſſary, here to be added and conſidered.

A Theoreme.

If fower right lines be in continuall proportion, and vpon the ſquare of the firſt, as a baſe, be erected a rectangle parallelipipedon, whoſe heith is the fourth line: that rectangle parallelipipedon is equal to the Cube made of the ſecond line. And if vpon the ſquare of the fourth line, as a baſe, be erected a rectangle parallelipipedon, whoſe heith is the firſt line, that parallelipipedon is equall to the Cube made of the third line.

Conſtruction.

Suppoſe A B, C D, E F, and G H to be fower right lines in cōtinuall proportion: and vpō the ſquare of A B (which let be A I) as a baſe, let be erected a rectāgle parallelipipedō, hauing his heith I K, equall to G H, the fourth line. And let that parallelipipedon be A K. Of the ſecond line C D, let a Cube be made: whoſe ſquare baſe, let be noted with C Q: and let his heith be noted by Q L: & let the whole Cube be ſignified by C L. I ſay that A K is equall to C L. Let the like cōſtruction be for the cube of the third line: that is, vpon the ſquare of G H (which ſuppoſe to be G N) let a rectangle parallelipipedon be e-



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rected, hauyng his heith N O, equall to A B, the first line: which parallelipipedon let be noted with G O. And suppose the cube of the third line (E F) to be E M: whose squarebase, let be noted by E R: and hys heith by R M. I say now (secondly) that G O is equal to E M. For the first part consider, that A I (the square base of A K) is to C Q, the square base of C L, as A B is to the third line E F, by the 2. Corollary of the 20. of the sixth. But as A B, is to E F, so (by alternate proportion) is C D to G H, to C D. The cubes roote, is Q L, the same cubes heith equall: and to G H is I K (by construction) equall: wherefore, as A I is to C Q, so is Q L to I K. The bases therefore and heithes of A K and C L, are reciprocally in proportion: wherefore by the second part of this 34. proposition, A K and C L are equall. For proofe of the second part of my theoreme, I say, that as A B, C D, E F, and G H, are in continuall proportion forward, so are they backward in continuall proportion, as by the fourth of the fift nay be proued. Wherefore now considering G H to be as first, and so A B to be the fourth: the square base G N, is to the square base E R, as G H is to C D, by the 2. corollary of the 20. of the sixth: But as G H is to C D, so is E F to A B, by alternate proportion: to the Cubik roote E F, is R M (the heith of the same Cube E M) equall. And to A B, is the heith N O equall, by construction: wherfore as G N is to E R, so is I M to N O. Therfore by the second part of this 34. proposition, G O is equall to E M. If fowre right line: (therefore) be in continuall proportion &c. as in the proposition: which was required to be demonstrated.

Demonstratiõ of the first part.

Demonstratiõ of the second part.

A Corollary logisticall.

Of my former Theoreme it followeth: Any two numbers being geuen, betwene which two we would haue two other numbers, middle, in continuall proportion: That is we multiply the square of the first number geuen, by the other geuen number (as if it were the fourth): the roote Cubik of that ofcome or product, shall be the second number sought. And farther, if we multiply the square of the other number geuen, by the first geuen number, the roote Cubike of that ofcome shall be the thirde number sought.

For (by my Theoreme) those rectangle parallelipipedõs made of the squares of the first & fourth, multiplied by the fourth & the first, accordingly, are equall to the Cubes made of the second & third numbers: which we make our two middle proportionals. Wherefore of those parallelipipedons (as Cubes) the Cubik rootes, by good and vsuall arte sought and found, geue the very two middle numbers desired. And where those numbers, are not by logisticall consideratiõ accounted Cubik numbers, ye may vse the logistical secret of approching nere to the precise veryte: so that therof most easily you shall perceaue, that your fayle is of the sence neuer to be perceaued: it is to wete, as in a lyne of an inch long, not to want or exceede the thousand thousand part: or farther yu may (infinitely approche at pleasure. O Mechanicall frend, be of good comfort, put to thy hand: Labor improbus, omnia vincit.

To finde two middle proportionals betwene two numbers geuẽ.

Note the practise of approching to precienes in Cubik rootes.

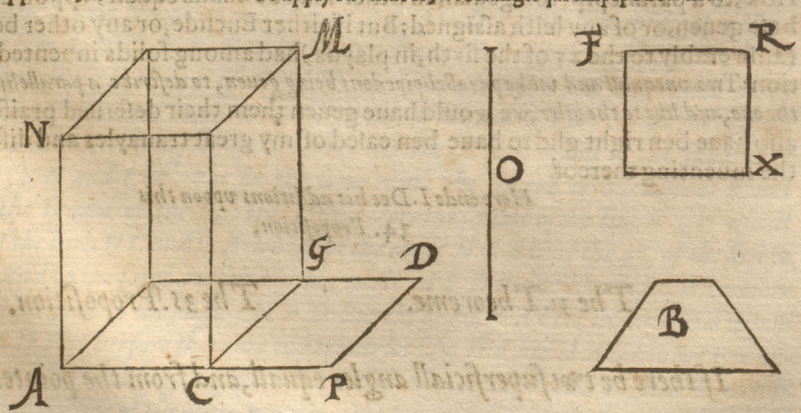
A Probleme. 1.

Vppon a right lined playne superficies geuen, to apply a rectangle paralelipipedon geuen.

Or we may thus expresse the same thing.

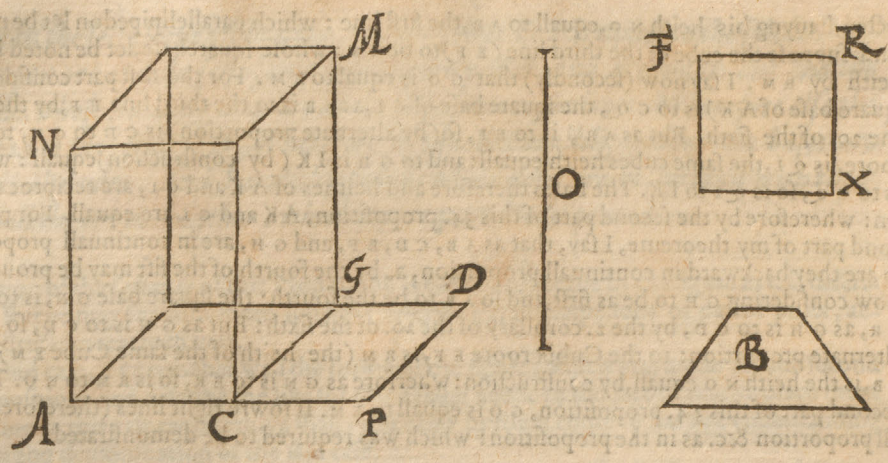
Vppon a right lined playne superficies geuen, to erect a rectangle paralelipipedon, equall to a rectangle parallelipipedon geuen.

Suppose the right lined playne superficies geuen to be B: and the rectangle parallelipipedon geuen to be A M. Vppon B, as a base must A M be applyed: that is, a rectangle parellipipedon must be erected vppon B, as a base, whiche shall be equall to A M. By the laste of the second, to the right lyned figure B, let an equall square be made: which suppose to be F R X. *produce one side of the base of the parallelipipedon



*This is the way to apply any square geuen, to a line also geuẽ, sufficiently extended.

rallelipipedõ AM, which let be AC, extended to the point P. Let the other side of the sayde base, concurring with AC, be CG. As CG is to FR (the side of the square FRX) so let the same FR be to a line cut of from CP sufficiently extended: by the 11. of the sixth: and let that third proportionall line be CP. Let the rectangle parallelogramme be made perfect, as CD. It is euident, that CD, is equall to the square FRX by the 17. of the sixth: and by construction FRX, is equall to B. Wherfore CD, is equall to B. By the 12. of the sixth, as CP, is to AC, so let AN (the heith of AM) be to the right line O. I say that a solide perpendicularly erected vppon the base B, hauinge the heith of the line O, is equall to the parallelipipedon AM. For CD is to AG, as CP is to AC by the firste of the sixth, and B is proued equall to CD: Wherfore by the 7. of the fifth, B is to AG as CP is to AC: But as CP is to AC, so is AN to O, by construction: Wherefore B is to AG as AN is to O. So than the bases B and AG are reciprocally in proportion with the heithes AN and O. By this 34 therefore, a solide erected perpendicularly vppon B as a base, hauing the height O, is equall to AM. Wherefore vppon a right lyned playn superficies geuen, we haue applied a rectangle parallelipipedon geuen: Which was requisite to be donne.



A Probleme 2.

A rectangle parallelipipedon being geuen to make an other equall to it of any heith assigned.

Suppose the rectangle parallelipipedon geuen to be A, and the heith assigned to be the right line B: Now must we make a rectangle parallelipipedon, equal to A: Whose heith must be equall to B. According to the manner before vsed, we must frame our cõstruction to a reciprokall proportiõ betwene the bases and heithes. Which will be done if, as the heith assigned beareth it selfe in proportion to the heith of the parallelipipedon giuen: so, one of the sides of the base of the parallelipipedon giuen, be to a fourth line, by the 12. of the sixth found. For that line founde, and the other side of the base of the geuen parallelipipedon, contayne a parallelogramme, which doth serue for the base, (which onely, we wanted) to vse with our giuen heith: and so is the Probleme to be executed.

Note.

Euclide in the 27. of this eleuenth hath taught, how, of a right line geuẽ, to describe a parallepipedõ, like, & likewise situated, to a parallelipipedõ geuẽ: I haue also added, How, to a parallepipedon geuen, an other may be made equall, vppon any right lined base geuen, or of any heith assigned: But if either Euclide, or any other before our time (answerably to the 25. of the sixth, in playns) had among solids inuented this proposition: *Two vnequall and vnlike parallelipipedons being geuen, to describe a parallelipipedon equall to the one, and like to the other*, we would haue geuen them their deserued praise: and I would also haue ben right glad to haue ben eased of my great trauayles and discourses about the inuenting thereof.

A probleme worth the searching for.

Here ende I. Dee his additions vppon this 34. Proposition.

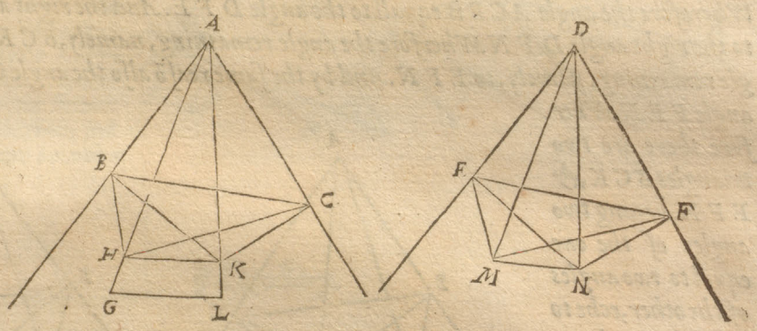
The 30. Theoreme. The 35. Proposition.

If there be two superficiall angles equall, and from the pointes of those angles

gles be eleuated on high right lines, comprehending together with thoſe right lines which containe the ſuperficiall angles, equall angles, eche to his coreſpõdent angle, and if in eche of the eleuated lines be takẽ a point at all auentures, and from thoſe pointes be drawen perpendicular lines to the ground playne ſuperficieces in which are the angles geuen at the beginning, and from the pointes which are by thoſe perpendicular lines made in the two playne ſuperficieces be ioyned to thoſe angles which were put at the beginning right lines: thoſe right lines together with the lines eleuated on high ſhall contayne equall angles.

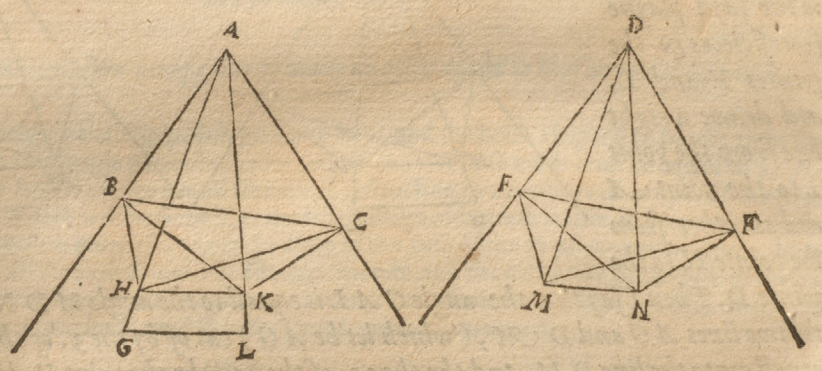
SVppoſe that theſe two rectiline ſuperficiall angles B A C, and E D F be equall the one to the other: and from the pointes A and D let there be eleuated vpward theſe right lines A G and D M, comprehendinge together with the lines put at the beginninge equall angles, ech to his correſpondent angle, that is, the angle M D E to the angle G A B, and the angle M D F to the angle G A C, and take in the lines A G and D M pointes at all auẽtures and let the ſame be G and M. And (by the 11. of the eleuẽth) from the pointes G and M draw vnto the ground playne ſuperficieces wherein are the angles B A C and E D F perpendicular lines G L and M N and let them fall in the ſayd playne ſuperficieces in the pointes N and L, and drawe a right line from the point L to the point A and an other from the pointe N to the pointe D. Then I ſay that the angle G A L is equall to the angle M D N. Frõ the greater of the two lines A G and D M, (which let be A G) cut of by the 3. of the firſt the line A H equall vnto the line D M. And (by the 31. of the firſt) by the point H, drawe vnto the line G L a parallel line, and let the ſame be H K. Now the line G L is erected perpendicularly to the grounde playne ſuperficies B A L: Wherfore alſo (by the 8. of the eleuenth) the line H K is erected perpẽdicularly to the ſame grounde plaine ſuperficies B A C. Drawe (by the 12. of the firſt) frõ the pointes K and N vnto the right lines A B, A C, D F, & D E perpẽdicular right lines, and let the ſame be K C, N F, K B, N E. And drawe theſe right lines H C, C B, M F, F E. Now foraſmuch as (by the 47. of the firſt) the ſquare of the line H A is equall to the ſquares of the lines H K and K A, but vnto the ſquare of the line K A are equall the ſquares of the lines K C and C A: Wherefore the ſquare of the line H A is equall to the ſquares of the lines H K, K C and C A. But by the ſame vnto the ſquares of the lines H K and K C is equall the ſquare of the line H C: Wherefore the ſquare of the line H A is equall to the ſquares of the lines H C and C A: wherfore the angle H C A is (by the 48. of the firſt) a right angle. And by the ſame reaſon alſo the angle M F D is a right angle. Wherefore the angle H C A is equall to the angle M F D. But the angle H A C is (by ſuppoſitiõ) equal to the angle M D F. Wherfore there are two triangles M D F and H A C hauing two angles of the one equall to twoo angles of the other, eche to his correſpondent angle, and one ſide of the one equall to one ſide of the other, namely, that ſide which ſubtendeth one of the equall angles,

Conſtruction.



Demonſtration.

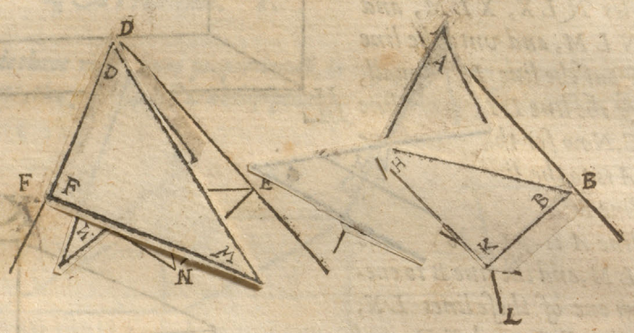
angles, that is, the side H A is equall to the side D M by construction. Wherefore the side remayning are (by the 26. of the first) equall to the sides remayning. Wherefore the side A C is equall to the side D F. In like sort may we proue that the side A B is equall to the side D E, if ye draw a right line from the point H to the point B, and an other from the point M to the point E. For forasmuch as the square of the line A H is (by the 47. of the first) equall to the squares of the lines A K and K H, and (by the same) vnto the square of the line A K are equall the squares of the lines A B and B K. Wherefore the squares of the lines A B, B K, and K H are equall to the square of the line A H. But vnto the squares of the lines B K and K H is equall the square of the line B H (by the 47. of the first) for the angle H K B is a right angle, for that the line H K is erected perpendicularly to the ground playne superficies: Wherefore the square of the line A H is equall to the squares of the lines A B and B H. Wherefore (by the 48. of the first) the angle A B H is a right angle. And by the same reason the angle D E M is a right angle. Now the angle B A H is equall to the angle E D M, for it is so supposed, and the line A H is equall to the line D M. Wherefore (by the 26. of the first) the line A B is equall to the line D E. Now forasmuch as the line A C is equall to the line D F, and the line A B to the line D E, therefore these two lines A C and A B are equall to these two lines F D and D E. But the angle also C A B is by supposition equall to the angle F D E. Wherefore (by the 4. of the first) the base B C is equall to the base E F, and the triangle to the triangle, and the rest of the angles to the reste of the angles. Wherefore the angle A C B is equall to the angle D F E. And the right angle A C K is equal to the right angle D F N. Wherfore the angle remayning, namely, B C K, is equall to the angle remayning, namely, to E F N. And by the same reaso also the angle C B K is equal to the angle F E N. Wherefore there are two triangles B C K, & E F N, hauing two angles of the one equal to two angles of the other, eche to his correspondent angle, and one side of the one equall to one side of the other, namely, that



side that lieth betwene the equall angles, that is the side B C is equall to the side E F: Wherefore (by the 26. of the first) the sides remaininge are equall to the sides remayning. Wherfore the side C K is equall to the side F N: but the side A C is equall to the side D F. Wherefore these two sides A C and C K are equall to these two sides D F and F N, and they contayne equall angles: Wherefore (by the 4. of the first) the base A K is equall to the base D N. And forasmuch as the line A H is equall to the line D M, therefore the square of the line A H is equall to the square of the line D M. But vnto the square of the line A H are equall the squares of the lines A K and K H (by the 47. of the first) for the angle A K H is a right angle. And to the square of the line D M are equall the squares of the lines D N and N M, for the angle D N M is a right angle. Wherefore the squares of the lines A K and K H are equall to the squares of the lines D N and N M: of which two, the square of the line A K is equall to the square of the line D N (for the line A K is proued equall to the line A N). Wherefore the residue, namely, the square of the line K H is equal to the residue, namely, to the square of the line N M. Wherefore the line H K is equall to the line M N. And forasmuch as these two lines H A and A K are equall to these two lines M D and D N, the one to the other, and the base

base H K is equall to the base M N: therfore (by the 8. of the first) the angle H A K is equall to the angle M D N. If therefore there be two superficiall angles equall, and frō the pointes of those angles be eleuated on high right lines, comprehending together with those right lines which were put at the beginning, equall angles, ech to his coresponden angle, and if in ech of the erected lines be taken a point at all aduentures, and from those pointes be drawen perpendicular lines to the plaine superficieces in which are the angles geuen at the beginning, and from the pointes which are by the perpendicular lines made in the two plaine superficieces be ioyned right lines to those angles which were put at the beginning, those right lines shall togeher with the lines eleuated on high make equall angles: which was required to be proued.

Becauſe the figures of the former demonſtration are ſomewhat hard to conceaue as they are there drawen in a plaine, by reaſon of the lines that are imagined to be eleuated on high, I haue here ſet othe figures, wherein you muſt erecte perpendicularly to the ground ſuperficieces the two triangles B F K, and E M N, and then eleuate the triangles D F M, & A C H, in ſuch ſort that the angles M and H of theſe triangles, may concurre with the angles M and H of the othe erected triangles. And then imagining only a line to be drawen fron the point G of the line A G to thepoint L in the ground ſuperficies, compare it with the former conſtruction & demonſtration, and it will make it very eaſye to conceaie.



¶ *Corollary.*

By this it is manifeſt, that if there be two rectiline ſuperficiall angles equall, and vpon thoſe angles be eleuated on high equall right lines containing together with the right lines put at the beginning equall angles: perpendicular lines drawen from thoſe eleuated lines to the ground plaine ſuperficieces wherein are the angles put at the beginning, are equall the one to the other. For it is manifeſt, that the perpendicular lines H K, & M N, which are drawen from the endes of the equall eleuated lines A H, and D M, to the ground ſuperficieces, are equall.

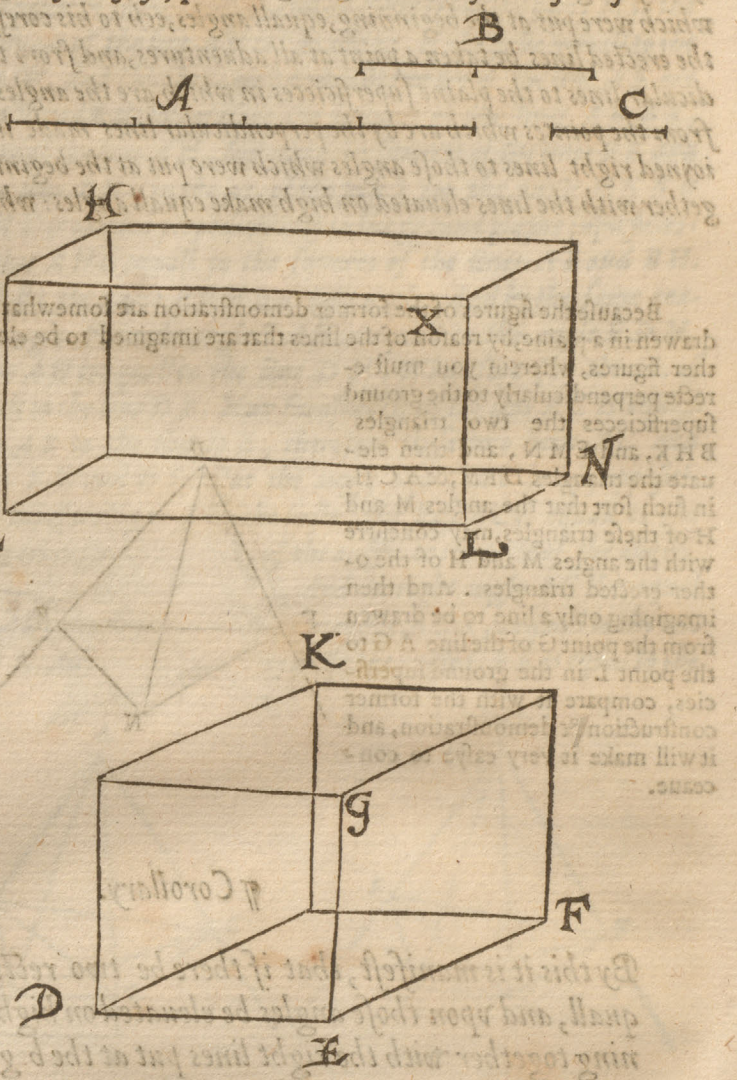
¶ *The 31. Theoreme.* *The 36. Propoſition.*

If there be three right lines proportionall: a Parallelipipedon deſcribed of thoſe three right lines, is equall to the Parallelipipedon deſcribed of the middle line, ſo that it conſiſte of equall ſides, and alſo be equiangle to the foreſayd Parallelipipedon.

Vppoſe that theſe three lines A, B, C, be proportionall, as A is to B, ſo let B be to C. Then I ſay, that the Parallelipipedon made of the lines A, B, C, is equall to the Parallelipipedon made of the line B, ſo that the ſolide made of the line B conſiſt of equall

Construction.

*quall sides, and be also equiangle to the solide made of the lines A,B,C. Describe (by the 23. of the eleuenth) a solide angle E contained vnder three superficiall angles, that is, D E G, G E F, and F E D: and (by the 3. of the first) put vnto the line B euery one of these lines D E, G E, & E F, equall: and make perfecte the solide E K. And vnto the line A let the line L M be equall. And (by the 26. of the eleuēth) vnto the right line L M, and at the point in it L, describe vnto the solide angle E an equall solide angle, cōtained vnder these plaine superficiall angles N L X, X L M, and N L M, and vnto the line B put the line L X equall, & the line L N to the line C. Now for that as the line A is to the line B, so is the line B to the line C: but the line A is equall to the line L M, and the line B to euery one of these lines L X, E F, E G, and E D, and the line C to the line L N. Wherefore as L M is to E F, so is D E to L N: So then the sides about the equal angles M L N, & D E F, are reciprokall: Wherfore (by the 14. of the sixt) the parallelogrāme M N is equall to the parallelogramme D F. And forasmuch as two plaine superficiall angles, namely, D E F and N L M are equall the one to the other, and vpon them are erected vpward equall right lines, L X and E G, comprehending with the right lines put at the beginning equall angles the one to the other. Wherefore * perpendicular lines drawen from the pointes X and G to the plaine superficieces wherin are the angles N L M, and D E F, are (by the Corollary of the former Proposition) equall the one to the other: and those perpendiculars are the altitudes of the Parallelipipedons L H and E K, by the 4. definition of the sixt. Wherfore the solides L H and E K, are vnder one and the selfe same altitude. But Parallelipipedons consisting vpon equall bases, and being vnder one and the selfe same altitude, are (by the 31. of the eleuenth) equall the one to the other. Wherefore the solide L H is equall to the solide E K. But the solide L H is described of the lines A,B,C, and the solide E K is described of the line B. Wherefore the Parallelipipedon described of the lines A,B,C, is equall to the Parallelipipedon made of the line B, which consisteth of equall sides, and is also equiangle to the foresaid Parallelipipedon. If therfore there be three right lines proportionall, a Parallelipipedō described of those three right lines is equall to the Parallelipipedō described of the middle line, so that it consist of equall sides, and also be equiangle to the foresaid Parallelipipedon: which was required to be proued.*



Demonstration.

* *It is euidēt that those perpendiculars are all one with the standing lines of the solides, if their solide angles be made of superficiall right angles onely.*

The construction and demonstration of this Proposition, and of the next Proposition following, may easily be conceaued and vnderstanded by the figures described in the plaine belonging to them. But ye may for the more full sight of them, describe such bodies of pasted paper, hauing their sides proportionall, as is required in the Propositions.

¶ *New inuentions (coincident) added by Master Iohn Dee.*

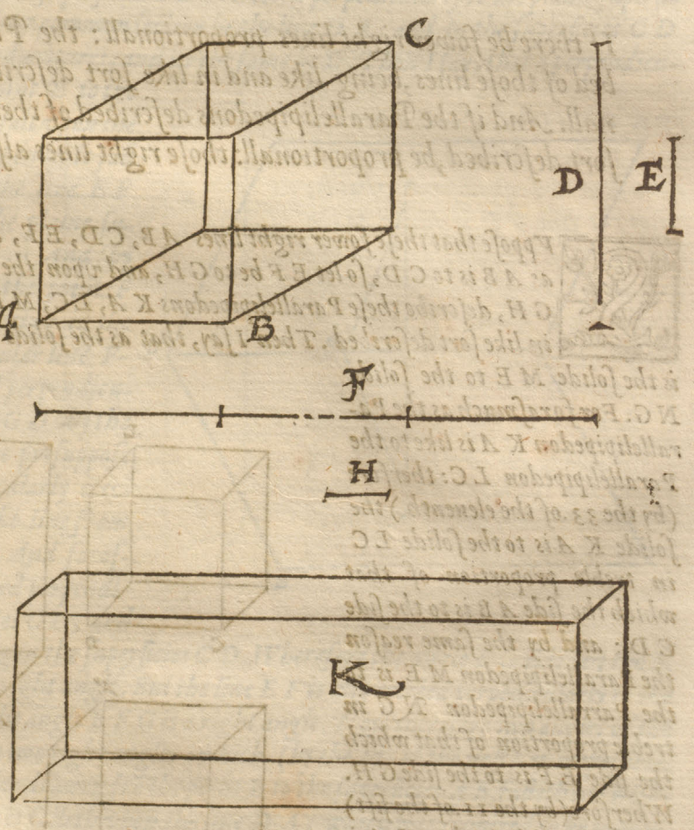
✻ *A Corollary.* 1.

Hereby it is euident, that if three right lines be proportionall: the Cube produced of the middle line, is equall to the rectangle Parallelipipedon made of those three lines.

For a Cube is a Parallelipipedon of equall sides: and also rectangled: as we suppose the Parallelipipedon, made of the three lines to be likewise rectangled.

¶ *A Probleme.* 1.

A Cube being geuen, to finde three right lines proportionall, in any proportion geuen betwene two right lines: of which three lines, the rectangle Parallelipipedon produced, shall be equall to the Cube geuen.



Suppose A C to be the Cube geuen: whose roote, suppose to be A B. Let the proportion geuen, be that which is betwene the two right lines D and E. I say now, three right lines are to be found, proportionall, in the proportion of D to E, of which, the rectangle Parallelipipedon produced, shall be equall to A C. By the 12. of the sixt let a line be found, which to A B haue that proportion that D hath to E. Let that line be F: and by the same 12. of the sixth, let an other line be found, to which, A B, hath that proportion that D hath to E: and let that line found be H. Let a rectangle Parallelipipedon mathematically be produced of the three right lines F, A B, and H, which suppose to be K: I say now, that F, A B, and H, are three right lines found proportionall in the proportion of D to E, of which, the rectangle Parallelipipedon K, produced, is equall to A C the Cube geuen. First it is euident that F, A B, and H, are proportionall in the proportion of D to E. For, by construction, as D is to E, so is F to A B: and by construction likewise, as D is to E, so is A B to H. Wherefore F is to A B, and A B is to H, as D is to E. So then it is manifest, F, A B, and H, to be proportionall in the proportion of D to E, and A B to be the middle line. By my former Corollary, therefore, the rectangle parallelipipedon made of F, A B, and H, is equall to the Cnbe made of A B. But A C, is (by supposition) the Cube made of A B: and of the three lines F, A B, and H, the rectangle parallelipipedon produced, is K, by construction: Wherfore, K, is equall to A C: A Cube being geuen, therefore, three right lines are found, proportionall in

any proportion gene betwene two right lines, of which three right lines the rectangle parallelipipedon produced, is equall to the Cube geuen. Which ought to be done.

✻ A Probleme. 2.

A rectangle Parallelipipedon being geuen, to finde three right lines proportionall: of the which, the rectangle Parallelipipedon produced, is equall to the rectangle Parallelipipedon geuen.

Doubling of the Cube. &c.

** Demonstration of possibilitie in the Problem.*

An other argument to comfort the studious.

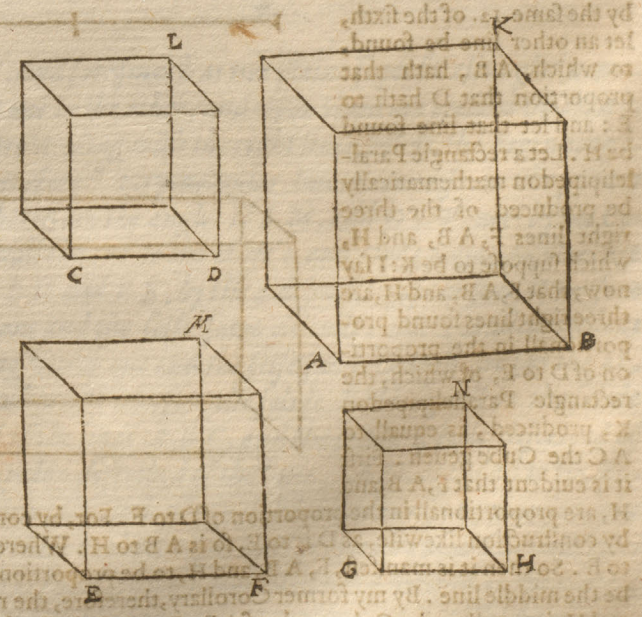
Listen to this new deuise, you couragious Mathematiciens: consider, how nere this crepeth to the famous Probleme of doubling the Cube. What hope may (in maner) any young beginner cóceiue, by one meanes or other, at one time or other, to execute this Probleme? * Seing to a Cube may infinitely infinite Parallelipipedons be found equall: all which Parallelipipedons shall be produced of three right lines proportionall, by the former Probleme: but to any rectangle Parallelipipedon geuen, some one Cube is equall: as is easie to demonstrate: We can not doubt, but vnto our rectangle Parallelipipedon geuen, many other rectangle Parallelipipedons are also equall, hauing their three lines of production, proportionall. In the former Probleme, infinitely infinite Parallelipipedons may be found of three proportionall lines produced, equall to the Cube geuen: it is to wete, the three lines to be of all proportions, that a man can deuise betwene two right lines: and here any one will serue: where also is infinite varietie: though all of one quantitie: as before in the Cube. I leaue as now, with thys marke here set vp to thoote at. Hit it who can.

¶ The 32. Theoreme. The 37. Proposition.

If there be fower right lines proportionall: the Parallelipipedons described of those lines, being like and in like sort described, shall be proportionall. And if the Parallelipipedons described of them, being like and in like sort described, be proportionall: those right lines also shall be proportionall.

Vppose that these fower right lines A B, C D, E F, and G H, be proportionall, as A B is to C D, so let E F be to G H, and vpon the lines A B, C D, E F, and G H, describe these Parallelipipedons K A, L C, M E, and N G, being like and in like sort described. Then I say, that as the solide K A is to the solide L C, so is the solide M E to the solide N G. For forasmuch as the Parallelipipedon K A is like to the Parallelipipedon L C: therfore (by the 33. of the eleuenth) the solide K A is to the solide L C in treble proportion of that which the side A B is to the side C D: and by the same reason the Parallelipipedon M E is to the Parrallelipipedon N G in treble proportion of that which the side E F is to the side G H. Wherfore (by the 11. of the fift) as the Parallelipipedon K A is to the Parallelipipedon L C, so is the Parallelipipedon M E to the Parallelipipedon N G.

Demonstration of the first part.



But now suppose, that as the Parallelipipedon K A is to

the

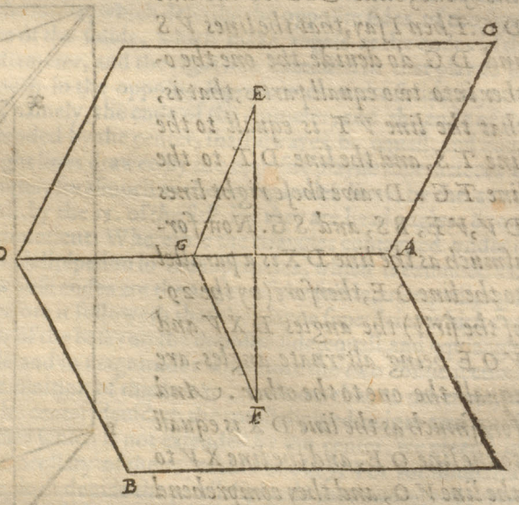
the Parallelipipedon L C, ſo is the Parallelipipedon M E to the Parallelipipedon N G. Then I ſay, that as the right line A B is to the right line C D, ſo is the right line E F to the right line G H. For againe foraſmuch as the ſolide K A is to the ſolide L C in treble proportion of that which the ſide A B is to the ſide C D, and the ſolide M E alſo is to the ſolide N G in treble proportion of that which the line E F is to the line G H, and as the ſolide K A is to the ſolide L C, ſo is the ſolide M E to the ſolide N G. Wherefore alſo as the line A B is to the line C D, ſo is the line E F to the line G H. If therefore there be fower right lines proportionall: the Parallelipipedons deſcribed of thoſe lines, being like & in like ſort deſcribed, ſhall be proportionall. And if the Parallelipipedons deſcribed of them, and being like and in like ſort deſcribed, be proportionall: thoſe right lines alſo ſhall be proportionall. which was required to be proued.

Demonſtration of the ſecond part, which is the conuerſe of the firſt part.

¶ The 33. Theoreme. The 38. Propoſition.

If a plaine ſuperficies be erected perpendicularly to a plaine ſuperficies, and from a point taken in one of the plaine ſuperficieces be drawen to the other plaine ſuperficies, a perpendicular line: that perpendicular line ſhall fall vpon the common ſection of thoſe plaine ſuperficieces.

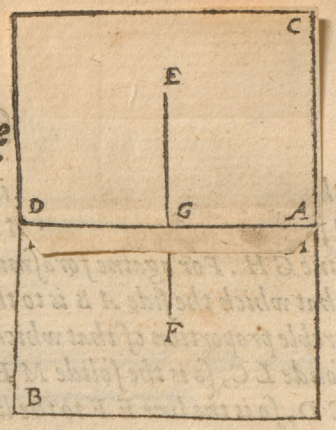
Vppoſe that the plaine ſuperficies C D be erected perpẽdicularly to the plaine ſuperficies A B, and let their common ſection be the line D A: and in the ſuperficies C D take a point at all aduentures, and let the ſame be E. Then I ſay, that a perpendicular line drawen from the point E to the plaine ſuperficies A B, ſhall fall vpon the right line D A. For if not, then let it fall without the line D A, as the line E F doth, and let it fall vpon the plaine ſuperficies A B in the point F. And (by the 12. of the firſt) from the point F draw vnto the line D A, being in the ſuperficies A B a perpendicular line F G, which line alſo is erected perpendicularly to the plaine ſuperficies C D: by the third diffinitiõ: by reaſon we preſuppoſe C D and A B to be perpendicularly erected ech to other. Draw a right line from the point E to the point G. And foraſmuch as the line F G is erected perpendicularly to the plaine ſuperficies C D, and the line E G toucheth it being in the ſuperficies C D. Wherefore the angle F G E is (by the 2. definition of the eleuenth) a right angle. But the line E F is alſo erected perpẽdicularly to the ſuperficies A B: wherefore the angle E F G is a right angle. Now therefore two angles of the triangle E F G, are equall to two right angles: which (by the 17. of the firſt) is impoſſible. Wherfore a perpendicular line drawen frõ the point E to the ſuperficies A B, falleth not without the line D A. Wherefore it falleth vpon the line D A: which was required to be proued.



Demonſtration leading to an impoſſibilitie.

¶ Note.

Campane maketh this as a Corollary, following vpon the 13: and very well, with ſmall ayde of other Propoſitions, he proueth it: whoſe demonſtratiõ there, *Fluſſas* hath in this place, and none other: though he ſayth that *Campane* of ſuch a Propoſitiõ, as of *Euclides*, maketh no mention.

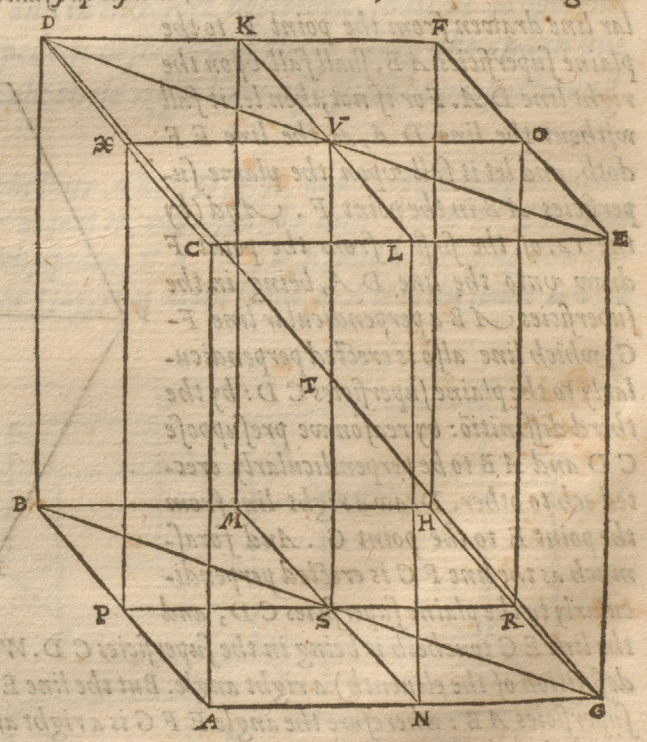


In this figure ye may more fully see the former Propoſition and demonſtration if ye erecte perpendicularly vnto the ground plaine ſuperficies A B the ſuperficies C D, and imagine a line to be extended from the point E to the point F, in ſtede whereof ye may extend if ye will a thred.

¶ The 34. Theoreme. The 39. Propoſition.

If the oppoſite ſides of a Parallelipipedon be deuided into two equall partes, and by their common ſections be extended plaine ſuperficieces: the commō ſection of thoſe plaine ſuperficieces, and the diameter of the Parallelipipedon ſhall deuide the one the other into two equall partes.

SVppoſe that A F be a Parallelipipedon, and let the oppoſite ſides thereof C F and A H be deuided into two equall partes in the pointes K, L, M, N, and likewiſe let the oppoſite ſides A D and G F be deuided into two equall partes in the pointes X, P, O, R, and by thoſe ſections extend theſe two plaine ſuperficieces K N & X R, and let the common ſection of thoſe plaine ſuperficieces be the line V S, and let the diagonall line of the ſolide A B be the line D G. Then I ſay, that the lines V S and D G do deuide the one the other into two equall partes, that is, that the line V T is equall to the line T S, and the line D T to the line T G. Drawe theſe right lines D V, V E, B S, and S G. Now foraſmuch as the line D X is a parallel to the line O E, therfore (by the 29. of the firſt) the angles D X V and V O E being alternate angles, are equall the one to the other. And foraſmuch as the line D X is equall to the line O E, and the line X V to the line V O, and they comprehend equall angles: Wherefore the baſe D V is equall to the baſe V E (by the 4. of the firſt) and the triangle D X V is equall to the triangle V O E, and the reſt of the angles to the reſt of the angles. Wherefore the angle X V D is equall to the angle O V E. Wherefore D V E is one right line, and by the ſame reaſon B S G is alſo one right line, and the line B S is equall to the line S G. And foraſmuch as the line C A is equall to the line D B, and is vnto it a parallel, but the line C A is equall to the line G E, and is vnto it alſo a parallel: wherfore (by the firſt common ſentence) the line D B is equall to the line G E, & is alſo a parallel vnto it: but the right lines D E and B G do ioyne theſe parallel lines together: Wherefore (by the 33. of the firſt) the line D E is a parallel vnto the line B G. And in either of theſe lines are



Conſtruction.

Demonſtration.

are taken pointes at all aduentures, namely, D, V, G, S, and a right line is drawen from the point D to the point G, and an other from the point V to the point S. Wherefore (by the 7. of the eleuenth) the lines D G and V S are in one and the selfe same plaine superficies. And forasmuch as the line D E is a parallel to the line B G, therefore (by the 24. of the first) the angle E D T is equall to the angle B G T, for they are alternate angles, and likewise the angle D T V is equall to the angle G T S. Now then there are two triangles, that is, D T V and G T S, hauing two angles of the one equall to two angles of the other, and one side of the one equall to one side of the other, namely, the side which subtendeth the equall angles, that is, the side D V to the side G S, for they are the halfes of the lines D E and B G: Wherefore the sides remayning are equall to the sides remayning. Wherfore the line D T is equall to the line T G, and the line V T to the line T S. If therefore the opposite sides of a Parallelipipedon be deuided into two equall partes, and by their sections be extended plaine superficieces, the common section of those plaine superficieces, and the diameter of the Parallelipipedon, do deuide the one the other into two equall partes: which was required to be demonstrated.

A Corollary added by *Flussas*.

Euery playne superficies extended by the center of a parallelipipedon, diuideth that solide into two equall partes: and so doth not any other playne superficies not extended by the center.

For euery playne extended by the center, cutteth the diameter of the parallelipipedon in the center into two equall partes. For it is proued, that playne superficieces which cutte the solide into two equall partes, do cut the dimetient into two equall partes in the center. Wherefore all the lines drawen by the center in that playne superficies shall make angles with the dimetient. And forasmuch as the diameter falleth vpon the parallel right lines of the solide, which describe the opposite sides of the sayde solide, or vpon the parallel playne superficieces of the solide, which make angels at the endes of the diameter: the triangles contayned vnder the diameter, and the right line extended in that playne by the center, and the right line, which being drawen in the opposite superficieces of the solide, ioyneth together the endes of the foresayde right lines, namely, the ende of the diameter, and the ende of the line drawen by the center in the superficies extended by the center, shall alwayes be equall, and equiangle, by the 26. of the first. For the opposite right lines drawen by the opposite playne superficieces of the solide do make equall angles with the diameter, forasmuch as they are parallel lines, by the 16. of this booke. But the angles at the céter are equall, by the 15. of the first, for they are head angles: & one side is equall to one side, namely, halfe the dimetient. Wherefore the triangles contayned vnder euery right line drawen by the center of the parallelipipedon in the superficies, which is extended also by the sayd center, and the diameter thereof, whose endes are the angles of the solide, are equall, equilater, & equiangle (by the 26. of the first). Wherfore it followeth that the playne superficies which cutteth the parallelipipedon, doth make the partes of the bases on the opposite side, equall, and equiangle, and therefore like, and equall both in multitude, and in magnitude: wherefore the two solide sections of that solide, shalbe equall and like, by the 8. diffinition of this booke. And now that no other playne superficies, besides that which is extended by the center, deuideth the parallelipipedon into two equall partes, it is manifest: if vnto the playne superficies which is not extended by the center, we extend by the center a parallel playne superficies (by the Corollary of the 15. of this booke). For forasmuch as that superficies which is extended by the center, doth deuide the parallelipipedō into two equall parts: it is manifest, that the other playne superficies (which is a parallel to the superficies which deuideth the solide into two equall partes) is in one of the equall partes of the solide: wherefore seing that the whole is euer greater then his partes, it must nedes be that one of these sections is lesse then the halfe of the solide, and therefore the other is greater.

For the better vnderstanding of this former proposition, & also of this Corollary added by *Flussas*, it shalbe very nedefull for you to describe of pasted paper or such like matter a parallelipipedō or a Cube, and to deuide all the parallelogrāmes therof into two equall parts, by drawing by the céters of the sayd parallelogrammes (which centers are the poynts made by the cutting of diagonall lines drawen frō the opposite angles of the sayd parallelogrāmes) lines parallels to the sides of the parallelogrāmes: as in the former figure described in a plaine ye may see, are the sixe parallelogrāmes D E, E H, H A, A D, DH, and C G, whom these parallel lines drawen by the céters of the sayd parallelogrāmes, namely, X O, O R, P R, and P X, do deuide into two equall parts: by which fower lines ye must imagine a playne superficies to be extended, also these parallel lynes K L, L N, N M, and M K, by which fower lines likewise ye must imagine a playne superficies to be extended ye: may if ye will put within your body made thus of pasted paper, two superficieces made also of the sayd paper, hauing to their limites lines equall to the foresayde parallel lines: which superficieces must also be deuided into two equall partes by parallel

lines drawen by their centers, and muſt cut the one the other by theſe parallel lines. And for the diameter of this body, extēd a thred from one angle in the baſe of the ſolide to his oppoſite angle, which ſhall paſſe by the center of the parallelipipedon, as doth the line D G in the figure before deſcribed in the playne. And draw in the baſe and the oppoſite ſuperficies vnto it, Diagonall lines, from the angles from which is extended the diameter of the ſolide: as in the former deſcription are the lines B G and D E. And when you haue thus deſcribed this body, compare it with the former demonſtration, and it will make it very playne vnto you, ſo your letters agree with the letters of the figure deſcribed in the booke. And this deſcription will playnely ſet forth vnto you the corollary following that propoſition. For where as to the vnderſtanding of the demonſtration of the propoſitiou the ſuperficieces put within the body wete extended by parallel lynes drawen by the cēters of the baſes of the parallelipipedon: to the vnderſtanding of the ſayd Corollary, ye may extende a ſuperficies by any other lines drawen in the ſayde baſes, ſo that yet it paſſethrough the middeſt of the thred, which is ſuppoſed to be the center of the parallelipipedon.

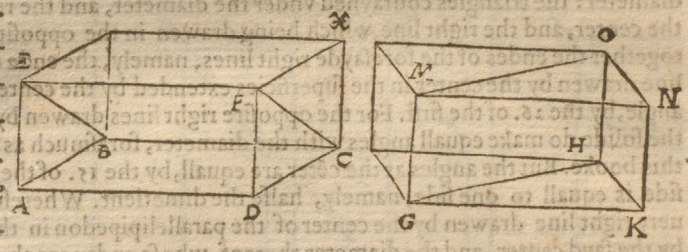
¶ The 35. Theoreme. The 40. Propoſition.

If there be two Priſmes vnder equall altitudes, & the one haue to his baſe a parallelogramme, and the other a triangle, and if the parallelogramme be double to the triangle: thoſe Priſmes are equall the one to the other.

Conſtruction.

Demonſtration.

Vppoſe that theſe two Priſmes A B C D E F, G H K M O N, be vnder equall altitudes, and let the one haue to his baſe the parallelogramme A C, and the other the triangle G H K, and let the parallelogramme A C be double to the triangle G H K. Then I ſay, that the Priſme A B C D E F is equall to the Priſme G H K M O N. Make perfecte the Parallelipipedons A X & G O. And foraſmuch as the parallelogramme A C is double to the triangle G H K, but the parallelogramme G H is alſo (by the 41. of the firſt) double to the triangle G H K, wherefore the parallelogramme A C is equall to the parallelogramme G H. But Parallelipipedons conſiſting vpon equall baſes and vnder one and the ſelfe ſame altitude, are equall the one to the other (by the 31. of the eleuenth). Wherefore the ſolide A X is equall to the ſolide G O. But the halfe of the ſolide A X is the Priſme A B C D E F, and the halfe of the ſolide G O is the Priſme G H K M O N. Wherfore the Priſme A B C D E F is equall to the Priſme G H K M O N. If therefore there be two Priſmes vnder equall altitudes, and the one haue to his baſe a parallelogramme, & the other a triangle, and if the parallelogramme be double to the triangle: thoſe Priſmes are equall the one to the other: which was required to be proued.



This Propoſition and the demonſtration thereof are not hard to conceaue by the former figures: but ye may for your fuller vnderſtanding of thē take two equall Parallelipipedons equilater and equiangle the one to the other deſcribed of paſted paper or ſuch like matter, and in the baſe of the one Parallelipipedon draw a diagonall line, and draw an other diagonall line in the vpper ſuperficies oppoſite vnto the ſaid diagonall line drawen in the baſe. And in one of the parallelogrammes which are ſet vpon the baſe of the other Parallelipipedon draw a diagonall line, and drawe an other diagonall line in the parallelogramme oppoſite to the ſame. For ſo if ye extend plaine ſuperficieces by thoſe diagonall lines there will be made two Priſmes in ech body. Ye muſt take heede that ye put for the baſes of eche of theſe Parallelipipedons equall parallelogrāmes. And then note thē with letters according to the letters of the figures before deſcribed in the plaine. And cōpare thē with the demonſtration, and they will make both it and the Propoſition very clere vnto you. They will alſo geue great light to the Corollary following added by *Fluſſas*.

By

A Corollary added by *Flussas.*

By this and the former propositions it is manifest, that Prismes and solides contayned vnder two poligonon figures equall, like, and parallels, and the rest parallelogrammes: may be compared the one to the other after the selfe same maner that parallelipipedons are.*

* *Which of some are called sided Columnes.*

For forasmuch as (by this proposition and by the second Corollary of the 25. of this booke) it is manifest, that euery parallelipipedon may be resolued into two like, and equal Prismes, of one and the same altitude, whose base shalbe one and the selfe same with the base of the parallelipipedon or the halfe thereof, which Prismes also shalbe contayned vnder the selfe same sides with the parallelipipedō, the sayde sides beyng also sides of like proportion: I say that Prismes may be compared together after the like maner that their Parallelipipedons are. For if we would deuide a Prisme like vnto his soli e by the 25. of this booke, ye shall finde in the Corollaryes of the 25. propositiō, that that which is set forth touching a parallelipipedon, followeth not onely in a Prisme, but also in any sided columne whose opposite bases are equall, and like, and his sides parallelogrammes.

If it be required by the 27. proposition vpon a right line geuen to describe a Prisme like and in like sorte situate to a Prisme geuen: describe first the whole parallelipipedon whereof the prisme geuen is the halfe (which thing ye see by this 40. proposition may be done). And vnto that parallelipipedō describe vpon the right line geuen by the sayd 27. proposition an other parallelipipedon like: and the halfe thereof shalbe the prisme which ye seeke for, namely, shalbe a prisme described vpon the right line geuen, and like vnto the prisme geuen.

In deede Prismes can not be cut according to the 28. proposition. For that in their opposite sides can be drawen no diagonall lines: howbeit by that 28. proposition those Prismes are manifestly confirmed to be equall and like, which are the halues of one and the selfe same parallelipipedon.

And as touching the 29. proposition, and the three following it, which proueth that parallelipipedons vnder one and the selfe same altitude, and vpon equall bases, or the selfe same bases, are equal: or if they be vnder one and the selfe same altitude, they are in proportion the one to the other, as their bases are: to apply these comparisons vnto Prismes, it is to be required, that the bases of the Prismes compared together, be either all parallelogrammes, or all triangles. For so one and the selfe altitude remayning, the comparison of thinges equall is euer one and the selfe same, and the halfes of the bases are euer the one to the other in the same proportion, that their wholes are. Wherfore Prismes which are the halues of the parallelipipedons, and which haue the same proportion the one to the other that the whole parallelipipedons haue, which are vnder one and the selfe same altitude: must needes cause that their bases beyng the halues of the bases of the parallelipipedōs are in the same proportiō the one to the other, that their whole parallelipipedons are. If therefore the whole parallelipipedons be in the proportion of the whole bases, their halues also (which are Prismes) shalbe in the proportion either of the wholes if their bases be parallelogrammes, or of the halues if they be triangles, which is euer all one by the 15. of the fiueth.

And forasmuch as by the 33. proposition, like parallelipipedons which are the doubles of their Prismes are in treble proportion the one to the other that their sides of like proportion are, it is manifest, that Prismes being their halues (which haue the one to the other the same proportion that their wholes haue, by the 15 of the fiueth) and hauing the selfe same sides that their parallelipipedons haue, are the one to the other in treble proportion of that which the sides of like proportion are.

And for that Prismes are the one to the other in the same proportion that their parallelipipedons are, and the bases of the Prismes (being all either triangles or parallelogrāmes) are the one to the other in the same proportion that the bases of the parallelipipedons are, whose altitudes also are alwayes equall, we may by the 34. proposition conclude, that the bases of the prismes and the bases of the parallelipipedons their doubles (being ech the one to the other in one and the selfe same proportion) are to the altitudes, in the same proportion that the bases of the double solides, namely, of the parallelipipedons are. For if the bases of the equall parallelipipedōs be reciprokall with their altitudes, then their halues which are Prismes shall haue their bases reciprokall with their altitudes.

By the 36. proposition we may conclude, that if there be three right lines proportionall, the angle of a Prisme made of these three lines (being common with the angle of his parallelipipedon which is double) doth make a prisme, which is equall to the Prisme described of the middle line, and contayning the like angle, consisting also of equall sides. For as in the parallelipipedon, so also in the Prisme, this one thing is required, namely, that the three dimensions of the proportionall lines do make an angle like vnto the angle contayned of the middle line taken three tymes. Now then if the solide angle of the Prisme be made of those three right lines, there shall of them be made an angle like to the angle of the parallelipipedon which is double vnto it. Wherefore it followeth of necessitie, that the Prismes which are alwayes the halues of the Parallelipipedons, are equiangle the one to the other, as also are their doubles, although they be not equilater: and therefore those halues of equall solides are equall the one to the other: namely, that which is described of the middle proportionall line is equall to that which is described of the three proportionall lines.

By the 37. proposition also we may conclude the same touching Prismes which was concluded

touching Parallelipipedons. For forasmuch as Prismes, described like & in like sort of the lines geuen, are the halues of the Parallelipipedons which are like and in like sort described, it followeth that these Prismes haue the one to the other the same proportion that the solides which are their doubles haue. And therfore if the lines which describe them be porportionall, they shalbe proportionall, and so conuersedly according to the rule of the sayd 37. proposition.

But forasmuch as the 39. proposition supposeth the opposite superficiall sides of the solide to be parallelogrammes, and the same solide to haue one diameter, which thinges a Prisme can not haue, therefore this proposition can by no meanes be applyed to Prismes.

Sided Columnes.

But as touching solides whose bases are two like, equall, and parallel poligonon figures, and their sides are parallelogrammes, forasmuch as by the second Corollary of the 25. of this booke it hath bene declared, that such solides are composed of Prismes, it may easely be proued that their nature is such, as is the nature of the Prismes, whereof they are composed. Wherefore a parallelipipedon being by the 27. proposition of this booke described, there may also be described the halfe thereof, which is a Prisme: and by the description of Prismes, there may be composed a solide like vnto a solide geuen composed of Prismes.

So that it is manifest, that that which the 29. 30. 31. 32. 33. 34. and 37. propositions set forth touching parallelipipedons, may well be applyed also to these kyndes of solides.

The end of the eleuenth booke of Euclides Elementes.

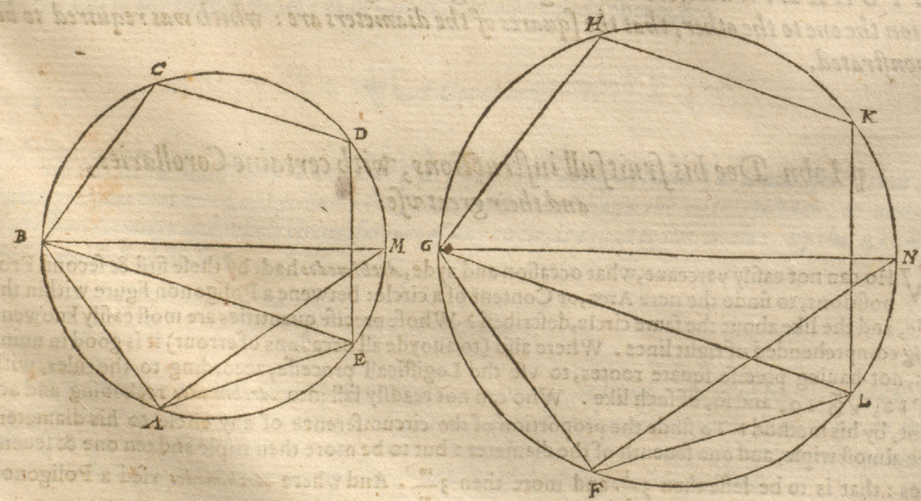
¶ The twelueth booke of Euclides Elementes.

IN THIS TVVELVETH BOOKE, EVCLIDE ſetteth forth the paſſions and proprieties of Pyramids, Priſmes, Cones, Cylinders, and Spheres. And compareth Pyramids, firſt to Pyramids, then to Priſmes: ſo likewiſe doth he Cones, and Cylinders. And laſtly he compareth Spheres the one to the other. But before he goeth to the treatie of thoſe bodies, he proueth that, like Poligonon figures inſcribed in circles, and alſo the circles thẽ ſelues are in proportion the one to the other, as the ſquares of the diameters of thoſe circles are. Becauſe that was neceſſary to be proued, for the confirmation of certayne paſſions and proprieties of thoſe bodies.

¶ *The 1. Theoreme.* *The 1. Propoſition.*

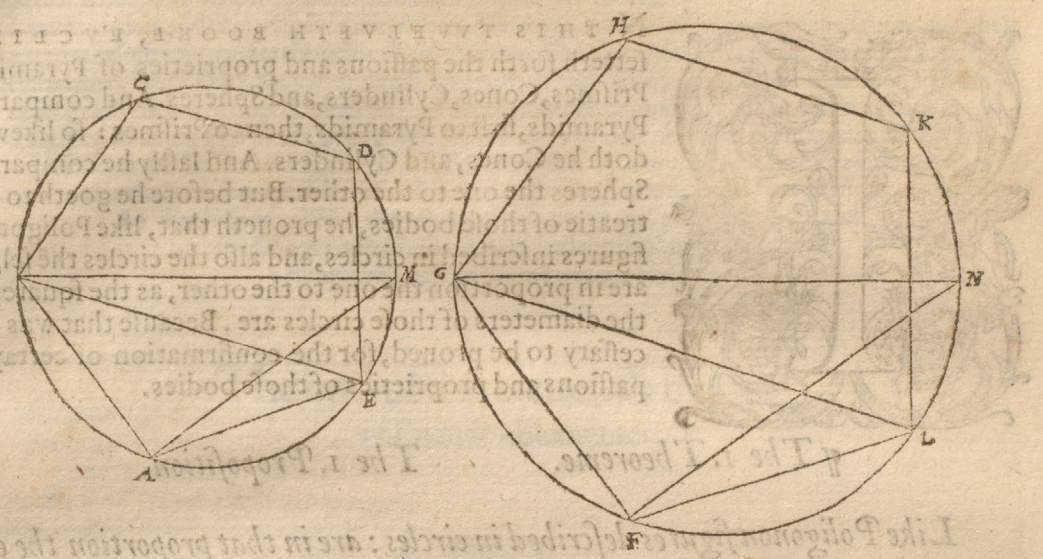
Like Poligonon figures deſcribed in circles: are in that proportion the one to the other, that the ſquares of their diameters are.

VPpoſe that there be two circles ABCDE, and FGHKL, and in them let there be deſcribed like Poligonon figures, namely, ABCDE, and FGHKL, and let the diameters of the circles be BM, and GN. Then I ſay, that as the ſquare of the line BM is to the ſquare of the line GN, ſo is the Poligonon figure ABCDE to the Poligonon figure FGHKL. Drawe theſe right lines BE, AM, GL, and FN. And foraſmuch as the Poligonon figure ABCDE is like to the Poligonon figure FGHKL, therefore the angle BAE is equall to the angle *Conſtruction.* *Demonſtration.*



GFL, and as the line BA is to the line AE, ſo is the line GF to the line FL (by the definition of like Poligonon figures). Now therefore there are two triangles BAE and GFL, hauing

hauing one angle of the one equall to one angle of the other, namely, the angle B A E equall to the angle G F L, and the ſides about the equall angles are proportionall. Wherefore (by the firſt definition of the ſixt) the triangle A B E is equiangle to the triangle F G L. Wherefore the angle A E B is equall to the angle F L G. But (by the 21. of the third) the angle A E B is equall to the angle A M B, for they conſiſte vpon one and the ſelfe ſame circumference: and

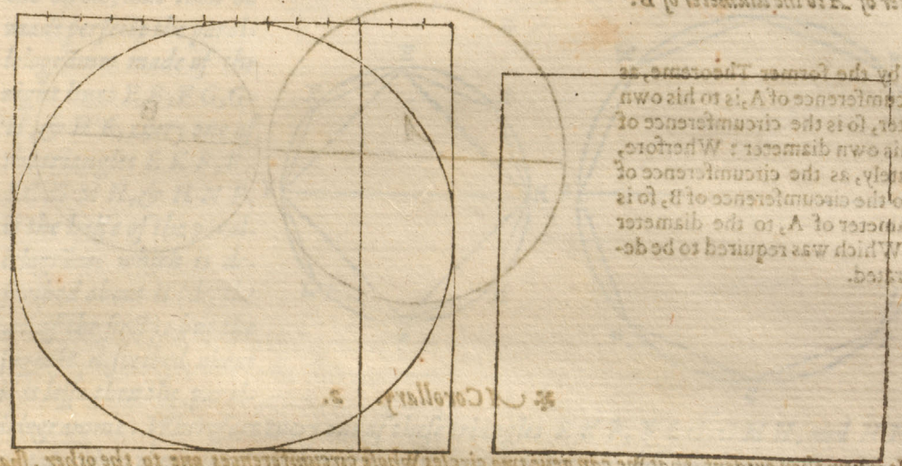


by the ſame reaſon the angle F L G is equall to the angle F N G. Wherfore the angle A M B is equall to the angle F N G. And the right angle B A M, is (by the 4. petition) equall to the right angle G F N. Wherefore the angle remayning, is equall to the angle remayning. Wherefore the triangle A M B is equiangle to the triangle F N G. Wherefore proportionally as the line B M is to the line G N, ſo is the line B A to the line G F. But the ſquare of the line B M is to the ſquare of the line G N in double proportion of that which the line B M is to the line G N (by the Corollary of the 20. of the ſixt). And the Poligonon figure A B C D E is to the Poligonon figure F G H K L in double proportion of that which the line B A is to the line G F (by the 20. of the ſixt). Wherefore (by the 11. of the fift) as the ſquare of the line B M is to the ſquare of the line G N, ſo is the Poligonon figure A B C D E, to the Poligonon figure F G H K L. Wherefore like Poligonon figures deſcribed in circles, are in that proportion the one to the other, that the ſquares of the diameters are: which was required to be demonſtrated.

¶ Iohn Dee his fruitfull inſtructions, with certaine Corollaries, and their great vſe.

WHo can not eaſily perceaue, what occaſion and ayde, *Archimedes* had, by theſe firſt & ſecond Propoſitions, to finde the nere Area, or Content of a circle: betwene a Poligonon figure within the circle, and the like about the ſame circle, deſcribed? Whoſe preciſe quantities are moſt eaſily knowen: being comprehended of right lines. Where alſo (to auoyde all occaſions of errour) it is good in numbers, not hauing preciſe ſquare rootes, to vſe the Logiſticall proceſſe, according to the rules, with $\sqrt{3}.12$, $\sqrt{3}.19$, and ſo, of ſuch like. Who can not readily fall into *Archimedes* reckoning and account, by his method? To finde the proportion of the circumference of any circle to his diameter, to be almoſt triple, and one ſeuenth of the diameter: but to be more then triple and ten one & ſeuentithes: that is to be leſſe then $3\frac{1}{7}$ and more then $3\frac{10}{71}$. And where *Archimedes* vſed a Poligonon figure of 96. ſides: he that, for exerciſe ſake, or for earneſt deſire of a more nerenes, will vſe Polygonon figures of 384. ſides (or more) may well trauaile therein, till either wearines cauſe him ſtay, or els he finde his labour fruitles. In deede *Archimedes* concluded proportion, of the circumference to the diameter,

meter, hath hitherto ſerued the vulgare and Mechánicall workemen: wherewith, who ſo is not contented, let his owne Methodicall trauaile ſatiſfie his deſire: or let him procure other therto. For, narrower termes (of greater and leſſe) found, and appointed to the circumference, will alſo winne to the Area of the circle a nearer quantitie: ſeing, it is well demóſtrated of *Archimedes*, that a triangle rectangle, of whoſe two ſides (contayning the right angle) one is equall to the ſemidiameter of the circle, and the other to the circumference of the ſame, is equall to the Area of that circle. Vpon which two Theoremes, it followeth, that the ſquare made of the diameter, is in that proportion to the circle (very neare) in which, 14, is to 11. Wherefore euery circle is $\frac{11}{14}$ eleuen fowertenthes (well neare) of the ſquare about him deſcribed. The one ſide, then, of that ſquare, deuide into 14. equall partes: and from that point which endeth the eleuenth part, drawe to the oppoſite ſide, a line, parallel to the other ſides, and ſo make perfecte the parallelogramme. Then, by the laſt Propoſition of the ſecond booke, vnto that parallelogramme (whoſe one ſide hath thoſe 11. equall partes), make a ſquare equall. Then is it euident, that ſquare to be equall to the circle, about which the firſt ſquare is deſcribed. As ye may here beholde in theſe figures. 2. 3. *The ſquaring of the circle.*



Gentle frend, the great deſire, which I haue, that both with pleaſure and alſo profite, thou mayeſt ſpend thy time in theſe excellent ſtudies, doth cauſe me here to furniſhe thee ſomewhat (extraordinarily) about the circle: not onely by pointing vnto thee, the wellſpring of *Archimedes*, his ſo much wondred at, and iuſtly commended trauaile (in the former 3. Theoremes, here repeated), but alſo to make thee more apt, to vnderſtand and practiſe this and other bookes following, where, vſe of the circle may be had in any conſideration: as in Cones, Cylinders, and Spheres, &c.

¶ *A Corollary.* 1.

By Archimedes *ſecond Theoreme (as I haue here alleaged them) it is manifeſt, that a parallelogramme contained, either vnder the ſemidiameter and halfe the circumference, or vnder the halfe ſemidiameter and whole circumference of any circle, is equall to the circle: by the 41. of the firſt: and firſt of the ſixt.*

¶ *A Corollary.* 2.

Likewiſe it is euident, that the parallelogramme contayned vnder the ſemidiameter, and halfe of any portion of the circumference of a circle geuen, is equall to that ſector of the ſame circle, to which the whole portion of the circumference geuen, doth belong. Or you may vſe the halfe ſemidiameter, and the whole portion of the circumference: as ſides of the ſaid parallelogramme.

The farther winning, and inferring, I committe to your ſkill, care, and ſtudye. But in an other ſort will I geue you newe ayde, and inſtruction here.

¶ *A Theoreme.*

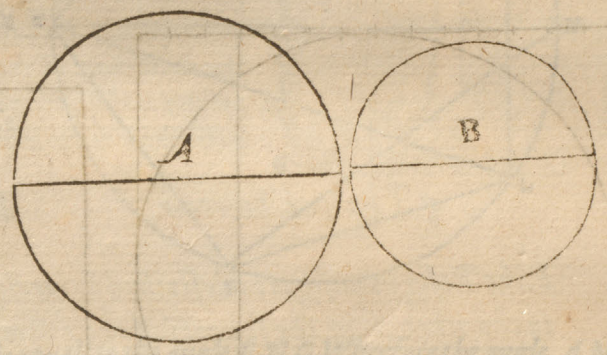
¶ *A Theoreme.*

Of all circles, the circumferences to their owne diameters, haue one and the ſame proportion: in what one circle ſoeuer, they are aſſigned.

That is (as *Archimedes* hath demonſtrated) almoſt, as 22. to 7: or nearer, if nearer be fouud: vntill the very preciſe proportion be demonſtrated. Which, what ſoeuer it be, in all circumferences to their proper diameters, will be demonſtrated one and the ſame.

✻ *A Corollary.* 1.

Wherefore if two circles be propounded, which ſuppoſe to be A and B, as the circumference of A is to the circumference of B, ſo is the diameter of A to the diameter of B.



For by the former Theoreme, as the circumference of A, is to his own diameter, ſo is the circumference of B, to his own diameter: Wherfore, alternately, as the circumference of A, is to the circumference of B, ſo is the diameter of A, to the diameter of B: Which was required to be demonſtrated.

✻ *A Corollary.* 2.

It is now then euident, that we can geue two circles whoſe circumferences one to the other, ſhall haue any proportion geuen in two right lines.

The great Mechanicall vſe (beſides Mathematicall conſiderations) which, theſe two Corollaryes may haue in Wheeles of Milles, Clockes, Cranes, and other engines for water workes, and for warres, and many other purpoſes, the earneſt and wittie Mechanicien will ſoone boult out, & gladly practiſe.

¶ *John Dee.*

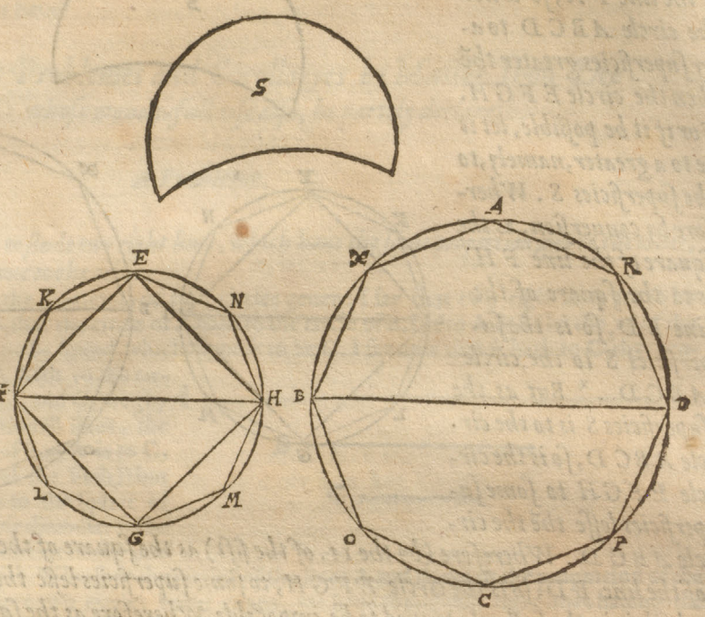
¶ *The 2. Theoreme.* *The 2. Propoſition.*

Circles are in that proportion the one to the other, that the ſquares of their diameters are.

Demonſtration leading to an impoſſibilitie.
Two caſes in this propoſition.
The firſt caſe.

SVppoſe that there be two circles A B C D, and E F G H, and let their diameters be B D and F H. Then I ſay, that as the ſquare of the line D B is to the ſquare of the line F H, ſo is the circle A B C D to the circle E F G H. For if the circle A B C D be not vnto the circle E F G H, as the ſquare of the line B D is to the ſquare of the line F H: then the ſquare of the line B D ſhall be to the ſquare of the line F H, as the circle A B C D is to a ſuperficies, either leſſe then the circle E F G H, or greater. First let the ſquare of the line B D be to the ſquare of the line F H, as the circle A B C D is to a ſuperficies leſſe then the circle E F G H, namely, to the ſuperficies S. Deſcribe (by the 6. of the fourth) in the circle E F G H a ſquare E F G H. Now this ſquare thus deſcribed is greater then the halfe of the circle E F G H. For if by the pointes E, F, G, H, we drawe right lines touching

touching the circle, the ſquare E F G H, is the halfe of the ſquare deſcribed about the circle, but the ſquare deſcribed about the circle, is greater then the circle. Wherefore the ſquare E F G H, which is inſcribed in the circle, is greater then the halfe of the circle E F G H. Deuide the circumferences E F, F G, G H, and H E, into two equall partes in the pointes K, L, M, N. And drawe theſe right lines E K, K F, F L, L G, G M, M H, H N, and N E. Wherefore euery one of theſe triangles E K F, F L G, G M H, and H N E, is greater then the halfe of the ſegmẽt of the circle which is deſcribed about it. For if by the pointes K, L, M, N, be drawen lines touching the circle, and then be made perfecte the parallelogrãmes made of the right lines E F, F G, G H, & H E, euery one of the triangles E K F, F L G, G M H, & H N E, is the halfe of the parallelogrãme which is deſcribed about it (by the 41. of the firſt): but the ſegmẽt deſcribed about it is leſſe then the parallelogramme. Wherefore euery one of theſe triangles E K F, F L G, G M H, and H N E, is greater then the halfe of the ſegment of the circle which is deſcribed about it. Now then deuiding the circumferences remaining into two equall partes, and drawing right lines from the pointes where thoſe diuiſions are made, & ſo continually doing this, we ſhall at the length (by the 1. of the tenth) leaue certaine ſegmentes of the circle, which ſhall be leſſe then the exceſſe, wherby the circle E F G H excedeth the ſuperficies S. For it hath bene proued in the firſt Propoſition of the tenth booke, that two vnequall magnitudes being geuen, if from the greater be taken away more then the halfe, and likewiſe againe from the reſidue more then the halfe, and ſo continually, there ſhall at the length be left a certaine magnitude which ſhall be leſſe then the leſſe magnitude geuen. Let there be ſuch ſegmentes left, & let the ſegmentes of the circle E F G H, namely, which are made by the lines E K, K F, F L, L G, G M, M H, H N, and N E, be leſſe then the exceſſe, whereby the circle E F G H excedeth the ſuperficies S. Wherefore the reſidue, namely, the Poligonon figure E K F L G M H N, is greater then the ſuperficies S. Inſcribe in the circle A B C D a Poligonon figure like to the Poligonon figure E K F L G M H N, and let the ſame be A X B O C P D R. Wherefore (by the Propoſition next going before) as the ſquare of the line B D is to the ſquare of the line F H, ſo is the Poligonon figure A X B O C P D R to the Poligonon figure E K F L G M H N. But as the ſquare of the line B D is to the ſquare of the line F G, ſo is the circle A B C D ſuppoſed to be to the ſuperficies S. Wherefore (by the 11. of the fift) as the circle A B C D is to the ſuperficies S, ſo is the Poligonon figure A X B O C P D R to the Poligonon figure E K F L G M H N. Wherefore alternately (by the 16. of the fift) as the circle A B C D is to the Poligonon figure deſcribed in it, ſo is the ſuperficies S to the Poligonon figure E K F L G M H N. But the circle A B C D is greater then the Poligonon figure deſcribed in it. Wherefore alſo the ſuperficies S is greater then the Poligonon figure E K F L G H M N: but it is alſo leſſe: which is impoſſible. Wherefore as the ſquare of the line B D is to the ſquare of the line F H,

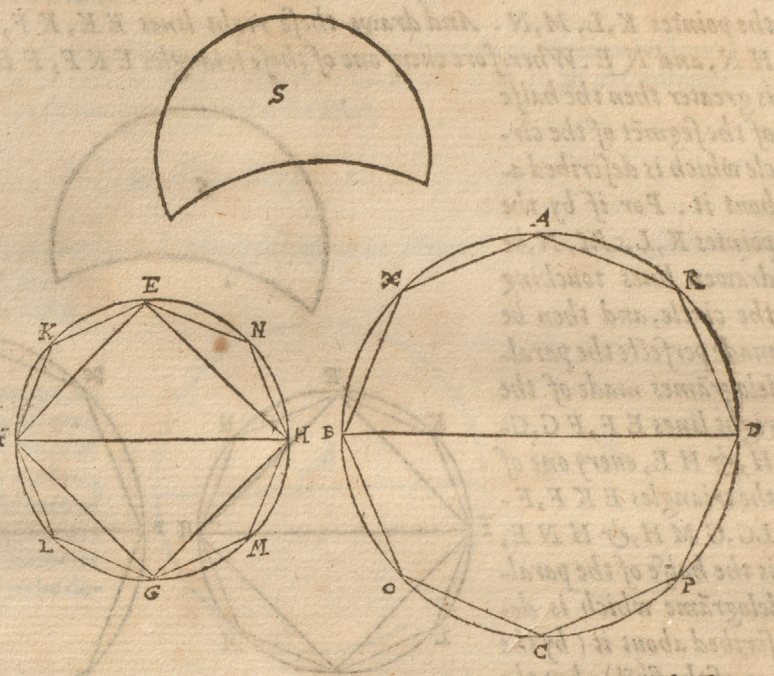


That a ſquare within any circle deſcribed is bigger than halfe the circle.

That the Iſoſceles triangles, without the ſquare, are greater then halfe the ſegments wherin they are.

so is not the circle A B C D to any superficies lesse then the circles E F G H.

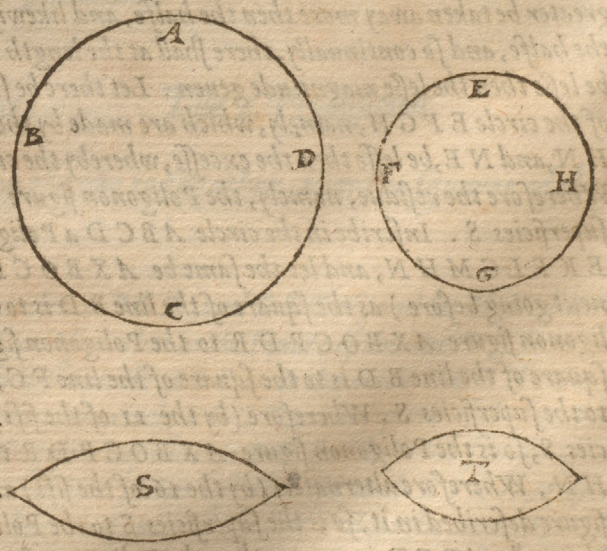
Second case. *In like sort also may wproue, that as the square of the line F H is to the square of the line B D, so is not the circle E F G H to any superficies lesse then the circle A B C D. I say, namely, that as the square of the line B D is to the square of the line F H, so is not the circle A B C D to any superficies greater thē then the circle E F G H. For if it be possible, let it be to a greater, namely, to the superficies S. Wherfore by conuersion, as the square of the line F H is to the square of the line B D, so is the superficies S to the circle A B C D.* * *But as the superficies S is to the circle A B C D, so is the circle E F G H to some superficies lesse thē the circle A B C D. Wherefore (by the 11. of the fift) as the square of the line F H is to the square of the line B D, so is the circle E F G H, to some superficies lesse then the circle A B C D: which is in the first case proued to be impossible. Wherefore as the square of the line B D is to the square of the line F H, so is not the circle A B C D to any superficies greater then the circle E F G H. And it is also proued that it is not, to any lesse. Wherefore as the square of the line B D is to the square of the line F H, so is the circle A B C D to the circle E F G H. Wherefore circles are in that proportion the one to the other, that the squares of their diameters are: which was required to be proued.*



* This Assūpt is afterward at the end of the demōstration proued.

¶ An Assumpt.

I say now, that the superficies S being greater then the circle E F G H, as the superficies S is to the circle A B C D, so is the circle E F G H to some superficies lesse then the circle A B C D. For, as the superficies S is to the circle A B C D, so let the circle E F G H be to the superficies T. Now I say, that the superficies T is lesse then the circle A B C D. For for that as the superficies S is to the circle A B C D, so is the circle E F G H to the superficies T, therefore alternately (by the 16. of the fift) as the superficies S is to the circle E F G H, so is the circle A B C D to the superficies T. But the superficies S is greater then the circle E F G H (by suppositiō). Wherefore also the circle A B C D is greater then the superficies T (by the 14. of the fift). Wherefore as the superficies S is to the circle A B C D,



A B C D, ſo is the circle E F G H to ſome ſuperficies leſſe then the circle A B C D: which was required to be demonſtrated.

¶ A Corollary added by *Fluſſas*.

Circles haue the one to the other, that proportion, that like Poligonon figures and in like ſort deſcribed in them haue. For, it was by the firſt Propoſition proued, that the Poligonon figures haue that proportiõ the one to the other, that the ſquares of the diameters haue, which proportion likewiſe, by this Propoſition the circles haue.

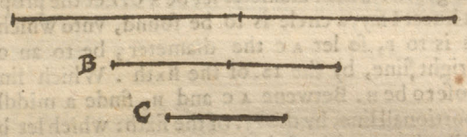
¶ *Very needefull Problemes and Corollaryes by Maſter Ihon Dee inuented: whoſe wonderfull vſe alſo, he partely declareth.*

A Probleme. 1.

Two circles being geuẽ: to finde two right lines, which haue the ſame proportion, one to the other, that the geuen circles haue, one to the other.

Suppoſe A and B, to be the diameters of two circles geuen: I ſay that two right lines are to be foũde, hauing that proportiõ, that the circle of A hath to the circle of B. Let to A & B (by the 11 of the ſixth) a third proportionall line be found, which ſuppoſe to be C. I ſay now that A hath to C, that proportion which the circle of A hath to the circle of B. For foraſmuch as A, B, and C, are (by conſtruction) three proportionall lines, the ſquare of A is to the ſquare of B, as A is to C, (by the Corollary of the 20. of the ſixth): but as the ſquare of the line A is to the ſquare of the line B, ſo is the circle whoſe diameter is the line A, to the circle whoſe diameter is the line B, by this ſecond of the eleuẽth. Wherfore the circles of the lines A and B, are in the proportion of the right lines A and C. Therefore two circles being geuen, we haue found two right lines hauing the ſame proportion betwene thẽ, that the circles geuen, haue one to the other: which ought to be done.

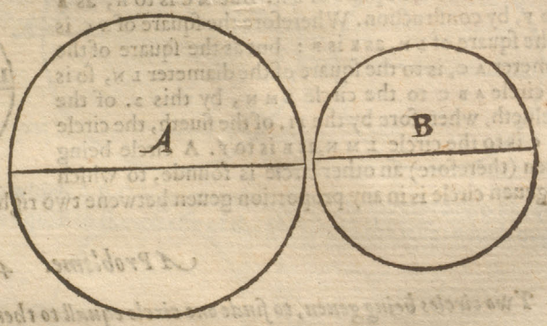
Conſtruction.



Demonſtration.

A Probleme. 2.

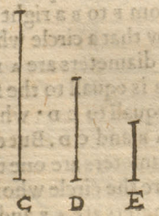
Two circles being geuen, and a right line: to finde an other right line, to which the line geuẽ ſhall haue that proportion, which the one circle hath to the other.



Conſtruction

Suppoſe two circles geuẽ: which let be A & B, & a right line geuẽ, which let be C: I ſay that an other right line is to be founde, to which the line C ſhall haue that proportion that the circle A, hath to the circle B. As the diameter of the circle A, is to the diameter of the circle B, ſo let the line C be to a fourth line, (by the 12. of the ſixth) let that fourth line be D. And, by the 11. of the ſixth, let a thirde line proportionall be found, to the lines C & D, which let be E: I ſay now, that the line C hath to the line E, that proportion which the circle A, hath to the circle B. For (by conſtruction) the lines C, D, and E, are proportionall: therefore the ſquare of C, is to the ſquare of D, as C is to E, by the Corollary of the 20. of the ſixth. But by conſtruction, as the diameter of the circle A, is to the diameter of the circle B, ſo is C, to D: wherefore as the ſquare of the diameter of the circle A, is to the ſquare of the diameter of the circle B, ſo is the ſquare of the line C to the ſquare of the line D, by the 22. of the ſixth. But as the ſquare of the diameter of A, the circle, is to the ſquare of the diameter of the circle B, ſo is the circle A, to the circle B, by the ſecond of the twelfth: wherefore by 11. of the fiueth, as the circle A, is to

Demonſtration.



the circle B, ſo is the ſquare of the line C, to the ſquare of the line D. But it is proued, that as the ſquare of the line C, is to the ſquare of the line D, ſo is the line C to the line E. Wherefore by the 11. of the fiueth, as the circle A, is to the circle B, ſo is the line C to the line E. Two circles being geuen therefore, and a right line, we haue found a right line, to which the right line geuen, hath that proportion, which the one circle hath to the other. Which ought to be done.

Note.

Diffreence betwene the firſt probleme and the ſecond.

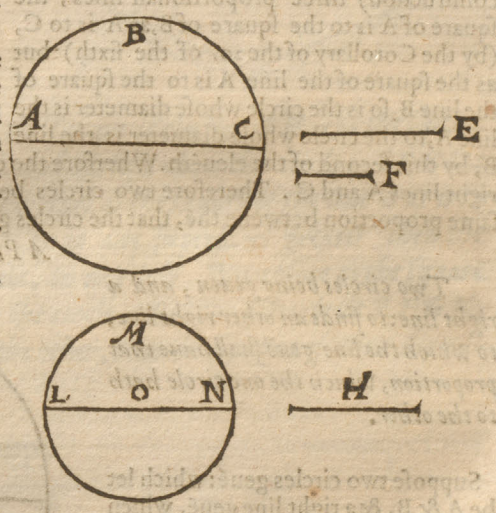
The difference betwene this Probleme, and that next before, is this: there, although we had two circles geuen, and two lines were found in that proportion the one to the other, in which the geuen circles were,: and here likewiſe are two circles geué, and two lines alſo are had in the ſame proportiõ, that the geuẽ circles are: yet there we tooke at pleaſure the firſt of the two lines, wherunto we framed the ſecond proportionally, to the circles geuen. But here the firſt of the two lines, is aſſigned, poynted, and determined to vs: and not our choyſe to be had therein, as was in the former Probleme.

A Probleme. 3.

A circle being geuen, to finde an other circle, to which the geuen circle is in any proportion geuẽ in two right lines.

Conſtruction.

Suppoſe the circle A B C geuen, and therefore his ſemidiameter is geuen: whereby his diameter alſo is geuen: which diameter let be A C. Let the proportion geuen, be that which is betwene E, F, two right lines. I ſay, a circle is to be found, vnto which A B C hath that proportion that E hath to F. As E is to F, ſo let A C the diameter, be to an other right line, by the 12. of the ſixth. Which line ſuppoſe to be H. Betwene A C and H, finde a middle proportionall line, by the 13. of the ſixth: which let be L N. By the 10. of the firſt, deuide L N, into two equall partes: and let that be done in the point O. Now vpon O L, (O being made the center) deſcribe a circle: which let be L M N.



Demonſtration.

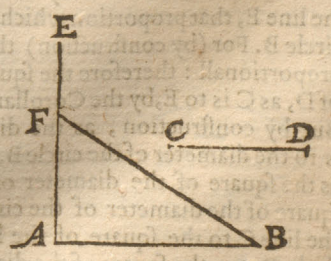
I ſay that A B C, is to L M N, as E is to F. For ſeing that A C, L N, and H, are three right lines in continuall proportion (by conſtruction) therefore (by the Corollary of the 20. of the ſixth) as A C is to H, ſo is the ſquare of A C to the ſquare of L N. But A C is to H, as E is to F, by conſtruction. Wherefore the ſquare of A C is to the ſquare of L N, as E is F: but as the ſquare of the diameter A C, is to the ſquare of the diameter L N, ſo is the circle A B C to the circle L M N, by this 2. of the twelueth, wherefore by the 11. of the fiueth, the circle A B C is to the circle L M N, as E is to F. A circle being geuen (therefore) an other circle is founde, to which the geuen circle is in any proportion geuen betwene two right lines: which ought to be done.

A Probleme. 4.

Two circles being geuen, to finde one circle equall to them both.

Conſtruction.

Suppoſe the two circles geué, haue their diameters A B. & C D. I ſay that a circle muſt be found equall to the two circles whoſe diameters are A B and C D: vnto the line A B, at the point A, erect a perpendicular line A E: from which (ſufficiently produced) cut a line equall to C D, which let be A F. By the firſt petition draw from F to B a right line: ſo is F A B made a triangle rectangle. I ſay now that a circle whoſe diameter is F B, is equal to the two circles whoſe diameters are A B and C D. For by the 47. of the firſt, the ſquare of F B is equall to the ſquares of A B & A F. Which A F is (by conſtruction) equall to C D: wherefore the ſquare of F B is equall to the ſquares of A B and C D. But circles are one to an other, as the ſquares of their diameters are one to the other, by this ſecond of the twelueth. Therefore the circle whoſe diameter is F B is equall to the circles whoſe diameters are A B and C D. Therefore two circles being geuen we haue found a circle equall to them both. Which was required to be done.



Demonſtration.

A Corollary

✻ A Corollary. 1.

Hereby it is made euident, that in all triangles rectangle, the circles, semicircles, quadrants, or any other portions of circles described vpon the subtendent line, is equall to the two circles, semicircles, quadrants, or any two other like portions of circles, described on the two lines comprehending the right angle, like to like being compared.

For like partes haue that proportion betwene them selues, that their whole magnitudes haue, of which they are like partes, by the 15. of the fifth. But of the whole circles, in the former probleme it is euident: and therefore in the fornamed like portions of circles, it is a true consequent.

✻ A Corollary. 2.

By the former probleme, it is also manifest, vnto circles three, fower, fiue, or to how many soeuer one will geue, one circle may be geuen equall.

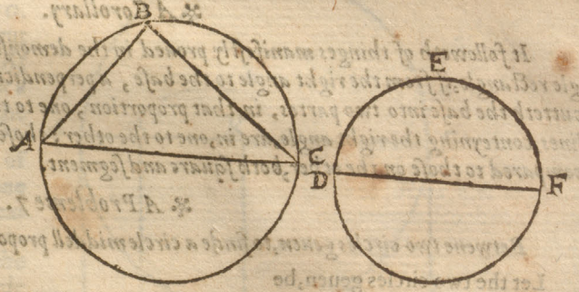
For if first, to any two, by the former probleme, you finde one equall, and then vnto your found circle and the third of the geuen circles, as two geuen circles, finde one other circle equall, and then to that second found circle, and to the fourth of the first geuen circles, as two circles, one new circle be found equall, and so proceede till you haue once cuppled orderly, euery one of your propoûded circles (except the first and second already doone) with the new circle thus found: for so the last found circle is equall to all the first geuen circles. If ye doubt, or sufficiently vnderstand me not: helpe your selfe by the discourse and demonstration of the last proposition in the second booke, and also of the 31. in the sixth booke.

¶ A Probleme 5.

Two vnequall circles being geuen, to finde a circle equall to the excesse of the greater to the lesse.

Suppose the two vnequal circles geué, to be A B C & D E F, & let A B C be the greater: whose diameter suppose to be A C: & the diameter of D E F suppose to be D F. I say a circle must be found equal to that excesse in magnitude, by which A B C is greater thé D E F. By the first of the fourth, in the circle A B C. Apply a right line equall to D F: whose one end let be at C, and the other let be at B. Fró B to A draw a right line. By the 30. of the third, it may appeare, that A B C is a right angle: and thereby A B C, the triangle is rectangled: wherfore by the first of the two corollaries, here before, the circle A B C is equall to the circle D E F, (For B C by constructi-on is equall to D F) and more ouer to the circle whose diameter is A B. That circle therefore whose diameter is A B, is the circle conteyning the magnitude, by which A B C is greater then D E F. Wherefore two vnequal circles being geuen, we haue found a circle equall to the excesse of the greater to the lesse: which ought to be doone.

Construction.



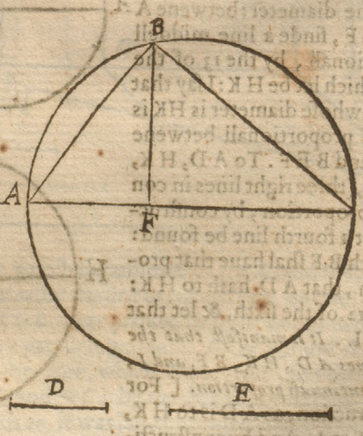
Demonstration.

A Probleme. 6.

A Circle being geuen to finde two Circles equall to the same: which found Circles, shall haue the one to the other, any proportion geuen in two right lines.

Suppose A B C, a circle geuen: and the proportion geuen, let it be that, which is betwene the two right lines D and E. I say, that two circles are to be found equall to A B C: and with al, one to the other, in the proportiõ of D to E. Let the diameter of A B C be A C. As D is to E, so let A C be deuided, by the 10. of the sixth, in the poynt F. At F, to the line A C let a perpẽdicular be drawne F B, and let it mete the circũferẽce at the poynt B. From the poynt B to the points A and C, let right lines be drawne: B A and B C. I say that the circles whose diametes are the lines B A and B C are equall to the circle A B C: and that those circles hauing to their diameters the lines B A and B C are one to the other in the proportion of the line D to the line E. For, first that they are equal, it is e-

Construction.

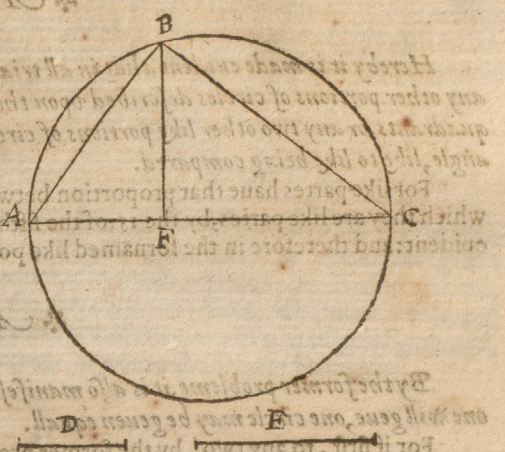


uident: by reaſon that A B C is a triangle rectangle: wherfore by the 47. of the firſt the ſquares of B A and B C are equall to the ſquare of A C: And ſo by this ſecond it is manifeſt, the two circles to be equall to the circle A B C.

Secondly as D is to E, ſo is A F to F C: by conſtruction.

" *And as the line A F is to the line F C, ſo is the ſquare of the line B A to the ſquare of the line B C.* [Which thing, we will briefely proue thus. The parallelogramme contayned vnder A C and A F, is equall to the ſquare of B A: by the Lemma after the 32. of the tenth booke: and by the ſame Lemma or Aſſumpt, the parallelogramme contayned vnder A C and F C, is equall to the ſquare of the line B C. Wherfore as the firſt parallelogramme hath it ſelfe to the ſecond: ſo hath the ſquare of B A (equall to the firſt parallelogramme) it ſelfe, to the ſquare of B C, equall to the ſecond parallelogramme. But both the parallelogrames haue one heigth, namely, the line A C: and baſes, the lines A F and F C: wherefore as A F is to F C, ſo is the parallelogramme contayned vnder A C, A F, to the parallelogramme contayned vnder A C, F C, by the firſt of the ſixth. And therefore as A F is to F C, ſo is the ſquare of B A to the ſquare of B C.] And as the ſquare of B A is to the ſquare of B C: ſo is the circle whoſe diameter is B A, to the circle whoſe diameter is B C, by this ſecond of the twelfth. Wherfore the circle whoſe diameter is B A, is to the circle whoſe diameter is B C, as D is to E. And before we proued them equall to the circle A B C. Wherefore a circle being geuen, we haue found two circles equall to the ſame: which haue the one to the other any proportion geuen in two right lines. Which ought to be done.

Note this well: for it is of great vſe.



Note.

An other way of demonſtratiõ of the firſt Probleme of this addition.

Here may you perceiue an other way how to execute my firſt probleme, for if you make a right angle conteyned of the diameters geuẽ, as in this figure ſuppoſe them B A and B C: and then ſubtend the right angle with the line A C: and from the right angle, let fall a line perpendicular to the baſe A C: that perpendicular at the point of his fall, deuideth A C into A F and F C, of the proportion required.

✻ *A Corollary.*

Note this properties of a triangle rectangle.

It followeth of thinges manifeſtly proued in the demonſtration of this probleme, that in a triangle rectangle, if from the right angle to the baſe, a perpendicular be let fall: the ſame perpendicular cutteth the baſe into two partes, in that proportion, one to the other, that the ſquares of the right lines, conteyning the right angle, are in, one to the other: thoſe on the one ſide the perpendicular, being compared to thoſe on the other, both ſquare and ſegment.

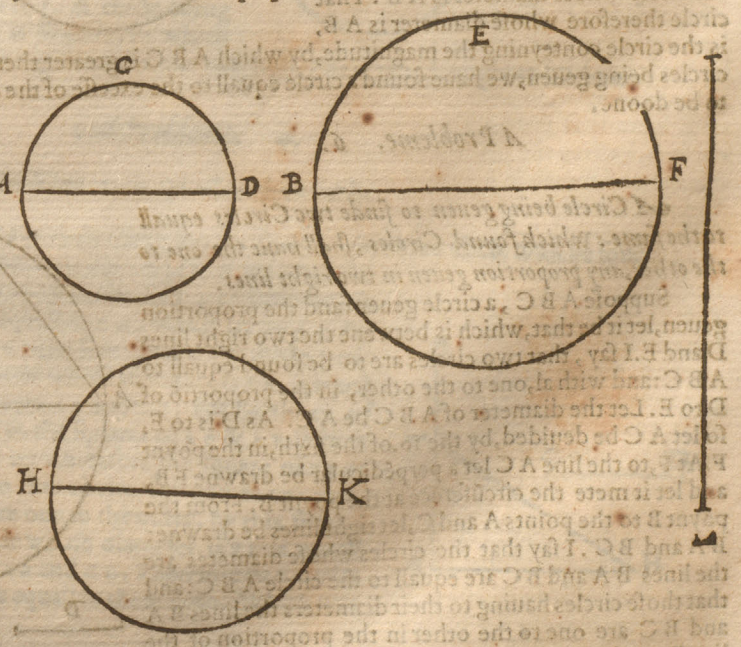
✻ *A Probleme 7.*

Betwene two circles geuen, to finde a circle middell proportionall.

Conſtruction.

Let the two circles geuen, be A C D and B E F: I ſay, that a circle is to be foũd, which betwene A C D and B E F is middell proportionall. Let the diameter of A C D, be A D, and of B E F, let B F be the diameter: betwene A D and B F, finde a line middell proportionall, by the 13. of the ſixth: which let be H K: I ſay that a circle, whoſe diameter is H K is middell proportionall betwene A C D and B E F. To A D, H K, and B F, (three right lines in continuall proportion, by conſtruction) let a fourth line be found: to which B F ſhal haue that proportion, that A D hath to H K: by the 12. of the ſixth, & let that line be L. *It is manifeſt that the fower lines A D, H K, B F, and L, are in continuall proportion.* [For by cõſtruction, as A D is to H K, ſo is B F to L. And by conſtructi-

Demonſtration.



on

on, as A D is to H K, so is H K to B F: wherefore H K is to B F, as B F is to L: by the 11. of the fifth, wherefore the 4. lines are in continuall proportion.] Wherefore as the first is to the third, that is A D to B F, so is the square of the first to the square of the second: that is, the square of A D, to the square of H K: by the corollary of the 20. of the sixth. And by the same corollary, as H K is to L, so is the square of H K to the square of B F. But by alternate proportion, the line A D is to B F, as H K is to L: wherefore the square of A D is to the square of H K, as the square of H K is to the square of B F. Wherefore the square of H K, is middell proportionall, betwene the square of A D and the square of B F. But as the squares are one to the other, so are the circles (whose diameters produce the same squares) one to the other, by this secõd of the twelfth: wherfore the circle whose diameter is the line H K, is middel propottional, betwene the circles whose diameters are the lines A D and B F. Wherefore betwene two circles geuen, we haue found a circle middell proportionall: which was requisite to be doone.

¶ A Corollary.

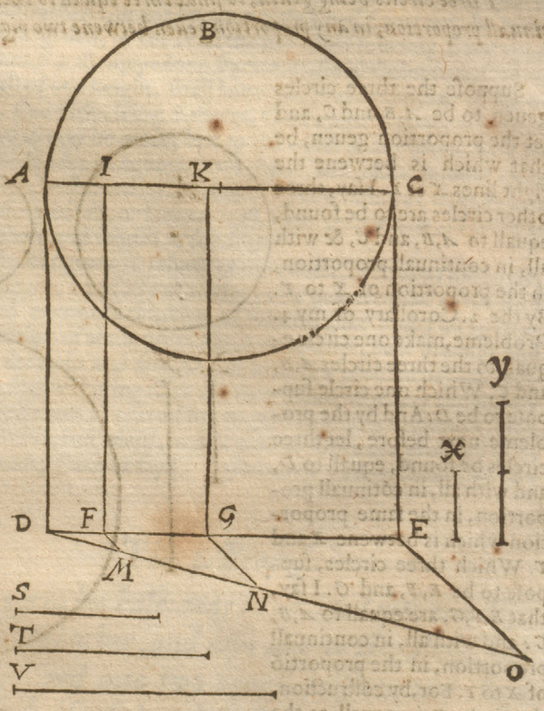
Hereby it is manifest, three lines or more, being in continuall proportion; that the circles hauing those lines to their diameters, are also in continuall proportion.

As of three, our demonstration hath already proued: so of fower, will the proufe go forward: if you adde a fifth line in continuall proportion to the fower geuen: as we did to the three, adde the fourth: namely, the line L. And so, if you haue 6, by putting to one more, the demonstration will be easie and plaine. And so of as many as you will.

✻ A Probleme. 8.

To a circle being geuen, to finde three circles equall: which three circles shall be in continuall proportion, in any proportion geuen betwene two right lines.

Suppose the circle geuen to be *A B C*: and the proportion geuen to be that which is betwene the lines *X* and *Y*. I say, that three circles are to be geuen, which three, together, shall be equall to the circle *A B C*: and withall in continuall proportion, in the same proportion which is betwene the right lines *X* and *Y*. Let the diameter of *A B C*, be *A C*. Of *A C*, make a square: by the 46. of the first: which which let be *A C D E*. From the point *D* drawe a line, sufficiently long (any way, * without the square): which let be *D O*. At the point *D*, and from the line *D O*, cut a line equall to *X*: which let be *D M*. At the point *M*, and from the line *M O*, cut a line equall to *Y*: which let be *M N*. At the point *N*, to the two lines *D M* and *M N*, set a third line proportionall, by the 12. of the sixt: which let be *N O*. From *E* (one of the angles of the square *A C D E*, next to *D*) draw a right line to *O*: making perfecte the triangle *D E O*. Now from the pointes *M* and *N*, drawe lines, to the side *D E*, parallel to the side *E O*: by the 31. of the first: which let be *M F* and *N G*. Wherfore, by the 2. of the sixt, the side *D E*, is proportionally cut in the pointes *F* and *G*, as *D O* is cut in the pointes *M* and *N*: therfore, as *D M* is to *M N*, so is *D F* to *F G*: and as *M N* is to *N O*, so is *F G* to *G E*. Wherefore, seing *D M*, *M N*, and *N O*, are, by construction, continually proportioned, in the proportion of *X* to *Y*: So likewise, are *D F*, *F G*, and *G E*, in continuall proportion, in the proportion of *X* to *Y*, by the 11. of the fift. From the pointes *F* and *G*, to the oppoſite side *A C*, let right lines be drawen parallel to the other sides: which lines, suppose to be *F I*, and *G K*: making thereby, three parallelogrammes *D I*, *F K*, and *G C*, equall to the whole square *A C D E*. Which three parallelogrammes, by the first of the sixt, are one to an other, as their bases, *D F*, *F G*, and *G E*, are. But *D F*, *F G*, and *G E*, were proued to be in continuall proportion, in the proportion of *X* to *Y*: Wherefore, the three parallelogrammes *D I*, *F K*, and *G C*, by the 11. of the fifth, are also in continuall proportion, and in the same, which *X* is in, to *Y*. Let three squares be made, equall to the three parallelogrammes *D I*, *F K*, and *G C*: by the last of the second: Let the sides of those squares be, orderly, *S*, *T*, and *V*. Forasmuch as, it was last concluded that the three paral-



* *Though I say, without the square, yet you must thinke, that it may be also within the square: & that diuersly. Wherfore this Probleme may haue diuerse cases, so, but, briefly to auoyde all, may thus be said: cut any side of that square into 3. parts, in the proportion of X to Y.*

Note the maner of the drift in this demonstration and construction, mixtly: and with no determination to the construction: as commõly is in problemes which is here of me so vsed, for an example to young studẽtes of Variety in art.

lelogrammes, *D I*, *F K*, and *G C* (which are equall to the ſquare *A C D E*) are alſo in continuall proportion, in the proportion of *X* to *Y*, therefore their equalls, namely, the three ſquares of *S*, *T*, & *V*, are alſo equall to the whole ſquare *A C D E*, and in continuall proportion, in the proportion of *X* to *Y*. Wherefore the three circles, whoſe diameters are *S*, *T*, and *V*, are equall to the circle, whoſe diameter is *A C*, the ſide of the ſquare *A C D E*, and alſo in continuall proportion, in the proportion of *X* to *Y*: by this ſecond of the twelfth. But, by conſtruction, *A C* is the diameter of the circle *A B C*. Wherefore we haue found three circles, equall to *A B C*: namely, the circle, whoſe diameter is *S*: and the circle, whoſe diameter is *T*: and the circle, whoſe diameter is *V*: which three circles, alſo, are in continuall proportion, in the proportion of *X* to *Y*. Wherefore to a circle being geuen, we haue found three circles equall in any proportion, geuen, betwene two right lines: which was requiſite to be done.

¶ *A Corollary.*

Hereby, it is euident, that a circle geuen, we may finde circles 4, 5, 6, 10, 20, 100, 1000, *or how many ſoeuer ſhall be appointed, being in continuall proportion, in any proportion, geuen betwene two right lines: which circles, all together, ſhall be equall to the circle geuen.*

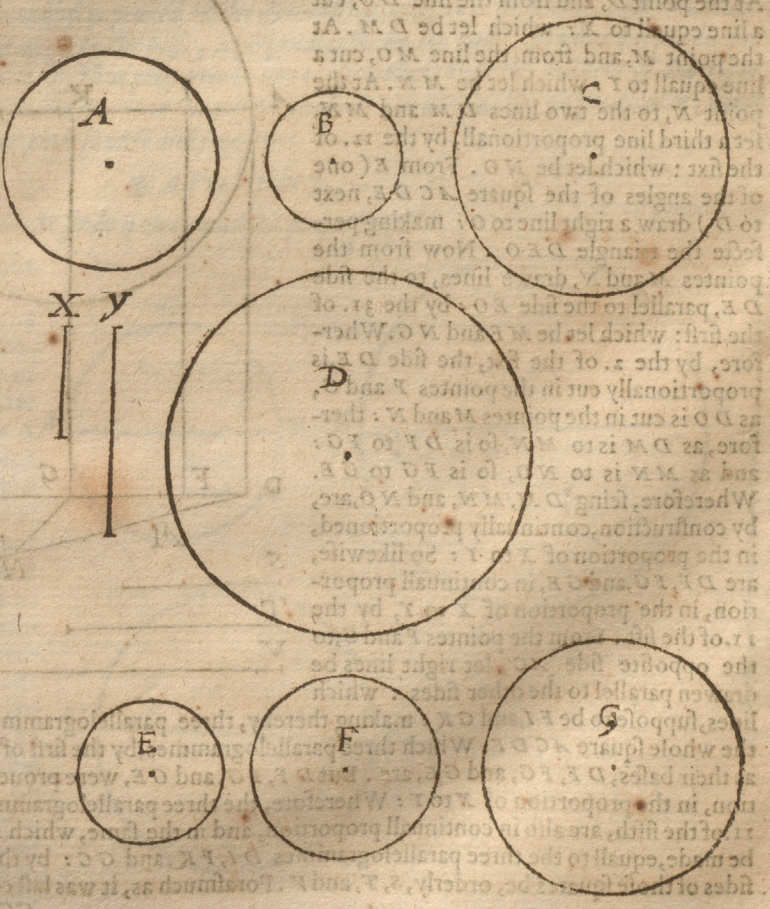
For, euermore deuiding the one ſide of the chiefe ſquare (which is made of the diameter of the circle geuen) into ſo many partes, as circles are to be made: ſo that betwene thoſe partes be continued the proportion geuen betwene two right lines: and from the pointes of thoſe diuiſions, drawe parallels, perpendiculars to the other ſide of the ſaid chiefe ſquare: making ſo many parallelogrammes of the chiefe ſquare, as are circles to be made: and to thoſe parallelogrammes (orderly) making equall ſquares: it is manifeſt that the ſides of thoſe ſquares, are the diameters of the circles required to be made.

✻ *A Probleme. 9.*

Three circles being geuen, to finde three equall to them: which three found circles ſhall be in continuall proportion, in any proportion geuen betwene two right lines.

Conſtruction.

Suppoſe the three circles geuen to be *A*, *B*, and *C*, and let the proportion geuen, be that which is betwene the right lines *X* & *Y*. I ſay, three other circles are to be found, equall to *A*, *B*, and *C*, & with all, in continuall proportion, in the proportion of *X* to *Y*. By the 2. Corollary of my 4. Probleme, make one circle equall to the three circles *A*, *B*, and *C*. Which one circle ſuppoſe to be *D*: And by the probleme next before, let three circles be found, equall to *D*, and with all, in cõtinuall proportion, in the ſame proportion which is betwene *X* and *Y*. Which three circles, ſuppoſe to be *E*, *F*, and *G*. I ſay, that *E*, *F*, *G*, are equall to *A*, *B*, *C*: and with all, in continuall proportion, in the proportiõ of *X* to *Y*. For, by cõſtruction, the circle *D* is equall to the circles *A*, *B*, & *C*: and by conſtruction likewiſe, the circles *E*, *F*, and *G*, are equall to the ſame circle *D*: Wherfore the three circles *E*, *F*, & *G*, are equall to the three circles *A*, *B*, & *C*: and by conſtruction *E*, *F*, and



Demonſtration.

and *G*, are in continuall proportion, in the proportion of the line *X* to the line *Y*. Wherefore *E*, *F*, and *G*, are equall to *A*, *B*, and *C*: and in continuall proportion, in the proportion of *X* to *Y*. Three circles, therefore, being geuen, we haue found three circles, equall to them, and alſo in continuall proportion, in any proportion geuen, betwene two right lines. Which was requiſite to be done.

¶ *A Corollary.* 1.

It is hereby very manifeſt, that vnto 4.5.6.10.20. 100. *or how many circles ſoeuer, ſhall be geuen, we may finde* 3.4.5.8.10. *or how many ſoeuer, ſhall be appointed: which, all together, ſhall be equall to the circles geuen, how many ſoeuer they are: and with all, our found circles, to be in continuall proportion, in any proportion aſſigned betwene two right lines geuen.*

For, euermore, by the Corollary of the 4. Probleme, reduce all your circles to one: and by the Corollary of my 8. Probleme, make as many circles as you are appointed, equall to the circles geuen, and continuall in proportion, in the ſame, wherein, the two right lines geuen, are: And ſo haue you performed, the thing required.

Note.

What incredible fruite in the Science of proportions may hereby grow, no mans tounge can ſufficiently expreſſe. And ſory I am, that vtterly leyſure is taken from me, ſomewhat to ſpecifie in particular hereof.

¶ *The key of one of the chiefe treaſure houſes, belonging to the State Mathematicall.*

THat, which in theſe 9. Problemes, is ſaid of circles, is much more ſayd of ſquares, by whoſe meanes, circles, are thus handled. And therefore ſeing to all Polygonon right lined figures, equall ſquares 1.
may be made, by the laſt of the ſecond: and contrariwiſe, to any ſquare, a right lined figure may be made 2.
equall, and withall, like to any right lined figure geuen, by the 25. of the ſixt. And fourthly, ſeing vpon 3.
the ſaid plaine figures, as vpō baſes, may Priſmes, Parallelipipedons, Pyramids, ſided Columnes, Cones, 4
and Cylinders, be reared: which being * all of one height, ſhall haue that proportion, one to the other, that their baſes haue, one to the other. And fiftly, ſeing Spheres, Cones, and Cylinders are one to other in certaine knowen proportions: and ſo may be made, one to the other in any proportion aſſigned. And ſixtly, ſeing vnder euery one of the kindes of figures, both plaine, and ſolide, infinite caſes may chaunce, by the ayde of theſe Problemes, to be ſoluted and executed: How infinite (then) vpon 6.
infinite, is the number of practiſes, either Mathematicall, or Mechanicall, to be performed, of compariſons betwene diuers kindes, of plaines to plaines, and ſolides to ſolides?

* *Note and remember one heith in theſe ſolids.*

Farthermore, to ſpeake of playne ſuperficiall figures, in reſpect of the contēt, or Area of the circle, 7.
ſundry mixt line figures, Anular and Lunular figures: and alſo of circles to be geuen equall to the ſayd vnuſed figures: and in all proportions els: and euermore thinking of ſolides, (like high) ſet vpon a-
ny of thoſe vnuſed figures, (O Lord) in cōſideration of al the premiſſes, how infinite, how ſtraunge and 8.
incredible ſpeculations and practiſes, may (by the ayde and direction of theſe few problemes), fall redily into the imaginations and handes of them, that will bring their minde and intent wholy and fixedly to ſuch mathematicall diſcourſes? In theſe Elementes, I entend but to geue to young beginners ſome light, aide, and courage to exerciſe their owne witts, and talent, in this moſt pleaſant and profitable ſcience. All thinges may not, neither yet can, in euery place be ſayd. *Opportunitie, and Sufficiency, beſt are to be allowed.*

¶ *The 3. Theoreme.* *The 3. Propoſition.*

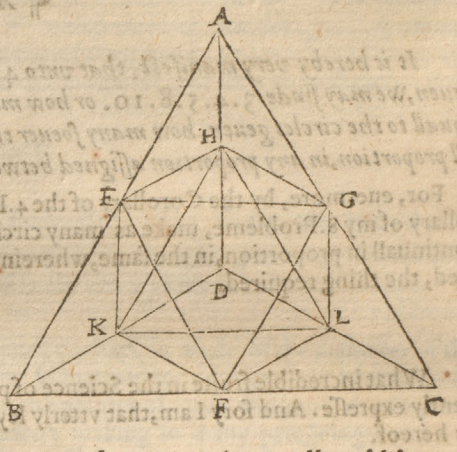
Euery Pyramis hauing a triangle to his baſe: may be deuided into two Pyramids equall and like the one to the other, and alſo like to the whole, hauing alſo triangles to their baſes, and into two equall priſmes: and thoſe two priſmes are greater then the halfe of the whole Pyramis.

Vppoſe that there be a Pyramis, whoſe baſe let be the triangle A B C, and his toppe the point D. Then I ſay, that the Pyramis A B C D may be deuided into two Pyramids equall and like the one to the other, and alſo like to the whole, hauing alſo triangles to their baſes, and into two equall priſmes, and thoſe two priſmes

Construction.

prismes are greater then the halfe of the whole Pyramis. Deuide (by the 10. of the first) the lines A B, B C, C A, A D, B D, & C D, into two equall partes in the pointes E, F, G, H, K, and L. And drawe these right lines E H, E G, G H, H K, K L, L H, E K, K F, and F G. Now

Demonstration of the first part, namely, that the whole Pyramids is deuided into two Pyramis equall and like the one to the other, and also to the whole, & hauing triangles to their bases.



forasmuch as the line A E is equall to the line E B, and the line A H to the line H D: therfore (by the 2. of the sixt) the line E H is a parallel to the line D B. And by the same reason, the line H K is a parallel to the line A B: Wherefore H E K B is a parallelogramme. Wherefore the line H K is equall to the line E B. But the line E B is equall to the line A E: Wherefore the line A E is equall to the line H K. And the line A H is equall to the line H D. Now therfore there are two lines A E and A H, equall to two lines K H and H D, the one to the other, and the angle E A H is (by the 29. of the first) equall to the angle K H D: Wherefore (by the 4. of the first) the base E H is equall to the base K D. Wherefore the triangle A E H, is equall and like to the triangle H K D. And by the same reason also the triangle A H G, is equall and like to the triangle H L D. And forasmuch as two right lines E H & H G touching the one the other, are parallels to two right lines K D and D L touching also the one the other, and not being in one and the selfe same plaine superficies with the two first: those lines (by the 10. of the eleuenth) containe equall angles. Wherefore the angle E H G is equall to the angle K D L. And forasmuch as two right lines E H and H G, are equall to two right lines K D & D L, the one to the other, and the angle E H G is (by the 10. of the eleuenth) equall to the angle K D L, therefore (by the 4. of the first) the base E G is equall to the base L K. Wherefore the triangle E H G is equall and like to the triangle K D L. And by the same reason also the triangle A E G is equall and like to the triangle H K L. Wherefore the Pyramis, whose base is the triangle A E G, and toppe the point H, is equall and like to the Pyramis, whose base is the triangle H K L, and toppe the point D.

And forasmuch as to one of the sides of the triangle A D B, namely, to the side A B, is drawen a parallel line H K, therefore the whole triangle A D B is equiangle to the triangle D H K, and their sides are proportionall (by the Corollary of the 2. of the sixt). Wherefore the triangle A D B is like to the triangle D H K. And by the same reason also the triangle D B C is like to the triangle D K L, and the triangle A D C to the triangle D H L. And forasmuch as two right lines B A and A C touching the one the other, are parallels to two right lines K H and H L, touching also the one the other, but not being in one and the selfe same superficies with the two first lines, therefore (by the 10. of the eleuenth) they containe equall angles. Wherefore the angle B A C is equall to the angle K H L. And as the line B A is to the line A C, so is the line K H to the line H L. Wherefore the triangle A B C is like to the triangle K H L. Wherfore the whole Pyramis whose base is the triangle A B C & top the point D, is like to the pyramis whose base is the triangle H K L, and toppe the point D. But the pryamis whose base is the triangle H K L and toppe the point D, is proued to be like to the pyramis whose base is the triangle A E G and toppe the poynt H. Wherefore also the pyramis whose base is the triangle A B C, and toppe the poynt D, is like to the pyramis whose base is the triangle A E G and toppe the poynt H (by the 21. of the sixth). Wherefore either of these pyramids A E G H and H K L D is like to the whole pyramis A B C D.

The conclusion of the first part.

Demonstratiõ of the second part, namely,

[And forasmuch as the lines B E, K H, are parallel lines and equall, as it hath bene proued, therefore the right lines B K and E H, which ioyne them together, are equall and parallels, by the 33. of the first. Againe forasmuch as the lines B E and F G are parallel lines and equall, therefore the lines E G and

and B F, which ioyne them together, are alſo equall and parallels: and by the ſame reaſon foraſmuch as the lines B G and K H are equall parallels, the lines F K and G H, which ioyne them together, are alſo equall parallels. Wherefore B E H K, B E G F, and K H G F, are parallelogrammes. And foraſmuch as their oppoſite ſides are equall, by the 34. of the firſt: therefore the triangles E H G, & B K F, are equiangle, by the 8. of the firſt: and therefore, by the 4. of the ſame, they are equall: and moreouer, by the 15. of the eleuenth, their ſuperficieces are parallels. Wherefore the ſolide B K F E H G is a Priſme, by the 11. definition of the eleuenth. Likewiſe foraſmuch as the ſides of the triangle H K L are equall and parallels to the ſides of the triangle G F C, as it hath before bene proued: It is manifeſt, that C F K L, F K H G, and C L H G, are parallelogrammes, by the 33. of the firſt. Wherefore the whole ſolide K L H F C G, is a Priſme, by the 11. definition of the eleuenth, and is contayned vnder the ſayd parallelogrammes C F K L, F K H G, and C L H G, and the two triangles H K L and G F C, which are oppoſite and parallels.]

that it is deuided moreouer into two equal Priſmes.

And foraſmuch as the line B F is equall to the line F C, therefore (by the 41. of the firſt) the parallelogramme E B F G is double to the triangle G F C. And foraſmuch as if there be two Priſmes of equall altitudes, and the one haue to his baſe a parallelogramme, and the other a triangle, and if the parallelogramme be double to the triangle, thoſe Priſmes are (by the 40. of the eleuenth) equall the one to the other: therefore the Priſme contained vnder the two triangles B K F and E H G, and vnder the three parallelogrammes E B F G, E B K H, and K H F G, is equall to the Priſme contained vnder the two triangles G F C, and H K L, and vnder the three parallelogrammes K F C L, L C G H, and H K F G.

Concluſion of the ſecond part.

And it is manifeſt, that both theſe Priſmes, of which the baſe of one is the parallelogramme E B F G, and the oppoſite vnto it the line K H, and the baſe of the other is the triangle G F C, and the oppoſite ſide vnto it the triangle K L H, are greater then both theſe Pyramids, whoſe baſes are the triangles A G E, and H K L, and toppes the pointes H & D. For if we drawe theſe right lines E F and E K, the Priſme whoſe baſe is the parallelogramme E B F G, and the oppoſite vnto it the right line H K, is greater then the Pyramis whoſe baſe is the triangle E B F, & toppe the point K. But the Pyramis whoſe baſe is the triangle E B F, and toppe the point K, is equall to the Pyramis whoſe baſe is the triangle A E G and toppe the point H, for they are contained vnder equall and like plaine ſuperficieces. Wherefore alſo the Priſme whoſe baſe is the parallelogramme E B F G and the oppoſite vnto it the right line H K, is greater then the Pyramis, whoſe baſe is the triangle A E G, and toppe the point H. But the priſme whoſe baſe is the parallelogramme E B F G, and the oppoſite vnto it the right line H K, is equall to the priſme, whoſe baſe is the triangle G F C, and the oppoſite ſide vnto it the triangle H K L: And the Pyramis whoſe baſe is the triangle A E G, and toppe the point H, is equall to the Pyramis, whoſe baſe is the triangle H K L, and toppe the point D. Wherefore the foreſaid two priſmes are greater then the foreſaid two Pyramids, whoſe baſes are the triangles A E G, H K L, and toppes the pointes H and D. Wherefore the whole Pyramis whoſe baſe is the triangle A B C, and toppe the point D, is deuided into two Pyramids equall and like the one to the other, and like alſo vnto the whole Pyramis, hauing alſo triangles to their baſes, and into two equall priſmes, and the two priſmes are greater then halfe of the whole Pyramis: which was required to be demonſtrated.

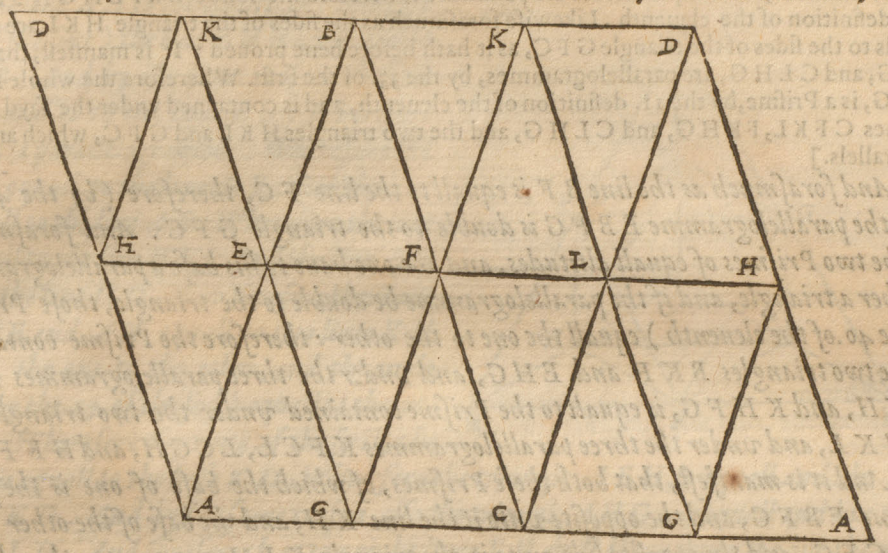
Demonſtratiõ of the laſt part that the two Priſmes are greater then the halfe of the whole Pyramis.

Concluſion of the laſt part.

Concluſion of the whole propoſition.

If ye will with diligence reade theſe fower bookes following of Euclide, which concerne bodyes, and clearely ſee the demonſtrations in them conteyned, it ſhall be requiſite for you when you come to any propoſition, which concerneth a body or bodies, whether they be regular or not, firſt to deſcribe of paſted paper (according as I taught you in the end of the definitions of the eleuenth booke) ſuch a body or bodyes, as are there required, and hauing your body, or bodyes thus deſcribed, when you haue noted it with letters according to the figure ſet forth vpõ a plaine in the propoſitiõ, follow the conſtructiõ required in the propoſition. As for example, in this third propoſitiõ it is ſayd that, *Euery pyramis hauing a triangle to his baſe, may be deuided into two pyramids. &c.* Here firſt deſcribe a pyramis of paſted paper haning his baſe triangled (it ſkilleth not whether it be equilater, or equiangled, or not, only in this propoſition is required that the baſe be a triangle. Then for that the propoſition ſuppoſeth the baſe of the pyramis to be the triangle A B C, note the baſe of your pyramis which you haue deſcribed with the letters A B C, and the toppe of your pyramis with the letter D: For ſo is required in the propoſition. And thus haue you your body ordered ready to the conſtruction. Now in the conſtruction it is required that ye deuide the lines, A B, B C, C A. &c, namely, the fixe lines which are the ſids of the fower triangles contayning

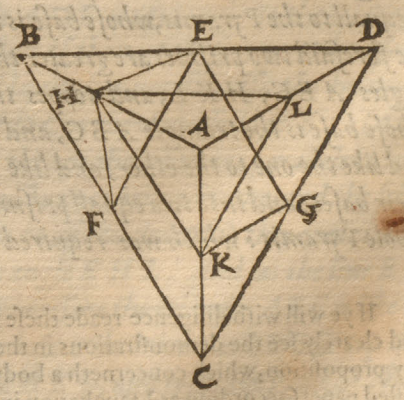
tayning the piramis, into two equall partes in the poyntet E, F, G, &c. That is, ye muſt deuide the line A B of your pyramis into two equall partes, and note the poynt of the deuiſion with the letter E, and ſo the line B C being deuided into two equall partes, note the poynt of the deuiſion with the letter F. And ſo the reſt, and this order follow ye as touching the reſt of the conſtruction there put, and when ye haue finiſhed the conſtruction, compare your body thus deſcribed with the demonſtration: and it will make it very playne and eaſy to be vnderſtãded. Whereas without ſuch a body deſcribed of matter, it is hard



for young beginners (vnleſſe they haue a very deepe imagination) fully to conceaue the demonſtration by the figue as it is deſcribed in a plaine. Here for the better declaration of that which I haue ſayd, haue I ſet a figure, whoſe forme if ye deſcribe vpon paſted paper noted with the like letters, and cut the lines B A, D A, D C, and folde it accordingly, it will make a Pyramis deſcribed according to the conſtruction required in the propoſition. And this order follow ye as touching all other propoſitions which concerne bodyes.

¶ An other demonſtration after *Campane* of the 3. propoſition.

Suppoſe that there be a Pyramis *A B C D* hauing to his baſe the triangle *B C D*, and let his toppe be the ſolide angle *A*: from which let there be drawne three ſubtended lines *A B*, *A C*, and *A D* to the three angles of the baſe, and deuide all the ſides of the baſe into two equall partes in the three poyntes *E*, *F*, *G*: deuide alſo the three ſubtẽded lines *A B*, *A C*, and *A D* in two equall partes in the three points *H*, *K*, *L*. And draw in the baſe theſe two lines *E F* and *E G*: So ſhall the baſe of the pyramis be deuided into three ſuperficieces: whereof two are the two triangles *B E F*, and *E G D*, which are like both the one to the other, and alſo to the whole baſe, by the 2. part of the ſecõd of the ſixth, & by the definitiõ of like ſuperficieces, & they are equal the one to the other, by the 8. of the firſt: the third ſuperficies is a quadrangled parallelogramme, namely, *E F G C*: which is double to the triangle *E G D*, by the 40. and 41. of the firſt. Now then agayne from the poynt *H* draw vnto the points *E* and *F* theſe two ſubtendent lines *H E* and *H F*: draw alſo a ſubtended line from the poynt *K* to the poynt *G*. And draw theſe lines *H K*, *K L*, and *L H*. Wherefore the whole pyramis *A B C D* is deuided into two pyramids, which are *H B E F*, and *A H K L*, and into two priſmes of which the one is *E H F G K C*, and is ſet vpon the quadrangled baſe *C F G E*: the other is *E G D H K L*, and hath to his baſe the triangle *E G D*. Now as touching the two pyramids *H B E F* and *A H K L*, that they are equall the one to the other, and alſo that they are like both the one to the other and alſo to the whole, it is manifeſt by the definition of equall and like bodyes, and by the 10. of the eleuenth, and by 2. part of the ſecond of the ſixth. And that the two Priſmes are equall it is manifeſt by the laſt of the eleuenth. And now that both the priſmes taken together are greater then the halfe of the whole pyramis, hereby it is manifeſt, for that either of them may be deuided into two pyramids, of which the one is a triangular pyramis equall to one of the two pyramids into which together with the two priſmes is deuided the whole pyramis, and the other is a quadrangled pyramis double to the other pyramis. Wherefore it is playne that the two priſmes taken together are three quarters of the whole



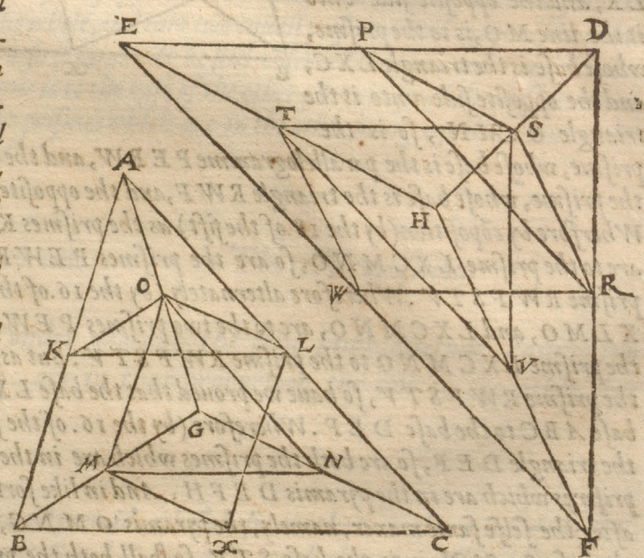
pyramis

pyramis deuided. But if ye are desirous to know the proportió betwen them, reade the 6. of this booke. But now here to this purpose it shall be sufficient to know, that the two prismes taken together do exceede in quantity the two partial pyramids, taken together, into which together with the two prismes the whole pyramis was deuided.

¶ The 4. Theoreme. The 4. Proposition.

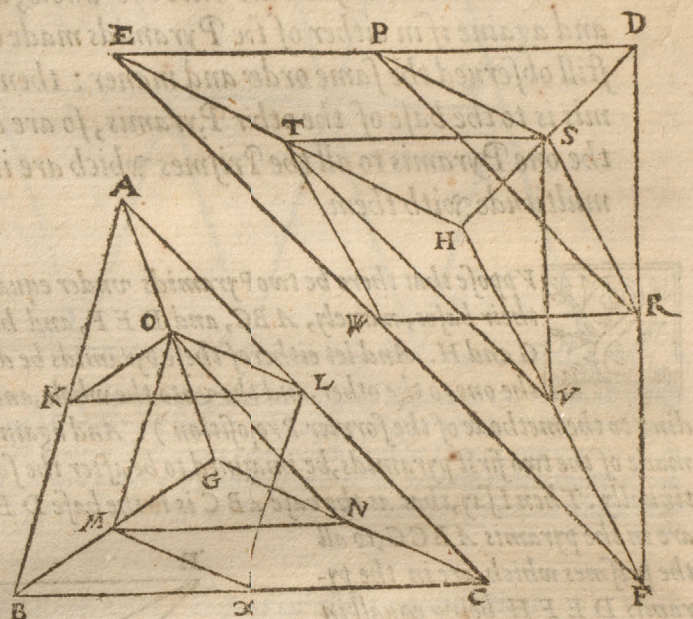
If there be two Pyramids vnder equall altitudes, hauing triangles to their bases, and either of those Pyramids be deuided into two Pyramids equall the one to the other, and like vnto the whole, and into two equall Prismes, and againe if in either of the Pyramids made of the two first Pyramids be still obserued the same order and maner: then as the base of the one Pyramis is to the base of the other Pyramis, so are all the Prismes which are in the one Pyramis to all the Prismes which are in the other, being equall in multitude with them.

Vppose that there be two Pyramids vnder equall altitudes, hauing triangles to their bases, namely, A B C, and D E F, and hauing to their toppes the pointes G and H. And let either of these pyramids be diuided into two pyramids equall the one to the other, and like vnto the whole, and into two equall prismes (according to the methode of the former Proposition). And againe, let either of those pyramids so made of the two first pyramids, be imagined to be after the same order deuided, and so do continually. Then I say, that as the base A B C is to the base D E F, so are all the prismes which are in the pyramis A B C G, to all the prismes which are in the pyramis D E F H being equall in multitude with them. For forasmuch as the line B X is equall to the line X C, and the line A L to the line L C: (For as we saw in the construction pertayning to the former Proposition, al the sixe sides of the whole pyramids, are ech deuided into two equall parts, the like of which construction is in this proposition also supposed): therefore the line X L is a parallel to the line A B, & the triangle A B C, is like to the triangle L X C, (by the Corollary of the second of the sixth): and by the same reason the triangle D E F is like to the triangle R W F. And forasmuch as the line B C is double to the line C X, and the line F E to the line F W: therefore as the line B C is to the line C X, so is the line E F to the line F W. And vpon the lines B C and C X are described rectiline figures like and in like sort set, namely, the triangles A B C and L X C, and vpon the lines E F and F W are also described rectiline figures, like and in like sort set, namely, the triangles D E F & R W F But if there be fower right lines proportionall, the rectiline figures described of them being like und in like sort set, shall also be proportionall (by the 22. of the sixt). Wherefore as the



triangle A B C is to the triangle L X C, ſo is the triangle D E F to the triangle R W F. Wherefore alternately (by the 16. of the fift) as the triangle A B C is to the triangle D E F, ſo is the triangle L X C to the triangle R W F. * But as the triangle L X C is to the triangle R W F, ſo is the priſme whoſe baſe is the triangle L X C, and the oppoſite ſide vnto it the triangle O M N, to the priſme whoſe baſe is the triangle R W F, and the oppoſite ſide vnto it the triangle S T V (by the Corollary of the 40. of the eleuenth). For theſe priſmes are vnder one & the ſelfe ſame altitude, namely, vnder the halfe of the altitude of the whole Pyramids, which Pyramids are ſuppoſed to be vnder one and the ſelfe ſame altitude: this is alſo proued in the Aſſumpt following). Wherefore (by the 11. of the fift) as the triangle A B C is to the triangle D E F, ſo is the priſme whoſe baſe is the triangle L X C, and the oppoſite ſide vnto it the triangle O M N, to the priſme whoſe baſe is the triangle R W F, and the oppoſite ſide vnto it the triangle S T V. And foraſmuch as there are two priſmes in the pyramis A B C G equal the one to the other, & two priſmes alſo in the pyramis D E F H equall the one to the other: therefore as the priſme, whoſe baſe is the parallelogrãme B K L X, and the oppoſite ſide vnto it the line M O, is to the priſme, whoſe baſe is the triangle L X C, and the oppoſite ſide vnto it the triangle O M N, ſo is the priſme, whoſe baſe is the parallelogramme P E R W, and the oppoſite vnto it the line S T, to the priſme, whoſe baſe is the triangle R W F, and the oppoſite ſide vnto it the triangle S T V. Wherfore by cõpoſition (by the 18. of the fift) as the priſmes K B X L M O, & L X C M N O, are to the priſme L X C M N O, ſo are the priſmes P E W R S T, and R W F S T V, to the priſme R W F S T V. Wherefore alternately (by the 15. of the fift) as the two priſmes K B X L M O, and L X C M N O, are to the two priſmes P E W R S T, and R W F S T V, ſo is the priſme L X C M N O to the priſme R W F S T V. But as the priſme L X C M N O is to the priſme R W F S T V, ſo haue we proued that the baſe L X C is to the baſe R W F, and the baſe A B C to the baſe D E F. Wherefore (by the 16. of the fift) as the triangle A B C is to the triangle D E F, ſo are both the priſmes which are in the pyramis A B C G, to both the priſmes which are in the pyramis D E F H. And in like ſort if we diuide the other pyramids after the ſelfe ſame maner, namely, the pyramis O M N G, and the pyramis S T V H: as the baſe O M N is to the baſe S T V, ſo ſhall both the priſmes that are in the pyramis O M N G, be to both the priſmes which are in the pyramis S T V H. But as the baſe O M N is to the baſe S T V, ſo is the baſe A B C to the baſe D E F. Wherefore (by the 11. of the fift) as the baſe A B C is to the baſe D E F, ſo are the two priſmes that are in the pyramis A B C G, to the two priſmes that are in the pyramis D E F H, and the two priſmes that are in the pyramis O M N G, to the two priſmes that are in the pyramis S T V H, and the fower priſmes to the fower priſmes. And ſo alſo ſhall it followe in the priſmes made by diuiding the two pyramids A K L O, and D P R S, and of all the other pyramids in generall, being equall in multitude.

* An Aſſũpt.



¶ An

And that as the triangle L X C is to the triangle R W F, so is the prisme whose base is the triangle L X C and the opposite side O M N, to the prisme whose base is the triangle R W F and the opposite side the triangle S T V, may thus be proued. For in the selfe same construction, imagine perpendicular lines to be drawne from the poyntes G and H to the two playne superficieces wherein are the triangles A B C and D E F. Now those perpendicular lines shall be equall the one to the other, for that the two pyramids are supposed to be of equall altitude. And forasmuch as two right lines, namely, G C, and the perpendicular line drawne from the poynt G, are deuided by two parallel playne superficieces, namely, A B C and O M N, therfore (by the 17. of the eleuenth) the partes of the lines deuided are proportionall. But the line G C is by the playne superficies O M N deuided into two equall partes in the poynt N. Wherefore also the perpendicular line drawne from the poynt G to the playne superficies wherein is the triangle A B C, is deuided into two equall partes by the superficies O M N: & by the same reason also the perpendicular line which is drawne from the poynt H to the playne superficies D E F is deuided into two equall partes, by the playne superficies S T V. And the perpendiculars drawne from the poyntes G and H to the playne superficieces A B C and D E F are equal. Wherefore also the perpendicular lines which are drawne from the triangles O M N and S T V to the playne superficieces A B C and D E F are equal the one to the other. Wherfore also the prismes whose bases are the triangles L X C and R W F and the opposite sides the triangles O M N and S T V, are of equall altitude. Wherefore also the parallelipipedons which are described of the forsayd prismes and being equall in altitude with them, are the one to the other, as the base of the one is to the base of the other. Wherefore also as the halfe of the bases of those parallelipipedons, namely, as the base L X C is to the base R W F, so are the forsayd prismes the one to the other. ¶ *If therfore there be two pyramis vnder equall altitudes, hauing triangles to their bases and either of those pyramids be deuided into two pyramids equall the one to the other and like vnto the whole, and into two equall prismes, and agayne if in either of the pyramids made of the two first pyramids be still obserued the same order and maner: then as the base of the one pyramis is to the base of the other piramis, so all the prismes which are in the one pyramis to all the prismes which are in the other being equall in multitude: which was required to be proued.*

An Assumpt.

Conclusion of the Whole.

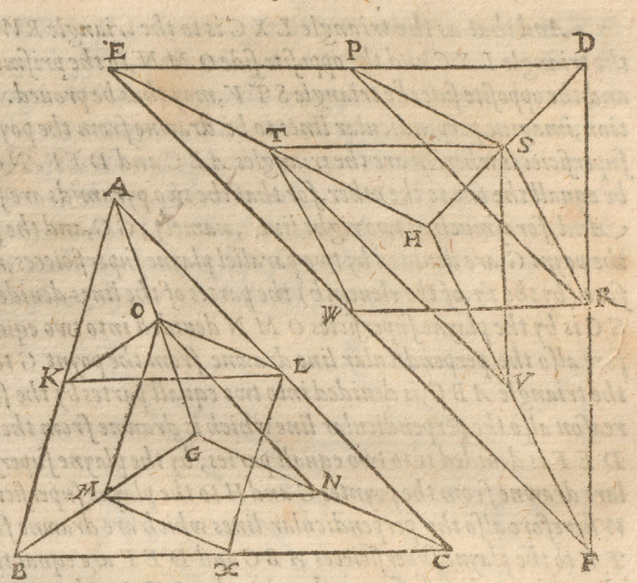
¶ *The 5. Theoreme.* *The 5. Proposition.*

Pyramids consisting vnder one and the selfe same altitude, hauing triangles to their bases: are in that proportion the one to the other that their bases are.

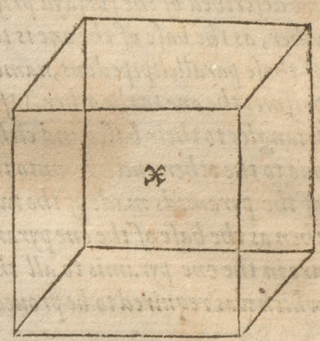
Vppose that these two Pyramids, whose bases are the triangles A B C & D E F, and toppes the pointes G and H, be vnder equall altitudes. Then I say, that as the base A B C is to the base D E F, so is the pyramis A B C G to the pyramis D E F H. For if the pyramis A B C G be not to the pyramis D E F H, as the base A B C is to the base D E F, then as the base A B C is to the base D E F, so is the pyramis A B C G to a solide, either lesse then the pyramis D E F H, or greater. First let it be to some lesse, and let the same be X. And (by the 3. of the twelfth) let the pyramis D E F H be deuided into two pyramids equall the one to the other, and like vnto the whole, and into two equall prismes. Now the two prismes are greater then the halfe of the whole pyramis.

Demonstration leading to an impossibilitie.

And againe (by the ſame) let the pyramids which are made of the diuiſion, be in like ſort diuided, and do this continually, vntill there remaine ſome pyramids made of the pyramis D E F H, which are leſſe then the exceſſe, whereby the pyramis D E F H excedeth the ſolide X. Let ſuch pyramids be taken, and for example ſake, let thoſe pyramids be D P R S, & S T V H. Wherfore the priſmes remayning which are in the pyramis D E F H, are greater then the ſolide X. Deuide (by the Propoſition next going before) the pyramis A B C G in like ſort, & as many times as the pyramis D E F H is deuided.



Wherefore (by the ſame) as the baſe A B C is to the baſe D E F, ſo are all the priſmes which are in the pyramis A B C G, to all the priſmes which are in the pyramis D E F H. But as the baſe A B C is to the baſe D E F, ſo is the pyramis A B C G to the ſolide X. Wherefore (by the 11. of the fift) as the pyramis A B C G, is to the ſolide X, ſo are the priſmes which are in the pyramis A B C G, to the priſmes which are in the pyramis D E F H. Wherefore alternately (by the 16. of the fift) as the pyramis A B C G is to the priſmes which are in it, ſo is the ſolide X, to the priſmes which are in the pyramis D E F H. But the pyramis A B C G is greater then the priſmes which are in it. Wherefore alſo the ſolide X is greater then the priſmes which are in the pyramis D E F H (by the 14. of the fift). But it is ſuppoſed to be leſſe: which is impoßible. Wherefore as the baſe A B C is to the baſe D E F, ſo is not the pyramis A B C G to any ſolide leße then the pyramis D E F H.



2. *I ſay moreouer, that as the baſe A B C is to the baſe D E F, ſo is not the pyramis A B C G, to any ſolide greater then the pyramis D E F H. For if it be poßible, let it be vnto ſome greater, namely, to the ſolide X. Wherefore (by conuerſion, by the Corollary of the 4. of the fift) as the baſe D E F is to the baſe A B C, ſo is the ſolide X to the pyramis A B C G. But as the ſolide X is to the pyramis A B C G, ſo is the pyramis D E F H to ſome ſolide leße then the pyramis A B C G, * as we haue before proued. Wherefore alſo (by the 11. of the fift) as the baſe D E F is to the baſe A B C, ſo is the pyramis D E F H, to ſome ſolide leſſe then the Pyramis A B C G: which thing we haue proued to be impoßible. Wherfore as the baſe A B C is to the baſe D E F, ſo is not the pyramis A B C G to any ſolide greater then the pyramis D E F H: and it is alſo proued that it is not in that proportion to any leſſe then the pyramis D E F H. Wherefore as the baſe A B C is to the baſe D E F, ſo is the pyramis A B C G to the pyramis D E F H. Wherefore pyramids conſiſting vnder one and the ſelfe ſame altitude, and hauing triangles to their baſes, are in that proportion the one to the other, that their baſes are: which was required to be demonſtrated.*

* *In the Aſſumpt following the ſecond propoſition of this booke.*

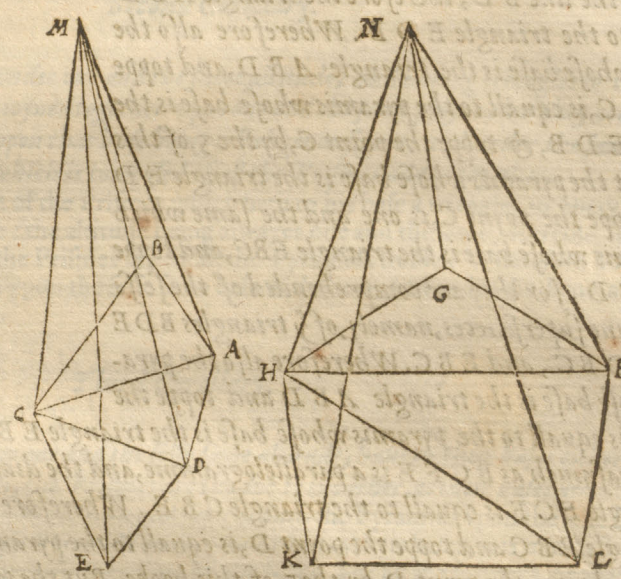
¶ *The*

¶ The 6. Theoreme. The 6. Proposition.

Pyramids consisting vnder one and the selfe same altitude, and hauing Poligonon figures to their bases: are in that proportion the one to the other, that their bases are.

Vppose that there be two Pyramids, hauing to their bases these Poligonon figures A B C E D, and F G H K L, and let their toppes be the pointes M and N, which let be of one and the selfe same altitude. Then I say, that as the base A B C E D is to the base F G H K L, so is the pyramis A B C E D M, to the pyramis F G H K L N. Diuide the base A B C E D into these triangles A B C, A C D, & C D E, and likewise the base F G H K L into these triangles F G H, F H L, and H K L. And imagine that vpon euery one of those triangles be set a pyramis of equall altitude with the two pyramids put at the beginning. And for that as the triangle A B C is to the triangle A D C, so is the pyramis A B C M to the pyramis A D C M (by the 5. of this boke). Wherfore, by composition (by the 18. of the fift) as the fower sided figure A B C D is to the triangle A C D, so is the pyramis A B C D M to the pyramis A C D M. But as the triangle A C D is to the triangle C D E, so is the pyramis A C D M to the pyramis C D E M. Wherefore of equalitie (by the 22. of the fift) as the base A B C D is to the base C D E, so is the pyramis A B C D M to the pyramis C D E M. Wherfore againe by composition (by the 18. of the fift) as the base A B C D E is to the base C D E, so is the pyramis A B C E D M to the pyramis C D E M. And by the same reason also as the base F G H K L is to the base H K L, so is the pyramis F G H K L N to the pyramis H K L N. And forasmuch as there are two pyramids C D E M and H K L N, hauing triangles to their bases, and being vnder one and the selfe same altitude, therefore (by the 5. of the twelfth) as the base C D E is to the base H K L, so is the pyramis C D E M to the pyramis H K L N. Now for that as the base A B C E D is to the base C D E, so is the pyramis A B C E D M to the pyramis C D E M. But as the base C D E is to the base H K L, so is the pyramis C D E M to the pyramis H K L N. Wherefore of equalitie (by the 22. of the fift) as the base A B C E D is to the base H K L, so is the pyramis A B C E D M to the pyramis H K L N. But also as the base H K L is the base F G H K L, so is the Pyramis H K L N to to the pyramis F G H K L N. Wherefore againe of equalitie (by the 22. of the fift) as the base A B C E D is to the base F G H K L, so is the pyramis A B C E D M to the pyramis F G H K L N. Wherefore pyramids consisting vnder one and the selfe same altitude, and hauing Polygonon figures to their bases, are in that proportion the one to the other, that their bases are: which was required to be proued.

Construction.



Demonstration.

The 7. Theoreme. The 7. Proposition.

Euery prisme hauing a triangle to his base, may be deuided into three pyramids equall the one to the other, hauing also triangles to their bases.

Demonstration.

SVppose that A B C D E F be a prisme, hauing to his base the triangle A B C, and the opposite side vnto it, the triangle D E F. Then I say that the prisme A B C D E F, may be deuided into three piramids equall the one to the other, and hauing triangles to their bases. Draw these right lines B D, E C, and C D. And forasmuch as A B E D is a parallelogramme, and his diameter is the line B D, therefore the triangle A B D is equall to the triangle E D B. Wherefore also the pyramis whose base is the triangle A B D, and toppe the poynt C, is equall to the pyramis whose base is the triangle E D B, & toppe the point C, by the 5. of this booke. But the pyramis whose base is the triangle E D B, and toppe the poynt C, is one and the same which the pyramis whose base is the triangle EBC, and toppe the poynt D, for they are comprehended of the selfe same playne superficieces, namely, of 3 triangles B D E D E C, D B C, and E B C. Wherefore also the pyramis whose base is the triangle A B D and toppe the poynt C, is equall to the pyramis whose base is the triangle E B C and toppe the point D. Againe forasmuch as B C F E is a parallelogramme, and the diameter thereof is E C, therefore the triangle E C F is equall to the triangle C B E. Wherefore also the pyramis whose base is the triangle E B C and toppe the poynt D, is equall to the pyramis, whose base is the triangle E C F and toppe the poynt D, by the 5. of this booke. But the pyramis whose base is the triangle B E C and toppe the poynt D, is proued to be equall to the pyramis whose base is the triangle A B D, and toppe the poynt C. Wherfore also the pyramis whose base is the triangle C E F and toppe the poynt D, is equall to the pyramis whose base is the triangle A B D & toppe the poynt C. Wherefore the prisme A B D E F is deuided into three equall pyramids hauing triangles to their bases. And forasmuch as the pyramis whose base is the triangle A B D and toppe the poynt C, is one & the selfe same with the pyramis whose base is the triangle C A B & toppe the poynt D (for they are contayned vnder the selfe same playne superficieces) but it hath beene proued that the pyramis whose base is the triangle A B D and toppe the poynt C, is the third pyramis of the prisme whose base is the triangle A B C and the opposite side vnto it the triangle D E F. Wherefore the pyramis whose base is the triangle A B C and toppe the poynt D is the third pyramis of the prisme, whose base is the triangle A B C, and opposite side the triangle D E F. Wherefore euery prisme hauing a triangle to his base, may be deuided into three pyramids equall the one to the other, hauing also triangles to their bases: which was required to be proued.

¶ Corollary.

Hereby it is manifest that euery pyramis is the third part of a prisme hauing one and the same base with it and also being vnder the selfe same altitude with it. For if the base of the prisme be any other rectiline figure the a triangle, that also may be deuided into prismes which shal haue triangles to their bases.

Here

Here *Campane* and *Flussas* adde certayne Corollaryes.

First Corollary.

Euery Prisme is treble to the Piramis, which hath the selfe same triangle to his base that the Prisme hath, and the selfe same altitude. As it is manifest by this proposition, where the Prisme is deuided into three equall Pyramids, of which, two are vpō one and the selfe same base, and vnder one and the selfe same altitude. But if the Prisme haue to his base a parallelogramme, and if the Pyramis haue to his base the halfe of the same parallelogramme, and their altitudes be equall, then agayne the Pyramis shalbe the third part of the Prisme. For it was manifest, by the 40. of the eleuēth, that Prismes, being vnder equall altitudes, and the one hauing to his base a triangle, and the other a parallelogramme double to the same triangle, are equall the one to the other. Wherof followeth the former conclusion.

Second Corollary.

If there be many Prismes vnder one and the same altitude, and hauing triangles to their bases, and if the triangular bases be so ioyned together vpon one and the same playne, that they compose a Poligonon figure: A pyramis set vpon that base being a Poligonon figure, and vnder the same altitude, is the third part of that solide, which is composed of all the Prismes added together.

For forasmuch as euery one of the Prismes which hath to his base a triangle, to euery one of the Pyramids set vpon the same base (the altitude being alwayes one and the same) is treble, it is manifest by the 12. of the fiueth, that all the Prismes are to all the Pyramids treble. Wherefore Parallelipipedons are treble to Pyramids set vpon the selfe same base with them, and vnder the same altitude, for that they contayne two Prismes.

Note. Sided Columnes (some time called prismes) are triple to pyramids, hauing one base and equall heith with them. Note: Parallelipipedons treble to pyramids of one base and heith with them.

Third Corollary.

If two Prismes being vnder one and the selfe same altitude, haue to their bases either, both triangles, or both parallelogramme., the Prismes are the one to the other, as their bases are.

For forasmuch as those Prismes are equemultiplices vnto the Pyramids vpon the selfe same bases, and vnder the same altitude, which Pyramids are in proportion as their bases, it is manifest (by the 15. of the fift), that the Prismes are in the proportion of the bases. For by the former Corollary, the Prismes are treble to the Pyramids set vpon the triangular bases.

Fourth Corollary.

Prismes are in sesquealtera proportion to Pyramids which haue the selfe same quadrangled base that the Prismes haue, and are vnder the selfe same altitude.

For, that Pyramis contayneth two Pyramids set vpon a triangular base of the same Prisme, for it is proued, that that Prisme is trebleto the Pyramis which is set vpon the halfe of his quadrangled base, vnto which the other which is set vpon the whole base is double, by the sixth of this booke.

Fiueth Corollary.

Wherefore we may in like sort conclude, that solides mencioned in the second Corollary (which solids Campane calleth sided Columnes) being vnder one and the selfe same altitude, are in proportion the one to the other, as their bases, which are poligonon figures.

For they are in the proportion of the Pyramids or Prismes, set vpon the selfe same bases, and vnder the selfe same altitude, that is, they are in the proportiō of the bases of the sayde Pyramids or Prismes. For those solids may be deuided into Prismes hauing the selfe same altitude, when as their opposite bases may be deuided into triangles, by the 20 of the sixth. Vpon which triangles Prismes beyng set, are in proportion as their bases.

By this 7. Proposition it playnely appeareth that *Euclide*, as it was before noted in the diffinitions, vnder the diffinition of a Prisme, comprehended also those kinds of solids, which *Campane* calleth sided Columnes. For in that he sayth, *Euery Prisme hauing a triangle to his base, may be deuided. &c.* he neded not (taking a Prisme in that sense which *Campane* and most men take it) to haue added that particle, *hauing to his base a triangle*. For by their sense, there is no Prisme, but it may haue to his base a triangle: and so it may seeme that *Euclide* ought without exceptiou haue sayd, that, euery prisme whatsoeuer, may

be deuided into three pyramids equall the one to the other, hauing also triangles to their bases. For so do *Campane* and *Flussas* put the proposition, leauing out the former particle *hauing to his base a triangle*, which yet is red in the Greeke copye, & not left out by any other interpreters knowne abroade except by *Campane* and *Flussas*. Yea and the Corollary following of this proposition added by *Theon* or *Euclide*, and amended by *M. Dee* semeth to confirme this sence.

Of this, is it made manifest, that euery pyramis is the third part of the prisme, hauing the same base with it, and equall altitude. For, and if the base of the prisme haue any other right lined figure (then a triangle) and also the superficies opposite to the base, the same figure: that prisme may be deuided into prismes, hauing triangled bases: and the superficieces to those bases opposite, also triangled alike and equally.

For there, as we see are put these wordes, *For and if the base of the prisme be any other right lined figure. &c.* whereof a man may well inferre that the base may be any other rectiline figure whatsoeuer, & not only a triangle or a parallelogramme, and it is true also in that sence, as it is plaine to see by the second corollary added out of *Flussas*, which corollary, as also the first of his corollaries, is in a maner all one with the Corollary added by *Theon* or *Euclide*. Farther *Theon* in the demonstration of the 10. proposition of this booke (as we shall afterward see) most playnely calleth not onely sided columnes prismes, but also parallelipipedons. And although the 40. proposition of the eleuenth booke may seme hereunto to be a let. For that it can be vnderstanded of those prismes onely which haue triangles to their like, equall, opposite, and parallel sides, or but of some sided columnes, and not of all: yet may that let be thus remoued away, to say that *Euclide* in that propositiō vsed *genus pro specie*, that is, the generall word for some special kinde therof: which thing also is not rare, not only with him, but also with other learned philosophers. Thus much I thought good by the way to note in farther defence of *Euclids* definition of a Prisme.

The 8. Theoreme. *The 8. Proposition.*

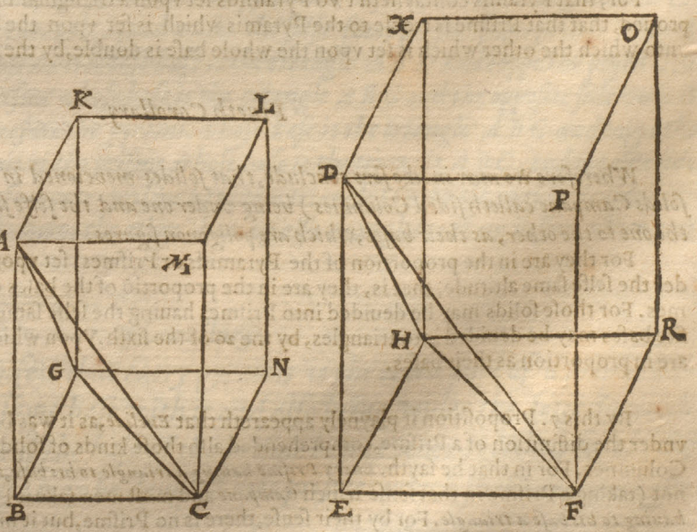
Pyramids being like & hauing triangles to their bases, are in treble proportion the one to the other, of that in which their sides of like proportion are.

Vppose that these pyramids whose bases are the triangles G B C and H E F and toppes, the poyntes A and D be like, and in like sort described, and let A B and D E be sides of like proportion. Then I say that the pyramis A B C G is to the pyramis D E F H in treble proportiō, of that in which the side A B is to the side D E. Make perfect the parallelipipedons, namely, the solides B C K L & E F X O. And forasmuch as the pyramis A B C G is like to the pyramis D E F H, therfore the angle A B C is equall

Construction.

Demonstration.

to the angle D E F, & the angle G B C to the angle H E F, and moreouer the angle A B G to the angle D E H, and as the line A B is to the line D E, so is the line B C to the line E F, and the line B G to the line E H. And for that as the line A B is to the line D E, so is the line B C to the line E F, and the sides about the equall angles, are proportionall, therefore the parallelogramme B M is like to the parallelogrāme E P: and by the same reason the parallelogramme B N is like



to the parallelogramme E R, and the parellelogramme B K is likevnto the parallelogramme E X. Wherefore the three parallelogrammes B M, K B and B N are like to the three parallelogrammes E P, E X, and E R. But the three parallelogrammes B M, K B, and B N are equall and like to the three oppoſite parallelagrammes, and the three parallelogrammes E P, E X, and E R are equall and like to the three oppoſite parallelogrammes. Wherefore the parallelipipedons B C K L and E F X O are comprehended vnder playne ſuperficieces like and equall in multitude. Wherefore the ſolide B C K L is like to the ſolide E F X O. But like parallelipipedons are (by the 33. of the eleuenth) in treble proportion the one to the other of that in which ſide of like proportion is to ſide of like proportion. Wherefore the ſolide B C K L is to the ſolide E F X O in treble proportion of that in which the ſide of like proportion A B is to the ſide of like proportion D E. But as the ſolide B C K L is to the ſolide E F X O, ſo is the pyramis A B C G to the pyramis D E F H (by the 15. of the fifth) for that the pyramis is the ſixth part of this ſolide: for the priſme, being the halfe of the parallelipipedon is treble to the pyramis. Wherefore the pyramis A B C G is to the pyramis D E F H in treble proportion of that in which the ſide A B is to the ſide D E. Which was required to be proued.

Corollary.

Hereby it is manifeſt that like pyramids hauing to their baſes poligonon figures, are in treble proportion the one to the other, of that in which ſide of like proportion, is to ſide of like proportion.

For if they be deuided into pyramids hauing triangles to their baſes (for like poligonō figures are deuided into like triangles, and equal in multitude, and the ſides are of like proportion) as one of the pyramids of the one, hauing a triangle to his baſe, is to one of the pyramids of the other, hauing alſo a triangle to his baſe, ſo alſo are all the pyramids of the one pyramis hauing triangles to their baſes to all the pyramids of the other pyramis hauing alſo triangles to their baſes. That is, the pyramis hauing to his baſe a poligonō figure, to the pyramis hauing alſo to his baſe a poligonō figure. But a pyramis hauing a triangle to his baſe, is to a pyramis hauing alſo a triangle to his baſe, & being like vnto it, in treble proportiō of that in which ſide of like proportiō is to ſide of like proportiō. Wherfore a pyramis hauing to his baſe a poligonō figure, is to a pyramis hauing alſo a poligonon figure to his baſe, the ſayd pyramids being like the one to the other, in treble proportion of that in which ſide of like proportion is to ſide of like proportion. Likewiſe Priſmes and ſided columnes, being ſet vpon the baſes of thoſe pyramids, and vnder the ſame altitude (foraſmuch as they are equemultiplices vnto the pyramids, namely, triples, by the corollary of the 7. of this booke) ſhal haue the former porportion that the pyramids haue, by the 15. of the fifth, and therefore they ſhall be in treble proportion of that in which the ſides of like proportion are.

An addition by Campane and Fluſſas.

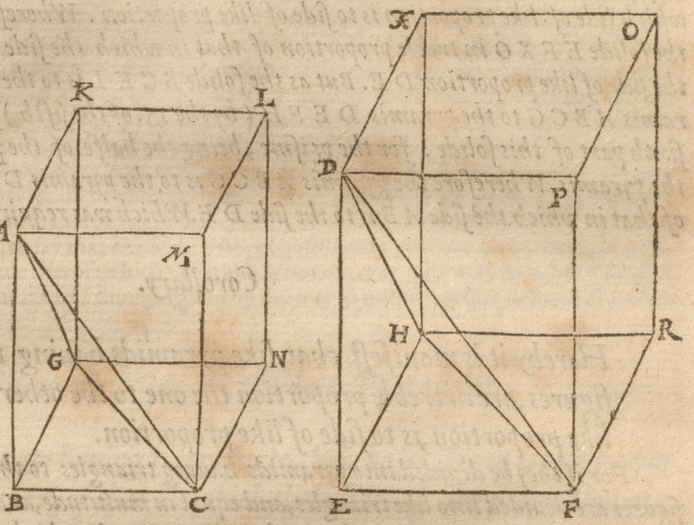
¶ The 9. Theoreme. The 9. Propoſition.

In equall pyramids hauing triangles to their baſes, the baſes are reciprokall to their altitudes. And pyramids hauing triangles to their baſes, whoſe baſes are reciprokall to their altitudes, are equall the one to the other.

Vppoſe that B C G A and E F H D be equall pyramids, hauing to their baſes the triangles B C G and E F H, and the tops the pointes A and D. Then I ſay that the baſes of the two pyramids B C G A and E F H D are reciprokall to their altitudes: that is, as the baſe B C G is to the baſe E F H, ſo is the altitude of the pyramis E F H D to the altitude of the pyramis B C G A. Make perfect the parallelipipedons, namely,

Demonstration of the first part.

namely, B G M L and E H P O. And forasmuch as the pyramis B C G A is equall to the pyramis E F H D, & the solide B G M L is sextuple to the pyramis B C G A. (For the parallelipipedon is duple to the Prisme set vpon the base of the Pyramis, & the Prisme is triple to the pyramis): and likewise the solide E H P O is sextuple to the pyramis E F H D. Wherefore the solide B G M L is equal to the solide E H P O. But in equall parallelipipedons, the bases are (by the 34. of the eleuēth) reciprokall to their altitudes. Wherfore as the base B N is to the base E Q, so is the altitude of the solide E H P O, to the altitude of the solide B G M L. But as the base B N is to the base E Q, so is the triangle G B C to the triangle H E F (by the 15. of the fifth, for the triangles G B C & H E F are the halues of the parallelogrammes B N and E Q). Wherfore (by the 11. of the fifth) as the triangle G B C is to the triangle H E F, so is the altitude of the solide E H P O to the altitude of the solide B G M L. But the altitude of the solide E H P O is one and the same with the altitude of the pyramis E F H D, and the altitude of the solide B G M L is one and the same with the altitude of the pyramis B C G A. Wherefore as the base G B C is to the base H E F, so is the altitude of the pyramis E F H D to the altitude of the pyramis B C G A. Wherefore the bases of the two pyramids B C G A and E F H D are reciprokall to their altitudes.



Demonstration of the second part, which is the conuerse of the first.

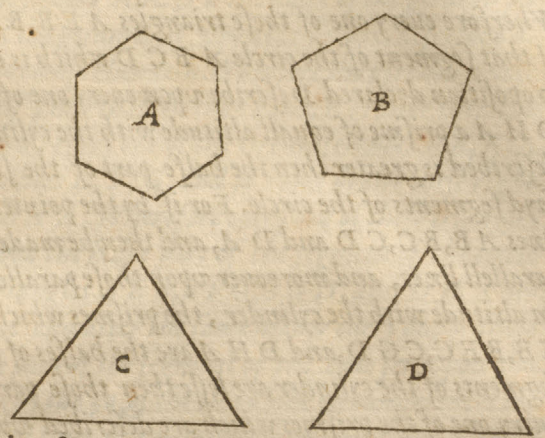
But now suppose that the bases of the pyramids B C G A and E F H D, be reciprokall to their altitudes, that is, as the base G B C is to the base H E F, so let the altitude of the pyramis E F H D be to the altitude of the pyramis B C G A. Then I say that the pyramis B C G A is equall to the pyramis E F H D. For (the selfe same order of construction remaining), for that as the base G B C is to the base H E F, so is the altitude of the pyramis E F H D to the altitude of the pyramis B C G A. But as the base G B C is to the base H E F, so is the parallelogramme G C to the parallelogramme H F. Wherefore (by the 11. of the fifth) as the parallelogramme G C is to the parallegoramme H F, so is the altitude of the pyramis E F H D to the altitude of the pyramis B C G A. But the altitude of the pyramis E F H D and of the solide E H P O, is one and the selfe same, and the altitude of the pyramis B C G A and of the solide B G M L, is also one and the same. Wherefore as the base G C is to the base H F, so is the altitude of the solide E H P O to the altitude of the solide B G M L. But parallelipipedons, whose bases are reciprokall to their altitudes are (by the 34. of the eleuenth) equall the one to the other. Wherefore the parallelipipedon B G M L is equall to the parallelipipedon E H P O. But the pyramis B C G A is the sixth part of the solide B G M L, and likewise the pyramis E F H D is the sixth part of the solide E H P O. Wherefore the pyramis B C G A is equall to the pyramis E F H D. Wherefore in equall pyramids hauing triangles to their bases, the bases are reciprokall to their altitudes. And pyramids hauing triangles to their bases, whose bases are reciprocall to their altitudes, are equall the one to the other: which was required to be demonstrated.

A Corollary

A Corrollary added by *Campane* and *Fluſſas*.

Hereby it is manifeſt that equall pyramids hauing to their baſes Poligonon figures, haue their baſes reciprokall with their altitudes. And Pyramids whoſe baſes being poligonon figures are reciprokall with their altitudes, are equall the one to the other.

Suppoſe that vpon the poligonon figures A and B, be ſet equall pyramids. Then I ſay that their baſes A and B are reciprokall with their altitudes. Deſcribe by the 25. of the ſixth, triangles equall to the baſes A and B. Which let be C and D. Vpon which let there be ſet pyramids equall in altitude with the pyramids A and B. Wherfore the pyramids C and D, being ſet vpō baſes equall with the baſes of the pyramids A and B, and hauing alſo their altitudes equall with the altitudes of the ſayd pyramids A and B, ſhall be equall by the 6. of this booke. Wherefore by the firſt part of this propoſition, the baſes of the pyramids, C to D are reciprokall with the altitudes of D to C. But in what proportion are the baſes C to D, in the ſame are the baſes A to B, foraſmuch as they are equall. And in what proportion are the altitudes of D to C, in the ſame are the altitudes of B to A, which altitudes are likewiſe equall. Wherefore by the 11. of the fifth, in what proportion the baſes A to B are, in the ſame reciprokally are the altitudes of the pyramids B to A. In like ſort by the ſecond part of this propoſition may be proued the conuerſe of this corollary. The ſame thing followeth alſo in a Priſme, and in a ſided columne, as hath before at large bene declared in the corollary of the 40. propoſition of the 11. booke. For thoſe ſolides are in proportiō the one to the other, as the pyramids or parallelipipedons, for they are either partes of equemultiplices or equemultiplices to partes.

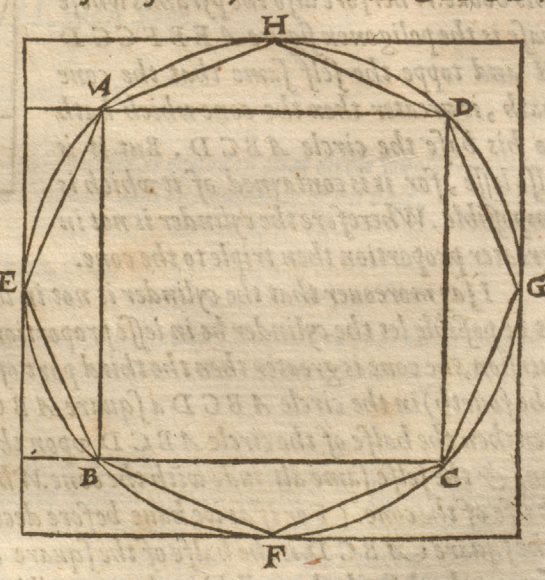


The 10. Theoreme. The 10. Propoſition.

Euery cone is the third part of a cilinder, hauing one and the ſelfe ſame baſe and one and the ſelfe ſame altitude with it.

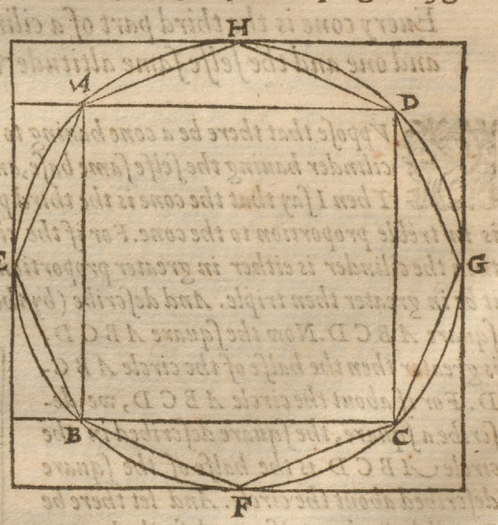
Vppoſe that there be a cone hauing to his baſe the circle A B C D, and let there be a cilinder hauing the ſelfe ſame baſe, and alſo the ſame altitude that the cone hath. Then I ſay that the cone is the third part of the cilinder, that is, that the cilinder is in treble proportion to the cone. For if the cilinder be not in treble proportion to the cone, then the cilinder is either in greater proportion then triple to the cone, or els in leſſe. Firſt let it be in greater then triple. And deſcribe (by the 6. of the fourth) in the circle A B C D a ſquare A B C D. Now the ſquare A B C D, is greater then the halfe of the circle A B C D. For if about the circle A B C D, we deſcribe a ſquare, the ſquare deſcribed in the circle A B C D is the halfe of the ſquare deſcribed about the circle. And let there be Parallelipipedon priſmes deſcribed vpon thoſe ſquares, equall in altitude with the cilinder. But priſmes are in that proportion the one to the other, that their baſes are (by the 32. of the eleuenth, and 5. Corollary of the 7. of this booke). Wherefore the priſme deſcribed vpon the ſquare A B C D is the halfe of the priſme deſcribed vpon the ſquare that is deſcribed about the circle. Now the clinder is leſſe then the priſme

Conſtruction. 1.



Parallelipipedons called Priſmes.

prisme which is made of the square described about the circle A B C D, being equal in altitude with it, for it contayneth it. Wherfore the prisme described vpon the square A B C D and being equall in altitude with the cylinder, is greater then half the cylinder. Deuide (by the 30. of the third) the circumferences A B, B C, C D and D A into two equall parts in the points E, F, G, H, And draw these right lines A E, E B, B F, F C, C G, G D, D H & H A. Wherfore euery one of these triangles A E B, B F C, C G D and D H A is greater then halfe of that segment of the circle A B C D which is described about it, as we haue before in the 2. proposition declared. Describe vpon euery one of these triangles A E B, B F C, C G D, and D H A a prisme of equall altitude with the cylinder. Wherefore euery one of these prismes so described is greater then the halfe part of the segment of the cylinder that is set vpon the sayd segments of the circle. For if by the pointes E, F, G, H, be drawen parallell lines to the lines A B, B C, C D and D A, and then be made perfect the parallelogrammes made by those parallell lines, and moreouer vpon those parallelogrāmes be erected parallelipipedons equall in altitude with the cylinder, the prismes which are described vpon eche of the triangles A E B, B F C, C G D, and D H A are the halfes of euery one of those parallelipipedons. And the segments of the cylinder are lesse then those parallelipipedons so described. Wherefore also euery one of the prismes which are described vpon the triangles A E B, B F C, C G D and D H A is greater then the halfe of the segment of the cylinder set vpon the sayd segment. Now therefore deuiding euery one of the circumferences remaining into two equall partes, and drawing right lines, and raysing vp vpon euery one of these triangles prismes equall in altitude with the cylinder, and doing this continually, we shall at the length (by the first of the tenth) leaue certaine segments of the cylinder which shalbe lesse then the excesse where by the cylinder excedeth the cone more then thrise. Let those segments be A E, E B, B F, F C, C G, G D, D H and H A. Wherfore the prisme remayning, whose base is the poligonon figure A E B F C G D H, and altitude the selfe same that the cylinder hath, is greater then the cone taken three tymes.* But the prisme whose base is the poligonon figure A E B F C G D H and altitude the selfe same that the cylinder hath, is treble to the pyramis whose base is the poligonon figure A E B F C G D A and altitude the selfe same that the cone hath, by the corollary of the 3. of this booke. Wherfore also the pyramis whose base is the poligonon figure A E B F C G D H and toppe the self same that the cone hath, is greater then the cone which hath to his base the circle A B C D. But it is also lesse, for it is contayned of it which is impossible. Wherefore the cylinder is not in greater proportion then triple to the cone.



* By this it is manifest that Euclide comprehended sided Columnes also vnder the name of a Prisme.

I say moreouer that the cylinder is not in lesse proportion then triple to the cone. For if it be possible let the cylinder be in lesse proportion then triple to the cone. Wherefore by conuersion, the cone is greater then the third part of the cylinder. Describe now (by the sixth of the fourth) in the circle A B C D a square A B C D. Wherefore the square A B C D is greater then the halfe of the circle A B C D vpon the square A B C D describe a pyramis hauing one & the selfe same altitude with the cone. Wherfore the pyramis so described is greater the halfe of the cone. (For if as we haue before declared we describe a square about the circle, the square A B C D is the halfe of the square described about the circle, and if vppon the squares be described parallelipipedons equall in altitude with the cone, which solides are also

called

*called prismes, the prisme or parallelipipedon described vpō the square A B C D is the halfe of the prisme which is described vpō the square described about the circle, for they are the one to the other in that proportiō that their bases are (by the 32. of the eleuēth, & 5. corollary of the 7. of this booke.) Wherfore also their third parts are in the self same proportion (by the 15. of the fift). Wherfore the pyramis whose base is the square ABCD is the halfe of the pyramis set vpon the square described about the circle. But the pyramis set vpon the square described about the circle is greater then the cone whome it comprehendeth. Wherfore the pyramis whose base is the square A B C D, and altitude the self same that the cone hath, is greater then the halfe of the cone.) Deuide (by the 30. of the third) euery one of the circumferences A B, B C, C D, and D A into two equall partes in the pointes E, F, G, and H: and drawe these right lines A E, E B, B F, F C, C G, G D, D H, and H A. Wherefore euery one of these triangles A E B, B F C, C G D, and D H A is greater then the halfe part of the segment of the circle described about it. Vppon euery one of these triangles A E B, B F C, C G D, and D H A describe a pyramis of equall altitude with the cone: and after the same maner euery one of those pyramids so described is greater then the halfe part of the segment of the cone set vpon the segment of the circle. Now therefore diuiding (by the 30, of the third) the circumferences remaining into two equall parts, & drawing right lines & rayſing vp vpon euery one of those triangles a pyramis of equall altitude with the cone, and doing this continually, we shal at the length (by the first of the tenth) leaue certayne segmentes of the cone, which shalbe lesse then the excesse whereby the cone excedeth the third part of the cylinder. Let those segmentes be A E, E B, B F, F C, C G, G D, D H, and H A. Wherefore the pyramis remayning, whose base is the poligonō figure A E B F C G D H and altitude the self same with the cone, is greater then the third part of the cylinder. But the pyramis whose base is the poligonon figure A E B F C G D H and altitude the self same with the cone, is the third part of the prisme whose base is the poligonō figure A E B F C G D H and altitude the self same with the cylinder. Wherfore * the prisme whose base is the poligonon figure A E B F C G D H, and altitude the self same with the cylinder, is greater then the cylinder whose base is the circle A B C D. But it is also lesse, for it is contayned of it, which is impossible. Wherfore the cylinder is not in lesse proportion to the cone then in treble proportion. And it is proued that it is not in greater proportion to the cone then in treble proportion, wherefore the cone is the third part of the cylinder. Wherfore euery cone is the third part of a cylinder, hauing one & the self same base, and one and the selfe same altitude with it: which was required to be demonstrated.*

* *A prisme hauing for his base a poligonon figure as we haue often before noted vnto you.*

¶ *Added by M. Iohn Dee.*

¶ *A Theoreme. 1.*

The superficies of euery vpright Cylinder, except his bases, is equall to that circle whose semidiameter is middell proportionall betwene the side of the Cylinder, and the diameter of his base.

¶ *A Theoreme. 2.*

The superficies of euery vpright, or Isosceles Cone, except the base, is equall to that circle, whose semidiameter is middell proportionall betwene the side of that Cone, and the semidiameter of the circle: which is the base of the Cone.

My entent in additions is not to amend *Euclides* Method, (which nedeth little adding or none at all). But my desire is somwhat to furnish you, toward a more general art Mathematical the *Euclides* Elemēts, (remayning in the termes in which they are written) can sufficiently helpe you vnto. And though *Euclides* Elementes with my Additions, run not in one Methodicall race toward my marke: yet in the meane space my Additions either geue light, where they are annexed to *Euclides* matter, or geue some ready ayde, and shew the way to dilate your discourses Mathematicall, or to inuent and practise thinges

Note: M. Dee his chiefe purpose in his additions.

Mechanically. And (in deede) if more leyſor had happened, many more ſtraunge matters Mathematicall had, (according to my purpoſe generall) bene preſently publiſhed to your knowledge: but want of due leaſour cauſeth you to want, that, which my good will toward you, moſt hartely doth wiſh you.

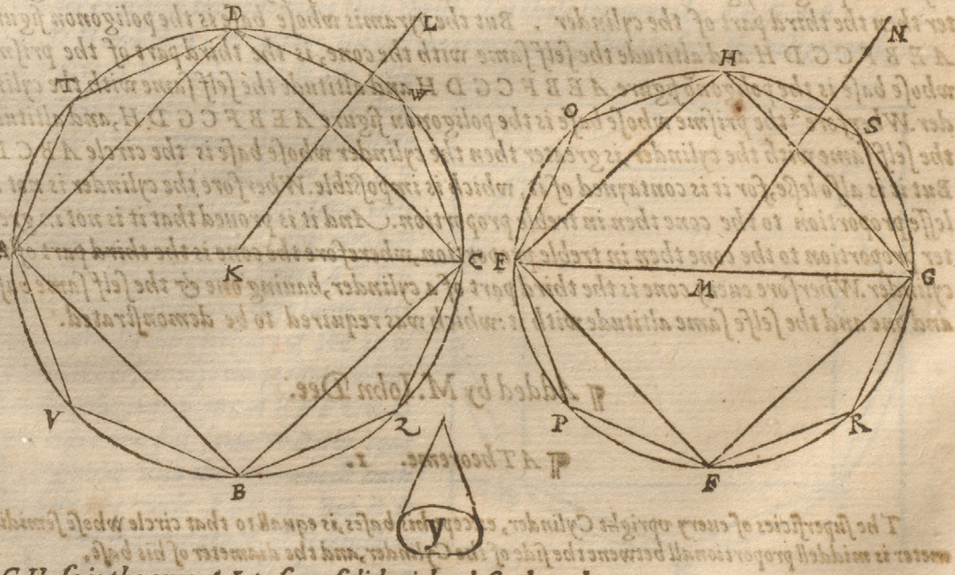
As concerning the two Theoremes here annexed, their veritie, is by *Archimedes*, in his booke of the Sphere and Cylinder manifeſtly demonſtrated, and at large: you may therefore boldly truſt to them, and vſe them, as ſuppoſitions, in any your purpoſes: till you haue alſo their demõſtrations. But if you well remember my inſtructions vpon the firſt propoſition of this booke, and my other additions vpon the ſecond, with the ſuppoſitions how a Cylinder and a Cone are Mathematically produced, you will not neede *Archimedes* demonſtration: nor yet be vtterly ignoraunt of the ſolide quantities of this Cylinder and Cone here compared: (the diameter of their baſe, and heith being knowne in any meaſure) neither can their croked ſuperficies remayne vnmeaſured. Whereof vndoubtedly great pleaſure and commoditie may grow to the ſincere ſtudent, and preciſe practiſer.

¶ The 11. Theoreme. The 11. Propoſition.

Cones and Cylinders being vnder one and the ſelfe ſame altitude, are in that proportion, the one other that their baſes are.

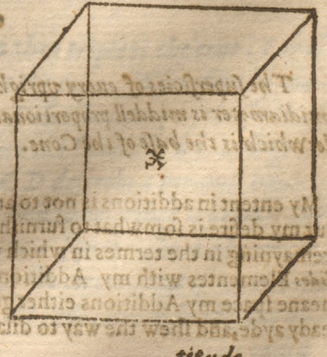
Demonſtration as touching Cones.

LEt there be take cones & cylindres vnder one and the ſelfe ſame altitude whoſe baſes let be the circles A B C D and E F G H, and axes the lines K L & M N, and let the diameters of their baſes be A C and E G. The I ſay that as the circle A B C D is to the circle E F G H, ſo is the cone A L to the cone E N, and alſo the cylinder A L to the cylinder E N. For if the cone A L be not to the cone E N as the circle A B C D is to the circle E F G H, then as the circle A B C D is to the circle E-



First caſe. 1.

F G H, ſo is the cone A L to ſome ſolide either leſſe then the cone E N, or greater. Firſt let it be vnto a leſſe, namely, to the ſolide X. And vnto that which the ſolide X is leſſe then the cone E N, let the ſolide T be equall. Wherefore the cone E N is equal to the ſolides X & T. Deſcribe (by the 6. of the fourth) in the circle E F G H a ſquare E F G H. Wherefore the ſquare is greater then halfe the circle. Rayſe vp vpon the ſquare E F G H a pyramis of equall altitude with the cone. Wherefore the pyramis, ſo rayſed vp is greater then halfe of the cone. For if we deſcribe a ſquare about the circle, and vpon that ſquare rayſe vp a pyramis of equall al-



titude

titude with the cone, the pyramis which is ſet vpon the ſquare deſcribed with in the circle is the halfe of the pyramis ſet vpon the ſquare deſcribed about the circle, for they are in proportion the one to the other as their baſes. But the cone is leſſe then the pyramis which is ſet vpon the ſquare deſcribed about the circle. Wherefore the pyramis whoſe baſe is the ſquare E F G H and top one and the ſame with the cone is greater then halfe of the cone: deuide (by the 30. of the third) the circumferences E F, F G, G H and H E into two equal partes in the points O, P, R, S, and draw theſe right lines H O, O E, E P, P F, F R, R G, G S, and S H. Wherefore euery one of theſe triangles H O E, E P F, F R G, and G S H is greater then halfe the ſegmẽt of the circle deſcribed about thẽ. Rayſe vp vpõ euery one of the triangles H O E, E P F, F R G and G S H a pyramis equall in altitude with the cone. Wherefore euery one of the pyramids ſo rayſed vp is greater then the halfe part of the ſegment of the cone deſcribed about it. Now then deuiding (by the 30. of the third) the circumferences remayning into two equall partes, and drawing right lines, and rayſing vp vppon euery one of thoſe triangles a pyramis of equall altitude with the cone, and thus doing continually, we ſhall at length by the firſt of the tenth leue certayne ſegmentes of the cone which ſhall be leſſe then the ſolide Y. Let thoſe ſegmentes be H O E, E P F, F R G, and G S H. Wherefore the pyramis remayning whoſe baſe is the poligonon figure H O E P F R G S & top the ſelfe ſame with the cone, is greater then the ſolide X. Inſcribe in the circle A B C D a poligonon figure like and in like ſort ſituate to the poligonon figure H O E P F R G S, and let the ſame be D T A V B Z C W, and vpon it rayſe a pyramis of equall altitude with the cone A L. Now for that as the ſquare of the line A C is to the ſquare of the line E G, ſo is the poligonon figure D T A V B Z C W to the poligonon figure H O E P F R G S (by the firſt of this booke): But as the ſquare of the line A C is to the ſquare of the line E G, ſo is the circle A B C D to the circle E F G H (by the ſecond of this booke). Wherefore (by the 11. of the fift) as the circle A B C D is to the circle E F G H, ſo is the poligonon figure D T A V B Z C W to the poligonon figure H O E P F R G S (this foloweth alſo of the corollary of the 2. of this booke): moreouer as the circle A B C D is to the circle E F G H, ſo is the cone A L to the ſolide X. And as the poligonon figure D T A V B Z C W, is to the poligonon figure H O E P R G S, ſo is the pyramis whoſe baſe is the poligonon figure D T A V B Z C W, and toppe the poynt L, to the pyramis whoſe baſe is the poligonon figure H O E P R S, and toppe the poynt N. Wherefore (by the 11. of the fift) as the cone A L is to the ſolide X, ſo is the pyramis whoſe baſe is the poligonõ figure D T A V B Z C W, and toppe the poynt L, to the pyramis whoſe baſe is the poligonon figure H E O P F R G S, and toppe the poynt N. Wherefore alternately (by the 16. of the fift) as the cone A L is to the pyramis which is in it, ſo is the ſolide X, to the pyramis which is in the cone E N. But the cone A L, is greater then the pyramis which is in it. Wherefore alſo the ſolide X is greater then the pyramis which is in the cone E N. But it is alſo leſſe by conſtruction. Which is impoſsible. Wherfore as the circle A B C D, is to the circle E F G H, ſo is not the cone A L to any ſolide leſſe then the cone E N.

Second caſe.

In like ſorte alſo may we proue, that as the circle E F G H, is to the circle A B C D, ſo is not the cone E N to any ſolide leſſe then the cone A L. Now I ſay that as the circle A B C D, is to the circle E F G H, ſo is not the cone A L to any ſolide greater then the cone E N. For if it be poſsible let it be vnto a greater, namely to the ſolide X. Wherefore by conuerſion, as the circle E F G H, is to the circle A B C D, ſo is the ſolide X to the cone A L: but as the ſolide X is to the cone A L, ſo is the cone E N, to ſome ſolide leſſe then the cone A L (as we may ſee by the aſſumpt put after the ſecond of this booke): Wherefore (by the 11. of the fift) as the circle E F G H is to the circle A B C G, ſo is the cone E N to ſome ſolide leſſe then the cone A L, which we haue proued to be impoſsible. Wherefore as the circle A B C D, is to the circle E F G H, ſo is not the cone A L to any ſolide greater then the cone E N. And it is alſo proued that it is not to any leſſe. Wherefore as the circle A B C D, is to the circle E F G H, ſo is the cone A L to the cone E N.

But as the cone is to the cone, ſo is the cylinder to the cylinder, (by the 15. of the fift) for

Demonstration touching cylinders.

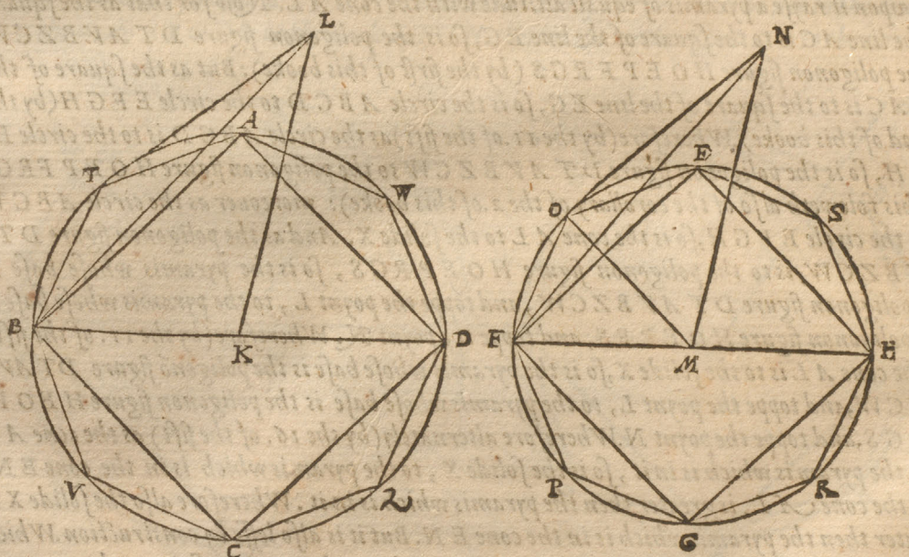
the one is in treble proportion to the other. Wherefore) by the 11. of the fift) as the circle A B C D is to the circle E F G H, so are the cylinders which are set vpon them the one to the other, the said cylinders being vnder equall altitudes with the cones. Cones therefore and cylinders, being vnder one & the self same altitude, are in that proportion the one to the other, that their bases are: which was required to be demonstrated.

¶ The 12. Theoreme. The 12. Proposition.

Like Cones and Cylinders, are in treble proportion of that in which the diameters of their bases are.

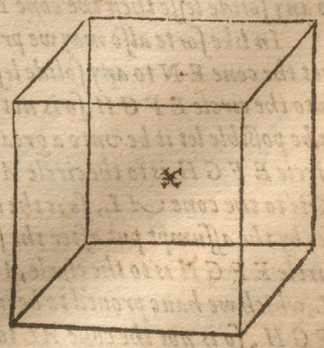
Vppose that these Cones and Cylinders, whose bases are the circles A B C D, and E F G H, and diameters of their bases B D and F H, be like, and let the axes of the Cones or Cylinders be K L and M N, and let the centres of their bases be the pointes K and M. Then I say, that the cone, whose base is the circle A B C D, & toppe the point L, is to the Cone, whose base is the circle E F G H, and toppe the point N, in treble proportion of that in which the diameter B D is to the diameter F H. For if the cone A B C D L be not to the cone E F G H N in treble proportion of that in which the diameter

First part which concerneth cones.



B D is to the diameter F H, the cone A B C D L shall be in treble proportion of that in which the diameter B D is to the diameter F H, either to some solide lesse then the cone E F G H N, or to some solide greater. First let it be vnto a lesse, namely, to the solide X. Describe (by the 6. of the fourth) in the circle E F G H, a square E F G H. Wherefore the square E F G H is greater then the halfe of the circle E F G H. Raise vp from the square E F G H a pyramis of equall altitude with the cone. Wherfore the pyramis so raised vp, is greater then the halfe part of the cone. Diuide now (by the 30. of the third) the circumferences E F, F G, G H, & H E, into two equall partes in the pointes O, P, R, S, and drawe these right lines E O, O F, F P, P G, G R, R H, H S, and S E. Wherefore euery one of these

First case. Construction.

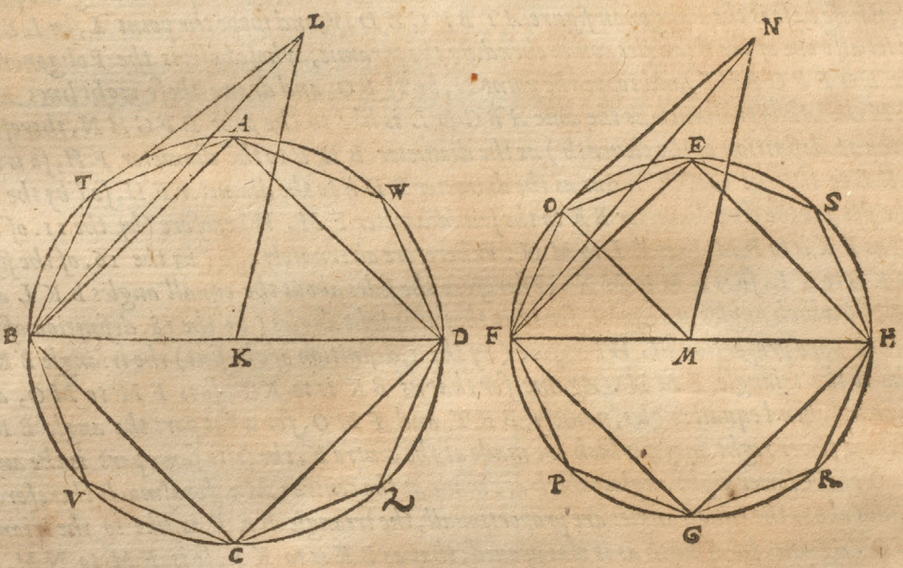


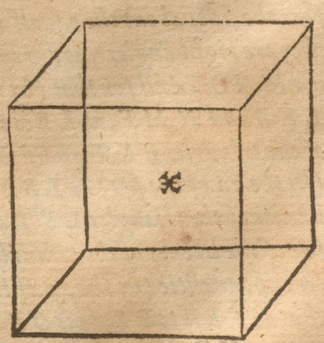
triangles

triangles E O F, F P G, G R H, and H S E, is greater then the halfe of the segment of the circle E F G H, described about ech of them. Erecte vpon euery one of these triangles E O F, F P G, G R H, and H S E, a pyramis, hauing one and the selfe same altitude with the cone. Wherefore euery one of the pyramids so raised vp, is greater then the halfe of the segment of the cone described about them. Now therefore diuiding the circumferences remayning into two equall partes, and drawing right lines, and raising vp vpon euery one of the triangles a pyramis, hauing one and the selfe same altitude with the cone, and, thus doing continually, we shall at the length (by the 1. of the tenth) leaue certaine segmentes of the cone, which shall be lesse then the excesse whereby the cone E F G H N excedeth the solide X. Let those segmentes left be E O, O F, F P, P G, G R, R H, H S, and S E. Wherefore the pyramis remayuing, whose base is the Poligonon figure E O F P G R H S, and toppe the point N, is greater then the solide X. Describe (by the 18. of the sixt) in the circle A B C D, vnto the Poligonon figure E O F P G R H S, a Poligonon figure like and in like sort situate, and let the same be A T B V C Z D W, and from it raise vp a pyramis, hauing one and the selfe same altitude with the cone A B C L. And let one of the triangles comprehending the pyramis, whose base is the Poligonon figure A T B V C Z D W, and toppe the point L, be L B T, and let also one of the triangles comprehending the pyramis, whose base is the Poligonon figure E O F P G R H S, and toppe the point N, be N F O, and drawe these right lines K T and M O. And forasmuch as the cone A B C D L is like to the cone E F G H N, therefore (by the 20. definition of the eleuenth) as the diameter B D is to the diameter F H, so is the axe K L to the axe M N: But as the diameter B D is to the diameter F H, so (by the 15. of the fift) is the semidiameter B K to the semidiameter F M. Wherefore (by the 11. of the fift) as B K is to F M, so is K L to M N. Wherefore alternately also (by the 16. of the fift) as B K is to K L, so is F M to M N. Wherefore the sides about the equall angles B K L and F M N (which angles are equall, for that they are right angles (by the 18. definition of the eleuenth) are proportionall. Wherefore (by the 1. definition of the sixt) the triangle B K L is like to the triangle F M N. Againe for that as B K is to K T, so is F M to M O, and they comprehend equall angles, namely, B K T and F M O, for what part the angle B K T is of those fower right angles which are made at the centre K, the selfe same part is the angle F M O of the fower right angles which are made at the centre M: forasmuch therefore as the sides about the equall angles are proportionall, the triangle B K T is like to the triangle F M O. Againe, forasmuch as it was proued, that as B K is to K L, so is F M to N M, but B K is equall to K T, and F M to M O, therefore as T K is to K L, so is O M to M N. Wherfore the sides about the equall angles T K L and O M N (which angles are equall, for that they are right angles) are proportionall. Wherefore the triangle L K T is like to the triangle M N O. And for that (by the 6. of the sixt) and by reason of the likenes of the triangles L K B and N M F, as L B is to B K, so is N F to F M, and againe by reason of the likenes of the triangles B K T and F M O, as K B is to B T, so is M F to F O, therefore of equalitie (by the 22. of the fift) as L B is to B T, so is N F to F O. Againe for that by reason of the likenes of the triangles L K T and N O M, as L T is to T K, so is N O to O M, and by reason of the likenes of the triangles T K B & O M F, as K T is to T B, so is M O to O F: therfore of equalitie (by the 22. of the fift) as L T is to T B, so is N O to O F. And it was proued that as T B is to B L, so is O F to F N. Wherefore againe of equalitie, as T L is to L B, so is O N to N F. Wherefore the sides of the triangles L T B & N O F, are proportionall. Wherfore (by the 5. of the sixt) the triangles L T B and N O F, are equiangle. Wherefore also they are like. Wherefore the pyramis, whose base is the triangle B K T, and toppe the point L, is like vnto the pyramis, whose base is the triangle F M O, and toppe the point N. For they are comprehended vnder like plaine superficieces, and equall in multitude. But pyramids being like, and hauing triangles to their bases, are (by the 8. of the twelfth) in treble proportion the one to the other of that in which sides of like proportion are. Wherefore the pyramis

Demonstration leading to an impossibilitie.

B K T L is to the pyramis F M O N, in treble proportion of that in which the line B K is to the line F M. And in like ſort if we draw right lines from the pointes A, W, D, Z, C, and V, to the point K, and likewiſe from the pointes E, S, H, R, G, and P, to the point M, and raiſe vp vpon the triangles pyramids, hauing the ſelfe ſame altitudes with the cones, we may proue that euery one of thoſe pyramids of one & the ſelfe ſame order, is to euery one of the pyramids of the ſelfe ſame order, in treble proportion of that in which the ſide of like proportion B K is to ſide of like proportion F M, that is, of that which the line B D hath to the line F H. But as one of the antecedentes is to one of the conſequentes, ſo are all the antecedentes to all the conſequẽtes (by the 12. of the fift). Wherfore as the pyramis B K T L is to the pyramis F M O N, ſo is the whole pyramis, whoſe baſe is the Poligonon figure A T B V C Z D W, and toppe the point L, to the whole pyramis, whoſe baſe is the Poligonon figure E O F P G R H S, and toppe the point N. Wherefore the pyramis, whoſe baſe is the Poligonon figure A T B V C Z D W, and toppe the point L, is to the pyramis, whoſe baſe is the Poligonon figure E O F P G R H S, and toppe the point N, in treble proportion of that in which the line B D is to the line F H.





And it is ſuppoſed alſo that the cone, whoſe baſe is the circle A B C D, and toppe the point L, is to the ſolide X, in treble proportion of that in which the line B D is to the line F H. Wherefore as the cone, whoſe baſe is the circle A B C D, and toppe the point L, is to the ſolide X, ſo is the pyramis, whoſe baſe is the Poligonon figure A T B V C Z D W, & toppe the point L, to the pyramis, whoſe baſe is the Poligonon figure E O F P G R H S, and toppe the point N. Wherefore alternately (by the 16. of the fift) as the cone, whoſe baſe is the circle A B C D, & toppe the point L, is to the pyramis which is in it, whoſe baſe is the Poligonõ figure A T B V C Z D W, & toppe the point L, ſo is the ſolide X, to the pyramis, whoſe baſe is the Poligonon figure E O F P G R H S, and toppe the point N. But the foreſaid cone is greater then the pyramis which is in it, for it containeth it. Wherefore the ſolide X is greater then the pyramis, whoſe baſe is the Poligonon figure E O F P G R H S, and toppe the point N. But it was ſuppoſed to be leſſe: which is impoßible. Wherfore the cone A B C D L is not to any ſolide leſſe then the cone E F G H N, in treble proportion of that in which the

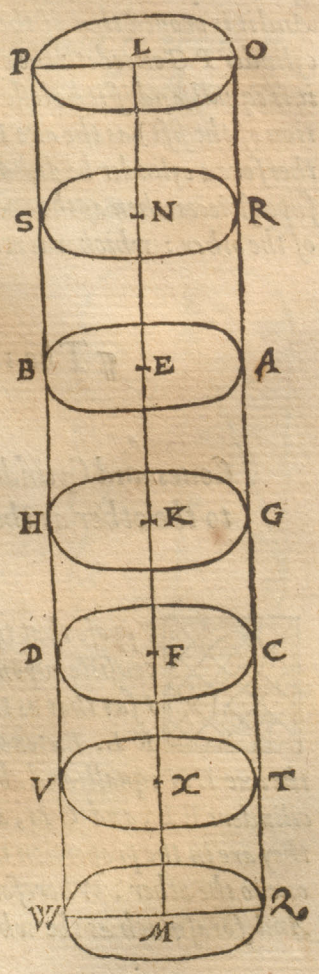
diameter B D is to the diameter F H. In like sort also may we proue, that the cone E F G H N is not to any solide lesse then the cone A B C D L, in treble proportion of that in which F H is to B D.

Now also I say, that the cone A B C D L is not to any solide greater then the cone E F G H N, in treble proportion of that in which the diameter B D is to the diameter F H. For if it be possible, let it be to a greater, namely, to the solide X. Wherefore by conuersion (by the Corollary of the 4. of the fift) the solide X is to the cone A B C D L, in treble proportion of that in which the diameter F H is to the diameter B D. But as the solide X is to the cone A B C D L, so is the cone E F G H N to some solide lesse then the cone A B C D L (as it is easie to see by the Assumpt put after the 2. Proposition). Wherefore also the cone E F G H N is vnto some solide lesse then the cone A B C D L, in treble proportion of that in which the diameter F H is to the diameter B D: which is proued to be impossible. Wherefore the cone A B C D L is not to any solide greater then the cone E F G H N in treble proportion of that in which the diameter B D is to the diameter F H, and it is also proued that it is not to any lesse. Wherefore the cone A B C D is to the cone E F G H N in treble proportion of that in which the diameter B D is to the diameter F H. Second case.

But as cone is to cone, so is cylinder to cylinder (by the 15. of the fift) for the cylinder is triple to the cone which is described on the one and selfe same base, and hauing one and the selfe same altitude with the cone. For it is proued (by the 10. of the twelfth) that euery cone is the third part of a cylinder, hauing one and the selfe same base with it, and one & the selfe same altitude. Wherefore the cylinder is vnto the cylinder in treble proportion of that in which the diameter B D is to the diameter F H. Wherfore like cones & cylinders are in treble proportion of that in which the diameters of their bases are: which was required to be proued. Second part which concerneth Cilinders.

The 13. Theoreme. The 13. Proposition.

If a Cylinder be diuided by a playne superficies being a parallell to the two opposite playne superficieces: then as the one Cylinder is to the other Cylinder, so is the axe of the one to the axe of the other.

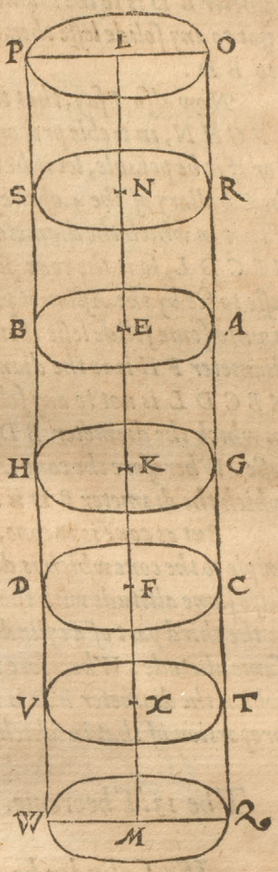


Construction.

Vppose that there be a cylinder A D, whose axe let be E F, and let the opposite bases be the circles A E B, and C F D, and let A D be diuided by the superfices G H being a parallell to the two opposite playne superficieces, A B, and C D. Then I say that as the cylinder B G is to the cylinder G D, so is the axe E K to the axe K F. Extend the axe E F on either side to the pointes L & M. And vnto the axe E K put as many axes equall as you will, namely, E N, & N L, & likewise vnto the axe F K put as many axes equal as you wil, namely, F X & X M. And by the points L, N, & X, M, extẽde playne superficieces parallels to the two superficieces A B, & C D (by the corollary of the 15. of the eleuẽth): & in the plaine superficieces thus extended by the pointes L, N, X, M, imagine to be be drawne these circles, namely, O P, R S, T V, and Z W, hauing to their centers the pointes L, N, M, X, and let them be equal to either of the circles A B, and C D: and vppon those circles imagine these cylinders P R, R B, D T, T W to

Demonſtration.

be ſet. Now foraſmuch as the axes L N, N E, and E K are equall the one to the other: Therefore the cylinders P R, R B, and B G are (by the 11. of the twelueth) in proportion the one to the other as their baſes are. But the baſes are equall. Wherefore alſo the cylinders P R, R B, and B G are equall the one to the other. And foraſmuch as the axes L N, N E, and E K are equal the one to the other, and the cylinders P R, R B, and B G are alſo equall the one to the other, and the multitude of the axes L N, N E, and E K is equall to the multitude of the cylinders P R, R B, and B G: therefore how multiplex the whole axe K L is to the axe E K, ſo multiplex is the whole cylinder P G to the cylinder B G. And (by the ſame reaſon) alſo how multiplex the whole axe M K is the axe K F, ſo multiplex is the whole cylinder W G to the cylinder G D. Wherfore if the axe K L be equal the axe K M, the cylinder P G is equall to cilinder G W. And if the axe K L be greater then the axe K M, the cylinder P G is greater then the cylinder G W. And if it be leſſe it is leſſe. Now therefore there are foure magnitudes, namely, the two axes E K, and K F, and the two cylinders B G, and G D, and vnto the axe E K, and to the cylinder B G, namely, to the firſt and the third are taken equemultiplices, namely, the axe K L, and the cylinder P G. And likewiſe vnto the axe G F, and vnto the cylinder G D, namely, the ſecond and the fourth, are taken other equemultiplices, namely, the axe K M, and the cylinder G W. And it is proued that if the axe K L, excede the axe K M, the cylinder P G excedeth the cylinder G W: and that if it be equall it is equall, and if it be leſſe it is leſſe. Wherfore (by the 6. definition of the fift) as the axe E K is to the axe K F, ſo is the cylinder B G to the cylinder G D. If therfore a cylinder be diuided by a plaine ſuperficies being a parallel to the two oppoſite plaine ſuperficieces: then as the one cylinder is to the other cilinder, ſo is the axe of the one to the axe of the other: which was required to be proued.



¶ The 14. Theoreme. The 14. Propoſition.

Cones and Cylinders conſiſting vpon equall baſes, are in proportion the one to the other as their altitudes.

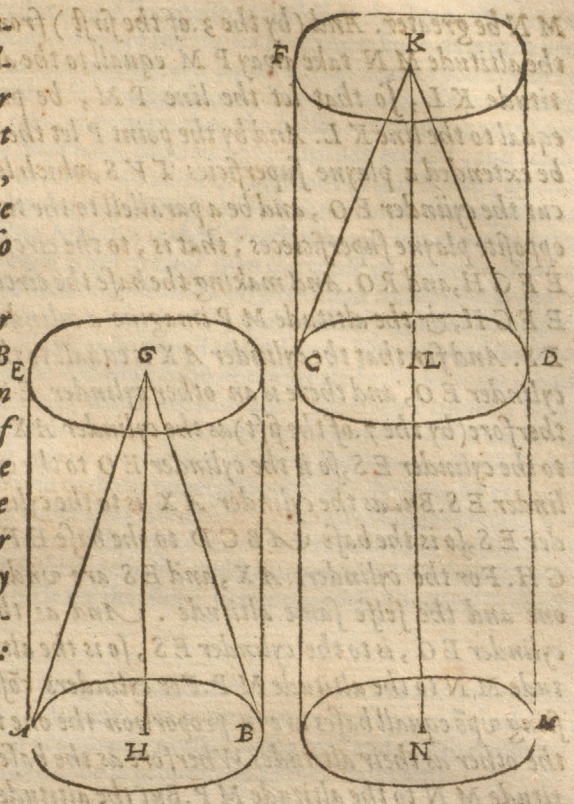
Conſtruction.

Demonſtration touching Cylinders.

SVppoſe that the cylinders F D, and E B, and the cones A G B, and C K D, do conſiſte vpon equall baſes, namely, vpon the circles A B, and C D. Then I ſay that as the cylinder E B is to the cylinder F D, ſo is the axe G H to the axe K L. Extende the axe K L directly to the poynte N, and vnto the axe G H, put the axe L N equall: and about the axe L N imagine a cylinder C M. Now foraſmuch as the cilinders E B, and C M, are vnder equall altitudes, therefore (by the 11. of the twelueth) they are in the proportion the one to the other as their baſes are. But the baſes are [illegible] one to the other. Wherefore alſo the cylinders E B, and C M are equall the one to the other. And foraſmuch as the whole cylinder F M, is diuided by a playne ſuperficies C D being a parallel

a parallell to either of the oppoſite plaine ſuperficieces: therefore (by the 13. of the twelueth) as the cylinder C M is to the cylinder F D, ſo is the axe L N to the axe L K. But the cylinder C M is equal to the cylinder E B, and the axe L N to the axe G H. Wherefore as the cylinder E B is to the cylinder F D, ſo is the axe G H to the axe K L.

But as the cylinder E B is to the cylinder F D, ſo (by the 15. of the fift) is the cone A B G to the cone C D K, for the cylinders are in treble proportion to the cones (by the 10. of the twelueth). Wherefore (by the 11. of the fift) as the axe G H is to the axe K L, ſo is the cone A B G to the cone C D K, & the cylinder E B to the cylinder F D. Wherfore cones & cylinders conſiſting vpon equal baſes are in proportion the one to the other as their altitudes: which was required to be demonſtrated.



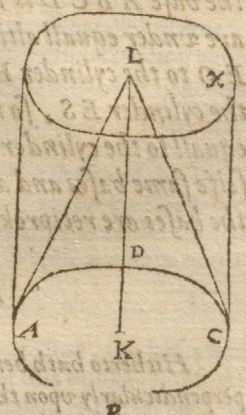
Demonſtration touching Cones.

¶ The 15. Theoreme. The 15. Propoſition.

In equall Cones and Cylinders, the baſes are reciprokall to their altitudes. And cones and Cylinders whoſe baſes are reciprokall to their altitudes, are equall the one to the other.

SVppoſe that theſe cones A C L, E G N or theſe cylinders A X, E O, whoſe baſes are the circles A B C D, E F G H, and axes K L, and M N (which axes are alſo the altitudes of the cones & cylinders) be equall the one to the other. The I ſay that the baſes of the cylinders X A & E O are reciprokal to their altitudes, that is, that as the baſe A B C D is to the baſe E F G H, ſo the altitude M N to the altitude K L. For the altitude K L is either equall to the altitude M N or not. Firſt let it be equall. But the cylinder A X is equal to the cylinder E O. But cones and cylinders conſiſting vnder one and the ſelfe ſame altitude, are in proportion the one to the other as their baſes are (by the 11. of the twelueth). Wherfore the baſe A B C D is equall to the baſe E F G H. Wherefore alſo they are reciprokal: as the baſe A B C D is to the baſe E F G H, ſo is the altitude M N to the altitude K L.

But now ſuppoſe that the altitude L K be not equall to the altitude M N, but let the altitude



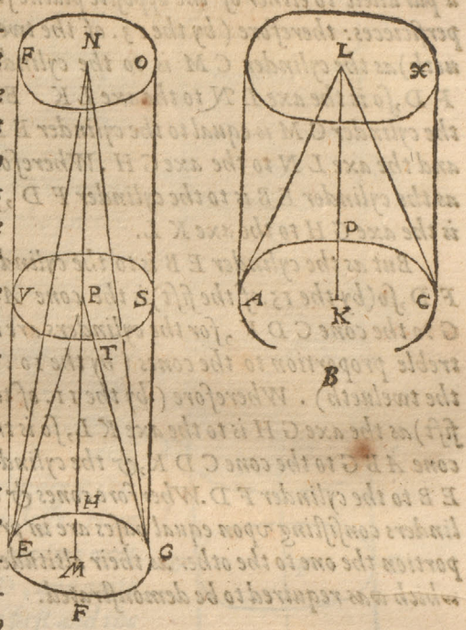
First part of the propoſitiõ demonſtrated touching Cones. Two caſes in this propoſition. The firſt caſe.

Second caſe. Conſtruction.

M N

M N be greater. And (by the 3. of the first) from the altitude M N take away P M equall to the altitude K L, so that let the line P M, be put equal to the line K L. And by the point P let there be extended a playne superficies T V S, which let cut the cylinder E O, and be a parallell to the two opposite playne superficieces, that is, to the circles E F G H, and R O. And making the base the circle E F G H, & the altitude M P imagine a cylinder E S. And for that the cylinder A X is equall to the cylinder E O, and there is an other cylinder E S, therfore (by the 7. of the fift) as the cylinder A X is to the cylinder E S, so is the cylinder E O to the cylinder E S. But as the cylinder A X is to the cylinder E S, so is the base A B C D to the base E F G H. For the cylinders A X, and E S are vnder one and the selfe same altitude. And as the cylinder E O, is to the cylinder E S, so is the altitude M N to the altitude M P. For cylinders cõsisting vpõ equall bases are in proportion the one to the other as their altitudes. Wherfore as the base A B C D is to the base E F G H, so is the altitude M N to the altitude M P. But the altitude P M is equall to the altitude K L. Wherefore as the base A B C D is to the base E F G H, so is the altitude M N to the altitude K L. Wherefore in the equall cylinders A X, and E O the bases are reciprokall to their altitudes.

Demonstration touching cylinders.



But now suppose that the bases of the cylinders A X, and E O be reciprokal to their altitudes, that is, as the base A B C D is to the base E F G H, so is the altitude M N to the altitude K L. Then I say that the cylinder A X is equall to the cylinder E O. For the selfe same order of constructiõ remayning, for that as the base A B C D is to the base E F G H, so is the altitude M N to the altitude K L, but the altitude K L is equall to the altitude P M. Wherefore as the base A B C D is to the base E F G H, so is the altitude M N to the altitude P M. But as the base A B C D is to the base E F G H, so is the cylinder A X to the cylinder E S, for they are vnder equall altitudes: and as the altitude M N is to the altitude P M, so is the cylinder E O to the cylinder E S (by the 14. of the twelueth). Wherefore also as the cylinder A X is to the cylinder E S, so is the cylinder E O to the cylinder E S. Wherefore the cylinder A X is equall to the cylinder E O (by the 9. of the fift). And so also is it in the cones which haue the selfe same bases and altitudes with the cylinders. Wherefore in equall cones and cylinders, the bases are reciprokall to their altitudes &c. which was required to be demonstrated.

Second part demonstrated.

A Corrollary added by *Campane* and *Flussas*.

Hitherto hath bene shewed the passions and proprieties of cones and cylinders whose altitudes fall perpendicularly vpon the bases. Now will we declare that cones and cilinders whose altitudes fall obliquely vpon their bases haue also the selfe same passions and proprieties which the foresayd cones and cilinders haue.

Forasmuch as in the tenth of this booke it was sayd, that euery Cone is the third part of a cilinder hauing one, and the selfe same base, & one & the selfe same altitude with it, which thing was demõstrated by a cilinder geuen, whose base is cut by a square inscribed in it, and vpon the sides of the square are described Isosceles triangles, making a poligonon figure, and againe vpon the sides of this poligonon figure are infinitely after the same maner described other Isosceles triangles taking away more thẽ the halfe, as hath oftẽtimes bene declared: therfore it is manifest, that the solides set vpon these bases, being

vnder

vnder the ſame altitude that the cilinder inclined is, and being alſo included in the ſame cilinder, do take away more then the halfe of the cilinder, and alſo more thē the halfe of the reſidue, as it hath bene proued in erected cylinders. For theſe inclined ſolides being vnder equall altitudes and vpon equall baſes with the erected ſolides are equall to the erected ſolides by the corollary of the 40. of the eleuenth. Wherfore they alſo in like ſort as the erected, take away more then the halfe. If therfore we cōpare the inclined cilinder, to a cone ſet vpon the ſelfe ſame baſe, and hauing his altitude erected, and reaſon by an argument leading to an impoſſibilitie by the demonſtration of the tenth of this booke, we may proue that the ſided ſolide included in the inclined cylinder is greater then the triple of his pyramis, and it is alſo equall to the ſame which is impoſſible. And this is the firſt caſe, wherein it was proued that the cilinder not being equall to the triple of the cone is not greater then the triple of the ſame. And as touching the ſecond caſe, we may after the ſame maner conclude that that ſided ſolide contayned in the cylinder is greater then the cylinder: which is very abſurd. Wherefore if the cylinder be neither greater then the triple of the cone, nor leſſe, it muſt nedes be equall to the ſame. The demonſtration of theſe inclined cylinders moſt playnely followeth the demonſtration of the erected cylinders: for it hath already bene proued, that pyramids, and ſided ſolides (which are alſo called generally Priſmes) being ſet vpon equall baſes and vnder one and the ſelfe ſame altitude, whether the altitude be erected or inclined, are equall the one to the other, namely, are in proportion as their baſes are by the 6. of this booke. Wherefore a cylinder inclined ſhall be triple to euery cone (although alſo the cone be erected) ſet vpon one and the ſame baſe with it, and being vnder the ſame altitude. But the cilinder erected was the triple of the ſame cone by the tenth of this booke. Wherefore the cilinder inclined is equall to the cilinder erected being both ſet vpon one and the ſelfe ſame baſe; and hauing one and the ſelfe ſame altitude. The ſame alſo cometh to paſſe in cones, which are the third partes of equall cilinders, & therefore are equall the one to the other.

Wherefore according to the eleuenth of this booke it followeth, that cylinders and cones inclined or erected, being vnder one and the ſelfe ſame altitude, are in proportion the one to the other as their baſes are. For foraſmuch as the erected are in proportion as their baſes are, and to the erected cilinders the inclined are equall: therefore they alſo ſhall be in proportion as their baſes are.

And therefore by the 12. of this booke like cones and cylinders being inclined are in triple proportion of that in which the diameters of the baſes are. For foraſmuch as they are equall to the erected which haue the proportion by the 12. of this booke, and their baſes alſo are equall with the baſes of the erected, therefore they alſo ſhall haue the ſame proportion.

Wherefore it followeth by the 13. of this booke, that a cylinder inclined, being cut by a playne ſuperficies parallel to the oppoſite playne ſuperficieces therof ſhall be cut according to the proportions of the axes. For ſuppoſe that vpon one and the ſelfe ſame baſe be ſet an erected cylinder and an inclined cylinder, being both vnder one and the ſelfe ſame altitude, which let be cut by a playne ſuperficies parallel to the oppoſite baſes. Now it is manifeſt that the ſections of the one cylinder are equall to the ſection of the other cylinder, for they are ſet vpon equall baſes, and vnder one and the ſelfe ſame altitude, namely, betwene the parallel playne ſuperficieces. And their axes alſo are by thoſe parallel playne ſuperficieces cut proportionally by the 17. of the eleuenth. Wherefore the inclined cylinders (being equall to the erected cylinders) ſhall haue the proportion of their axes, as alſo haue the erected. For in ech, the proportion of the axes is one and the ſame.

Wherefore inclined Cones and Cylinders being ſet vpon equall baſes, ſhall by the 14. of this booke be in proportion as their altitudes are. For foraſmuch as the inclined are equall to the erected which haue the ſelfe ſame baſes and altitude, and the erected are in proportion as their altitudes: therfore the inclined ſhall be in proportion the one to the other as the ſelfe ſame altitudes which are common to ech, namely, to the inclined and to the erected.

And therefore in equall cones and cylinders whether they be inclined or erected, the baſes ſhall be reciprokally proportionall with the altitudes, and contrariwiſe by the 15. of this booke. For foraſmuch as we haue oftentimes ſhewed that the inclined cones and cylinders are equall to the erected, hauing the ſelf ſame baſes and altitudes with them, and the erected vnto whome the inclined are equall, haue their baſes reciprokall proportionally with their altitudes, therefore it followeth, that the inclined (being equall to the erected) haue alſo their baſes and altitudes (which are common to eche) reciprokally proportionall. Likewiſe if their altitudes & baſes be reciprokally proportionall, they thēſelues alſo ſhall be equall, for that they are equall to the erected cylinders and cones ſet vpon the ſame baſes and being vnder the ſame altitude, which erected cylinders are equall the one to the other by the ſame 15. of this booke. Wherefore we may conclude, that thoſe paſſions & proprieties which in this twelfth booke haue bene proued to be in cones and cylinders whoſe altitudes are erected perpendicularly to the baſes, are alſo in thoſe cones and cylinders whoſe altitudes are ſet obliquely vpon their baſes. Howbeit this is to be noted that ſuch inclined cones or cylinders are not perfect round as are the erected:

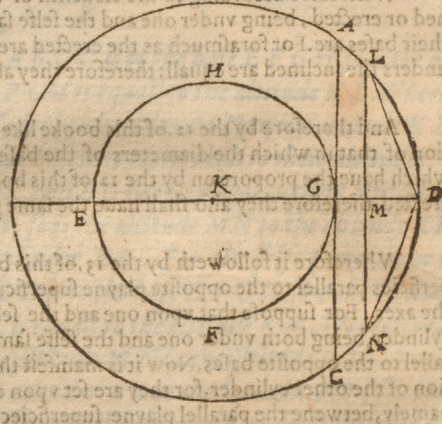
for

so that if they be cut by a playne superficies passing at right angles with their altitude, this section is a Conicall section, which is called *Ellipsis*, and shall not describe in their superficies a circle as it doth in erected cylinders & cones, but a certaine figure, whose lesse diameter is in cylinders equall to the dimetient of the base: that is, is one and the same with it. And the same thing happeneth also in cones inclined, being cut after the same maner.

The 1. Probleme. The 16. Proposition.

Two circles hauing both one and the selfe same centre being geuen, to inscribe in the greater circle a poligonon figure, which shall consist of equall and euen sides, and shall not touch the superficies of the lesse circle.

Construction. *Vppose that there be two circles A B C D, and E F G H hauing one & the selfe same centre, namely, K. It is required in the greater circle which let be A B C D to inscribe a poligonon figure which shalbe of equal and euen sides and not touch the circle E F G H. Drawe by the centre K a right line B D. And (by the 11. of the first) from the point G rayse vp vnto the right line B D a perpendicular line A G, and extend it to the point C. Wherefore the line A C toucheth the circle E F G H (by the 15. of the third). Now therfore if (by the 30. of the third) we diuide the circumference B A D into two equall partes, and againe the halfe of that into two equal partes, and thus do cōtinually, we shall (by the corollary of the 1. of the tenth) at the length leaue a certayne circumference lesse then the circumference A D. Let the circumference left be L D. And from the point L. Drawe (by the 12. of the first) vnto the line B D a perpendicular line* L M, and extende it to the point N. And draw these right lines L D and D N. And forasmuch as the angles D M L, and D M N are right angles, therfore (by the 3. of the third) the right line B D diuideth the right line L N into two equall parts in the pointe M. Wherfore (by the 4. of the first) the rest of the sides of the triangles D M L, and D M N, namely, the lines D L, and D N shalbe equall. And forasmuch as the line A C is a parallell to the L N (by the 28. of the first): But A C toucheth the circle E F G H, wherfore the line L M toucheth not the circle E F G H, and much lesse do the lines L D, and D N touch the circle E F G H. If therefore there be applied right lines equall to the line L D continually into the circle A B C D (by the 1. of the fourth) there shalbe described in the circle A B C D a poligonon figure which shalbe of equall, and* euen sides, and shall not touch the lesse circle, namely, E F G H: (by the 14. of the third or by the 29.) which was required to be done.



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* Note this L M, because of K Z in the next proposition, and here the point M: for the point Z in the next demonstration.

* For that the sections were made by the number two: that is, by taking halues, and of the residue the halfe: and so, to L D, being an halfe, and a residue which shall be a comon measure backe againe, to make sides of the poligonon figure.

¶ Corollary.

Hereby it is manifest that a perpendicular line drawen from the poynt L to the line B D toucheth not one of the circles.

¶ An Assumpt added by *Flussas*.

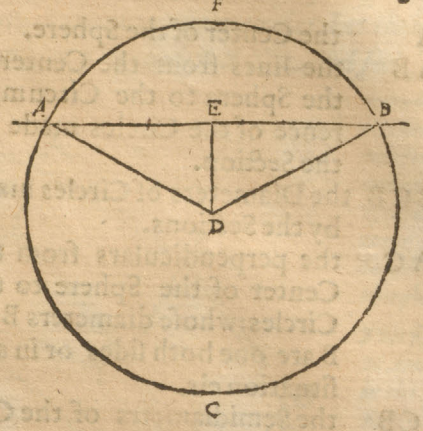
If a Sphere be cut of a playne superficies, the common section of the superficieces, shall be the circumference of a circle.

Suppose

Suppoſe that the ſphere A B C be cut by the playne ſuperficies A E B, and let the centre of the ſphere be the poynt D. And from the poynt D, let there be drawne vnto the playne ſuperficies A E B a perpendicular line (by the 11. of the eleuenth) which let be the line D E. And from the poynt E draw in the playne ſuperficies A E B vnto the common ſection of the ſayd ſuperficies and the ſphere, lines how many ſo euer, namely, E A and E B. And draw theſe lines D A and D B. Now foraſmuch as the right angles D E A and D E B are equall (for the line D E is erected perpendicularly to the playne ſuperficies). And the right lines D A & D B which ſubtend thoſe angles are by the 12. definirion of the eleuenth equall: which right lines moreouer (by the 47. of the firſt) do contayne in power the ſquares of the lines D E, E A, and D E, E B: if therfore from the ſquares of the lines D E, E A, and D E, E B ye take away the ſquare of the line D E which is cõmon vnto them, the reſidue namely the ſquares of the lines E A & E B ſhall be equal. Wherfore alſo the lines E A and E B are equall. And by the ſame reaſon may we proue, that all the right lines drawne from the poynt E to the line which is the cõmon ſection of the ſuperficies of ſphere and of the playne ſuperficies, are equall. Wherefore that line ſhall be the circumference of a circle, by the 15. definition of the firſt. *But if it happen the plaine ſuperficies which cutteth the ſphere, to paſſe by the centre of the ſphere, the right lines drawne from the centre of the ſphere to their common ſection, ſhall be equall by the 12. definition of the eleuenth. For that common ſection is in the ſuperficies of the ſphere. Wherefore of neceſſitie the playne ſuperficies comprehended vnder that line of the common ſection ſhall be a circle, and his centre ſhall be one and the ſame with the centre of the ſphere.

Conſtruction.

Demonſtration.



** The circles ſo made: or ſo conſidered in the ſphere, are called the greateſt circles: All other, not hauing the center of the ſphere, to be their center alſo, are called leſſe circles. Note theſe deſcriptions.*

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Euclide hath among the definition of ſolides omitted certayne, which were eaſy to conceaue by a kinde of Analogie. As a ſegment of a ſphere, a ſector of a ſphere, the vertex, or toppe of the ſegment of a ſphere: with ſuch like. But that (if nede be) ſome farther light may be geuẽ, in this figure next before, vnderſtand a ſegment of the ſphere A B C to be that part of the ſphere contayned betwen the circle A B, (whoſe center is E) and the ſpherical ſuperficies A F B. To which (being a leſſe ſegment) adde the cone A D B (whoſe baſe is the former circle: and toppe the center of the ſphere) and you haue D A F B a ſector of a ſphere, or ſolide ſector (as I call it). D E extended to F, ſheweth the top or vertex of the ſegment, to be the poynt F: and E F is the altitude of the ſegment ſpherical. Of ſegmentes, ſome are greater thẽ the halfe ſphere, ſome are leſſe. As before A B F is leſſe, the remanent, A B C is a ſegment greater then the halfe ſphere.

¶A Corollary added by the ſame *Fluſſas.*

By the foreſayd aſſumpt it is manifeſt, that if from the centre of a ſphere the lines drawne perpendicularly vnto the circles which cutte the ſphere, be equall: thoſe circles are equall. And the perpendicular lines ſo drawne fall vpon the centres of the ſame circles.*

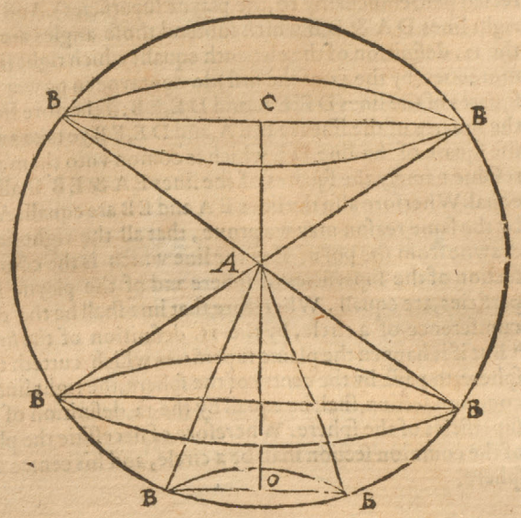
For the line which is drawne frõ the centre of the ſphere to the circumference, containeth in power, the power of the perpendicular line, and the power of the line which ioyneth together the endes of thoſe lines. Wherfore frõ that ſquare or power of the line from the center of the ſphere to the circumference or cõmon ſectiõ drawne, which is the ſemidiameter of the ſphere, taking away the power of the perpendicular, which is cõmon to them round about, it followeth, that the reſidues how many ſo euer they be, be equall powers, and therefore the lines are equall the one to the other. Wherefore they will deſcribe equall circles, by the firſt definition of the third. And vpon their centers fall the perpendicular lines by the 9. of the third.

** An other Corollary.* *And thoſe circles vpon which falleth the greater perpendicular lines are the leſſe circles. For the powers of the lines drawne from the centre of the ſphere to the circumference being alwayes one and equall, to the powers of the perpendicular lines and alſo to the powers of the lines drawne from the centres of the circles to their circumference, the greater that the powers of the perpendicular lines taken away from the power contayning them both are, the leſſe are the powers and therefore the lines remayning, which are the ſemidiameters of the circles and therefore the leſſe are the circles which they deſcribe. Wherefore if the circles be equall, the perpendicular lines falling from the centre of the ſphere vpon thẽ, ſhall alſo be equall. For if they ſhould be greater or leſſe, the circles ſhould be vnequall as it is before manifeſt. But we ſuppoſe the perpendiculars to be equall.

* An other Corollary. * Alſo the perpendicular lines falling vpon thoſe baſes are the leaſt of all, that are drawne from the centre of the ſphere: for the other drawne

from the centre of the ſphere to the circumference of the circles, are in power equall both to the powers of the perpendiculars and to the powers of the lines ioyning theſe perpendiculars and theſe ſubtendent lines together: making triangles rectangle round about: as moſt eaſily you may conceaue of the figure here annexed.

A the Center of the Sphere.
AB the lines from the Center of the Sphere to the Circumference of the Circles made by the Section.
BCB the Diameters of Circles made by the Sections.
AC the perpendiculars from the Center of the Sphere to the Circles: whoſe diameters B C-B are one both ſides or in any ſituation els.
CB the Semidiametrs of the Circles made by the Sections.
AO a perpendicular longe then AC: and therefore the Semidiameter O B is leſſe.
ACB, & AOB triangles rectangle.



¶ The 2. Probleme. The 17. Propoſition.

Two ſpheres conſiſting both about one & the ſelfe ſame cẽtre, being geuẽ, to inſcribe in the greater ſphere a ſolide of many ſides (which is called a Polyhedron) which ſhall not touch the ſuperficies of the leſſe ſphere.

Conſtruction.

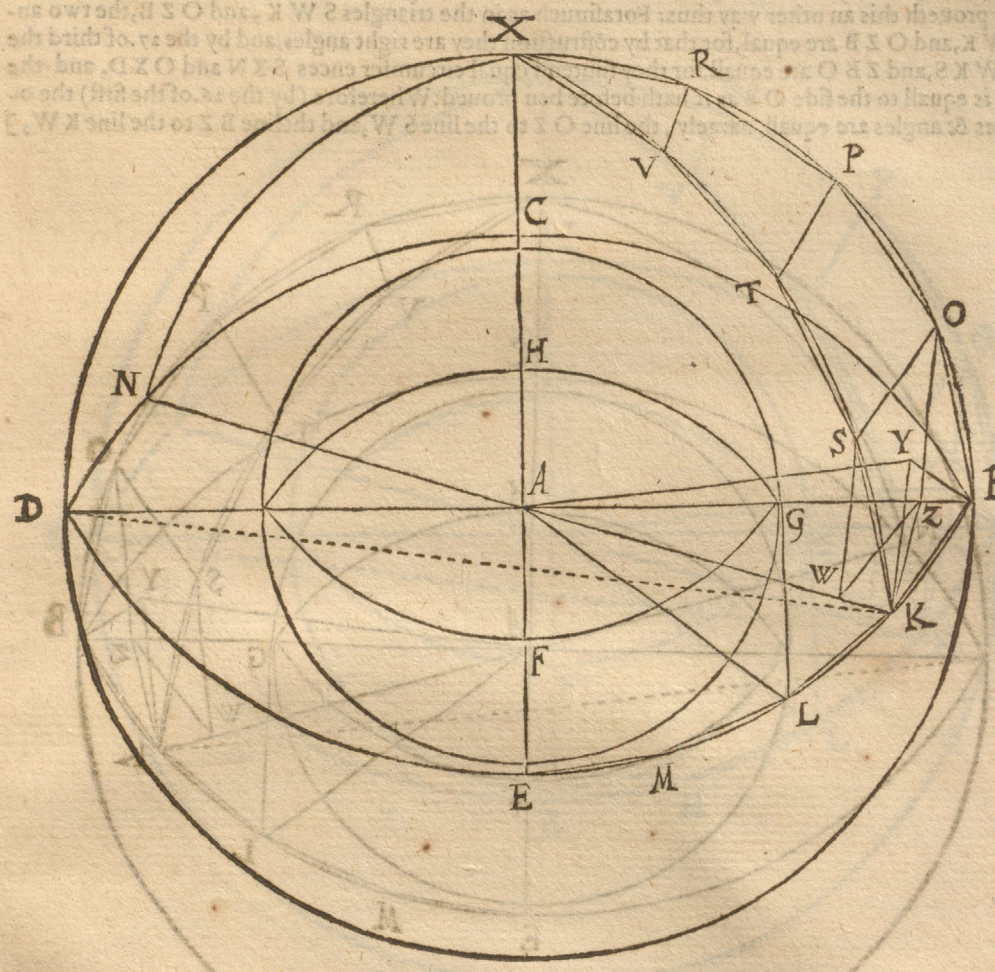
Vppoſe that there be two ſpheres about one & the ſelfe ſame cẽtre, namely, about A. It is required in the greater ſphere to inſcribe a Polyhedron, or a ſolide of many ſides, which ſhal not with his ſuperficies touch the ſuperficies of the leſſe ſphere. Let the ſpheres be cut by ſome one plaine ſuperficies paſsing by the center A. Then ſhall their ſectiõs be circles. [For (by the 12. definition of the eleuenth) the Diameter remaining fixed, and the ſemicircle being turned round about maketh a ſphere. Wherefore in what poſitiõ ſo euer you imagine the ſemicircle to be, the playne ſuperficies which paſſeth by it ſhal make in the ſuperficies of ỹ ſphere a circle. And it is manifeſt that is alſo a greater circle, for the diameter of the ſphere which is alſo the diameter of the ſemicircle, and therefore alſo of the circle is (by the 15. of the third) greater then all the right lines drawne in the circle or ſphere] which circles ſhall haue both one center being both alſo in that one playne ſuperficies, by which the ſpheres were cut. Suppoſe that that ſection or circle in the greater ſphere be B C-D E, and in the leſſe, be the circle F G H. Drawe the diameters of thoſe two circles in ſuch ſorte that they make right angles, and let thoſe diameters be B D, and C E. And let the line A G, being part of the line A B, be the ſemidiameter of the leſſe ſphere and circle, as A B is the ſemidiameter of the greater ſphere and greater circle: both the ſpheres and circles hauing one and the ſame center. Now two circles that is, B C D E, and F G H conſiſting both about one, and the ſelfe ſame centre being geuen, let there be deſcribed (by the propoſition nexte going before) in the greater circle B C D E a poligonon figure conſiſting of equall and euen ſides, not touching the leſſe circle F G H. And let the ſides of that figure in the fourth part of the circle, namely, in B E, be B K, K L, L M, and M E. And draw a right line from the point*

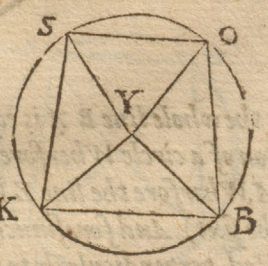
* This is alſo proued in the Aſsumpt before added out of Fluſſas.

Note what a greater or greateſt circle in a Spere is.

First part of the Conſtruction.

point K to the point A, and extende it to the point N. And (by the 12. of the eleuenth) from the A rayse vp to the superficies of the circle B C D E a perpẽdicular line A X, and let it light vppon the superficies of the greater sphere in the point X. And by the line A X, and by either of these lines B D, and K N extend playne superficieces. Now by that which was before spokẽ, those plaine superficieces shal in the * superficies of ỹ sphere make two greater circles. Let their semicircles consisting vpon the diameters B D, and K N be B X D. and K X N. And forasmuch as the line X A is erected perpendicularly to the playne superficies of the circle B C D E. Therfore al the plaine superficieces which are drawne by the line X A are erected perpendicularly to the superficies of the circle B C D E (by the 18. of the eleuenth). Wherefore the semicircles B X D, and K X N are erected perpendicularly to the playne superficies of the circle B C D E. And forasmuch as the semicircles B E D, B X D, and K X N are equall, for they consist vpon equall diameters B D, and K N: therefore also the fourth parts or quarters of those circles, namely, B E, B X and K X are equal the one to the other. Wherefore how many sides of a poligonon figure there are in the fourth parte or quarter B E, so many also are there in the other fourth partes or quarters B X, and K X, equall to the right lines B K, K L, L M, and M E. Let those sides be described, and let them be B O, O P, P R, R X, K S, S T, T V, and V X: and drawe these right lines S O, T P, and V R. And from the pointes O and S. Drawe to the playne superficies of the circle B C D E perpendicular lines, which perpendicular lines will fall vpon the common sections of the plaine superficieces, namely, vpon the lines B D, & K N, (by the 38. of the eleuenth)





Note.
* You know full well, that in the superficies of the sphere, onely the circumferences of the circles ure: but by those circumferences the limitatiõ and aſsigning of circles is vsed: und so, the circumference of a circle, vsually called a circle, which in this place can not offend.

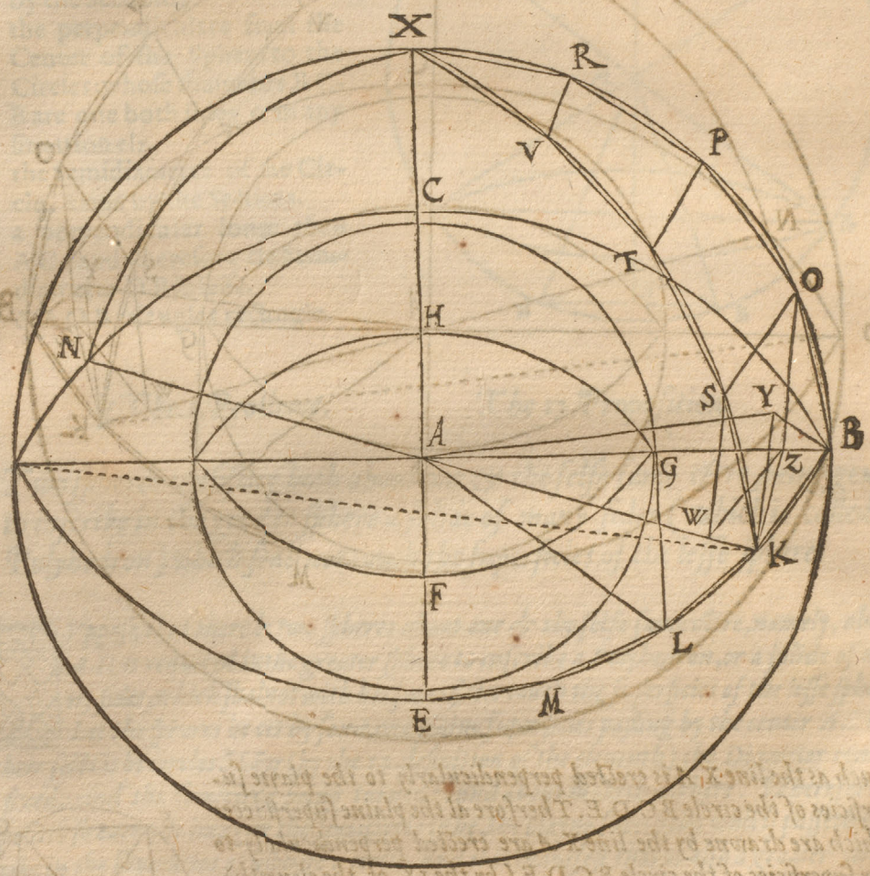
This figure is restored by M. Dee his diligence. For in the greeke and Latine Euclides, the line G L, the line A G, and the line K Z, (in which three lynes the chiefe pinch of both the demonstrations doth stand), are vntruely drawen: as by comparing, the studious may perceaue.

Note.
You must imagine the right line A X, to be perpẽdicular vpon the diameters B D and C E: though here A C the semidiater, seme to be part of A X. And so in other pointes, in this figure, and many other, strengthen your imagination, according to the tenor of constructions: though in the delineatiõ in plaine, sense be not satisfied.

Note.
B O equall to B K, in respect of M. Dee his demonstration following.

eleuenth) for that the playne ſuperficieces of the ſemicircles B X D, K X N are erected perpendicularly to the playne ſuperficies of the circle B C D E. Let thoſe perpendicular lines be G Z, † and S W. And drawe a right line from the point Z to the point W. And foraſmuch as in the equall ſemicircles B X D and K X N the right lines B O, and K S are equall, from the ends wherof are drawne perpẽdicular lines O Z, and S W, therfore (by the corollary of the 35. of the eleuẽth) the line O Z is equall to the line S W, & the line B Z is equal to the line K W. [Fluſſas proueth this an other way thus: Foraſmuch as in the triangles S W K, and O Z B, the two angles S W K, and O Z B are equal, for that by cõſtruction they are right angles, and by the 27. of third the angles W K S, and Z B O are equall for they ſubtend equal circumferences S X N and O X D, and the ſide S K is equall to the ſide O B as t hath before ben proued. Wherefore (by the 26. of the firſt) the other ſides & angles are equall, namely, the line O Z to the line S W, and the line B Z to the line K W.]

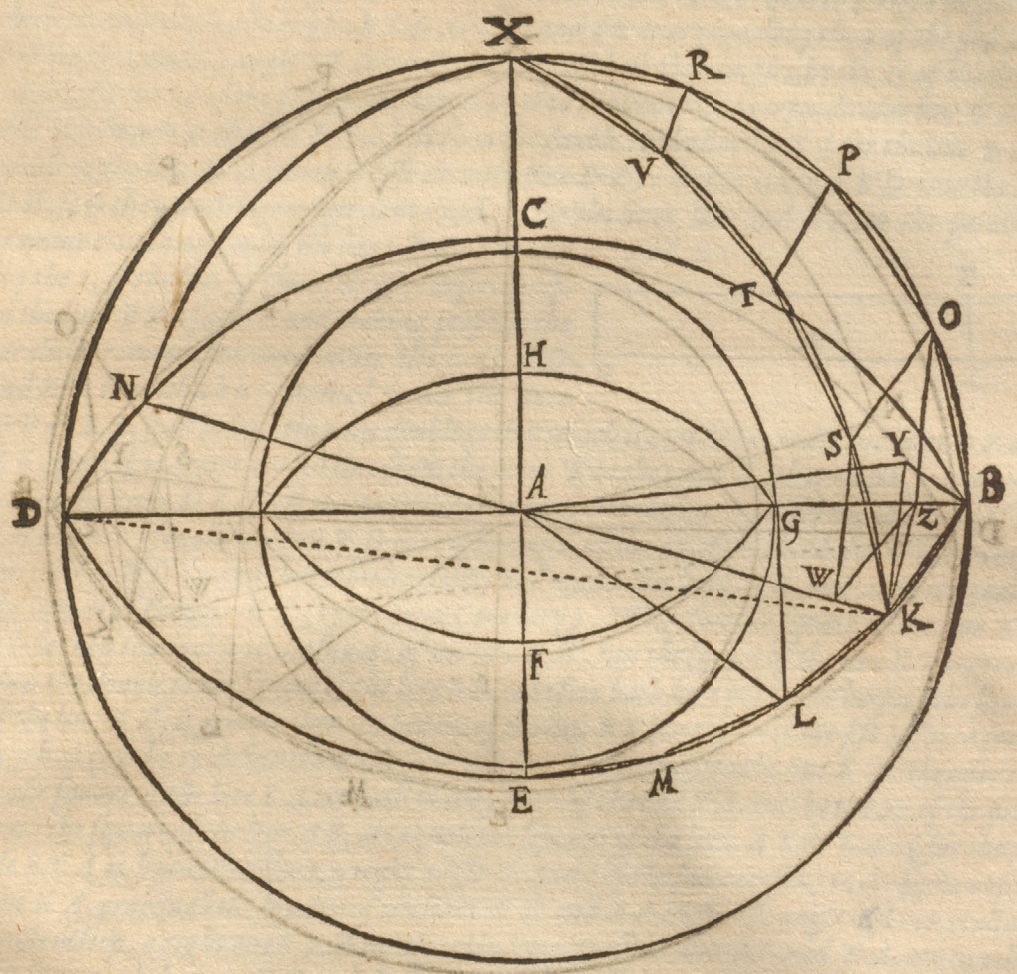
† Note this point Z that you may the better vnderſtand M. Dee his demõſtration.



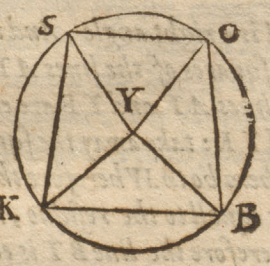
But the whole line B A is equall to the whole line K A: by the definition of a circle. Wherfore the reſidue Z A is equall to the reſidue W A. Wherfore the line Z W is a parallel to the line B K (by the 2. of the ſixt). And foraſmuch as ether of theſe lines O Z, & S W is erected perpendicularly to the playne ſuperficies of the circle B C D E, therefore the line O Z is a parallell to the line S W, (by the 6. of the eleuẽth) and it is proued that it is alſo equal vnto it. Wherfore the lines W Z and S O are alſo equall and parallels (by the 7. of the eleuenth, and the 33. of the firſt, and by the 3. of the firſt) And foraſmuch as W Z is a parallell to S O: But Z W is a parallell is to K B: Wherefore S O is alſo a parallell to K B, (by the 9. of the eleuenth). And the lines B O, and K S do knit them together. Wherefore the fower ſided figure B O K S is in one and the ſelfe ſame playne ſuperficies.

For

For (by the 7. of the eleuenth) if there be any two parallell right lines, and if in either of them be taken a point at allauentures, a right line drawn by these points is in one, & the selfe same playne superficies with the parallels. And by the same reason also euery one of the fower sided figures S O P T, and T P R V is in one and the selfe same playne superficies. And the triangle V R X is also in one and the selfe same plaine superficies (by the 2. of the eleuenth). Now if we imagine right lines drawne frō the pointes O, S, P, T, R, V, to the point A, there shalbe descri-



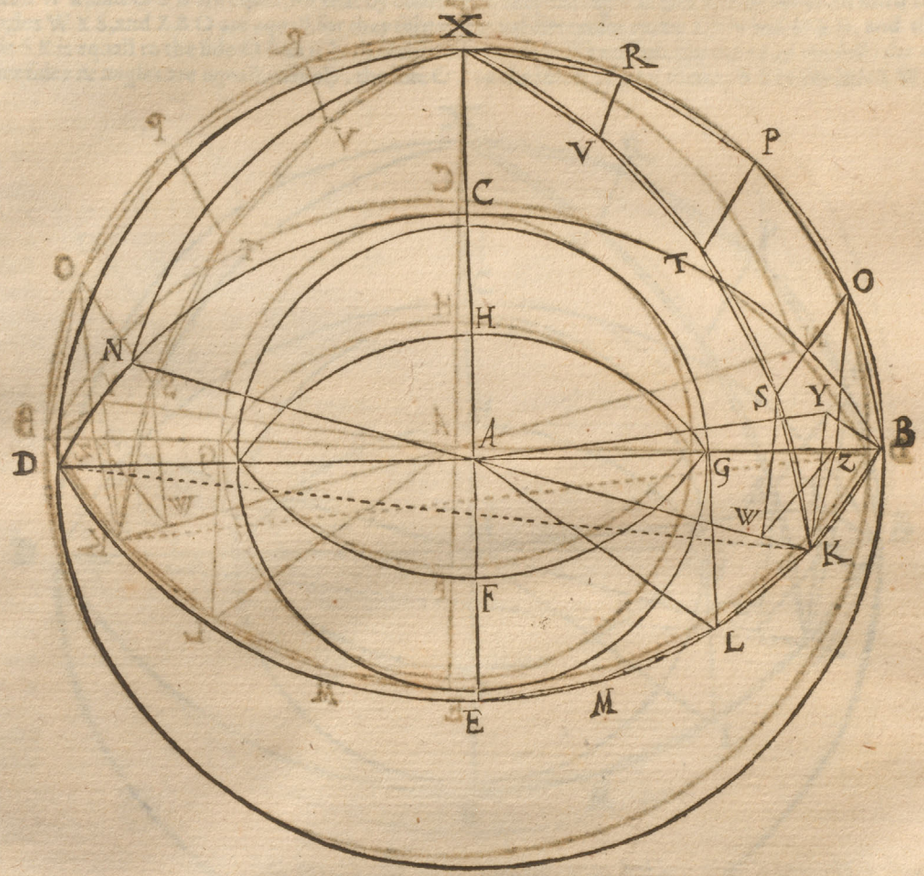
bed a Polyhedrō or a solide figure of many sides, betwene the circūferences B X and K X composed of pyramids, whose bases are the fower sided figures B K O S, S O P T, T P R V, and the triangle V R X, and toppe the point A. And if in euery one of the sides K. L, L M and M E we vse the selfe same construction that we did in B K, and moreouer in the other three quadrants or quarters, and also in the other halfe of the sphere, there shall then be made a Polyhedron or solide figure consisting of many sides described in the sphere, which Polyhedron is made of the pyramids whose bases are the foresayd fower sided figures, and the triangle V R X, and others which are in the selfe same order with them: and common toppe to them all in the point A.



Now I say that the forsayd polihedron solide of many sides toucheth not the superficies of the lesse sphere in which is the circle F G H. Draw (by the 11. of the eleuenth) frō the poynt A to the playne superficies of the fower sided figure K B O S a perpendicular line A Y, and let it fall vpon the playne superficies in the point Y. And drawe these right lines B Y & Y K. And forasmuch as the line A Y is erected perpendicularly to the playne superficies of B K O S, Second part of the construction.

Second part of the demonstration.

therefore the same A Y is erected perpendicularly to all the right lines that touch it, and are in the plaine superficies of the fower sided figure (by the 2. definition of the eleuenth). Wherefore the line A Y is erected perpendicularly to either of these lines B Y, and Y K. And forasmuch as (by the 15. definition of the first) the line A B is equall to the line A K, therfore the square of the line A B is equall to the square of the line A K. And to the square of the line A B are equall the squares of the lines A Y, and Y B (by the 47. of the first) for the angle B-

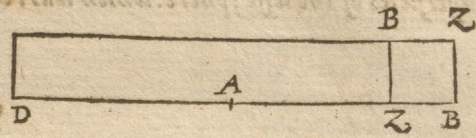


Y A is a right angle. And to the square of the line A K are equal the squares of the lines A Y and Y K. Wherefore the squares of the lines A Y, and Y B are equal to the squares of the lines A Y, and Y K: take away the square of the line A Y which is common to them both. Wherefore the residue, namely, the square of the B-Y is equall to the residue, namely, to the square of the line Y K. Wherefore the line B Y is equall to the line Y K. In like sort also may we proue that right lines drawne from the point Y to the poyntes O, and S, are equall to either of the lines B Y, and Y K.

Wherefore making the center the poynt Y, and the space either the line B Y or the line Y K describe a circle, and it shal passe by the poyntes O, and S, and the fower sided figure K-B O S shalbe inscribed in the circle. And forasmuch as the line K B is greater then the line W Z (by the 2. of the sixt, because A K is greater then A W), but the line W Z is equall to the line S O. Wherfore the line B K is greater thē the line S O. But the line B K is equal to either

of

of the lines K S, and B O by construction. Wherefore either of the lines K S, and B O is greater then the line S O. And forasmuch as in the circle is a fower sided figure K B O S, and the sides B K, B O and K S are equal, and the side O S is lesse then any one of them, and the line B Y is drawne from the centre of the circle: therefore the square of the line K B is greater then the double of the square of the line B Y (by the 12. of the second) (for that it subtendeth an angle greater then a right angle contayned of the two equall lines B Y, and Y K, which angle B Y K is an obtuse angle. For the 4. angles at the cēter Y are equal to 4. right angles: of which three, namely, the angles B Y K, K Y S, and B Y O are equall by the 4. of the first, and the fourth namely, ỹ angle S Y O is lesse then any of those three angles, by the 25. of the first.) Drawe (by the 12. of the first) from the point K to the line B Z, a perpendicular line ✝ K Z. And forasmuch as the line B D is lesse then the double to the line D Z (for the line B D is double to the line D A, which is lesse then the line D Z): but as the line B D is to the line D Z, so is the parallelogramme contained vnder the lines D B and B Z, to the parallelogramme contained vnder the lines D Z and Z B (by the 1. of the sixt): therefore if ye describe vpon the line B Z a square, and making perfecte the parallelogramme contained vnder the lines Z D and Z B, that which is contained vnder the lines D B & B Z, shall be lesse then the double to that which is contained vnder the lines D Z and Z B. And if ye drawe a right line from the point K to the point D, that which is contained vnder the lines D B and B Z, is equall to the square of the line B K (by the Corollary of the 8. of the sixt) for the angle B K D is a right angle, by the 31. of the third, for it is in the semicircle B E D): and that which is contained vnder the lines D Z and Z B, is equall to the square of the line K Z (by the same Corollary). Wherefore the square of the line K B, is lesse then the double to the square of the line K Z. But the square of the line K B is greater then the double to the square of the line B Y, as before hath bene proued. Wherfore the square of the line K Z, is greater then the square of the line B Y (by the 10. of the fift). And forasmuch as (by the 15. definition of the first) the line B A is equall to the line K A: therefore also the square of the line B A is equall to the square of the line K A. But (by the 47. of the first) vnto the square of the line A B, are equall the squares of the lines B Y & Y A: (for the angle B Y A is, by construction, a right angle). And (by the same reason) to the square of the line K A, are equall the squares of the lines K Z and Z A: (for the angle K Z A is also by construction, a right angle). Wherefore the squares of the lines B Y and Y A, are equall to the squares of the lines K Z and Z A: Of which the square of the line K Z is greater then the square of the line B Y, as hath before bene proued. Wherefore the residue, namely, the square of the line Z A, is lesse then the square of the line Y A. Wherfore the line Y A is greater then the line A Z. * Wherefore the line A Y is much greater then the line A G. But the line A Y falleth vpon one of the bases of the Polihedron, and the line A G falleth vpon the superficies of the lesse sphere. Wherefore the Polihedron toucheth not the superficies of the lesse sphere.



✝ Which of necessity shall fall vpon Z, as M. Dee proueth it: and his prose is set after at this marke ✝ following.

I. Dee. * But A Z is greater thē A G, as in the former propositiō, K M was euident to be greater then K G: so may it also be made manifest that K Z doth neyther touch nor cut the circle F G H.

An other proue that the line A Y is greater thē the line A G.

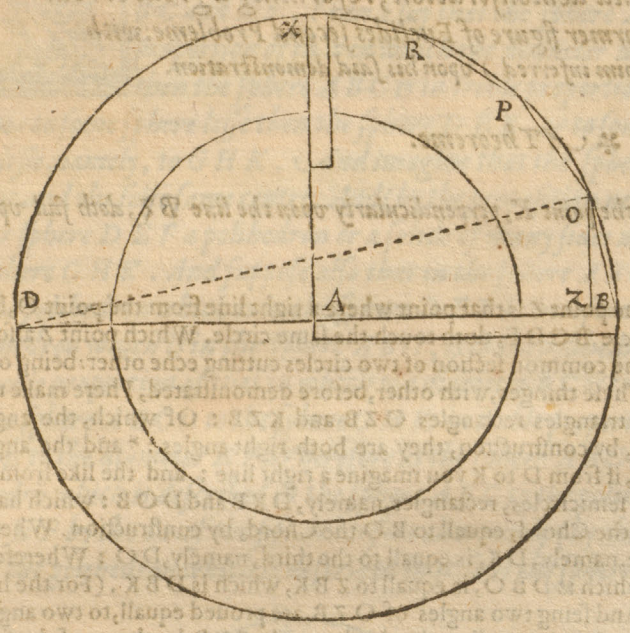
An other and more ready demonstration to proue that the line A Y is greater then the line A G. Raise vp (by the 11. of the first) from the poynt G to the line A G a perpendicular line G L. And draw a right line frō the point A to the poynt L. Now thē deuiding (by the 30. of the third) the circumference E B into halues, & agayne that halfe into halues, & thus doing continually, we shall at the length by the corollary of the first of the tenth, leaue a certayne circumference which shal be lesse then the circumference of the circle B C D which is subtended of a line equall to the line G L. Let the circumference left be K B. Wherfore also the right line K B is lesse then the right line G L. And forasmuch as the fower sided figure B K O S is

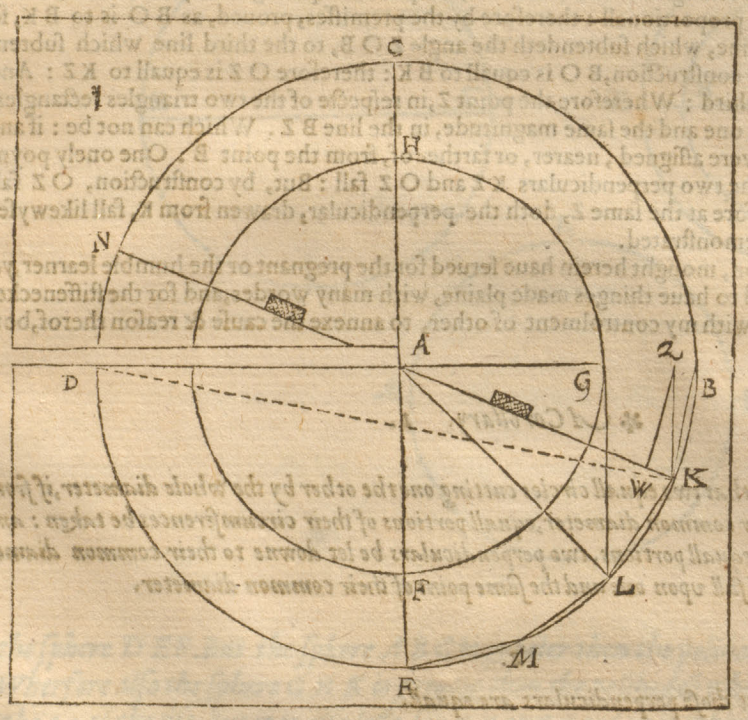
in a circle, and the lines O B, B K, and K S are equall, and the line O S is leſſe, therefore the angle B Y K is an obtuſe angle. Wherefore the line B K is greater then the line B Y. But the line G L is greater then the line K B. Wherefore the line G L is much greater then the line B Y. Wherefore alſo the ſquare of the line G L is greater then ſquare of the line B Y. And foraſmuch as (by the 15. definition of the firſt) the line A L is equall to the line A B, therefore the ſquare of the line A L is equall to the ſquare of the line A B. But vnto the ſquare of the line A L are equall the ſquares of the lines A G and G L, and to the ſquare of the line A B are equall the ſquares of the lines B Y and Y A. Wherefore the ſquares of the lines A G and G L are equall to the ſquares of the lines B Y and Y A, of which the ſquare of the line B Y is leſſe then the ſquare of the line G L. Wherefore the reſidue, namely, the ſquare of the line Y A is greater thē the ſquare of the line A G. Wherfore alſo the line A Y is greater thē the line A G. Wherfore two ſpheres conſisting both about one and the ſelfe ſame center, being geuen, there is inſcribed in the greater ſphere a polihedron or ſolide of many ſides which toucheth not the ſuperficies of the leſſe ſphere: which was required to be done.

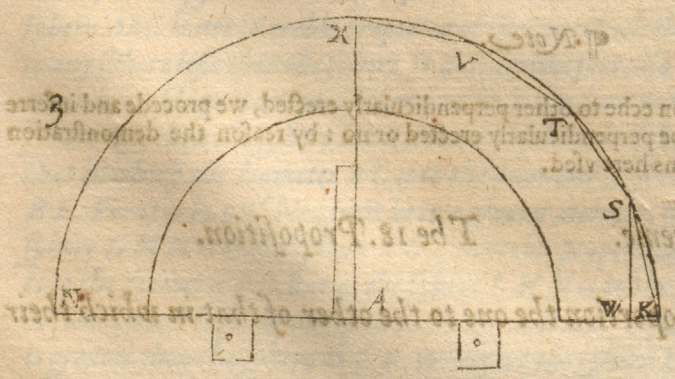
¶ Corollary.

And if in the other ſphere, namely, in the leſſe ſphere be inſcribed a Polihedron or ſolide of many ſides like to the polihedron inſcribed in the ſphere B C D E, then the polihedron inſcribed in the ſphere B C D E is to the polihedron inſcribed in the other ſphere in treble proportion of that in which the diameter of the ſphere B C D E is to the diameter of the other ſphere. For thoſe ſolides being deuided into pyramids equall in number and equall in order, the pyramids ſhall be like. But like pyramids are (by the 8. of the twelfth) the one to the other in treble proportion of that in which ſide of like proportion is to ſide of like proportion. Wherfore the pyramis whoſe baſe is the fower ſided figure K B O S and toppe the poynt A is to that pyramis which is of like order in the other ſphere, in treble proportion of that in which ſide of like proportiō is to ſide of like proportiō, that is of that in which the ſide A B which is drawne frō the cēter of the ſphere which is about the cēter A, is to the ſide which is drawn frō the cēter of the other ſphere. And in like ſort alſo euery one of ỹ pyramids which is in ỹ ſphere which is about the cētre A is to euery one of the pyramids of the ſelfe ſame order in the other ſphere in treble proportiō of that in which the ſide A B is to the ſide which is drawne from the center of the other ſphere. But as one of the antecedentes is to one of the conſequentes, ſo are all the antecedents to all the conſequentes by the 12. of the fifth. Wherefore the whole polihedron ſolide of many ſides which is in the ſphere which is about the center A, is to the whole polihedron or ſolide of many ſides which is in the other ſphere, in treble proportion of that in which the ſide A B is to the ſide which is drawne from the center of the other ſphere, that is, of that which the diameter B D is to the diameter of the other ſphere, by the 15. of the fifth: which was required to be demonſtrated.

¶ Maſter







M.Dee his deuiſe, to helpe the imagination to young ſtudẽts in Geometry· and to make his demonſtration more euident as concerning the errors by hym corrected in Euclides figure, by the ignorant, miſlined.

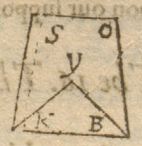
I. Dee.

This figure is anſwerable to the firſte plaine: which, cutting the two Spheres by their common center A, made two concentricall circles (hauing the ſame center with the two Spheres) namely B C D E, and F G H. Vppon which, if you aptly reare perpendicularly, the ſecond figure contayning two concentricall circles, (to the firſt equall) and make the pointes noted with like letters to agree, and afterward vppon the ſecõd figure, ſet on the third figure being here for the better handling made a ſemicircle: which vppon the firſt figure muſt alſo be erected perpendicularly: And laſtly if you take the little quadrangled figure B O K S, and make euery point to touch, his like: & then reade the conſtruction & wey the demonſtratiõ (twiſe or thriſe being red ouer) ſhall you in this delineatiõ in apt paſtborde, or like matter framed, finde al things in this probleme very euident.

I neede not warne you, that the line A Y may eaſely be imagined, or with a fine thred ſupplyed: or of the right lines imaginable betwene P and T, and betwene R and V, I neede ſay nothing, truſting that the great exerciſe paſt, by that tyme you are orderly come to this place, will haue made you ſufficient perfect to ſupply any farther thinge herein to be conſidered.

The little fowercornerd peeces remaying to the ſemicircle, are to be let through the firſt ground playne: therby to ſtay this ſemicircle the better in his apt place and ſituation: which it will the more aptly doo, if ye do abate ſlauntingly, the contrary araſſes of the ſlitt of it, and of the ſlitt of the ſecond figure, into which it is to be let: abating thẽ alike much: a litle will ſerue. Experience, by aduiſe, will teach ſufficiently.

4



¶ Maſter Dee his aduiſe and demonſtration, reforming a great errour in the deſignation of the former figure of Euclides ſecond Probleme: with two Corollaries (by him inferred) vpon his ſaid demonſtration.

✣ A Theoreme.

✣ *That a right line drawen from the point K, perpendicularly vpon the line B Z, doth fall vpon the point Z: We will, thus, make euident.*

BY the premiſſes, it is manifeſt, that the point Z is that point where a right line from the point O, being perpendicularly let fall to the circle B C D E, doth touch the ſame circle. Which point Z alſo is proued to be in the right line B A D, the common ſection of two circles cutting eche other: being one to the other perpendicularly erected. Theſe thinges, with other, before demonſtrated, I here make my ſuppoſitions. Conſider now the two triangles rectangles O Z B and K Z B: Of which, the angle O Z B is equall to the angle K Z B. For, by conſtruction, they are both right angles: * and the angle Z B O is equall to the angle Z B K. For, if from D to K you imagine a right line: and the like from D to O: you haue two triangles in equall ſemicircles, rectangles, namely, D K B and D O B: which haue the diameter B D common: and B K, the Chord, equall to B O the Chord, by conſtruction. Wherefore (by the 47. of the firſt) the third ſide, namely, D K, is equall to the third, namely, D O: Wherefore (by the 5. of the ſixt) the angle Z B O, which is D B O, is equall to Z B K, which is D B K. (For the line Z B, by conſtruction, is part of D B). And ſeing two angles of O Z B, are proued equall, to two angles of K Z B, of neceſſitie the third, namely, Z O B, is equall to the third, namely, Z K B, by the 32. of the firſt. Wherefore the two triangles rectangles O Z B and K Z B, are proued equiangled. By the fourth, therefore, of the ſixt, their ſides are proportionall: therefore by the premiſſes, proued, as B O is to B K, ſo is O Z to K Z, and the third line, which ſubtendeth the angle Z O B, to the third line which ſubtendeth the angle Z K B. But, by conſtruction, B O is equall to B K: therefore O Z is equall to K Z: And the third alſo is equall to the third: Wherefore the point Z, in reſpecte of the two triangles rectangles, O Z B and K Z B, determineth one and the ſame magnitude, in the line B Z. Which can not be: if any other point, in the line B Z, were aſſigned, nearer, or farther of, from the point B. One onely poynt therefore, is that, at which the two perpendiculars K Z and O Z fall: But, by conſtruction, O Z falleth at Z the point, and therefore at the ſame Z, doth the perpendicular, drawen from K, fall likewyſe: Which was required to be demonſtrated.

* *This, as an aſſumpt, is preſently proued.*

Although a briefe monition, mought herein haue ſerued for the pregnant or the humble learner, yet for them that are well pleaſed to haue thinges made plaine, with many wordes, and for the ſtiffenecked buſie body, it was neceſſary, with my controlment of other, to annexe the cauſe & reaſon therof, both inuincible and alſo euident.

✣ A Corollary. 1.

Hereby it is manifeſt, that two equall circles cutting one the other by the whole diameter, if from one and the ſame end of their common diameter, equall portions of their circumferences be taken: and from the pointes ending thoſe equall portions, two perpendiculars be let downe to their common diameter, thoſe perpendiculars ſhall fall vpon one and the ſame point of their common diameter.

2.

Secondly it followeth that thoſe perpendiculars are equall.

¶ Note.

From circles in our firſt ſuppoſition eche to other perpendicularly erected, we procede and inferre now theſe Corollaries, whether they be perpendicularly erected or no: by reaſon the demonſtration hath a like force, vpon our ſuppoſitions here vſed.

¶ The 16. Theoreme. The 18. Propoſition.

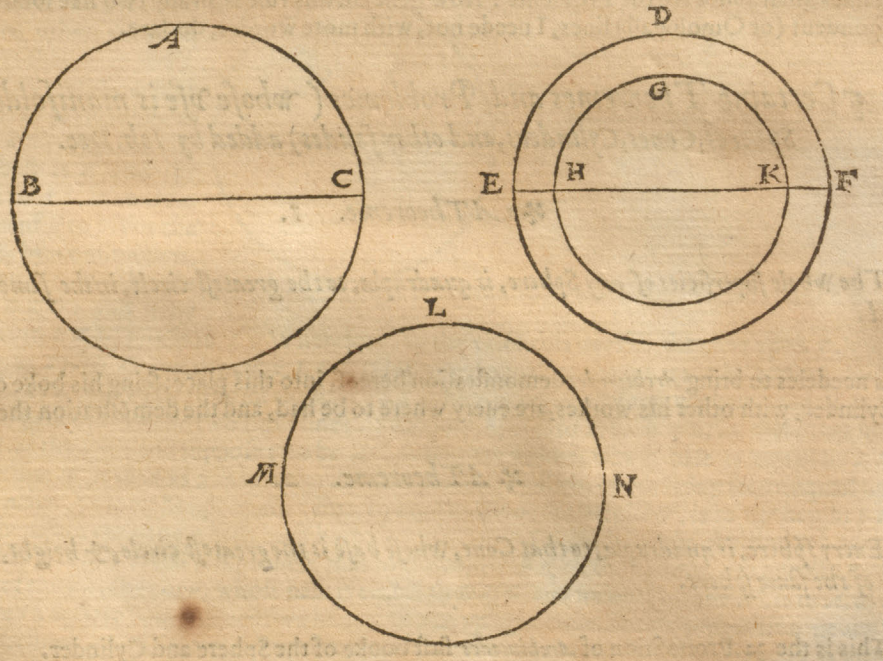
Spheres are in treble proportion the one to the other of that in which their diameters are.

Suppoſe

SVppoſe that there be two ſpheres A B C and D E F, and let their diameters be B C and E F. Then I ſay that the ſphere A B C is to the ſphere D E F in treble proportion of that in which the diameter B C is to the diameter E F. For if not, then the ſphere A B C is in treble proportion of that in which B C is to E F, either to ſome ſphere leſſe then the ſphere D E F, or to ſome ſphere greater. Firſt let it be vnto a leſſe, namely, to G H K. And imagine that the ſpheres D E F and G H K be both about one and the ſelfe ſame centre. And (by the propoſition next going before) deſcribe in the greater ſphere D E F a polihedron or a ſolide of many ſides not touching the ſuperficies of the leſſe ſphere G H K. And ſuppoſe alſo that in the ſphere A B C be inſcribed a polihedron like to the polihedron which is in the ſphere D E F. Wherefore (by the corollary of the ſame) the polihedron which is in the ſphere A B C, is to the polihedron which is in the ſphere D E F in treble proportion of that in which the diameter B C is to the diameter E F. But by ſuppoſition the ſphere A B C is to the ſphere G H K in treble proportion of that in which the diameter B C is to the diameter E F. Wherefore as the ſphere A B C is to the ſphere G H K, ſo is the polihedrō which is deſcribed in the ſphere A B C to the polihedrō which is deſcribed in the ſphere D E F by the 11. of the fift. Wherfore alternately (by the 16. of the fift) as the ſphere A B C is to the polihedron which is deſcribed in it, ſo is the ſphere G H K to the polihedron which is in

Two caſes in this propoſition.

The firſt caſe. Demonſtration leading to an impoſſibilitie.



the ſphere D E F. But the ſphere A B C is greater then the polihedrō which is deſcribed in it. Wherfore alſo the ſphere G H K is greater then the polihedrō which is in the ſphere D E F by the 14. of the fift. But it is alſo leſſe, for it is contayned in it, which impoſſible. Wherefore the ſphere A B C is not in treble proportiō of that in which the diameter B C, is to ỹ diameter E F, to any ſphere leſſe then the ſphere D E F. In like ſort alſo may we proue that the ſphere D E F is not in treble proportion of that in which the diameter E F is to the diameter B C, to any ſphere leſſe then the ſphere A B C. Now I ſay that the ſphere A B C is not in treble proportiō of that in which the diameter B C is to the diameter E F to any ſphere greater thē the ſphere D E F. For if it be poſſible, let it be to a greater, namely, to L M N. Wherfore by conuerſion the ſphere L M N is to the ſphere A B C in treble proportion of that in which the diameter E F is to the diameter B C. But as the ſphere L M N is to the ſphere A B C, ſo is the ſphere D E F to ſome ſphere leſſe thē the ſphere A B C, * as it hath before bene proued, for the ſphere L M N is greater then the ſphere D E F. Wherfore the ſphere D E F is in treble proportiō of that in which

Second caſe.

* *As it is eaſie to gather by the aſſumpt put after the ſecond of this booke.*

which the diameter EF is to the diameter BC to some sphere lesse thē the sphere ABC, which is proued to be impossible. Wherefore the sphere A B C is not in treble proportion of that in which BE is to E F to any sphere greater thē the sphere D E F. And it is also proued that it is not to any lesse. Wherefore the sphere A B C is to the sphere D E F in treble proportion of that in which the diameter B C is to the diameter E F: which was required to be demonstrated.

A Corrollary added by *Flussas.*

Hereby it is manifest, that spheres are the one to the other, as like Polihedrons and in like sort described in them are: namely, eche are in triple proportion of that in which the diameters.

✲ *A Corollary added by M. Dee.*

It is then euident, how to geue two right lines, hauing that proportion betwene them, which, any two spheres geuen, haue the one to the other.

Note: a generall rule.

For, if to their diameters, as to the first and second lines (of fower in continuall proportion) you adioyne a third and a fourth line in continuall proportion (as I haue taught before): The first and fourth lines, shall aunswere the Probleme. How generall this rule is, in any two like solides, with their correspondent (or Omologall) lines, I neede not, with more wordes, declare.

¶ *Certaine Theoremes and Problemes (whose vse is manifolde, in Spheres, Cones, Cylinders, and other solides) added by Ioh. Dee.*

✲ *A Theoreme. 1.*

The whole superficies of any Sphere, is quadrupla, to the greatest circle, in the same sphere contayned.

It is needeles to bring *Archimedes* demonstration hereof, into this place: seing his boke of the Sphere and Cylinder, with other his workes, are euery where to be had, and the demōstration therof, easie.

✲ *A Theoreme. 2.*

Euery sphere, is quadrupla, to that Cone, whose base is the greatest circle, & height, the semidiameter of the same sphere.

This is the 32. Proposition of *Archimedes* first booke of the Sphere and Cylinder.

✲ *A Probleme. 1.*

A Sphere being geuen, to make an vpright Cone, equall to the same: or in any other proportion, geuen, betwene two right lines.

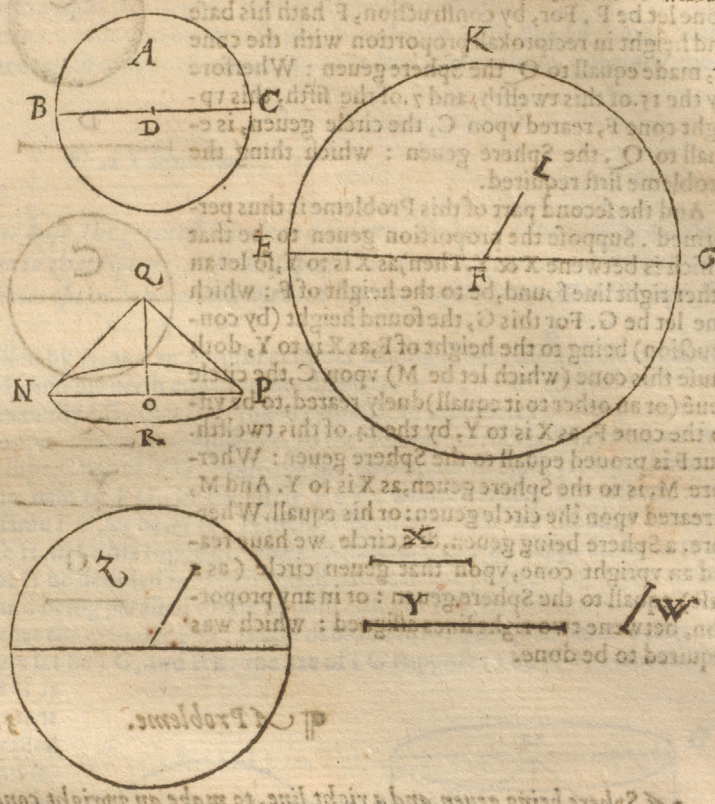
Construction.

Suppose the Sphere geuen, to be *A*, his diameter being *B C*, and center *D*: with a line equall to the semidiameter *B D* (which let be *N O*) describe a circle *N R P*: whose diameter let be *N P*, and center *O*, it is euident, that *N R P* is equall to the greatest circle in *A*, contayned. At the center *O*, let a perpendicular be reared equall to *B D* (the semidiameter of *A*) which suppose to be *O Q*: It is now plaine that to the Cone, whose base is the circle *N R P*, and height *O Q*, the Sphere *A*, is quadrupla: by the 2. Theoreme here, and by construction. Take a line equall to *N P*, which let be *F E*: and with the semidiameter *F E* (making the point *F* center) describe a circle: which suppose to be *E K G*, and diameter *E G*. At the center *F*, reare a line perpendicular to *E K G*, by the 12. of the eleuenth: and make it equall to *O Q*. Let that line be *F L*. I say that the Cone, whose base is the circle *E K G*, and height the line *F L*, is equall to *A*. For seing *F E*, the semidiameter of *E K G*, is equall to *N P* (the diameter of *N R P*) by construction: *E G*, the diameter of *E K G*, shall be double to *N P*. Wherfore the square of *E G*, is quadrupla to the square of *N P*: by the 4. of the second. But as the square of *E G*, is to the

Demonstration.

square

ſquare of *N P*, ſo is the circle *E K G* to the circle *N R P*, by the 2. of this twelueth. Wherefore the circle *E K G*, is quadruple to the circle *N R P*. And *F L*, the height is (by conſtruction) equall to *O Q* the height. Wherefore the cone, whoſe baſe is the circle *E K G*, and height *F L*, is quadruple to the cone, whoſe baſe is *N R P*, and the height *O Q*: by the 11. of this twelfth. But vnto the ſame cone whoſe baſe is *N R P*, and height *O Q*, the Sphere *A* is likewiſe proued quadrupla. Wherefore the cone whoſe baſe is *E K G* and height *F L*, is equall to the Sphere *A*: by the 7. of the fift. To a Sphere being geuen therefore, we haue made an vpright cone equall.



The ſecond part of the Probleme two wayes executed. 1.

And as concerning the other part of this Probleme, it is now eaſie to execute, and that two wayes: I meane to *A* the ſphere geuen, to make an vpright cone in any proportion geuen betwen two right lines. For, let the proportion geuen, be that which is betwene *X* and *Y*. By the order of my additions, vpon the 2. of this twelfth booke: to the circle *E K G* make an other circle in that proportion that *X* is to *Y*: which let be *Z*. Vpon the center of *Z*, reare a line perpendicular and equall to *F L*. I ſay that the cone, whoſe baſe is *Z*, and the height equall to *F L*, is to *A*, in the proportion of *X* to *Y*. For the cone vppon *Z*, by conſtruction, hath height equall to the height of the cone *L E K G*: and *Z*, by conſtruction, is to *E K G*, as *X* is to *Y*: Wherefore, by the 11. of this twelfth, the cone vpon *Z*, is to the cone *L E K G*, as *X* is to *Y*. But the cone *L E K G* is proued equall to the ſphere *A*. Wherfore the cone vpon *Z*, is to *A*, as *X* is to *Y*, by the 7. of the fift. To a Sphere geuen therefore, we haue made a cone in any proportion geuen, betwene two right lines. Secondly, as *X* is to *Y*, ſo to *F L*, let there be a fourth line, by the 12. of the ſixt: and ſuppoſe it to be *W*. I ſay that a cone, whoſe baſe is equall to *E K G*, and height the line *W*, is to *A*, as *X* is to *Y*. For, by the 14. of this twelfth, cones being ſet on equall baſes, are one to the other, as their heightes are: But, by conſtruction, the height *W*, is to the height *F L*, as *X* is to *Y*. Wherefore the cone which hath his baſe equall to *E K G*, and height the line *W*, is to the cone *L E K G*, as *X* is to *Y*. And it is proued, that to the cone *L E K G*, the Sphere *A* is equall: Wherfore, by the 7. of the fift, the cone, whoſe baſe is equall to *E K G*, and height the line *W*, is to *A*, as *X* is to *Y*. Therefore a Sphere being geuen, we haue made an vpright cone, in any proportion geuen betwene two right lines. And before, we made an vpright cone, equall to the Sphere geuen. Wherfore a Sphere being geuen, we haue made an vpright cone, equall to the ſame, or in any other proportion, geuen betwene two right lines. I call that an vpright cone, whoſe axe is perpendicular to his baſe. 2. *An vpright Cone.*

¶ *A Corollary.*

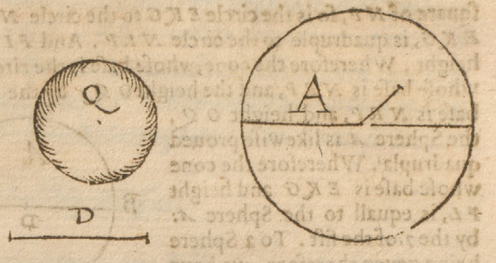
Of the firſt part of the demonſtration, it is euident: A Sphere being propounded, that a Cone, whoſe baſe hath his ſemidiameter, equall to the diameter therof, and height equall to the ſemidiameter of the ſame Sphere, is equall to that ſphere propounded.

¶ *A Probleme.* 2.

A Sphere being geuen, and a circle, to reare an vpright Cone, vpon that circle (as a baſe) equall to the Sphere geuen: or in any proportion betwene two right lines aſſigned.

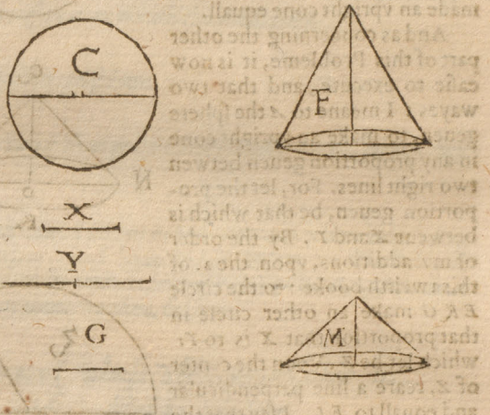
Suppoſe the Sphere geuen, to be Q: and the circle geuen to be C: By the firſt Probleme make an vpright cone equall to Q the Sphere geuen: which cone ſuppoſe to be A: and (by the 2. Probleme of my additions vpon the ſecond of this twelfth booke) as C the circle geuen, is to the baſe of A, ſo let

the height of A, be to a line found: which let be D. Then it is euident, that the cone, which hath for his base C, the circle geuen, and height the line D, last found, shall be equall to Q the Sphere geuen: which cone let be F. For, by construction, F hath his base and height in reciprokall proportion with the cone A, made equall to Q the Sphere geuen: Wherfore by the 15. of this twelfth, and 7. of the fifth, this vpright cone F, reared vpon C, the circle geuen, is equall to Q, the Sphere geuen: which thing the Probleme first required.



The second part of the Probleme.

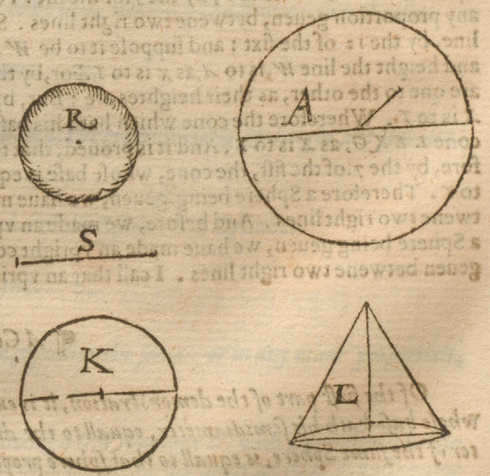
And the second part of this Probleme is thus performed. Suppose the proportion geuen to be that which is betwene X & Y. Then, as X is to Y, so let an other right line found, be to the height of F: which line let be G. For this G, the found height (by construction) being to the height of F, as X is to Y, doth cause this cone (which let be M) vpon C, the circle geuē (or an other to it equall) duely reared, to be vnto the cone F, as X is to Y, by the 14. of this twelfth. But F is proued equall to the Sphere geuen: Wherfore M, is to the Sphere geuen, as X is to Y. And M, is reared vpon the circle geuen: or his equall. Wherfore, a Sphere being geuen, & a circle, we haue reared an vpright cone, vpon that geuen circle (as a base) equall to the Sphere geuen: or in any proportion, betwene two right lines assigned: which was required to be done.



¶ A Probleme. 3.

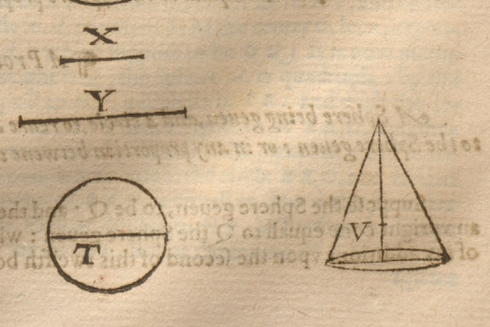
A Sphere being geuen, and a right line, to make an vpright cone, equall to the Sphere geuen, or in any other proportion geuen betwene two right lines: which made cone, shall haue his height equall to the right line geuen.

Suppose the Sphere geuen, to be R: and the right line geuen, to be S. To R the Sphere geuen, make an vpright cone, equall: by the first Probleme: which cone suppose to be A. Then as S, the line geuen, is to the height of A, so let the base of A, be to an other circle, which let be K, by my additions, vpon the second Proposition of this twelfth booke. I say that an vpright cone, hauing his height, equall to S, the right line geuen, and his base K, is equall to the Sphere geuen. Let this cone be noted by L: for by construction, thys cone L, and A, haue theyr heightes and bases reciprokall in proportiō: Wherfore this cone L, and the cone A, are equall, (by the 15. of the twelfth). But A is equall to the Sphere geuen by construction. Wherefore L is equall to the Sphere geuen. And the height of L, is equall to the right line geuen, by construction: which ought to be done.



The second part of the Probleme.

For the second part: finde a circle, which shall haue to the base of L, any proportion appointed in right lines: as the proportion of X to Y: which, by my additions, vpō the second of this booke, ye haue learned to do. Then, with the height, equall to the heigth of L reared vpon this last found circle, which let be T, as a base, you shall satisfie the Probleme. Let that Cone be V. For this last cone V, is to L, as his base is to the base of L, by the 11. of this twelfth. But L is proued equall to the Sphere geuen: Wherfore by the 7. of the fift, this last cone V, hath to R, the



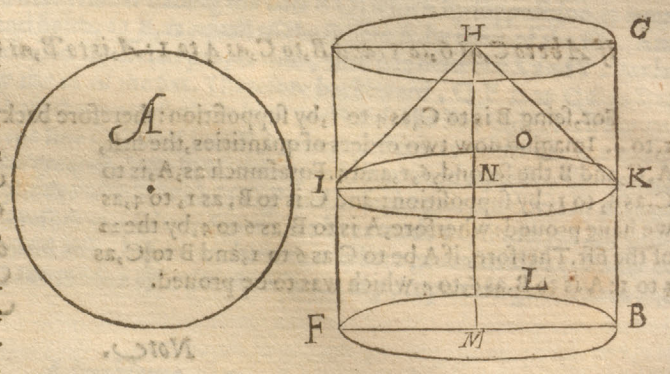
the Sphere geuen, that proportion which is betwene X and Y assigned: and forasmuch as the height of this cone V, is equall to the height of L: and the height of L, equall to S, the right line geuen (by construction): it is euident, that a Sphere being geuen; & a right line, we haue made an vpright cone, equall to the Sphere geuen, or in any other proportion geuen betwene two right lines: which made cones, haue their height equall to the right line geuen: which ought to be done.

Vnwilling I am to vse thus many wordes, in matters so plaine and easie. But this (I thinke) can not hinder them, that by nature are not so quicke of inuention, as to lead euery thing, generally spoken, to a particular execution.

¶ A Theoreme. 3.

Euery Cylinder which hath his base, the greatest Circle in a Sphere, & heith equall to the diameter of that Sphere, is Sesquialtera to that Sphere. Also the superficies of that Cylinder, with his two bases is Sesquilatera to the superficies of the Sphere: and without his two bases, is equall to the superficies of that Sphere

Suppose, a sphere to be signified by A, and an vpright cylinder hauing to his base a circle equall to the greatest circle in A contayned, and his heith equall to the diameter of A, let be signified by F G. I say that F G, is sesquialter to A: Secondly I say that the croked cylindricall superficies of F G, together with the superficieces of his two opposit bases, is sesquialtera to the whole superficies sphericall of A. Thirdly I say that the cylindricall superficies of F G, omitting his two opposite bases, is equall to the superficies of the spere A. Let the base of F G, be the circle F L B: whose center, suppose M, and diameter F B. And the axe of the same F G, let be, M H. Which is his heith (for we suppose the cylinder to be vpright): and suppose H, to be his toppe or vertex. Forasmuch as, by supposition M H is equall to the diameter of A. Let M H be deuided into two equall partes in the point N, by a playne superficies passing by the point N, and being parallell to the opposit bases of F G. By the thirtenth of this twelfth booke, it then foloweth, that the cylinder F G, is also deuided into two equall parts: being cylinders: which two equall cylinders let be I G, and F K: the axe of I G suppose to be H N: and of F K the axe to be N M. And for that, F G, is an vpright cylinder, and at the poynt N, cut by a playne Superficies parallell to his opposite bases, the common section of that playne superficies and the cylinder F G, must be a * circle, equall, to his base F L B, and haue his center, the point N. Which circle, let be I O K: And seing that F L B is, by supposition, equall to the greatest circle in A, I O K, also, shall be equall to the greatest circle, in A, contained: Also, by reason M H, is by supposition, equal to the diameter of A: and N H, by construction, half of M H, it is manifest that N H is equall to the semidiameter of A.



* *This may easely be demonstrated, as in the 17. proposition the section of a sphere was proued to be a circle.*

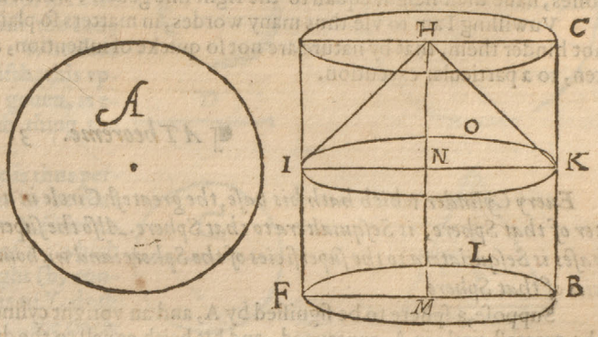
If, therefore, you suppose a cone to haue the circle I O K to his base: and N H to his heith, the sphere A, shall be to that Cone, quadrupla, by the 2. Theoreme. Let that cone be H I O K. Wherefore A, is quadrupla to H I O K. And the Cylinder I G hauing the same base, with H I O K (the circle I O K) and the same heith, (the right line N H) is triple to the cone H I O K: by the 10. of this twelfth booke. But to I G, the whole cylinder F G, is double, as is proued: Wherefore F G, is triple and triple, to the cone H I O K, that is, sextuple. And A is proued quadrupla to the same H I O K. Wherefore F G is to H I O K, as 6. to 1: and A, is to H I O K, as 4. to 1: * Therfore F G is to A, as 6, to 4: which in the least termes, is, as 3 to 2. but 3 to 2, is the termes of sesquialtera proportion. Wherefore the cylinder F G, is to A sesquialtera in proportion. Secondly, forasmuch as the superficies of a cylinder (his two oppositе bases excepted) is equall to that circle whose semidiameter is middell proportionall betwene the side of the cylinder, and the diameter of his base: (as vnto the 10. of this booke, I haue added.) But of F G, the side B G, being parallell and equall to the axe M H, must also be equall to the diameter of A. And the base F L B, being (by supposition) equall to the greatest circle in A contained, must haue his diameter (F B) equal to the sayd diameter of A. The middle proportional therfore betwene B G and F B, being equall eche to other, shall be a line, equall to either of them.

* *For taking away all doubt, this, as a Lemma, afterward is demõstrated.*

[As if you set B G and F B together, as one line, and vpon that line composed, as a diameter make a semicircle: and from the center, to the circumference draw a line perpendicular to the sayd diameter: by the 13. of the sixth, that perpendicular, is middel proportional betwene F B and B G, the semidiameters: and he him selfe also a semidiameter: and therfore by the definition of a circle, equall to F B, and likewise, to B G.] And a circle, hauing his semidiameter, equall to the diameter F B, is quadruple to the circle F L B. [For the square of euery whole line, is quadruple to the square of his halfe line, as may

A Lemma (as it were) presently demonstrated.

be proued by the 4. of the ſecond: and by the ſecond of this twelfth, circles are one to the other, as the ſquares of their diameters, are.] Wherfore the ſuperficies cylindricall of F G, alone, is quadrupla to his baſe F L B. But if a certayne quantity be dupla to one thing, and an other, quadrupla to the ſame one thing, thoſe two quantities together are ſextupla to the ſame one thing. Therefore ſeing the baſe, oppoſite to F L B, (being equall to to F L B) added to F L B, maketh that cõpound, double to F L B: that double added to the cylindricall ſuperficies of F G, doth make a ſuperficies ſextupla to F L B. And the ſuperficies of A, is quadrupla to the ſame F L B, by the firſt Theoreme. Therefore the cylindricall ſuperficies of F G, with the ſuperficieces of his two baſes is to the ſuperficies F L B, as 6 to 1. and the ſuperficies of A to F L B, is as 4 to 1. Wherfore the cylindricall ſuperficies of F G, & his two baſes, together, are to the ſuperficies of A, as 6 to 4: that is, in the ſmalleſt termes, as 3 to 2. Which is proper to ſeſquialtera proportion. Thirdly, it is already made euident that the ſuperficies cylindrical, of F G (onely by it ſelf) is quadrupla to F L B. And alſo it is proued, that the ſuperficies of the ſphere A, is quadrupla to the ſame F L B, Wherefore by the 7. of the fifth, the cylindricall ſuperficies of F G, is equall to the ſuperficies of A. Therfore, euery cylinder, which hath his baſe the greateſt circle in a ſphere, and heith equal to the diameter of that ſphere, is ſeſquialtera to that ſpere: Alſo the ſuperficies of that cylinder with his two baſes, is ſeſquialtera, to the ſuperficies of the ſphere: and without his two baſes is equall to the ſuperficies of the ſphere: which was to be demonſtrated.

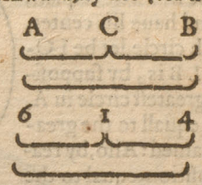


3.

The Lemma.

If A be to C, as 6, to 1: and B, to C, as 4 to 1: A, is to B, as 6, to 4.

For, ſeing, B is to C, as 4 to 1, by ſuppoſition: therefore backward, by the 4. of the fifth, C is to B, as 1, to 4. Imagine now two orders of qnantities, the firſt, A, C, and B the ſecond, 6, 1, and 4. Foraſmuch as, A, is to C, as 6, to 1, by ſuppoſition: and C is to B, as 1, to 4, as we haue proued: wherfore, A is to B, as 6 to 4, by the 22 of the fift. Therfore, if A be to C as 6 to 1, and B to C, as 4 to 1: A is to B, as 6, to 4. which was to be proued.



Note.

Sleight things (ſome times) lacking euidẽt prouſe, brede doubt or ignorance. And, I nede not warne you, how gen: rall, this demonſtration is: for if you put in the place of 6 and 4, any other numbers, the like manner of concluſion will follow. So likewiſe, in place of 1. any other one number may be, as, if A be to C as 6 to 5: and B vnto C, be as 7 to 5: A, ſhall be to B, as 6 to 7. &c.

A Probleme. 4.

To a Sphere geuen, to make a cylinder equall, or in any proportion geuen betwene two right lines.

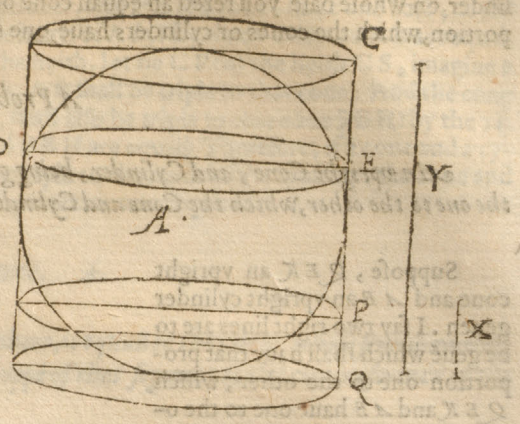
Conſtruction.

Suppoſe the geuen Sphere to be A: and the proportion geuen to be that betwene X and Y. I ſay that a cylinder is to be made, equall to A: or els in the ſame proportion to A, that is betwene X to Y. Let a cylinder be made (ſuch one as the Theoreme next before ſuppoſed) that ſhall haue his baſe equall to the greateſt circle in A, and height equall to the diameter of A: Let that cylinder be the vpright cylinder B C. Let the one ſide of B C, be the right line Q C. Deuide Q C into three equal parts: of which, let Q E containe two, and let the third part be C E. By the point E ſuppoſe a plaine (parallel to the baſes of B C) to paſſe through the cylinder B C, cutting the ſame by the circle D E. I ſay that the cylinder B E is equall to the Sphere A. For ſeing B C, being an vpright cylinder, is cut by a plaine, parallel to his baſes, by conſtruction: therefore as the cylinder D C, is to the cylinder B E, ſo is the axe of D C, to the axe of B E, by the 13. of this twelfth. Wherefore as the axe is to the axe, ſo is cylinder to cylinder. But axe is to axe, as ſide to ſide, namely, C E to Q E, becauſe the axe is parallel to any ſide

Demonſtration.

of an

of an vpright cylinder: by the definition of a cylinder. And the circle of the ſection, is parallel to the baſes, by conſtruction. Wherefore in the parallelogramme (made of the axe, and of two ſemidiameters, on one ſide parallels, one to the other, being coupled together by a line drawen betwene their endes in their circumferences, which line is the ſide Q C) it is euident, that the axe of B C is cut in like proportion, that the ſide Q C is cut. Wherfore the cylinder D C, is to the cylinder B E, as E C is to Q E. Wherefore, by compoſition, the cylinders D C and B E, that is, whole B C, are to the cylinder B E, as C E and Q E (the whole right line Q C) are to Q E. But by cõſtruction, Q C is of 3. ſuch partes, as Q E containeth 2. Wherefore the cylinder B C, is of 3. ſuch partes, as B E contayneth 2. Wherefore B C the cylinder, is to B E, as 3. to 2: which is ſeſquialtera proportion. But by the former Theoreme, B C is ſeſquialtera to the Sphere A: Wherefore, by the 7. of the fift, B E is equall to A. Therefore to a Sphere geuen, we haue made a cylinder equall.



Or thus more briefely omitting all cutting of the Cylinder.

Foraſmuch as B C is an vpright Cylinder: his ſides are equall to his axe or heith: therefore the two cylinders, whereof one hath the heith Q C and the other the heith Q E, hauing both their baſes, the greateſt circle in the Sphere A, are one to the other as Q C is to Q E, by the 14. of this twelfth, but Q C is to Q E, as 3. to 2, by conſtruction: and 3. to 2, is in Seſquialtera proportion: therefore the cylinder B C hauing his heith Q C, & his baſe the greateſt circle in A cõteyned, is Seſquialtera to the cylinder which hath his baſe the greateſt circle in A conteyned, and heith the line Q E. But by the former Theoreme, B C, is alſo Seſquialtera to A: wherfore the cylinder hauing the baſe B Q (which by ſuppoſition, is equal to the greateſt circle in A conteyned) and heith, Q E, is equall to the ſphere A, by the 7. of the fift. And now it can not be hard, to geue a cylinder, to A, in that proportion, which is betwene X and Y. For let the ſide Q E, be to Q P, as Y is to X, by the 12. of the ſixt. Therefore backeward, Q P is to Q E as X to Y. Wherefore the cylinder hauing the baſe the greateſt circle in A and heith the line Q P, is to the cylinder hauing the ſame baſe, and heith the line Q E, as X is to Y, by the 14. of this twelfth: but the cylinder hauing the heith Q E, & his baſe the greateſt circle in A, conteyned, is proued equall to the Sphere A. Wherefore by the 7. of the fift, the cylinder whoſe heith is Q P and baſe the greateſt circle in A, conteyned, is to the ſphere A, as X to Y. Therefore to a ſphere geuen: we haue made a cylinder, in any proportion geuen betwene two right lines, and alſo, before we haue to a ſphere geuen, made a cylinder equall. Therefore to a ſphere geuen, we haue made a cylinder equall, or in any proportion geuen betwene two right lines.

The ſecond part of the Probleme.

A Probleme. 5.

A Sphere being geuen, and a circle, vpon that circle as a baſe, to rere a cylinder, equall to the ſphere geuen: or in any proportion, geuen betwene two right lines.

A Probleme. 6.

A Sphere being geuen, and a right line, to make a cylinder, equall to the ſphere geuen, or in any other proportion, betwene two right lines, geuen.

In this 5. and 6. probleme, firſt make a cylinder equall to the ſphere geuen, by the 4. probleme: and then by the order of the 2. and 3. problemes, in cones, execute theſe accordingly in cylinders.

A Probleme. 7.

Two vnlike Cones or Cylinders, being geuen, to finde two right lines, which haue the ſame proportion one to the other, that the two geuen cones or cylinders haue one to the other.

Vpon one of their baſes rere a cone (if cones be compared) or a cylinder (if cylinders be compared) equall to the other: by the order of the ſecond and third problemes: and the heith of the cone or cy-

linder, on whose base you rered an equall cone or cylinder, with the new heith found, haue that proportion, which the cones or cylinders haue, one to the other, by the 14. of this twelfth booke.

A Probleme. 8.

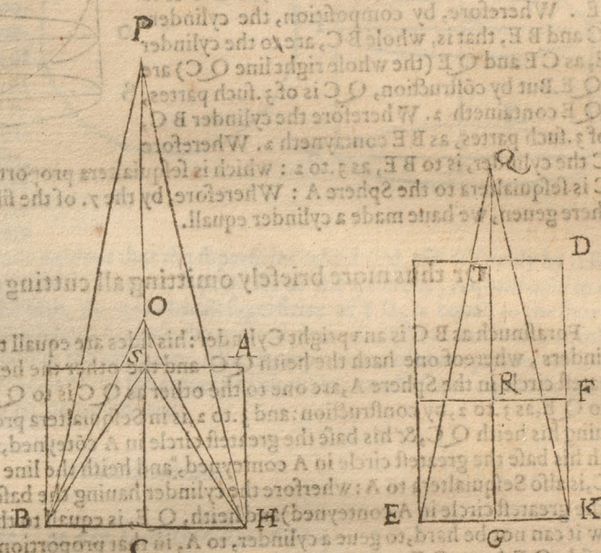
An vpright Cone, and Cylinder, being geuen: to finde two right lines, hauing that proportion, the one to the other, which the Cone and Cylinder haue, one to the other.

Construction.

Suppose, $Q E K$ an vpright cone and $A B$ an vpright cylinder geuen. I say two right lines are to be geué which shall haue that proportion one to the other, which $Q E K$ and $A B$ haue one to the other. Vpon the base $B H$, erecte a Cone, equall to $Q E K$: by the order of the secód probleme: which let be $O B H$, and his heith let be $O C$, and let the heith of $A B$, (the cylinder), be $C S$, produce $C S$ to P: so that $C P$, be tripla to $C S$, & make perfect the cone $P B H$. I say that $P C$ and $O C$ haue that proportion, which $A B$ hath to $Q E K$.

Demonstration.

For, by construction, $O B H$ is equall to $Q E K$: and $P B H$ is equal to $A B$, as we will proue, (Assumpt wise). And $P B H$, and $O B H$ are vpon one base, namely $B H$: wherfore by the 14 of this twelfth, $P B H$ and $O B H$ are one to the other as their heithes $P C$, and $O C$, are one to the other, wherefore the cylinder and cone equall to $P B H$ and $O B H$ are as $P C$ is to $O C$: by the 7. of the fifth. But $A B$ the cylinder $Q E K$ the cone, are equall to $P B H$ and $O B H$: by construction: wherefore $A B$ the cylinder, is to $Q E K$ the cone, as $P C$, is to $O C$. Wherefore we haue found two right lines hauing that proportion that a cone and a cylinder geuen, haue one to the other. Which thing we may execute vpon the base of the cone geuen, as we did vpon the base of the cylinder geuen, on this maner. Vpon the base of the cone $Q E K$, which base, let be $E K$, erect a cylinder, equall to $A B$, by the order of my second probleme. Which cylinder let be $E D$, and $G T$ his heith, and let the heith of the cone $Q E K$, be $Q G$. Take the line $G R$, the third part of $Q G$, (by the 9. of the sixth): and with a playne passing by R, parallel to $E K$, cut of the cylinder $E F$: which shall be equall to the cone $Q E K$, by the assumpt following: I say now, that $A B$, the cylinder, is to $Q E K$ the cone, as $G T$, is to $G R$. For the cylinder $E D$ is to the cylinder $E F$, as $G T$ is to $G R$, by the 14. of the twelfth: and to $E D$ is the cylinder $A B$ equall: by construction: and to $E F$, we haue proued the cone $Q E K$, equall, wherefore by the 7. of the fifth, $A B$ is to $Q E K$, as $G T$ is to $G R$. Wherfore an vpright cone, & a cylinder being geuen, we haue found two right lines hauing the same proportion betwene them, which the cone and the cylinder, haue one to the other: which was requisite to be done.



An other way of executing this probleme.

An assumpt.

If a cone, and a cylinder, being both on one base, are equall one to the other: the heith of the cone is tripla to the heith of the cylinder. And if a cone and a cylinder being both on one base, the heith of the cone be tripla to the heith of the cylinder, the cone and the cylinder are equall.

We will vse the cylinder A B & the cone P B H in the former probleme: with their base & heithes so noted as before. I say if P B H be equall to A B, that C P the heith of P B H, is tripla to C S the heith of the cylinder A B. Suppose vpon the base B H, a cone to be rered of the heith of C S, which let be S-B H: it is manifest that A B is tripla to that cone S B H, by the 10. of this twelfth. Wherfore a cone equal to A B the cylinder is tripla to S B H the cone, by the 7. of the fifth, but P B H is supposed equall to A B. Therefore P B H is tripla to S B H, therefore the heith of P B H shall be tripla to the heith of S B H by the 14. of the twelfth. But the heith of P B H, is C P: and of S B H, the heith is C S: wherefore C P is tripla to C S. And C S is the heith of the cylinder A B by supposition. Therefore a cone and a cylinder, being

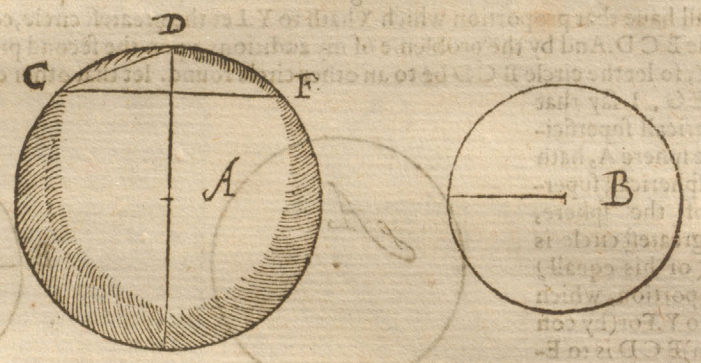
being both on one baſe,and equall, the heith of the cone is tripla to the heith of the cylinder. And the ſecond part,as eaſely may be confirmed. For if A B a cylinder, and P B H a cone, haue one baſe, as the circle about B H: and the heith of P B H be tripla to the heith of A B, I ſay that P B H, and A B are equal. The heith of A B let be (as afore) C S: and of P B H, the heith, let be C P: of the heith C S, imagine a Cone vpon the ſame baſe B H: by the 10. of this twelfth, A B ſhall be triple to that cone. And the cone P B H hauing heith C P, (by ſuppoſition) tripla to C S, ſhall alſo be tripla to that cone S B H: by the 14. of this twelfth. Wherefore by the 7. of the fifth A B and P B H are equall. Therefore, if a cone and a cylinder being both on one baſe, the heith of the cone be tripla to the heith of the cylinder, the cone and the cylinder are equall. So haue we demonſtrated both partes: as was required.

The conuerſe of the aſſūpt.

¶ *A Theoreme. 4.*

The ſuperficies of the ſegment or protion of any ſphere, is equall to the circle, whoſe ſemidiameter, is equall to that right line which is drawne from the toppe of that ſegment to the circumference of the circle, which is the baſe of that portion or ſegment.

As in the Sphere A, a Segment being cut of by the circle, whoſe diameter is C E: & the ſame circle being the baſe of the Segment, whoſe top alſo is D: the croked ſuperficies ſpherical of the ſame Segment, is equall to a circle whoſe Semidiameter is equall to the right line D C. As is the circle B.



This hath *Archimedes* demonſtrated in this firſt booke of the Sphere and Cylinder, in his 40. and 41 propoſitions: and I remitte them thether, that will herein demonſtratiuely be certified: I would wiſh all Mathematiciens, as well of verities eaſy, as of verities rare and obſcure, to ſeeke the cauſes demonſtratiue, the finall fruite thereof, is perfection in this art.

✻ *Note.*

Beſides all other vſes and commodities, that are of the Croked ſuperficieces of the Cone, Cylinder, and Sphere, ſo eaſely and certaynely, of vs to be dealt with all: this is not the leaſt, that a notable Error, which among Sophiſticall brablers, and vngeometricall Maſters and Doctors, hath a long time bene vpholden: may moſt euidently, hereby be confuted, and vtterly rooted out of all mens fantaſies for euer. The Error is this, *Curui, ad rectum, nulla eſt proportio*, that is: Betwene croked and ſtraight, is no proportion. This error, in lines, ſuperficieces, and ſolides, may with more true demonſtrations be ouerthrowne, then the fauourers of that fond fantaſie are able, with argument, either probable or Sophiſticall to make ſhew or pretence to the contrary. In lines, I omitte (as now) *Archimedes* two wayes, for the finding of the proportion of the circles circumference to a ſtraight line. I meane, by the inſcription and circumſcription of like poligonon figures, and that other, by ſpirall lines. And I omitte likewiſe (as now) in ſolides, of a parallelipipedon, equall to a Sphere, Cone, or Cylinder: or any ſegment or ſector of the ſayd ſolides. And onely, here require you to conſider in this twelfth booke, the wayes brought to your knowledge, how to the croked ſuperficies of a cone and cylinder, and of a ſphere, (the whole, any ſegment or ſector thereof) a playne and ſtraight ſuperficies may be geuen equall: Namely, a Circle to be geuen equall, to any of the ſayd croked ſuperficieces aſſigned, and geuen. And then farther by my Additions vpon the ſecond propoſition, you haue meanes to proceede in all proportions, that any man can in right lines geue, or aſſigne. Therfore, *Curui ad rectum, proportio omnimoda poteſt dari.* One thing it is, to demonſtrate, that betwene a croked line and a ſtraight: or a croked ſuperficies and a playne or ſtraight ſuperficies, &c. there is proportion. And an other thing it is, to demonſtrate a particular and ſpeciall kinde of proportion, being betwene a croked ſuperficies and a ſtraight or playne ſuperficies. For this alſo confirmeth the firſt. This ſhort warning will cauſe you to auoyde the ſayd error, and make you alſo hable to cure them, which are infected therewith.

A great error commonly maintained.

„ *Betwene*
„ *ſtraight*
„ *and croked*
„ *all maner*
„ *of proportiō may be*
„ *geuen.*

✻ *A Theoreme. 5.*

Any two Spheres being geuen, as the Spherical Superficies of the one, is to the Sphirical Superficies

ficies of the other: So is the greateſt Circle conteyned in that one, to the greateſt Circle conteyned in that other. And as greateſt Circle, is to greateſt Circle, ſo is Sphericall ſuperficies, to Sphericall ſuperficies.

For the ſuperficies of euery ſphere is quadrupla to his greateſt circle, by my firſt Theoreme: wherfore, of two geuen ſpheres, as the ſphericall ſuperficies of the one, is to his greateſt circle, ſo is the ſphericall ſuperficies of the other, to his greateſt circle: therfore by alternate proportion, as ſpherical ſuperficies is to ſpherical ſuperficies, ſo is greateſt circle to greateſt circle. And therfore alſo as greateſt circle, is to greateſt circle, ſo is ſpherical ſuperficies to ſphericall ſuperficies: which was to be demõſtrated.

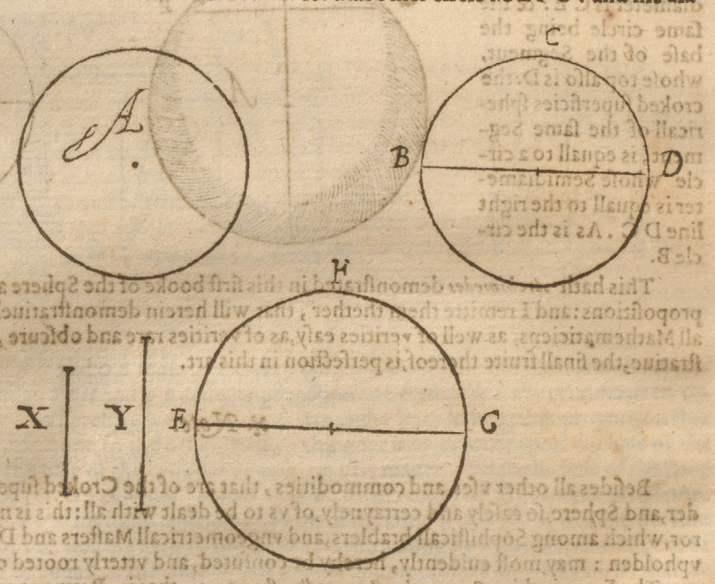
✻ A Probleme. 9.

A Sphere being geuen, to geue an other Sphere, to whoſe Sphericall ſuperficies, the ſuperficies Spherical of the Sphere geuen ſhall haue any proportion, betwene two right lines geuen.

Conſtruction.

Suppoſe A, to be a ſphere geuen, and the proportion geuen, to be that, which is betwene the right lines X and Y. I ſay that a ſphere is to be geuen to whoſe ſpherical ſuperficies, the ſuperficies ſpherical of A, ſhall haue that proportion which X hath to Y. Let the greateſt circle, conteyned in A the ſphere be the circle B C D. And by the probleme of my additions, vpon the ſecond propoſition of this booke, as X is to Y, ſo let the circle B C D be to an other circle found. let that other circle be E F G: and his diameter E G. I ſay that the ſpherical ſuperficies of the ſphere A, hath to the ſpherical ſuperficies of the ſphere, whoſe greateſt circle is E F G, (or his equall) that proportion, which X hath to Y. For (by conſtruction) B C D is to E F G, as X is to Y: and by the theoreme next before, as B C D is to E F G, ſo is the ſpherical ſuperficies of A (whoſe greateſt circle is B C D, by ſuppoſition) to the ſpherical ſuperficies, of the ſphere, whoſe greateſt circle is E F G. wherefore, by the 11. of the fifth, as X is to Y: So is the ſpherical ſuperficies of A, to the ſpherical ſuperficies of the ſphere, whoſe greateſt circle is E F G: wherefore, the ſphere whoſe diameter is E G, (the diameter alſo of E F G) is the ſphere, to whoſe ſpherical ſuperficies, the ſpherical ſuperficies of the ſphere A, hath that proportion which X hath to Y. A ſphere being geuen therefore, we haue geuen an other ſphere, to whoſe ſpherical ſuperficies, the ſuperficies ſpherical of the ſphere geuẽ hath any proportion geuen, betwene two right lines: which ought to be done.

Demonſtration.



A Probleme. 10.

A Sphere being geuen, and a Circle leſſe then the greateſt Circle, in the ſame Sphere conteyned, to coapt in the Sphere geuen, a Circle equall to the Circle geuen.

The diffinitiõ of a circle coapted in a ſphere.

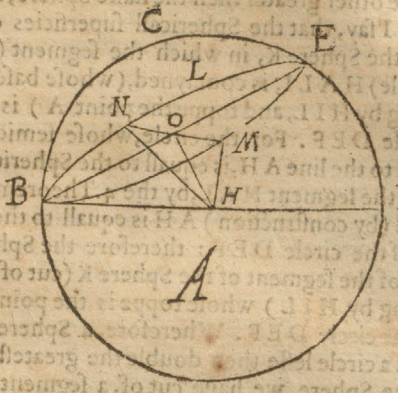
Conſtruction.

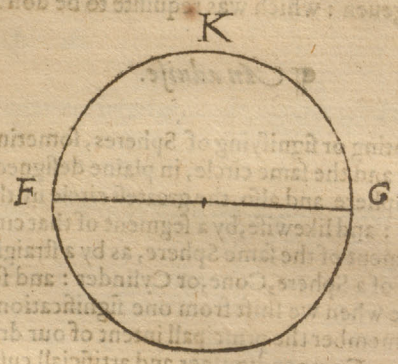
Suppoſe A, to be the ſphere geuen: and the circle geuen leſſe then the greateſt circle in A conteyned, to be F K G. I ſay, that in the Sphere A, a circle, equall to the circle F K G, is to be coapted. Firſt vnderſtand, what we meane here, by coapting of a circle in a Sphere. We ſay, that circle to be coapted in a Sphere, whoſe whole circumferẽce is in the ſuperficies of the ſame Sphere. Let the greateſt circle in the Sphere A conteyned, be the circle B C D. Whoſe diameter ſuppoſe to be B D, and of the circle F K G, let F G be the diameter. By the 1. of the fourth, let a line equall to F G, be coapted in the circle B C D. Which line coapted, let be B E. And by the line B E, ſuppoſe a playne to paſſe, cutting the Sphere A, and

to be

to be perpendicularly erected to the ſuperficies of B C D. Seing that the portion of the playne remayning in the ſphere, is called their common ſection: the ſayd ſection ſhall be a circle, as before is proued. *And the common ſection of the ſayd playne, and the greateſt circle B C D, (which is B E by ſuppoſition) ſhall be the diameter of the ſame circle*, as we will proue. For, let that circle be B L E M. Let the center of the ſphere A, be the point H: which H, is alſo the céter of the circle B C D, becauſe B C D is the greateſt circle in A conteyned. From H, the center of the ſphere A, let a line perpendicularly be let fall to the circle B L E M. Let that line be H O: and it is euident that H O ſhall fall vpon the common ſection B E, by the 38. of the eleuenth. And it deuideth B E, into two equall parts, by the ſecond part of the third propoſition of the third booke: by which poynt O, all other lines drawne in the circle B L E M, are, at the ſame pointe O, deuided into two equall parts. As if from the poynt M, by the point O, a right line be drawne one the other ſide comming to the circumference, at the poynt N: it is manifeſt that N O M is deuided into two equall partes at the poynt O: by reaſon, if from the center H, to the poyntes N and M, right lines be drawne, H N and H M, the ſquares of H M, and H N are equall: for that all the ſemidiameters of the ſphere are equal: and therefore their ſquares are equall one to the other: and the ſquare of the perpendicular H O, is common: wherefore the ſquare of the third line M O is equall to the ſquare of the third line N O: and therefore the line M O to the line N O. So therefore is N M equally deuided at the poynt O. And ſo may be proued of all other right lines, drawne in the circle B L E M, paſſing by the poynt O, to the circumference one both ſides. Wherefore O is the center of the circle B L E M: and therefore B E paſſing by the poynt O is the diameter of the circle B L E M. Which circle (I ſay) is equal to F K G: for by conſtruction B E is equall to F G: and B E is proued the diameter of B L E M, and F G is by ſuppoſition the diameter of the circle F K G: wherefore B L E M is equall to F K G the circle geuen: and B L E M is in A the ſphere geuē. Wherfore we haue in a ſphere geuen coapted a circle equall to a circle geuen: which was to be done.

Demōſtration.





This is manifeſt: if you conſider the two triangles rectangles, H O M and H O N: and then with all, vſe the 47. of the firſt of Euclide.

✻ *A Corollary.*

Beſides our principall purpoſe, in this Probleme, euidently demonſtrated, this is alſo made manifeſt: that if the greateſt circle in a Sphere, be cut by an other circle, erected vpon him at right angles, that the other circle is cut by the center, and that their common ſection is the diameter of that other circle: and therefore that other circle deuided is into two equall partes. 1. 2. 3.

✻ *A Probleme. 11.*

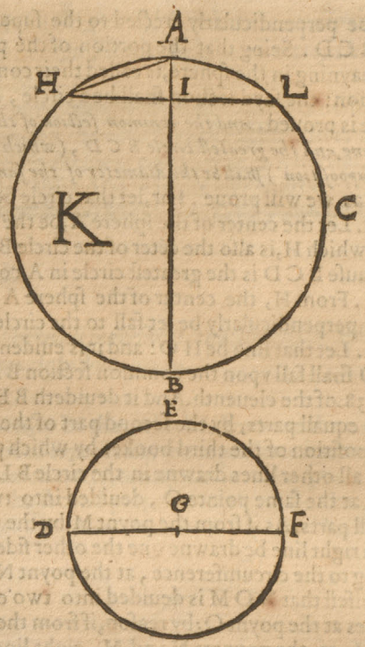
A Sphere being geuen, and a circle, leſſe then double the greateſt circle in the ſame Sphere contained, to cut of, a ſegment of the ſame Sphere, whoſe Sphericall ſuperficies, ſhall be equall to the circle geuen.

Conſtruction.

Suppoſe K to be a Sphere geuen, whoſe greateſt circle let be A B C: and the circle geuen, ſuppoſe to be D E F. I ſay, that a ſegment of the Sphere K, is to be cut of, ſo great, that his Spherical ſuperficies, ſhall be equall to the circle D E F. Let the diameter of the circle A B C, be the line A B. At the point A, in the circle A B C, coapt a right line equall to the ſemidiameter of the circle D E F (by the firſt of the fourth). Which line ſuppoſe to be A H. From the point H, to the diameter A B, let a perpendicular line be drawen: which ſuppoſe to be H I. Produce H I to the other ſide of the circumference, and let it come to the circumference at the point L. By the right line H I L (perpendicular to A B) ſuppoſe a plaine ſuperficies to paſſe, perpendicularly erected vpon the circle A B C: and by this

plaine ſuperficies, the Sphere to be cut into two ſegmentes: one leſſe then the halfe Sphere, namely, H A L I: and the other greater then the halfe Sphere, namely, H B L I. I ſay, that the Spherical ſuperficies of the ſegment of the Sphere K, in which the ſegment (of the greateſt circle) H A L I, is contayned, (whoſe baſe is the circle paſſing by H I L, and toppe the point A) is equall to the circle D E F. For the circle, whoſe ſemidiameter is equall to the line A H, is equall to the Spherical ſuperficies of the ſegment H A L, by the 4. Theoreme here added. And (by conſtruction) A H is equall to the ſemidiameter of the circle D E F: therefore the Spherical ſuperficies of the ſegment of the Sphere K (cut of by the circle paſſing by H I L) whoſe toppe is the point A, is equall to the circle D E F. Wherefore, a Sphere being geuen, and a circle leſſe then louble the greateſt circle in the ſame Sphere, we haue cut of, a ſegment of the ſame Sphere, whoſe Spherical ſuperficies, is equall to the circle geuen: which was requiſite to be done.

Demonſtratiõ



¶ *An aluiſe.*

In noting or ſignifying of Spheres, ſometimes we vſe by one and the ſame circle, in plaine deſigned, to repreſent a Sphere and alſo the greateſt circle in the ſame contained: and likewiſe, by a ſegment of that circle, ſignifie a ſegment of the ſame Sphere, as by a ſtraight line, we often ſignifie the circle, which is the baſe of a ſegment of a Sphere, Cone, or Cylinder: and ſo in ſuch like. Wherin, conſider our ſuppoſitions: and take heede when we ſhift from one ſignification to an other, in one and the ſame deſignation: and withall remember the principall intent of our drift: and ſuch light thinges, can not either trouble or offend thee. Compendiouſnes and artificiall cuſtome, procureth ſuch meanes: ſufficient, to ſtirre vp imagination Mathematicall: or to informe the practiſer Mechanicall.

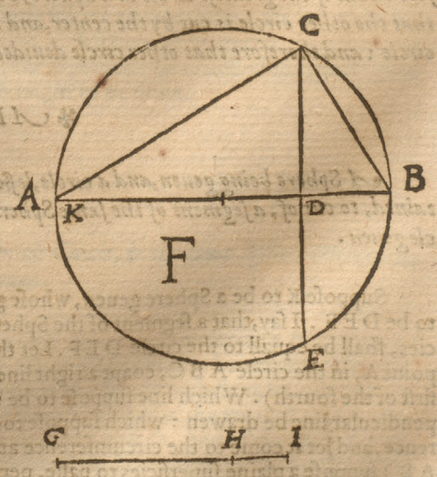
✻ *A Probleme.* 12.

To cut a Sphere geuen, into two ſuch ſegmentes, that the Spherical ſuperficieces of the ſegmentes ſhall haue one to the other, any proportion geuen betwene two right lines.

Conſtruction.

Suppoſe F to be a Sphere geuen: and the proportion geuen, to be that, which is betwene G H and H I. I ſay, that the Sphere I, is to be cut into two ſuch ſegmentes; that the Spherical ſuperficies of thoſe ſegmentes, ſhall haue that proportion, one to the other, which, the right line G H, hath to the right line H I. Suppoſe A B C E to be a greateſt circle, in the Sphere F, contained: and his diameter, to be A B. Deuide A B into two ſuch partes, as G I is diuided into, in the point H (by the 10. of the ſixt) Let thoſe partes be A D, and D B. So that, as G H is to H I, ſo is A D to D B. By the point D, let a plaine ſuperficies paſſe, cutting the Sphere F, and the diameter A B: So, that vnto that cutting plaine, the diameter A B, be perpendicular: and the Sphere alſo thereby deuided into two ſegmentes, whoſe cõmon baſe ſuppoſe to be the circle C E, hauing the center, the point D: and the toppe of the one to be the point A, and the toppe of the other to be the point B: and the ſegmentes themſelues, to be noted by E A C, and E B C: Drawe from the two toppes, A and B, to C (a point in the circumference of their common baſe) two right lines A C and B C. I ſay now, that the Spherical ſuperficies of the ſegment E A C, hath to the Spherical ſuperficies of the ſegment E B C, the ſame proportion, which G H hath to H I. For, foraſmuch as circles haue that proportion, one to the other, that the ſquares of their diameters haue one to the other (by the 2. of this twelfth). *And the ſquares of theyr ſemidiameters, haue the ſame proportion one to the other, which the ſquares of theyr diameters haue.* ¶ For like partes haue

Demonſtratiõ



This in maner of a Lemma, is preſently proued.

that

that proportion one to the other, that the whole magnitudes, whose like partes they are, haue the one to the other: by the 15. of the fift. But the square of euery diameter is quadruple to the square of his semidiameter: as hath often before, bene proued: therefore, circles haue one to an other, that proportion, that the squares of their semidiameters haue one to the other]. Wherefore, seing A C and B C are semidiameters of two circles, whereof eche is equall to the Sphericall superficies of the segmentes, betwene whose toppes and circumference of their base, they aredrawen: by the 4. Theoreme of these additions: it followeth that both those circles, whose semidiameters they are: and also those Sphericall superficieces, which are equall to those circles, haue the one to the other, the same proportion, which the square of A C hath to the square of B C. But A C is drawen betwene the circumference of the base, and toppe of the segment Sphericall, E A C, by construction and likewise B C is drawen betwene the toppe, and circumference of the base, of the Sphericall segment E B C, by construction: Wherefore the Sphericall superficies of the segment E A C, is to the Sphericall superficies of the segment E B C, as the square of A C is to the square of B C. But the square of A C is to the square of B C, as A D is to D B: by the Corollary of the Probleme of my additions vpon the second of this twelfth: And A D is to D B, as G H is to H I: by construction. Wherefore the Spherical superficies of the segment E A C, is to the Spherical superficies of the segment E B C, as G H is to H I. We haue therfore, cut the Sphere geuen, into two such segmentes, that the Spherical superficieces of the segmentes, haue one to the other any proportion geuen betwene two right lines: which vas to be done.

¶ *A Corollary. 1.*

Here it appeareth demonstrated, that, circles are one to the other, as the squares of their semidiameters are, one to the other.

Wherby (as occasion shall serue) you may, by force of the former argument, vse other like partes of the diameter, as well as halues.

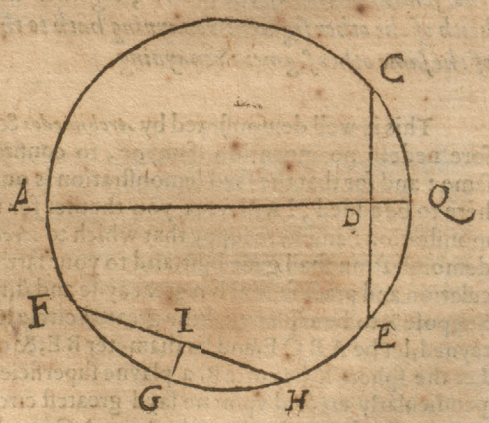
¶ *A Corollary. 2.*

It is also euident, that the Spherical superficieces of the two segmentes of any Sphere, to whose common base, the diameter (passing to their two toppes) is perpendicular, haue that proportion the one to the other, that the portions of the sayd diameter, haue the one to the other: that superficies and that portion of the diameter on the one side of the common base, being compared to that superficies, and that portion of the diameter, on the other side of the common base.

¶ *A Corollary. 3.*

It likewise euidently followeth, that the two Spherical superficieces of two segmentes of a Sphere: which two segmentes are equall to the Sphere, are in that proportion the one to the other, that their axes (perpendicularly erected to their bases) are in, one to the other: where soeuer in the Sphere those segmentes be taken.

I say that the Spherical superficies of the segment C A E, and the Spherical superficies of the segment F G H, hauing their axes A D and G I (perpendicular to their bases): are in proportion one to the other, as A D is to G I: if the segment of the Sphere containing C A E with (the segment of the same Sphere) F G H, be equall to the whole Sphere. For seing the diameter (or axe) A D, extended to the other pole or toppe, opposite to A (which opposite toppe, let be Q) doth make with the segment C A E, the complement of the whole Sphere: and by supposition, the segment F G H, with the segment C A E, are equall to the whole Sphere: Wherefore from equall, taking C A E (the segment common) remayneth the segment C Q E, equall to the segment F G H. And therby, *Axe, Base, Solitie*, and superficies Spherical of the segment F G H, must (of necessitie) be equall to



Note here of Axe, base, & soliditie, more then I nede to bring any farther proofe for.

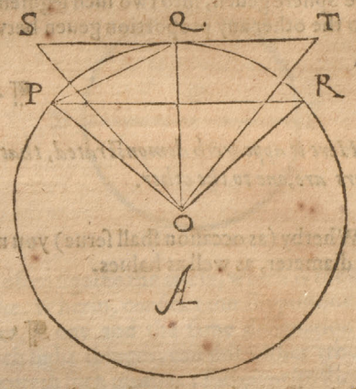
the Axe, Base, Soliditie, and superficies Spherical of the segment C Q E: Wherefore, by the second Corollary here, and the 7. of the fift, our conclusion is inferred, the superficies Spherical, of the segment C A E, to be, to the superficies Spherical of the segment F G H, as A D is to G I.

A Theoreme. 6.

To any solide sector of a Sphere, that vpright Cone is equall, whose base is equall to the conuex Spherical superficies of that sector, and heith equall to the semidiameter of the same Sphere.

Note.

Hereof the demonstration in respect of the premises: and the common argument of inscription and circumscription of figures is easy: and nevertheless, if your owne witte will not helpe you sufficiently: you may take helpe at *Archimedes* hand, in his first booke & last proposition of the sphere and cylinder. Whether if ye haue recourse, you shal perceaue how your Theoreme here amendeth the common translation there: and also our delineation geueth more luely shew of the chiefe circumstances necessary to the construction, then there you shall finde. Of the sphere here imagined to be A, we note a solide sector by the letters P Q R O. So that P Q R doth signifie the spherical superficies, to that solide sector belonging: (*which is also common to the segment of the same sphere P R Q*) and therefore a line drawne from the toppe of that segment, (which toppe suppose to be Q,) is the semidiameter of the circle, which is equall to the spherical superficies of the sayd solide sector, or segment: as before is taught. Let that line be Q P. By Q draw a line contingent: which let be S Q T. At the poynt Q from the line Q S, cut a line equall to P Q which let be S Q. And vnto S Q, make Q T equall, then draw the right lines O S O T and O Q. About which O Q (as an axe fastened) if you imagine the triangle O S T, to make an *halfe circular reuolution, you shall haue the vpright cone O S T: (whose heith is O Q, the semidiameter of the sphere, and base the circle, whose diameter is S T,) equall to the solide sector P Q R O.



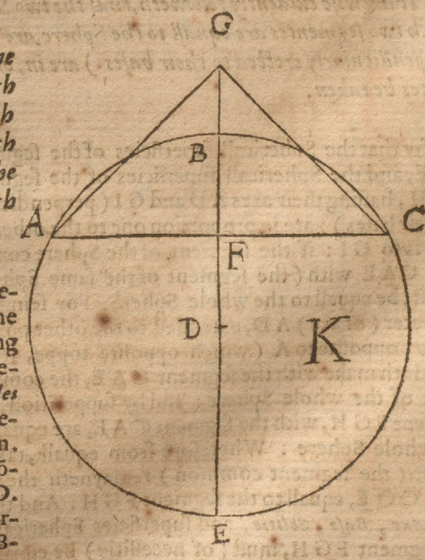
**I say haife a circular reuolution: for that suffiseth in the whole diameter S T, to describe a circle by: if it be moued about his center Q &c.*

A Theoreme. 7.

To any segment, or portion of a Sphere, that cone is equall, which hath that circle to his base, which is the base of the segmẽt, and heith, a right line, which vnto the heith of the segmẽt hath that proportiõ, which the semidiameter of the Sphere, together with the heith of the other segment remayning hath to the heith of the same other segment remayning.

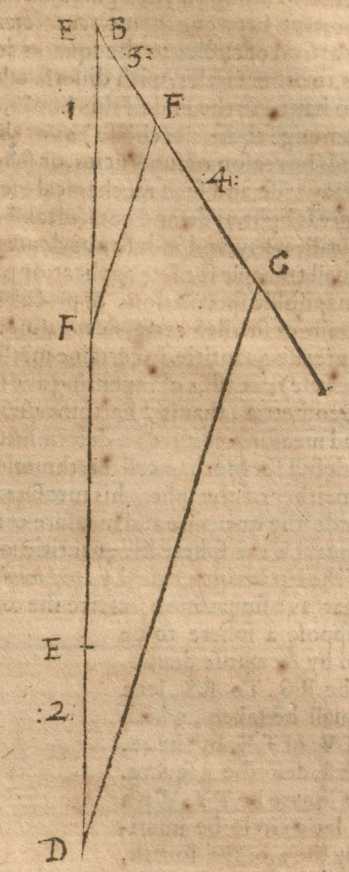
Lib. 2. prop. 2. de Sphera & Cylindro.

This is well demonstrated by *Archimedes* & therefore nedeth no inuentiõ of myne, to confirme the same: and for that the sayd demonstration is ouer long here to be added, I will refere you thether for the demonstration: and here supply that which to *Archimedes* demonstration shall geue light, and to your farther speculation and practise, shal be a great ayde and direction. Suppose K to be a sphere: & the greatest circle K in cõteyned, let be A B C E, and his diameter B E, & céter D. Let the sphere K, be cutte by a playne superficies, perpendicularly erected vpon the sayd greatest circle A B C E: let the section be the circle about A C: And let the



segmentes

ſegmentes of the ſphere be the one that wherein is A-B C, whoſe toppe is B: and the other let be that wherein is A E C and his toppe, let be E: I ſay that a cone which hath his baſe the circle about A C, & heith a line which to B F (the heith of the ſegment, whoſe toppe is B,) hath that proportion that a line compoſed of D E, the ſemidiameter of the ſphere, and E F (the heith of the other remayning ſegment, whoſe toppe is E) hath to E F, (the heith of that other ſegment remayning), is equall to the ſegment of the ſphere K, whoſe toppe is B. To make this cone, take my eaſy order thus. Frame your worke for the finding of the fourth proportionall line: by making E F the firſt: and a line compoſed of D-E and E F, the ſecond: and the third, let be B F: then by the 12. of the ſixth, let the fourth proportionall line be found: which let be F G: vpon F the center of the baſe of the ſegment, whoſe toppe is B, erect a line perpendicular equall to F G found: and drawe the lines G A and G C: and ſo make perfect the cone G A C. I ſay, that the cone G A C, is equall to the ſegment (of the ſphere K) whoſe toppe is B. In like maner, for the other ſegmẽt whoſe toppe is E, to finde the heith due for a cone equal to it: by the order of the Theoreme you muſt thus frame your lines: let the firſt be B F: the ſecond D B and B F, compoſed in one right line, and the third muſt be E F: where by the 12. of the ſixth, finding the fourth, it ſhall be the heith, to rere vpon the baſe, (the circle about A C,) to make an vpright cone, equall to the ſegment, whoſe toppe is E.



Logiſtically.

The Logiſticall finding hereof is moſt eaſy: the diameter of the ſphere being geuen, and the portions of the diameter in the ſegmentes conteyned (or axes of the ſegmentes) being knowne. Then order your numbers in the rule of proportion, as I here haue made moſt playne, in ordring of the lines: for the ſought heith will be the producte.

✣ A Corollary. 1.

Hereby, and other the premiſes it is euident that to any ſegment of a Sphere, whoſe whole diameter is knowne and the Axe of the ſegment geuen, An vpright cone may be made equall: or in any proportion, betwene two right lines aſſigned: and therefore alſo a cylinder may to the ſayd ſegment of the Sphere, be made equall, or in any proportion geuen, betwene two right lines. Note.

A Corollary. 2.

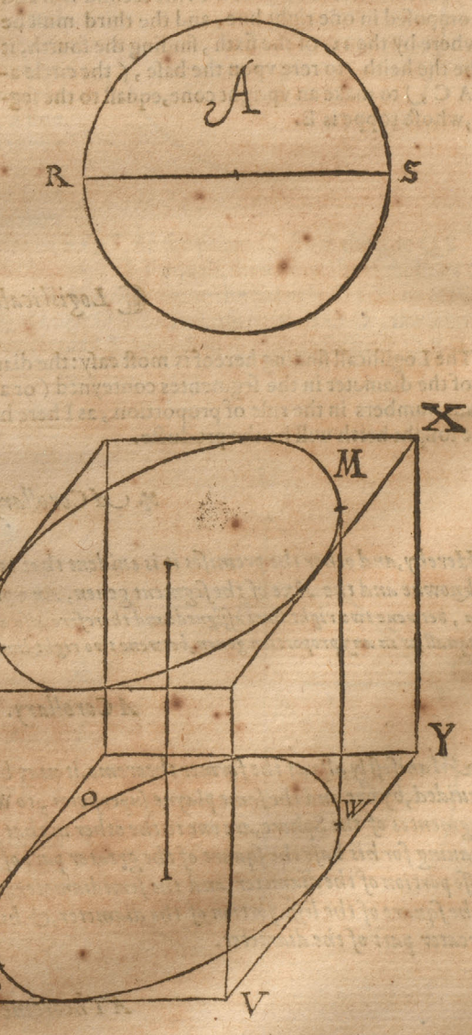
Manifeſtly alſo, of the former theoreme, it may be inferrid that a Sphere, and his diameter being deuided, by one and the ſame playne ſuperficies, to which the ſayd diameter is perpendicular: the two ſegmentes of the Sphere, are one to the other in that proportion, in which a rectangle parallelipipedon hauing for his baſe the ſquare of the greater part of the diameter, and his heith a line compoſed of the leſſe portion of the diameter, and the ſemidiameter: to the rectangle parallelipipedon, hauing for his baſe the ſquare of the leſſe portion of the diameter, & his heith a line compoſed of the ſemidiameter & the greater part of the diameter.

A Theoreme. 8.

Euery Sphere, to the cube, made of his diameter, is (in miner) as 11. *to* 21.

As vpon the first and second propositiõs of this booke, I began my additions with the circle (being the chiefe among playne figures) and therein brought manifold considerations, about circles: as of the proportion betwene their circumferences and their diameters: of the content or Area of circles: of the proportion of circles to the squares described of their diameters: & of circles to be geuen in al proportions, to other circles: with diuerse other most necessary problemes (whose vse is partly there specified): So haue I in the end of this booke, added some such Problemes & Theoremes, about the sphere (being among solides the chiefe) as of the same, either in it selfe considered, or to cone and cylinder, compared (by reason of superficies, or soliditie, in the hole, or in part:) such certaine knowledge demonstratiue may arise, and such mechanical exercise thereby be deuised, that (sure I am) to the sincere & true student great light, ayde, and comfortable courage (farther to wade) will enter into his hart: and to the Mechanicall, witty, and industrous deuiser, new maner of inuentions, & executions in his workes will (with small trauayle for fete application) come to his perceiueraunce and vnderstanding. Therefore, euen as manifolde speculations & practises may be had with the circle, his quantitie being not knowne in any kinde of smallest certayne measure: So likewise of the sphere many Problemes may be executed and his precise quantitie, in certaine measure, not determined, or knowne: yet, becawse, both one of the first (humane) occasiõs of inuenting and stablishing this Arte, was measuring of the earth (and therfore called Geometria, that is, Earthmeasuring), and also the chiefe and generall end (in deede) is measure: and measure requireth a determination of quantitie in a certayne measure by nũber expressed: It was nedefull for Mechanicall earthmeasures, not to be ignorant of the measure and contents of the circle, neither of the sphere his measure and quantitie, as neere as sense can imagine or wish. And (in very deede) the quantitie and measure of the circle, being knowne, maketh not onely, the cone and cylinder, but also the sphere his quantitie to be as precisely knowne, and certayne. Therefore seing in respect of the circles quantitie (by *Archimedes* specified) this Theoreme is noted vnto you: I wil, by order, vpon that (as a suppositiõn) inferre the conclusion of this our Theoremes.

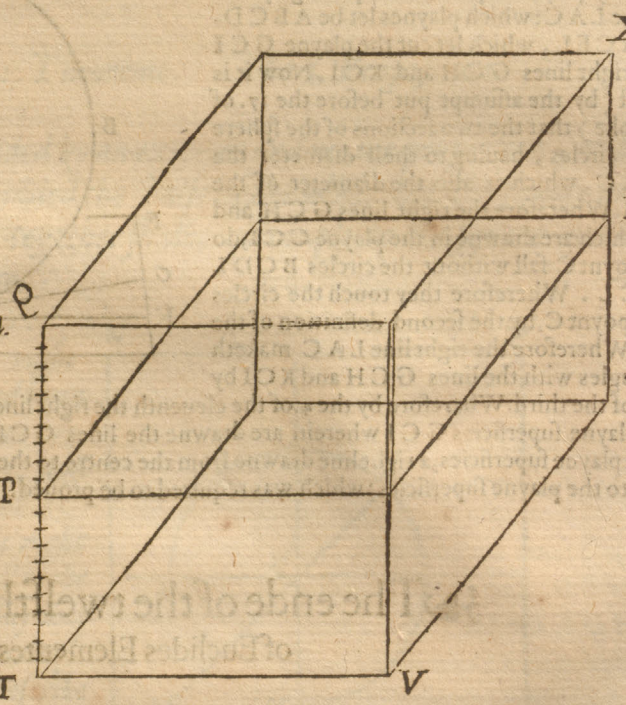
Suppose a sphere to be signified by A: whose diameter let be R S. To R S, let a line equall be taken, which let be T V: of T V, by the 46. of the first, describe a square. Let that square be T Y. With in T Y let a circle be inscribed: by the 2. of the fourth, which circle suppose to be O Z W. That O Z W is equall to the greatest circle in the sphere A conteyned, it is euident by the diameter, equal to T V. If vpõ the square T Y, as a base, be erected, a parallelipipedõ rectãgle, whose heith is equall to T V, it is euident that that parallelipipedon is a cube. Which let be done: and that cube produced, let be noted by T X. Likewise, if vpon the circle O Z W, as a base, and of the heith equall to the line T V, a cylinder be erected, it is manifest that the cylinder hath his base equall to the greatest circle, in the sphere A, conteined: & heith, a line equall to the diameter of the same sphere A. Which cylinder let be produced and noted by Z M. I say now that the sphere A, is to the cube T X, (in maner) as the number 11. is to the number 21. For seing the cube T X, was produced of his base, (the square T Y), being brought into the heith of a line equall to T V: & likewise seing the cylinder Z M,



is produced, of his baſe (the circle O Z W) being brought into a line, equal to the ſaid T V: it followeth, ſeing their heithes is all one, that the cube T X ſhall be to the cylinder Z M, as the baſe of T X, (which is the ſquare T Y) is to the baſe of Z M, that is the circle O Z W. But the ſquare T Y, is to the circle O Z W, as the number 14. is to the nũber 11. (in maner), by *Archimedes* demõſtratiõ: wherfore, the cube T X is to the cylinder Z M, as the number 14, is to the number 11. (well nere). And by my third Theoreme (here added) the cylinder Z M, is to the Sphere A, in ſeſquialtera proportion: that is, as 3. to 2. Wherefore the cylinder Z M, hauing the ſame 11. equall partes (which he conteyneth in reſpect of the cube T X, being 14. of the ſame partes) deuided into 3. equall portions, euery one of thoſe portions is $3\frac{2}{3}$. And allowing to the Sphere A, two of thoſe portions: it is euident, that the Sphere A ſhall be $7\frac{1}{3}$ ſuch partes as are 14. in the cube T X: and 11. in the cylinder Z M. Wherefore the Sphere A, is to the cube T X, as $7\frac{1}{3}$ to 14. The fraction being reduced, maketh $\frac{22}{3}$: and the number 14. being brought to the ſame name, and denomination of thirds, maketh $\frac{42}{3}$. Put away now theyr common denominator: and then remayneth, for the Sphere A, 22. ſuch partes, as the cube T X hath 42. And then depreſſing them, to the ſmalleſt termes: for the Sphere A, you ſhall haue 11. ſuch partes as the cube T X conteyneth 21. Wherefore euery Sphere, to the cube made of his diameter is, as 11. to 21. which was requiſite to be demonſtrated.

Note. 1.

Wherfore if you deuide the one ſide (as T Q) of the cube T X into 21. equall partes, and where 11. partes do end, reckening from T, ſuppoſe the point P: and by that point P, imagine a plaine (paſſing parallel to the oppoſite baſes) to cut the cube T X: and therby, the cube T X, to be deuided into two rectangle parallelipipedons, namely, T N, and P X: It is manifeſt, * T N, to be equall to the Sphere A, by conſtruction: and the 7. of the fift. 21.



* *A rectangle parallelipipedon geuen, equall to a Sphere geuen.*

Note. 2.

Secondly, the whole quantitie, of the Sphere A, being cõtayned in the rectangle parallelipipedon T N, you may eaſilie tranſforme the ſame quantitie, into other parallelipipedons rectangles, of what height, and of what parallelogramme baſe you liſt: by my firſt and ſecond Problemes vpon the 34. of this booke. And the like may you do, to any aſſigned part of the Sphere A: by the like meanes deuiding the parallelipipedon T N: as the part aſſigned doth require. As if a third, fourth, fifth, or ſixth, part of the Sphere A, were to be had in a parallelipipedon, of any parallelogramme baſe aſſigned, or of any heith aſſigned: then deuiding T P, into ſo many partes (as into 4. if a fourth part be, to be tranſformed: or into fiue, if a fifth part, be to be tranſformed &c.) and then proceede, as you did with cutting of T N, from T X. And that I ſay of parallelipipedons, may in like ſort (by my ſayd two problemes, added to the 34 of this booke) be done in any ſided columnes, pyramids, and priſmes: ſo that in pyramids and ſome priſmes you vſe the cautions neceſſary, in reſpect of their quantities, compared to the quantities of bodyes hauing parallel, equall, and oppoſite baſes: whoſe partes thoſe pyramids or priſmes are: as before in their propoſitions, is by *Euclide* demonſtrated. And finally, ſeing, in theſe preſent additions, you haue the wayes and orders how to geue to a Sphere, or any ſegment of the ſame, Cones, or Cylinders equall, or in any proportion betwene two right lines, geuen: with many other moſt neceſſary ſpeculations and practiſes about the Sphere: I truſt that I haue ſufficiently fraughted your imagination, for your honeſt and profitable ſtudie herein, and alſo geuen you ready matter, wherewith to ſtop the mouthes of the malycious, ignorant,

To a Sphere, or to any part of a Sphere aſſigned: as a third, fourth, fifth &c. to geue a parallelipipedon equall.

Sided Columes Pyramids, and priſmes to be geuen equall to a Sphere, or to any certayne part thereof.

To a Sphere or any ſegment, or ſector of the ſame, to geue a cone or cylinder equall or in any proportion aſſigned.

Farther vse of Spherical Geometrie.

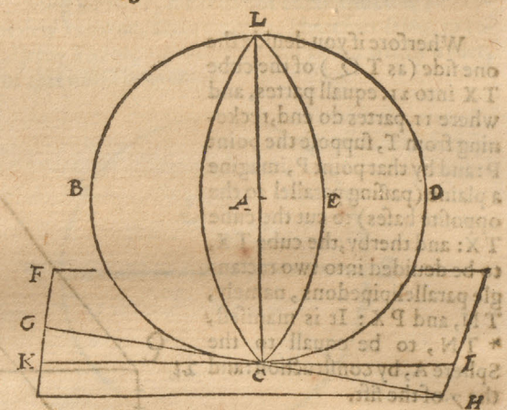
rant, and arrogant, despisers of the most excellent discourses, trauayles, and inuentions mathematicall. Seing aswel the heauenly spheres, & sterres their spherical soliditie, with their conuexe spherical superficies, to the earth at all times respecting, and their distances from the earth, as also the whole earthly Sphere and globe it selfe, and infinite other cases, concerning Spheres or globes, may hereby with as much ease and certainety be determined of, as of the quantitie of any bowle, ball, or bullet, which we may gripe in our handes (reason, and experience, being our witnesses): and without these aydes, such thinges of importance neuer hable of vs, certainely to be knowne, or attayned vnto.

Here ende *M. Iohn Dee* his additions vpon the last proposition of the twelfth booke.

A proposition added by *Flussas*.

If a Sphere touche a playne superficies: a right line drawne from the center to the touche, shall be erected perpendicularly to the playne superficies.

Suppose that there be a Sphere B C D L: whose centre let be the poynt A. And let the playne superficies G C I touch the Spere in the poynt C, and extend a right line from the centre A to the poynt C. Then I say that the line A C is erected perpendicularly to the playne G I C. Let the sphere be cutte by playne superficieces passing by the right line L A C: which playnes let be A B C D-L and A C E L, which let cut the playne G C I by the right lines G C H and K C I. Now it is manifest (by the assumpt put before the 17. of this booke) that the two sections of the sphere shall be circles, hauing to their diameter the line L A C, which is also the diameter of the sphere. Wherefore the right lines G C H and K C I which are drawne in the playne G C I, do at the poynt C fall without the circles B C D L and E C L. Wherefore they touch the circles in the poynt C, by the second definition of the third. Wherefore the right line L A C maketh right angles with the lines G C H and K C I by the 16. of the third. Wherefore by the 4. of the eleuenth the right line A C is erected perpendicularly to to the playne superficies G C I wherein are drawne the lines G C H and K C I. If therefore a Sphere touch a playne superficies, a right line drawne from the centre to the touche, shall be erected perpendicularly to the playne superficies: which was required to be proued.



The ende of the twelfth booke of Euclides Elementes.

¶ The thirtenth booke of Euclides Elementes.

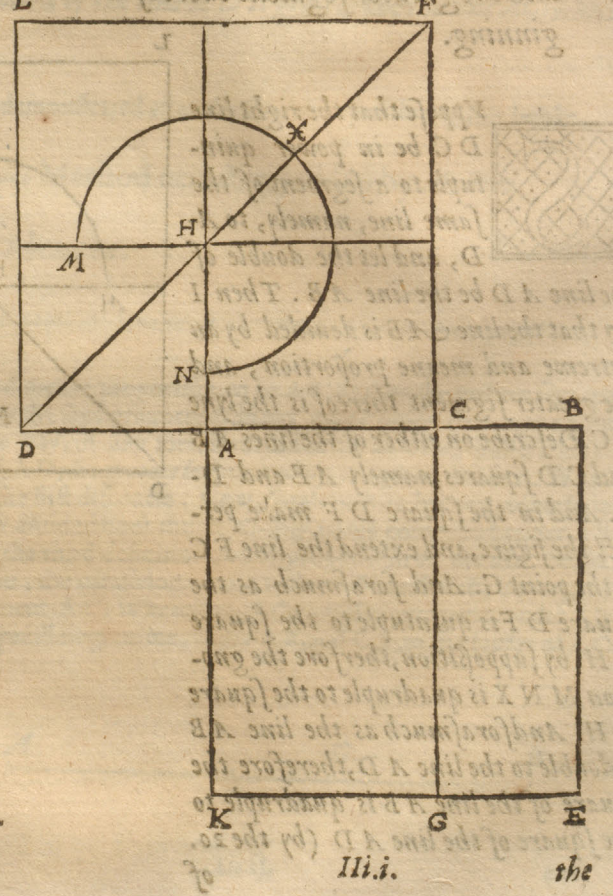
IN THIS THIRTENTH BOOKE are ſet forth certayne moſt wonderfull and excellent paſsions of a lyne deuided by an extreme and meane proportion: a matter vndoubtedly of great and infinite vſe in Geometry, as ye ſhall both in thys booke, and in the other bookes following moſt euidently perceaue. It teacheth moreouer the compoſition of the fiue regular ſolides, and how to inſcribe them in a Sphere geuen, and alſo ſetteth forth certayne compariſons of the ſayd bodyes both the one to the other, and alſo to the Sphere, wherein they are deſcribed.

The argument of the thirtenth booke.

The 1. Theoreme. The 1. Propoſition.

If a right line be deuided by an extreme and meane proportion, and to the greater ſegment, be added the halfe of the whole line: the ſquare made of thoſe two lines added together ſhalbe quintuple to the ſquare made of the halfe of the whole lyne.

SVppoſe that the right line A B be deuided by an extreme and meane proportiõ in the point C. And let the greater ſegment therof, be A C. And vnto A C, adde directly a ryght line A D, and let A D be equall to the halfe of the line A B. Then I ſay that the ſquare of the line C D is quintuple to the ſquare of the line D A. Deſcribe (by the 46. of the firſt) vpon the lines A B and D C ſquares, namely, A E & D F. And in the ſquare D F, deſcribe and make complete the figure. And extend the line F C, to the point G. And foraſmuch as the line A B is deuided by an extreme and meane proportion in the point C, therefore that which is contayned vnder the lines A B and B C is equall to the ſquare of the line A C. But that which is contayned vnder the lines A B and B C, is the parallelogramme C E, and the ſquare of



Conſtruction.

Demonſtration.

the line A C is the ſquare H F. Wherefore the parallelogramme C E is equall to the ſquare H F. And foraſmuch as the line B A, is double to the line A D, by conſtruction: but the lyne B A is equall to the line K A, and the line A D, to the lyne A H: therefore alſo, the lyne K A, is double to the line A H. But as the lyne K A is to the line A H, ſo is the parallelogramme C-K to the parallelogramme C H: Wherefore the parallelogramme C K is double to the parallelogramme C H. And the parallelogrammes L H and C H are double to the parallelogramme C H (for ſupplementes of parallelogrammes are by the 43. of the firſt equall the one to the other). Wherefore the parallelogramme C K is equall to the parallelogrammes L H & C H. And it is proued that the parallelogramme C E is equall to the ſquare F H. Wherefore the whole ſquare A E is equall to the gnomon M X N. And foraſmuch as the line B A, is double to the line A D, therefore the ſquare of the line B A is, by the 20. of the ſixth, quadruple to the ſquare of the line D A, that is, the ſquare A E to the ſquare D H. But the ſquare A E is equall to the gnomō M X N, wherefore the gnomō M X N, is alſo quadruple to the ſquare D H. Wherefore the whole ſquare D F is quintuple to the ſquare D H. But the ſquare D F, is the ſquare of the line C D, and the ſquare D H is the ſquare of the line D A. Wherefore the ſquare of the line C D, is quintuple to the ſquare of the line D A. If therefore a right line be deuided by an extreame and meane proportion, and to the greater ſegment, be added the halfe of the whole line: the ſquare made of thoſe two lines added together ſhalbe quintuple to the ſquare made of the halfe of the whole line: Which was required to be demonſtrated.

Thys propoſition is an other way demonſtrated after the ſiueth propoſition of this booke.

The 2. Theoreme. The 2. Propoſition.

If a right line, be in power quintuple to a ſegment of the ſame line: the double of the ſayd ſegment is deuided by an extreame and meane proportion, and the greater ſegment thereof is the other part of the line geuen at the beginning.

This propoſition is the conuerſe of the former.

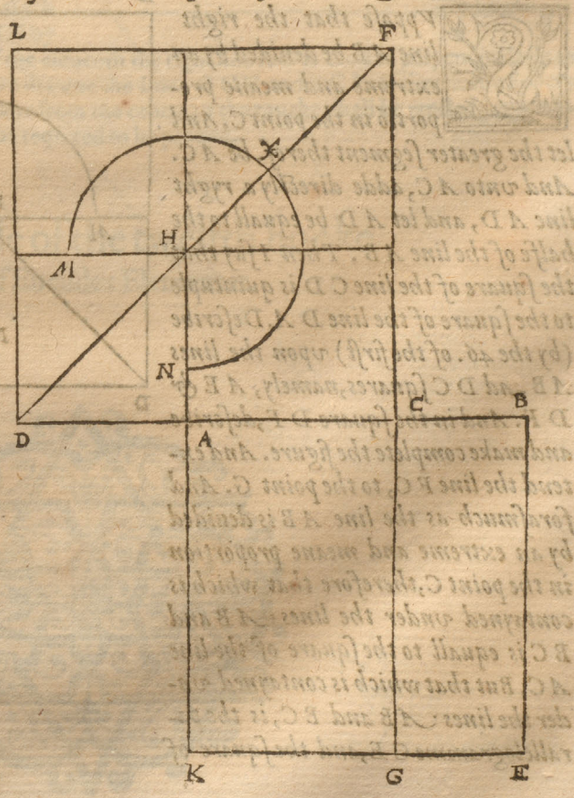
VPpoſe that the right line D C be in power quintuple to a ſegment of the ſame line, namely, to A-D, and let the double of the line A D be the line A B. Then I ſay that the line A B is deuided by an extreme and meane proportion, and the greater ſegment thereof is the lyne A C.

Conſtruction.

Deſcribe on either of the lines A B and C D ſquares, namely A E and D-F. And in the ſquare D F make perfect the figure, and extend the line F C to the point G.

Demonſtration.

And foraſmuch as the ſquare D F is quintuple to the ſquare D H, by ſuppoſition, therfore the gnomon M N X is quadruple to the ſquare D H. And foraſmuch as the line A B is double to the line A D, therefore the ſquare of the line A B is quadruple to the ſquare of the line A D (by the 20. of



*of the ſixt), that is, the ſquare A E to the ſquare D H. And it is proued that the gnomen M. N X is quadruple to the ſquare D H. Wherefore the gnomon M N X is equall to the ſquare A E. And foraſmuch as the line A B is double to the line A D, but the line A B is equall to the line A K, and the line A D to the line A H: therefore the line A K is double to the line A. H: wherefore alſo (by the firſt of the ſixth) the parallelogramme A G is double to the parallelogramme C H. But the parallelogrammes L H and C H are double to the parallelogramme C H (by the 43. of the firſt): wherefore the parallelogramme A G is equall to the parallelogrammes L H and C H. And it is proued that the whole gnomon M N X is equall to the whole ſquare A E. Wherefore the reſidue H F is equall to the parallelogramme C E. And C E is that which is contained vnder the lines A B and C B, for the line A B is equall to the line B E, and H F is the ſquare made of the line A C. Wherefore that which is contayned vnder the lines A B and B C, is equall to the ſquare of the line A C. Wherfore as the line A B is to the line A C, ſo is the line A C to the line C B. * But the line A B is greater then the line A C, wherefore the line A C is greater then the line C B. Wherefore the line A B is deuided by an extreme and meane proportion, and the greater ſegment thereof is the line A C. If therfore a right line be in power quintuple to a ſegment of the ſame line, the double of the ſayd ſegment is deuided by an extreame & meane proportion, and the greater ſegment thereof is the other part of the line geuen at the beginning: Which was required to be proued.*

** An Aſſũpt.*

** Now, that the double of the line A D (that is A B) is greater then the line A C may thus be proued. For if not, then if if it be poſsible let the line A C be double to the line A D, wherefore the ſquare of the line A C is quadruple to the ſquare of the line A D. Wherefore the ſquares of the lines A C and A D are quintuple to the ſquare of the line A D. And it is ſuppoſed that the ſquare of the line D C is quintuple to the ſquare of the line A D, wherefore the ſquare of the line D C is equall to the ſquares of the lines A C and A D: which is impoſſible (by the 4. of the ſecond). Wherefore the line A C is not double to the line A D. In like ſorte alſo may we proue that the double of the line A D is not leſſe then the line A C, for this is much more abſurd: wherefore the double of the line A D is greater thẽ the line AC: which was required to be proued.*

** The Aſſũpt proued.*

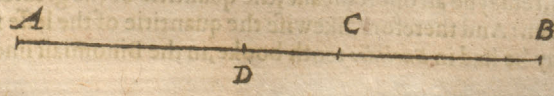
This propoſition alſo is an other way demonſtrated after the ſiueth propoſition of this booke.

Two Theoremes, (in *Euclides* Method neceſſary) added by *M. Dee.*

A Theoreme. 1.

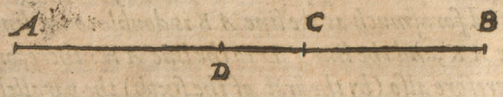
A right line can be deuided by an extreame and meane proportion, but in one onely poynt.

Suppoſe a line diuided by extreame and meane proportion, to be A B. And let the greater ſegment be A C. I ſay, that A B can not be deuided by the ſayd proportion, in any other point then in the point C. If an aduerſary woulde contend that it may, in like ſort, be deuided in an other point: let his other point, be ſuppoſed to be D: making A D, the greater ſegment of his imagined diuiſion. Which A D, alſo, let be leſſe then our A C: for the firſt diſcourſe. Now, foraſmuch as by our aduerſaries opinion, A D, is the greater ſegment, of his diuided line: the parallelogramme conteyned vnder A B, and D B, is equall to the ſquare of A D, by the third definition and 17. propoſition of the ſixth Booke. And by the ſame definition and propoſition, the parallelogramme vnder A B, and C B, conteyned, is equall to the ſquare of our greater ſegment A C. Wherefore, as the parallelogramme, vnder A B, and D B, is to the ſquare of A D: ſo is the parallelogramme, vnder A B, and C B, to the ſquare of A C. For proportion of equality, is concluded in them both. But, foraſmuch as D B, is (by *ſuppoſition) greater thẽ C B, the parallelogrãme vnder A B, and D B, is greater then the parallelogramme vnder A C,



** Becauſe A C is ſuppoſed greater then A D: therefore his reſidue is leſſe, then the reſidue of A D, by the common ſentence. Wherefore, by the ſuppoſition: D B is greater then B C.*

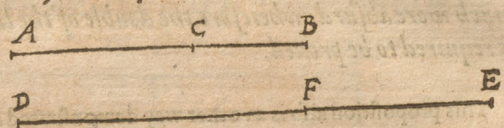
and C B: by the firſt of the ſixth (for A B is their equall heith.) Wherefore, the ſquare of A D, ſhalbe greater then the ſquare of A C: by the 14. of the fifth. But the line A D, is leſſe then the line A C, by ſuppoſition: wherefore the ſquare of A D is leſſe then the ſquare of A C. And it is concluded alſo to be greater then the ſquare of A C: Wherefore the ſquare of A D, is both greater, then the ſquare of A C: and alſo leſſe. Which is a thing impoſſible. The ſquare therefore of A D, is not equall to the parallelogramme vnder A B, and D B. And therefore by the third definition of the ſixth, A B is not deuided by an extreame and meane proportion, in the point D: as our aduerſary imagined. And (Secondly) in like ſort will the inconueniency fall out: if we aſſigne A D, our aduerſaries greater ſegment, to be greater then our A C. Therefore ſeing neither on the one ſide of our point C: neither on the other ſide of the ſame point C, any point can behad, at which the line A B can be deuided by an extreame and meane proportion, it followeth of neceſſitie, that A B can be deuided by an extreame and meane proportion in the point C, onely. Therefore, a right line can be deuided by an extreame and meane proportion, but in one, onely point: which was requiſite to be demonſtrated.



A Theoreme. 2.

What right line ſo euer, being deuided into two partes, hath thoſe his two partes, proportionall, to the two ſegmentes of a line deuided by extreame and meane proportion: is alſo it ſelfe deuided by an extreame and meane proportion: and thoſe his two partes, are his two ſegments, of the ſayd proportion.

Suppoſe, A B, to be a line deuided by an extreame and meane proportion in the point C, and A C to be the greater ſegment. Suppoſe alſo the right line D E, to be deuided into two partes, in the point F: and that the part D F, is to F E, as the ſegment A C, is to C B: or D F, to be, to A C, as F E is to C B. For ſo theſe partes are proportionall, to the ſayd ſegmentes. I ſay now, that D E is alſo deuided by an extreame and meane proportion in the point F. And that D F, F E, are his ſegmentes of the ſayd proportion. For, ſeing, as A C, is to C B: ſo is D F, to F E: (by ſuppoſition). Therfore, as A C, and C B (which is A B) are to C B: ſo is D F, and F E, (which is D E) to F E: by the 18. of the fifth. Wherefore (alternately) as A B is to D E: ſo is C B, to F E. And therefore, the reſidue A C, is to the reſidue D F, as A B is to D E, by the fifth of the fift. And then alternately, A C is to A B, as D E, is to D F. Now therefore backward, A B is to A C, as D E is to D F. But as A B is to A C, ſo is A C to C B: by the third definition of the ſixth booke. Wherefore D E is to D F, as A C is to C B: by the 11. of the fifth. And by ſuppoſition, as A C is to C B, ſo is D F to F E: wherefore by the 11. of the fifth, as D E is to D F: ſo is D F to F E. Wherefore by the 3. definition of the ſixth, D E is deuided by an extreame and meane proportion, in the point F. Wherefore D F, and F E are the ſegmentes of the ſayd proportion. Therefore, what right line ſo euer, being deuided into two partes, hath thoſe his two partes, proportionall to the two ſegmentes of a line deuided by extreame and meane proportion: is alſo it ſelfe deuided by an extreme and meane proportion, and thoſe his two partes are his two ſegmentes, of the ſayd proportion: which was requiſite to be demonſtrated.



Note.

The chiefe line in all Euclides Geometrie. What is ment here by, A ſection in one onely point.

Many wayes, theſe two Theoremes, may be demonſtrated: which I leaue to the exerciſe of young ſtudentes. But vtterly to want theſe two Theoremes, and their demonſtrations: in ſo principall a line, or rather the chiefe piller of *Euclides* Geometricall pallace, was hetherto, (and ſo would remayne) a great diſgrace. Alſo I thinke it good to note vnto you, what we meane, *by one onely poynt*. We meane, that the quantities of the two ſegmentes, can not be altered, the whole line being once geuen. And though, from either end of the whole line, the greater ſegment may begin: And ſo as it were the point of ſection may ſeeme to be altered: yet with vs, that is no alteration: foraſmuch as the quantities of the ſegmentes, remayne all one. I meane, the quantitie of the greater ſegment, is all one: at which end ſo euer it be taken: And therefore, likewiſe the quantitie of the leſſe ſegment is all one. &c. The like conſideration may be had in *Euclides* tenth booke, in the Binomiall lines, &c.

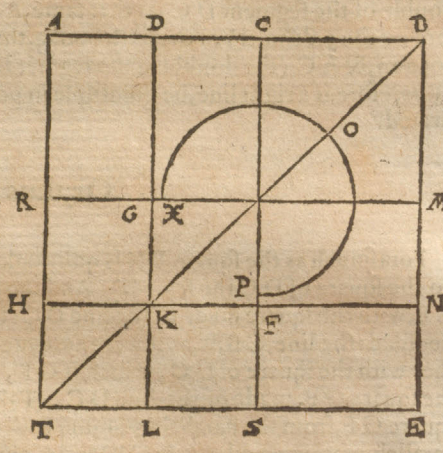
Iohn Dee. 1569. Decemb. 18.

The 3. Theoreme. The 3. Proposition.

If a right line be deuided by an extreme and meane proportion, and to the leſse ſegment be added the halfe of the gerater ſegment: the ſquare made of thoſe two lines added together, is quintuple to the ſquare made of the half line of the greater ſegment.

*SVppoſe that the right line A B be deuided by an extreme and meane proportion in the point C. And let the greater ſegment thereof be A C. And deuide A C into two equall partes in the point D. Then I ſay that the ſquare of the line B D, is quintuple to the ſquare of the line D C. Deſcribe (by the 46. of the firſt) vpon the line A B a ſquare A E. And deſcribe and make perfect the figure (that is diuide the lyne A T like vnto the diuiſion of the line AB, by the 10. of the ſixth, in the pointes R, H, by which pointes drawe (by the 31. of the firſt) vnto the line A B parallel lines R M and H N. So likewiſe draw by the pointes D, C, vnto the line B E theſe parallel lines D L and C S, & draw the diameter B T). And foraſmuch as the line A C is double to the line D C, therefore the ſquare of A C, is quadruple to the ſquare of D C, by the 20. of the ſixth, that is, the ſquare R S to the ſquare F G. And foraſmuch as that which is contayned vnder the lines A B and B C is equall to the ſquare of the line A C, and that which is contayned vnder the lines A B and B C is equall to the parallelogramme C E, & the ſquare of the line A C is the ſquare R S: wherefore the parallelogramme C E is equall to the ſquare R S. But the ſquare R S is quadruple to the ſquare F G: wherefore the parallelogramme C E alſo is quadruple to the ſquare F G. Agayne foraſmuch as the line A D is equall to the line D C, therfore the line H K is equall the line K F, wherefore alſo the ſquare G F is equall to the ſquare H L: wherefore the line G K is equall to the line K L, that is, the line M N to the line N E: wherefore the parallelogramme M F is equall to the parallelogramme F E. But the parallelogramme M F is equall to the parallelogramme C G, wherfore the parallelogramme C G is alſo equal to the parallelogramme F E. Put the parallelogramme C N, common to the both: * Wherefore the gnomon X O P is equall to the parallelogramme C E. But the parallelogramme C E is proued to be quadruple to G F the ſquare, wherefore the gnomon X O P is quadruple to the ſquare G F. Wherefore the ſquare D N is quintuple to the ſquare F G. And D N is the ſquare of the line D B, and G F the ſquare of the line D C. Wherefore the ſquare of the line D B is quintuple to the ſquare of the line D C. If therefore a right line be deuided by an extreme and meane proportion, and to the leſſe ſegment be added the halfe of the greater ſegment: the ſquare made of thoſe two lines added together, is quintuple to the ſquare made of the halfe line of the greater ſegmẽt. Which was required to be demonſtrated.*

Conſtruction.



Demonſtration.

*Note, how C E and the gnomon X O P, are proued equall, for it ſerueth in the conuerſe demonſtrated by M. Dee, here next after.

Ye ſhall finde this propoſition an other way demonſtrated after the fiueth propoſition of this booke.

Here foloweth M. Dee, his additions.

¶ A Theoreme. 1.

If a right line, geuen, be quintuple in powr, to the powre of a ſegment of him ſelf: the double of that

This proposition, is the conuerse of the former.

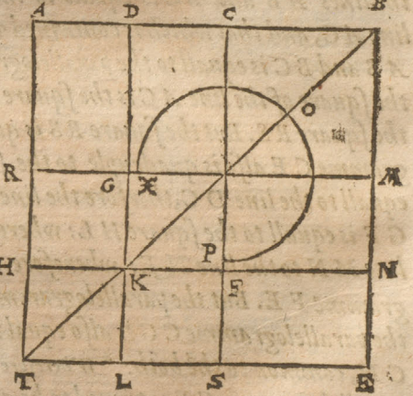
segment, and the other part remayning, of the first geuen line, make a line, diuided by extreme and meane proportion: and that double of the segment, is the greater part thereof.

Forasmuch as, this, is the conuerse of Euclides third proposition: we will vse the same suppositions and constructions there specified: so farre, as they shall serue our purpose. Beginning therefore at the conclusion, we must infer the part of the proposition, before graunted. It was concluded, that the square of the line D B, is quintuple, to the square of the line D C, his owne segment. Therefore D N (the square of D B) is quintuple, to G F, (the square of D C). But the square of A C (the double of D C) which is R S, is quadruple to G F, (by the second Corollary of the 20. of the sixth): and therefore R S, with G F, are quintuple to G F: and so it is euident, that the square D N, is equall to the square R S, together with the square G F. Wherefore, from those two equalles, taking, the square G F, (common to them both): remayneth the square R S, equall to the Gnomon X O P. But to the Gnomon X O P, the parallelogramme * C E, is equall: Wherefore the square of the line A C, which is R S, is equall to the parallelogrãme C E. Which parallelogamme is cõtained vnder B E, (equall to A B:) and C B, the part remayning of the first line geuen: which was D B. And the line A B, is made of the double of the segment D C, and of C B, the other part of the line D B, first geuen. Wherefore the double of the segment D C, with C B, the part remayning (which altogether, is the whole line A B) is to A C, (the double of the segment D C) as that same, A C, is to C B: by the second part of the 16. of the sixth. Therfore by the 3. definitiõ of the sixth booke, the whole line A B, is deuided by an extreme and meane proportion, & A C, (the double of the segmẽt D C) being middell proportionall, is the greater part therof. Wherefore, if a right line, be quintuple in power, &c. (as in the proposition) which was to be demonstrated.

** As we haue noted the place of the peculer profe therof in the demõtration of the.*

Or, thus it may be demonstrated.

Forasmuch as the square, DN is quintuple to the square GF, (I meane the square of DB the line geuẽ, to the square of D C the segmẽt): And the same square D N, is equall to the parallelogrãme vnder A B, C B, with the square made of the line D C: by the sixth of the second: (for vnto the line A C, equally deuided: the line, C B, is, as it were adioyned). Wherefore the parallelogramme vnder A B, C B, together with the square of D C, which is G F, is quintuple to the square G F, made of the line D C. Taking then, that square G F, from the parallelogramme vnder A B, C B: that parallelogramme (vnder A B, C B) remayning alone, is but quadruple to the sayd square of the line D C. But, (by the 4. of the second, or the second Corollary of the 20. of the sixth) R S, the square of the line A C, is quadrupla to the same square G F. Wherfore by the 7. of the fifth, the square of the line A C, is equall to the parallelogramme vnder A-B, C B, and so, by the second part of the 16. of the sixth: A-B, A C, and C B, are three lines in continuall proportion. And seing A B is greater thẽ A C, the same A C, the double of the line D C, shall be greater then the part B C, remayning: Wherfore by the 3. definition of the sixth, A B, (composed or made of the double of D C, and the other part of D B remaining) is deuided by an extreme and middel proportion: and also his greater segment is A C the double of the segment D C. Wherfore, If a right line be quintuple in power &c. as in the proposition: which was to be demonstrated.



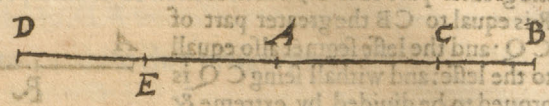
✻ A Theoreme. 2.

If a right line, deuided by an extreme and meane proportion, be geuen, and to the great segment therof, be directly adioyned a line equal to the whole line geuen; that adioyned line, and the said greater segment, do make a line diuided by extreme and meane proportion, whose greater segment is the line adioyned.

Suppose the line geuen, deuided by extreame and meane proportion, to be A B deuided in the point C, and his greater segment, let be A C: vnto A C directly adioyne a line equall to A B: let that be A D: I say, that A D, together with A C, (that is D C) is a deuided by extreme and middel proportion, whose greater segment is A D, the line adioyned. Deuide A D, equally in the point E. Now, forasmuch as A E, is the halfe of A D, (by construction,) it is also, the halfe of A B (equall, to A D, by construction): Wherfore by the 1. of the thirteenth, the square of the line composed of A C and A E (which line is E C) is quintuple to the square of the line A E. Wherefore the double of A E, and the line A C

composed

compoſed, (as in one right line) is a line deuided by extreme and meane proportion, by the conuerſe of this third (by me demonſtrated): and the double of A E, is the greater ſegment. But D C is the line compoſed of the double of A E, & the line A C: and with all, A D is the double of A E. Wherfore, D C, is a line deuided by extreme and meane proportion, and A D, is his greater ſegment. If a right line, therefore, deuided by extreme and meane proportion, be geuen, and to the greater ſegment thereof, be directly adioyned a line equall to the whole line geuen, that adioyned line, and the ſayd greater ſegment, do make a line diuided by extreame and meane proportion, whoſe greater ſegment, is the line adioyned: Which was required to be demonſtrated.



Two other briefe demonſtrations of the ſame.

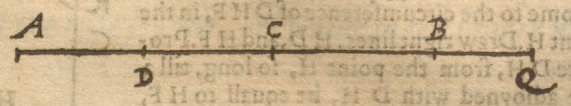
Foraſmuch as, A D is to A C: as A B, is to A C (becauſe A D is equall to A B, by conſtruction): but as A B is to A C, ſo is A C to C B: by ſuppoſition. Therefore by the 11. of the fifth, as A C, is to C B, ſo is A D to A C.* Wherefore, as A C, and C B, (which is A B) is to C B: ſo is A D, and A C (which is D C) to A C. Therefore, euerſedly, as A B, is to A C: ſo is D C to A D. And it is proued, A D, to be to A C: as A C is to C B. Wherefore as A B is to A C, and A C, to C B: ſo is D C, to A D, and A D, to A C. But A B, A C, and C B are in continuall proportion, by ſuppoſition: Wherfore D C, A D, and A C, are in continuall proportion. Wherefore, by the 3. definition of the ſixth booke, D C, is deuided by extreme and middell proportion, and his greateſt ſegment, is A D. Which was to be demonſtrated. Note from the marke *, how this hath two demonſtrations. One I haue ſet in the margent by.

** Therefore, by my ſecond Theoreme added vpon the ſecond propoſition. D C is deuided by extreame and meane proportion in the point A. And becauſe A C is bigger then C B: therfore D A is greater then A C: wherefire if a right line &c. as in the propoſition. Which was to be demonſtrated.*

¶ *A Corollary.* 1.

Vpon Euclides third propoſition demonſtrated, it is made euident: that, of a line deuided by extreame and meane proportion, if you produce the leſſe ſegment, equally to the length of the greater: the line therby adioyned, together with the ſayd leſſe ſegment, make a new line deuided by extreame and middle proportion: Whoſe leſſe ſegment, is the line adioyned.

For, if A B, be deuided by extreme and middell proportion in the point C, A C, being the greater ſegment, and C B be produced, from the poynt B, making a line, with C B, equall to A C, which let be C Q: and the line thereby adioyned, let be B Q: I ſay that C Q, is a line alſo deuided by an extreame and meane proportion, in the point B: and that B Q (the line adioyned) is the leſſe ſegment. For by the thirde, it is proued, that halfe A C, (which, let be, C D) with C B, as one line, compoſed, hath his powre or ſquare, quintuple to the powre of the ſegment C D: Wherfore, by the ſecond of this booke, the double of C D, is deuided by extreme and middell proportion · and the greater ſegment thereof, ſhalbe C B. But, by conſtruction, C Q, is the double of C D, for it is equall to A C. Wherefore C Q is deuided by extreme and middle proportion, in the point B: and the greater ſegment thereof ſhalbe, C B. Wherefore B Q, is the leſſe ſegment, which is the line adioyned. Therefore, a line being deuided, by extreme and middell proportion, if the leſſe ſegment, be produced equally to the length of the greater ſegment, the line thereby adioyned together with the ſayd leſſe ſegment, make a new line deuided, by extreme & meane proportion, whoſe leſſe ſegment, is the line adioyned. Which was to be demonſtrated.

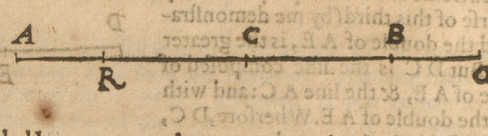


¶ *A Corollary.* 2.

If from the greater ſegment, of a line diuided, by extreme and middle proportion, a line, equall to the leſſe ſegment be cut of: the greater ſegment, thereby, is alſo deuided by extreme and meane proportion, whoſe greater ſegment, ſhall be now that part of it, which is cut of.

For, taking from A C, a line equall to C B: let A R remayne. I ſay, that A C, is deuided by an extreme and meane proportion in the point R: and that C R, the line cut of, is the greater ſegment. For it is proued in the former Corollary that C Q is deuided by extreme and meane proportion in the point B. But A C, is equall to C Q, by conſtruction: and C R is equall to C B by conſtruction: Wherefore the

residue, A R is equall to B Q the residue. Seing therfore the whole A C is equall to the whole C Q: and the greater part of A C, which is C-R is equal to C B the greater part of C Q: and the lesse segmẽt also equall to the lesse: and withall seing C Q is proued to be diuided by extreme & meane proportion in the point B, it



foloweth of necessity, that, A C, is diuided by extreme and meane proportion in the point R. And seing C B, is the greater segment of C Q: C R shall be the greater segment of A C. Which was to be demonstrated.

❊ *A Corollary.* 3.

It is euident thereby, a line being diuided by extreme and meane proportion, that the line whereby the greater segment excedeth the lesse, together with the lesse segment, do make a line diuided by extreme and meane proportion: whose lesse segment, is the sayd line of exceesse, or difference betwene the segments.

Iohn Dee.

¶ *Two new wayes, to deuide any right line geuen by an extreme and meane proportion: demonstrated and added by M. Dee.*

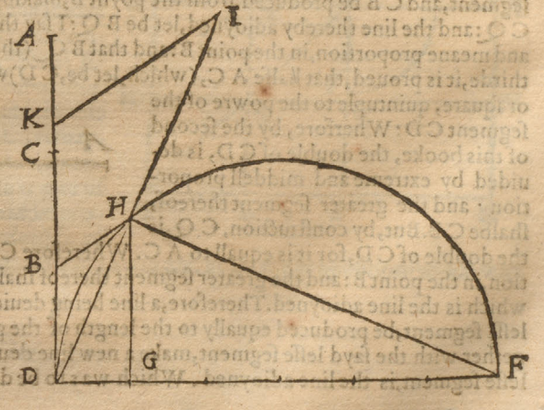
A Probleme.

To deuide by an extreme and meane proportion, any right line geuen, in length and position.

Construction.

**Though I say, perpẽdicular: yet you may perceue how infinite other positiõs will serue: so that D I and A D make an angle for a triangle to haue his sides proportionally cut. &c.*

Suppose a line geuen in length and position, to be A B. I say that A B is to be deuided by an extreme, and meane proportion. Deuide A B into two equall parts as in the point C. Produce A B directly, from the point B, to the point D: making B D, equal to B C. To the line A D, and at the point D, let a line be drawen *perpendicular: by the 11. of the first, which let be D F: (of what length you will). From D F and at the point D, cut of the sixth parte of D F: by the 9. of the sixth. And let that sixth part, be the line D G. Vppon D F, as a diameter, describe a semicircle: which let be D H-F. From the point G, rere a line perpendicular to D F, which suppose to be G H: and let it come to the circumference of D H F, in the point H. Draw right lines, H D, and H F. Produce D H, from the point H, so long, till a line adioyned with D H, be equall to H F, which let be D I, equall to H F. From the point H, to the point B, (the one ende of our line geuen) let a right line be drawen: as H-B. From the point I, let a line be drawen, to the line A B: so that it be also parallel to the line H B. Which parallel line suppose to be I-K: cutting the line A B, at the point K. I say that A B, is deuided by an extreme & meane proportion, in the point K. For the triangle D K I, hauing H B, parallel to I K, hath his sides D K and D I, cut proportionally, by the 2. of the sixth. Wherefore as I H is to H D: so is K B, to B D. And therfore compoundingly, (by the 18. of the fiueth) as D I, is to D H: so is D K to D B. But by construction D I is equall to H F: wherefore by the 7. of the fifth, D I is to D H, as H F is to D H. Wherefore by the 11. of the fifth, D K is to D B, as H F is to D H. Wherefore the square of D K is to the square of D B, as the square of H F, is to the square of D H: by the 22. of the sixth. But the square of H F, is to the square of D H: as the line G F is to the line G D: by my corrollary vpon the 6, probleme of my additions to the second proposition of the twelfth. Wherefore by the 11. of the fifth, the square of D K is to the square of D B, as the line G F is to the line G D. But by construction, G F is quintuple to G D. Wherefore the square of D K is quintuple to the square of D B: and therefore, the double of D B, is deuided by an extreme and meane proportiõ, and B K is the greater segment therof, by the 2. of this thirteenth. Wherefore seing A B is the double of D B by construction: the line A B is deuided by an extreme and meane



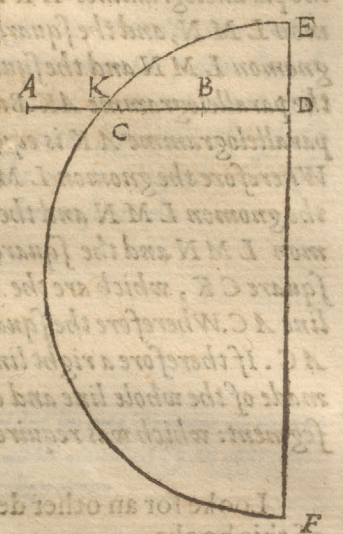
Demonstration.

p roportion:

proportion: and his greater ſegment,is the line B K. Wherefore A B is deuided by an extreme and meane proportion, in the point K. We haue therefore deuided by extreme and meane proportion any line geuen in length and poſition. Which was requiſite to be done.

The ſecond way to execute this probleme.

Suppoſe the line geuen to be *A B*.Deuide *A B* into two equall parts: as ſuppoſe it to be done in the point *C*. Produce *A B* from the point *B*: adioyning a line equall to *B C*, which let be *B D*. To the right line *A D*, and at the point *D*, erect a perpendicular line equall to *B D*, let that be *D E*. Produce *E D* tró the point *D* to the point *F*: making *D F* to contayne fiue ſuch equall partes, as *D E* is one. Now vpon *E F* as a diameter, deſcribe a ſemicircle which let be *E K F*: and let the point where the circumference of *E K F*, doth cut the line *A B*, be the point *K*. I ſay that *A B*, is deuided in the point *K*, by an extreme and meane proportion. For by the 13. of the ſixth *E D*, *D K*, & *D F*, are three lines in continuall proportion, (*D K* being the middle proportionall). Wherefore by the corollary of the 20. of the ſixth, as *E D* is to *D F*, ſo is the ſquare of *E D*, to the ſquare of *D K*, but by conſtruction, *E D*, is ſubquintuple to *D F*. Wherefore the ſquare of *E D*, is ſubquintuple to the ſquare of *D K*. And therefore the ſquare of *D K*, is quintuple to the ſquare of *E D*. And *B D* is equall to *E D*, by conſtruction, therefore the ſquare of *D K*, is quintuple to the ſquare of *B D*. Wherefore the double of *B D*, is deuided by an extreme and meane proportion: whoſe greater ſegment is *B K*, by the ſecond of this thirteenth. But by conſtruction, *A B*, is the double of *B D*: Wherefore *A B*, is diuided by extreme and meane proportion, and his greater ſegment, is *B K*: and thereby, *K*, the point of the diuiſion. We haue therefore deuided by extreme and meane proportion, any right line geuen, in length and poſition. Which was to be done.



Note.

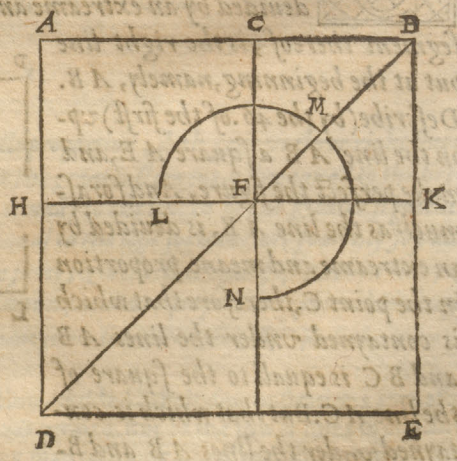
Ech of theſe wayes, may well be executed: But in the firſt, you haue this auantage: that the diameter is taken at pleaſure. Which in the ſecond way, is euer iuſt thriſe ſo long, as the line geuen to be deuided.

✠ Iohn Dee.

¶ The 4. Theoreme. The 4. Propoſition.

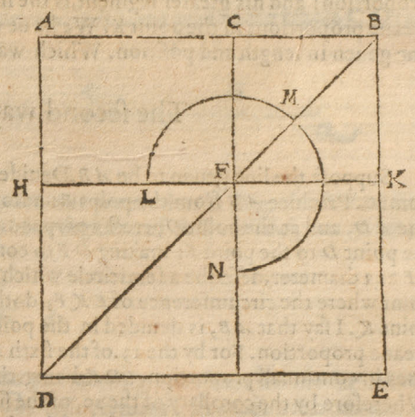
If a right line be deuided by an extreame and meane proportion: the ſquares made of the whole line and of the leſſe ſegmẽt, are treble to the ſquare made of the greater ſegment.

Vppoſe that the right line A B, be deuided by an extreame & meane proportiõ, in the point C. And let the greater ſegment thereof be A C. Then I ſay, that the ſquares made of the lines A B, and B C, are treble to the ſquare of the line A C. Deſcribe (by the 46. of the firſt) vpon the line A B, a ſquare A D E B. And make perfect the figure. Now foraſmuch as the line A B, is deuided by an extreame and meane proportion, in the point C: and the greater ſegmẽt thereof, is the line A C, therefore that which is contayned vnder the lines A B and B C is equall to the ſquare of the line A C. But that which is contayned vnder the lines A B and C B is the pa-



Demonſtration.

rallelogramme A K, and the ſquare of the line A-C is the ſquare F D. Wherefore the parallelogramme A K is equall to the ſquare F D. And the parallelogramme A F is equall to the parallelogramme F E, put the ſquare C K common to them both: wherfore the whole parallelogrãme A K is equall to the whole parallelogramme C-E. Wherefore the parallelogrammes C E and A-K are double to the parallelogramme A K. But the parallelogrammes A K and C E, are the gnomon L M N, and the ſquare C K. Wherefore the gnomon L M N and the ſquare C K, are double to the parallelogramme A K. But it is proued that the parallelogramme A K is equal to the ſquare D F. Wherefore the gnomon L M N and the ſquare C K are double to the ſquare D F. Wherefore the gnomon L M N and the ſquares C K and D F, are treble to the ſquare D F. But the gnomon L M N and the ſquares C K and D F, are the whole ſquare A E together with the ſquare C K, which are the ſquares of the lines A B and B C. And D F is the ſquare of the line A C. Wherefore the ſquares of the lines A B and B C, are treble to the ſquare of the line A C. If therefore a right line be deuided by an extreame and meane proportion, the ſquares made of the whole line and of the leſſe ſegment, are treble to the ſquare made of the greater ſegment: which was required to be proued.



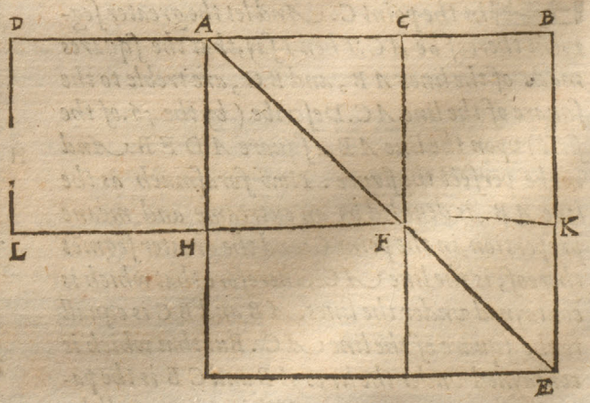
Looke for an other demonſtration of this propoſition after the fifth propoſition of this booke.

¶ The 5. Theoreme. The 5. Propoſition.

If a right line be deuided by an extreame and meane proportion, and vnto it be added a right line, equall to the greater ſegment, the whole right line is deuided by an extreame and meane proportion, and the greater ſegment thereof, is the right line geuen at the beginning.

I. Dee. This is moſt euident of my ſecond Theoreme, added to the third propoſitiõ. For to adde to a whole line, a line equall to the greater ſegmẽt: & to adde to the greater ſegment a line equall to the whole line, is all one thing, in the line produced. By the whole line, I meane the line diuided by extreme and meane proportion.

Vppoſe that the right line A B be deuided by an extreame and meane proportion in the point C, and let the greater ſegment thereof be A C. And vnto the line A B, adde the line A D equall to the line A C. Then I ſay that the line D B, is deuided by an extreame and meane proportion in the point A: and the greater ſegment thereof, is the right line put at the beginning, namely, A B. Deſcribe (by the 46. of the firſt) vpon the line A B a ſquare A E, and make perfect the figure. And foraſmuch as the line A B, is deuided by an extreame and meane proportion in the point C, therefore that which is contayned vnder the lines A B and B C is equall to the ſquare of the line A C. But that which is contayned vnder the lines A B and B-C is



C is the parallelogramme C E, and the ſquare of the line A C is the ſquare C H. Wherefore the parallelogramme C E is equall to the ſquare C H. But vnto the ſquare C H is equall the ſquare D H, by the firſt of the ſixth: and vnto the parallelogramme C E, is equall the parallelogramme H E. Wherefore the parallelogramme D H is equall to the parallelogramme H E. Adde the parallelogramme H B, common to them both. Wherefore the whole parallelogramme D K is equall to the whole ſquare A E. And the parallelogramme D K, is that which is contayned vnder the lines B D and D A, for the line A D is equall to the line D L, & the ſquare A E is the ſquare of the line A B. Wherfore that which is contayned vnder the lines A D and D B is equall to the ſquare of the line A B. Wherefore as the line D B is to the line B A, ſo is the line B A to the line A D, by the 17. of the ſixth. But the line D B is greater then the line B A. Wherefore the line B A is greater then the line A D. Wherefore the line B D is deuided by an extreame and meane proportion in the point A, and his greater ſegment is the line A B. If therefore a right line be deuided by an extreame and meane proportion, and vnto it be added a right line, equall to the greater ſegment: the whole right line is deuided by an extreame and meane proportion, and the greater ſegment therof, is the right line geuen at the beginning: which was required to be demonſtrated.

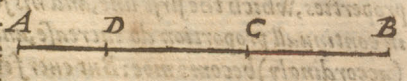
This propoſition is agayne afterward demonſtrated.

A Corollary added by *Campane*.

Hereby it is manifeſt that if from the greater ſegment of a line deuided by an extreame & meane proportion, be taken away the leſſe ſegment: the ſayd greater ſegment ſhall be deuided by an extreame and meane proportion, and the greater ſegment thereof, ſhall be the line taken away.

This is before demonſtrated moſt euidently and briefly by M. Dee, after the 3. propoſition.

As let the line A B, be deuided by an extreame and meane proportion, in the point C. And let the greater ſegment, be the line A C. From A C take C B: making the reſidue A D. I ſay that A C is alſo deuided by an extreame and meane proportion in the point D, and that his greater portion is D C. For, by the definition (of a line ſo deuided) A B, is to A C, as A C is to C B. But as A C is to C B, ſo is A C to D C, by the 7. of the fifth (for D C, by conſtruction is equall to C B) wherefore, by the 11. of the fifth, as A B is to A C, ſo is A C to C D: and therefore by the 19. of the fifth, as A B is to A C, ſo is the reſidue C B, to the reſidue A D. But C B is to A D, as D C is to A D (by the 7. of the fifth) for D C is by conſtruction equall to C B. Wherefore, A C is to D C, as D C is to A D, and ſo, by the definition of a line deuided by an extreame and meane proportion, it appeareth, A C in the point D, to be deuided, by an extreame and meane proportion: which was to be proued.



Two Corollaries (added by M. Dee) following chiefely vpon the veritie, and demonſtration of his Additions, vnto the 3. porpoſition annexed, and partely vpon this fifth, by Euclide demonſtrated.

A Corollary. 1.

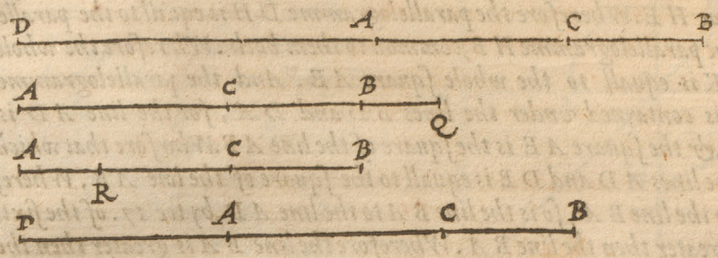
As any line being deuided by an extreame and middle proportion, doth geue vs three right lines, in continuall proportion: So, either by adioyning directly to the greater ſegment, a line equall to the firſt whole line: or (Secondly) by producing the leſſe ſegment, equally to the length of the greater ſegment: or (thirdly) by cutting from the greater ſegment, a part equal to the leſſe ſegment: or (fourthly) by adioyning, directly to the greater ſegment, a line equall to the ſame ſegment: it is manifeſt, that in euery of theſe fower wayes, we haue two lines deuided by an extreame and meane proportion: (it is to weete, one geuen, and the other made) and with all, in euery way, we haue fower lines in continuall proportion.

1.
2.
3.
4.

Note.

Of theſe two lines, (by extreame and meane proportion deuided,) their demonſtrations, are after *Euclides* 3, propoſition added: and here in this fifth by *Euclide* proued. But of the fower lines in continuall proportion, ſeing, the demonſtration is moſt eaſy for any man to frame. I will here, but note the lines

vnto you: as in euery of the fower places, the constructions haue them lettred and specified. As in my
Note 4. 1. Proportionall lines. first way added after the third proposition, D C, A D, A C, and C B: are fower lines in continuall propor-



2. tion. And in the second way, A R, C Q (equall to A C) C B, and B Q: are fower lines in continuall pro-
3. portion. And in the third way, A B, A C, C R, (equall to C B) and A R, are the fower lines in continuall
4. proportion. And in the fourth vay (by *Euclide* declared) D B, A B, A C (equall to A D) and C B, are fow-
Note 1. er lines in continuall proportion. So, that in the first way you haue A D, and A C, middle proportionals
two mid- 2. betwene D C, and C B. In the second way, you haue C Q, and C B, betwene A B, and B Q. In the third
dle propor- 3. way, you haue A C, C R, betweie A B and A R: and in the fourth way, you haue A B, and A C, betwene
tionals. 4. D B, and C B.

A Corollary. 2.

Note 4. wayes of progression, in the proportion of a line diuided by extreme and middle proportion. *It is also manifest, that yoi may by any of the fower wayes, here specified, proceede infinitely, in the proportion of a line deuided by extreame & middle proportion: And in the first, and fourth wayes, encreasing continually the quantities of the lines made: but in the second and third wayes, diminishing continually the quantities of the sayd whole lines, made (and thereby their segmentes). And yet, neuertheless reteyning in euery line made (by any of the wayes) and in his segments, all, and the same properties, which the first line, and his segmentes haue. After which rate of Progression, as the termes in continuall proportion do encrease, and are moe in number: So, likewise, do the middle proportionalls, (accordingly) become moe: But euer fewer in number, by two, then the termes of the Progression are.*

¶ What Resolution is.

What resolution and composition is, hath before bene taught in the beginning of the first booke. *Resolution, is the assumption or taking of the thing which is to be proued, as graunted, and by thinges which necessirily follow it, to passe vnto some truth graunted.*

¶ What Composition is.

Composition, is an assumption or taking of a thing graunted, and by thinges which of necessity follow it, to passe vnto the finding out of the thing sought or to be proued.

Resolution of the first Theoreme.

Suppose that a certaine right line A B, be diuided by an extreame & meane proportion in the point C, & let the greater segmẽt therof be A C, vnto which adde a line equal to the halfe of the line A B, and let that line be A D. Then I say that the square of the line C D is quintuple to the square of A D. For forasmuch as the square of the line C D is quintuple to the square of A D: but the square of the line C D is (by the 4. of the second) equall to that which is composed of the squares of the lines C A, & A D, together with that

which

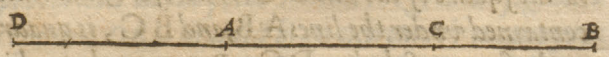
which is contayned vnder the lines C A,and A D twiſe. Wherfore that which is compoſed of the ſquares of the lines C A,& A D,together with that which is cõtained vnder the lines C-A,& A D twiſe is quintuple to the ſquare of the line A D.Wherfire,that which is compoſed of the ſquare of the line C A together with that,which is contayned vnder the lines C A, and A D twiſe is quadruple to the ſquare of the line A D.But vnto that, which is contayned vnder the lines C A, and A D , twiſe,is equall that which is contayned vnder the lines C A, and A B,for the line A B is double to the line A D . And vnto the ſquare of the line A-C, is equall that which is cõtayned vnder the lines A B& B C,for the line A B, is by ſuppoſition diuided by an extreme and meane proportion,in the point C .Wherefore, that, which is contayned vnder the lines A B, and A C, together with that which is contayned vnder the lines A B,and B C,is quadruple to the ſquare of the line A D.But that , which is compoſed of that which is contayned vnder the lines A B,and A C,together with that which is contayned vnder the lines A B,and B C,is the ſquare of the line AB(by the 2. of the ſecond.)Wherfore the ſquare of the line A B is quadruple to the ſquare of the line A D . And ſo is it in deede: for the line A B is double to the line A D: as was at the firſt ſuppoſed.

Compoſition of the firſt Theoreme.

Now foraſmuch as the ſquare of the line A B is quadruple to the ſquare of the line A D, but the ſquare of the line A B,is that which is contayned vnder the lines A B,and A C,together with that,which is contayned vnder the lines B A , and B C. Wherefore that which is contayned vnder the lines B A,and A C, together with that which is contayned vnder the lines B A,and B C is quadruple to the ſquare of the line A D . But that which is contayned vnder the lines B A,and A C,is equall to that which is contayned vnder the lines D A , and A C twiſe(by the 1 .of the ſixth),and that which is contayned vnder the lines A B,and B C is equall to the ſquare of the line A C, by the definition of a line diuided by extreme and meane proportion.Wherefore the ſquare of the line A C,together with that, which is contayned vnder the lines D A,and A C twiſe,is quaduple to the ſquare of the line D A.Wherfore that which is compoſed of the ſquares of the lines D A,and A C,together with that which is contayned vnder the lines D A,and AC,twiſe,is quintuple to the ſquare of the line D A.But that which is compoſed of the ſquares of the lines D A , and A C , together with that which is contayned vnder the lines D A,and A C twiſe,is equall to the ſquare of the line C D (by the 4.of the ſecond).Wherefore the ſquare of the line C D is quintuple to the ſquare of the line A D: which was required to be demonſtrated.

Reſolution of the 2. Theoreme.

Suppoſe that a certayne right line,C D,be quintuple to a ſegmẽt of the ſame line,namely, to D A: and let the double of the line D A,be AB.Then I ſay that the line A B is diuided by an extreme and meane proportion in the point C: and the greater ſegmẽt therof is A C, which is the reſt of the right line put at the beginning.For foraſmuch as the line A B is diuided by an extreame and meane proportion in the poynt C , and the greater ſegment thereof is the line A C,therefore that which is contained vnder the lines A B,and BC,is equall to the ſquare of the line A C . But that which is contayned vnder the lines B A, and A C , is equal to that which is contayned vnder the lines D A,and A C twiſe: for the line B A,is double to the line A D . Wherefore that which is contayned vnder the lines A B,and B C together with that which is cõtayned vnder the lines B A,and A C,which is the ſquare of the line A B(by the 2. of the ſecond) is equall to



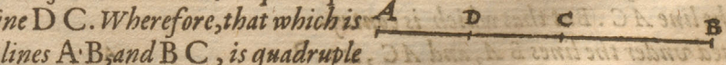
that which is contayned vnder ỹ lines D A, & A C, twise together with the ſquare of the line A C. But the ſquare of the line AB, is quadruple to the ſquare of the line D A. Wherfore that which is contayned vnder the lines D A, and A C, twiſe, together with the ſquare of the line A C, is quadruple to the ſquare of the line A D. Wherefore the ſquares of the lines D A, and A C, together with that which is contayned vnder the lines D A, and A C, twiſe, which is the ſquare of the line D C, are quintuple to the ſquare of the line D A. And ſo are they in deede by ſuppoſition.

Compoſition of the 2. Theoreme.

Now foraſmuch as the ſquare of the line C D, *is quintuple to the ſquare of the line* D A, *But the ſquare of the line* C D, *is that which is cõpoſed of the ſquares of ỹ lines* D A, *&* A C, *together with that which is cõtained vnder the lines* D A, *&* A C, *twiſe: Wherfore ỹ ſquares of the line* D A, *&* A C *together with that which is cõtayned vnder the lines* D A, *&* A C, *twiſe, are quintuple to the ſquare of the line* D A. *Wherfore, by diuiſiõ, that which is cõtained vnder the lines* D A, *and* A C, *twiſe together with the ſquare of the line* C A, *is quadruple to the ſquare of the line* A D. *And the ſquare of the line* A B, *is quadruple to the ſquare of the line* A D. *Wherefore that which is contayned vnder the lines* D A, *and* A C *twiſe, which is that, which is contayned vnder the lines* B A, *and* A C *once, together with the ſquare of the line* A C, *is equall to the ſquare of the line* A B. *But the ſquare of* B A, *is that which is contayned vnder the lines* B A, *and* A C, *together with that which is contayned vnder the lines* B A, *and* B C. *Wherfore that which is contayned vnder the lines* BA, *&* A-C, *together with that which is cõtayned vnder the lines* AB, *&* BC, *is equall to that which is contayned vnder the lines* B A, *and* A C *together with the ſquare of the line* A C. *Now then taking away that which is common to them both, namely, that which is cõtayned vnder the lines* B A, *and* A C, *the reſidue, namely, that which is contayned vnder the lines* A B, B C *is equall to the ſquare of the line* A C. *Wherefore as the line* B A, *is to the line* A C, *ſo is the line* A C *to the line* C B. *But the line* B A *is greater then the line* A C, *wherefore the line* A C *alſo is greater then the line* C B. *Wherefore the line* A B *is diuided by an extreame, and meane proportion in the poynt* C, *and the greater ſegment thereof is the line* A C, *which was required to be demonſtrated.*

Reſolution of the 3. Theoreme.

Suppoſe that a certayne right line A B, *be diuided by an extreame, and meane proportion in the point* C: *and let the greater ſegment thereof be the line* A C, *and let the halfe of the line* A C, *be the line* C D. *Then I ſay that the ſquare of the* B D *is quintuple to the ſquare of the line* C D. *For foraſmuch as the ſquare of the line* BD, *is quintuple to the ſquare of the line* C D. *But the ſquare of the line* D B, *is that which is contayned vnder the lines* A B, *and* B C, *together with the ſquare of the line* D C *(by the 6. of the ſecond). Wherefore that which is contayned vnder the lines* A B, *and* B C, *together with the ſquare of the line* D C, *is quintuple to the ſquare of the line* D C. *Wherefore, that which is contayned vnder the lines* A B, *and* B C, *is quadruple to the ſquare of the line* D C. *But vnto that which is contayned vnder the lines* A B, *and* B C, *is equall the ſquare of the line* A C: *for the line* A B, *is diuided by an extreame and meane proportion in the point* C. *Wherefore the ſquare of the line* A C *is quadruple to the ſquare of the line* D C: *and ſo is it in deede, for the line* A C *is double to the line* D C.



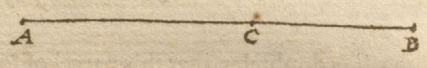
Com-

Compoſition of the 3. Theoreme.

Foraſmuch as the line A C *is double to the line* D C, *therefore the ſquare of the line* A C *is quadruple to the ſquare of the line* D C *(by the 20. of the ſixth). But vnto the ſquare of the line* A C, *is equall that which is contayned vnder the lines* A B, *and* B C, *by ſuppoſition: wherefore that which is contayned vnder the lines* A B, *and* B C, *is quadruple to the ſquare of the line* C D. *Wherefore, that which is contayned vnder* A B, *and* B C, *tother with the ſquare the line* D C, *which is the ſquare of the line* D B *(by the 6. of the ſecond) is quintuple to the ſquare of the line* D C: *which was required to be demonſtrated.*

Reſolution of the 4. Theoreme.

Suppoſe that a certayne right line A B, be diuided by an extreme and meane proportion in the point C. And let the greater ſegment thereof be A C. Then I ſay that the ſquares of the lines A B, and B C, are treble to the ſquare of the line A C. For foraſmuch as the ſquares of the lines A B, and B C, are treble to the ſquare of the line A C, but the ſquares of the lines A B, and B C, are that which is contayned vnder A B, and B C, twiſe together with the ſquare of the line A C (by the 7. of the ſecond). Wherefore that which is contayned vnder the lines A B, and B C, twiſe, together with the ſquare of the line A C, is treble to the ſquare of the line A C. Wherefore, that which is contayned vnder the lines A B, & B C, twiſe, is double to the ſquare of the line A C. Wherefore that which is contayned vnder the lines A B, and B C, once, is equall to the ſquare of the line A C. And ſo it is in deede. For the line A B is diuided by an extreme, and meane proportion in the point C.



Compoſition of the 4. Theoreme.

Foraſmuch therefore as the line A B, is diuided by an extreme and meane proportion in the poynt C, and the greater ſegment thereof is the line A C, therfore that which is contayned vnder the lines A B, and B C is equall to the ſquare of the line A C. Wherfore that which is cõtayned vnder the lines A B, and B C twiſe is double to the ſquare of A C. Wherfore that which is contayned vnder the lines A B, and B C, twiſe, together with the ſquare of the line A C, is treble to the ſquare of the line A C. But that which is contayned vnder the lines A B, and B C, twiſe, together with the ſquare of the line A C, is the ſquares of the lines A B, and BC (by the 7. of the ſecond). Wherefore the ſquares of the lines A B, and B C, are treble to the ſquare of the line A C: which was required to be demonſtrated.

Reſolution of the 5. Theoreme.

Suppoſe that a certaine right line A B, be diuided by an extreme and meane proportion in the point C. And let the greater ſegment therof be the line A C. And vnto the line A B, adde a line equall to the line A C, and let the ſame be A D. Then I ſay that the line D B, is diuided by an extreme and meane proportion in the point A. And the greater ſegment therof is the line A B. For foraſmuch as the line D B is diuided by an extreme & meane proportion in the point A, and the greater ſegment thereof is the line A B, therfore as the line D B, is to the line B A, ſo is the line B A, to the line A D: but the line A D, is equall to the line A C: wherefore as

the

the line D B, is to the line B A, ſo is the line B A to the line A C. Wherfore by conuerſiõ as the line B D is to the line D A, ſo is the line A B to the line B C (by the corollary of the 19. of the fifth): wherfore by diuiſion, by the 17. of the fifth, as the line B A, is to the line A D, ſo is the line A C, to the line C B. But the line A D is equall to the line A C. Wherfore as the line B A, is to the line A C, ſo is the line A C to the line C B. And ſo it is indeede, for the line A B is, by ſuppoſition, diuided by an extreme and meane proportion in the point C.

Compoſition of the 5. Theoreme.

Now foraſmuch as the line A B, is diuided by an extreme and meane proportion in the point C: therefore as the line B A is to the line A C, ſo is the line A C to the line C B: but the line A C is equall to the line A D. Wherefore as the line B A is to the line A D, ſo is the line A C to the line C B. Wherfore by compoſition (by the 18. of the fifth) as the line B D is to the line D A, ſo is the line A B to the line B C. Wherefore by conuerſion (by the corollary of the 19. of the fifueth) as the line D B is to the line B A, ſo is the line B A to the line A C: but the line A C is equall to the line A D. Wherefore as the line D B is to the line B A, ſo is the line B A to the line A C. Wherfore the line D B, is deuided by an extreme and meane proportion in the point A: and his greater ſegment is the line A B: which was required to be demonſtrated.

An Aduiſe, by Iohn Dee, added.

SEing, it is doubteles, that this parcel of *Reſolution and Compoſition*, is not of *Euclides* doyng: it can not iuſtly be imputed to *Euclide*, that he hath, therby, eyther ſuperfluitie or any part diſproportioned in his whole *Compoſition Elementall*. And though, for one thing, one good demonſtration well ſuffiſeth: for ſtabliſhing of the veritie: yet, of one thing diuerſly demonſtrated: to the diligent examiner of the diuerſe meanes, by which, that varietie ariſeth, doth grow good occaſions of inuenting demonſtrations, where matter is more ſtraunge, harde, and barren. Alſo, though reſolution were not in all *Euclide* before vſed: yet thankes are to be geuen to the Greke Scholie writter, who did leaue both the definition, and alſo, ſo ſhort and eaſy examples of a Method, ſo auncient, and ſo profitable. The antiquity of it, is aboue 2000. yeares: it is to wete, euer ſince Plato his time, and the profite, therof ſo great, that thus I finde in the Greeke recorded. *Μεθοδοι δὲ ὅμως παραδίδονται: καλλίση μὲν ἡ διὰ τῆς αναλύσεως, ἐπ' ἀρχὴν ὁμολογούμενον, ἀνάγουσα τὸ ζητούμενον: ἣν καὶ ὁ Πλάτων (ὡς φασι) Λεοδάμαντι παρέδωκεν. ἀφ' ἧς καὶ ἐκεῖνος πολλῶν καὶ γεωμετρίαν εὑρεῖν ἱστορῆται γενέσθαι.* Proclus hauing ſpoken of ſome by nature, excellent in inuenting demonſtrations, pithy and breif ſayeth: Yet are there Methods geuen [for that purpoſe]. And in dede, that, the beſt, which, by Reſolution, reduceth the thing inquired of, to an vndoubted principle. Which Method, Plato, taught Leodamas (as is reported) And he is regiſtred, thereby, to haue bene the inuenter of many things in Geometry.

* *Proclus in the Greeke: in the 58. page.*

And, verely, in Problemes, it is the chief ayde for winning and ordring a demonſtration: firſt by Suppoſition, of the thing inquired of, to be done: by due and orderly Reſolution to bring it to a ſtay, at an vndoubted veritie. In which point of Art, great abundance of examples, are to be ſeen, in that excellent and mighty Mathematicien, Archimedes: & in his expoſitor, Eutocius, in Menæchmus likewiſe: and in Diocles booke, de Pyrijs: and in many other. And now, for as much as, our *Euclide* in the laſt ſix Propoſitions of this thirteenth booke propoundeth, and concludeth thoſe Problemes, which were the ende, Scope, and principall purpoſe, to which all the premiſſes of the 12. bookes, and the reſt of this thirteenth, are directed and ordered: It ſhall be artificially done, and to a great commodity, by Reſolution, backward, from theſe 6. Problemes, to returne to the firſt definition of the firſt booke: I meane, to the definition of a point. Which, is nothing hard to do. And I do counſaile all ſuch, as deſire to atteine, to the profound knowledge of Geometrie, Arithmeticke, or any braunche of the ſciences Mathematicall, ſo by Reſolution, (diſcreatly and aduiſedly) to reſolue, vnloſe, vnioynt and diſſeauer euery part of any worke Mathematicall, that, therby, aſwell, the due placing of euery verity, and his proofe: as alſo, what is either ſuperfluous, or wanting, may euidently appeare. For ſo to inuent, & there with to order their writings, was the cuſtome of them, who in the old time, were moſt excellent. And I (for my part) in writing any Mathematicall concluſion, which requireth great diſcourſe, at length haue found, (by experience) the commoditie of it, ſuch: that to do other wayes, were to me a confuſion, and an vnmethodicall heaping of matter together: beſides the difficulty of inuenting the matter to be diſpoſed and ordred. I haue occaſion, thus to geue you friendely aduiſe, for your behofe: becauſe ſome, of late, haue inueyed againſt *Euclide*, or *Theon* in this place, otherwiſe than I would wiſh they had.

¶ The

The 6. Theoreme. The 6. Propoſition

If a rationall right line be diuided by an extreme and meane proportion: eyther of the ſegments, is an irrationall line of that kinde, which is called a reſiduall line.

SVppoſe that A B, beyng a rationall line be deuided by an extreme and meane proportion in the point C, and let the greater ſegment thereof be A C. Then I ſay that eyther of the lines A C, and C B, is an irrationall line of that kinde, which is called a reſiduall. Extend the line A B, to the point D: and let the line A D, be equall to halfe of the line A B. Conſtruction.

Demonſtration.

Now foras much as the right line A B, is diuided by an extreme & meane proportion in the point C, and vnto the greater ſegmẽt AC is added a line A D, equall to the halfe of the right line A B: therfore (by the 1. of the thirtenth) the ſquare of the line C D, is quintuple to the ſquare of the line A D. Wherefore the ſquare of the line C D, hath to the ſquare of the line A D, that proportion that nũber hath to nũber. Wherfore the ſquare of the line C D is commẽſurable to the ſquare of the line A D. But the ſquare of the line D A is rationall, for the line D A is rationall, forasmuch as it is the halfe of the rationall line A B. Wherefore the ſquare of the line C D, is rationall. Wherefore alſo the line C D, is rationall. And foraſmuch as the ſquare of the line C D, hath not to the ſquare of the line A D, that proportion that a ſquare number hath to a ſquare number, therfore (by the 9. of the tenth) the line C D, is incommenſurable in length, to the line A D. Wherefore the lines C D, and D A are rationall commenſurable in power only. Wherfore the line A C is a reſiduall line, by the 73. of the tenth. Againe, foraſmuch as the line A B, is deuided by an extreme and meane proportion, and the greater ſegment thereof is A C, therfore that which is cõtayned vnder the lines AB, & BC, is equall to the ſquare of the line A C. Wherefore the ſquare of the line A C, applyed to the rationall line A B, maketh the bredth B C. But the ſquare of a reſiduall line, applyed to a rationall line maketh the bredth a firſt reſiduall line (by the 97. of the tenth). Wherefore the line C B, is a firſt reſiduall line. And it is proued that the line A C, is alſo a reſiduall line. If therefore a rationall right line be diuided by an extreme and meane proportion, either of the ſegments, is an irrationall line of that kinde, which is called a reſiduall line, which was required to be demonſtrated.

¶ A Corollary added by *Campane*.

Hereby it is manifeſt, that if the greater ſegment be a rationall line: the leſſe ſegment ſhalbe a reſiduall line.

For if the greater ſegment A C, of the right line A C B be diuided into two equall partes in the point D, the ſquare of the line D B, ſhalbe quintuple to the ſquare of the line D C, (by the 3. of this booke.) And foraſmuch as the line C D, (beyng the halfe of the rationall line ſuppoſed A C) is rationall by the 6. diffinition of the tenth: And vnto the ſquare of the line D C, the ſquare of the line D B is commenſurable, (for it is quintuple vnto it) wherfore the ſquare of the line D B, is rationall. Wherfore alſo the line D B is rationall. And foraſmuch as the ſquares of the lines D B & D C, are not in proportion, as a ſquare nũber is to a ſquare number: therefore the lines D B and D C are incommenſurable in length (by the 9. of the tenth.) Wherefore they are commenſurable in power only: Wherefore by the 73. of the tenth, the line B C, which is the leſſe ſegment, is a reſiduall line.

The 7. Theoreme. The 7. Propoſition.

If an equilater Pẽtagon haue three of his angles, whether they follow in or-

der, or not in order, equall the one to the other: that Pentagon ſhalbe equiangle.

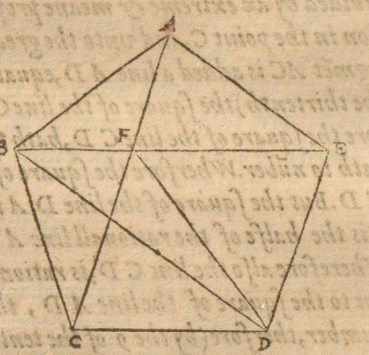
Two caſes in this propoſition.

Conſtruction.

The firſt caſe.

Demonſtration.

SVppoſe that A B C D E, be an equilater pentagon. And let the angles of the ſayd Pentagon, namely, firſt, three angles folowing in order, which are at the points A,B,C, be equal the one to the other. Then I ſay that the Pentagon A B C D E is equiangle. Draw theſe right lines A C, B E, and F D. Now foraſmuch as theſe two lines C B, and B A, are equall to theſe two lines B A, and A E, the one to the other, and the angle C B A is equall to the angle B A E: therefore (by the 4. of the firſt) the baſe A C is equall to the baſe B E, and the triangle A B C is equall to the triangle A B E, and the reſt of the angles are equal to the reſt of the angles, vnder which are ſubtended equall ſides. Wherefore the angle B C A is equall to the angle B E A, and the angle A B E to the angle C A B. Wherefore alſo the ſide A F, is equall to the ſide B F (by the 6. of the firſt). And it was proued that the whole line A C is equal to the whole line B E. Wherefore the reſidue C F is equall to the reſidue F E. And the line C D, is equall to the line D E. Wherefore theſe two lines F C, and C D are equall to theſe two lines F E, & E D, and the baſe F D, is common to them both. Wherefore the angle F C D, is equall to the angle F E D (by the 8. of the firſt). And it is proued that the angle B C A, is equal to the angle A E B. Wherefore the whole angle B C D is equall to the whole angle A E D. But the angle B C D, is ſuppoſed to be equall to the angles A, and B. Wherefore the angle A E D, is equall to the angles A and B. In like ſort alſo may we proue that the angle C D E, is equall to the angles A, and B. Wherefore the Pentagon A B C D E is equiangle.



The ſecond caſe.

But now ſuppoſe that three angles, which folow not in order, be equall the one to the other, namely, let the angles A, C, D, be equall. Then I ſay that in this caſe alſo the Pentagon A B C D E is equiangle. Draw a right line from the point B, to the point D. Now foraſmuch as theſe two lines B A, and A E, are equall to theſe two lines B C, and C D, and they comprehende equall angles, therefore (by the 4. of the firſt) the baſe B E, is equall to the baſe B D. And the triangle A B E, is equall to the triangle B D C, and the reſt of the angles are equall to the reſt of the angles, vnder which are ſubtended equall ſides: wherefore the angle A E B, is equall to the angle C D B. And the angle B E D, is equall to the angle B D E, (by the 5. of the firſt) for the ſide B E, is equall to the ſide B D. Wherefore the whole angle A E D, is equal to the whole angle C D E. But the angle C D E, is ſuppoſed to be equall to the angles A, and C. Wherefore the A E D, is equall to the angles A, and C. And by the ſame reaſon alſo the angle A B C, is equall to the angles A, C, and D. Wherefore the Pentagon A B C D E is equiangle. If therefore an equilater Pentagon haue three of his angles, whither they follow in order, or not in order, equall the one to the other: that Pentagon ſhalbe equiangle: which was required to be proued.

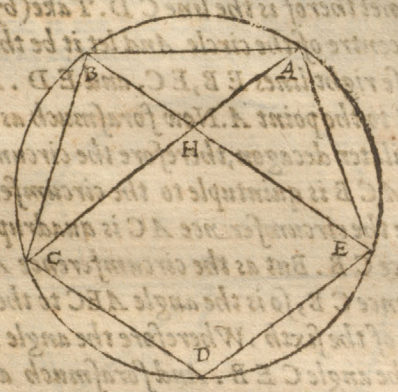
The 8. Probleme. The 8. Propoſition.

If in an equilater & equiangle Pẽtagon two right lines do ſubtend two of the angles following in order: thoſe lines doo diuide the one the other by an extreme and meane proportion: and the greater ſegments of thoſe lines are ech equall to the ſide of the Pentagon.

Suppoſe

SVppoſe that A B C D E be an equilater and equiangle Pentagon. And let two right lines A C, and B E, ſubtend the two angles A, and B, which follow in order. And let them cut the one the other in the point H. Then I ſay that either of thoſe lines is diuided by an extreme & meane proportiõ in the point H: And that eche of the greater ſegments of thoſe lines are equal to the ſide of the Pentagon. Circumſcribe (by the 14. of the fourth) about the Pentagõ A B C D E, a circle A B C D E. And foraſmuch as theſe two right lines E A, and A B, are equall to theſe two right lines A B, and B C, and they contayne equall angles: therefore (by the 4. of the firſt) the baſe B E, is equal to the baſe A C: and the triangle A B E, is equall to the triangle A B C, and the angles remayning, are equall to the angles remayning, the one to the other, vnder which are ſubtended equall ſides. Wherefore the angle B A C, is equall to the angle A B E. Wherfore the angle A H E is double to the angle B A H, (by the 32. of the firſt) for it is an outward angle of the triangle A B H. And the angle E A C is double to the angle B A C (by the laſt of the ſixth). For the circumference E D C is double to the circumference C B. Wherefore the angle H-A E is equall to the angle A H E. Wherefore alſo the right line H E, is (by the 6. of the firſt) equall to the right line E A, that is to the line A B. And foraſmuch as the right line B A is equall to the right line A E, therefore the angle A B E is equall to the angle A E B. But it is proued that the angle A B E is equal to the angle B A H: wherefore alſo the angle B E A is equall to the angle B A H. And in the two triangles A B E, and A B H, the angle A B E is common to them both, wherefore the angle remayning, namely, B A E is equall to the angle remayning, namely, to A H B (by the corollary of the 32. of the firſt). Wherfore the triangle A B E, is equiangle to the triangle A B H. Wherefore proportionally as the line E B, is to the line B A, ſo is the line A B to the line B H (by the 4. of the ſixth). But the line B A is equall to the line E H. Wherefore as the line B E is to the line E H, ſo is the line E H to the line H B. But the line B E is greater then the line B A: wherefore the line E H, alſo is greater then the line H-B. Wherefore the line B E, is diuided by an extreme and meane proportion in the point H (by the 3. diffinition of the ſixth) and his greater ſegment E H is equall to the ſide of the Pentagon. In like ſort alſo may we proue that the line A C is diuided by an extreme and meane proportion in the point H, and that his greater ſegment C H, is equall to the ſide of the Pentagon. (For the whole line A C is equall to the whole line B E, and it hath bene proued that the parts taken away B H, and A H are equall. wherfore the reſidue C H is equall to the reſidue E H (by the 19. of the fifth). If therefore in an equilater and equiangle Pentagon two right lines do ſubtend two of the angles following in order: thoſe lines doo diuide the one the other by an extreme and meane proportion: and the greater ſegments of thoſe lines are eche equal to the ſide of the Pentagon: which was required to be demonſtrated.

Conſtruction. Demonſtration.



¶ The 9. Theoreme. The 9. Propoſition.

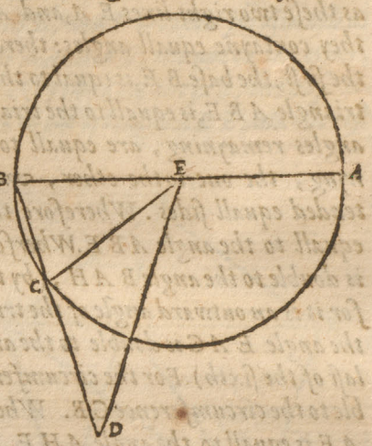
If the ſide of an equilater hexagon, and the ſide of an equilater decagon or tẽangled figure, which both are inſcribed in one & the ſelfe ſame circle, be added together: the whole right line made of them is a line diuided by an extreame and meane proportion, and the greater ſegment of the ſame is

the ſide of the hexagon.

Suppoſe that there be a circle A B C. And let the ſide of a decagon or tenangled figure inſcribed in the circle A B C, be B C, and let the ſide of an hexagon or ſixe angled figure inſcribed in the ſame circle, be C D. And let the lines B C and C D be ſo ioyned together directly that they both make one right line, namely, B D. Then I ſay that the line B D is diuided by an extreame and meane proportion in the point C: and that the greater ſegmẽt therof is the line C D. Take (by the 1. of the third) the centre of the circle. And let it be the point E: and draw theſe right lines E B, E C, and E D. And extend the line B E to the point A. Now foraſmuch as B C is the ſide of an equilater decagon, therefore the circumference or ſemicircle A C B is quintuple to the circumference C B. Wherefore the circumference A C is quadruple to the circumference C B. But as the circumference A C is to the circumference C B, ſo is the angle A E C to the angle C E B, by the laſt of the ſixth. Wherefore the angle A E C is quadruple to the angle C E B. And foraſmuch as the angle E B C is equall to the angle E C B (by the 5. of the firſt,) for the line E B is equall to the line E C, by the diffinition of a circle, therefore the angle A E C is double to the angle E C B, by the 32. of the firſt. And foraſmuch as the right line E C is equall to the right line C D, by the corollary of the 15. of the fourth (for either of them is equall to the ſide of the hexagon inſcribed in the circle A B C) therefore the angle C E D is equall to the angle C D E: wherefore the angle E C B is double to the angle E D C, by the 32. of the firſt. But it is proued that the angle A E C is double to the angle E C B, wherefore the angle A E C is quadruple to the angle E D C. And it is proued that the angle A E C is quadruple to the angle B E C. Wherefore the angle E D C is equall to the angle B E C. And the angle E B D is common to the two triangles B E C and B E D: wherefore the angle remayning B E D is equall to the angle remayning E C B, by the corollary of the 32. of the firſt. Wherefore the triangle E B D is equiangle to the triangle E B C. Wherfore, by the 4. of the ſixt, proportionally, as the line B D is to the line B E, ſo is the line B E to the line B C. But the line E B is equall to the line C D. Wherefore as the line B D is to the D C, ſo is the line D C to the line C B. But the line B D is greater then the line D C: wherefore alſo the line D C is greater then the line C B. Wherefore the right line B D is diuided by an extreame and meane proportion in the point C: and his greater ſegment is D C. If therefore the ſide of an equilater hexagon, and the ſide of an equilater decagon or tenangled figure, which both are inſcribed in one and the ſelfe ſame circle, be added together: the whole right line made of them, is a line diuided by an extreame and meane proportion, and the greater ſegment of the ſame is the ſide of the hexagon: which was required to be proued.

Conſtruction.

Demonſtration.



A Corollary added by *Fluſſas*.

This Corollary is the 3. propoſition of the 14. booke after Campane.

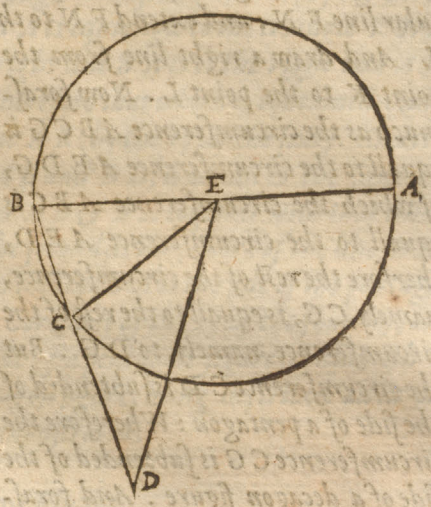
Hereby it is manifeſt, that the ſide of an exagon inſcribed in a circle being cut by an extreame and meane proportion, the greater ſegment thereof is the ſide of the decagon inſcribed in the ſame circle. For if from the right line D C be cut of a right line equall to the line C B, we may thus reaſon, as the whole D B is to the whole D C, ſo is the part taken away D C to the part taken away C B: wherefore by the 19. of the fifth, the reſidue is to the reſidue as the whole is to the whole. Wherefore the line D C is cutte like vnto the line D B: and therefore is cut by an extreame and meane proportion.

Campane putteth the conuerſe of this propoſition after this maner.

If a line be diuided by an extreme and meane proportion, of what circle the greater ſegment is the ſide of an equilater Hexagon, of the ſame ſhall the leſſe ſegment be the ſide of an equilater Decagon. And of what circle the leſſe ſegment is the ſide of an equilater Decagon, of the ſame is the greater ſegment the ſide of an equilater Hexagon.

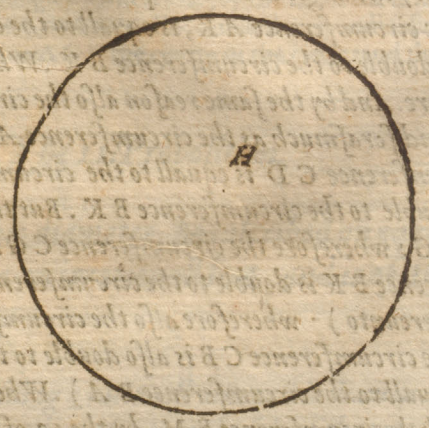
For the former figure remayning, ſuppoſe that the line B D be diuided by an extreme and meane proportion in the point C: and let the greater ſegment therof be D C. Thé I ſay that of what circle the line D C is the ſide of an equilater Hexagon, of the ſame circle is the line C B the ſide of an equilater decagon: and of what circle, the line B C is the ſide of an equilater Decagon, of the ſame is the line D C the ſide of an equilater Hexagon. For if the line D C be the ſide of an Hexagon inſcribed in the circle, then by the corollary of the 15. of the fourth, the line D C is equal to the line BE. And foraſmuch as the proportiõ of the line B D to the line D C is as the proportion of the line D-C to the line C B, by ſuppoſition: therfore (by the 7. of the fifth) the proportion of the line B D to the line B E, is as the proportion of the line B E to the line B C. Wherefore (by the 6. of the ſixth) the two triangles D E B, E B C are equiangle (for the angle B is common to eche triangle). Wherefore the angle D is equall to the angle C E B: for they are ſubtended of ſides of like proportion. And foraſmuch as the angle A E C is quadruple to the angle D (by the 32. of the firſt twiſe taken, and by the 5. of the ſame) therefore the ſame angle A E C is quadruple to the angle C E B. Wherefore (by the laſt of the ſixth) the circumference A C is quadruple to to the circumferẽce C B. Wherfore the line B C is the ſide of a decagon inſcribed in the circle A C B.

Demonſtration of the firſt part.



But now if the line B C be the ſide of a decagon inſcribed in the circle A B C, the line C D ſhalbe the ſide of an Hexagon inſcribed in the ſame circle. For let D C be the ſide of an Hexagon inſcribed in the circle H. Now by the firſt part of this propoſition the line B C ſhalbe the ſide of a decagon inſcribed in the ſame circle. Suppoſe that in the two circles A C-B and H be inſcribed equilater decagons, all whoſe ſides ſhalbe equall to the line C B. And foraſmuch as euery equilater figure inſcribed in a circle is alſo equiangle, therefore bothe the decagons are equiangle. And foraſmuch as al the angles of the one taken together are equall to al the angles of the other taken together, as it is eaſy to be proued by that which is added after the 32. of the firſt, therefore one of theſe decagons is equiangle to the other: and therfore the one is like to the other by the diffinition of like ſuperficieces. And for that if there be two like rectiline figures inſcribed in two circles, the proportion of the ſides of like proportion of thoſe figures, ſhalbe as the proportion of the Diameters of thoſe circles, as it is eaſy to proue by the corollary of the 20. of the ſixth, and firſt of this booke: but the ſides of the like decagons inſcribed in the two circles A B C and H are equall: therefore theyr Diameters alſo are equall. Wherefore alſo theyr ſemidiameters are equall. But the ſemidiameters and the ſide of the Hexagon are equall, by the corollary of the 15. of the fourth. Wherefore the line D C is the ſide of an hexagon in the circle A B C, as alſo it is the ſide of an hexagon inſcribed in the circle F, which is equall to the circle A B C: which was required to be proued.

Demonſtration of the ſecond part.



¶ *The 10. Theoreme.* *The 10. Propoſition.*

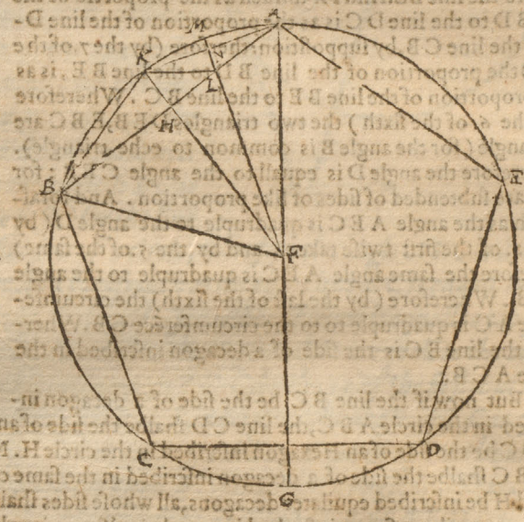
If in a circle be deſcribed an equilater Pentagon, the ſide of the Pentagon containeth in power both the ſide of an hexagon and the ſide of a decagon,

being all described in one and the selfe same circle.

Construction.

SVppose that A B C D E be a circle. And in the circle A B C D E describe (by the 11. of the fourth) a pentagon figure A B C D E. Then I say, that the side of the pentagon figure A B C D E containeth in power both the side of an hexagon figure, and of a decagon figure, being described in the circle A B C D E. Take (by the 1. of the third) the centre of the circle, and let the same be F. And drawing a right line from the point A to the point F, extend it to the point G. And drawe a right line from the point F to the point B. And from the point F drawe (by the 12. of the first) vnto the line A B a perpendicular line F H: and extend it to the point K. And drawe the right lines A K and K B. And againe, from the point F draw (by the same) vnto the line A K, a perpendicular line F N: and extend F N to the point M, which line let cut the line A B in the point L. And draw a right line from the point K to the point L. Now forasmuch as the circumference A B C G is equall to the circumference A E D G, of which the circumference A B C is equall to the circumference A E D, therfore the rest of the circumference, namely, C G, is equall to the rest of the circumference, namely, to D G. But the circumference C D is subtended of the side of a pentagon: Wherefore the circumference C G is subtended of the side of a decagon figure. And forasmuch as the line B H is equall to the line H A (by the 3. of the third) and the line F H is common to them both, & the angles at the point H are right angles. Wherefore (by the 4. of the first) the angle A F K is equall to the angle K F B. Wherefore also (by the 26. of the third) the circumference A K, is equall to the circumference K B. Wherefore the circumference A B is double to the circumference B K. Wherefore the right line A K is the side of a decagon figure. And by the same reason also the circumference A K is double to the circumference K M. And forasmuch as the circumference A B is double to the circumference B K, but the circumference C D is equall to the circumference A B: wherefore the circumference C D is double to the circumference B K. But the circumference C D is double to the circumference C G: wherefore the circumference C G is equall to the circumference B K. But the circumference B K is double to the circumference K M (for that the circumference K A is double thereunto): wherefore also the circumference C G is double to the circumference K M. But the circumference C B is also double to the circumference B K (for the circumference C B is equall to the circumference B A). Wherefore the whole circumference G B is double to the whole circumference B M, by the 12. of the fift. Wherefore also the angle G F B is double to the angle B F M, by the last of the sixt. But the angle G F B is double to the angle F A B (by the 32. of the first, or 20. of the third). For the angle F A B is equall to the angle A B F. Wherefore the angle B F L is equall to the angle F A B, by the 15. of the fift. And the angle A B F is common to the two triangles A B F and B F L. Wherefore (by the Corollary of the 32. of the first) the angle remayning A F B is equall to the angle remayning B L F. Wherefore the triangle A B F is equiangle to the triangle B F L. Wherefore (by the 4. of the sixt) proportionally as the right line A B is to the right line B F, so is the same right line F B to the

Demonstration.



the right line B L: wherefore that which is contained vnder the lines A B and B L is equall to the ſquare of the line B F, by the 17. of the ſixt. Againe foraſmuch as the line A L is equall to the line L K [for by the laſt of the ſixt, the angle K F L is equall to the angle A F L, which equall angles are contained vnder the lines F K, F L, and F A, F L, & the line F K is equall to the line F A, and the line F L is common to them both. Wherefore, by the 4. of the firſt, the line A L is equall to the line L K] and the line L N is common to them both, & maketh right angles at the point N, and (by the 3. of the third) the baſe A N is equall to the baſe K N. Wherfore alſo the angle L K N is equall to the angle L A N. But the angle L A N is equall to the angle K B L (by the 5. of the firſt). Wherefore the angle L K N is equall to the angle K B L. And the angle K A L is common to both the triangles A K B and A K L. Wherefore the angle remaining A K B is equall to the angle remayning A L K (by the Corollary of the 32. of the firſt). Wherefore the triangle K B A is equiangle to the triangle K L A. Wherefore (by the 4. of the ſixt) proportionally as the right line B A is to the right line A K, ſo is the ſame right line K A to the right line A L. Wherefore that which is contained vnder the lines B A and A L is equall to the ſquare of the line A K (by the 17. of the ſixt). And it is proued that that which is contained vnder the lines A B and B L, is equall to the ſquare of the line B F. Wherefore that which is contained vnder A B & B L together with that which is contained vnder B A and A L (which by the 2. of the ſecond, is the ſquare of the line B A) is equall to the ſquares of the lines A F and A K. But the line B A is the ſide of the pentagon figure, and A F the ſide of the hexagon figure (by the Corollary of the 15. of the fourth), and A K the ſide of the decagon figure. Wherefore the ſide of a pentagon figure, containeth in power both the ſide of an hexagon figure, and of a decagon figure, being deſcribed all in one and the ſelfe ſame circle: which was required to be demonſtrated.

¶ A Corollary added by *Fluſſas*.

A perpendicular line from any angle drawen to the baſe of a pentagon, paſſeth by the centre.

For if we drawe a right line from the poynt A to the poynt C, and an other from the poynt A to the point D: thoſe right lines ſhall be equall, by the 4. of the firſt: and therefore in the triangle A C D the angles at the points C and D are, by the 5. of the firſt, equall. But the angles made at the point where the line A G cutteth the line C D, are by ſuppoſition right angles: wherefore, by the 26. of the firſt, the line C D is by the line A G diuided into two equall partes, and it is alſo diuided perpendicularly: wherefore by the corollary of the firſt of the third in the line A G is the centre of the circle: and therefore the line A G paſſeth by the centre.

¶ *The 11. Theoreme.* *The 11. Propoſition.*

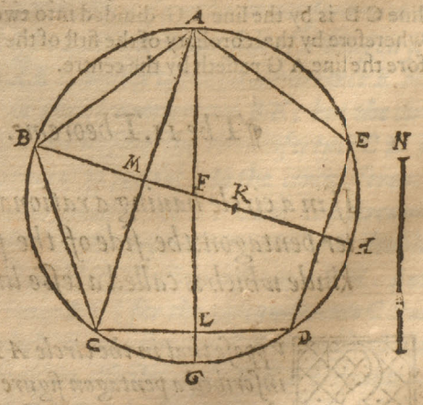
If in a circle hauing a rationall line to his diameter be inſcribed an equilater pentagon: the ſide of the pentagon is an irrationall line, and is of that kinde which is called a leſſe line.

Vppoſe that in the circle A B C D E hauing a rationall line to his diameter, be inſcribed a pentagon figure A B C D E. Then I ſay that the ſide of the pentagon figure A B C D E, namely, the ſide A B, is an irrational line of that kinde which is called a leſſe line. Take (by the 1. of the third) the centre of the circle, and let the ſame be the point F, and draw a right line from the point A to the point F, and an other from the point F to the point B, and extend thoſe lines to the pointes G and H. And draw a right line from the point A to the point C. And from the ſemidiameter F H take the fourth part (by the 9. of the ſixt) and let the ſame be F K: But the line F H is rationall (for that it is the halfe of the diameter which is ſuppoſed to be rationall), wherefore alſo the Conſtruction.

line

Demonſtration.

line F K is rationall. And the line or ſemidiameter B F is rationall. Wherefore the whole line B K is rationall. And foraſmuch as the circumference A C G is equall to the circumference A D G, of which the circumference A B C is equall to the circumference A E D, wherefore the reſidue C G is equall to the reſidue G D. Now if we drawe a right line from the point A to the point D, it is manifeſt that the angles A L C and A L D are right angles. For foraſmuch as the circumference C G is equall to the circumference G D, therefore (by the laſt of the ſixth) the angle C A G is equall to the angle D A G. And the line A C is equall to the line A D, for that the circumferences which they ſubtend are equall, and the line A L is common to them both, therefore there are two lines A C and A L equall to two lines A D and A L, and the angle C A L is equall to the angle D A L. Wherefore (by the 4. of the firſt) the baſe C L is equall to the baſe L D, and the reſt of the angles to the reſt of the angles, and the line C D is double to the line C L. And by the ſame reaſon may it be proued, that the angles at the point M are right angles, and that the line A C is double to the line C M. Now foraſmuch as the angle A L C is equall to the angle A M F, for that they are both right angles, and the angle L A C is common to both the triangles A L C and A M F: wherefore the angle remayning, namely, A C L, is equal to the angle remayning A F M, by the corollary of the 32. of the firſt. Wherefore the triangle A C L is equiangle to the triangle A M F. Wherefore proportionally, by the 4. of the ſixth, as the line L C is to the line C A, ſo is the line M F to the line F A. And in the ſame proportion alſo are the doubles of the antecedents L C and M F (by the 15. of the fifth). Wherefore as the double of the line L C is to the line C A, ſo is the double of the line M F to the line F A. But as the double of the line M F is to the line F A, ſo is the line M F to the halfe of the line F A, by the 15. of the fifth, wherefore as the double of the line L C is to the line C A, ſo is the line M F to the halfe of the line F A, by the 11. of the fifth. And in the ſame proportion, by the 15. of the fifth, are the halues of the conſequents, namely, of C A and of the halue of the line A F. Wherefore as the double of the line L C is to the halfe of the line A C, ſo is the line M F to the fourth part of the line F A. But the double of the line L C is the line D C, and the halfe of the line C A is the line C M, as hath before bene proued, and the fourth part of the line F A is the line F K (for the line F K is the fourth part of the line F H by conſtruction). Wherfore as the line D C is to the line C M, ſo is the line M F to the line F K. Wherfore by compoſition (by the 18. of the fifth) as both the lines D C and C M are to the line C M, ſo is the whole line M K to the line F K. Wherefore alſo (by the 22. of the ſixt) as the ſquares of the lines D C and C M are to the ſquare of the line C M, ſo is the ſquare of the line M K to the ſquare of the line F K. And foraſmuch as (by the 8. of the thirtenth) a line which is ſubtended vnder two ſides of a pentagon figure, as is the line A C, being diuided by an extreame & meane proportion, the greater ſegment is equall to the ſide of the pentagon figure, that is, vnto the line D C: and (by the 1. of the thirtenth) the greater ſegment hauing added vnto it the halfe of the whole, is in power quintuple to the ſquare made of the halfe of the whole: and the halfe of the whole line A C is the line C M. Wherefore the ſquare that is made of the lines D C and C M, that is, of the greater ſegment and of the halfe of the whole, as of one line, is quintuple to the ſquare of the line C M, that is, of the halfe of the whole. But as the ſquare made of the lines D C and C M, as of one line, is to the ſquare of the line C M, ſo is it proued, that the ſquare of the line M K is to the ſquare of the line F K. Wherefore the ſquare of the line M K is quintuple to the ſquare of the line F K. But the ſquare of the line K F is rationall, as



hath

hath before bene proued, wherefore alſo the ſquare of the line M K is rationall, by the 9. diffinition of the tenth, for the ſquare of the line M K hath to the ſquare of the line K F that proportion that number, hath to number, namely, that 5. hath to 1. and therefore the ſayd ſquares are commenſurable, by the 6. of the tenth. Wherefore alſo the line M K is rationall. And foraſmuch as the line B F is quadruple to the line F K (for the ſemidiameter BF is equal to the ſemidiameter F H), therfore the line B K is quintuple to the line F K. Wherefore the ſquare of the line B K is 25. times ſo much as the ſquare of the line K F, by the corollary of the 20 of the ſixt. But the ſquare of the line M K, is quintuple to the ſquare of the F K, as is proued. Wherfore the ſquare of the line B K is quintuple to the ſquare of the line K M. Wherfore ỹ ſquare of the line B K, hath not to ỹ ſquare of the line K M, that proportiõ that a ſquare number hath to a ſquare number, by the corollary of the 25. of the eight. Wherefore (by the 9. of the tenth) the line B K is incommenſurable in length to the line K M, and either of the lines is rationall. Wherefore the lines B K and K M are rationall commenſurable in power onely. But if frõ a rationall line be taken away a rationall line being commenſurable in power onely to the whole, that which remayneth is irrationall, and is (by the 73. of the tenth) called a reſiduall line. Wherefore the line M B is a reſiduall line. And the line conueniently ioyned vntoit, is the line M K. Now I ſay that the line B M is a fourth reſiduall line. Vnto the exceſſe of the ſquare of the line B K aboue the ſquare of the line K M, let the ſquare of the line N beequall (which exceſſe how to finde out, is taught in the aſſumpt put after the 13 propoſition of the tenth). Wherefore the line B K is in power more then the line K M by the ſquare of the line N. And foraſmuch as the line K F is commenſurable in length to the line F B, for it is the fourth part thereof, therefore (by the 16. of the tenth) the whole line K B is commenſurable in length to the line F B. But the line F B is commenſurable in length to the line B H, namely, the ſemidiameter to the diameter: wherefore the line B K is commenſurable in length to the line B H, by the 12. of the tenth. And foraſmuch as the ſquare of the line B K is quintuple to the ſquare of the line K M, therefore the ſquare of the line B K hath to the ſquare of the line K M that proportion that fiue hath to one. Wherefore by conuerſion of proportion (by the corollary of the 19. of the fifth) the ſquare of B K hath to the ſquare of the line N, that proportion that fiue hath to fower: & therfore it hath not that proportiõ that a ſquare number hath to a ſquare number, by the corollary of the 25. of the eight. Wherfore, the line B K is incommenſurable in length to the line N (by the 9. of the tenth). Wherfore the line B K is in power more then the line K M, by the ſquare of a line incommenſurable in length to the line B K. Now then foraſmuch as the whole line B K is in power more then the line conueniently ioyned, namely, then K M, by the ſquare of a line incõmenſurable in length to the line B K, and the whole line B K is commenſurable in length to the rationall line geuen B H: therefore the line M B is a fourth reſiduall line, by the diffinition of a fourth reſiduall line. But a rectangle parallelogramme contayned vnder a rationall line and a fourth reſidual line, is irrationall, and the line which contayneth in power the ſame parallelogramme is alſo irrationall, and is called a leſſe line (by the 94. of the tenth). But the line A B contayneth in power the parallelogramme contayned vnder the lines H B and B M (for if we drawe a right line from the point A to the point H, the triangle A B H ſhall be like to the triangle A B M, by the 8. of the ſixth. For from the right angle B A H is drawen to the baſe B H a perpendicular line. And therefore as the line B H is to the line B A, ſo is the line A B to the line B M. this followeth alſo of the corollary of the ſayd 8. of the ſixth. Wherefore the line A B which is the ſide of the pentagon figure, is an irrationall line of that kinde which is ealled a leſſe line. If therefore in a circle hauing a rationall line to his diameter be inſcribed an equilater pentagon, the ſide of the pentagon is an irrationall line, and is of that kinde which is called a leſſe line: which was required to be demonſtrated.

¶ The 12. Theoreme. The 12. Propoſition.

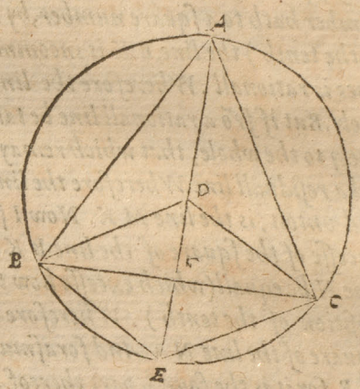
If in a circle be deſcribed an equilater triangle: the ſquare made of the ſide of

the triangle, is treble to the ſquare made of the line, which is drawen from the centre of the circle to the circumference.

Conſtruction.

Demonſtration.

SVppoſe that ◺ A B C be a circle, and in it deſcribe an equilater triangle ◺ A B C. Then I ſay that the ſquare made of the ſide of the triangle A B C, is treble to the ſquare made of the line drawen from the center of the circle ◺ A B C to the circumference. Take (by the 1. of the third) the centre of the circle, and let the ſame be D. And draw a right line from the point A to the poynt D, and extend it to the point E. And draw a right line from the point B to the poynt E. Now foraſmuch as the triangle ◺ A B C is equilater, therefore eche of theſe three circumferences A B, A C, & B E C is the third part of the whole circumference of the circle A B C: wherefore the circumference B E is the ſixth part of the circūference of the circle [for the circumferēce of the ſemicircle A B E is equall to the circumference of the ſemicircle A C E, from which taking away equal circumferences A B and A C, the circumference remayning B E ſhalbe equal to the circumference remayning E C]: wherefore the right line B E is the ſide of an equilater hexagon figure deſcribed in the circle. Wherefore it is equall to the line drawen from the centre of the circle to the circumference, that is vnto the line D E (by the corollary of the 15. of the ſixth). And foraſmuch as the line A E is double to the line D E, therefore the ſquare of the line A E is quadruple to the ſquare of the line D E (by the 4. of the ſecond): that is, to the ſquare of the line B E. But the ſquare of the line A E is equall to the ſquares of the lines A B, and B E (by the 47. of the firſt) for the angle A B E is (by the 31. of the third) a right angle. Wherfore the ſquares of the line A B & B E are quadruple to the ſquare of the line B E. Wherefore taking away the ſquare of the line B E, the ſquare of the line A B ſhalbe treble to the ſquare of B E: but the line B E is equall to the line D E. Wherefore the ſquare of the line A B is treble to the ſquare of the line D E. Wherefore the ſquare made of the ſide of the triangle, is treble to the ſquare made of the line drawen frō the centre of the circle to the circumference: which was required to be proued.



⁂ A Corollary added by *Campane.*

Hereby it is manifeſt, that the line B C, which is the ſide of the equilater triangle, diuideth the ſemidiameter D E into two equall parts. For let the poynt of the diuiſion be F. And ſuppoſe a line to be drawen from the poynt D to the B, and an other from the poynt D to the poynt C. Now it is manifeſt (by the 4. of the firſt) that the line B F is equall to the line F C, and therefore (by the 3. of the third) all the angles at the poynt F are right angles. Wherefore (by the 47. of the firſt) the ſquare of the line B D is equall to the ſquares of the lines B F and F D, and by the ſame the ſquare of the line B E is equall to the ſquares of the lines B F, and F E: but the line B D is equall to the line B E (as hath before bene proued). Wherefore by the common ſentence the twoo ſquares of the two lines B F and F D are equall to the two ſquares of the lines B F, and F E. Wherefore taking away the ſquare of the line B F which is cōmon to them both: the reſidue, namely, the ſquare of the line D F ſhalbe equall to the reſidue, namely, to the ſquare of the line F E. Wherfore alſo the line F D is equal to the line F E. Wherfore hereby it is manifeſt that a perpendicular line drawen from the centre of a circle to the ſide of an equilater triangle inſcribed in it, is equall to halfe of the line drawen from the centre of the ſame circle, to the circumference thereof.

⁂ A Corollary added by *Fluſſas.*

The ſide of an equilater triangle is in power ſeſquitertia to the perpendicular line which is drawē from one of the angles to the oppoſite ſide. For of what parts the line A B contayneth in power 12. of

ſuch

ſuch parts the line B F which is the halfe of A B contayned in power 3. Wherefore the reſidue, namely, the perpendicular line A F contayneth in power of ſuch parts 9. (for the ſquares of the lines AF, and B F are by the 47. of the firſt equall to the ſquare of the line A B). Now 12. to 9. is ſeſquitertia: wherfore the power of the line A B is to the power of the line A F in ſeſquitertia proportion.

This Corollary is the 11. propoſition of the 14. booke after Campane.

Moreouer the ſide of the triangle is the meane proportionall betwene the diameter and the perpendicular line: For (by the Corollary of the 8. of the ſixth) the line AE is to the line AB as the line A B is to the line A F.

Farther the perpendicular line drawen from the angle diuideth the baſe into two equal parts and paſſeth by the center. For if there ſhould be drawen any other right line frō the point A to the poynt F, thē that which is drawen by the point D, two right lines ſhould include a ſuperficies, which is impoſſible. Wherefore the contrary followeth, namely, that the line, which being drawen from the angle paſſeth by the center, is a perpendicular line to the baſe (by the 3 of the third).

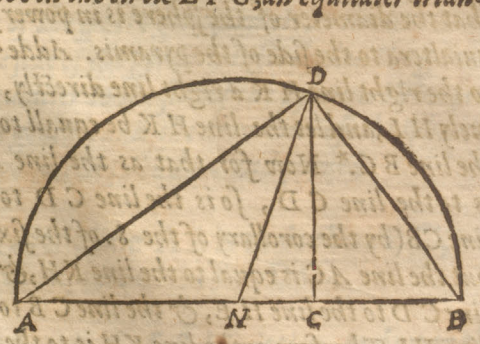
This Corollary is the 3. Corollary after the 17. propoſition of the 14. booke after Campane.

The 1. Probleme. The 13. Propoſition.

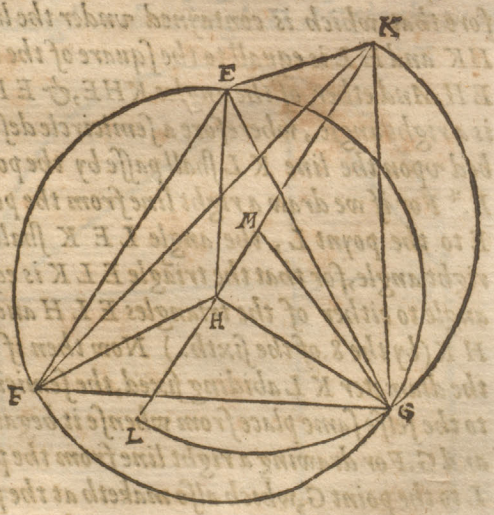
To make a Pyramis, and to comprehend it in a ſphere geuen: and to proue that the diameter of the ſphere is in power ſeſquialtera to the ſide of the Pyramis.*

* *By the name of a Pyramis both here & in this booke following, Vnderſtand a Tetrahedron.*

VPpoſe that the diameter of the ſphere geuen be A B, and diuide A B in the poynt C, ſo that the line A C be double to the line B C (by the 9. of the ſixth). And vpon the line A B, making the center the point N, deſcribe a ſemicircle A D B. And (by the 11. of the firſt) from the point C rayſe vp vnto the line A B a perpendicular line C D. And drawe a right line from the point D to the point A. And deſcribe a circle E F G hauing his ſemidiameter equall to the line C D. And deſcribe in the circle E F G, an equilater triangle E F G (by the 2. of the fourth). And (by the 1. of the third) take the cētre of the circle, and let the ſame be the point H. And draw theſe right lines E H, H F, & H G. And (by the 12. of the eleuenth) frō the point H rayſe vp vnto the playne ſuperficies of the circle E F G a perpēdicular line H K, & let the line H K be equal to the right line A C. And draw theſe right lines K E, K F, & K G. Now foraſmuch as the line H K is erected perpēdicularly to the plaine ſuperficies of the circle E F G, therfore (by the 2. definition of the eleuenth) it maketh right angles with all the right lines that touch it, and which are in the ſelfe ſame ſuperficies of the circle E F G. But it toucheth euery one of theſe right lines, H E, H F and H G: Wherefore the right line H K is erected perpendicularly to euery one of theſe lines H E, H F, and H G. And foraſmuch as the line A C is equal to the line H K, and the line C D to the line H E, and they comprehende right angles, therfore the baſe D A is equall to the baſe K E (by the 4 of the firſt). And by the ſame reaſon ech of the lines K F, and K G is equall to the line D A. Wherefore the three lines K E, K F, and K G are equall the one to the other. And foraſmuch as the line A C is double to the line C B, (by conſtruction): therefore the line A B is treble to the line B C: but as the line A B is to the

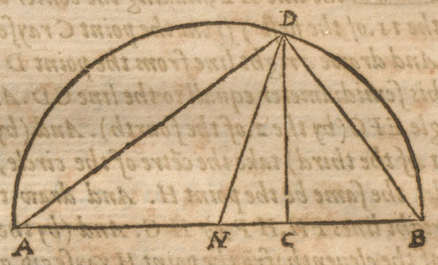


First part of the demonſtration.



This Assumpt is againe at the end of the demonstratiõ of the proposition proued.

line B C, so is the square of the line A D to the square of the line D C. [Whiche may thus be proued. It is manifest (by the Corollary of the 8. of sixth) that the line C D is the meane proportionall betwene the lines A C and C B. Wherfore (by the corollary of the 20. of the same) the square of the line A C is to the square of the line C D, as the line A C is to the line C B: wherefore by composition (by the 18. of the fiueth) the squares of the lines A C and C D are to the square of the line C D, as the line A B is to the line B C. But the squares of the lines A C and C D are (by the 47. of the first) equall to the square of the line A D: wherefore (by the 7. of the fiueth) the square of the line A D is to the square of the line D C, as the line A B is to the line B C.] Wherefore the square of the line A D is treble to the square of the line D C. And forasmuch as the line K E is equall to the line A D (as it hath ben proued) and the line H E is put equall to the line C D: therefore the square of the line K E is triple to the square of the line H E. But vnto the square of the same line H E is (by the 12. of the thirteenth) the square of the line F E treble: wherefore the line E F is equall to the line K E. Now the lines K E, K F, and K G are equall the one to the other, as it hath before ben proued, and so also are the lines E F, F G and G E, for that they are the sides of an equilater triangle. Wherefore euery one of these lines E F, F G, and G E, is equall to euery one of the lines K E, K F and K G. Wherefore these fower triangles E F G, K E F, K F G and K G E are equilater. Wherefore there is made a Pyramis consisting of fower equall and equilater triangles, whose base is the triangle E F G and toppe the poynt K.



Second part of the Construction.

Second part of the demõstratiõ.

* Looke at the end of this demonstration for an other construction and demonstration of this second part after Flussat.

Now it is required to comprehende the same Pyramis in the sphere geuẽ, and to proue that the diameter of the sphere is in power sesquialtera to the side of the pyramis. Adde vnto the right line H K a right line directly, namely H L, and let the line H K be equall to the the line B C.* Now for that as the line A C is to the line C D, so is the line C D to the line C B (by the corollary of the 8. of the sixth) but the line A C is equal to the line K H, & the line C D to the line H E, & the line C B to the line H L. Therfore as the line K H is to the line H E, so is the line H E to the line H L. Wherefore that which is contayned vnder the lines H K and H L is equall to the square of the line E H. And ether of the angles K H E, & E H L is a right angle, wherefore a semicircle described vpon the line K L shall passe by the poynt E.* For if we draw a right line from the point E to the poynt L, the angle L E K shalbe a right angle, for that the triãgle E L K is equiangle to either of the triangles E L H and E H K (by the 8. of the sixth.) Now then if the

* Reade the two assumpts added by Campane (which are set at the end of the demonstration of this proposition) for the better vnderstanding of this reason.

the diameter K L abiding fixed, the semicircle be turned round about, vntil it returne vnto the selfe same place from whense it began to be moued, it shall also passe by the pointes F and G. For drawing a right line from the ponit F to the point L, and an other from the poynt L to the point G, which also maketh at the points F and G right angles, the pyramis shall be contayned in the sphere geuen. For the line K L being the diameter of the sphere is equall to the line A B, which is the diameter of the sphere geuen, for the line K H is put equall to the line A C, and the line H L to the line C B.

[In the ſemicircle A D B of the former figure drawe the line D N. And diuide the line K L into two equall parts in the point M. And draw a line from M to G. And foraſmuch as by conſtruction the line K H is equall to the line A C, and the line H L to the line C B: therefore the whole line A B is equal to the whole line K L. Wherefore alſo the halfe of the line K L, namely, the line L M, is equal to the ſemidiameter B N: wherefore taking away from thoſe equall lines, equall parts B C and L H, the reſidues N C, and M H ſhalbe equall. Wherefore in the two triangles M H G and N C D, the two ſides about the equall right angles D C N and G H M, namely, the ſides D C, C N, and G H, H M, are equall: wherfore the baſes M G and N D are equall (by the 4. of the firſt.) And by the ſame reaſon may it be proued that right lines drawen from the poynt M to the points E and F are equal to the line N D. But the right line N D is equall to the line A N, which is drawen from the centre to the circumference: wherefore the line M G is equall to the line M K, & alſo to the lines M E, M F and M L. Wherfore making the cẽtre the poynt M, and the ſpace M K or M G deſcribe a ſemicircle K G L: and the diameter K L abiding fixed let the ſayd ſemicircle K G L be moued rounde about vntill it returne to the ſame place from whence it began to be moued: and there ſhalbe deſcribed a ſphere about the centre M (by the 12. diffinition of the eleuenth) touching euery one of the angles of the Pyramis which are at the points K, E, F, G: for thoſe angles are equally diſtãt from the centre of the ſphere, namely, by the ſemidiameter of the ſayd ſphere, as hath before bene proued. Wherefore in the ſphere geuen whoſe diameter is the line K-L, or the line A B, is inſcribed a Tetrahedron E F G K.]

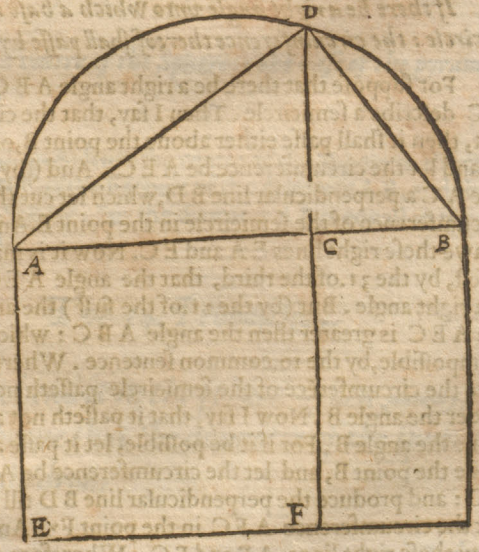
An other conſtruction and demonſtration of the ſecond part after Fluſſas.

Now I ſay, that the diameter of the ſphere is in power ſeſquialtera to the ſide of the Pyramis. For foraſmuch as the line A C is double to the line C B (by cõſtructiõ) therfore the line A B is treble to the line B C. Wherfore by conuerſion) by the corollary of the 19. of the ſiueth) the line A B is ſeſquialtera to the line A C. But as the line B A is to the line A C, ſo is the ſquare of the line B A, to the ſquare of the line A D. For if we draw a right line frõ the point B, to the point D, as the line B D is to the line A D, ſo is the ſame A D to the line A C, by reaſõ of the likenes of the triangles D A B, & D A C (by the 8. of the ſixth): & by reaſon alſo that as the firſt is to the third ſo is the ſquare of the firſt to the ſquare of the ſecõd (by the corollary of the 20. of the ſixth). Wherfore the ſquare of the line B A, is ſeſquialter to the ſquare of the line A D. But the line B A is equal to the diameter of the ſphere geuẽ, namely, to the line K L, as hath bene proued, & the line A D is equal to the ſide of the pyramis inſcribed in the ſphere. Wherfore ẏ diameter of the ſphere is in power ſeſquialter to the ſide of the pyramis. Wherfore there is made a pyramis comprehended in a ſphere geuen, and the diameter of the ſphere is ſeſquialtera to the ſide of the pyramis: which was required to be done and proued.

Third part of the demonſtration.

¶ An other demonſtration to proue that as the line A B is to the line B C, ſo is the ſquare of the line A D to the ſquare of the line D C.

Let the deſcription of the ſemicircle A D B be as in the firſt deſcription. And vpon the line A C deſcribe (by the 46. of the firſt) a ſquare E C, and make perfecte the parallelogrãme F B. Now foraſmuch as the triangle D A B is equiangle to the triangle D A C (by the 32. of the ſixt: therfore as the line B A is to the line A D, ſo is the line D A to the line A C, by the 4. of the ſixt. Wherefore that which is contained vnder the lines B A and A C, is equall to the ſquare of the line A D, by the 17. of the ſixt. And for that as the line A B is to the line B C, ſo is the parallelogramme E B to the parallelogramme F B, by the 1. of the ſixt: and the parallelogramme E B is that which is contained vnder the lines B A and A C (for the line E A is equall to the line A C): and the parallelogramme B F is that which is contained vnder the lines A C and B C. Wherefore as the line A B is to the line B C, ſo is that which is contained vnder the lines B A and

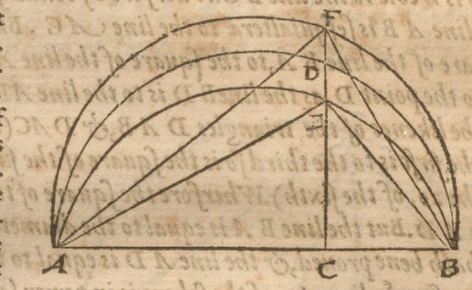


A C, to that which is contained vnder the lines A C and C B. But that which is contained vnder the lines B A and A C, is equall to the ſquare of the line A D, by the Corollary of the 8. of the ſixt, and that which is contained vnder the lines A C and C B, is equall to the ſquare of the line C D, for the perpendicular line D C is the meane proportionall betwene the ſegmentes of the baſe, namely, A C and C B, by the former Corollary of the 8. of the ſixt, for that the angle A D B is a right angle. Wherefore as the line A B is to the line B C, ſo is the ſquare of the line A D to the ſquare of the line D C, by the 11. of the fift: which was required to be proued.

¶ Two Aſſumptes added by *Campane.*

Firſt Aſſumpt.

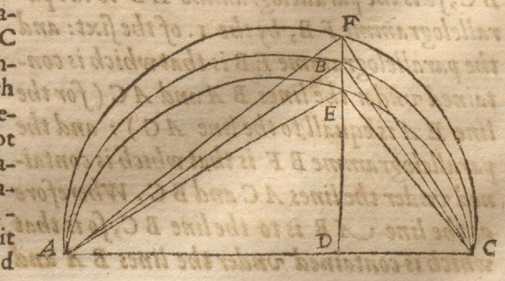
Suppoſe that vpon the line A B be erected perpendicularly the line D C, which line D C let be the meane proportionall betwene the partes of the line A B, namely, A C & C B: ſo that as the line A C is to the line C D, ſo let the line C D be to the line C B. And vpon the line A B deſcribe a ſemicircle. The I ſay, that the circumference of that ſemicircle ſhall paſſe by the point D, which is the end of the perpendicular line. But if not, then it ſhall either cut the line C D, or it ſhall paſſe aboue it, and include it not touching it. Firſt let it cut it in the point E. And drawe theſe right lines E B and E A. Wherefore by the 31. of the third, the whole angle A E B is a right angle. Wherefore by the firſt part of the Corollary of the 8. of the ſixt, the line A C is to the line E C, as the line E C is to the line C B. But by the 8. of the fift, the proportion of the line A C to the line E C, is greater then the proportion of the ſame line A C to the line C D (for the line C E is leſſe then the line C D). Now for that the line C B is to the line C B, as the line A C is to the line C B, and the line C D is to the line C B, as the line A C is to the line C D, therefore by the 13. of the fift, the proportion of the line E C to the line C B, is greater then the proportion of the line C D to the line C B. Wherefore by the 10. of the fift, the line E C is greater then the line D C, namely, the part greater then the whole: which is impoſſible. Wherefore the circumference ſhall not cut the line C D. Now I ſay, that it ſhall not paſſe aboue the line C D, and not touch it in the point D. For if it be poſſible, let it paſſe aboue it, and extend the line C D to the circūference, and let it cut it in the point F. And draw the lines F B and F A, and it ſhall followe as before that the line C D is greater then the line C F: which is impoſſible. Wherefore that is manifeſt which was required to be proued.



¶ *Second Aſſumpt.*

If there be a right angle vnto which a baſe is ſubtended, and if vpon the ſame be deſcribed a ſemicircle: the circumference thereof ſhall paſſe by the point of the right angle.

For ſuppoſe that there be a right angle A B C, vnto which ſubtend the baſe A C, and vpon the line A C deſcribe a ſemicircle. Then I ſay, that the circumference thereof ſhall paſſe by the point B. For if not, then it ſhall paſſe either aboue the point B, or vnder the point B. Firſt let it paſſe vnder the point B, and let the circumference be A E C. And (by the 12. of the firſt) from the point B drawe vnto the line A C a perpendicular line B D, which let cut the circumference of the ſemicircle in the point E. And drawe theſe right lines E A and E C. Now it is manifeſt, by the 31. of the third, that the angle A E C is a right angle. But (by the 21. of the firſt) the angle A E C is greater then the angle A B C: which is impoſſible, by the 10. common ſentence. Wherefore the circumferēce of the ſemicircle paſſeth not vnder the angle B. Now I ſay, that it paſſeth not aboue the angle B. For if it be poſſible, let it paſſe aboue the point B, and let the circumference be A F C: and produce the perpendicular line B D till it cut the circumference A F C in the point F. And draw theſe right lines A F and F C. Wherefore a-



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gaine, by the 31. of the third, the angle A F C is a right angle. But by ſuppoſition, the angle A B C is a right angle, and is, by the 21. of the firſt, greater then the right angle A F C: which againe by the foreſayd common ſentence, is impoſſible. Wherfore a ſemicircle deſcribed vpon the baſe A C, paſſeth neither vnder the point B, nor aboue it. Wherefore it paſſeth by it: which was required to be proued.

The conuerſe of this was added after the demonſtration of the 31. of the third, out of Pelitarius. And theſe two Aſſumptes of Campane are neceſſary, for the better vnderſtanding of the demonſtration of the ſecond part of this 13. Propoſition, wherein is proued that the pyramis is contained in the Sphere geuen.

¶ Certaine Corollaryes added by *Fluſſas.*

❊ *Firſt Corollary.*

The diameter of the Sphere is in power quadruple ſeſquialtera to the line which is drawen from the centre to the circumference of the circle which containeth the baſe of the pyramis.

For foraſmuch as it hath bene proued, that the diameter K L is in power ſeſquialter to the ſide E F: and it is proued alſo, by the 12. of this booke, that the ſide E F is in power triple to the line E H (which is drawen from the centre of the circle contayning the triangle E F G). But the proportion of the extremes, namely, of the diameter to the line E H, conſiſteth of the proportions of the meanes, namely, of the proportion of the diameter to the line E F, and of the proportion of the line E F to the line E H, by the 5. definition of the ſixt: which proportions, namely, triple, and ſeſquialter, added together, make quadruple ſeſquialter (as it is eaſie to proue by that which was taught in the declaration of the 5. definition of the ſixt booke). Wherefore the Corollary is manifeſt.

¶ *Second Corollary.*

Onely the line which is drawen from the angle of the pyramis to the baſe oppoſite vnto it, & paſſing by the center of the Sphere, is perpendicular to the baſe, and falleth vpon the centre of the circle which containeth the baſe.

This Corollary is the 15. propoſition of the 14. booke after Campane.

For if any other line (then the line K M H which is drawen by the cenrre of the Sphere to the centre of the circle) ſhould fall perpendicularly vpon the plaine of the baſe, then, from one and the ſelfe ſame point ſhould be drawen to one and the ſelfe ſame plaine two perpendicular lines, contrary to the 13. of the eleuenth: which is impoſſible. Farther if from the toppe K ſhould be drawen to the center of the baſe, namely, to the point H, any other right line not paſſing by the centre M, two right lines ſhoulde include a ſuperficies contrary to the laſt common ſentence: which were abſurde. Wherefore onely the line which is drawen by the center of the Sphere to the centre of the baſe, is perpendicular to the ſayd baſe. And the line which is drawen from the angle perpendicularly to the baſe, ſhall paſſe by the centre of the Sphere.

❊ *Third Corollary.*

The perpendicular line which is drawen from the centre of the Sphere to the baſe of the pyramis, is equall to the ſixt part of the diameter of the Sphere.

This Corollary Campane putteth as a Corollary after the 17. propoſition of the 14. booke.

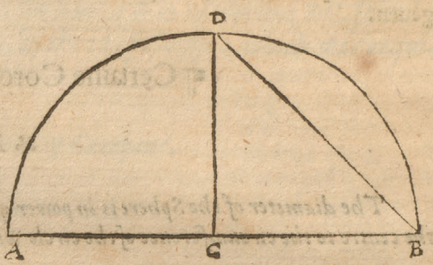
For it is before proued, that the line M H (which is drawen from the centre of the Sphere to the centre of the baſe) is equall to the line N C: which line N C is the ſixt part of the diameter A B, and therfore the line M H is the ſixt part of the diameter of the Sphere. For the diameter A B is equall to the diameter of the Sphere, as hath alſo before bene proued.

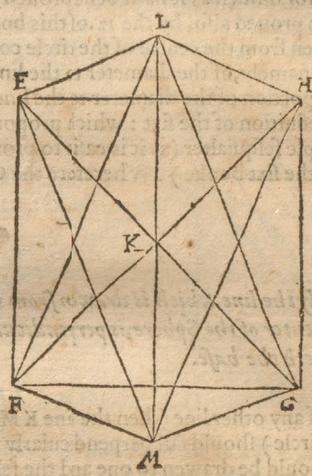
¶ *The 2. Probleme.* *The 14. Propoſition.*

To make an octohedron, and to cõprehend it in the ſphere geuen, namely, that wherein the pyramis was comprehended: and to proue that the diameter of the ſphere is in power double to the ſide of the octohedron.

Take

Construction. TAke the diameter of the former sphere geuen, which let be the line A B: and diuide it (by the 10. of the first into two equall partes in the point C. And describe vpon the line A B a semicircle A D B. And (by the 11. of the first) frō the point C rayse vp vnto the line A B a perpendicular line C D. And draw a right line from the point D to the point B. And describe a square E F G H hauing euery one of his sides equall to the line B D. and draw the diagonal lines F H & E G, cutting the one the other in the point K. And (by the 12. of the eleuenth) from the point K (namely, the point where the lines F H and E G cut the one the other) rayse vp to the playne superficies, wherein the square E F G H is, a perpendicular line K L, and extend the line K L on the other side of the playne superficies to the point M. And let eche of the lines K L and K M be put equall to one of these lines K E, K F, K H or K G. And draw these right lines L E, L F, L G, L H, M E, M F, M G, and M H. Now forasmuch as the line K E is (by the corollary of the 34. of the first) equall to the line K H, and the angle* E K H is a right angle, therefore the square of H E is double to the square of E K, by the 47. of the first: Agayne forasmuch as the line L K is equall to the line K E by position, and the angle L K E is, by the second diffinition of the eleuenth, a right angle: therefore the square of the line E L is double to the square of the line E K: and it is proued that the square of the line H E is double to the square of the line E K. Wherefore the square of the line L E is equall to the square of the line E H. Wherefore also the line L E is equall to the line E H. And by the same reason the line L H is also equall to the line H E. Wherefore the triangle L H E is equilater. In like sort may we proue that euery one of the rest of the triangls whose bases are the sides of the square E F G H, and toppes the pointes L and M, are equilater. The sayd eight triangles also are equall the one to the other, for euery side of eche is equall to the side of the square E F G H. Wherfore there is made an octohedron cōtained vnder eight triangles whose sides are equall. Now it is required to comprehend it in the sphere geuen, and to proue that the diameter of the sphere is in power double to the side of the octohedron.





Frist part of the demonstration.

* For the 4. angles at the point K are equall to fower right angles by the Corollary of the 15. of the first: and those 4. angle, are equall the one to the other by the 8. of the first: and therefore ech is a right angle.

Second part of the demōstratiō. Forasmuch as these three lines L K, K M, and K E are equall the one to the other, therfore a semicircle described vpon the line L M shall passe also by the point E. And by the same reason, if the semicircle be turned round about, vntill it returne vnto the selfe same place from whence first it began to be moued, it shal passe by the pointes F, G, H, and the octohedron shall be comprehended in a sphere. I say also that it is comprehended in the sphere geuen. For forasmuch as the line L K is equal to the line K M, by position, and the line K E is common to them both. and they contayne right angles by the 3. diffinition of the eleuenth, therefore (by the 4. of the first) the base L E is equall to the base E M. And forasmuch as the angle L E M is a right angle, by the 31. of the third, for it is in a semicircle, as hath bene proued, therefore the square of the line L M is double to the square of the line L E by the 47. of the first. Againe forasmuch as the line A C is equall to the line B C, therefore the line A B is double to the line B C, by the diffinition of a circle. But as the line A B is to the line B C, so is the square of the

line

*line A B to the ſquare of the line B D, by the corollaries of the 8. and 20. of the ſixt. Wherefore the ſquare of the line A B is double to the ſquare of the line B D. And it is proued that the ſquare of the line L M is duble to the ſquare of the line L E. Wherefore the ſquare of the line B D is equall to the ſquare of the line L E. For the line E H which is equall to the line L F, is put to be equall to the line D B. Wherefore the ſquare of the line A B is equall to the ſquare of the line L M. Wherefore the line A B is equall to the line L M. And the line A B is the diameter of the ſphere geuē, wherefore the line L M is equall to the diameter of the ſphere geuen. Wherefore the octoedron is contayned in the ſphere geuen: * and it is alſo proued that the diameter of the ſphere is in power double to the ſide of the octohedron. Wherefore there is made an octohedron, and it is comprehended in the ſphere geuen, wherein was comprehended the Pyramis: and it is proued that the diameter of the ſphere is in power double to the ſide of the octohedrn: which was required to be doone, and to be proued.*

* *For the ſquare of the line A B, which is proued equall to the ſquare of the line L M, is double to the ſquare of the line B D, which is alſo equall to the ſquare of the line L E.*

Certayne Corollaries added by *Fluſſas.*

✻ *Firſt Corollary.*

The ſide of a Pyramis is in power ſeſquitertia to the ſide of an octohedron inſcribed in the ſame Sphere.

For foraſmch as the diameter is in power double to the ſide of the octohedron, therefore of what partes the diameter contayneth in power 6. of the ſame, the ſide of the octohedron cōtayneth in power 3. but of what partes the diamete contayneth 6. of the ſame, the ſide of the pyramis contayneth 4. by the 13. of this booke. Wherefore of what partes the ſide of the pyramis contayneth 4. of the ſame the ſide of the octohedron contayneth 3.

Second Corollary.

An octohedron is deuided into two equall and like Pyramids.

The common baſes of theſe Pyramids are ſet vpon euery ſquare contayned of the ſides of the octohedron, vpon which ſquare are ſet the 8. triangles of the octohedron: which pyramids are by the 8. diffinition of the eleuenth equall and like. And the foreſayd ſquare common to thoſe Pyramids, is the halfe of the ſquare of the diameter of the ſphere, for it is the ſquare of the ſide of the octohedron.

This Corollary is the 16. propoſition of the 14. booke after Campane.

Third Corollary.

The three diameters of the octohedron, do cutte the one the other perpendicularly into two equall parts, in the center of the ſphere which contayneth the ſayd octohedron.

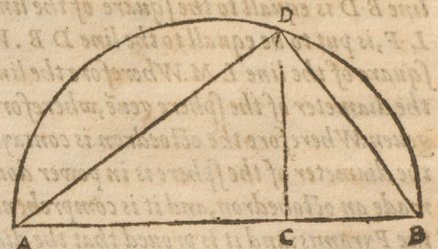
As it is manifeſt by the three diameters E G, F H, and L M which cutte the one the other in the center K equally and perpendicularly.

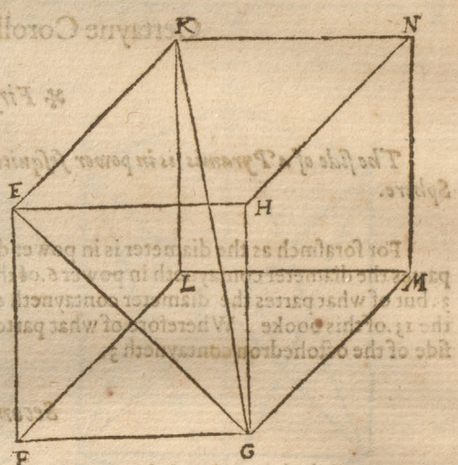
¶ *The 3. Prbleme. The 15. Propoſition.*

To make a ſolide called a cube, and to comprehend it in the ſphere geuen, namely, that Sphere wherein the former two ſolides were comprehended: and to proue that the diameter of the ſphere, is in power treble to the ſide of the cube.

TAke the diameter of the ſphere geuen, namely, A B, and diuide it in the point C. So that let the line A C be double to the line C B by the 9. of the ſixt. And vpon the

line A B describe a semicircle A D B. And (by the 11. of the first) from the poynt C rayse vp vnto the line A B a perpẽdicular line C D. And draw a right line D B. And describe a square E F G H, hauing euery one of his sides equall to the line D B, And from the pointes E, F, G, H, rayse vp (by the 12. of the eleuenth) vnto the playne superficies of the square E F G H perpendicular lines E K, F L, G M, and H N: and let euery one of the lines E K, F L, G M, and H N be put equall to one of the lines E F, F G, G H, or H E, which are the sides of the square, and draw these right lines K L, L M, M N, and N K. Wherfore there is made a cube namely F N which is contayned vnder six equall squares. Now it is required to comprehend the same cube in the sphere geuen, and to proue that ỹ diameter of the sphere is in power double to the side of the cube. Draw these right lines K G and E G. And forasmuch as the angle K E G is a right angle, for that the line K E is erected perpendicularly to the playne superficies E G, and therefore also to the right line E G, by the 2. diffinitiõ of the eleuenth, wherefore a semicircle described vpon the line K G shall* passe by the poynt E. Agayne forasmuch as the line F G is erected perpendicularly to either of these lines F L and F E, by the diffinition of a square, & by the 2. diffinition of the eleuenth, therefore the line F G is erected perpendicularly to the playne superficies F K, by the 4. of the eleuenth. Wherefore if we draw a right line from the point F to the point K, the line G F shall be erected perpendicularly to the line K F, by the 2. diffinition of the eleuenth. And by the same reason agayne a semicircle described vpon the line G K shall passe also by the point F. And likewise shall it passe by the rest of the pointes of the angles of that cube. If now the diameter K G abiding fixed the semicircle be turned round about vntill it returne into the selfe same place from whence it began first to be moued, the cube shalbe comprehẽded in a sphere. I say also that it is comprehended in the sphere geuen.





First part of the demonstration.

Second part of the Constructiõ. Second part of the demonstration.

* By the 2. Assumpt of the 13. of this booke.

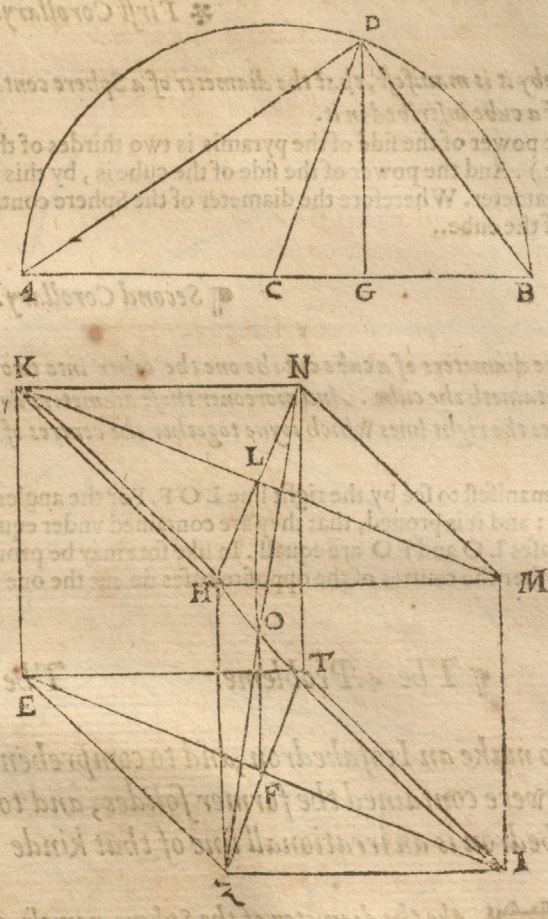
For forasmuch as the line G F is equall to the line F E and the angle F is a right angle, therefore the square of the line E G is by the 47. of the first double to the square of the line E F. But the line E F is equall to the line E K. Wherefore the square of the line E G is double to the square of the line E K. Wherfore the squares of E G and E K that is the square of the line G K, by the 47. of the first, are treble to the square of the line E K. And forasmuch as the line A B is treble to the line B C, but as the line A B is to the line B C, so is the square of the line A B to the square of the line B D, by the corollaries of the 8. and 20. of the sixt. Wherefore the square of the line A B is treble to the square of the line B D. And it is proued that the square of the line G K is treble to the square of the line K E, and the line K E is put equall to the line B D. Wherefore the line K G is also equall to the line A B. And the line A B is the diameter of the sphere geuen. Wherefore the line K G is equall to the diameter of the sphere geuen. Wherfore the cube is cõprehẽded in the sphere geuen: and it is also proued that the diameter of the Sphere is in power treble to the side of the cube: which was required to be doone, and to be proued.

Third part of the demonstration.

An other

❋ An other demonſtration after *Fluſſas*.

Suppoſe that the diameter of the Sphere geuen in the former Propoſitions, be the line A B. And let the center be the point C, vpon which deſcribe a ſemicircle A D B. And from the diameter A B cut of a third part B G, by the 9. of the ſixt. And from the point G raiſe vp vnto the line A B a perpendicular line D G, by the 11. of the firſt. And draw theſe right lines D A, D C, and D B. And vnto the right line D B put an equall right line Z I: and vpon the line Z I deſcribe a ſquare E Z I T. And frō the pointes E, Z, I, T, erecte vnto the ſuperficies E Z I T perpendicular lines E K, Z H, I M, T N (by the 12. of the eleuenth): and put euery one of thoſe perpendicular lines equall to the line Z I. And drawe theſe right lines K H, H M, M N, and N K, ech of which ſhall be equall and parallels to the line Z I, and to the reſt of the lines of the ſquare, by the 33. of the firſt. And moreouer they ſhall containe equall angles (by the 10. of the eleuenth): and therefore the angles are right angles, for that E Z I T is a ſquare: wherfore the reſt of the baſes ſhall be ſquares. Wherfore the ſolide E Z I T K H M N being cōtained vnder 6. equall ſquares, is a cube, by the 21. definition of the eleuenth. Extend by the oppoſite ſides K E and M I of the cube, a plaine K E I M: and againe by the other oppoſite ſides N T and H Z, extend an other plaine H Z T N. Now foraſmuch as ech of theſe plaines deuide the ſolide into two equall partes, namely, into two Priſmes equall and like (by the 8. definition of the eleuenth): therfore thoſe plaines ſhall cut the cube by the centre, by the Corollary of the 39. of the eleuenth. Wherefore the cōmon ſection of thoſe plaines ſhall paſſe by the centre. Let that common ſection be the line L F. And foraſmuch as the ſides H N and K M of the ſuperficieces K E I M and H Z T N do diuide the one the other into two equall partes, by the Corollary of the 34. of the firſt, and ſo likewiſe do the ſides Z T and E I: therefore the common ſection L F is drawen by theſe ſections, and diuideth the plaines K E I M and H Z T N into two equall partes, by the firſt of the ſixt: for their baſes are equall, and the altitude is one and the ſame, namely, the altitude of the cube. Wherefore the line L F ſhall diuide into two equall partes the diameters of his plaines, namely, the right lines K I, E M, Z N, and N T, which are the diameters of the cube. Wherfore thoſe diameters ſhall concurre and cut one the other in one and the ſelfe ſame poynt, let the ſame be O. Wherfore the right lines O K, O E, O I, O M, O H, O Z, O T, and O N, ſhall be equall the one to the other, for that they are the halfes of the diameters of equall and like rectangle parallelogrāmes. Wherefore making the centre the point O, and the ſpace any of theſe lines O E, or O K. &c. a Sphere deſcribed, ſhall paſſe by euery one of the angles of the cube, namely, which are at the pointes E, Z, I, T, K, H, M, N, by the 12. definition of the eleuenth, for that all the lines drawen from the point O to the angles of the cube are equall. But the right line E I containeth in power the two equall right lines E Z, and Z I, by the 47. of the firſt. Wherefore the ſquare of the line E I is double to the ſquare of the line Z I. And foraſmuch as the right line K I ſubtendeth the right angle K E I (for that the right line K E is erected perpendicularly to the plaine ſuperficies of the right lines E Z and Z T (by the 4. of the eleuēth): therefore the ſquare of the line K I is equall to the ſquares of the lines E I and E K, but the ſquare of the line E I is double to the ſquare of the line E K (for it is double to the ſquare of the line Z I, as hath bene proued, and the baſes of the cube are equall ſquares). Wherefore the ſquare of the line K I is triple to the ſquare of the line K E, that is, to the ſquare of the line Z I. But the right line Z I is equall to the right line D B, by poſition, vnto whoſe ſquare the ſquare of the diameter A B is triple, by that which was demonſtrated in the 13. Propoſition of this booke. Wherefore the diameters K I & D B are equall. Wherefore there is deſcribed a cube K I, and it is comprehended in the Sphere geuen wherin the other



ſolides were contained, the diameter of which Sphere is the line A B. And the diameter K I or A B of the ſame Sphere, is proued to be in power triple to the ſide of the cube, namely, to the line D B, or Z I.

¶ Corollaryes added by *Fluſſas*.

❋ *Firſt Corollary.*

Hereby it is manifeſt, that the diameter of a Sphere containeth in power the ſides both of a pyramis and of a cube inſcribed in it.

For the power of the ſide of the pyramis is two thirdes of the power of the diameter (by the 13. of this booke). And the power of the ſide of the cube is, by this Propoſition, one third of the power of the ſayd diameter. Wherefore the diameter of the Sphere contayneth in power the ſides of the pyramis and of the cube..

¶ *Second Corollary.*

All the diameters of a cube cut the one the other into two equall partes in the centre of the ſphere which containeth the cube. And moreouer thoſe diameters do in the ſelfe ſame point cut into two equall partes the right lines which ioyne together the centres of the oppoſite baſes.

As it is manifeſt to ſee by the right line L O F. For the angles L K O, and F I O, are equall, by the 29. of the firſt: and it is proued, that they are contained vnder equall ſides: Wherefore (by the 4. of the firſt) the baſes L O and F O are equall. In like ſort may be proued, that the reſt of the right lines which ioyne together the centres of the oppoſite baſes do cut the one the other into two equall partes in the centre O.

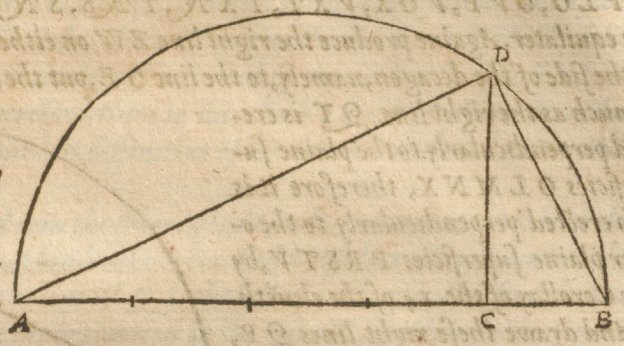
¶ *The 4. Probleme.* *The 16. Propoſition.*

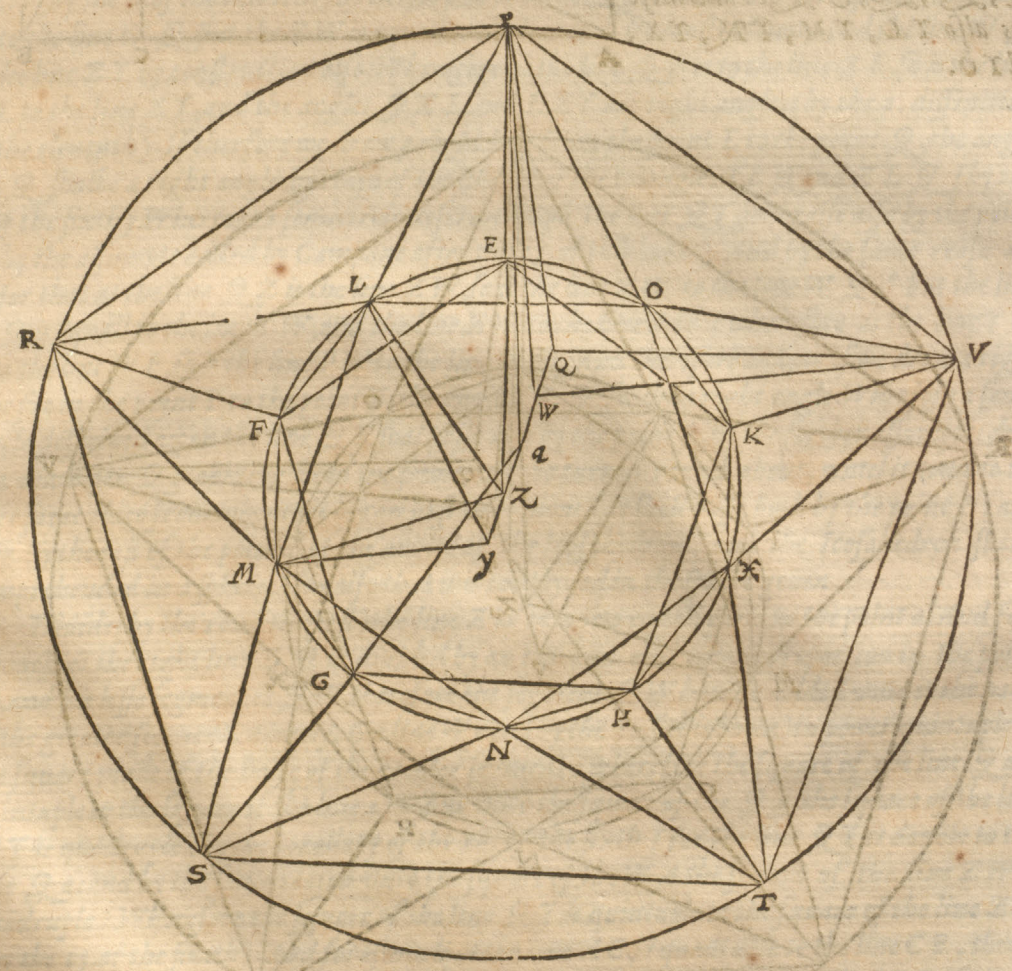
To make an Icoſahedron, and to comprehend it in the Sphere geuen, wherein were contained the former ſolides, and to proue that the ſide of the Icoſahedron is an irrationall line of that kinde which is called a leſſe line.

First part of the conſtruction.

TAke the diameter of the Sphere, namely, the line A B: and deuide it in the point C, ſo that let the line A C be quadruple to the line C B, by the 9. of the ſixt. And deſcribe vpō the line A B a ſemicircle A D B. And (by the 11. of the firſt) from the point C raiſe vp vnto the line A B a perpendicular line C D. And draw a right line from the point D to the point B. And deſcribe a circle E F G H K whoſe line from the centre (which let be the point Z) to the circumference, let be equall to the line D B. And in the circle E F G H K deſcribe (by the 11. of the fourth) an equilater and equiangle Pentagon figure E F G H K. And deuide the circumferences E F, F G, G H, H K, and K E, into two equall partes in the pointes L, M, N, X, O. Draw alſo theſe right lines L M, M N, N X, X O, and O L: and moreouer theſe lines O E, E L, L F, F M, M G, G N, N H, H X, X K, and K O, and they ſhall be the ſides of an equilater decagon inſcribed in the circle E F G H K, by the 29. of the third. Wherefore the figure L M N X O is an equilater pentagon, by the 29. of the third, and the right line E O is the ſide of a decagon or ten angled figure. Raiſe vp (by the 12. of the eleuenth) from the pointes E, F, G, H, K, and the centre Z, vnto the plaine ſuperficies of the circle, perpendicular lines E P, F R, G S, H T, K V, and Z W, and let ech of them be put equall to the line drawen from the centre of the circle E F G H K, to the circumference, namely, to the line Z E. Wherefore right lines drawen from W to P, from W to V, from W to T, from W to S, from W to R, ſhall be equall and parallels to right lincs drawen from Z to E, from Z to K, from Z to H, from Z to G, and from Z to F, by the 6. and 7. of the eleuenth, and 33. of the firſt. Wherefore the plaine ſuperficieces E F G H K, and P R S T V, which are extended by thoſe parallel lines, are parallel

rallel ſuperficieces, by the 15. of the eleuenth. Wherefore making the centre the point W, and the ſpace W P, or W V, deſcribe a circle, and it ſhall paſſe by the pointes T, S, R, and ſhall be equall to the circle E F G H K. For the ſemidiameters of eche are equall. And drawe theſe right lines P R, R S, S T, T V, V P, and they ſhall make a pentagon, whoſe ſides ſhall be equall to the ſides of the Pentagon O L M N X, by the 29. of the firſt. For ech of them doth ſubtend two ſides of the decagon, or the fift part of equall circles. From the vpper pointes P, R, S, T, V, draw theſe lines P O, P L, R L, R M, S M, S N, T N, T X, V X, V O: which ſhall ſubtend right angles cõtained vnder the ſides of the decagon E L - F M G N H X K O, and the perpendicular lines P E, R F, S G, T H, V K.



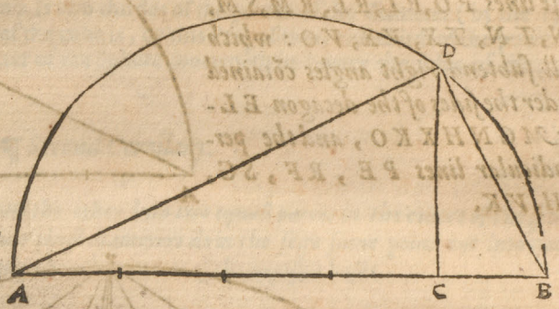


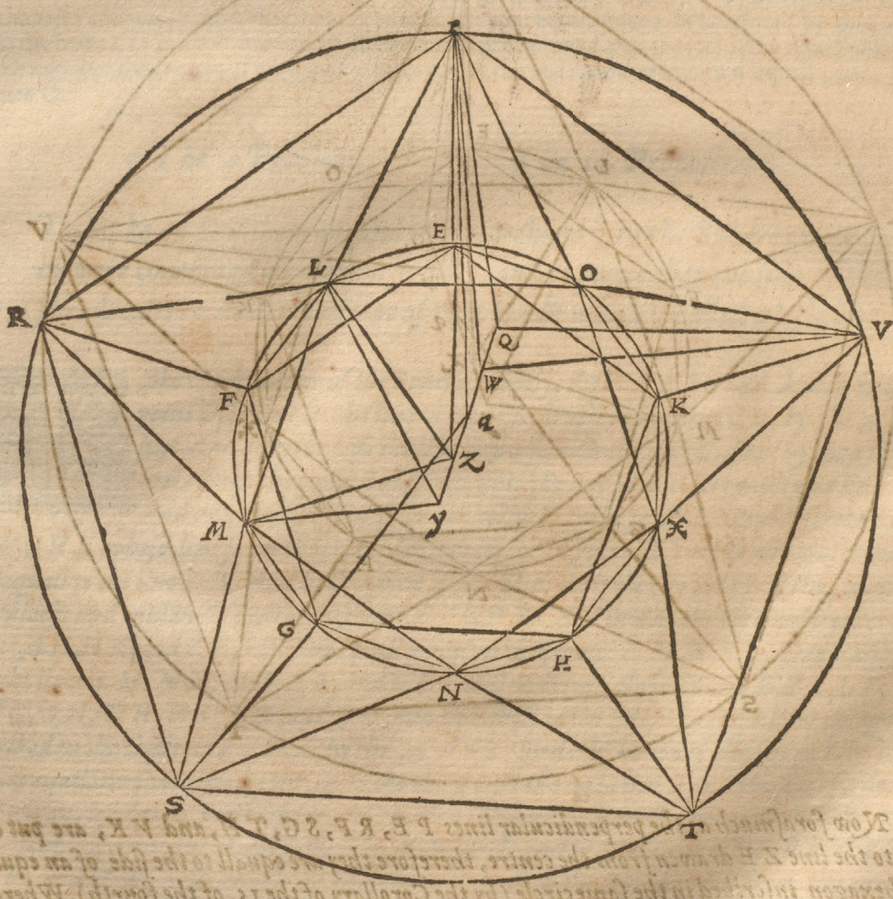
Now foraſmuch as the perpendicular lines P E, R F, S G, T H, and V K, are put equall to the line Z E drawen from the centre, therefore they are equall to the ſide of an equilater hexagon inſcribed in the ſame circle (by the Corollary of the 15. of the fourth). Wherefore the right lines P O, P L, V O, and V X (which ſubtend the right angles contained vnder thoſe perpendicular lines and the ſides of the decagon) containe them in power, by the 47. of the firſt. But the ſide of a pentagon (namely, the ſide L O or P V) containeth in power the ſides of an hexagon and of a decagon inſcribed in one and the ſelfe ſame circle, by the 10. of

First part of the demonſtration.

this booke. Wherefore the ſubtending lines P O, P L, V O, V X, T X, T N, S N, S M, R M, R L, containe in power the ſelfe ſame ſquare that the ſides of the pentagon O L M N X containe, or that the ſides of the pentagon P R S T V containe: and therefore thoſe ſubtending lines are equall to the ſides of the foreſaid pentagons. Wherefore the triangles contained of thoſe ſubtending lines and of the ſides of the pentagons, and which are ten in number, namely, P L O, O V P, V O X, V X T, T X N, T N S, S N M, S M R, R M L, and R L P, are equilater. Againe produce the right line Z W on either ſide to the points Q & Y: and vnto the ſide of the decagon, namely, to the line O E, put the lines Z Y and W Q equal. And foraſmuch as the right line Q Y is erected perpendicularly to the plaine ſuperficies O L M N X, therefore it is alſo erected perpendicularly to the other plaine ſuperficies P R S T V, by the Corollay of the 14. of the eleueth. And drawe theſe right lines Q P, Q V, Q T, Q S, & Q R: and theſe lines alſo Y L, Y M, Y N, Y X, and Y O.

Second part of the cõſtruction.





Second part of the demõſtratiõ.

Now foraſmuch as the lines Q P, Q V, Q T, Q S, and Q R do eche ſubtend right angles contayned vnder the ſides of an equilater hexagon & of an equilater decagon inſcribed in the circle P R S T V or in the circle E F G H K (which two circles are equall) therfore the ſayd lines are eche equal to the ſide of the pentagon inſcribed in the foreſayd circle by the 10.

of

of this booke, and are equall the one to the other, by the 4. of the first, (for all the angles at the poynt W which they subtend are right angles). Wherefore the fiue triangles Q P V, Q P R, Q R S, Q S T, and Q T V, which are contayned vnder the sayd lines Q V, Q P, Q R, Q S, Q T, and vnder the sides of the pentagon V P R S T, are equilater, and equal to the ten former triangles. And by the same reason the fiue triangles opposite vnto them, namely, the triangles Y M L, Y M N, Y N X, Y X O, and Y O L, are equilater and equal to the said ten triangles. For the lines Y L, Y M, Y N, Y X, and Y O do subtend right angles cõtayned vnder the sides of an equilater hexagon and of an equilater decagõ inscribed in the circle E F G H K, which is equall to the circle P R S T V. Wherefore there is described a solide contayned vnder 20. equilater triangles. Wherefore by the last diffinition of the eleuenth there is described an Icosahedron.

Now it is required to comprehend it in the sphere geuen, and to proue that the side of the Icosahedron is an irrationall line of that kinde which is called a lesse line. Forasmuch as the line Z W is the side of an hexagon, & the line W Q is the side of a dceagon, therfore the line Z Q is diuided by an extreme and meane proportion in the point W, and his greater segmẽt is Z W (by the 9. of the thirtẽth). Wherfore as the line Q Z is to the line Z W, so is the line Z W to the line W Q. But the Z W is equall to the line Z L by construction, and the line W Q to the line Z Y by construction also: Wherefore as the line Q Z is to the line Z L, so is the line Z L to the line Z Y, and the angles Q Z L. and L Z Y are right angles (by the 2. diffinition of the eleuenth): If therfore we draw a right line from the poynt L to the poynt Q, the angle Y L Q shalbe a right angle, by reasõ of the likenes of the triangles Y L Q and Z L Q (by the 8. of the sixth). Wherfore a semicircle described vpõ the line Q Y, shal passe also by the point L (by the assumpts added by Campane after the 13. of this booke). And by the same reasõ also, for that as the line Q Z is the line Z W, so is the line Z W to the line W Q, * but the line Z Q is equall to the line Y W, and the line Z W to the line P W: wherefore as the line Y W is to the line W P, so is the line P W to the line W Q. And therefore agayne if we draw a right line from the poynt P to the point Y, the angle Y P Q shalbe a right angle. Wherfore a semicircle described vpon the line Q Y shal passe also by the point P, by the former assumpts: & if the diameter Q Y abiding fixed the semicircle be turned round about, vntil it come to the selfe same place from whence it began first to be moued, it shall passe both by the point P, and also by the rest of the pointes of the angles of the Icosahedron, and the Icosahedron shalbe comprehended in a sphere. I say also that it is contayned in the sphere geuen.

* For the line Q W, is equall to the line Y Z, & the line Z W, is commõ to them both. This part is againe afterward demonstrated by Flussas.

Diuide (by the 10. of the first) the line Z W into two equall parts in the point a. And forasmuch as the right line Z Q is diuided by an extreme and meane proportion in the point W, and his lesse segment is Q W, therefore the segment Q W hauing added vnto it the halfe of the greater segment, namely, the line W a, is (by the 3. of this booke) in power quintuple to the square made of the halfe of the greater segment: wherefore the square of the line Q a is quintuple to the square of the line a W. But vnto the square of the Q a, the square of the line Q Y is quadruple (by the corollary of the 20. of the sixth) for the line Q Y is double to the line Q a: and by the same reason vnto the square of the W A the square of the line Z W is quadruple. Wherefore the square of the line Q Y is quintuple to the square of the line Z W (by the 15. of the fiueth). And forasmuch as the line A C, is quadruple to the line C B, therefore the line A B is quintuple to the line C B. But as the line A B is to the line B C, so is the square of the line A B to the square of the line B D (by the 8 of the sixth, and corollary of the 20. of the same). Wherfore the square of the line A B is quintuple to the square of the line B D. And it is is proued that the square of the line Q Y is quintuple to the square of the line Z W. And the line B D is equall to the line Z W, for either of them is by position equall to the line which is drawen from the centre of the circle E F G H K to the circumference. Wherefore the line A B is equall to the Y Q. But the line A B is the diameter of the sphere geuen: Wherefore the line Y Q, which is proued to be the diameter of the sphere contayning the Icosahedron

ſahedron, is equall to the diameter of the ſphere geuen. Wherefore the Icoſahedron is contayned in the ſphere geuē. Now I ſay that the ſide of the Icoſahedron is an irrationall line of that kinde which is called a leſſe line. For foraſmuch as the diameter of the ſphere is rational, and is in power quintuple to the ſquare of the line drawen frō the centre of the circle O L M N X: wherefore alſo the line which is drawen from the centre of the circle O L M N X is rationall: wherefore the diameter alſo being cōmenſurable to the ſame line (by the 6. of the tenth) is rationall. But if in a circle hauing a rationall line to his diameter be deſcribed an equilater pentagon, the ſide of the pentagon is (by the 11. of this booke) an irrationall line, of that kinde which is called a leſſe line. But the ſide of the pentagon O L M N X is alſo the ſide of the Icoſahedron deſcribed, as hath before ben proued. Wherfore the ſide of the Icoſahedrō is an irrationall line of that kinde which is called a leſſe line. Wherefore there is deſcribed an Icoſahedron and it is contayned in the ſphere geuen, and it is proued that the ſide of the Icoſahedron is an irrationall line of that kind which is called a leſſe line. Which was required to be done, and to be proued.

A Corollary.

Hereby it is manifeſt that the diameter of the ſphere, is in power quintuple to the line which is drawen from the centre of the circle to the circumference, on which the Icoſahedron is deſcribed. And that the diameter of the ſphere: is compoſed of the ſide of an hexagon, and of two ſides of a decagon deſcribed in one and the ſelfe ſame circle.

Fluſſas proueth the Icoſahedron deſcribed, to be cōtayned in a ſphere, by drawing right lines from the poynt, a, to the poyntes P and G after this maner.

Foraſmuch as the lines Z W, W P are put equal to the line drawen from the centre to the circumference, and the line drawen from the centre to the circumference is double to the line a W, by conſtruction: therefore the line W P is alſo double to the ſame line a W. Wherefore the ſquare of the line W P is quadruple to the ſquare of the line a W (by the corollary of the 20. of the ſixth). And thoſe lines P W and W a contayning a right angle P W a (as hath before bene proued) are ſubtended of the right line a P. Wherefore (by the 47. of the firſt) the line a P contayneth in power the lines P W, and W a. Wherefore the right line a P is in power quintuple to the line W a. Wherefore the right lines a P, and a Q being quintuple to one and the ſame line W a, are (by the 9. of the fiueth) equall. In like ſorte alſo may we proue that vnto thoſe lines a P and a Q, are equall the reſt of the lines drawen from the poynt a to the reſt of the angles R, S, T, V. For they ſubtend right angles contayned of the line W a, and of the lines drawen from the centre to the circumference. And foraſmuch as vnto the line W a is equall the line V a, which is likewiſe erected perpendicularly vnto the other plaine ſuperficies O L M N X: therefore lines drawen from the point a to the angles O, L, M, N, X, and ſubtending right angles at the point Z contayned vnder lines drawen frō the centre to the circumference, and vnder the line a Z, are equal not onely the one to the other, but alſo to the lines, drawen frō the ſayde poynt a, to the former angles at the poynts, P, R, S, T, V. For the lines drawen frō the centre to the circumference of ech circle are equall, & the line a W is equall to the line a Z. But the line a P is proued equal to the line a Q, which is the halfe of the whole Q Y. Therefore the reſidue a Y is equall to the foreſayd lines a P, a Q &c. Wherefore making the centre the poynt a, and the ſpace one of thoſe lines a Q, a P, &c. extende the ſuperficies of a ſphere, & it ſhal touch the 12. angles of the Icoſahedron, which are at the pointes O, L, M, N, X, P, K, S, T, V, Q, Y: which ſphere is deſcribed, if vpō the diameter Q Y, be drawen a ſemicircle, and the ſayd ſemicircle be moued about, till it returne vnto the ſame place from whenſe it began firſt to be moued.

¶ A Corollary added by *Fluſſas*.

The oppoſite ſides of an Icoſahedron are parallels. For the diameters of the ſphere do fall vpon the oppoſite angles of the Icoſahedron: as it was manifeſt by the right line Q Y. If therefore there be imagined to be drawen the two diameters P N, and O M they ſhall concurre in the point F: wherefore the right lines which ioyne them together, P V, and L N, are in one and the ſelfe ſame playne ſuperficies, by

the

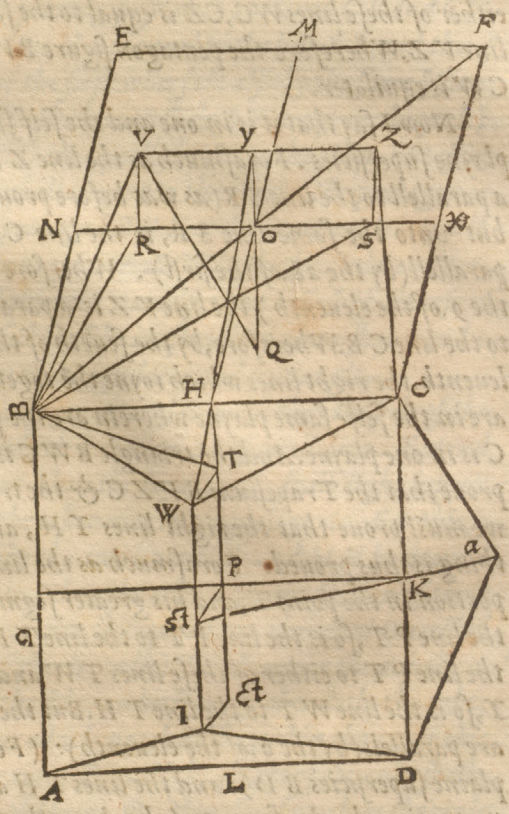
the 1.of the eleuenth. And forasmuch as the alternate angles at the endes of the diameters are equall (by the 8.of the first): for the triangles contayned vnder equall semidiameters and the side of the Icosahedron are equiangle: therefore (by the 28.of the first) the lines P V and L N are paralles.

¶ The 5.Probleme. The 17.Proposition.

To make a Dodecahedron, and to comprehend it in the sphere geuen, wherin were comprehended the foresayd solides: and to proue that the side of the dodecahedron is an irrationall line of that kind which is called a residuall line.

First part of the construction.

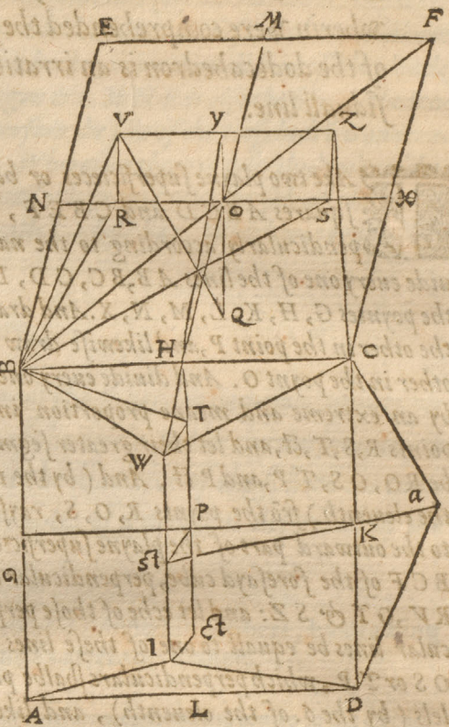
TAke two playne superficieces or bases of the foresayde cube, which let be the two squares A B C D and C B E F, cutting the one the other in the line B C perpendicularly according to the nature of a cube. And (by the 10.of the first) diuide euery one of the lines A B, B C, C D, D A, E F, E B, and F C into two equall partes in the poyntes G, H, K, L, M, N, X. And drawe these right lines G K and H L, cutting the one the other in the point P, and likewise draw the right lines M H and N X cutting the one the other in the poynt O. And diuide euery one of these right lines N O, O X, H P, and L P, by an extreme and meane proportion in the points R, S, T, ☐, and let their greater segments be R O, O S, T P, and P ☐. And (by the 12.of the eleuenth) frō the poynts R, O, S, rayse vp to the outward part of the playne superficies E B C F of the foresayd cube, perpendicular lines R V, O Y & S Z: and let eche of those perpēdicular lines be equall to one of these lines R O, O S or T P, which perpendiculars shalbe parallels (by the 6.of the eleuenth), and likewise from the pointes T, P, ☐, rayse vp vnto the outward part of the playne superficies A B C D of the sayd cube, these perpendicular lines T W, P ☐, and ☐ I, eche of which perpendicular lines put equall also to the line O S, or O R or T P: and the sayd perpendiculars shalbe parallels (by the foresayd 6.of the eleuenth): And draw these right lines V H, H W, B W, W C, C Z and C B. Now I say that the pentagon figure V B W C Z is equilater and in one and the self same plaine superficies, and moreouer is equiangle. Draw these right lines T B, R B, and S B. And forasmuch as the right line N O is diuided by an extreme and meane proportion in the poynt R, and his greater segment is the line R O, therefore the square of the lines N O and N R are treble to the square of the line R O (by the 4.of this booke). But the line O N is equall to the line N B, and the line O R to the line R V. Wherefore the squares of the lines B N and R N are treble to the square of the line R V. But vnto the squares of the lines B N and N R is equall the square of the line B R (by the 47.of the first). Wherefore the square of the line B R is treble to the square of the line R V. Wherefore the squares of the lines B R and R V are quadruple to the square of the line R V. But vnto the squares of the lines B R and R V is equall the square of the line



The pentagon V B W C Z, proued equilater.

B V (by the 47. of the first): for the angle B R V is a right angle (by the 2. diffinition of the eleuenth). Wherefore the square of the line B V is quadruple to the square of the line V R. Wherefore the line B V is double to the the line R V (by the Corollary of the 20. of the sixth). And the line Z V is also double to the line R V (for that the line S R is double to the line O-R, that is, to the line RV which is equall to the line OS). Wherfore the line BV is equall to the line V Z. And forasmuch as the two lines B N, and N R are equall to the two lines B H and H-T, namely, the wholes and the lesse segmẽts, and they comprehend right angles, namely, of the squares, B O, and B P, therefore (by the 4. of the first) the bases B R and B T are equall. And for-asmuch as the lines B R, and B T are equall, and the two lines R V, and T W are also by construction equall, and the angles B R V, and B T W are by supposition right angles: therefore againe (by the 4. of the first) the bases B V and B W are equall: but the line B V, is proued equall to the line V Z. Wherfore the line BW is also equall to the line V Z. In like sort also may we proue that either of these lines WC, CZ is equal to the same line V Z. Wherefore the pentagon figure B V Z-C W is equilater.*

* *The line S R is equal & parallel to the line Z V (by the 33. of the first) for they ioyne together equall and parallel lines, and are in the same superficies with them, by the 7. of the eleuenth.*



The pentagon V B W C R, proued to be in one and the selfe same playne superficies.

Now I say that it is in one and the self same playne superficies. Forasmuch as the line Z V is a parallell to the line S R (as was before proued) but vnto the same line S R, is the line C B a parallell (by the 28. of the first). Wherfore (by the 9. of the eleuenth) the line V Z is a parallell to the line C B. Wherefore, by the seuẽth of the eleuenth, the right lines which ioyne thẽ together are in the selfe same playne wherein are the parallell lines. Wherefore the Trapesium B V Z-C is in one playne. And the triangle B W C is in one playne (by the 2. of the eleuenth). Now to proue that the Trapesium B V Z C & the triangle B W C are in one and the self same plaine, we must proue that the right lines Y H, and H W are made directly one right line. which thing is thus proued. Forasmuch as the line H P is diuided by an extreme and meane proportion in the point T, and his greater segment is the line P T, therefore as the line H P is to the line P T, so is the line P T to the line T H. But the line H P is equall to the line H O, and the line P T to either of these lines T W and O Y. Wherefore as the line H O is to the line O-Y, so is the line W T to the line T H. But the lines H O and T W being sides of like proportion are parallels (by the 6. of the eleuenth): (For either of them is erected perpendicularly to the plaine superficies B D): and the lines T H and O Y are parallels, which are also sides of like proportion, by the same 6. of the eleuenth, (For either of them is also erected perpendicularly to the playne superficies B F.) But when there are two triangles, hauing two sides proportionall to two sides, so set vpon one angle, that their sides of like proportiõ are also parallels (as the triangles Y O H and H T W are), whose two sides, O H & H T, being in the two bases of the cube making an angle at the point H, the sides remayning of those triangles shal (by the 32. of the sixth) be in one right line. Wherfore the lines Y H & H W make both one right line. But euery right line is (by the 1. of the eleuenth) in one & the self same plaine superficies. Wherefore if ye draw a right line from B to Y, there shalbe made a triangle B W Y, which shalbe in one and the selfe same plaine (by the 2. of the eleuenth). And therefore the whole

pentagon

pentagon figure V B W C Z is in one and the selfe same playne superficies.

Now also I say that it is equiangle. For forasmuch as the right line N O is diuided by an extreame and meane proportion in the point R, and his greater segment is O R, therefore as both the lines N O and O R added together is to the line O N, so (by the 5. of this booke) is the line O N to the line O R. But the line O R is equall to the line O S. Wherefore as the line S N is to the line N O, so is the line N O to the line O S. Wherfore the line S N is diuided by an extreme and meane proportion in the point O, and his greater segment is the line N O. Wherefore the squares of the lines N S and S O are treble to the square of the line N O (by the 4. of this booke). But the line N O is equall to the N B, and the line S O to the line S Z: wherfore the squares of the lines N S and Z S are treble to the square of the line N B: wherfore the squares of the lines Z S, S N and N B, are quadruple to the square of the line N B. But vnto the squares of the lines S N & N B (by the 47. of the first) is equal the square of the line S B: wherefore the squares of the lines B S and S Z, that is, the square of the line B Z, by the 47. of the first, (for the angle Z S B is a right angle by positiõ) is quadruple to the square of the line N B. Wherfore the line B Z is double to the line B N (by the Corollary of the 20. of the sixth). But the line B C is also double to the line B N. Wherefore the line B Z is equall to the line B C. Now forasmuch as these two lines B V and V Z are equall to these two lines B W and W C, and the base B Z is equall to the base B C, therefore (by the 8. of the first) the angle B V Z is equall to the angle B W C. And in like sort (by the 8. of the first) may we proue that the angle V Z C is equall to the angle B W C (prouing first that the lines C B and C V are equal: which are proued equal by this, that the line N S is equal to the line X R, and therfore the line C R is equal to the line B S, by the 47. of the first: wherfore also by the same ꝯ line C V is equal to the line B Z, that is, to the line B C (for the lines B C & B Z are proued equal.) Wherefore the three angles B W C, B V Z, and V Z C are equall the one to the other. But if in an equilater pentagon figure there be thre angles equall the one to the other, the pentagon is (by the 7. of the thirteth) equiangle: wherfore the pentagon B V Z C W is equiangle. And it is also proued that it is equilater. Wherfore the pentagon B V Z C W is both equilater & equiangle. And it is made vpon one of the sides of the cube, namely, vpon B C. * If therefore vpon euery one of the twelue sides of the cube be vsed the like construction, there shal then be made a dodecahedron contayned vnder twelue pentagons equilater and equiangle.

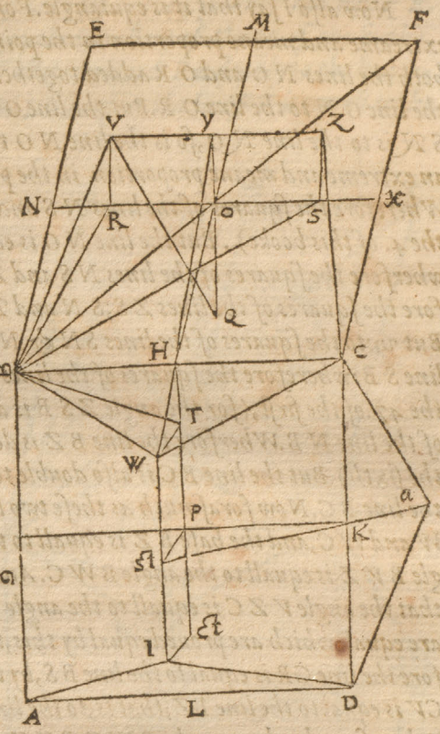
The pentagon V B W C Z, is proued equiangle.

** Looke for a farther construction after Flussas at the ende of the demonstration.*

Now it is required to comprehend it in the sphere geuen, and to proue that the side of the dodecahedron is an irrationall line of that kinde which is called a residuall line. Extend the line Y O, and let the line extended be Y Q: now then the line Y Q shall light vppon the diameter of the cube, and shall diuide the one the other into two equall parts. For this is manifest to se by the 39. of the eleuenth. (For if by the two lines N X and M H be drawen two playnes perpendicularly to the bases, and cutting the cube, the common section of those playnes shalbe the line Y O produced: for their common section is from the poynt O erected perpendicularly to the plaine E B C F, by the 19. of the eleuenth). Let them cut the one the other in the point Q wherefore Q is the centre of the sphere which comprehendeth the cube, and Y Q is the halfe diameter of the sphere by that which was demõstrated in the 15. of this booke: wherefore the right lines drawen from the centre Q to all the angles of the cube shalbe equall. And draw a right line from the point V to the point Q. Now forasmuch as the right line N S is diuided by an extreme and meane proportion in the point O, and his greater segment is the line N O, as hath before ben proued, therefore the squares of the lines N S and S O are treble to the square of the line N O, by the 4. of this booke. But the line N S is equal to the line Y Q (for the line N O is equal to the line O Q as hath before ben proued, & the line Y O to the line O S) being both lesse segmentes: but the line O S is equall to the line Y V, for the line R O is equall thereunto: wherefore the squares of the lines Q Y and Y V are treble to the square of the line N O. But vnto the squares of the lines Q Y & Y V the square of the line V Q is equall (by the 47. of the first): wherefore the square of the line V Q is treble to the

That the dodecahedron is contayned in the sphere geuen.

ſquare of the line NO. But the ſemidiameter of the ſphere cõprehẽding the ſaid cube is in power treble to the half of the ſide of the cube. For we haue before (in the 15. of this booke) taught how to make a cube, and to comprehende it in a ſphere, and haue proued that the diameter of the ſphere is in power treble to the ſide of the cube. Now in what proportiõ ỹ whole is to the whole, in the ſame is the halfe to the halfe (by the 15. of the fifth). But the line NO is the half of the ſide of the cube. Wherefore the line V Q is equall to the ſemidiameter of the ſphere cõprehẽding the cube. But the point Q is the centre of the ſphere cõprehending the cube. Wherefore the point V, which is one of the angles of the dodecahedron, toucheth the ſuperficies of the ſphere geuen. In like ſort alſo may we proue, that euery one of the reſt of the angles of the dodecahedron toucheth the ſuperficies of the ſphere. Wherefore the dodecahedron is comprehended in the ſphere geuen.



That the ſide of the dodecahedron is a reſiduall line.

Now I ſay, that the ſide of the dodecahedron is an irrationall line of that kinde which is called a reſiduall line. For foraſmuch as the line NO is diuided by an extreme and meane proportion in the point R, and his greater ſegment is the line OR, and the line OX is alſo diuided by an extreme and meane proportion in the point S, and his greater ſegment is the line OS. Wherefore the whole line NX is diuided by an extreme and meane proportion, and his greater ſegment is the line RS. (For for that as the line ON is to the line OR, ſo is the line OR to the line NR, and in the ſame proportion alſo are their doubles (for the partes of eque-multiplices haue one and the ſelfe ſame proportion with the whole, by the 15. of the fifth). Wherefore as the line NX is to the line RS, ſo is the line RS to both the lines NR and SX added together. But the line NX is greater then the line RS, by both the lines NR and SX added together. Wherefore the line NX is diuided by an extreme and meane proportion, and his greater ſegment is the line RS. But the line RS is equall to the line VZ, as hath before bene proued. Wherefore the line NX is diuided by an extreme and meane proportion, and his greater ſegment is the line VZ. And foraſmuch as the diameter of the Sphere is rationall, and is in power treble to the ſide of the cube, by the 15. of this booke, therefore the line NX, being the ſide of the cube, is rationall. But if a rationall line be diuided by an extreme and meane proportion, either of the ſegmentes is (by the 6. of this booke) an irrationall line of that kinde which is called a reſiduall line. Wherefore the line VZ being the ſide of the dodecahedron, is an irrationall line of that kinde which is called a reſiduall line. Wherfore there is made a dodecahedron, and it is cõprehended in the Sphere geuen, wherein the other ſolides were contained, and it is proued that the ſide of the dodecahedron is a reſiduall line: which was required to be done, and alſo to be proued.

¶ Corollary.

Hereby it is manifeſt, that the ſide of a cube being diuided by an extreme and

and meane proportion, the greater ſegment thereof is the ſide of the dodecahedron. As it was manifeſt by the line V Z which was proued to be the greater ſegment of the right line N X, namely, of the ſide of the cube.

A further conſtruction of the dodecahedron after *Fluſſas*.

Foraſmuch as it hath bene proued that the pentagon B V Z C W is equilater and equiangle and toucheth one of the ſides of the cube. Let vs ſhow alſo by what meanes vpon eche of the 12. ſides of the cube may in like ſort be applyed pentagons ioyning one to the other, and compoſing the 12. baſes of the dodecahedron. Draw in the former figure theſe right lines A I, I D, I L, & K. Now foraſmuch as the line P L was in the point & diuided like vnto the lines P H, O N. or O X, and vpon the pointes T, P, &, were erected perpendicular lines equall vnto the line O Y, and the reſt: namely, vnto the greater ſegmēt: and the lines T W and & I were proued parallels, therefore the lines W I and T & are parallels, by the 7. of the eleuenth, and 33. of the firſt. Wherefore alſo, by the 9. of the eleuenth, the lines W I and D C are parallels. Wherefore by the 7. of the eleuenth C W I D is a playne ſuperficies. And the triangle A I D is a playne ſuperficies, by the 2. of the eleuenth. Now it is manifeſt that the right lines I D, & I A are equall to the right line W C. For the right lines A L & L & (which are equall to the right lines B H, & H T) do make the ſubtēded lines A & and B T equall by the 4. of the firſt. And agayne foraſmuch as the lines B T and T W contayne a right angle B T W, as alſo doo the right lines A & and & I contayne the right angle A & I (for the right lines W T, and I & are erected perpendicularly vnto one and the ſelfe ſame playne A B C D by ſuppoſition). And the ſquares of the lines B T and T W are equall to the ſquares of the lines A &, and & I (for it is proued that the line B T is equall to the line A &, and the line T W to the line & I). And vnto the ſquares of the lines B T and T W is equall the ſquare of the line B W, by the 47. of the firſt: likewiſe by the ſame vnto the ſquares of the lines A & and & I is equall the ſquare of the line A I. Wherefore the ſquare of the line B W is equall to the ſquare of the line A I, wherefore alſo the line B T is equall to the line A I. And by the ſame reaſon are the lines I D and W C equall to the ſame lines. Now foraſmuch as the lines A I and I D, and the lines A L and L D are equall, and the baſe I L is common to them both, the angles A L I and D L I ſhalbe equall, by the 8. of the firſt: and therefore they are right angles, by the 10. diffinition of the firſt. And by the ſame reaſon are the angles W H B, and W H C right angles. And foraſmuch as the two lines H T and T W are equall to the two lines L & and & I, and they contayne equall angles, that is, right angles by ſuppoſition, therefore the angles W H T, and I L &, are equall by the 4. of the firſt. Wherefore the playne ſuperficies A I D is in like ſort inclined to the playne ſuperficies A B C D, as the playne ſuperficies B W C is inclined to the ſame playne A B C D, by the 4. diffinition of the eleuenth. In like ſort may we proue that the playne W C D I is in like ſort inclined to the playne A B C D, as the playne B V Z C is to the playne E B C F. For that in the triangles Y O H and K P K which conſiſt of equall ſides (eche to his correſpondent ſide), the angles Y H O, and K K P, which are the angles of the inclination, are equall. And now if the right line K K be extended to the point a, and the pentagon C W I D a be made perfect, we may, by the ſame reaſon, proue that that playne is equiangle and equilater, that we proued the pentagon B V Z C W to be equaliter and equiangle. And likewiſe if the other playnes B W I A and A I D be made perfect, they may be proued to be equall and like pentagons and in like ſort ſituate, and they are ſet vpon theſe common right lines B W, W C, W I, A I, and I D. And obſeruing this methode, there ſhall vpon euery one of the 12. ſides of the cube be ſet euery one of the 12. pentagons which compoſe the dodecahedron.

Draw in the former figure theſe lines, & A, & L, & D.

¶ Certayne Corollaryes added by *Fluſſas*.

⁂ *Firſt Corollary.*

The ſide of a cube, is equall to the right line which ſubtendeth the angle of the pentagon of a dodecahedron contayned in one and the ſelfe ſame ſphere with the cube.

For the angles B W C and A I D, are ſubtended of the lines B C and A D. Which are ſides of the Cube.

¶ *Second Corollary.*

In a dodecahedron there are ſixe ſides euery two of which are parallels and oppoſite, whoſe ſections into two equall partes, are coupled by three right lines, which in the center of the ſphere which contayneth the dodecahedron, deuide into two equall partes and perpendicularly both themſelues and alſo the ſides.

For vpon the ſixe baſes of the cube are ſet ſixe ſides of the dodecahedron, as it hath bene proued (by the lines Z V, W I &c.) which are cutte into two equall partes by right lines, which ioyne together the centers of the baſes of the cube, as the line Y O produced, and the other like. Which lines coupling together the centers of the baſes are three in number, cutting the one the other perpendicularly (for they are parallels to the ſides of the cube) and they cutte the one the other into two equall partes in the center of the ſphere which contayneth the cube (by that which was demonſtrated in the 15. of this booke). And vnto theſe equall lines, ioyning together the centers of the baſes of the cube, are without the baſes added equall partes O Y, P ſ, and the other like, which by ſuppoſition are equall to halfe of the ſide of the dodecahedron. Wherefore the whole lines, which ioyne together the ſectoins of the oppoſite ſides of the dodecahedron, are equall, and they cut thoſe ſides into two equall partes and perpendicularly.

Third Corollary.

A right line ioyning together the poynts of the ſections of the oppoſite ſides of the dodecahedron into two equall partes, being diuided by an extreame and meane proportion: the greater ſegment thereof ſhalbe the ſide of the cube, and the leſſe ſegment the ſide of the dodecahedron contayned in the ſelfe ſame ſphere.

For it was proued that the right line Y Q is diuided by an extreame and meane proportion in the poynt O, and that his greater ſegment O Q is halfe the ſide of the cube, and his leſſe ſegment O Y is halfe of the ſide V Z (which is the ſide of the dodecahedron). Wherefore it followeth (by the 15. of the fifth) that their doubles are in the ſame proportion. Wherefore the double of the line Y Q which ioyneth the poynt oppoſite vnto the line Y, is the whole: and the greater ſegment is the double of the line O Q which is the ſide of the cube: & the leſſe ſegment is the double of the line Y O, which is equall to the ſide of the dodecahedron, namely, to the ſide V Z.

¶ The 6. Probleme. The 18. Propoſition.

To finde out the ſides of the foreſayd fiue bodies, and to compare them together.

TAke the diameter of the Sphere geuen, and let the ſame be A B, and diuide it in the point C, ſo that let the line A C be equall to C B, by the 10. of the firſt: and in the point D, ſo that let A D be double to D B, by the 9. of the ſixt. And vpon the line A B deſcribe a ſemicircle A E B. And from the pointes C and D, raiſe vp (by the 11. of the firſt) vnto the line A B perpendicular lines C E and D F. And draw theſe right lines A F, F B, and B E. Now foraſmuch as the line A D is double to the line D B, therefore the line A B is treble to the line D B. Wherefore the line B A is ſeſquialter to the line A D (for it is as 3. to 2.). But as the line B A is to the line A D, ſo is the ſquare of the line B A to the ſquare of the line A F (by the 6. of the ſixt, or by the Corollary of the ſame, and by the Corollary of the 20. of the ſame): for the triangle A F B is equiangle to the triangle A F D. Wherefore the ſquare of the line B A is ſeſquialter to the ſquare of the line A F. But the diameter of a ſphere is in power ſeſquialter to the ſide of the pyramis, by the 13. of this booke, and the line A B is the diameter of the ſphere. Wherefore the line A F is equall to the ſide of the pyramis.

The ſide of a pyramis.

Againe foraſmuch as the line A B is treble to the line B D: but as the line A B is to the line B D, ſo is the ſquare of the line A B to the ſquare of the line F B, by the Corollaries of the 8. and 20. of the ſixt. Wherefore the ſquare of the line A B is treble to the ſquare of the line F B. But the diameter of a ſphere is in power treble to the ſide of the cube (by the 15. of this booke) and the diameter of the ſphere is the line A B. Wherefore the line B F is the ſide of the cube.

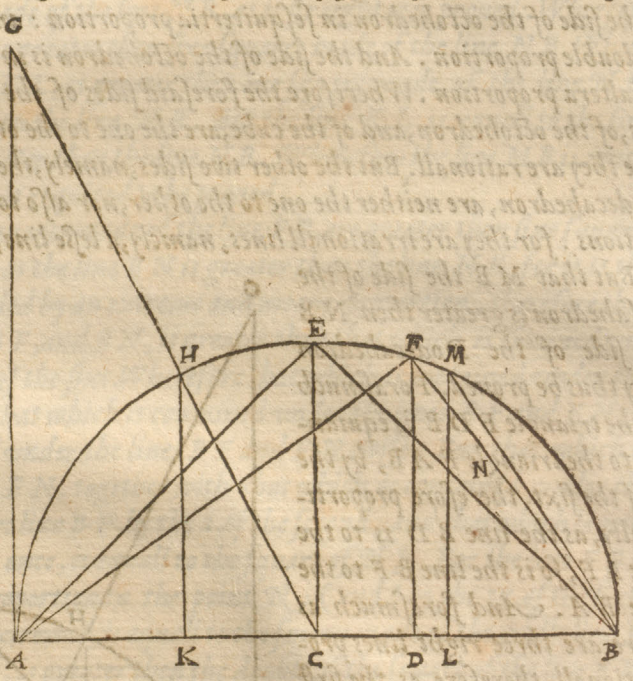
The ſide of a cube.

And foraſmuch as the line A C is equall to the line C B, therefore the line A B is double to the line C B. But as the line A B is to the line C B, ſo is the ſquare of the line A B to the ſquare of the line B E (by the former Corollaries). Therefore the ſquare of the line A B is double to

the

the ſquare of the line B E. But the diameter of the ſphere is in power double to the ſide of the octohedron, and A B is the diameter of the ſphere geuen: wherefore the line B E is the ſide of the octohedron. Raiſe vp (by the 11. of the firſt) from the point A vnto the right line A B a perpendicular line A G. And put the line A G equall to the line A B. And drawe a right line from the point G to the point C. And let the line G C cut the circumference in the point H. And (by the 12. of the firſt) from the point H drawe vnto the line A B a perpendicular line H K. Now foraſmuch as the line G A is double to the line A C (for G A is equall to A B). But as G A is to A C, ſo is H K to K C (by the 28. of the firſt, and Corollary of the 2. of the ſixt): wherefore the line H K is double to the line K C. Wherefore the ſquare of the line H K is quadruple to the ſquare of the line K C, by the Corollary of the 20. of the ſixt. Wherefore the ſquares of the lines H K and K C, which are all one with the ſquare of the line H C, by the 47. of the firſt, is quintuple to the ſquare of the line K C. But the line H C is equall to the line C B, by the definition of a circle. Wherfore the ſquare of the line B C is quintuple to the ſquare of the line C K. And foraſmuch as the line A B is double to the line B C, of which the line A D is double to the line D B. Wherfore the reſidue, namely, B D, is double to the reſidue, namely, to D C (by the 19. of the fift). VVherefore the line B C is treble to the line C D. Wherefore the ſquare of the line B C is nonecuple to the ſquare of the line C D, by the Corollary of the 20. of the ſixt. But the ſquare of B C is onely quintuple to the ſquare of C K. Wherefore the ſquare of C K is greater thē the ſquare of C D, by the 10. of the fift. Wherefore alſo the line C K is greater then the line D C. Vnto the line C K put (by the 2. of the firſt) an equall line C L. And from the point L raiſe vp vnto the line A B a perpendicular line L M. And drawe a right line from the point M to the point B. Now foraſmuch as the ſquare of the line C B is quintuple to the ſquare of the line C K, and the line A B is double to the line B C, and the line K L is double to the line C K: therefore the ſquare of the line A B is quintuple to the ſquare of the line K L, by the 15. of the fift. But the diameter of a ſphere is in power quintuple to the line which is drawen from the centre of the circle to the circumference on which the Icoſahedron is deſcribed, by the Corollary of the 16. of the ſixt. And the line A B is the diameter of the ſphere: wherefore the line K L is the ſemidiameter of the circle on which the Icoſahedron is deſcribed. Wherefore the line K L is the ſide of an hexagon figure deſcribed in the ſame circle, by the Corollary of the 15. of the fourth. And foraſmuch as the diameter of the ſphere is made of the ſide an hexagon figure, and of two ſides of a decagon being ech of them deſcribed in one and the ſelfe ſame circle (by the Corollary of the 16. of this booke): and the line A B is the diameter of the ſphere, and the line K L is the ſide of the hexagon, and the line A K is equall to the line L B: wherefore either of the lines A K and L B is the ſide of a decagon deſcribed in the circle on which the Icoſahedron is deſcribed (that is, in the circle whoſe ſemidiameter is the line K L). And foraſmuch as the line L B is the ſide of a decagon, and the line M L of an hexagon (for M L is equall to K L, for that it is equall to K H, by the 14. of the third, for they are equally diſtant

The ſide of an octohedron.



distant from the centre, and ech of the lines H K and K L is double to K C). Wherefore the line M B is the side of a pentagon, by the 10. of this booke. But the side of the pentagon is also the side of the Icosahedron, by that which was demonstrated in the 10 of this booke. Wherefore the line M B is the side of an Icosahedron.

The side of an Icosahedron.

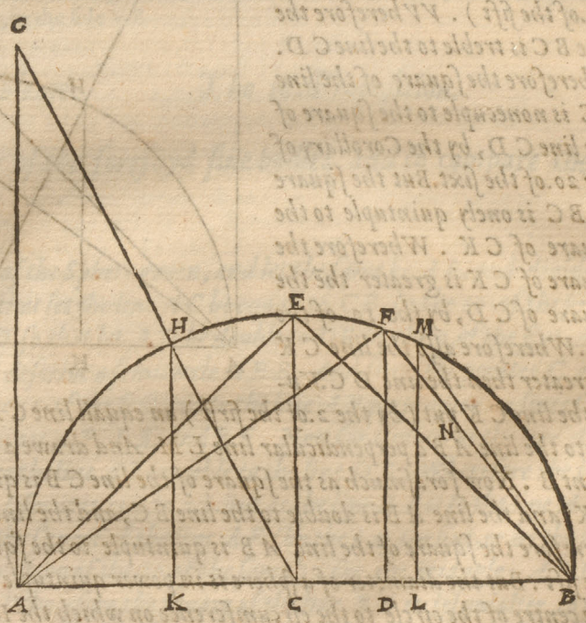
The sides of a dodecahedron.

And forasmuch as the line F B is the side of a cube, let it be diuided by an extreme and meane proportion in the point N, and let the greater segment therof be N B. Wherefore the line N B is the side of a Dodecahedron, by the Corollary of the 17. of this booke.

Comparison of the fiue sides of the foresayd bodies.

And forasmuch as it hath bene proued, by the 13. of this booke, that the diameter of the sphere is in power sesquialter to A F the side of the pyramis, and is in power double to B E the side of the octohedron, by the 14. of the same, and is in power treble to F B the side of the cube, by the 15. of the same. Wherefore it followeth, that of what partes the diameter of the sphere containeth sixe, of such partes the side of the pyramis containeth fower: and the side of the octohedron three: and the side of the cube two. Wherefore the side of the pyramis is in power to the side of the octohedron in sesquitertia proportion: and is in power to the side of the cube in double proportion. And the side of the octohedron is in power to the side of the cube in sesquialtera proportion. Wherefore the foresaid sides of the three figures, that is, of the pyramis, of the octohedron, and of the cube, are the one to the other in rationall proportions Wherfore they are rationall. But the other two sides, namely, the sides of the Icosahedron and of the Dodecahedron, are neither the one to the other, nor also to the foresaid sides, in rationall proportions: for they are irrationall lines, namely, a lesse line, and a residuall line.

The side of the Icosahedron proued greater then the side of the dodecahedron.



But that M B the side of the Icosahedron is greater then N B the side of the Dodecahedron may thus be proued. Forasmuch as the triangle F D B is equiangle to the triangle F A B, by the 8. of the sixt, therefore proportionally, as the line B D is to the line B F, so is the line B F to the line B A. And forasmuch as there are three right lines proportionall, therefore as the first is to the third, so is the square made of the first to the square made of the second, by the Corollary of the 20. of the sixt. Wherfore as the line D B is to the line B A, so is the square of the line D B to the square of the line B F. Wherefore (by conuersion, by the Corollary of the 4. of the fiueth) as the line A B is to the line B D, so is the square of the line F B to the square of the line B D. But the line A B is treble to the line B D, as hath before bene proued. Wherefore the square of the line F B is treble to the square of the line B D. But the square of the line A D is quadruple to the square of the line D B, by the Corollary of the 20. of the sixt, for the line A D is double to the line D B. Wherefore the square of the line A D is greater then the square of the line F B, by the 10. of the fift. Wherefore also the line A D is greater then the line F B. Wherefore the line A L is much greater then the line F B. And the line A L being diuided by an extreme & meane proportion, his greater segment is the line K L, by the 9. of this boke: (for the line L K is the side of an hexagon, and the line K A is the side of a decagon inscribed in one and the same circle, as hath before bene proued): and the line F B being diuided

by an

by an extreme and meane proportion, his greater ſegment is N B. Wherefore the line K L is greater then the line N B. (* For two lines diuided by an extreme and meane proportion, are euery way like proportionall). But the line K L is equall to the line L M. Wherefore the line L M is greater then the line N B. But the line M B is greater then the line L M. Wherfore the line M B being the ſide of the Icoſahedron, is much greater then the line N B being the ſide of the Dodecahedron: which was required to be done, and to be proued.

* This Aſſumpt is afterward proued in the 14. booke and 4. propoſition.

An other way to proue that the line M B is greater then the line N B. Foraſmuch as the line A D is double to the line D B, therefore the line A B is treble to the line D B. But as A B is to B D, ſo is the ſquare of the line A B to the ſquare of the line B F, by the 8. of the ſixt (for the triangle F A B is equiangle to the triangle F D B). Wherefore the ſquare of the line A B is treble to the ſquare of the line B F. And it is before proued, that the ſquare of the line A B is quintuple to the ſquare of the line K L. Wherefore fiue ſquares made of the line K L, are equall to three ſquares made of the line F B. But three ſquares made of the line F B, are greater then ſixe ſquares made of the line N B, as is ſtraight way proued. Wherfore fiue ſquares made of the line K L, are greater then ſixe ſquares made of the line N B. Wherfore alſo one ſquare made of the line K L, is greater then one ſquare made of the line N B. Wherefore the line K L is greater then the line N B. But the line K L is equall to the line L M. Wherefore the line L M is greater then the line N B. Wherefore the line M B is much greater then the line N B: which was required to be proued.

An other demõmonſtration to proue that the ſide of the Icoſahedrõ is greater then the ſide of the dodecahedron.

But now let vs proue that three ſquares made of the line F B, are greater then ſixe ſquares made of the line N B. Foraſmuch as the line B N is greater then the line N F, for it is the greater ſegment of the line B F diuided by an extreme and meane proportion, therefore that which is contained vnder the lines B F, and B N, is greater then that which is cõtained vnder the lines B F and F N, by the 1. of the ſixt. Wherefore that which is contained vnder the lines B F and B N, together with that which is contained vnder the lines B F and F N, is greater then that which is contained vnder the lines B F and F N twiſe. But that which is contained vnder the lines B F and F N, together with that which is contained vnder the lines B F and B N, is the ſquare of the line B F, by the 2. of the ſecond, and that which is contained vnder the lines B F and F N once, is equall to the ſquare of N B. For the line F B is diuided by an extreme and meane proportion in the point N: (and (by the 17. of the ſixt) that which is contained vnder the extremes, is equall to the ſquare made of the midle line). Wherefore the ſquare of the line F B, is greater then the double of the ſquare of the line B N. Wherefore one of the ſquares made of the line B F, is greater then two ſquares made of the line B N. Wherefore alſo three ſquares made of the line F B, are greater then ſixe ſquares made of the line B N: which was required to be proued.

That 3. ſquares of the line F B are greater thẽ 6. ſquares of the line N B.

A Corollary.

Now alſo I ſay that beſides the fiue foreſayd ſolides there can not be deſcribed any other ſolide cõprehẽded vnder figures equilater & equiangle the one to the other. For of two triangles, or of any two other playne ſuperficieces can not be made a ſolide angle (for, that is cõtrary to the diffinition of a ſolide angle). Vnder three triangles is contayned the ſolide angle of a pyramis: vnder fower, the ſolide angle of an octohedrõ: vnder fiue, the ſolide angle of an Icoſahedrõ: of ſixe, equilater & equiangle triangles ſet to one point can not be made a ſolide angle. For foraſmuch as the angle of an equilater triangle is two third partes of a right angle, the ſixe angles of the ſolide ſhalbe equall to fower right angles, which is impoſſible. For euery ſolide angle is (by the 21. of the eleuẽth) contayned vnder playne angles leſſe thẽ fower right angles. And by the ſame reaſon can not be made a ſolide angle contained vnder more thẽ ſixe playne ſuperficiall angles of equilater triangles. Vnder three ſquares is contained the angle of a cube. Vnder fower ſquares it is impoſſible that a ſolide angle ſhould be contayned: for then

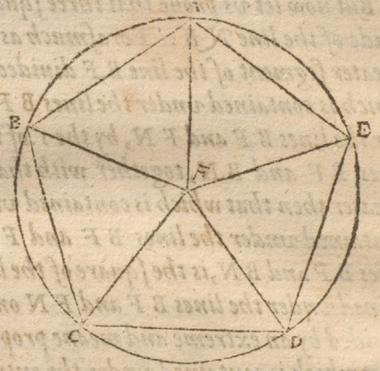
That there can be no other ſolide beſids theſe fiue, contayned vnder equilater and equiangle baſes.

agayne it ſhould be contayned vnder fower right angles. Wherefore much leſſe can any ſolide angle be contayned vnder more ſquares then fower. Vnder three equilater and equiangle pentagons is contayned the ſolide angle of a dodecahedron. But vnder fower it is impoſſible. For foraſmuch as the angle of a pentagon is a right angle and the fift part more of a right angle, the fower angles ſhalbe greater then fower right angles: which is impoſſible. And therfore much leſſe can a ſolide angle be compoſed of more pentagons then fower. Neither can a ſolide angle be contayned vnder any other equilater and equiangle figures of many angles, forthat that alſo ſhould be abſurd. For the more the ſides increaſe, the greater are the angles which they contayne, and therfore the farther of are the ſuperficiall angles contayned of thoſe ſides from compoſing of a ſolide angle. Wherefore beſides the foreſayd fiue figures there can not be made any ſolide figure contayned vnder equall ſides and equall angles: which was required to be proued.

An Aſſumpt.

That the angle of an equilater and equiangle Pentagon is one right angle and a fiueth part ouer: which thing was alſo before proued in the corollary of the 32. of the firſt.

But now that the angle of an equilater and equiangle pentagon is a right angle and a fifth part more of a right angle, may thus be proued. Suppoſe that A B C D E be an equilater and equiangle pentagon. And (by the 14. of the fourth) deſcribe about it a circle A B C D E. And take (by the 1. of the third) the center thereof, and let the ſame be F. And draw theſe right lines F A, F B, F C, F D, F E. Wherefore thoſe lines do diuide the angles of the pentagon into two equall partes in the poyntes A, B, C, D, E, by the 4. of the firſt. And foraſmuch as the fiue angles that are at the poynt F are equall to fower right angles, by the corollary of the 15. of the firſt, and they are equall the one to the other by the 8. of the firſt: therfore one of thoſe angles, as for example ſake, the angle A F B is a fifth part leſſe then a right angle. Wherfore the angles remayning, namely, F A B, & A B F, are one right angle and a fifth part ouer. But the angle F A B is equall to the angle F B C. Wherefore the whole angle A B C being one of the angles of the pentagon is a right angle and a fifth part more then a right angle: which was required to be proued.



¶A Corollary added by *Fluſſas*.

Now, let vs teach, how thoſe fiue ſolides, haue eche like inclinations of theyr baſes.

The ſides of the angle of the inclination of the ſuperficieces of the Tetrahedrō are proued rationall.

Firſt let vs take a Pyramis, and diuide one of the ſides thereof into two equall parts: and from the two angles oppoſite vnto that ſide, draw perpēdiculars, which ſhall fall vpon the ſection, by the corollary of the 12. of the thirteenth, and at the ſayd poynt of diuiſion (as may eaſily be proued). Wherfore they ſhal containe the angle of the inclination of the plaines, by the 4. diffinition of the eleuenth, which angle is ſubtended of the oppoſite ſide of the pyramis. Now foraſmuch as the reſt of the angles of the inclination of the playnes of the Pyramis, are contayned vnder two perpēdicular lines of the triangles, and are ſubtended of the ſide of the Pyramis, it foloweth, by the 8. of the firſt, that thoſe angles are equall. Wherefore (by the 5. diffinition of the eleuēth) the ſuperficieces are in like ſort inclined the one to the other.

The ſides of the angle of the inclination of the ſuperficieces of the cube proued rationall.

One of the ſides of a Cube being diuided into two equall parts, if from the ſayd ſection be drawen in two of the baſes thereof, two perpendicular lines, they ſhalbe parallels and equall to the ſides of the ſquare which cōtayne a right angle. And foraſmuch as all the angles of the baſes of the Cube are right angles: therefore thoſe perpendiculars falling vpon the ſection of the ſide common to the two baſes, ſhall contayne a right angle (by the 10. of the eleuenth): which ſelfe angle is the angle of inclination (by the 4. diffinition of the eleuenth) and is ſubtended of the diameter of the baſe of the Cube. And by the

the ſame reaſon may we proue that the reſt of the angles of the inclination of the baſes of the cube are right angles. Wherefore the inclinations of the ſuperficieces of the cube the one to the other, are equal (by the 5. diffinition of the eleuenth).

That the plaines of an octohedron are in like ſort inclined.

In an Octohedron take the diameter which coupleth the two oppoſite angles. And from thoſe oppoſite angles draw to one and the ſelfe ſame ſide of the Octohedron, in two baſes thereof, two perpendicular lines, which ſhall diuide that ſide into two equall parts and perpendicularly (by the Corollary of the 12. of the thirtenth). Wherefore thoſe perpendiculars ſhall contayne the angle of the inclination of the baſes (by the 4. diffinition of the eleuẽth): and the ſame angle is ſubtended of the diameter of the Octohedrõ. Wherfore the reſt of the angles after the ſame maner deſcribed in the reſt of the baſes, being comprehended and ſubtended of equall ſides, ſhall (by the 8. of the firſt) be equall the one to the other. And therefore the inclinations of the playnes in the Octohedron, ſhal (by the 5. diffinition of the eleuenth) be equall.

That the plaines of an Icoſahedron are in like ſort inclined.

In an Icoſahedron let there be drawen from the angles of two of the baſes, to one ſide common to both the ſayd baſes perpendiculars, which ſhall contayne the angle of the inclination of the baſes (by the 4. diffinition of the eleuenth): which angle is ſubtended of the right line which ſubtendeth the angle of the pentagon which contayneth fiue ſides of the Icoſahedron, by the 16. of this booke: for it coupleth the twoo oppoſite angles of the triangles which are ioyned together. Wherefore the reſt of the angles of the inclination of the baſes being after the ſame maner found out, they ſhalbe contayned vnder equall ſides, and ſubtended of equall baſes, and therefore (by the 8. of the firſt) thoſe angles ſhalbe equall. Wherfore alſo al the inclinations of the baſes of the Icoſahedron the one to the other ſhalbe equall, by the 5. diffinition of the eleuenth.

That the plaines of a Dodecahedron are in like ſort inclined.

In a Dodecahedron, from the two oppoſite angles of two next pentagons draw to theyr common ſide perpendicular lines, paſſing by the centres of the ſayd pentagons, which ſhal, where they fal, diuide the ſide into two equall parts by the 3. of the third. (For the baſes of a Dodecahedron are contayned in a circle) And the angle contayned vnder thoſe perpendicular lines is the inclination of thoſe baſes (by the 4. diffinition of the eleuenth). And the foreſayd oppoſite angles are coupled by a right line equal to the right line which coupleth the oppoſite ſections into two equall parts of the ſides of the dodecahedrõ (by the 33. of the firſt). For they couple together the halfe ſids of the dodecahedrõ, which halfes are parallels and equall, by the 3. corollary of the 17. of this booke: which coupling lines alſo are equall, by the ſame corollary. Wherefore the angles being contayned of equal perpendicular lines, and ſubtended of equall coupling lines, ſhall (by the 8. of the firſt) be equal. And they are the angles of the inclinations. Wherefore the baſes of the dodecahedron are in like ſort inclined the one to the other (by the 5. diffinition of the eleuenth).

Fluſſas after this teacheth how to know the rationality or irrationality of the ſides of the triangles, which contayne the angles of the inclinations of the ſuperficieces of the foreſayd bodies.

The ſides of the angle of the inclination of the ſuperficieces of the Tetrahedrõ are proued rationall.

In a Pyramis the angle of the inclinatiõ is contayned vnder two perpẽdicular lines of the triangles, and is ſubtended of the ſide of the Pyramis Now the ſide of the pyramis is in power ſeſquitertia to the perpendicular line, by the corollary of the 12. of this booke: and therfore the triangle cõtained of thoſe perpẽdicular lines and the ſide of pyramis, hath his ſides rational & commenſurable in power the one to the other.

The ſides of the angle of the inclination of the ſuperficieces of the cube proued rationall.

Foraſmuch as the twoo ſides of a Cube (or right lines equall to them) ſubtended vnder the diameter of one of the baſes, doo make the angle of the inclination: and the diameter of the cube is in power ſeſquialter to the diameter of the baſe, which diameter of the baſe is in power double to the ſide (by the 47. of the firſt): therefore thoſe lines are rationall and commenſurable in power.

The ſides of the angle &c. of the octohedron proued rationall.

In an Octohedron, whoſe two perpendiculars of the baſes contayne the angle of the inclination of the Octohedron, which angle alſo is ſubtended of the diameter of the Octohedron, the diameter is in power double to the ſide of the Octohedron, & the ſide is in power ſequitertia to the perpẽdiclar line, by the 12. of this booke: wherfore the diameter thereof is in power duple ſuperbipartiens tertias to the perpendicular line. Wherfore alſo the diameter and the perpẽdicular line are rationall and commenſurable (by the 6. of the tenth.)

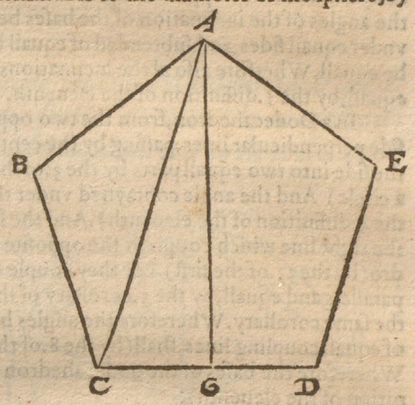
As touching an Icoſahedron, it was proued in the 16. of this booke, that the ſide thereof is a leſſe line, when the diameter of the ſphere is rationall. And foraſmuch as the angle of the inclination of the baſes thereof, is contayned of the perpendicular lines of the triangles, and ſubtended of the right line which ſubtendeth the angle of the Pentagon which contayneth fiue ſides of the Icoſahedron: and vnto the perpendicular lines the ſide is commenſurable (namely, is in power ſeſquitertia vnto them, by the Corollary of the 12. of this booke): therefore the perpendicular lines which contayne the angles are irrationall lines, namely, leſſe lines (by the 105. of the tenth booke.) And foraſmuch as the diameter contayneth in power both the ſide of the Icoſahedron, and the line which ſubtendeth the foreſayd angle, if from the power of the diameter which is rationall, be taken away the power of the ſide of the Icoſahedron which is irrationall, it is manifeſt that the reſidue which is the power of the ſubtending line ſhalbe irrationall. For if it ſhoulde be rationall, the number which meaſureth the whole power of the diameter, and the part taken away of the ſubtending line, ſhould alſo, by the 4. common ſentence of

The ſides of the angle &c. of the Icoſahedron proued irrationall.

the ſeuenth meaſure the reſidue, namely, the power of the ſide: which is irrationall for that it is a leſſe line, which were abſurd. Wherefore it is manifeſt that the right lines which compoſe the angle of the inclination of the baſes of the Icoſahedron are Irrationall lines. For the ſubtending line hath to the line contayninge, a greater proportion, then the whole hath to the greater ſegment.

The angle of the inclination of the baſes of a dodecahedron, is contayned vnder two perpendiculars of the baſes of the dodecahedron, and is ſubtended of that right line, whoſe greater ſegment is the ſide of a Cube inſcribed in the dodecahedron, which right line is equall to the line which coupleth the ſections, into two equal parts, of the oppoſite ſides of the dodecahedron. And this coupling line we ſay is an irrationall line, for that the diameter of the ſphere contayneth in power both the coupling line, and the ſide of the dodecahedron: but the ſide of the dodecahedron is an irrationall line, namely, a reſiduall line (by the 17. of this booke). Wherefore the reſidue namely, the coupling line is an irrationall line, as it is eaſy to proue by the 4. cómon ſentence of the ſeuéth. And that the perpédicular lines which contayne the angle of the inclination are irrationall, is thus proued.

Suppoſe that there be a Pentagon A B C D E, and draw in it the perpendicular line, A G, and let the line ſubtending the angle of the pentagon be A C. Now foraſmuch as the right line A C is the ſide of the cube, and C D the ſide the Dodecahedron inſcribed in one and the ſelfe ſame ſphere, by the 2. Corollary of the 17. of this booke: but the line A C is commenſurable to the diameter of the ſphere, by the 15. of this booke, and is therefore rationall, by the 6. diffinitiõ of the tenth: & the right line C D which is the ſide of the dodecahedron is irrationall (by the 17. of this booke). Wherfore the line C G which is the halfe of the line C D is irrationall by the 103. of the 10. boke. And the right line A C contayneth in power the two right lines A G and G C (by the 47. of the firſt). If therfore from the power of the right line A C being rationall, be také away the power of the line C G being irrationall, the power remayning, namely, the power of the line A G, ſhall of neceſſitie be irrationall. For if the power of the line A G taken away ſhould be rationall, and the whole power of the line A C is rationall, the reſidue, namely, the power of the line C G ſhould be alſo rationall, and ſhould be meaſured by the ſelfe ſame numbers, by the 4. common ſentence of the ſeuenth. But it is proued that the line C G is irrationall, for it is the halfe of the whole reſiduall line C D (by the 17. of the thirtenth): which is impoſſible. Wherefore the perpendicular line A G is irrationall. Wherfore the angle of the inclination of the dodecahedron, which is contayned vnder two perpendicular lines of the pentagon, and is ſubtended of a right line, which coupleth the ſections into two equall parts of the oppoſite ſides of the dodecahedron (by the 2. corollary of the 18. of this booke) which line we haue proued to be irrationall (for that it is equall to the two lines A C, and C D by the 4. corollary of the 17. of this booke) is contayned vnder irrationall right lines.



The ſides of the angle &c. of the dodecahedrõ proued irrationall.

How to know whether the angle of the inclinatiõ be a right angle, an acute angle, or an oblique angle.

By the proportion of the ſubtending line (of the foreſayd angles of inclination) to the lines which containe the angle, is found out the obliquitie of the angle. For if the ſubtending line be in power double to the line which contayneth the angle, then is the angle a right angle (by the 48. of the firſt.) But if it be in power leſſe then the double it is an acute angle (by the 13. of the ſecond). But if it be in power more then the double, or haue a greater proportion then the whole hath to the greater ſegmẽt, the angle ſhalbe an obtuſe angle (by the 12. of the ſecond and 4. of the thirtenth). By which may be proued that the ſquare of the whole is greater then the double of the ſquare of the greater ſegment.

This is to be noted that that which *Fluſſas* hath here taught touching the inclinations of the baſes of the fiue regular bodies, Hypſicles teacheth after the 5. propoſition of the 15. booke. Where he confeſſeth, that he receiued it of one Iſidorus, and ſeking to make the mater more cleare, he endeuored himſelfe to declare, that the angles of the inclination of the ſolides are geuen, and that they are either acute or obtuſe, according to the nature of the ſolide: although *Euclide* in all his 15. bookes hath not yet ſhewed, what a thing geuen is. Wherefore *Fluſſas* framing his demõſtration vpon an other ground procedeth after an other maner, which ſemeth more playne, and more aptly hereto be placed then there. Albeit the reader in that place ſhal not be fruſtrate of his alſo.

(.˙.)

The ende of the thirtenth Booke of Euclides Elementes.

¶ The fourteenth booke of Euclides Elementes.

IN this booke, which is commonly accompted the 14. booke of *Euclide* is more at large intreated of our principall purpose: namely, of the comparison and proportion of the fiue regular bodies (customably called the 5. figures or formes of *Pythagoras*) the one to the other, and also of their sides together, eche to other: which thinges are of most secret vse, and inestimable pleasure, and commoditie to such as diligently search for them, and attayne vnto them. Which thinges also vndoubtedly for the woorthines and hardnes thereof (for thinges of most price are most hardest) were first searched, and found out of Philosophers, not of the inferior or meane sort, but of the depest and most grounded Philosophers, and best exercised in Geometry. And albeit this booke with the booke following, namely, the 15. booke, hath bene hetherto of all men for the most part, and is also at this day numbred and accompted amõgst *Euclides* bookes, and supposed to be two of his, namely, the 14. and 15. in order: as all exemplars (not onely new and lately set abroade, but also old monumentes written by hand) doo manifestly witnes: yet it is thought by the best learned in these dayes, that these two bookes are none of *Euclides*, but of some other author, no lesse worthy, nor of lesse estimation and authoritie, notwithstanding, then *Euclide*. *Apollonius* a man of deepe knowledge a great Philosopher and in Geometrie maruelous (whose wõderful bookes writtẽ of the sections of cones, which exercise & occupy the wittes of the wisest and best learned, are yet remayning) is thought, and that not without iust cause, to be the author of them, or as some thinke *Hypsicles* him selfe. For what can be more playnely, then that which he him selfe witnesseth in the preface of this booke. *Basilides of Tire* (sayth Hypsicles) *and my father together, scanning, and peysing a writing or booke of Apollonius, which was of the comparison of a dodecahedron to an Icosahedron inscribed in one and the selfe same sphere, and what proportion these figures had the one to the other, found that Apollonius had fayled in this matter. But afterward* (sayth he) *I found an other copy or booke of Apollonius, wherein the demonstration of that matter was full and perfect, and shewed it vnto them, whereat they much reioyced.* By which wordes it semeth to be manifest that *Apollonius* was the first author of this booke, which was afterward set forth by *Hypsicles*. For so his owne wordes after in the same preface seme to import.

The argument of the fourteenth booke.

The Preface of *Hypsicles* before the fourteenth booke.

FRend Protarchus, *whẽ that* Basilides *of Tire came into* Alexandria, *hauing familiar frendship with my father by reason of his knowledge in the mathematicall sciences, he remayned with him a long time, yea euen all the time of the pestilence. And sometime reasoning betwene themselues of that which* Apollonius *had written touching the comparison of a dodecahedron and of an Icosahedron inscribed in one and the selfe same sphere, what proportion such bodies haue the one to the other, they iudged that* Apollonius *had somewhat erred therein. Wherefore they (as my father declared vnto me) diligently weighing it, wrote it perfectly. Howbeit afterward I happened to finde an other booke written of* Apollonius, *which contayned in it the*

right demonſtration of that which they ſought for: which when they ſaw, they much reioyſed. As for that which Apollonius *wrote, may be ſene of all men, for that it is in euery mans hand. And that which was of vs more diligently afterward written agayne, I thought good to ſend and dedicate vnto you, as to one whome I thought worthy commendation, both for that deepe knowledge which I know you haue in all kindes of learning, and chiefely in Geometrie, ſo that you are able redily to iudge of thoſe thinges which are ſpoken, and alſo for the greate loue and good will which you beare towardes my father and me. Wherfore vouchſafe gently to accept this, which I ſend vnto you. But now is it time to end our preface, and to begin the matter.*

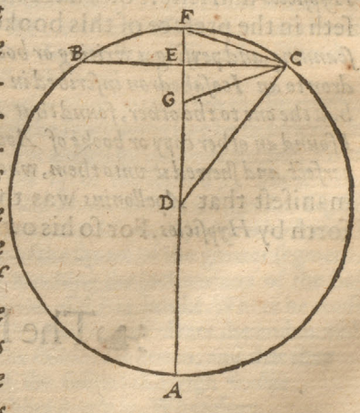
¶ The 1. Theoreme. The 1. Propoſition.

First propoſition after Fluſſas.

A perpendicular line drawen from the centre of a circle to the ſide of a Pentagon deſcribed in the ſame circle: is the halfe of theſe two lines, namely, of the ſide of an hexagon figure, and of the ſide of a decagon figure being both deſcribed in the ſelfe ſame circle.

Conſtruction.

Vppoſe that the circle be A B C. And let the ſide of an equilater Pentagon deſcribed in the circle A B C, be B C. And (by the 1. of the third) take the centre of the circle, and let the ſame be D. And (by the 12. of the firſt) from the point D draw vnto the line B C a perpendicular line D E. And extend the right line D E directly to the point F. Then I ſay, that the line D E (which is drawen from the centre to B C the ſide of the pentagon) is the halfe of the ſides of an hexagon and of a decagon taken together and deſcribed in the ſame circle. Draw theſe right lines D C and C F. And vnto the line E F put an equall line G E. And draw a right line from the point G to the point C. Now foraſmuch as the circumference of the whole circle is quintuple to the circũference B F C (which is ſubtended of the ſide of the pentagon) and the circumference A C F is the halfe of the circumference of the whole circle, and the circumference C F (which is ſubtended of the ſide of the decagon) is the halfe of the circumference B C F: therefore the circumference A C F is quintuple to the circumference C F (by the 15. of the fift). Wherefore the circumference A C is qradruple to the circumference F C. But as the circumference A C is to the circumference F C, ſo is the angle A D C to the angle F D C, by the laſt of the ſixt. Wherefore the angle A D C is quadruple to the angle F D C. But the angle A D C is double to the angle E F C, by the 20. of the third: Wherefore the angle E F C is double to the angle G D C. But the angle E F C is equall to the angle E G C, by the 4. of the firſt. Wherfore the angle E G C is double to the angle E D C. Wherefore the line D G is equall to the line G C (by the 32. and 6. of the firſt). But the line G C is equall to the line C F, by the 4. of the firſt. Wherfore the line D G is equall to the line C F. And the line G E is equall to the line E F (by conſtruction). Wherefore the line D E is equall to the lines E F and F C added together. Vnto the lines E F and F C adde the line D E. Wherefore the lines D F and F C added together, are double to the line D E. But the line D F is equall to the ſide of the hexagon: and F C to the ſide of the decagon.



Demonſtration.

Wherefore

Wherefore the line D E is the halfe of the side of the hexagon, and of the side of the decagon being both added together and described in one and the selfe same circle.

*It is manifest * by the Propositions of the thirteenth booke, that a perpendicular line drawen from the centre of a circle to the side of an equilater triangle described in the same circle, is halfe of the semidiameter of the circle. Wherefore by this Proposition, a perpendicular drawen from the centre of a circle to the side of a Pentagon, is equall to the perpendicular drawen from the centre to the side of the triangle, and to halfe of the side of the decagon described in the same circle.*

* *This is manifest by the 12. propositiō of the thirteenth booke as Campane well gathereth in a Corollary of the same.*

¶ The 2. Theoreme. The 2. Proposition.

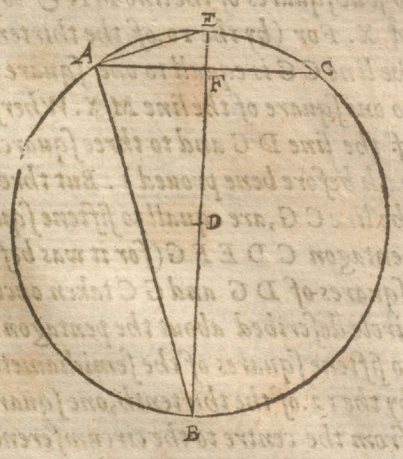
One and the selfe same circle comprehendeth both the Pentagon of a Dodecahedron, and the triangle of an Icosahedron, described in one and the selfe same Sphere.

The 4. popositiō after Flussas.

THis *Theoreme is described of* Aristeus *in that booke whose title is,* The comparison of the fiue figures, *and is described of* Apollonius *in his second edition of the cōpariſon of a Dodecahedron to an Icosahedron, which is,* * *that as the superficies of a Dodecahedrō is to the superficies of an Icosahedron, so is the Dodecahedron to an Icosahedron, for that a perpendicular line drawen from the centre of a sphere to the pentagon of a dodecahedron and to the triangle of an Icosahedron is one and the selfe same. Now must we also proue that one and the selfe same circle comprehendeth both the pentagon of a Dodecahedron, and also the triangle of an Icosahedron described in one and the selfe same sphere, first this being proued:*

* *This is afterward proued in the 4. proposition.*

If in a circle be described an equilater pentagon, the squares which are made of the side of the pentagon, and of that right line which is subtended vnder two sides of the pentagon, are quintuple to the square of the semidiameter of the circle. Suppose that A B C be a circle. And let the side of a pentagon in the circle A B C, be A C. And take (by the 1. of the third) the centre of the circle and let the same be D. And (by the 12. of the first) from the point D draw vnto the line A C a perpendicular line D F. And extend the line D F on either side to the pointes B and E. And draw a right line from the point A to the point B. Now I say, that the squares of the lines B A and A C are quintuple to the square of the line D E. Drawe a right line from the point A to the point E. Wherefore the line A E is the side of a decagon figure. And forasmuch as the line B E is double to the line D E: therefore the square of the line B E is quadruple to the square of D E (by the 20. of the sixt). But vnto the square of the line B E, are equall the squares of the lines B A and A E (by the 47. of the first, for the angle B A E is a right angle, by the 31. of the third). Wherefore the squares of the lines B A and A E, are quadruple to the square of the line D E. Wherfore the squares of the lines A B, A E, and D E, are quintuple to the square of the line D E. But the squares of the lines D E and A E, are equall to the square of the line A C (by the 10. of the thirteenth). Wherefore the squares of the lines B A and A C, are quintuple to the square of the line D E.



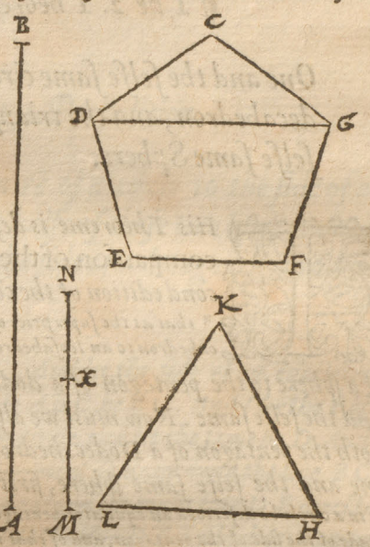
This Assumpt is the 3. proposition after Flussas. Construction of the Assumpt.

Demonstration of the Assumpt.

This being thus proued, now is to be demonstrated that one and the selfe same circle cōprehendeth both the pentagon of a dodecahedron, & the triangle of an Icosahedron described in one & the self same circle. Take the diameter of the sphere, & let the same be A B. And in the same sphere describe a dodecahedron, & also an Icosahedron. And let one of the pētagons of the dodecahedron be C D E F G, & let one of the triangles of the Icosahedron be K L H. Now I say that the semidiameters of the circles which are described about them are equall, that

Construction of the proposition.

that is, that one and the selfe same circle contayneth both the pentagon C D E F G, and the triangle K L H. Draw a right line from the point D to the point G. Wherfore the line D G is the side of a cube (by the corollary of the 17. of the thirteth). Take a certayne right line M N. And let the square of the line A B be quintuple to the square of the line M N, by the assumpt put after the 6. propositiõ of the teth. But the diameter of a sphere is in power quintuple to the square of the semidiameter of the circle, on which is described the Icosahedrõ (by the corollary of the 16. of the thirteenth). Wherefore the line M N is the semidiameter of the circle on which is described the Icosahedron. Diuide (by the 30. of the sixth) the line M N by an extreame and meane proportion in the poynt X. And let the greater segment thereof be M X. Wherefore the line M X is the side of a decagon described in the same circle (by the corollary of the 9. of the thirteenth). And forasmuch as the square of the line A B is quintuple to the square of the line M N: But the square of the line B A is treble to the square of the line D G (by the corollary of the 15. of the thirteenth). Wherfore three squares of the line D G are equall to fiue squares of the line M N.* But as thre squares of the line D G are to fiue squares of the line M N, so are three squares of the line C G to fiue squares of the line M X. Wherfore three squares of ỹ line C G are equll to fiue squares of the line M X. But fiue squares of the line C G are equal to fiue squares of the line M N & to fiue squares of the M X. For (by the 10. of the thirteenth) one square of the line C G is equall to one square of the line M N & to one square of the line M X. Wherfore fiue squares of the line C G are equall to thre squares of the line D G and to three squares of the line C G (as it is not hard to proue, marking what hath before bene proued). But three squares of the line D G, together with three squares of the line C G, are equall to fiftene squares of the semidiameter of the circle described about the pentagon C D E F G (for it was before proued in the assumpt put in this proposition) that the squares of D G and G C taken once, are quintuple to the square of the semidiameter of the circle described about the pentagon C D E F G). And fiue squares of the line K L are equall to fiftene squares of the semidiameter of the circle described about the triangle K L H. (For by the 12. of the thirteenth, one square of the line L K is triple to one square of the line drawne from the centre to the circumference). Wherefore fiftene squares of the line drawne from the centre to the circumference (of the circle which contayneth the pentagon C D E F G) are equall to fiftene squares of the line drawne from the centre to the circumference of the circle which contayneth the triangle K L H): wherefore one of the squares which is drawne from the centre to the circumference of the one circle, is equall to one of the squares which is drawne from the centre to the circumference of the other circle. Wherefore the diameter is equall to the diameter, wherefore one and the selfe same circle comprehendeth both the pentagon of a dodecahedron and the triangle of an Icosahedron described in one and the selfe same circle: which was required to be proued.



Demonstration of the proposition.

* Here is required an Assumpt which is afterward proued in the 4. propositiõ of this boke: namely, that lines deuided by an extreme & meane proportion, are euery way proportionall: which graunted then followeth the reason aptly. For the line M N is by supposition deuided by an extreme and meane proportion, & his greater segment is the line M X, and the line D G deuided by an an extreame and meane proportion, his greater segmẽt is the line C G, by the 8. of the thirteenth.

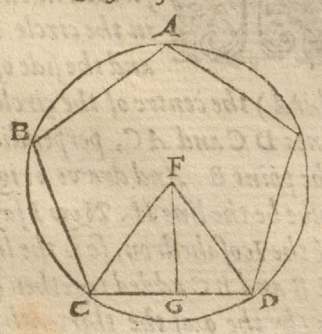
¶ The 3. Theoreme. The .3 Proposition.

The 5. proposition after Flussas.

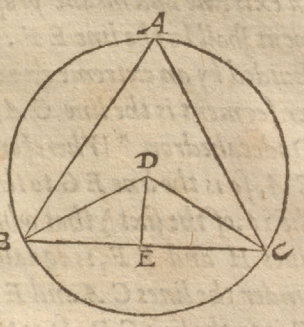
If there be an equilater and equiangle pẽtagon, aud about it be described a circle, and from the centre to one of the sides be drawne a perpendicular line, that which is contayned vnder one of the sides and the perpendicular line

line thirty times, is equall to the ſuperficies of the dodecahedron.

SVppoſe that A B C D be an equilater and equiangle pentagon. And about the ſame pentagon, deſcribe (by the 14. of the fourth) a circle. And let the centre ther- Conſtruction.
of be the poynt F. And from the poynt F draw (by the 12. of the firſt) vnto the line C D a perpendicular line F G. Now I ſay that that which is contayned vnder the lines C D and G F thirty times, is equall to 12. pentagons of the ſame quantitie that the pentagon A B C D is. Draw theſe right lines C F and F D. Now foraſmuch as that Demonſtration.
which is contayned vnder the lines C D and FG is double to the triangle C D F (by the 41. of the firſt): therefore that which is contayned vnder the lines C D and F G fiue times is equall to ten of thoſe triangles. But ten of thoſe triangles are two pentagos, and ſixe times ten of thoſe triangles are all the pentagons. Wherefore that which is contayned vnder the lines C D and F G thirty times is equall to 12. pentagons But 12 pentagons are the ſuperficies of dodecahedron. Wherfore that which is contayned vnder the lines C D and F G thirty times is equall to the ſuperficies of the dodecahedron.



In like ſort alſo may we proue that if there be an equilater triangle, as for example, the The 5. propoſition after Fluſſas.
triangle A B C, and about it be deſcribed a circle, and the centre of the circle be the point D, and the perpendicular line be the line D E: that which is contayned vnder the lines B C and D E thirty times, is equall to the ſuperficies of the Icoſahedron. For agayne foraſmuch as that which is contayned vnder the lines D E and B C is double to the triangle D B C (by the 41. of the firſt): therefore two triangles are equall to that which is contayned vnder the lines D E and B C, and three of thoſe triangles contayne the whole triangle. Wherefore ſixe ſuch triangles as D B C is, are equall to that which is contayned vnder the lines D E and B C thriſe. But ſixe ſuch triangles as D B C is, are equall to two ſuch triangles as A B C is. Wherefore that which is contained vnder the lines D E and B C thriſe, is equall to two ſuch triangles as A B C is. But two of thoſe triangles take ten times contayneth the whole Icoſahedron. Wherfore that which is contayned vnder the lines D E & BC thirty times, is equall to twenty ſuch triangles as the triangle A B C is, that is, to ỹ whole ſuperficies of the Icoſahedrō. Demonſtration.



** Wherefore as the ſuperficies of the dodecahedron is to the ſuperficies of the Icoſahedron, ſo is that which is contayned vnder the lines C D and F G to that which is contayned vnder the lines B C and D E.*

* This is the reaſon of the Corollary following.

¶ Corollary.

By this it is manifeſt, that as the ſuperficies of the Dodecahedron is to the ſuperficies of the Icoſahedron, ſo is that which is contained vnder the ſide of the Pentagon, and the perpēdicular line which is drawen from the centre of the circle deſcribed about the Pentagon to the ſame ſide, to that which is contained vnder the ſide of the Icoſahedron and the perpendicular line which is drawen from the centre of the circle deſcribed about the triangle to the ſame ſide: ſo that the Icoſahedron and Dodecahedron be both deſcribed in one and the ſelfe ſame Sphere.

A Corollary which alſo Fluſſas putteth as a Corollary after the 5 propoſition in his order.

¶ The 4. Theoreme. The 4. Propoſition.

The 6. poposition after Flussas.

This being done, now is to be proued, that as the ſuperficies of the Dodecahedron is to the ſuperficies of the Icoſahedron, ſo is the ſide of the cube to the ſide of the Icoſahedron.

Couſtruction.

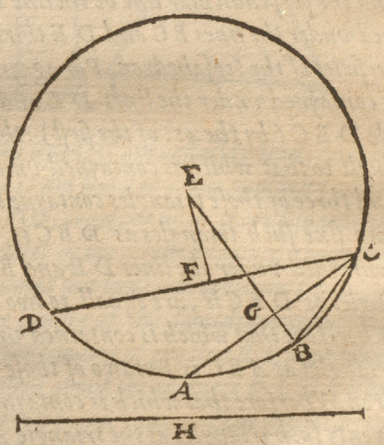
TAke (by the 2. Theoreme of this booke) a circle containing both the pentagon of a Dodecahedron, and the triangle of an Icoſahedron, being both deſcribed in one and the ſelfe ſame ſphere, and let the ſame circle be D B C. And in the circle D B C deſcribe the ſide of an equilater triangle, namely, C D, and the ſide of an equilater pentagon, namely, A C. And take (by the 1. of the third) the centre of the circle, and let the ſame be E. And from the point E drawe vnto the lines D C and A C, perpendicular lines E F and E G. And extend the line E G directly to the point B. And drawe a right line from the point B to the point C. And let the ſide of the cube be the line H. Now I ſay, that as the ſuperficies of the Dodecahedron is to the ſuperficies of the Icoſahedron, ſo is the line H to the line C D. Foraſmuch as the line made of the lines E B and B C added together (namely, of the ſide of the hexagon, and of the ſide of a decagon) is (by the 9. of the thirtenth) diuided by an extreme and meane proportion, and his greater ſegment is the line B E: and the line E G is alſo (by the 1. of the fouretenth) the halfe of the ſame line, and the line E F is the halfe of the line B E (by the Corollary of the 12. of the thirtenth). Wherefore the line E G being diuided by an extreme and meane proportion, his greater ſegment ſhall be the line E F. And the line H alſo being diuided by an extreme & meane proportion, his greater ſegment is the line C A, as it was proued† in the Dodecahedron. * Wherefore as the line H is to the line C A, ſo is the line E G to the line E F. Wherefore (by the 16. of the ſixt) that which is contained vnder the lines H and E F, is equall to that which is contained vnder the lines C A and E G. And for that as the line H is to the line C D, ſo is that which is contained vnder the lines H and E F, to that which is contained vnder the lines C D and E F (by the 1. of the ſixt). But vnto that which is contained vnder the lines H and E F, is equall that which is contained vnder the lines C A and E G. Wherefore (by the 11. of the fift) as the line H is to the line C D, ſo is that which is contained vnder the lines C A and E G, to that which is contained vnder the lines C D and E F, that is (by the Corollary next going before) as the ſuperficies of the Dodecahedron is to the ſuperficies of the Icoſahedron, ſo is the line H to the line C D.*

Demonſtration.

* This is not hard to proue by the 15. 16. and 19. of the fiueth.

† In the Corollary of the 17. of the thirteth.

* Here againe is required the Aſſumpt which is afterward proued in this 4. propoſition.



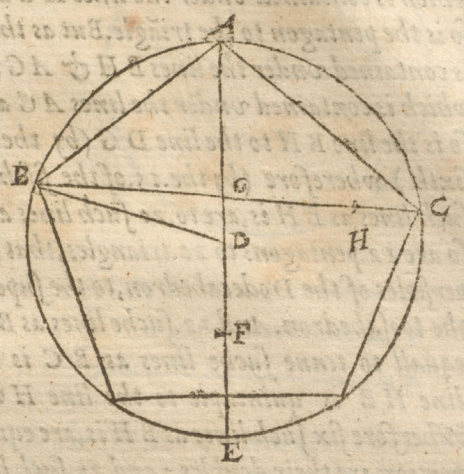
An other demonſtration to proue that as the ſuperficies of the Dodecahedron is to the ſuperficies of the Icoſahedron, ſo is the ſide of the cube to the ſide of the Icoſahedron. †

† But firſt the Aſſumpt following, the conſtruction whereof here beginneth, is to be proued.

ET there be a circle A B C. And in it deſcribe two ſides of an equilater pentagon (by the 11. of the fift) namely, A B and A C: and draw a right line from the point B to the point C. And (by the 1. of the third) take the centre of the circle, and let the ſame be D. And draw a right line from the point A to the point D, and extend it directly to the point E, and let it cut the line B C in the point G. And let the line D F be halfe to the line D A, and let the line

the line G C be treble to the line H C, by the 9. of the sixt. Now I say, that that which is contained vnder the lines A F and B H, is equall to the pentagon inscribed in the circle A B G. Draw a right line from the point B to the point D. Now forasmuch as the line A D is double to the line D F, therefore the line A F is sesquialter to the line A D. Againe, forasmuch as the line G C is treble to the line C H, therefore the line G H is double to the line C H. Wherefore the line G C is sesquialter to the line H G. Wherefore as the line F A is to the line A D, so is the line G C to the line G H. Wherefore (by the 16. of the sixt) that which is contained vnder the lines A F & H G, is equall to that which is contained vnder the lines D A and G C. But the line G C is equall to the line B G (by the 3. of the third). Wherfore that which is contained vnder the lines A D and B G, is equall to that which is contained vnder the lines A F and G H. But that which is contained vnder the lines A D and B G, is equall to two such triangles as the triangle A B D is (by the 41. of the first). Wherefore that which is contained vnder the lines A F and G H, is equall to two such triangles as the triangle A B D is. Wherefore that which is contained vnder the lines A F and G H fiue times, is equall to ten triangles. But ten triangles are two pentagons. Wherefore that which is contained vnder the lines A F and G H fiue times, is equall to two pentagons. And forasmuch as the line G H is double to the line H C, therefore that which is contained vnder the lines A F and G H, is double to that which is contained vnder the lines A F and H C (by the 1. of the sixt). Wherefore that which is contained vnder the lines A F and C H twise, is equall to that which is contained vnder the lines A F and G H once. Take eche of those parallelogrammes fiue times. Wherefore that which is contained vnder the lines A F and H C ten times, is equall to that which is contained vnder the lines A F & G H fiue times, that is, to two pentagons. Wherefore that which is contained vnder the lines A F and H C fiue times, is equall to one pentagon. But that which is contained vnder the lines A F and H C fiue times, is equall (by the 1. of the sixt) to that which is contained vnder the lines A F and H B, for the line H B is quintuple to the line H C (as it is easie to see by the construction) and they are both vnder one & the selfe same altitude, namely, vnder A F. Wherfore that which is contained vnder the lines A F and B H, is equall to one pentagon.

The Assumpt, which also Flussas putteth as an Assumpt after the 6. propositiõ. Demonstration of the Assumpt.

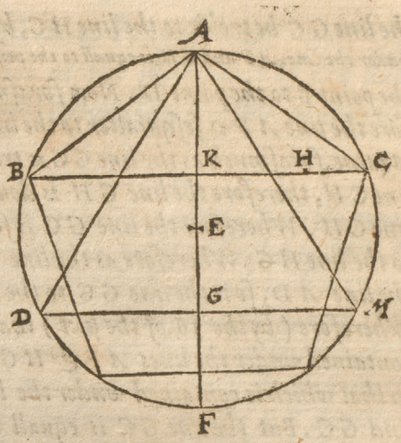


This being proued, now let there be drawne a Circle comprehending both the Pentagon of a Dodecahedron, and the triangle of an Icosahedron, being both described in one and the selfe same Sphere.

LEt the circle be A B C. And in it describe as before, two sides of an equilater pentagon, namely B A and A C: and draw a right line from the point B to the point C: and take the centre of the circle and let the same be E. And from the point A to the point E draw a right line A E: and extend the line A E to the point F. And let it cut the line B C in the point K. And let the line A E be doble to the line E G, & let the line C K be treble to the line C H, by the. 9. of the sixth. And frõ the point G raise vp (by the. 11. of the first) vnto the line A F a perpendicular line G M: and extend the line G M directly to the point D. Wherfore the line M D is the side of an equiliter triãgle, by the corollary of the. 12. of the thirteenth: draw these right lines A D and A M. Wherfore A D M is an equilater triangle. And for as much as that which is contained vnder the lines A G and B H is equal to the pentagon (by the former assumpt) and that which

Construction pertaining to the second demonstration of the 4. propositiõ.

Second demonstration of the 4. proposition.



is cõtained vnder the lines A G and G D is equal to the triangle A D M: therefore as that which is contained vnder the lines A G and H B is to that which is contained vnder the lines A G and G D, so is the pentagon to the triãgle. But as that which is contained vnder the lines B H & A G is to that which is contained vnder the lines A G and G D, so is the line B H to the line D G (by the.1.of the sixth) wherefore (by the.15.of the fifth) as 12. such lines as B H is, are to.20 such lines as D G is, so are 12.pentagons to 20.triangles, that is the superficies of the Dodecahedron, to the superficies of the Icosahedron. And 12.suche lines as B H is, are equall to tenne suche lines as B C is (for the line H B is quintuple to the line H C): and the line B C is sextuple to the line C H. Wherfore six such lines as B H is, are equal to fiue such lines as B C are: and in the same proportion are their doubles: and 20.such lines as the line D G is, are equal to.10.such lines as the line D M is: for the line D M is double to the line D G. Wherfore as 10.such lines as B C is, are to 10.such lines as D M is, that is, as the line B C is to the line D M, so is the superficies of the Dodecahedron to the superficies of the Icosahedron. But the line B C is the side of the cube, and the line D M the side of the Icosahedron: wherefore (by the 11.of the fifth) as the superficies of the Dodecahedron is to the superficies of the Icosahedron, so is the line B C to the line D M, that is, the side of the cube to the side of the Icosahedron.

The 7. proposition after Flussas.

Now we will we proue that a right line being deuided by an extreme and meane proportiõ, what proportiõ the line cõtaining in power the squares of the whole line and of the greater segment, hath to the line containing in power the squares of the whole line and of the lesse segment, the same proportion hath the side of the cube to the side of the Icosahedron, being both described in one and the selfe same sphere.

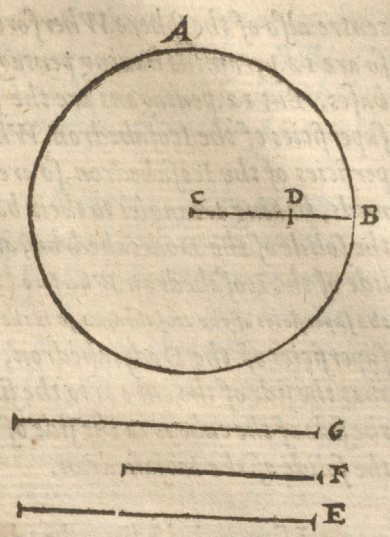
Construction.

Demonstration.

Vppose that A B be a circle containing both the pentagon of a Dodecahedron & the triangle of an Icosahedron described bothe in one and the selfe same sphere. Take the centre of the circle, and let the same be C. And from the point C extend to the circumference a right line at all auentures, and let the same B C. And (by the 30.of the sixth) deuide the line B C by an extreme and meane proportion in the point D, and let the greater segment therof be C D. Wherfore the line C D is the side of a Decagõ described in the same circle (by the corollary of the 9.of the thirteenth). Take the side of an Icosahedron, and let the same be the line E, and the side of a Dodecahedron, and let the same be the line F, and the side of a cube & let the same be the line G. Wherfore the line E is the side of an equilater triangle, and F of an equaliter pentagon described in one and the selfe same circle. And the line G being deuided by an extreme and meane proportion, his greater segment is the line F, by the corollary of the 17.of the thirteth. Now forasmuch as the line E is the side of an equilater triangle, but (by the 12.of the thirtenth) the side of an equilater triangle is in power treble to the line B C, (which is drawne from the center to the circumference) therefore the square of the line E is treble to the square of the line B C: but the squares of the line B C and B D are (by the 4.of the thirtenth) treble to the square of the line C D. Wherfore as the square of the line E is to the square of the line C B, so are the squares of the lines C B and B D to the square of the line C D. Wherefore alternately (by the 16.of

the

the fifth) as the ſquare of the line E is to the ſquares of the lines C B and B D, ſo is the ſquare of the line C B to the ſquare of the line C D.* But as the ſquare of the line B C is to the ſquare of the line C D, ſo is the ſquare of the line G (the ſide of the cube) to the ſquare of the line F, the ſide of Dodecahedron. For the line F is the greater ſegmēt of the line G (as was before proued.) Wherfore (by the .11. of the fift) as the ſquare of the line E is to the ſquares C B and B D, ſo is the ſquare of the line G, to the ſquare of the line F. Wherefore alternately (by the 16. of the fifth) & alſo by conuerſion (by the corollary of the 4. of the fift) as the ſquare of the line G, is to the ſquare of the line E, ſo is the ſquare of the line F, to the ſquares of the lines CB & BD. But vnto the ſquare of the line F are equal the ſquares of the lines BC & CD, for the ſide of a pentagon cōtaineth in power both the ſide of a ſixe angled figure, and the ſide of a ten angled figure (by the 10. of the thirteenth.) Wherfore as the ſquare of the line G, is to the ſquare of the line E, ſo are the ſquares of the lines B C and C D to the ſquares of the lines C B and B D. But as the ſquares of the lines C B and C D are to the ſquares of the lines C B & B D,† ſo (any right line what ſo euer it be, being diuided by an extreme and meane proportion) is the line containing in power the ſquares of the whole line, and of the greater ſegmēt, to the line containing in power the ſquares of the whole line, and of the leſſe ſegment: wherfore (by the 11. of the fifth) as the ſquare of the line G (the ſide of the cube) is to the ſquare of the line E, ſo (any right line being deuided by an extreme and meane proportion) is the line containing in power the ſquares made of the whole line, and of the greater ſegmēt, to the line containing in power the ſquares made of the whole line, and of the leſſe ſegment: but the line G is the ſide of the Cube, and the line E of the Icoſahedron (by ſuppoſition.) If therfore a right line be deuided by an extreeme and meane proportion, as the line cōtaining in power the ſquares of the whole line, and of the greater ſegment, is to the line containing in power the ſquares of the whole line and of the leſſe ſegment: ſo is the ſide of the cube to the ſide of the Icoſahedron, being both deſcribed in one and the ſelfe ſame ſphere.



*Here againe is required the Aſſumpt afterward proued in this 4. propoſition.

†As may by the Aſſumpt afterward in this propoſitiō be plainely proued.

Now will we proue that as the ſide of the Cube is to the ſide of the Icoſahedron, ſo is the ſolide of the Dodecahedron to the ſolide of the Icoſahedron.

The 8. propoſition after Fluſſas.

Forasmuche as equal circles comprehend both the pentagon of a Dodecahedron, and the triangle of an Icoſahedron, being both deſcribed in one and the ſelfe ſame ſphere, by the 2. of this booke: but in a ſphere equal circles are equally diſtant from the centre (for the perpendicular lines drawn from the centre of the ſphere to the plaine ſuperficieces of the circles are equal, and do fall vpon the centres of the circles. †Wherfore perpendicular lines drawne from the centre of the ſphere, to the centre of the circle, comprehending bothe the triangle of an Icoſahedron, and the pentagon of a Dodecahedron are equal: wherefore the pyramides, whoſe baſes are the pentagons of the Dodecahedron, are of equal altitude with the piramides whoſe baſes are the triangles of the Icoſahedron. But piramids of equal altitude, are in that proportion the one to the other, that their baſes are (by the 5. of the twelfth) wherefore as the pentagon is to the triangle, ſo is the pyramis whoſe baſe is the pentagon of the Dodecahedron and toppe the centre of the ſphere, to the pyramis whoſe baſe is the triangle and top the

†By the Corollary added by Fluſſas after his Aſſumpt put after the 17. propoſition of the 12. booke.

centre also of the sphere. Wherfore (by the 15. of the fifth) as 12. pẽtagons are to 20. triangles, so are 12. pyramids hauing pentagons to theyr bases to 20. pyramids hauing triãgles to their bases. But 12. pentagons are the superficies of the Dodecahedron, and 20. triangles are the superficies of the Icosahedron. Wherefore as the superficies of the Dodecahedron is to the superficies of the Icosahedron, so are 12. pyramids hauing pentagons to their bases to 20. pyramids, hauing triangles to their bases. But 12. pyramids hauing pentagons to their bases, are the solide of the Dodecahedron, and 20. pyramids hauing triangles to their bases are the solide of the Icosahedron. Wherfore (by the 11. of the fifthe) as the superficies of the Dodecahedron is to the superficies of the Icosahedron so is the solide of the Dodecahedron to the solide of the Icosahedron. But as the superficies of the Dodecahedron, is to the superficies of the Icosahedron, so haue we proued that the side of the cube is to the side of the Icosahedron. Wherfore, by the 11. of the fifth, as the side of the cube is to the side of the Icosahedron, so is the solide of the Dodecahedron to the solide of the Icosahedron.

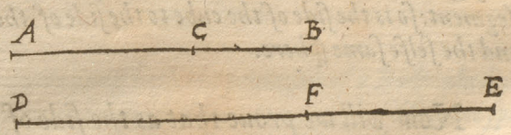
Corollary of the 8. after Flussas.

If two right lines be diuided by an extreame and meane proportion, they shall euery way be in like proportion: which thing is thus demonstrated.

This Assumpt is the 3. propositiõ after Flussas, and is it which diuers times hath bene taken as graunted in this booke, and once also in the last proposition of the 13. booke: as we haue before noted.

Demonstration.

LEt the line A B be (by the 30. of the sixth) diuided by an extreame and meane proportion in the poynt C, and let the greater segment thereof be the line C A. And likewise also let the line D E be diuided by an extreame and meane proportion in the poynt F, and let the greater segment thereof be the line D F. Then I say that as the whole line A B is to the greater segment thereof A C, so is the whole line D E to the greater segment thereof D F. For forasmuch as that which is contayned vnder the lines A B and B C is equall to the square of the line A C (by the diffinition of a line diuided be an extreame and meane proportion): and that which is contayned vnder the lines D E and E F is also equall to the square of the line D F (by the same diffinition): therefore as that which is contayned vnder the lines A B and B C is to the square of the line A C, so is that which is contayned vnder the lines D E and E F to the square of the line D F. For in eche is the proportion of equalitie. Wherfore as that which is contayned vnder the lines A B and B C fower times, is to the square of the line A C, so is that which is contaynd vnder the lines D E and E F fower times to the square of the line D F (by the 15. of the fifth). Wherfore by composition (by the 18. of the fifth) as that which is contayned vnder the lines A B and B C fower times, together with the square of the line A C, is to the square of the line A C, so is that which is contayned vnder the lines D E and E F fower times, together with the square of the line D F, to the square of the line D F. Wherefore as the square which is made of the lines A B and B C added together and made one line (which square by the 8. of the second is equall to that which is contayned vnder the lines A B and B C fower times together with square of the line A C) is to the square of the line A C, so is the square made of the lines D E & E F added together and made one line (which square is also, by the same, equal to that which is contayned vnder the lines D E and E F fower times together with the square of the line D F) to the square of the line D F. Wherefore also as the lines A B & B C added together are to the line AC, so are the lines D E & E F added together to the line D F (by the 22. of the sixt). Wherefore by cõposition (by the 18. of the fifth) as both the lines A B & B C added the one to the other, together with the line A C, that is, as two such lines as A B is, are to the line A C, so are both the lines D E and E F added the one to the other together with the line D F, that is two such



lines

lines as D E is to the line D F. And in the ſame proportion are the halues of the antecedents by the 15. of the fifth. Wherefore as the line A B is to the line A C, ſo is the line D E to the line D F. (And therefore by the 19. of the fifth, as the line A B is to the line B C, ſo is the line D F to the line F E. Wherefore alſo by diuiſion by the 17. of the fifth, as the line A C is to the line C B, ſo is the line D F to the line D E).

Now that we haue proued, that, any right line whatſoeuer being diuided by an extreame and meane proportion, what proportion the line contayning in power the ſquares made of the whole line and of the greater ſegment added together, hath to the line contayning in power the ſquares made of the whole line and of the leſſe ſegment added together, the ſame proportion hath the ſide of the cube to the ſide of the Icoſahedron: Now alſo that we haue proued, †that as the ſide of the cube is to the ſide of the Icoſahedron, ſo is the ſuperficies of the Dodecahedron to the ſuperficies of the Icoſahedron, being both deſcribed in one and the ſelfe ſame ſphere: and moreouer ſeing that we haue proued, †that as the ſuperficies of the Dodecahedron is to the ſuperficies of the Icoſahedrō, ſo is the Dodecahedrō to the Icoſahedron, for that both the pentagon of the Dodecahedron, and the triangle of the Icoſahedron are comprehended in one and the ſelfe ſame circle: All theſe thinges I ſay being proued, it is manifeſt, that if in one and the ſelfe ſame ſphere be deſcribed a Dodecahedron, and an Icoſahedron, they ſhall be in proportion the one to the other, as, a right line whatſoeuer being diuided by an extreame and meane proportion, the line contayning in power the ſquares of the whole line and of the greater ſegment added together, is to the line containing in power the ſquares of the whole line and of the leſſe ſegment added together. For for that as the Dodecahedron is to the Icoſahedron, ſo is the ſuperficies of the Dodecahedron to the ſuperficies of the Icoſahedron, that is, the ſide of the cube to the ſide of the Icoſahedron: but as the ſide of the cube is to the ſide of the Icoſahedron, ſo, any right line what ſo euer being diuided by an extreame and meane proportion, is the line contayning in power the ſquares of the whole line and of the greater ſegment added together, to the line contayning in power the ſquares of the whole line and of the leſſe ſegment added together. Wherefore as a Dodecahedron is to an Icoſahedron deſcribed in one and the ſelfe ſame ſphere, ſo, any right line what ſo euer being diuided by an extreame and meane proportion, is the line contayning in power the ſquares of the whole line & of the greater ſegment added together, to the line contayning in power the ſquares of the whole line and of the leſſe ſegment added together.*

* *In the 4. ſection of this propoſition.*

† *In the 1. and 3. ſection of the ſame propoſitiō.*
† *In the 5. ſectiō of the ſame propoſition.*

A Corollary.

(?¿)

The ende of the fourtenth Booke

of Euclides Elementes
after Hypſicles.

¶ The fourteenth booke of Euclides Elementes after Flussas.

THE Or that the fouretenth Booke, as it is set forth by Flussas, containeth in it moe Propositions then are found in Hypsicles, & also some of those Propositions which Hypsicles hath, are by him somewhat otherwise demonstrated, I thought my labour well bestowed for the readers sake to turne it also all whole, notwithstanding my trauaile before taken in turning the same booke after Hypsicles. Where note ye, that here in this 14. booke after Flussas, and in the other bookes following, namely, the 15. and 16. I haue in alleadging of the Propositions of the same 14. booke, followed the order and number of the Propositions, as Flussas hath placed them.

¶ *The first Proposition.*

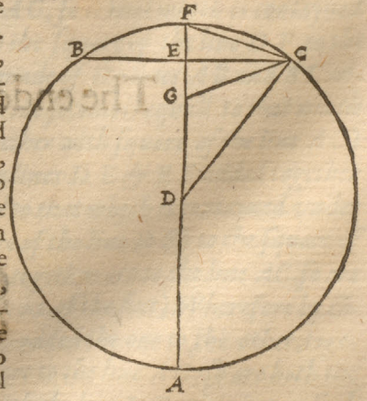
The first proposition after Campane.

A perpendicular line drawen from the centre of a circle, to the side of a Pentagon inscribed in the same circle: is the halfe of these two lines taken together, namely, of the side of the hexagon, and of the side of the decagon inscribed in the same circle.

Construction.

TAke a circle A B C, and inscribe in it the side of a pentagon, which let be B C, and take the centre of the circle, which let be the point D: and frō it draw vnto the side B C a perpendicular line D E: which produce to the point F. And vnto the line E F put the line E G equall. And draw these right lines C G, C D, and C F. Then I say, that the right line D E (which is drawen from the centre to B C the side of the pentagon) is the halfe of the sides of the decagon and hexagon, taken together. Forasmuch as the line D E is a perpendicular vnto the line B C: therefore the sections B E and E C shall be equall (by the 3. of the third): and the line E F is common vnto them both, and the angles F E C and F E B, are right angles, by supposition. Wherefore the bases B F and F C are equall (by the 4. of the first). But the line B C is the side of a pentagon, by construction. Wherefore F C which subtendeth the halfe of the side of the pentagon, is the side of the decagon inscribed in the circle A B C. But vnto the line F C is, by the 4. of the first, equall the line C G, for they subtend right angles C E G, and C E F, which are contained vnder equall sides. Wherefore also the angles C G E, and C F E, of the triangle C F G, are equall, by the 5. of the first. And forasmuch as the arke F C is subtended of the side of a decagon, the arke C A shall be quadruple to the arke C F: Wherefore also the angle C D A shall be quadruple to the angle C D F (by the last of the sixt). And forasmuch as the same angle C D A, which is set at the centre, is double to the angle C F A, which is set at the circumference, by the 20. of the third: therefore the angle C F A, or C F D, is double to the angle C D F, namely, the halfe of quadruple. But vnto the angle C F D or C F G, is proued equall the angle C G F: Wherefore the outward angle C G F, is double to the angle C D F. Wherefore the angles C D G and D C G, shall be equall. For vnto those two angles the angle C G F is equall, by the 32. of the first. Wherefore the sides G C and G D, are equall, by the 6. of the first. Wherefore also the

Demonstration.



the line G D is equall to the line F C, which is the ſide of the decagon. But vnto the right line F E is equall the line E G, by conſtruction. Wherefore the whole line D E is equall to the two lines C F and F E. Wherefore thoſe lines taken together (namely, the lines D F and F C) ſhall be double to the line D E. Wherefore the line D E (which is drawen from the centre perpendicularly to the ſide of the pentagon) ſhal be the halfe of both theſe lines taken together, namely, of D F the ſide of the hexagon, and C F the ſide of the decagon. For the line D F which is drawen from the centre, is equall to the ſide of the hexagon, by the Corollary of the 15. of the fourth. Wherefore a perpendicular line drawen from the center of a circle, to the ſide of a pentagon inſcribed in the ſame circle: is the halfe of theſe two lines taken together, namely, of the ſide of the hexagon, and of the ſide of the decagon inſcribed in the ſame circle: which was required to be proued.

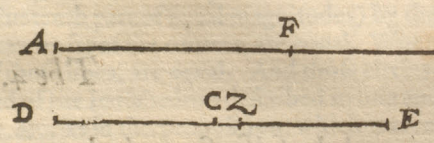
✻ A Corollary.

If a right line drawen perpendicularly from the centre of a circle to the ſide of a pentagon, be diuided by an extreme and meane proportion: the greater ſegment ſhall be the line which is drawen from the ſame centre to the ſide of an equilater triangle inſcribed in the ſame circle. For, that line (drawen to the ſide of the triangle) is (by the Corollary of the 12. of the thirteenth) the halfe of the line drawen from the centre to the circumference, that is, of the ſide of the hexagon: Wherefore the reſidue ſhall be the halfe of the ſide of the decagon. For the whole line is the halfe of the two ſides, namely, of the ſide of the hexagon, and of the ſide of the decagon. But of the ſide of a decagon and of an hexagon taken together, the greater ſegment is the ſide of the hexagon (by the 9. of the thirteenth). Wherefore the greater ſegment of their halfes ſhall be the halfe of the hexagon, by the 15. of the fift: which halfe is the perpendicular line drawen from the centre to the ſide of the triangle, by the Corollary of the 12. of the thirteenth.

¶ The ſecond Propoſition.

If two right lines be diuided by an extreme and meane proportion: they ſhall be diuided into the ſelfe ſame proportions. *The 2. propoſition after Campane.*

VPpoſe that theſe two right lines A B and D E be eche cut by an extreme and meane proportion in the pointes F and Z. Then I ſay, that theſe two lines are diuided into the ſelfe ſame proportions, that is, that the line A B is in the point F diuided in like ſort as the line D E is in the point Z. For if they be not in like ſort cut, let one of them, namely, D E, be cut like vnto the line A B in the point C. So that let the line D E be to D C the greater part, as the greater part D C is to C E the leſſe part, by the 3. definitiõ of the ſixt. But (by ſuppoſition) the line D E is to the line D Z, as the line D Z is to the line Z E. Wherefore the right line D E is diuided by an extreme and meane proportion in two pointes C and Z. But the proportion of D E to D C the leſſe line, is greater then the proportion of the ſame D E to D Z the greater line, by the 2. part of the 8. of the fift. But as D E is to D C, ſo is D C to C E: Wherefore the proportion of D C to C E, is greater then the proportion of D Z to Z E. And foraſmuch as D Z is greater then D C, the proportion of D Z to C E ſhall be greater then the proportion of D C to C E, by the 8. of the fift. Wherefore the proportion of D Z to C E, is much greater then the proportion of D Z to Z E. Wherefore one and the ſelfe ſame magnitude, namely, D Z, hath to C E the greater line, a greater proportion then it hath to Z E the leſſe line, contrary to the ſecond part of the 8. of the fift: which is impoſſible. Wherfore the right lines A B & D E, are not cut vnlike. Wherfore they are cut like, and into the ſelfe ſame proportions. And the ſame demonſtration alſo will ſerue, if the point C fall in any other place. For alwaies ſome one of them ſhall be the greater. If therefore two right lines be cut by an extreme and meane proportion: they ſhall be cut into the ſelfe ſame proportions: which was required to be proued.



Demonſtration leading to an impoſſibilitie.

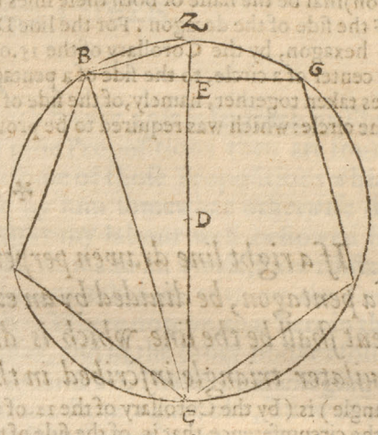
¶ The third Propoſition.

If in a circle be deſcribed an equilater Pentagon: the ſquares made of the ſide of the Pentagon and of the line which ſubtendeth two ſides of the *The 4. propoſition after Campane.*

Pentagon, theſe two ſquares (I ſay) taken together, are quintuple to the ſquare of the line drawen from the centre of the circle to the circũference.

Conſtruction.

SVppoſe that in the circle B C G the ſide of a Pentagon be B G: and let the line B C ſubtend two ſides thereof. And let the line B G be diuided into two equall partes by a right line drawen from the centre D: namely, by the diameter C D E produced to the point Z. And drawe the right line B Z. Then I ſay, that the right lines B C and B G, are in power quintuple to the right line D Z, which is drawen from the centre to the circumference.



Demonſtration.

For foraſmuch as (by the 47. of the firſt) the ſquares of the lines C B and B Z, are equall to the ſquare of the diameter C Z: therefore they are quadruple to the ſquare of the line D Z, by the 20. of the ſixt (for the line C Z is double to the line D Z). Wherefore the right lines C B, B Z, and Z D, are in power quintuple to the line Z D. But the right line B G containeth in power the two lines B Z and Z D, by the 10. of the thirteenth. For D Z is the ſide of an hexagon, & B Z the ſide of a decagon. Wherefore the lines B C and B G (whoſe powers are equall to the powers of the lines C B, B Z, Z D) are in power quintuple to the line D Z. If therefore in a circle be deſcribed an equilater Pentagon: the ſquares made of the ſide of the Pentagon and of the line which ſubtendeth two ſides of the Pentagon, theſe two ſquares (I ſay) taken together, are quintuple to the ſquare of the line drawen from the centre of the circle to the circumference.

¶ A Corollary.

This Corollary Campane alſo putteth after the 4. propoſition in his order.

If a Cube and a Doderahedron be contained in one and the ſelfe ſame Sphere: the ſide of the Cube, and the ſide of the Dodecahedron, are in power quintuple to the line which is drawen from the centre of the circle which containeth the Pentagon of the Dodecahedron. For it was proued in the 17. of the thirteenth, that the ſide of the Cube ſubtendeth two ſides of the Pentagon of the Dodecahedron, where the ſayd ſolides are contained in one and the ſelfe ſame Sphere. Wherfore the ſide of the Cube ſubtending two ſides of the Pentagon, and the ſide of the ſame Pentagon, are contained in one and the ſelfe ſame circle. Wherefore, by this Propoſition, they are in power quintuple to the line which is drawen from the centre of the ſame circle which containeth the Pentagon of the Dodecahedron.

The 4. Propoſition.

The 5. propoſition after Campane.

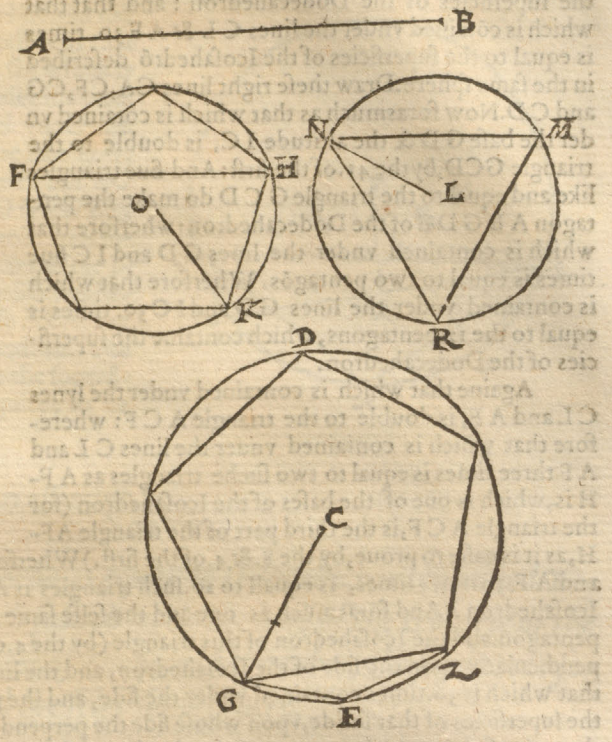
One and the ſelfe ſame circle containeth both the Pentagon of a Dodecahedron, and the triangle of an Icoſahedron deſcribed in one and the ſelfe ſame ſphere.

Conſtruction.

LEt the diameter of the ſphere geuen be A B, and let the baſes of the Icoſahedron and Dodecahedron deſcribed in it, be the triangle M N R, and the pentagon F K H, and about them let there be deſcribed circles, by the 5. and 14. of the fourth. And let the lines drawne from the centres of thoſe circles to the circumferences be L N and O K. Then I ſay that the lines L N and O K are equal, and therfore one and the ſelfe ſame circle containeth both thoſe figures. Let the right line A B, be in power quintuple to ſome one right line, as to the line C G (by the Corollary of the 6. of the tenth.) And making the cẽtre the poynt C, & the ſpace C G, deſcribe a circle D Z G. And let the ſide of a pentagon inſcribed in that circle (by the 11. of the fourth) be the line Z G. And let E G (ſubtending halfe of the arke Z G) be the ſide of a Decagon inſcribed in that circle. And by the 30. of the ſixt, diuide the line C G by an extreme & meane proportion in the poynt I. Now foraſmuche as in the 16. of the thirteenth, it was proued, that this line C G (vnto whome the diameter A B of the ſphere is in power quintuple) is the line which is drawne from the centre of the circle, which containeth fiue angles of the Icoſahedron, and the ſide of the pentagon deſcribed

Demonſtration.

in



in that circle D Z G, namely the line Z G is ſide of the Icoſahedron deſcribed in the Sphere, whoſe diameter is the line A B: therefore the right line Z G, is equal to the line MN, which was put to be the ſide of the Icoſahedrō, or of his triāgular baſe. Moreouer, by the 17. of the thirteenth, it was manifeſt that the right line FH (which ſubtendeth the angle of the pentagon of the Dodecahedron inſcribed in the foreſayde ſphere) is the ſide of the Cube, inſcribed in the ſelf ſame ſphere. (For vpon the angles of the cube, were made the angles of the Dodecahedron.) Wherefore the diameter A B is in power triple to F-H, the ſide of the Cube (by the 15. of the thirteenth). But the ſame line A B is (by ſuppoſition) in power quintuple to the line C G. Wherefore fiue ſquares of the line C G, are equal to thre ſquares of the line F H: (for eche is equal to one and the ſelf ſame ſquare of the line A B). And forasmuche as E G the ſide of the Decagon, cutteth the right line C G by an extreme and meane proportion (by the corollary of the 9. of the thirteenth): Likewiſe the line H K, cutteth the line F H, the ſide of the Cube by an extreeme and meane proportion (by the Corollary of the 17. of the thirteenth): therfore the lines C G and F H, are deuided into the ſelf ſame proportions, by the ſecond of this booke: and the right lines C I and E G, which are the greater ſegmentes of one and the ſelfe ſame line C G, are equal: And forasmuche as fiue ſquares of the line C G are equal to thre ſquares of the lines F H: therefore fiue ſquares of the line G E, are equal to thre ſquares of the line H K (for the lines G E and H K are the greater ſegmēts of the lines C G and F H). Wherefore fiue ſquares of the lines CG & GE are equal to thre ſquares of the lines FH & HK, by the 12 of the fift. But vnto the ſquares of the lines C G and G E, is equal the ſquare of the line Z G, by the 10. of the thirtēth: and vnto the line Z G the line M N was equal: wherfore fiue ſquares of the line M N, are equall to three ſquares of the lines F H, H K. But the ſquares of the lines F H and H K, are quintuple to the ſquare of the line O K (which is drawne from the centre) by the third of this booke. Wherfore thre ſquares of the lines F H and H K make 15. ſquares of the line O K. And forasmuch as the ſquare of the line M N is triple to the ſquare of the line L N (which is drawne from the centre) by the 12. of the thirteenth, therfore fiue ſquares of the line M N are equal to 15. ſquares of the line L N. But fiue ſquares of the line M N are equal vnto thre ſquares of the lines F H and H K. Wherefore one ſquare of the line L N is equall to one ſquare of the line O K (being eche the fiuetenth part of equal magnitudes) by the 15. of the fifth. Wherfore the lines L N and O K, which are drawne from the centers, are equal. Wherefore alſo the circles N R M, and F K H which are deſcribed of thoſe lines, are equal. And thoſe circles contayne (by ſuppoſition) the baſes of the Dodecahedron and of the Icoſahedron deſcribed in one and the ſelfe ſame ſphere. Wherfore one and the ſelfe ſame circle. &c. as in the propoſition: which was required to be proued.

The 5. Propoſition.

If in a circle be inſcribed the pentagon of a Dodecahedron, and the triangle of an Icoſahedron, and from the centre to one of theyr ſides, be drawne a perpendicular line: That which is contained 30. times vnder the ſide, & the perpendicular line falling vpon it, is equal to the ſuperficies of that ſolide, vpon whoſe ſide the perpendicular line falleth.

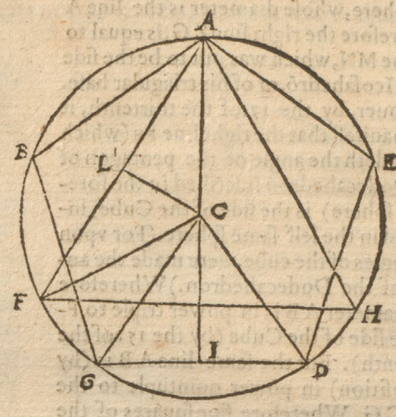
This is the 6. and 7. propoſitions after Campane.

Conſtruction.

Vppoſe that in the circle A G E, be deſcribed the pentagon of a Dodecahedron, which let be AB G D E, and the triangle of an Icoſahedron deſcribed in the ſame ſphere, which let be A F H. And let the centre be the poynt C. From which draw perpendicularly the line C I to the ſide of the Pentagon, and the line C L to the ſide of the triangle. Then I ſay that the rectangle figure contained vnder the lines C I and G D 30. times, is equal to

the ſuperficies of the Dodecahedron: and that that which is cõtained vnder the lines C L & A F 30. times is equal to the ſuperficies of the Icoſahedrõ deſcribed in the ſame ſphere. Draw theſe right lines C A, C F, C G and C D. Now forasmuch as that which is cõtained vnder the baſe G D & the altitude I C, is double to the triangle G C D, by the 41. of the firſt: And fiue triangles like and equal to the triangle G C D do make the pentagon A B G D E of the Dodecahedron: wherfore that which is contained vnder the lines G D and I C fiue times is equal to two pentagõs. Wherfore that which is contained vnder the lines G D and I C 30. times is equal to the 12. pentagons, which containe the ſuperficies of the Dodecahedron.

Demonſtration.



Againe that which is contained vnder the lynes C L and A F, is double to the triangle A C F: wherefore that which is contained vnder the lines C L and A F three times is equal to two ſuche triangles as A F H is, which is one of the baſes of the Icoſahedron (for the triangle A C F, is the third part of the triangle A F H, as it is eaſie to proue, by the 8. & 4. of the firſt.) Wherfore that which is cõtained vnder the lines C L and A F, 30 times times, is equall to 20. ſuch triangles as A F H is, which containe the ſuperficies of the Icoſahedron. And forasmuch as one and the ſelfe ſame ſphere containeth the Dodecahedron of this pentagon, and the Icoſahedron of this triangle (by the 4. of this booke:) and the line C L falleth perpendicularly vpon the ſide of the Icoſahedron, and the line C I vpon the ſide of the Dodecahedron: that which is 30. times contained vnder the ſide, and the perpendicular line falling vpon it, is equal to the ſuperficies of that ſolide, vpon whoſe ſide the perpendicular falleth. If therefore in a circle &c. as in the propoſition: which was required to be demonſtrated.

A Corollary.

This Corollary Campane alſo addeth after the 7. propoſition in his order.

The ſuperficieces of a Dodecahedron and of an Icoſahedron deſcribed in one and the ſelfe ſame ſphere, are the one to the other, as that which is contained vnder the ſide of the one and the perpendicular line drawne vnto it from the centre of his baſe, to that which is contained vnder the ſide of the other, and the perpendicular line drawne to it from the centre of his baſe. For as thirtye times is to thirty times, ſo is once to once by the 15. of the fifth.

The 6. Propoſition.

The 5. propoſition after Campane.

The ſuperficies of a Dodecahedron, is to the ſuperficies of an Icoſahedron deſcribed in one and the ſelfe ſame ſphere, in that proportion, that the ſide of the Cube is to the ſide of the Icoſahedron contained in the ſelf ſame ſphere.

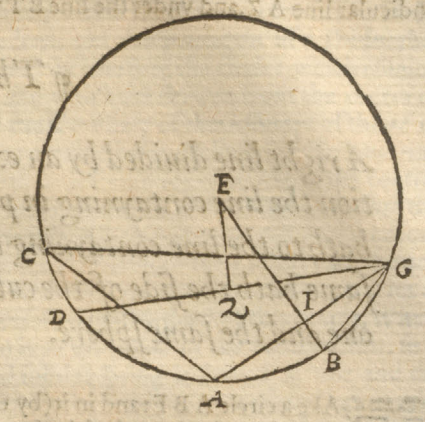
Conſtruction.

SVppoſe that there be a circle A B G, & in it (by the 4. of this boke) let there be inſcribed the ſides of a Dodecahedron and of an Icoſahedron contained in one and the ſelfe ſame ſphere. And let the ſide of the Dodecahedron be A G, and the ſide of the Icoſahedron be D G. And let the centre be the poynt E: from which drawvnto thoſe ſides, perpendicular lines E I and E Z. And produce the line E I to the poynt B, and draw the line B G. And let the ſide of the cube contained in the ſelf ſame ſphere be G C. Then I ſay that the ſuperficies of the Dodecahedron is to the ſuperficies of the Icoſahedron, as the line C G, is to the line G D. For forasmuche as the line E I being diuided by an extreme and meane proportion, the greater ſegment therof ſhall be the line E Z, by the corollary of the firſt of this booke: and the line C G beirg diuided by an extreme and meane proportion, his greater ſegment is the line A G, by the corollary of the 17. of the thirteenth: Wherefore the right lines E I and C G are cut proportionally by the ſecond of this booke. Wherefore as the line C G, is to the line A G, ſo is the line E I to the line E Z. Wherefore that which is contained vnder the extreames C G and E Z, is equall to that which is contayned vnder the meanes A G and E I. (by the 16. of the ſixth.) But as that which is contained vnder the lines C G and E Z is to that which is contained vnder the lines D G and E Z, ſo (by the firſt of the ſixth) is the line C G to the

Demonſtration.

line

line D G, for both thoſe parallelogrames haue one and the ſelfe ſame altitude, namely the line E Z. Wherfore as that which is contained vnder the lines E I and A G (which is proued equal to that which is contained vnder the lines C G and E Z) is to that which is contained vnder the lines D G and E Z, ſo is the line C G to the line D G. But as that which is contained vnder the lines E I and A G is to that which is contained vnder the lines D G and E Z, ſo (by the corollary of the former propoſition) is the ſuperficies of the Dodecahedron, to the ſuperficies of the Icoſahedron. Wherfore as the ſuperficies of the Dodecahedron is to the ſuperficies of the Icoſahedron, ſo is C G the ſide of the cube, to G D the ſide of the Icoſahedron. The ſuperficies therefore of a Dodecahedron is to the ſuperficies. &c. as in the propoſition, which was required to be proued.



An aſſumpt.

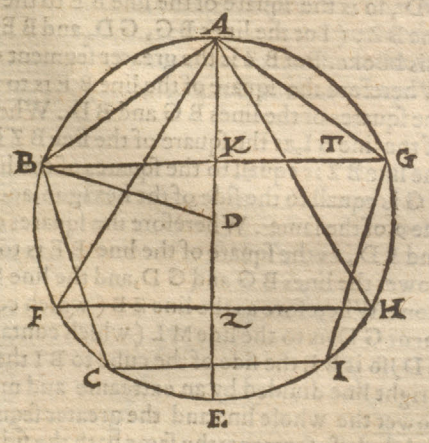
The Pentagon of a Dodecahedron, is equall to that which is contained vnder the perpendicular line which falleth vpon the baſe of the triangle of the Icoſahedron, and fiue ſixth partes of the ſide of the cube, the ſayd three ſolides being deſcribed in one and the ſelfe ſame ſphere.

This Aſſumpt Campane alſo hath after the 8 propoſition, in his order.

Suppoſe that in the circle A B E G, the pentagon of a Dodecahedron be A B C I G, and let two ſides thereof A B and A G be ſubtended of the right line B G. And let the triangle of the Icoſahedron inſcribed in the ſelfe ſame ſphere, by the 4. of this booke, be A F H. And let the centre of the circle be the poynt D, and let the diameter be A D E, cutting F H, the ſide of the triangle in the poynt Z, and cutting the line B G in the poynt K. And draw the right line B D. And from the right line K G cut of a third part T G, by the 9. of the ſixth. Now the line B G ſubtending two ſides of the Dodecahedron, ſhalbe the ſide of the cube inſcribed in the ſame ſphere, by the 17. of the thirteenth: and the triangle of the Icoſahedron of the ſame ſphere ſhalbe A F H by the 4. of this booke. And the line A Z which paſſeth by the centre D ſhall fall perpendiculary vpon the ſide of the triangle. For foraſmuch as the anges G A E & B A E are equall (by the 27. of the third, for they are ſet vpon equall circumferences): therefore the baſes B K and K G are (by the 4. of the firſt) equall. Whereore the line B T contayneth 5. ſixth partes of the line B G. Then I ſay that that which is contayned vnder the lines A Z and B T, is equall to the pentagon A B C I G. For foraſmuch as the line A Z is ſeſquialter to the line A D (for the line D E is diuided into two equall partes in the poynt Z, by the corollary of the 12. of the thirteenth). Likeviſe by conſtruction, the line K G is ſeſquialter to the line K T: therefore as the line A Z is to the line A D, ſo is the line K G to the line K T. Wherefore that which is contayned vnder the extreames A Z and K T, is equall to that which is contayned vnder the meanes A D and K G, by the 16. of the ſixth. But vnto the line K G is the line B K proued equall. Wherefore that which is contayned vnder the lines A Z and K T is equall to that which is contayned vnder the lines A D and B K. But that which is contayned vnder the lines A D and B K is (by the 41. of the firſt) double to the triangle A B D. Wherefore that which is contayned vnder the lines A Z and I T is double to the ſame triangle A B D. And foraſmuch as the pentagon A B C I G contayneth fiue triangles equall to the triangle A B D, and that which is contayned vnder the lines A Z and K T contayneth two ſuch triangles: therefore the pentagon A B C I G is duple ſeſquialter to the rectangle paralldogramme contayned vnder the lines A Z and K T. And foraſmuch as by conſtruction the line B T is duple ſeſquialter to the right line K T: but by 1. of the ſixth, that which is cõteyned vnder the lines A Z and B T is to that which is contayned vnder the lines A Z and K T, as the baſe B T is to the baſe K T: therefore that which is contayned vnder the lines A Z and B T is duple ſeſquialter to that which is contayned vnder the line A Z & K T. But vnto that which is contayned vnder the lines A Z and K T the pentagon A B C I G is proued duple ſeſquialter. Wherfore the pentagon A B C I G of the Dodecahedron is equall to that which is contayned vnder the per-

Conſtruction.

Demonſtration.



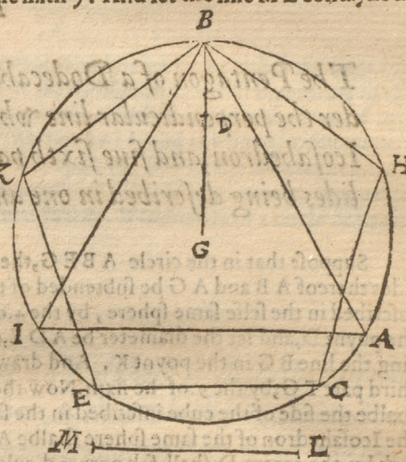
pendicular line A Z, and vnder the line B T which is fiue sixe partes of the line B G.

¶ The 7. Proposition.

The 9. proposition after Campane.

A right line diuided by an extreame and meane proportion: what proportion the line contayning in power the whole line and the greater segment, hath to the line contayning in power the whole and the lesse segment: the same hath the side of the cube to the side of the Icosahedron contayned in one and the same sphere.

Construction. TAke a circle A B E: and in it (by the 11. of the fourth) inscribe an equilater pentagon B Z E C H: and (by the second of the same) an equilater triangle A B I. And let the centre thereof be the poynt G. And drawe a line from G to B. And diuide the line G B by an extreame and meane proportion in the poynt D (by the 30. of the sixth). And let the line M L contayne in power both the whole line G B and his lesse segment B D (by the corollary of the 13. of the tenth). And draw the right line B E subtending the angle of the pentagon, which shall be the side of the cube (by the corollary of the 17. of the thirteenth): and the line B I shall be the side of the Icosahedron, and the line B Z the side of the Dodecahedron by the 4. of this booke. Then I say that B E the side of the cube is to B I the side of the Icosahedron, as the line contayning in power the lines B G & G D is to the line contayning in power the lines G B and B D.



Demonstration. For forasmuch as (by the 12. of the thirteenth) the line B I is in power triple to the line B G, and (by the 4. of the same) the squares of the line G B & B D are triple to the square of the line G D. Wherefore (by the 15. of the fifth) the square of the line B I is to the squares of the lines G B & B D (namely, triple to triple), as the square of the line B G is to the square of the line G D (namely, as one is to one). But as the square of the line B G is to the square of the G D, so is the square of the line B E to the square of the line B Z. (For the lines B G, G D, and B E, B Z are in one and the same proportion, by the second of this booke. For B Z is the greater segment of the line B E, by the corollary of the 17. of the thirteenth). Wherefore the square of the line B E is to the square of the line B Z, as the square of the line B I is to the squares of the lines B G and B D. Wherefore alternately the square of the line B E is to the square of the line B I, as the square of the line B Z is to the squares of the lines G B and B D. But the square of the line B Z is equall to the squares of the lines B G and G D (by the 10. of the thirteenth). For the line B G is equall to the side of the hexagon, and the line G D to the side of the decagon, by the corollary of the 9. of the same. Wherefore the squares of the lines B G and G D, are to the squares of the lines G B and B D, as the square of the line B E is to the square of the line B I. But the line Z B contayneth in power the lines B G and G D, and the line M L contayneth in power the lines G B and B D by construction. Wherefore as the line Z B (which contayneth in power the whole line B G and the greater segment G D) is to the line M L (which contayneth in in power the whole line G B and the lesse segment B D) so is B E the side of the cube to B I the side of the Icosahedron, by the 22. of the sixth. Wherefore a right line diuided by an extreame and meane proportion: what proportion the line contayning in power the whole line and the greater segment, hath to the line contayning in power the whole line and the lesse segment: the same hath the side of the cube to the side of the Icosahedron cõtayned in one and the same sphere: which was required to be proued.

¶ The 8. Proposition.

This Campane putteth as a Corollary in the 9. proposition after his order.

The solide of a Dodecahedron is to the solide of an Icosahedron: as the side of a Cube is to the side of an Icosahedron, all those solides being described in one and the selfe same Sphere.

Forasmuch

FOraſmuch as in the 4.of this booke, it hath bene proued, that one and the ſelf ſame circle containeth both the triangle of an Icoſahedron, and the pentagon of a Dodecahedron deſcribed in one and the ſelfe ſame Sphere: Wherefore the circles, which cõtaine thoſe baſes, being equall, the perpendiculars alſo which are drawen from the centre of the Sphere to thoſe circles, ſhall be equall (by the Corollary of the Aſſumpt of the 16 of the twelfth). And therefore the Pyramids ſet vpon the baſes of thoſe ſolides haue one and the ſelfe ſame altitude: For the altitudes of thoſe Pyramids concurre in the centre. Wherefore they are in proportion as their baſes are, by the 5.and 6.of the twelfth. And therefore the pyramids which compoſe the Dodecahedron, are to the pyramids which compoſe the Icoſahedron, as the baſes are, which baſes are the ſuperficieces of thoſe ſolides. Wherefore their ſolides are the one to the other, as their ſuperficieces are. But the ſuperficies of the Dodecahedron is to the ſuperficies of the Icoſahedron, as the ſide of the cube is to the ſide of the Icoſahedron, by the 6.of this booke. Wherfore by the 11.of the fifth, as the ſolide of the Dodecahedron is to the ſolide of the Icoſahedron, ſo is the ſide of the cube to the ſide of the Icoſahedron, all the ſaid ſolides being inſcribed in one and the ſelfe ſame Sphere. Wherefore the ſolide of a Dodecahedron is to the ſolide of an Icoſahedron: as the ſide of a cube is to the ſide of an Icoſahedron, all thoſe ſolides being deſcribed in one and the ſelf ſame Sphere: which was required to be proued.

✻ A Corollary.

The ſolide of a Dodecahedron is to the ſolide of an Icoſahedron, as the ſuperficieces of the one are to the ſuperficieces of the other, being deſcribed in one and the ſelfe ſame Sphere: Namely, as the ſide of the cube is to the ſide of the Icoſahedron, as was before manifeſt: for they are reſolued into pyramids of one and the ſelfe ſame altitude.

This Corollary is the 9. propoſition after Campane.

¶ The 9. Propoſition.

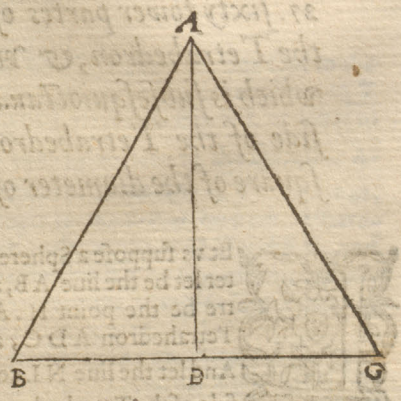
If the ſide of an equilater triangle be rationall: the ſuperficies ſhall be irrationall, of that kinde which is called Mediall.

The 12. propoſition after Campane.

SVppoſe that A B G be an equilater triangle: and from the point A draw vnto the ſide B G a perpendicular line A D: and let the line A B be rationall. Then I ſay, that the ſuperficies A B G is mediall. Foraſmuch as the line A B is in power ſeſquitertia to the line A D (by the Corollary of the 12.of the thirteenth): of what partes the line A B containeth in power 12, of the ſame partes the line A D containeth in power 9: wherefore the reſidue B D containeth in power of the ſame partes 3. (For the line A B cõtaineth in power the lines A D and B D, by the 47.of the firſt). Wherfore the lines A D and D B are rationall and commenſurable to the rationall line ſet A B, by the 6. of the tenth. But foraſmuch as the power of the line A D is to the power of the line D B in that proportion that 9. a ſquare number is to 3. a number not ſquare: therfore they are not in the proportion of ſquare numbers, by the Corollary of the 25.of the eight. And therfore they are not commenſurable in length, by the 9. of the tenth. Wherefore that which is contained vnder the lines A D and D B, which are rationall lines commenſurable in power onely, is mediall, by the 22. of the tenth. But that which is contained vnder the lines A D and D B, is double to the triangle A B D, by the 41.of the firſt. Wherefore that which is contained vnder the lines A D and D B, is equall to the whole triangle A B G (which is double to the triangle A B D, by the 1.of the ſixt). Wherefore the triangle A B G is mediall. If therfore the ſide of an equilater triangle be rationall: the ſuperficies ſhall be irrationall, of that kinde which is called Mediall: which was required to be proued.

Conſtruction.

Demonſtration.



✻ A Corollary.

If an Octohedron and a Tetrahedron be inſcribed in a Sphere whoſe diameter is rationall: their ſuperficieces ſhall be mediall. For thoſe ſuperficieces conſiſte of equilater triangles, whoſe ſides are commenſurable to the diameter which is rationall, by the 13.and 14.of

The 13. propoſition after Campane.

14. of the thirteenth, and therefore they are rationall. But they are commensurable in power onely to the perpendicular line, and therefore they containe a mediall triangle, as it was before manifest.

¶ The 10. Proposition.

The 14. proposition after Campane.

If a Tetrahedron and an Octohedron be inscribed in one and the self same Sphere: the base of the Tetrahedron shall be sesquitertia to the base of the Octohedron, and the superficieces of the Octohedron shall be sesquialtera to the superficieces of the Tetrahedron.

Demonstration of the first part.

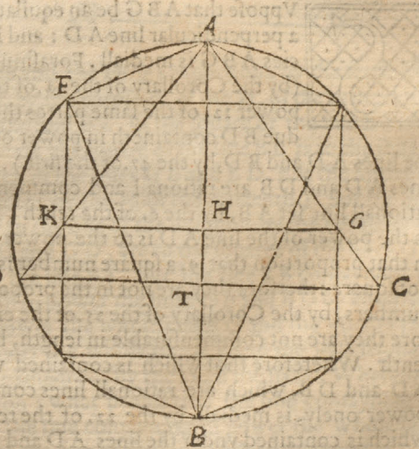
FOrasmuch as the diameter of the Sphere is in power sesquialtera to the side of the Tetrahedron (by the 13. of the thirteenth) and the same diameter is in power duple to the side of the Octohedron (by the 14. of the same booke): therefore of what partes the diameter containeth in power sixe, of the same the side of the Tetrahedron contayneth in power 4, and of the same the side of the Octohedron containeth in power 3. Wherefore the power of the side of the Tetrahedron, is to the power of the side of the Octohedron in the same proportion that 4. is to 3: which is sesquitertia. And like triangles (which are the bases of the solides) described of those sides, shall haue the one to the other the same proportion that the squares made of those sides haue. For both the triangles are the one to the other, and also the squares are the one to the other, in double proportion of that in which the sides are, by the 20. of the sixth. Wherefore of what partes one base of the Tetrahedron was 4: of the same are fower bases of the Tetrahedron 16: likewise of what partes of the same one base of the Octohedron was 3: of the same are 8. bases of the Octohedron 24. Wherfore the bases of the Octohedron are to the bases of the Tetrahedron, in that proportion that 24. is to 16: which is sesquialtera. If therefore a Tetrahedron and an Octohedron be inscribed in one and the selfe same Sphere: the base of the Tetrahedron shall be sesquitertia to the base of the Octohedron, and the superficieces of the Octohedron shall be sesquialtera to the superficieces of the Tetrahedron: which was required to be proued.

Demonstration of the second part.

¶ The 11. Proposition.

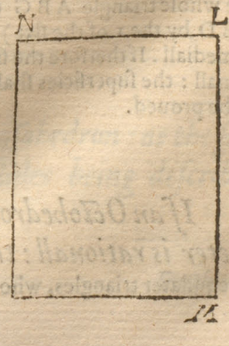
The 17. proposition after Campane.

A Tetrahedron is to an Octohedron inscribed in one and the selfe same Sphere, in proportion, as the rectangle parallelogrãme contained vnder the line, which containeth in power 27. sixty fower partes of the side of the Tetrahedron, & vnder the line which is subsesquioctaua to the same side of the Tetrahedron, is to the square of the diameter of the Sphere.



First part of the construction.

LEt vs suppose a Sphere, whose diameter let be the line AB, and let the centre be the point H. And in it let there be inscribed a Tetrahedron ADC, and an Octohedron AEKBG. And let the line NL containe in power $\frac{27}{64}$ of AC the side of the Tetrahedron. And let the line ML be in lẽgth subsesquioctaua to the same side. Thẽ I say, that the Tetrahedron ACD is to the Octohedron AEB, as the rectangle parallelogramme contayned vnder the lines NL and LM, is to the square of the line AB. Forasmuch as the line drawen frõ the angle A by the centre H perpẽdicularly vpon the base of the Tetrahedron, falleth vpon the cẽter T of the circle which containeth that base, and maketh the right line HT the sixth part of the diameter AB (by the Corollary of the 13. of the thirteenth): therefore the line HA (which is drawen from the centre to the circũference) is triple to the line HT: and therefore the whole line AT is



First part of the Demonstration.

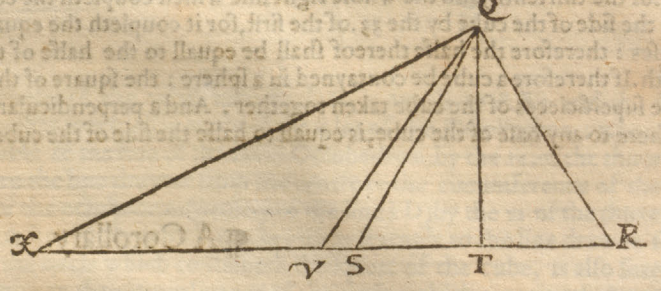
to

to the line A H, as 4. is to 3. Let the Tetrahedron A D C be cut by a plaine G H K paſſing by the centre H, and being parallel vnto the baſe D T C, by the Corollary of the 15. of the eleuenth. Now then the triangle A D C of the Tetrahedron, ſhall be cut by the right line K G, which is parallel to the line D C, by the 16. of the eleuenth. Wherfore as the line A T is to the line A H, ſo is the line A C to the line A G (by the 2. of the ſixth). Wherefore the line A C is to the line A G ſeſquitertia, that is, as 4. to 3. And foraſmuch as the triangles A D C, A K G, and the reſt which are cut by the plaine K H G, are like the one to the other, by the 5. of the ſixth: the pyramids A D C and A K G, ſhall be like the one to the other, by the 7. definition of the eleuenth. Wherefore they are in triple proportion of that in which the ſides A C and A G are, by the 8. of the twelfth. But the proportion of the ſides A C to A G is, as the proportion of 4. to 3. Now then, if, by the 2. of the eight, ye finde out 4. of the leſt numbers in continuall proportion, and in that proportion that 4. is to 3: which ſhall be 64. 48. 36. and 27: it is manifeſt, by the 15. definition of the fifth, that the extremes 64. to 27. are in triple proportion of that in which the proportion geuen 4. to 3. is: Or the quantitie of the proportion of 4. to 3. (which is 1. and $\frac{1}{3}$) being twiſe multiplied into it ſelfe, there ſhall be produced the proportion of 64. to 27. Wherefore the Pyramis or Tetrahedron A D C is to the pyramis A K G, as 64. is to 27: which is triple to the proportion of 4. to 3. And foraſmuch as the line A C is vnto the line A G in length ſeſquitertia: of what partes the line A C containeth in power 64: of the ſame partes doth the line A G containe in power 36. For (by the 2. of the ſixth) the proportion of the powers or ſquares, is duple to the proportion of the ſides which are as 64. is to 48.

Second part of the conſtruction.

Now then vpon the line R S which let be equall to the line A G, let there be an equilater triangle Q R S deſcribed (by the firſt of the firſt). And from the angle Q, draw to the baſe R S a perpendicular line Q T. And extend the line R S to the poynt X. And as 27. is to 64. (ſo by the corollary of the 6. of the tenth) let the line R S be to the line R X. And diuide the line R X into two equall partes in the poynt V, and draw the line Q V. And foraſmuch as the line R S is equall to the line A G, of what partes the line A C contayneth in power 64. of the ſame part the line R S contayneth in power 36. for it is proued that the line A G contayneth in power 36. of thoſe partes: And of what partes the line R S contayneth in power 36 of the ſame partes the line Q T contayneth in power 27. by the corollary of the 12. of the thirteenth. Wherfore of what partes the line A C contayneth in power 64. of the ſame parts the line Q T contayneth in power 27. Wherefore the right line Q T ſhall be equall to the right line L N by ſuppoſition. Againe foraſmuch as the line R S is put equall to the line A G: and of what partes the line R S contayneth in length 27. of the ſame parts is the line R X put to contayne in length 64. and of what partes the line R X contayneth in length 64. of the ſame the line A C (which is in length ſeſquitertia to the line A G or R S) contayneth 36. Wherefore the line R V (which is the halfe of the line R X) containeth in length of the ſame partes 32. of which the line A C contayned in length 36. Wherefore the line R V is to the line A C ſubſeſquioctaua: and therefore the line R V is equall to the line L M which is alſo ſubſeſquioctaua to the ſame line A C. And foraſmuch as the line N L is equall to the line Q T, and the line L M to the line R V (as before hath bene proued) the rectangle parallelogramme contayned vnder the lines Q T and R V, ſhall be equall to the rectangle parallelogramme, contayned vnder the line N L which is in power $\frac{27}{64}$ to the ſide A C, and vnder the line L M, which is in length ſubſeſquioctaua to the ſame ſide A C. But that which is contayned vnder the lines Q T and R V is double to the triangle Q V R by the 41. of the firſt: and to the ſame triangle Q V R is the triangle Q X R duple by the firſt of the ſixth. Wherefore the whole triangle Q X R is equall to that which is contayned vnder the lines Q T and R V, and therefore is equall to the parallelogramme M N. And foraſmuch as the line R X by ſuppoſition contayneth in length 64. of thoſe partes of which the line R S contayneth 27: and the triangles Q R X, and Q R S are, by the firſt of the ſixth, in the proportion of their baſes, that is, as 64. is to 27: but as 64. is to 27. ſo is the pyramis or tetrahedron A D C to the pyramis A K G: wherefore as the parallelogramme N M or the triangle Q R X, is to the triangle Q R S, ſo is the pyramis A D C to the pyramis A K G. And foraſmuch as the ſemidiameter A H is the altitude of the pyramis A K G, and alſo of the two equall and like pyramids of the octohedron which haue their common baſe in the ſquare of the octohedron (by the corollary of the 14. of the thirteenth): therefore as the baſe of the pyramis A K G (which is the triangle Q R S) is to two ſquares of the octohedron, that is, to the ſquare of the diameter A B, which is equall to thoſe ſquares (by the 47. of the firſt), ſo is the pyramis A K G to the octohedron A E B, by the 6. of the twelfth. And foraſmuch as the parallelogramme M N is to the baſe Q R S, as the pyramis A D C is to the pyramis A K G, and the baſe Q R S is to the ſquare of the line B E, as the pyramis A K G is to the octohedron A E B: therefore by proportion of equality taking away the meanes (by the 22. of the fifth) as the parallelogramme N M is to the ſquare of the line B E, ſo is the pyramis A D C to the octo-

Second part of the Demonſtration.



hedron A E B inſcribed in one and the ſelfe ſame ſphere. But the parallelogramme N M is contayned vnder the line N L which by ſuppoſition is in power $\frac{27}{64}$ to A C the ſide of the tetrahedron A D C, and vnder the line L M which is alſo by ſuppoſition in length ſubſeſquioctaua to the ſame line A C. Wherfore a tetrahedron & an octohedron inſcribed in one and the ſelfe ſame ſphere, are in proportiõ, as the rectangle parallelogramme contayned vnder the line, which contayneth in power 27. ſixty fower parts of the ſide of the Tetrahedron, and vnder the line which is ſubſeſquioctaua to the ſame ſide of the Tetrahedron, is to the ſquare of the diameter of the ſphere: which was required to be proued.

¶ The 12. Propoſition.

The 18. propoſition after Campane.

If a cube be contayned in a ſphere: the ſquare of the diameter doubled, is equall to all the ſuperficieces of the cube taken together. And a perpendicular line drawne from the centre of the ſphere to any baſe of the cube, is equall to halfe the ſide of the cube.

Demonſtration of the firſt part.

For foraſmuch as (by the 15. of the thirteenth) the diameter of the ſphere is in power triple to the ſide of the cube: therefore the ſquare of the diameter doubled is ſextuple to the baſe of the ſame cube. But the ſextuple of the power of one of the ſides contayneth the whole ſuperficies of the cube. For the cube is compoſed of ſixe ſquare ſuperficieces (by the 21. diffinition of the eleuenth) whoſe ſides therefore are equall: wherefore the ſquare of the diameter doubled is equall to the whole ſuperficies of the cube. And foraſmuch as the diameter of the cube, and the line which falleth perpendicularly vpõ the oppoſite baſes of the cube, do cut the one the other into two equall partes in the centre of the ſphere which containeth the cube (by the 2. corollary of the 15. of the thirteenth) and the whole right line which coupleth the centres of the oppoſite baſes, is equall to the ſide of the cube by the 33. of the firſt, for it coupleth the equall and parallel ſemidiameters of the baſes: therefore the halfe thereof ſhall be equall to the halfe of the ſide of the cube by the 15. of the fifth. If therefore a cube be contayned in a ſphere: the ſquare of the diameter doubled is equall to all the ſuperficieces of the cube taken together. And a perpendicular line drawne from the centre of the ſphere to any baſe of the cube, is equall to halfe the ſide of the cube: which was required to be proued.

Demonſtration of the ſecond part.

¶ A Corollary.

The Corollary of the 8. propoſition after Campane.

If two thirds of the power of the diameter of the ſphere be multiplyed into the perpendicular line equall to halfe the ſide of the cube, there ſhall be produced a ſolide equall to the ſolide of the cube. For it is before manifeſt that two third partes of the power of the diameter of the ſphere are equall to two baſes of the cube. If therefore vnto eche of thoſe two thirds be applyed halfe the altitude of the cube, they ſhall make eche of thoſe ſolides equall to halfe of the cube, by the 31. of the eleuenth: for they haue equall baſes. Wherefore two of thoſe ſolides are equall to the whole cube.

You ſhall vnderſtand (gentle reader) that *Campane* in his 14. booke of *Euclides* Elementes hath 18. propoſitiõs with diuers corollaries following of them. Some of which propoſitions and corollaries I haue before in the twelfth and thirteenth bookes added out of Fluſſas as corollaries (which thing alſo I haue noted on the ſide of thoſe corollaries, namely, with what propoſition or corollary of Campanes 14. booke they doo agree). The reſt of his 18. propoſitions and corollaries are contained in the twelue former propoſitions and corollaries of this 14. booke after Fluſſas: where ye may ſee on the ſide of eche propoſition and corollary with what propoſition and corollary of Campanes they agree. But the eight propoſitions following together with their corollaries, Fluſſas hath added of him ſelfe, as he him ſelfe affirmeth.

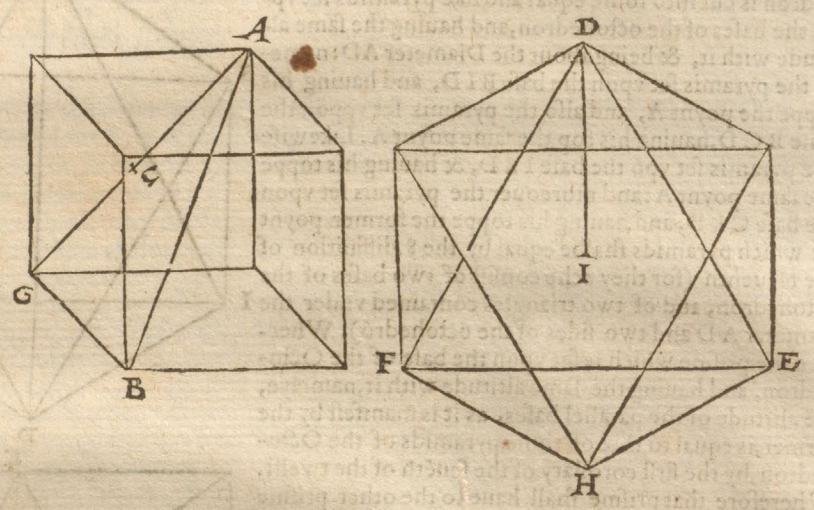
The

The 13. Propoſition.

One and the ſelf ſame circle containeth both the ſquare of a cube, and the triangle of an Octohedron deſcribed in one and the ſelfe ſame ſphere.

VPpoſe that there be a cube A B G, and an Octohedron D E F deſcribed in one and the ſelfe ſame ſphere, whoſe diameter let be A B, or D H. And let the lines drawne from the cẽtres (that is the ſemidiameters of the circles which ctõaine the baſes of thoſe ſolides) be C A and I D. Then I ſay that the lines C A and I D are equal. Foraſmuche as A B the diameter of the ſphere which containeth the cube, is in power triple to B G the ſide of the cube (by the 15. of the thirteenth) vnto which ſide, A G the diameter of the baſe of the cube, is in power double (by the 47. of the firſt): which line A G is alſo the diameter of the circle, which cõtaineth the baſe (by the 9. of the fourth:) therfore A B the diameter of the ſphere is in power ſeſquialter to the line A G: namely, of what partes the line A B, containeth in power 12. of the ſame the line A G, ſh al containe in power 8. And therfore the right line A C whiche is drawn from the cẽtre of the circle to the circumference, conteineth in power of the ſame partes 2. Wherefore the diameter of the ſphere is in power ſextuple to the lyne which is drawne from the centre to the circumference of the circle whiche containeth the ſquare of the cube But the Diameter of the ſelfe ſame Sphere whych containeth the Octohedron, is one and the ſelfe ſame with the diameter of the cube, namely, D H, is equall to A B: and the ſame diameter is alſo the diameter of the ſquare which is made of the ſides of the Octohedron: wherefore the ſaide diameter is in power double to the ſide of the ſame Octohedron, by the 14. of the thirteenth. But the ſide D F is in power triple to the line drawne from the centre to the circumference of the circle which containeth the triangle of the octohedron (namely to the line I D) by the 12. of the thirteenth. Wherfore the ſelfe ſame diameter A B or D H, which was in power ſextuple to the line drawne from the centre to the circumference of the circle which containeth the ſquare of the cube, is alſo ſextuple to the line I D drawne from the centre to the circumference of the circle, which containeth the triangle of the Octohedron: Wherefore the lines drawne from the centres of the circles to the circumferences which containe the baſes of the cube and of the octohedron are equal. And therfore the circles are equal, by the firſt diffinition of the third. Wherfore one and the ſelfe ſame circle containeth &c. as in the propoſition: which was required to be proued.

Demonſtration.



A Corollary.

Hereby it is manifeſt, that perpendiculars coupling together in a ſphere, the centres of the circles which containe the oppoſite baſes of the cube and of the Octohedron, are equal. For the circles are equal, by the ſecond corollary of the aſſumpt of the 16. of the twelfth: and the lines which paſſing by the centre of the ſphere, couple together the centres of the baſes, are alſo equal, by the firſt corollary of the ſame. Wherfore the perpendicular which coupleth together the oppoſite baſes of the Octohedron, is equal to the ſide of the cube. For either of them is the altitude erected.

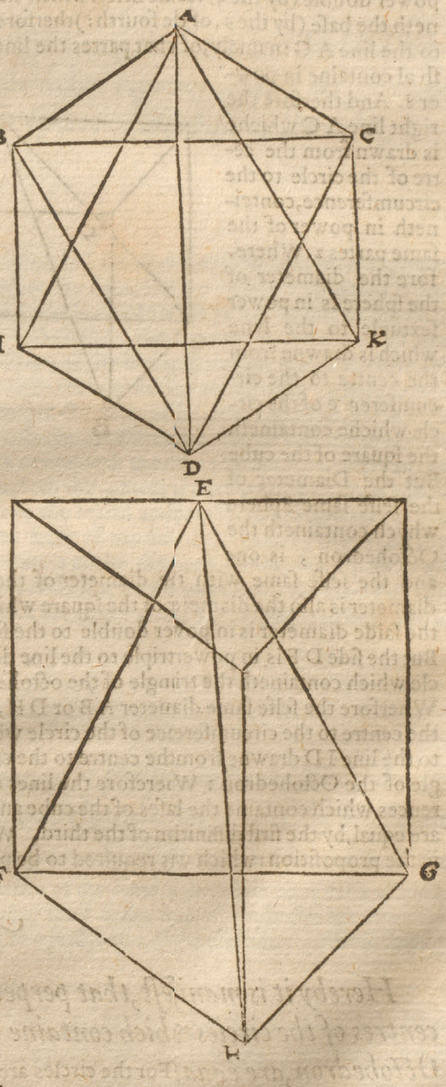
The 14. Propoſition.

An Octohedron is to the triple of a Tetrahedron contained in one and the

ſelfe ſame ſphere, in that proportion that t heir ſides are.

Conſtruction. SVppoſe that there be an octohedron A B C D, and a Tetrahedron E F G H: vpon whoſe baſe F G H erect a Priſme, which is done by erecting from the angles of the baſe perpendicular lines equal to the altitude of the Tetrahedron: which priſme ſhalbe triple to the Tetrahedron E F G H, by the firſt corollary of the 7. of the twelfth. Then I ſay that the octohedron A B C D is to the priſme which is triple to the Tetrahedron, E F G H, as the ſide B C is to the ſide F G.

Demonſtration. For forasmuch as the ſides of the oppoſite baſes of the octohedron, are right lines touching the one the other, and are parellels to other right lines touching the one the other, for the ſides of the ſquares which are cõpoſed of the ſides of the octohedrõ, are oppoſite: Wherfore the oppoſite plaine triangles, namely, A B C & K I D, ſhalbe parallels, and ſo the reſt by the 15. of the eleuenth. Let the diameter of the Octohedron, be the line A D. Now then the whole Octohedron is cut into foure equal and like pyramids ſet vpon the baſes of the octohedron, and hauing the ſame altitude with it, & being about the Diameter AD: namely the pyramis ſet vpon the baſe B I D, and hauing his toppe the poynt A, and alſo the pyramis ſet vppon the baſe B C D, hauing his top the ſame poynt A. Likewiſe the pyramis ſet vpõ the baſe I K D, & hauing his toppe the ſame poynt A: and moreouer the pyramis ſet vpon the baſe C K D, and hauing his toppe the former poynt A: which pyramids ſhalbe equal by the 8. diffinition of the eleuenth (for they eche conſiſt of two baſes of the octohedron, and of two triangles contained vnder the diameter A D and two ſides of the octohedrõ). Wherfore the priſme which is ſet vpon the baſe of the Octohedron, and hauing the ſame altitude with it, namelye, the altitude of the parallel baſes, as it is manifeſt by the former, is equal to thre of thoſe pyramids of the Octohedron, by the firſt corollary of the ſeuẽth of the twelft. Wherefore that priſme ſhall haue to the other priſme vnder the ſame altitude, compoſed of the 4. pyramids of the whole octohedron, the proportion of the triangular baſes, by the 3. corollary of the ſame. And forasmuch as 4. pyramids are vnto 3. pyramids in ſeſquitercia proportion, therefore the trianguler baſe of the priſme which containeth 4. pyramids, is in ſeſquitertia proportion to the baſe of the priſme which containeth thre pyramids of the ſame octohedron, and are ſet vpon the baſe of the Octohedron and vnder the altitude thereof: that is, in ſeſquitercia proportion to the baſe of the Octohedron. But the baſe of the ſame octohedron is in ſeſquitertia proportion to the baſe of the pyramis, by the tenth of this booke: Wherefore the triangular baſes, namely, of the priſme which cõtaineth four pyramids of the octohedron, and is vnder the altitude thereof, are equal to the triangular baſes of the priſme, which containeth three pyramids vnder the altitude of the pyramis E F G H. But the priſme of the octohedron is equal to the octohedron: and the priſme of the pyramis E F G H is proued triple to the ſame pyramis E F G H. Now then the priſmes ſet vpõ equal baſes, are the one to the other as their altitudes are (by the corollary of the 25. of the eleuenth) namely, as are the parallelipidedons their doubles, by the corollary of the 31. of the eleuenth. But the altitude of the Octohedron is equal to the ſide of the cube contained in the ſame ſphere, by the corollary of the 13. of this booke. And the ſide of the cube is in power to the altitude of the Tetrahedon in that proportion that 12. is to 16, by the 18. of the thirteenth: And the ſide of the octohedron is to the ſide of the pyramis in that proportion that 18. is to 24. (by the ſame 18. of the thirtẽth) which proportion is one & the ſelf ſame with the proportiõ of 12. to 16. Wherfore that priſme which is equal to the Octohedron, is to the priſme which is triple to the Tetrahedron, in that proportiõ that the altitudes, or that the ſides are. Wherfore an octohedrõ is to the triple of a Tetrahedron cõtained in one and the ſelfe ſame ſphere, in that proportion that their ſides are: which was required to be demonſtrated.



A Corollary.

The sides of a Tetrahedron & of an Octohedrō are proportionall with their altitudes. For the sides & alitudes were in power sesquitercia. Moreouer the diameter of the sphere is to the side of the Tetrahedron, as the side of the Octohedron is to the side of the cube, namely, the powers of eche is in sesquialte proportion, by the 18. of the thirteenth.

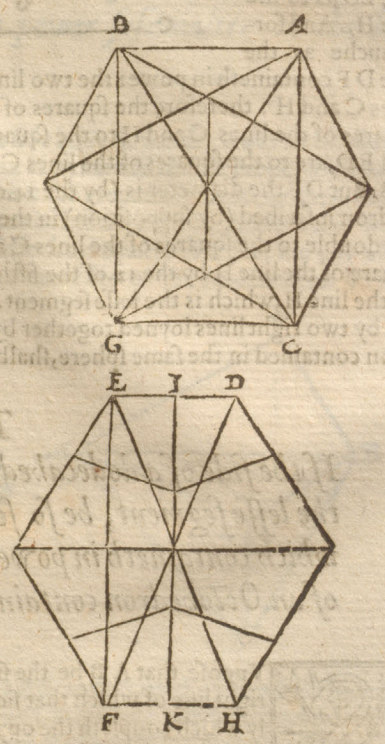
The 15. Proposition.

If a rational line containing in power two lines, make the whole and the greater segment, and again containing in power two lines, make the whole and the lesse segment: the greater segment shalbe the side of the Icosahedron, and the lesse segment shalbe the side of the Dodecahedron contayned in one and the selfe same sphere.

SVppose that A G be the diameter of the sphere which containeth the Icosahedron A B G C. And let B G subtend the sides of the pentagon described of the sides of the Icosahedron (by the 16. of the thirteth.) Moreouer vpon the same diameter A G, or D F equal vnto it, let ther be described a dodecahedron D E F H, by the 17. of the thirtenth, whose opposites sides E D and F H let be cut into two equal partes in the poynts I and K, and draw a line from I to K. And let the line E F couple two of the opposite angles of the bases which are ioyned together. Thē I say that A B the side of the Icosahedron is the greater segment which the diameter A G containeth in power together with the whole line, and line I D is the lesse segment, which the same diameter A G or D F containeth in power together with the whole. For forasmuche as the oppoſite sides A B and G C of the Icosahedron being coupled by the diameters A G and B C, are equal & parallels, by the 2. corollary of the 16. of the thirteth: the right lines B G & A C which couple thē together are equal & parallels by the 33. of the first. Moreouer the angles B A C & A B G being subtended of equa diameters, shall by the 8. of the first be equal, & by the 29 of the first, they shal be right angles. Wherfore the right line A G cōtaineth in power the two lines A B and B G, by the 47. of the first. And forasmuch as the line B G subtendeth the angle of the pentagon composed of the sides of the Icosahedron, the greater segment of the right line B G, shalbe the right line A B, by the 8. of the thirteenth: which line A B, together with the whole line B G, the line A G containeth in power. And forasmuch as the line I K coupling the oppoſite and parallel sides E D and F H of the Dodecahedron, maketh at those poyntes right angles, by the 3. corollary of the 17. of the thirtenth: the right line E F which coupleth together equal and parallel lines E I & F K, shalbe equal to the same line I K, by the 33. of the first. Wherfore the angle D E F shalbe a right angle by the 29. of the first. Wherefore the diameter D F cōtaineth in power the two lines E D and E F. But the lesse segment of the line I K is E D the side of the Dodecahedron, by the 4. corollary of the 17. of the thirtenth. Wherfore the same line E D is also the lesse segment of the line E F (which is equal vnto the line I K): wherfore the diameter D F containing in power the two lines E D and E F (by the 47. of the first) containe thin power E D the side of the dodecahedron, the lesse segment, together with the whole. If therfore a rational line A G or D F containing in power two lines A B and B G, doo make the whole line and the greater segment, and againe containing in power two lines E F and E D, do make the whole line and the lesse segment: the greater segment A B, shall be the side of the Icosahedron, and the lesse segment E D shall be the side of the Dodecahedron contained in one and the selfe same sphere.

Construction.

Demonstration.



The 16. Proposition.

If the power of the side of an Octohedron be expreſsed by two right lines

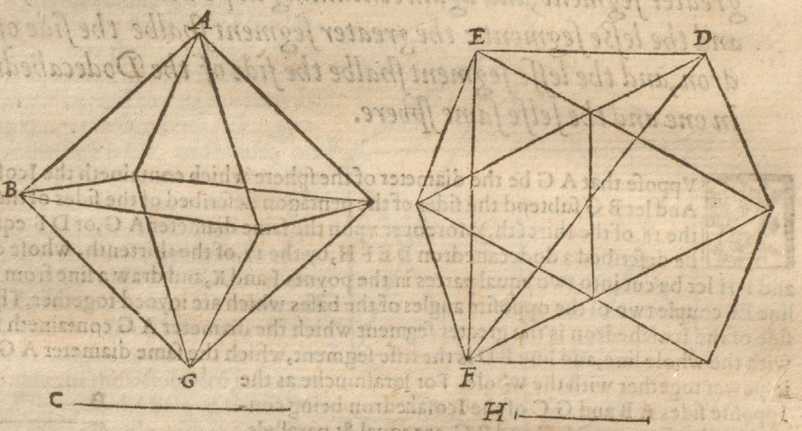
ioyned together by an extreme and meane proportion: the ſide of the Icoſahedron contained in the ſame ſphere, ſhalbe duple to the leſſe ſegment.

Conſtruction.

LEt A B the ſide of the Octohedron A B G containe in power the two lines C and H, which let haue that proportion that the whole hath to the greater ſegment (by the corollarye of the firſt propoſition added by *Fluſſas* after the laſt propoſitiō of the ſixth booke). And let the Icoſahedron contained in the ſame ſphere be D E F, whoſe ſide let be D E, and let the right line ſubtending the angle of the pentagon made of the ſides of the Icoſahedron be the line E F. Then I ſay that the ſide E D is in power double to the line H the leſſe of thoſe ſegmentes.

Demonſtration.

Foraſmuch as by that which was demonſtrated in the 15. of this booke, it was manifeſt that E D the ſide of the Icoſahedron is the greater ſegment of the line E F, and that the diameter D F containeth in power the two lines E D and E F, namely, the whole and the greater ſegment: but by ſuppoſition the ſide A B cōtaineth in power the two lines C & H ioined together in the ſelf ſame proportiō. Wherefore the line E F is to the line E D, as the line C, is to the line H, by the 2. of this boke. And alternatly by the 16. of the fiueth, the line E F is to the line C, as the line E D, is to the line H. And foraſmuche as the line D F containeth in power the two lines E D and E F, and the line A B containeth in power the two lines C and H: therefore the ſquares of the lines E F and E D are to the ſquare of the line D F, as the ſquares of the lines C and H to the ſquare of the line A B. And alternately, the ſquares of the lines E F and E D, are to the ſquares of the lines C and H, as the ſquare of the line D F is to the ſquare of the line A B. But D F the diameter is (by the 14. of the thirteenth) in power double to A B the ſide of the octohedron inſcribed (by ſuppoſition) in the ſame ſphere. Wherefore the ſquares of the lines E F and E D, are double to the ſquares of the lines C and H. And therfore one ſquare of the line E D is double to one ſquare of the line H by the 12. of the fifth. Wherfore E D the ſide of the Icoſahedron is in power duple to the line H, which is the leſſe ſegment. If therfore the power of the ſide of an octohedron be expreſſed by two right lines ioyned together by an extreme and meane proportion: the ſide of the Icoſahedron contained in the ſame ſphere, ſhalbe duple to the leſſe ſegment.



The 17. Propoſition.

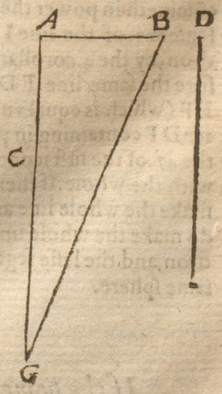
If the ſide of a dodecahedron, and the right line, of whome the ſaid ſide is the leſſe ſegment, be ſo ſet that they make a right angle: the right line which containeth in power halfe the line ſubtending the angle, is the ſide of an Octohedron contained in the ſelfe ſame ſphere.

Conſtruction.

VPpoſe that A B be the ſide of a Dodecahedron, and let the right line of which that ſide is the leſſe ſegment be A G, namely which coupleth the oppoſite ſides of the Dodecahedron, by the 4. corollary of the 17. of the thirteenth: and let thoſe lines be ſo ſet that they make a right angle at the point A. And draw the right line B G. And let the line D containe in power halfe the line B G (by the firſt propoſition added by *Fluſſas* after the laſte of the ſixth). Then I ſay that the line D is the ſide of an Octohedron contayned in the ſame ſphere.

Demonſtration.

Forasmuche as the line A G maketh the greater ſegment G C the ſide of the cube contained in the ſame ſphere (by the ſame 4. corollary of the 17. of the thirteenth): and the ſquares of the whole line A G. and of the leſſe ſegment A B are triple to the ſquare of the greater ſegment G C, by the 4. of the thirteenth: Moreouer the diameter of the ſphere, is in power triple to the ſame line G C the ſide of the cube (by the 15. of the thirteenth:



Wherefore

Wherfore the line B G is equal to the diameter. For it containeth in power the two lines A B and A G (by the 47. of the firſt,) and therefore it containeth in power the triple of the line G C. But the ſide of the Octohedron contained in the ſame ſphere, is in power triple to halfe the diameter of the ſphere by the 14. of the thirteenth. And by ſuppoſition the line D containeth in power the halfe of the line B G. Wherefore the line D (containing in power the halfe of the ſame diameter) is the ſide of an octohedron. If therfore the ſide of a Dodecahedron and the right line of whome the ſaid ſide is the leſſe ſegment, be ſo ſet that they make a right angle: the right line which containeth in power halfe the line ſubtending the angle, is the ſide of an Octohedron contained in the ſelfe ſame ſphere: Which was required to be proued.

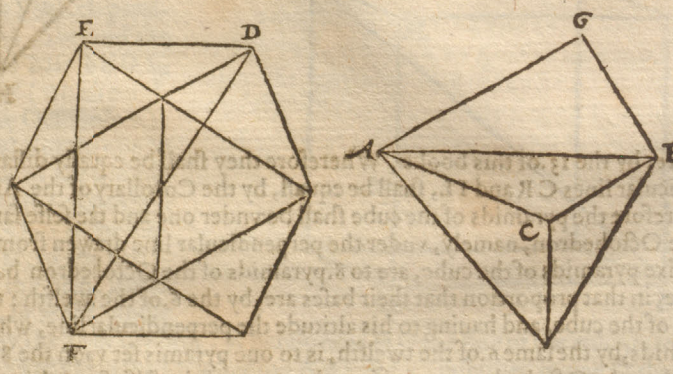
✻ A Corollary.

Vnto what right line the ſide of the Octohedron is in power ſeſquialter: vnto the ſame line the ſide of the Dodecahedron inſcribed in the ſame ſphere, is the greater ſegment. For the ſide of the Dodecahedron is the greater ſegment of the ſegment C G, vnto which D the ſide of the Octohedron is in power ſeſquialter, that is, is halfe of the power of the line B G, which was triple vnto the line C G.

¶ The 18. Propoſition.

If the ſide of a Tetrahedron containe in power two right lines ioyned together by an extreme and meane proportion: the ſide of an Icoſahedron deſcribed in the ſelfe ſame Sphere, is in power ſeſquialter to the leſſe right line.

Vppoſe that A B C be a Tetrahedron, and let his ſide be A B, whoſe power let be diuided into the lines A G and G B, ioyned together by an extreme and meane proportion: namely, let it be diuided into A G the whole line, and G B the greater ſegment (by the Corollary of the firſt Propoſition added by Fluſſas after the laſt of the ſixth). And let E D be the ſide of the Icoſahedron E D F contained in the ſelfe ſame Sphere. And let the line which ſubtendeth the angle of the Pentagon deſcribed of the ſides of the Icoſahedron be E F. *Conſtruction.*



Then I ſay, that E D the ſide of the Icoſahedron is in power ſeſquialter to the leſſe line G B. Foraſmuch as (by that which was demonſtrated in the 15. of this booke) the ſide E D is the greater ſegment of the line E F which ſubtendeth the angle of the Pentagon. But as the whole line E F is to the greater ſegment E D, ſo is the ſame greater ſegment to the leſſe (by the 30. of the ſixth): and by ſuppoſition, A G, was the whole line, and G B the greater ſegment: Wherefore as E F is to E D, ſo is A G to G B, by the ſecond of the foureteenth. And alternately, the line E F is to the line A G, as the line E D is to the line G B. And foraſmuch as (by ſuppoſition) the line A B containeth in power the two lines A G and G B: therefore (by the 48. of the firſt) the angle A G B is a right angle. But the angle D E F is a right angle, by that which was demonſtrated in the 15. of this booke. Wherefore the triangles A G B and F E D, are equiangle, by the 8. of the ſixth. Wherefore their ſides are proportionall: namely, as the line E D is to the line G B, ſo is the line F D to the line A B, by the 4. of the ſixth. But by that which hath before bene demonſtrated, F D is the diameter of the Sphere which containeth the Icoſahedron: which diameter is in power ſeſquialter to A B the ſide of the Tetrahedron inſcribed in the ſame Sphere, by the 13. of the thirteenth. Wherfore the line E D the ſide of the Icoſahedron, is in power ſeſquialter to G B the greater ſegment or leſſe line. If therefore the ſide of a Tetrahedron containe in power two right lines ioyned together by an extreme and meane proportion: the ſide of an Icoſahedron deſcribed in the ſelfe ſame Sphere, is in power ſeſquialter to the leſſe right line. *Demonſtration.*

¶ The

¶ The 19. Propoſition.

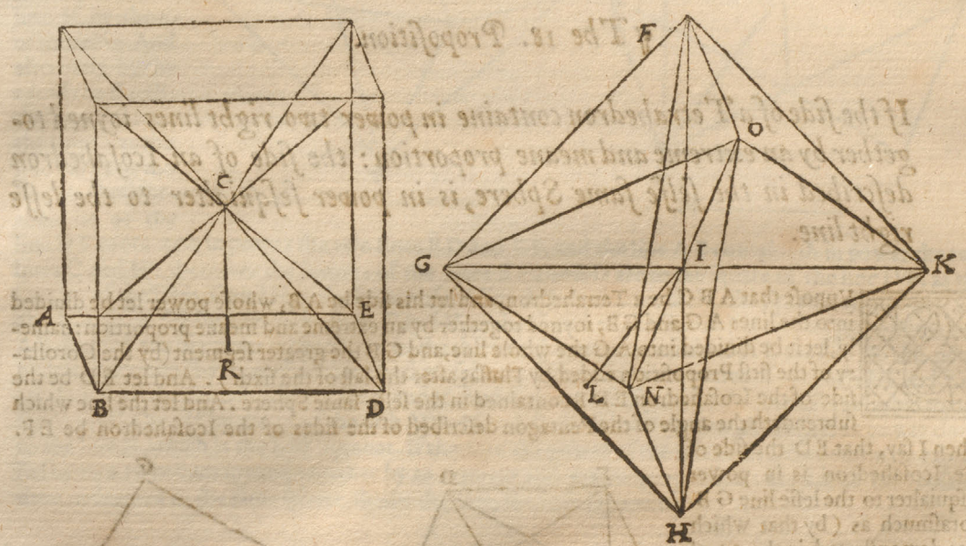
The ſuperficies of a Cube is to the ſuperficies of an Octohedron inſcribed in one and the ſelfe ſame Sphere, in that proportion that the ſolides are.

Conſtruction.

SVppoſe that A B C D E be a Cube, whoſe fower diameters let be the lines A C, B C, D C, and E C produced on ech ſide. Let alſo the Octohedron inſcribed in the ſelfe ſame Sphere be F G H K: whoſe three diameters let be F H, G K, and O N. Then I ſay, that the cube A B D is to the Octohedron F G H, as the ſuperficies of the cube is to the ſuperficies of the Octohedron. Drawe from the centre of the cube to the baſe A B E D, a perpendicular line C R. And from the centre of the Octohedron draw to the baſe G N H, a

Demonſtration.

perpendicular line I L. And foraſmuch as the three diameters of the cube do paſſe by the centre C, therefore, by the 2. Corollary of the 15. of the thirteenth, there ſhall be made of the cube ſixe pyramids, as thys pyramis A B D E C, equall to the whole cube. For there are in the cube ſixe baſes, vpon which fall equall perpendiculars from the centre, by the Corollary of the Aſſumpt of the 16. of the twelfth, for the baſes are contained in equall circles of the Sphere. But in the Octohedron the three diameters do make vpon the 8. baſes, 8. pyramids, hauing their toppes in the centre, by the 3. Corollary of the 14. of the thirteenth. Now the baſes of the cube and of the Octohedron are contained in equall circles of the



Sphere, by the 13. of this booke. Wherefore they ſhall be equally diſtant from the centre, and the perpendicular lines C R and I L, ſhall be equall, by the Corollary of the Aſſumpt of the 16. of the twelfth. Wherefore the pyramids of the cube ſhall be vnder one and the ſelfe ſame altitude with the pyramids of the Octohedron, namely, vnder the perpendicular line drawen from the centre to the baſes. Wherfore ſixe pyramids of the cube, are to 8. pyramids of the Octohedron being vnder one and the ſame altitude, in that proportion that their baſes are, by the 6. of the twelfth: that is, one pyramis ſet vpon ſixe baſes of the cube, and hauing to his altitude the perpendicular line, which pyramis is equall to the ſixe pyramids, by the ſame 6. of the twelfth, is to one pyramis ſet vpon the 8. baſes of the Octohedron, being equall to the Octohedron, and alſo vnder one and the ſelfe ſame altitude, in that proportion that ſixe baſes of the cube, which containe the whole ſuperficies of the cube, are to 8. baſes of the Octohedron, which containe the whole ſuperficies of the Octohedron. For the ſolides of thoſe pyramids are in proportion the one to the other, as their baſes are, by the ſelfe ſame 6. of the twelfth. Wherefore the ſuperficies of the cube is to the ſuperficies of the Octohedron inſcribed in one and the ſelfe ſame Sphere, in that proportion, that the ſolides are: which was required to be proued.

¶ The 20. Propoſition.

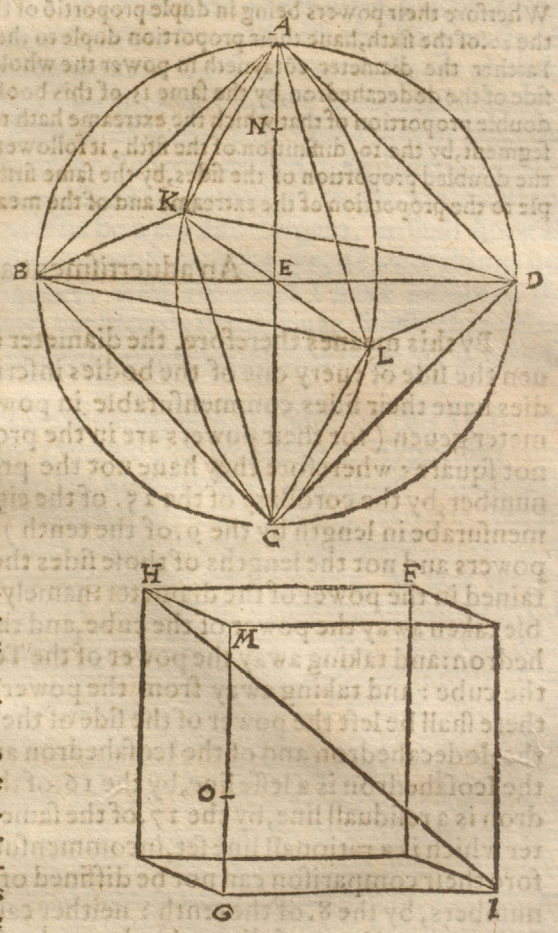
If a Cube and an Octohedron be contained in one & the ſelfe ſame Sphere: they ſhall be in proportion the one to the other, as the ſide of the Cube is to the

the ſemidiameter of the Sphere.

Conſtruction.

VPpoſe that the Octohedron A E C D B be inſcribed in the Sphere A B C D: and let the cube inſcribed in the ſame Sphere be F G H I M: whoſe diameter let be H I, which is equall to the diameter A C, by the 15.of the thirteenth: let the halfe of the diameter be A E. Then I ſay, that the cube F G H I M is to the Octohedron A E C D B, as the ſide M G is to the ſemidiameter A E. Foraſmuch as the diameter A C is in power double to B K the ſide of the Octohedron (by the 14.of the thirteenth) and is in power triple to M G the ſide of the cube (by the 15.of the ſame): therefore the ſquare B K D L ſhall be ſeſquialter to F M the ſquare of the cube. From the line A E cut of a third part A N, and frō the line M G cut of likewiſe a third part G O, by the 9.of the ſixth. Now then the line E N ſhall be two third partes of the line A E, and ſo alſo ſhall the line M O be of the line M G. Wherefore the parallelipipedon ſet vpon the baſe B K D L, and hauing his altitude the line E A, is triple to the parallelipipedon ſet vpon the ſame baſe, and hauing his altitude the line A N, by the Corollary of the 31.of the eleuenth: but it is alſo triple to the pyramis A B K D L which is ſet vpon the ſame baſe, and is vnder the ſame altitude (by the ſecond Corollary of the 7.of the twelfth). Wherefore the pyramis A B K D L is equall to the parallelipipedon, which is ſet vpon the baſe B K D L, and hath to his altitude the line A N. But vnto that parallelipipedō, is double the parallelipipedon which is ſet vppon the ſame baſe B K D L, and hath to his altitude a line double to the line E N, by the Corollary of the 31.of the firſt: and vnto the pyramis is double the Octohedron A B K L D C, by the 2.Corollary of the 14.of the thirteenth. Wherefore the Octohedron A B K D L C is equall to the parallelipipedon ſet vpon the baſe B K L D, & hauing his altitude the line E N (by the 15. of the fifth). But the parallelipipedon ſet vpon the baſe B K D L, which is ſeſquialter to the baſe F M, and hauing to his altitude the line M O, which is two third partes of the ſide of the cube M G, is equall to the cube F G: by the 2.part of the 34.of the eleuenth. (For it was before proued that the baſe B K D L is ſeſquialter to the baſe F M). Now then theſe two parallelipipedons, namely, the parallelipipedon which is ſet vpō the baſe B K D L (which is ſeſquialter to the baſe of the cube) and hath to his altitude the line M O (which is two third partes of M G the ſide of the cube) which parallelipipedon is proued equall to the cube, and the parallelipipedon ſet vpon the ſame baſe B K D L, and hauing his altitude the line E N (which parallelipipedon is proued equall to the Octohedron): theſe two parallelipipedons (I ſay) are the one to the other, as the altitude M O, is to the altitude E N (by the Corollary of the 31.of the eleuenth). Wherefore alſo as the altitude M O, is to the altitude E N, ſo is the cube F G H I M, to the Octohedron A B K D L C, by the 7.of the fifth. But as the line M O is to the line E N, ſo is the whole line M G to the whole line E A, by the 18.of the fifth. Wherefore as M G the ſide of the cube, is to E A the ſemidiameter, ſo is the line F G H I M to the Octohedron A B K D L C inſcribed in one & the ſelfe ſame Sphere. If therefore a cube and an Octohedron be contained in one and the ſelfe ſame Sphere: they ſhall be in proportion the one to the other, as the ſide of the cube is to the ſemidiameter of the Sphere; which was required to be demonſtrated.

Demonſtration.

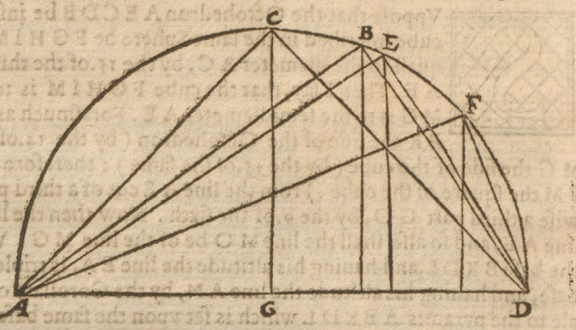


A Corollary.

Diſtinctly to notefie the powers of the ſides of the fiue ſolides by the power of the diameter of the ſphere.

The ſides of the tetrahedron and of the cube doo cut the power of the diameter of the ſphere into two ſquares which are in proportion double the one to the other. The octohedron cutteth the

power of the diameter into two equall ſquares. The Icoſahedron into two ſquares, whoſe proportion is duple to the proportion of a line diuided by an extreame and meane proportion, whoſe leſſe ſegmẽt is the ſide of the Icoſahedron. And the dodecahedron into two ſquares, whoſe proportion is quadruple to the proportion of a line diuided by an extreame and meane proportion, whoſe leſſe ſegment is the ſide of the dodecahedron. For A D the diameter of the ſphere, contayneth in power A B the ſide of the tetrahedron, and B D the ſide of the cube, which B D is in power halfe of the ſide A B. The diameter alſo of the ſphere contayneth in power A C and C D two equall ſides of the octohedron. But the diameter contayneth in power the whole line A E and the greater ſegment thereof E D, which is the ſide of the Icoſahedron, by the 15. of this booke.



Wherfore their powers being in duple proportiõ of that in which the ſides are, by the firſt corollary of the 20. of the ſixth, haue their proportion duple to the proportion of an extreame & meane proportiõ. Farther the diameter cõtayneth in power the whole line A F, and his leſſe ſegment F D, which is the ſide of the dodecahedron, by the ſame 15. of this booke. Wherefore the whole hauing to the leſſe, a double proportion of that which the extreame hath to the meane, namely, of the whole to the greater ſegment, by the 10. diffinition of the fifth, it followeth that the proportion of the power is double to the doubled proportion of the ſides, by the ſame firſt corollary of the 20. of the ſixth: that is, is quadruple to the proportion of the extreame and of the meane, by the diffinition of the ſixth.

An aduertiſment added by *Fluſſas*.

By this meanes therefore, the diameter of a ſphere being geuen, there ſhall be geuen the ſide of euery one of the bodies inſcribed. And foraſmuch as three of thoſe bodies haue their ſides commenſurable in power onely, and not in length, vnto the diameter geuen (for their powers are in the proportion of a ſquare number to a number not ſquare: wherefore they haue not the proportion of a ſquare number to a ſquare number, by the corollary of the 25. of the eight: wherefore alſo their ſides are incommenſurabe in length by the 9. of the tenth): therefore it is ſufficient to compare the powers and not the lengths of thoſe ſides the one to the other: which powers are contained in the power of the diameter: namely, from the power of the diameter, let there ble taken away the power of the cube, and there ſhall remayne the power of the Tetrahedron: and taking away the power of the Tetrahedron, there remayneth the power of the cube: and taking away from the power of the diameter halfe the power thereof, there ſhall be left the power of the ſide of the octohedron. But foraſmuch as the ſides of the dodecahedron and of the Icoſahedron are proued to be irrationall (for the ſide of the Icoſahedron is a leſſe line, by the 16. of the thirteenth: and the ſide of the dodecahedron is a reſiduall line, by the 17. of the ſame): therfore thoſe ſides are vnto the diameter which is a rationall line ſet, incommenſurable both in length and in power. Wherefore their compariſon can not be diffined or deſcribed by any proportion expreſſed by numbers, by the 8. of the tenth: neither can they be compared the one to the other: for irrational lines of diuers kindes are incõmẽſurable the one to the other: for if they ſhould be commenſurable, they ſhould be of one and the ſelfe ſame kinde, by the 103. and 105. of the tenth, which is impoſſible. Wherefore we ſeking to compare them to the power of the diameter, thought they could not be more aptly expreſſed, then by ſuch proportions, which cutte that rationall power of the diameter according to their ſides: namely, diuiding the power of the diameter by lines which haue that proportiõ, that the greater ſegment hath to the leſſe, to put the leſſe ſegment to be the ſide of the Icoſahedron: & deuiding the ſayd power of the diameter by lines hauing the proportion of the whole to the leſſe ſegment, to expreſſe the ſide of the dodecahedron by the leſſe ſegment: which thing may well be done betwene magnitudes incommenſurable.

The ende of the fourtenth Booke
of Euclides Elementes
after Fluſſas.

¶ The fiftenth booke of Euclides Elementes.

The argument of the 15. booke.

THis fiuetenth and laſt booke of Euclide, or rather the ſecond boke of *Appollonius* or *Hypſicles*, teacheth the inſcription and circumſcriptiō of the fiue regular bodies one within and about an other: a thing vndoutedly pleſant and delectable in minde to contemplate, and alſo profitable and neceſſary in act to practiſe. For without practiſe in act, it is very hard to ſe and conceiue the conſtructions and demonſtrations of the propoſitions of this booke, vnles a man haue a very depe, ſharpe, & fine imagination. Wherfore I would wiſh the diligent ſtudēt in this booke, (to make the ſtudy thereof more pleaſant vnto him) to haue preſently before his eyes, the bodyes formed & framed of paſted paper (as I taught after the diffinitions of the eleuenth booke.) And then to drawe and deſcribe the lines and diuiſions, and ſuperficieces, according to the conſtructions of the propoſitions. In which deſcriptions if he be wary and diligent, he ſhall finde all things in theſe ſolide matters, as clere and as manifeſt vnto the eye, as were things before taught only in plaine or ſuperficial figures. And although I haue before in the twelfth boke admoniſhed the reader hereof, yet bicauſe in this boke chiefly that thing is required, I thought it ſhould not be irkeſome vnto him, againe to be put in minde thereof.

Farther this is to be noted, that in the Greke exemplars are found in this 15. booke only 5. propoſitions, which 5. are alſo only touched and ſet forthe by Hypſicles: vnto which Campane addeth 8. and ſo maketh vp the number of 13. Campane vndoubtedly although he were very well lerned, and that generally in all kinds of learning, yet aſſuredly being brought vp in a time of rudenes, when all good letters were darkned, & barberouſnes had ouerthrowne and ouerwhelmed the whole world, he was vtterly rude and ignorant in the Greke tongue, ſo that certenly he neuer redde *Euclide* in the Greke, nor (of like) tranſlated out of the Greke: but had it tranſlated out of the Arabike tonge. The Arabians were men of great ſtudy, and induſtry, and commonly great Philoſophers, notable Phiſitions, and in mathematicall Artes moſt expert, ſo that all kinds of good learning flouriſhed and raigned amongſt them in a manner only. Theſe men turned whatſoeuer good author was in the Greke tonge (of what Art and knowledge ſo euer it were) into the Arabike tonge. And frō thence were many of thē turned into the Latine, and by that meanes many Greeke authors came to the handes of the Latines, and not from the firſt fountaine the Greke tonge, wherin they were firſt written. As appeareth by many words of the Arabike tonge yet remaining in ſuch bokes: as are *Zenith*, *nadir*, *helmuayn*, *helmuariphe*, and infinite ſuche other. Which Arabians alſo in tranſlating ſuch Greke workes, were accuſtomed to adde, as they thought good, & for the fuller vnderſtanding of the author, many things: as is to be ſene in diuers authors, as, namely, in *Theodoſius de Sphera*, where you ſee in the olde tranſlation (which was vndoubtedly out of the Arabike) many propoſitions, almoſt euery third or fourth leafe. Some ſuch copye of *Euclide*, moſt likely, did *Campanus* follow, wherein he founde thoſe propoſitiōs, which he hath more & aboue thoſe which are found in the Greke ſet out by Hypſicles: and that not only in this 15. boke, but alſo in the 14. boke, wherin alſo ye finde many propoſitions more thē are founde in the Greeke, ſet out alſo by Hypſicles. Likewiſe in the bookes before, ye ſhall finde many propoſitions added, and manye inuerted, and ſet out of order farre otherwiſe, then they are placed in the Greeke examplars. *Fluſſas* alſo a diligent reſtorer of *Euclide*, a man alſo which hath well deſerued of the whole Art of Geometrie, hath added moreouer in this booke (as alſo in the former

14. boke he added 8. propoſitiõs) 9. propoſitiõs of his owne, touching the inſcription, and circumſcription of theſe bodies, very ſingular vndoubtedly, and wittye. All which, for that nothing ſhould want to the deſirous louer of knowledge, I haue faithfully with no ſmall paines turned. And whereas *Fluſſas* in the beginning of the eleuenth booke, namely, in the end of the diffinitions there ſet, putteth two diffinitions, of the inſcription and circumſcription of ſolides or corporall figures, within or about the one the other, which certainely are not to be reiected: yet for that vntill this preſent 15. boke, there is no mention made of the inſcription or circumſcription of theſe bodyes, I thought it not ſo conuenient there to place them, but to referre thē to the beginning of this 15. booke: where they are in maner of neceſſitie required to the elucidation of the Propoſitions and demonſtrations of the ſame. The diffinitions are theſe.

Diffinition. 1.

A ſolide figure, is then ſaid to be inſcribed in a ſolide figure, when the angles of the figure inſcribed touche together at one time, either the angles of the figure circumſcribed, or the ſuperficieces, or the ſides.

Diffinition. 2.

A ſolide figure is then ſaid to be circumſcribed about a ſolide figure: when together at one time either the angles, or the ſuperficieces, or the ſides of the figure circumſcribed, touch the angles of the figure inſcribed.

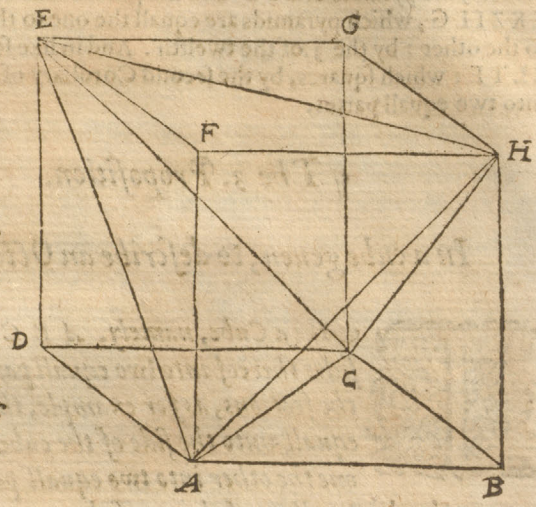
IN the fourth booke in the diffinitions of the inſcription or circumſcription of playne rectiline figures one with in or about an other, was required that all the angles of the figure inſcribed, ſhould at one time touch all the ſides of the figure circumſcribed: but in the fiue regular ſolides (to whome chefely theſe two diffinitions pertaine) for that the nomber of their angles, ſuperficieces, & ſides are not equal, one compared to an other: it is not of neceſſitie, that all the angles of the ſolide inſcribed ſhould together at one time touch either all the angles, or all the ſuperficieces, or all the ſides of the ſolide circumſcribed: but it is ſufficient, that thoſe angles of the inſcribed ſolide which touch, doe at one time together eche touch ſome one angle of the figure circumſcribed, or ſome one baſe, or ſome one ſide: ſo that if the angles of the inſcribed figure do at one time touche the angles of the figure circumſcribed, none of them may at the ſame time touche either the baſes or the ſides of the ſame circumſcribed figure: and ſo if they touch the baſes, they may touche neither angles nor ſides: and likewiſe if they touche the ſides, they may touch neither angles nor baſes. And although ſometimes all the angles of the figure inſcribed can not touch either the angles, or the baſes, or the ſides of the figure circumcribed, by reaſon the nomber of the angles, baſes or ſides of the ſaid figure circumſcribed, wanteth of the nomber of the angles of the figure inſcribed yet ſhall thoſe angles of the inſcribed figure which touch, ſo touch, that the void places left betwene the inſcribed and circumſcribed figures ſhal on euery ſide be equal and like. As ye may afrerwarde in this fiftenth booke moſt plainely perceiue.

¶ The

¶ The 1. Proposition. The 1. Probleme.

In a Cube geuen to describe † a trilater equilater Pyramis.

Vppose that the cube geuen be A B C D E F G H. In the same cube it is required to inscribe a Tetrahedron. Drawe these right lines A C, C E, A E, A H, E H, H C. Now it is manifest, that the triangles A E C, A H E, A H C, and C H E, are equilater, for their sides are the diameters of equall squares. Wherfore A E C H is a trilater equilater pyramis, or Tetrahedron, & it is inscribed in the cube geuē (by the first definition of this booke): which was required to be done.



† *In this proposition as also in all the other following, by the name of a pyramis vnderstand a tetrahedron: as I haue before admonished.*

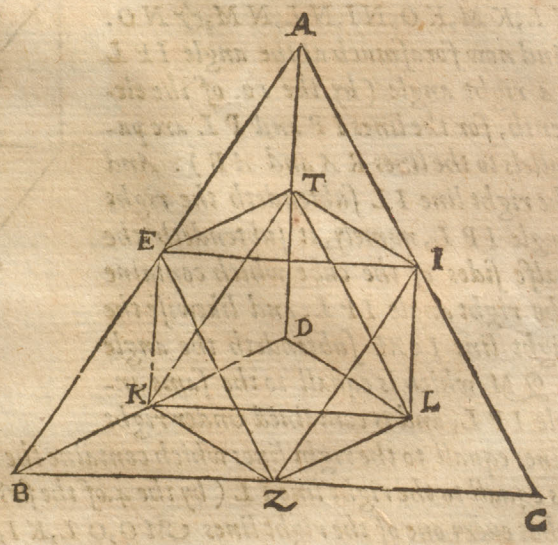
Construction.

Demonstration.

¶ The 2. Proposition. The 2. Probleme.

In a trilater equilater Pyramis geuen to describe an Octohedron.

Vppose that the trilater equilater pyramis geuē be A B C D, whose sides let be diuided into two equall partes in the pointes E, Z, I, K, L, T. And draw these 12. right lines E Z, Z I, I E, K L, L T, T K, E K, K Z, Z L, L I, I T, and T E. Which 12. right lines are, by the 4. of the first, equall. For they subtend equall plaine angles of the bases of the pyramis, and those equall angles are contained vnder equall sides, namely, vnder the halfes of the sides of the pyramis. Wherefore the triangles T K L, T L I, T I E, T E K, Z K L, Z L I, Z I E, Z E K, are equilater: and they limitate, and containe the solide T K L E Z I. Wherefore the solide T K L E Z I is an Octohedron: by the 23. definition of the eleuenth. And the angles of the same Octohedron do touch the sides of the pyramis A B C D in the pointes E, Z, I, T, K, L. Wherefore the Octohedron is inscribed in the pyramis (by the 1. definition of this booke). Wherefore in the trilater equilater pyramis geuen, is inscribed an Octohedron: which was required to be done.



Construction.

Demonstration.

⁂ A Corollary added by *Flussas*.

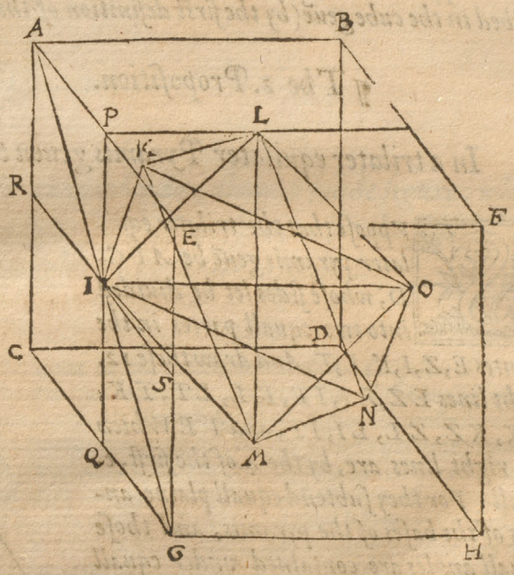
Hereby it is manifest, that a pyramis is cut into two equall partes, by euery

one of the three equall ſquares, which diuide ỹ Octohedron into two equall partes and perpendicularly. For the three diameters of thoſe ſquares do in the centre cut the one the other into two equall partes and perpendicularly, by the third Corollary of the 14. of the thirtenth, which ſquares, as for example, the ſquare E K L I, do diuide in ſunder the pyramids and the priſmes, namely, the pyramis K L T D and the priſme K L T E I A from the pyramis E K Z B, and the priſme E K Z I L G, which pyramids are equall the one to the other, and ſo alſo are the priſmes equall the one to the other: by the 3. of the twelfth. And in like ſort do the reſt of the ſquares, namely, K Z I T and Z L T E: which ſquares, by the ſecond Corollary of the 14. of the thirtenth, do diuide the Octohedron into two equall partes.

¶ The 3. Propoſition. The 3. Probleme.

In a cube geuen, to deſcribe an Octohedron.

Conſtruction. *TAke a Cube, namely, A B C D E F G H. And diuide euery one of the ſides thereof into two equall partes. And drawe right lines coupling together the ſections, as for example, theſe right lines, P Q and R S, which ſhall be equall vnto the ſide of the cube (by the 33. of the firſt) and ſhall diuide the one the other into two equall parts in the middeſt of the diameter A G in the point I (by the Corollary of the 34. of the firſt). Wherefore the point I is the centre of the baſe of the cube. And by the ſame reaſon may be found out the centres of the reſt of the baſes, which let be the pointes K, L, O, N, M. And drawe theſe right lines L I, I M, M O, O L, K I, K L, K M, K O, N I, N L, N M, & N O.*



Demonſtration. *And now foraſmuch as the angle I P L is a right angle (by the 10. of the eleuenth, for the lines I P and P L are parallels to the lines R A and A B). And the right line I L ſubtendeth the right angle I P L, namely, it ſubtendeth the halfe ſides of the cube which containe the right angle I P L, and likewiſe the right line I M ſubtendeth the angle I Q M which is equall to the ſame angle I P L, and is contained vnder right lines equall to the right lines which containe the angle I P L. Wherefore the right line I M is equall to the right line I L (by the 4. of the firſt). And by the ſame reaſon may we proue, that euery one of the right lines M O, O L, K I, K L, K M, K O, N I, N L, N M, and N O, which ſubtend angles equall to the ſelfe ſame angle I P L, and are cõtained vnder ſides equall to the ſides which containe the angle I P L, are equall to the right line I L. Wherefore the triangles K L I, K L O, K M I, K M O, and N L I, N L O, N M I, N M O, are equilater and equall: and they containe the ſolide I K L O N M. Wherefore I K L O N M is an Octohedron, by the 23. definition of the eleuenth. And foraſmuch as the angles thereof do altogether in the pointes I, K, L, O, N, M, touch the baſes of the cube which containeth it, it followeth that the Octohedron is inſcribed in the cube (by the firſt definition of this booke). Wherefore in the cube geuen, is deſcribed an Octohedron: which was required to be done.*

¶ A Corollary

¶ A Corollary added by *Flussas.*

Hereby it is manifeſt, that right lines ioyning together the centres of the oppoſite baſes of the cube, do cut the one the other into two equall parts, and perpendicularly, in the centre of the cube, or in the centre of the Sphere which containeth the cube.

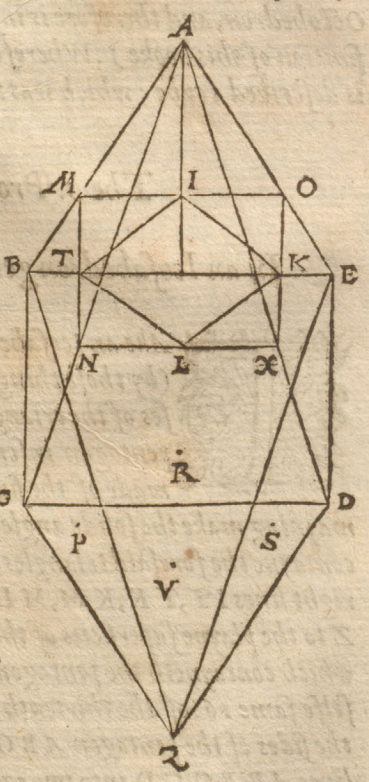
For foraſmuch as the right lines L M and I O which knit together the centres of the oppoſite baſes of the cube, do alſo knit together the oppoſite angles of the Octohedron inſcribed in the cube, it followeth (by the 3. Corollary of the 14. of the thirteenth) that thoſe lines L M and I O, do cut the one the other into two equall partes in a point. But the diameters of the cube do alſo cut the one the other into two equall partes, by the 39. of the eleuenth. Wherfore that point ſhall be the centre of the ſphere which containeth the cube. For making that point the centre, and the ſpace ſome one of the ſemidiameters, deſcribe a ſphere, and it ſhall paſſe by the angles of the cube: and likewiſe making the ſame point the centre, and the ſpace halfe of the line L M, deſcribe a ſphere, and it ſhall alſo paſſe by the angles of the Octohedron.

¶ *The 4. Propoſition.* *The 4. Probleme.*

In an Octohedron geuen, to deſcribe a Cube.

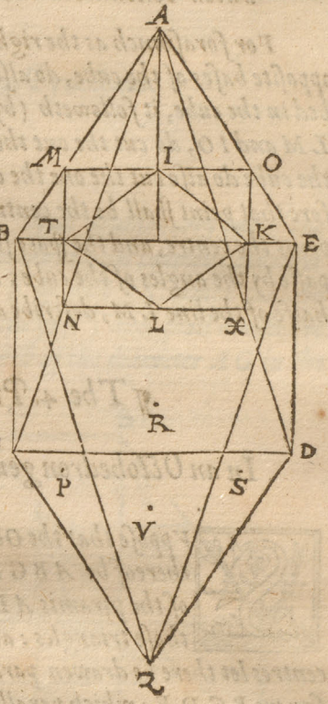
Vppoſe that the Octohedron geuen be A B G D E Z. And let the two pyramids thereof be A B G D E, and Z B G D E. And take the centres of the triangles of the pyramis A B G D E, that is, take the centres of the circles which containe thoſe triangles: and let thoſe centres be the pointes T, I, K, L. And by thoſe centres let there be drawen parallel lines to the ſides of the ſquare B G D E: which parallel right lines let be M T N, N L X, X K O, & O I M. And foraſmuch as thoſe parallel right lines do (by the 2. of the ſixth) cut the equall right lines A B, A G, A D, and A E, proportionally, therfore they concurre in the pointes M, N, X, O. Wherefore the right lines M N, N X, X O, and O M, which ſubtend equall angles ſet at the point A, & contained vnder equall right lines, are equall (by the 4. of the firſt). And moreouer, ſeing that they are parallels vnto the lines B G, G D, D E, E B, which make a ſquare, therefore M N X O is alſo a ſquare, by the 10. of the eleuenth. Wherefore alſo, by the 15. of the ſame, the ſquare M N X O is parallel to the ſquare B G D E. For all the right lines touch the one the other in the pointes of their ſections. From the centres T, I, K, L, drawe theſe right lines T I, I K, K L, L T. And drawe the right line A I C. And foraſmuch as I is the centre of the equilater triangle A B E, therefore the right line A I being extended, cutteth the right line B E into two equall partes (by the Corollary of the 12. of the thirteenth). And foraſmuch as M O is a parallel to B E, therefore the triangle A I O is like to the whole triangle A C E (by the Corollary of the 2. of the ſixth). And the right line M O is diuided into two equall partes in the point I (by the 4. of the ſixth). And by the ſame reaſon may we proue, that the right lines M N, N X, X O, are diuided into two equall partes

Conſtruction.



Demonſtration.

partes in the pointes T,L,K. Wherefore also againe, the bases T I,I K,K L, L T, which subtend the angles set at the pointes M,O,X,N, which angles are right angles, and are contained vnder equall sides, those bases, I say, are equall. And forasmuch as T I M is an Isosceles triangle, therefore the angles set at the base, namely, the angles M T I and M I T, are equal (by the 5. of the first). But the angle M is a right angle: wherefore eche of the angles M I T and M T I, is the halfe of a right angle. And by the same reason the angles O I K & O K I, are equall. Wherefore the angle remayning, namely, T I K, is a right angle (by the 13. of the first). For the right lines T I and I K are set vpon the line M O. And by the same reason may the rest of the angles, namely, I K L, K L T, L T I, be proued right angles, and they are in one and the self same plaine superficies, namely, M N X O (by the 7. of the eleuēth). Wherefore the right lines which ioyne together the centres of the plaine superficiall triangles which make the solide angle A, do make the square I T K L. And by the same reason may be proued, that the plaine superficiall triangles of the rest of the fiue solide angles of the Octohedron set at the pointes B, G, Z, D, E, do in the centres of their bases receaue squares, So that there are in number sixe squares, for euery Octohedron hath sixe solide angles: and those squares are equall: for their sides do containe equall angles of inclinations contained vnder equall sides, namely, vnder those sides which are drawen from the centre to the side of the equall triangles (by the 2. Corollary of the 18. of the thirtenth). Wherefore I T K L R P V S is a cube (by the 21. definition of the eleuenth) and hath his angles in the centres of the bases of the Octohedron, and therefore is inscribed in it (by the first definition of this booke). Wherefore in an Octohedron geuen is described a cube: which was required to be done.



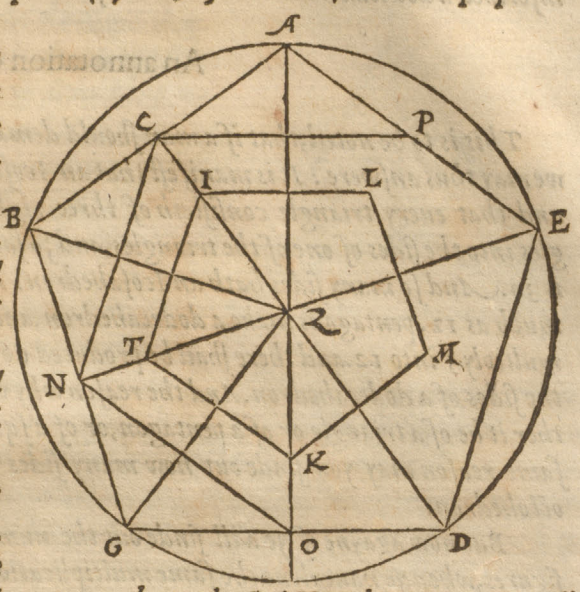
The 5. Proposition. The 5. Probleme.

In an Icosahedron geuen, to describe a Dodecahedron.

Construction. TAke an Icosahedron, one of whose solide angles let be Z. Now forasmuch as (by those thinges which haue bene proued in the 16. of the thirtenth) the bases of the triangles which contayne the angle of the Icosahedron doo make a pentagon inscribed in a circle, let that pentagon be A B G D E, which is made of the fiue bases of the triangles, whose playne superficiall angles remayning make the solide angle geuen, namely, Z. And take the centres of the circles which contayne the foresaid triangles, which centers let be the poyntes I,T,K,M,L: and draw these right lines I T, T K, K M, M L, L I. Now then a perpendicular line drawne from the poynt Z to the playne superficies of the pentagon A B G D E, shall fall vpon the centre of the circle which contayneth the pentagon A B G D E (by those thinges which haue bene proued in the selfe same 16. of the thirtenth). Moreouer perpendicular lines drawne from the centre to the sides of the pentagon A B G D E shall in the poyntes C, N, O where they fall cut the right lines A B, B G, G D into two equall partes (by the 3. of the third). Draw these right lines C N and N O. And forasmuch as the angles C B N and N G O are equall, and are contained vnder equall sides, therefore the base C N is equall to the base N O (by the 4. of the first). Moreouer

Demonstration.

ouer perpendicular lines drawne from the poynt Z to the baſes of the pentagon A B G D E, ſhall likewiſe cutte the baſes into two equall partes (by the 3. of the third). For the perpendiculars paſſe by the centre (by the corollary of the 12. of the thirtēth): Wherfore thoſe perpendicular lines ſhall fal vpō the points C, N, O. And now foraſmuch as the right lines Z I, I C are equall to the right lines Z T, T N, & alſo to the right lines Z K, K O (by reaſon of the likenes of the equall triangles): therefore the line I T is a parallell to the line C N, and ſo alſo is the line T K to the line N O (by the 2. of the ſixt). Wherfore the angles I T K, and C N O are equal (by the 11. of the eleuenth). Agayne foraſmuch as the triangles C B N, and N G O are Iſoſcels triangles, therefore the angles B C N and B N C are equall (by the 5. of the firſt). And by the ſame reaſon the angles G N O, and G O N are equall. And moreouer the angles B C N and B N C are equall to the angles G N O, and G O N, for that the triangles C B N and N G O are equall and like. But the three angles B N C, C N O, O N G, are equall to two right angles (by the 13. of the firſt): for that vpon the right line B G are ſet the right lines C N & O N. And the three angles of the triangle C B N, namely, the angles B N C, B C N or G N O (for the angle G N O is equall to the angle B C N as it hath bene proued) and N B C are alſo equall to two right angles (by the 32. of the firſt). Wherefore taking away the angles B N C & G N O, the angle remayning, namely, C N O is equall to the angle remayning, namely, to C B N. Wherfore alſo the angle I T K (which is proued to be equall to the angle C N O) is equall to the angle C B N. Wherefore I T K is the angle of a pentagon. And by the ſame reaſon may be proued that the reſt of the angles, namely the angles T I L, I L M, L M K, M K T, are equall to the reſt of the angles, namely to B A E, A E D, E D G, D G B. Wherefore I T K M L is an equilater and equiangle pentagon (by the 4. of the firſt) For the equall baſes of the pentagon I T K M L doo ſubtend equall angles ſet at the point Z, and comprehended vnder equall ſides. Moreouer it is manifeſt that the pentagon I T K M L is in one and the ſelfe ſame playne ſuperficies. For foraſmuch as the angles O N C and N C P are in one and the ſelfe ſame playne ſuperficies, namely in the ſuperficies A B G D E: But vnto the ſame playne ſuperficies the playne ſuperficieces of the angles K T I and T I L are parallels (by the 15. of the eleuenth). And the triangles K T I and T I L concurre: wherefore they are in one and the ſelfe ſame playne ſuperficies (by the corollary of the 16. of the eleuēth). And by the ſame reaſō ſo may we proue that the triangles I L M, L M K, M K T are in the ſelfe ſame playne ſuperficies wherein are the triangles K T I and T I L. Wherefore the pentagon I T K M L is in one and the ſelfe ſame playne ſuperficies. Wherefore the ſolide angle of the Icoſahedron, namely the ſolide angle at the poynt Z ſubtendeth an equilater and equiangle pentagon plaine ſuperficies, which pentagon hath his plaine ſuperficiall angles in the centres of the triangles which make the ſolide angle Z. And in like ſort may we proue that the other eleuen ſolide angles of the Icoſahedron, eche of which eleuen ſolide angles are equall and like to the ſolide angle Z (by the 16. of the thirtenth) are ſubtended vnto pentagons equall, and like, and in like ſort ſet to the pentagon I T K M L. And foraſmuch as in thoſe pētagons the right lines, which ioyne together the centers of the baſes, are common ſides, it followeth that thoſe 12. pentagons include a ſolide which ſolide is therefore a dodecahedron (by the 24. diffinition of the eleuenth): and is, by the firſt diffinition of this booke, deſcribed in the Icoſahedron, fiue



ſides whereof are ſet vpon the pentagon A B G D E. Wherefore in an Icoſahedron geuen is inſcribed a dodecahearon: which was required to be done.

An annotation of *Hypſicles*.

This is to be noted, that if a man ſhould demaund how many ſides an Icoſahedron hath, we may thus anſwere: It is manifeſt that an Icoſahedron is contayned vnder 20. triangles, and that euery triangle conſiſteth of three right lines. Now then multiply the 20. triangles into the ſides of one of the triangles, and ſo ſhall there be produced 60. the halfe of which is 30. And ſo many ſides hath an Icoſahedron. And in like ſort in a dodecahedron, foraſmuch as 12. pentagons make a dodecahedron, and euery pentagon contayneth 5. right lines, multiply 5 into 12. and there ſhall be produced 60. the halfe of which is 30. And ſo many are the ſides of a dodecahedron. And the reaſon why we take the halfe, is, for that euery ſide whether it be of a triangle or of a pentagon, or of a ſquare as in a cube, is taken twiſe. And by the ſame reaſon may you finde out how many ſides are in a cube, and in a pyrmis, and in an octohedron.

But now agayne if ye will finde out the number of the angles of euery one of the ſolide figures, when ye haue done the ſame multiplication that ye did before, diuide the ſame ſides, by the number of the plaine ſuperficieces which comprehend one of the angles of the ſolides As for example, foraſmuch as 5. triangles contayne the ſolide angle of an Icoſahedron, diuide 60. by 5. and there will come forth 12. and ſo many ſolide angles hath an Icoſahedron. In a dodecahedron, foraſmuch as three pentagons comprehend an angle, diuide 60. by 3. and there will come forth 20: and ſo many are the angles of a dodecahedron. And by the ſame reaſon may you finde out how many angles are in eche of the reſt of the ſolide figures.

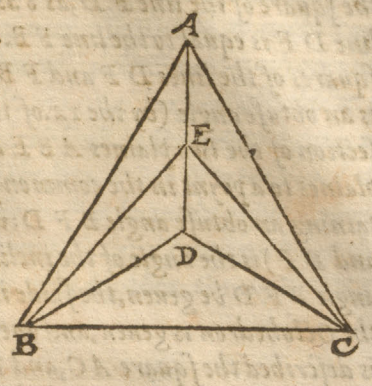
That which here followeth concerning the inclination of the plaines of the fiue ſolides, was before tought (though not altoge ther after the ſame maner) out of Fluſſas in the latter ende of the 13. booke.

If it be required to be knowne, how one of the plaines of any of the fiue ſolides being geuen, there may be found out the inclination of the ſayd plaines the one to the other, which contayne eche of the ſolides. This (as ſayth Iſidorus *our greate maſter) is found out after this maner. It is manifeſt that in a cube, the plaines which contayne it, doo cut the one the other by a right angle. But in a Tetrahedron, one of the triangles being geuen, let the endes of one of the ſides of the ſayd triangle be the centers, and let the ſpace be the perpendicular line drawne from the toppe of the triangle to the baſe, and deſcribe circumferences of a circle, which ſhall cutte the one the other: and from the interſection to the centers draw right lines, which ſhall containe the inclination of the plaines cõtayning the Tetrahedron. In an Octohedron, take one of the ſides of the triangle thereof, and vpon it deſcribe a ſquare, and draw the diagonall line, and making the centres, the endes of the diagonall line, and the ſpace likewiſe the perpendicular line drawne from the toppe of the triangle to the baſe, deſcribe circumferences: and agayne from the common ſection to the centres draw right lines, and they ſhall contayne the inclination ſought for. In an Icoſahedron, vpon the ſide of one of the triangles thereof, deſcribe a pentagon, and draw the line which ſubtendeth one of the angles of the ſayd pentagon, and making the centres the endes of that line, and the ſpace the perpendicular line of the triangle, deſcribe circumferences: and draw from the common interſection of the circumferences, vnto the centres right lines: and they ſhall contayne likewiſe the inclination of the plaines of the icoſahedron. In a dodecahedron, take one of the pentagons, and draw likewiſe the line which ſubtendeth one of the angles of the pentagon, and making the centres the endes of that line, and the ſpace, the perpendicular line drawne from the ſection into two equall partes of that line to the ſide of the pentagon, which is parallel vnto it, deſcribe circumferences: and from the point of the interſection of the circumferences draw vnto the centres right lines: and they ſhall alſo containe the inclination of the plaines of the dodecahedron. Thus did this moſt ſingular learned man reaſon, thinking the demonſtration in euery one of them to be plaine and cleare. But to make the demonſtration of them mani-*

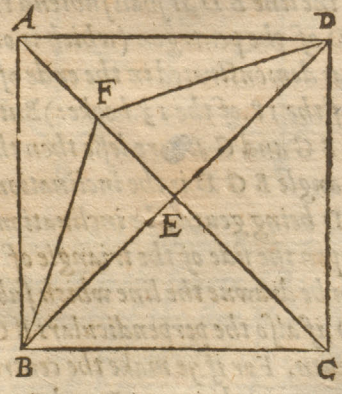
feſt

feſt, I think it good to declare and make open his wordes, and firſt in a Tetrahedron.

Suppoſe that there be a Pyramis or Tetrahedrō ABCD cōtained vnder 4. equilater triangles: & let the toppe therof be the point D. And (by the 10. of the firſt) diuide the ſide AD into two equall parts in the point E: & draw the lines BE and EC. And foraſmuch as ADB and ADC are equilater triangles, and the line AD is diuided into two equall partes, therefore the lines BE and EC fall perpendicularly vpon the line AD, by the 8. of the firſt. Now I ſay that the angle BEC is an acute angle. For foraſmuch as the line AC is double to the line AE (for by conſtruction the line AD, which is equal to the line AC, is diuided into two equall partes in the point E): therefore the ſquare of the line AC is quadruple to the ſquare of the line AE (by the corollary of the 20. of the ſixt). But the ſquare of the line AC is equall to the ſquares of the lines AE and EC (by the 47. of the firſt): and the ſquare of the line AC is to the ſquare of the line CE (ſeſquitertia) as 4. to 3: (for the ſquare of the line AC is proued quadruple to the ſquare of the line AE): wherefore the ſquare of the line BC (which is equall to the ſquare of the line AC) is leſſe then the ſquares of the two perpendiculars BE & EC (for it is vnto them in ſubſeſquialter proportiō, namely, as 4. to 6. or 2. to 3.) Wherefore (by the 13. of the ſecond) the angle BEC is an acute angle. Now foraſmuch as the line AD is the common interſection of the two plaines ABD, and ADC, and in either of thoſe plaines to one point of the common ſection are drawne perpendicular lines BE and EC which containe an acute angle BEC, therefore (by the 5. diffinition of the eleuenth) the angle BEC is the inclination of the plaines, and it is geuen. For the line BC, which is the ſide of the triangle, being geuen, and any one of the lines BE or EC, which is the perpendicular of the equilater triangle, being alſo geuen: make the centres the poyntes B and C, that is, the endes of one of the ſides, and the ſpace the perpendicular of the triangle, and deſcribe circumferences, and they ſhall cutte the one the other in the poynt E. And from the poynt E draw to the centres B and C right lines, and they ſhall containe the inclination of the plaines: and this is it which Iſidorus before ſayd. And now that making the centres the poynts B and C, and the ſpace the perpendicular of the triangle, the circles deſcribed ſhall cutte the one the other, it is manifeſt, for either of the lines BE and EC is greater then halfe of the line BC. Now if the centers were the poynts B and C, and the ſpace the halfe of the line BC, the circles deſcribed ſhall touch the one the other. But if the ſpace be leſſe then the halfe, they ſhal neither touch nor cut the one the other: but if it be greater, they ſhall vndoubtedly cut.



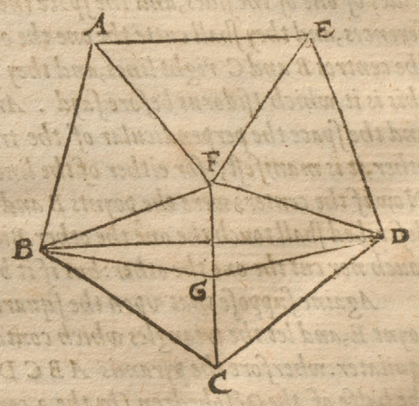
Againe ſuppoſe that vpon the ſquare ABCD be ſet a pyramis, hauing his altitude the poynt E, and let the triangles which containe it, be equilater: wherfore the pyramis ABCDE ſhalbe the halfe of the Octohedron (by the 2 corollary of the 14. of the thirteenth.) Deuide by the 10. of the firſt) one ſide of one of the triangles, namely, the line AE, into two equal partes in the poynt F: and draw the lines BF and DF: wherefore the lines BF and DF are equal and fal perpendicularly vpon the line AE (by the 4. and 8. of the firſt.) Then I ſay that the angle BFD, is an obtuſe angle. For draw the line BD. And foraſmuch as AC is a ſquare, and the diameter is BD: therefore the ſquare of the line BD is double to the ſquare of the line DA) by the 47. of the firſt.) But the ſquare of the line DA is to the ſquare of the line



The reaſon of this you ſhall moſt plainely ſee in that which is added out of Proclus after the 22. propoſition of the firſt booke.

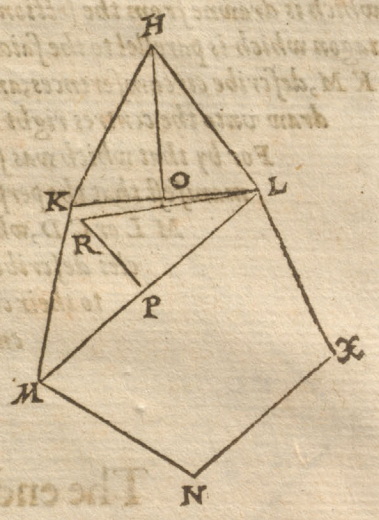
D F,as 4.is to 3.) as was in the former proued.) Wherefore the ſquare of the line D B is to the ſquare of the line F D,as 8.is to 3.(namely,as 2.to 1.and 4.to 3.added together),but the line D F is equal to the line F B. Wherefore the ſquare of the line D B,is greater then the ſquares of the lines D F and F B (for it is to them,as 8.is to 6.) Wherfore the angle B F D, is an obtuſe angle (by the 12.of the ſecond.) And foraſmuche as the line A E is the common ſection of the two plaines A B E and A D E cutting the one the other,and in either of thoſe plaines to a poynt in the common ſection are drawne perpendicular lines,B F and D F,containing an obtuſe angle B F D:wherfore the angle B F D (contained of the right lines B F and D F)is the angle of the inclination of the plaines A B E and A D E. If therefore the angle B F D be geuen,the ſaide inclination alſo is geuen. For foraſmuch as the triangle of the Octohedron is geuen,and one of the ſides of the Octohedron is the line A D, and vpon it is deſcribed the ſquare A C,and B D the diameter of that ſquare being geuen, and the lines B F and F D are the perpendiculars of that triangle: wherefore alſo the angle B F D is geuen.Now then if vpon the ſide of the triangle be deſcribed a ſquare:as the ſquare A C, and the diameter B D be drawne, if alſo making the centres the poyntes B and D,and the ſpace, the ſaid perpendicular of the triangle,we deſcribe circles, they ſhall cut the one the other in the poynt F.And the right lines which are drawne from the centres to the poynt F ſhal containe that inclination,which is comprehended vnder the angle B F D, which is the angle of the inclination of thoſe plaines. And it is manifeſt that either of the lines B F and F D,is greater then the halfe of the line. For for that,by the demonſtration, it was proued that the ſquare of the line B D is to the ſquare of the line F D,as 8.is to 3:therfore the ſquare of half the line B D,is to the ſquare of the line F D,as 2.is to thre (for the ſquare of halfe the line B D is the fourth part of the ſquare of the whole line B D,by the 4.of the ſecond).Wherefore either of the lines B F and F D,is greater then the line B D: wherfore the circles which are deſcribed by thoſe lines B F and F D,and hauing their centres the poynts B and D ſhall cut the one the other.And thus much touching the octohedron.

As touching the Icoſahedron, ſuppoſe an equilater pẽtagon ABCDE, & vpon it let there be ſet a pyramis hauing his toppe the poynt F: and let the triangles which cõtaine it, be equilater.Now thẽ the pyramis ABCDEF,ſhal be a part of the Icoſahedrõ.Let F C one ſide of one of the triangles be deuided into two equal partes in the poynt G.And draw the lines BG & GD,which ſhal be equal & fal perpẽdicularly vpon the line FC (as it is eaſie to ſe by the demõſtratiõ of the former).Thẽ I ſay that ỹ angle BGD is an obtuſe angle: which thing is manifeſt. For drawing the line B D,it ſhall ſubtend the obtuſe angle B C D of the pentagon (which is obtuſe,by that which was demonſtrated in the ende of the firſt corollary of the 18.of the 13.booke:)But the angle B G D is greater then the angle B C D, for the lines B G and G D are leſſe then the lines B C and C D: wherefore likewiſe as in the former the angle B G D is the inclination of the triangles B F C,and C F D.Wherfore the angle B G D being geuen,the inclination alſo of the plaines of the Icoſahedron ſhall be geuen. For if vpon the ſide of the triangle of the Icoſahedron be deſcribed an equilater pentagon, and then be drawne the line which ſubtendeth two ſides of the pentagon,as in this figure the line B D,if alſo the perpendiculars B G and G D of the triangles be drawne,the angle BGD ſhalbe geuen. For if ye make the centres the endes of the line which ſubtendeth two ſides of the pentagon,as the poynts B and D,and the ſpace the perpendicular of the triangle, and ſo deſcribe circles,they ſhall cut the one the other in the poynt G,and from the poynt of the in-

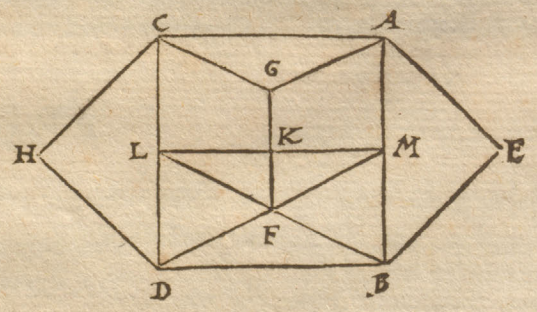


terſection

terſection G, drawe vnto the centres B and D right lines, and they ſhal containe the angle of the inclination B G D. And it is manifeſt, by the deſcription of the figure, that either of the lines B G and G D is greater then the line B D. Which thing may alſo thus be proued. Suppoſe an equilater H K L, and vpon K L (one of the ſides thereof) deſcribe an equilater pentagon K M N X L, and draw the line M L. And diuide (by the 10. of the firſt) the ſide K L into two equal parts in the poynt O, & draw the line H O, which ſhall be the perpendicular of the triangle H K L (by the 8. of the firſt.) Then I ſay that the line H O is greater then half of the line M L, which ſubtendeth the inclination of the plaines. For from the poynt K draw (by the 12. of the firſt) vnto the line M L a perpendicular line K P: and foraſmuche as the angle K L P is greater then the third part of a right angle, that is, then the angle K H O (For the angle K L M is two fiueth partes of a right angle, by the 4. of the firſt, and by the aſsumpt put after the firſt corollary of the 18. of the thirteenth booke, and the angle K H O is one third part of one right angle, for the whole angle K H L, wherof the angle K H O is the half, by the 4. of the firſt, is one third part of two right angles, by the 32. of the firſt:) vpon the line M L, and at the poynt L put vnto the angle K H O an equal angle P L R (by the 23. of the firſt.) Wherfore the triangles P L R & O H K, ſhalbe equiangle, by the corollary of the 32. of the firſt. Wherefore alſo the line P L ſhalbe the perpendicular of the equilater triangle deſcribed vpon the line R L. Wherefore (by the corollarye added by Fluſſas after the 12. propoſition of the thirtenth booke) the line R L is in power ſeſquitercia, that is, as 4. is to 3. to the perpendicular L P. But the line K L is greater then the line L R (by the 19. of the firſt. Wherfore y̆ ſquare of the line K-L hath to the ſquare of the line L P a greater proportion thē hath 4. to 3: but it hath to the ſquare of the line H O that proportion that 4. hath to 3. Wherfore the line K L hath to the line L P a greater proportion then it hath to the line H O. Wherfore the line H O is greater then the line L P, by the 10. of the fifth.



As concerning a Dodecahedron. Take one of the ſquares of the cube wheron the Dodecahedron is deſcribed (by the 17. of the thirteenth): and let the ſame be A B C D: and let the two plaines of the Dodecahedron ſet vpon it be A E B F G, and G F D H C. Then I ſay that here alſo is geuen the inclination of the two pentagons. Diuide (by the 10. of the firſt) the ſide F G into two equal partes in the poynt K. And from the poynt K draw vnto the line F G in either of the plaines A E B-F G and G F D H C perpendicular lines K L and K M (by the 11. of the firſt.) And draw the line M L. Firſt I ſay that the angle M K L is an obtuſe angle. For, by the diſcourſe of the demonſtration of the 17. propoſition of the 13. boke, where is taught the deſcription of the Dodecahedron, it is manifeſt, that the line drawne perpendicularly from the poynt K to the ſquare A B C D, is equal to halfe the ſide of the pentagon. Wherefore it is leſſe then halfe of the line M L. Wherefore the angle M K L is an obtuſe angle. Moreouer by the former diſcourſe of the 17. propoſition of the 13. booke, it was manifeſt that the ſquare of the line K L is equal to the ſquare of halfe the ſide of the cube, and to the ſquare of halfe the ſide of the pentagon. And foraſmuche as



the lines K L and K M are equal, and are eche greater then halfe of the line M L: wherfore the angle M K L being geuen, there ſhall alſo be geuen the inclination of the two plaines of the Dodecahedron. For foraſmuch as the ſide of the ſquare A B C D ſubtendeth two ſides of the pentagon geuen, the pentagon alſo is geuen, and therefore alſo is geuen the line M L. But there is alſo geuen either of the lines M K and K L: for they are drawne perpendicularly frō the ſection into two equal partes of the line A B, which ſubtendeth two ſides of the pentagon vnto the ſide of the pentagon, which is a parallel vnto the line A B: namely, to the ſide F G. Wherfore there is geuen the angle L K M, which is the angle of the inclination ſought for. And now touching Iſidorus *wordes, he ſayeth, that the pentagon being geuen, we muſt draw the line which ſubtendeth two ſides of the pentagon, which line is equal to the ſide of the cube: and making the centres the endes of that line, and the ſpace the perpendicular line, which is drawne from the ſection of the ſame line into two equal parts to the ſide of the pentagon which is parellel to the ſaid line, as in the former deſcription the line K L, or the line K M, deſcribe circumferences, and from the poynt of the interſection of the circumferences draw vnto the centres right lines which ſhall containe the angle of the inclination.*

For by that which was ſayd before, namely, touching the Icoſahedron, it is manifeſt that the perpendicular K L, is greater then halfe of the line M L or C D, which is equal vnto it. And therefore the circles deſcribed by thoſe perpendiculars, and hauing to their centres the end of the line C D, ſhall cut the one the other, as was before proued.

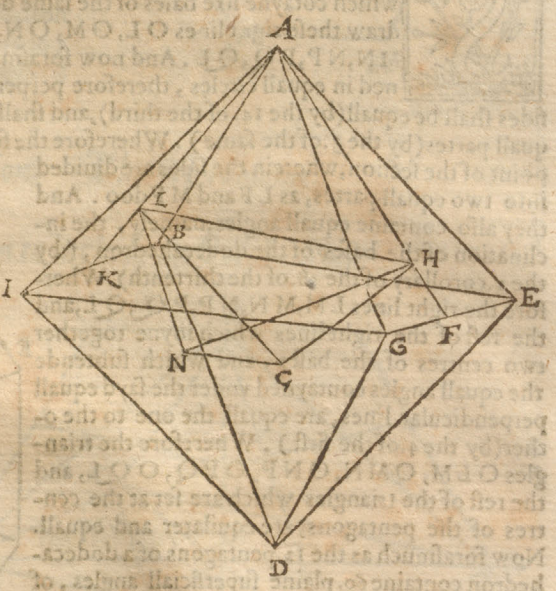
The ende of the fiuetenth Booke of Euclides Elementes after *Hypſicles.*

¶ *The 6. Propoſition.* *The 6. Probleme.*

In an Octohedron geuen, to inſcribe a trilater equilater Pyramis.

Conſtruction.

SVppoſe that the Octohedron wherein the Tetrahedron is required to be inſcribed, be A B G D E I. Take fower baſes of the Octohedron, that is, three which cloſe in the loweſt triangle E G D, namely, A E G, B E D, I G D: and let the fourth be A I B, which is oppoſite to the loweſt triangle before put, namely, to E G D. And take the centres of thoſe fower baſes, which let be the pointes H, C, N, L. And vpon the triangle H C N erecte a pyramis H C N L. Now foraſmuch as theſe two baſes of the Octohedron, namely, A G E and A B I are ſet vpon the right lines E G and B I which are oppoſite the one to the other, in the ſquare G E B I of the Octohedron, from the point A drawe by the centres of the baſes, namely, by the centres H, L, perpendicular lines A H F, A L K, cutting the lines E G and B I into two equall partes in the pointes F, K (by the Corollary of the 12. of the thirteenth). Wherfore a right line drawen frō the point F to the point K, ſhall be a parallel and equall to the ſides of the Octohedron, namely, to E B and G I (by the 33. of the firſt). And the right line H L which cutteth the equall ſides A F, A K, proportionally (for A H and A L are drawen from the centres of equall circles to the circumferences) is a parallel to the right line F K (by the 2. of the ſixth) and alſo to the ſides of the Octohedron, namely, to E B and I G (by the 9. of the eleuenth). Wherefore as the line A F is to the line A H, ſo is the line F K to the line H L (by the 4. of the ſixth): For the triangles A F K and A H L are like (by the Corollary of the 2. of the ſixth). But the line A F is in ſeſquialter proportion to the line A H: (for the ſide E G maketh H F the halfe of the right line A H, by the Corollary of the 12. of the thirteenth). Wherfore F K or G I the ſide of the Octohedron, is ſeſquialter to the right line H L. And by the ſame reaſon may we proue that the ſides of the Octohedron are ſeſquialter to the reſt of the right lines which make the pyramis H N C L, namely, to the right lines H N, N C, C L, L N, and C H: wherefore thoſe right lines are equall, and therefore the triangles which are deſcribed of them, namely, the triangles H C N, H N L, N C L, and C H L, which make the pyramis H N C L, are equall and equilater. And foraſmuch as the angles of the ſame pyramis, namely, the angles H, C, N, L, do end in the centres of the baſes of the Octohedron, therefore it is inſcribed in the ſame Octohedron, by the firſt definition of this booke. Wherefore in an Octohedron geuen, is inſcribed a trilater equilater pyramis: which was required to be done.



Demonſtration.

⁂ A Corollary.

The baſes of a Pyramis inſcribed in an Octohedron, are parallels to the baſes of the Octohedron. For foraſmuch as the ſides of the baſes of the Pyramis touching the one the other, are parallels to the ſides of the Octohedron which alſo touch the one the other, as for example, H L was proued to be a parallel to G I, and L C to D I, therefore, by the 15. of the eleuenth, the plaine ſuperficies which is drawen by the lines H L and L C, is a parallel to the plaine ſuperficies drawen by the lines G I and D I. And ſo likewiſe of the reſt.

⁂ Second Corollary.

A right line ioyning together the centres of the oppoſite baſes of the Octohedron, is ſeſquialter to the perpendicular line drawen from the angle of the inſcribed

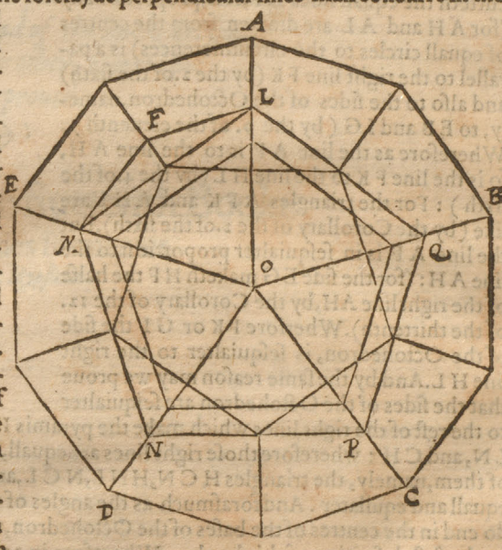
ſcribed pyramis to the baſe thereof. For foraſmuch as the pyramis and the cube which containeth it do in the ſelfe ſame pointes end their angles (by the 1. of this booke): therefore they ſhall both be incloſed in one and the ſelfe ſame Octohedron (by the 4. of this booke). But the diameter of the cube ioyneth together the centres of the oppoſite baſes of the Octohedron, and therefore is the diameter of the Sphere which containeth the cube and the pyramis inſcribed in the cube (by the 13. and 14. of the thirtenth): which diameter is ſeſquialter to the perpendicular which is drawen from the angle of the pyramis to the baſe thereof: for the line which is drawen from the centre of the ſphere to the baſe of the pyramis, is the ſixth part of the diameter (by the 3. Corollary of the 13. of the thirtenth). Wherefore of what partes the diameter containeth ſixe, of the ſame partes the perpendicular containeth fower.

¶ The 7. Propoſition. The 7. Probleme.

In a dodecahedron geuen, to inſcribe an Icoſahedron.

Conſtruction. Vppoſe that the dodecahedron geuen, be A B C D E. And let the centres of the circles which cõtayne ſixe baſes of the ſame dodecahedron be the points L, M, N, P, Q, O. And draw theſe right lines O L, O M, O N, O P, O Q, and moreouer theſe right lines L M, M N, N P, P Q, Q L. And now foraſmuch as equall and equilater pentagons are contayned in equall circles, therefore perpendicular lines drawne from their centres to the ſides ſhall be equall (by the 14. of the third), and ſhall diuide the ſides of the dodecahedron into two equall partes (by the 3. of the ſame). Wherefore the foreſayde perpendicular lines ſhall concurre in the

Demonſtration. point of the ſection, wherein the ſides are diuided into two equall partes, as L F and M F doo. And they alſo containe equall angles, namely, the inclination of the baſes of the dodecahedron, (by the 2. corollary of the 18. of the thirtenth). Wherfore the right lines L M, M N, N P, P Q, Q L, and the reſt of the right lines which ioyne together two centres of the baſes, and which ſubtende the equall angles contayned vnder the ſayd equall perpendicular lines, are equall the one to the other (by the 4. of the firſt). Wherefore the triangles O L M, O M N, O N P, O P Q, O Q L, and the reſt of the triangles which are ſet at the centres of the pentagons, are equilater and equall. Now foraſmuch as the 12. pentagons of a dodecahedron containe 60. plaine ſuperficiall angles, of which 60. euery thre make one ſolide angle of the dodecahedron, it followeth that a dodecahedron hath 20. ſolide angles: but eche of thoſe ſolide angles is ſubtẽded of ech of the triangles of the Icoſahedron, namely, of ech of thoſe triangles which ioyne together the centres of the pentagõs which make the ſolide angle, as we haue before proued. Wherefore the 20. equall and equilater triangles which ſubtende the 20. ſolide angles of the dodecahedron, and haue their ſides which are drawne from the centres of the pentagons common, doo make an Icoſahedron (by the 25. diffinition of the eleuenth): and it is inſcribed in the dodecahedron geuen (by the firſt diffinition of this booke) for that the angles thereof doo all at one time touch the baſes of the dodecahedron. Wherefore in a dodecahedron geuen, is inſcribed an Icoſahedron: which was required to be done.

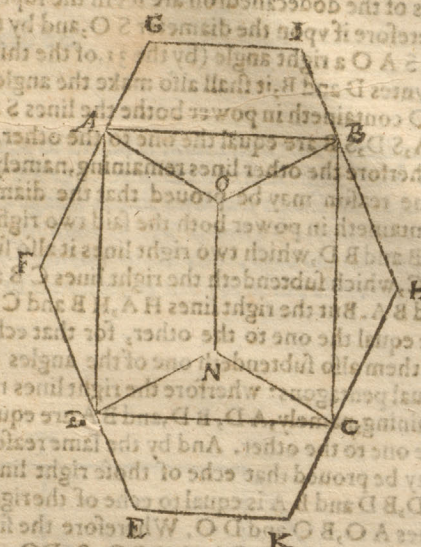


¶ The 8. Propoſition. The 8. Probleme.

In a dodecahedron geuen, to include a cube.

Conſtruction. DEſcribe (by the 17. of the thirtenth) a dodecahedron. And by the ſame, take the 12. ſides of the cube, eche of which ſubtend one angle of eche of the 12. baſes of the dodecahedron: for the ſide of the cube ſubtendeth the angle of the pentagon of the dodecahedron, by the 2. corollary of the 17. of the thirtenth. If therefore in the dodecahedron deſcribed (by the ſelfe ſame 17. propoſition) we draw the 12. right lines ſubtended vnder the foreſayd 12. angles, and ending in 8. angles of the dodecahedron, and concurring

ring togeather in ſuch ſort that they be in like ſort ſituate, as it was plainely proued in that propoſition, then ſhall it be manifeſt, that the right lines drawne in this dodecahedron from the foreſayd 8. angles thereof doo make the foreſayd cube, which therefore is included in the dodecahedron; for that the ſides of the cube are drawne in the ſides of the dodecahedron, and the angles of the ſame cube are ſet in the angles of the ſaid dodecahedron. As for example, take 4. pentagons of a dodecahedron, namely A G I B O, B H C N O, C K E D N and D F A O N. And draw theſe right lines A B, B G, C D, D A. Which fower right lines make a ſquare: for that eche of thoſe right lines doo ſubtend equall angles of equall pentagons, & the angles which thoſe 4. right lines cõtaine are right angles, as we proued in the conſtruction of the dodecahedron, in the 17. propoſitiõ before alledged. Wherfore the ſixe baſes being ſquares, do make a cube (by the 21. diffinition of the eleuenth) and for that the 8. angles of the ſayd cube are ſet in 8. angles of the dodecaheeron, therefore is the ſayd cube inſcribed in the dodecahedron (by the firſt diffinition of this booke). Wherefore in a dodecahedron is inſcribed a cube: which was required to be doone.



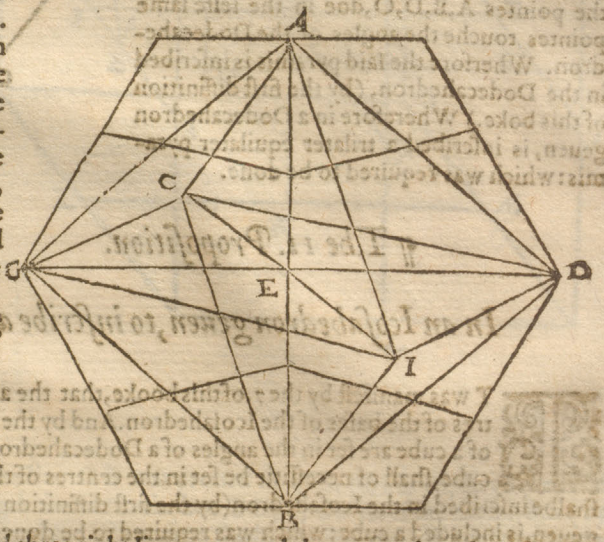
Demonſtration.

¶ The 9. Propoſition. The 9. Probleme.

In a Dodecahedron geuen to include an Octohedron.

Conſtruction.

VPpoſe that the dodecahedron geuen be A B G D. Now (by the 3. corellary of the 17. of the thirteth take the 6. ſides which are oppoſite the one to the other, thoſe 6. ſides, I ſaye whoſe ſections wherin they are deuided into two equal partes, are coupled by three right lines which in the centre of the ſphere, wherin the Dodecahedron is contained, doe cut the one the other perpendicularly. And let the poyntes wherin the forſayde ſides are cut into two equal partes be A, B, G, D, C, I. And let the foreſaid thre right lines ioyning togeather the ſaide ſections be A B, G D and C I. And let the centre of the ſphere be E. Now foraſmuch as (by the foreſaid correllary) thoſe thre right lines are equal, it foloweth (by the 4. of the firſt) that the right lines ſubteding the right angles which they make at the centre of the ſphere, whiche right angles are contained vnder the halues of the ſaid three right lines, are equal the one to the other: that is, the right lines A G, G B, B D, D A, C A, C G, C B, C D, and I A, I G, I B, I D are equal the one to the other. Wherfore alſo the 8. triangles C A G, C G B, C B D, C D A, I A G, I G B, I B D & I D A are equal and equilater. And therefore A G B D C I is an Octohedron by the 23. definition of the eleuẽth.) And the ſayd Octahedron is included in the dodecahedron (by the firſt definition of this booke:) for that all the angles thereof doe at one time touch the ſides of the dodecahedron. Wherefore in the dodecahedron geuen, is included an Octohedron: which was required to be done.



Demonſtration.

¶ The 10. Propoſition. The 10. Probleme.

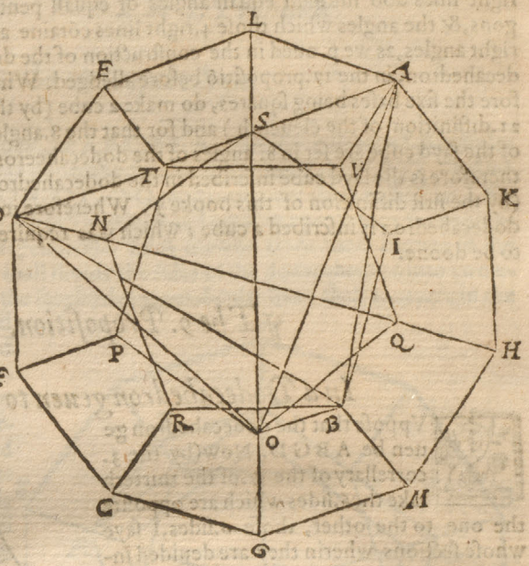
In a Dodecahedron geuen, to inſcribe an equilater trilater Pyramis.

Conſtruction.

SVppoſe that the Dodecahedron geuen, be A B C D, of which Dodecahedron take thre baſes meting at the poynt S, namely theſe thre baſes A L S I K, D N S L E and S I B R N: and of theſe thre baſes take the three angles at the poynts A, B, D: and draw theſe right lines A B, B D and D A: and let the diameter of the ſphere containing the dodecahedron be S O, and

Demonstration.

then draw thse right lines A O, B O and D O: Now foraſmuch as (by the 17. of the thirteenth) the angles of the dodecahedron are ſe in the ſuperficies of the ſphere deſcribed about the Dodecahedron: therefore if vpon the diameter S O, and by the poynt A, be deſcribed a ſemicircle, it ſhall make the angle S A O a right angle (by the 31. of the third.) And likewiſe if the ſame ſemicircle be drawne by the poyntes D and B, it ſhall alſo make the angles S B O, and S D O right angles. Wherefore the diameter S O containeth in power bothe the lines S A, A O, or the lines S B, B O, or els S D, D O, but the lines S A, S D, S B are equal the one to the other, for they eche ſubtend one of the angles of equal pentagōs. Wherfore the other lines remaining, namely, A O, B O, D O are equal the one to the other. And by the ſame reaſon may be proued that the diameter H D which ſubtendeth the two right lines H A, A D, containeth in power both the ſaid two right lines, and alſo containeth in power bothe the right lines H B and B D, which two right lnes it alſo ſubtendeth. And moreouer by the ſame reaſon the diameter A C, which ſubtendeth the right lines C B and B A, containeth in power both the ſaid right lines C B and B A. But the right lines H A, H B and C B are equal the one to the other, for that eche of them alſo ſubtendeth one of the angles of equal pentagons: wherfore the right lines remaining, namely, A D, B D, and B A are equal the one to the other. And by the ſame reaſon may be proued that eche of thoſe right lines A D, B D and B A is equal to ecle of the right lines A O, B O and D O. Wherefore the ſixe right lines AB, BD, DA, AO, BO, & DO are equal the one to the other. And therefore the triangles which are made of thē, namely, the triangles A B D, A O B, A O D and B O D are equal and equilater: which triangles therfore do make a pyramis ABDO, whoſe baſe is A-B D and toppe the poynt O. Eche of the angles of which pyramis, namely, the angles at the pointes A, B, D, O, doe in the ſelfe ſame pointes touche the angles of the Dodecahedron. Wherfore the ſaid pyranis is inſcribed in the Dodecahedron, (by the firſt diffinition of this boke.) Wherefore in a Dodecahedron geuen, is inſcribed a trilater equilater pyramis: which was required to be done.



¶ The 11. Propoſition. The 11. Probleme.

In an Icoſahedron geuen, to inſcribe a cube.

IT was manifeſt by the 7. of this booke, that the angles of a Dodecahedron are ſet in the centres of the baſes of the Icoſahedron. And by the 8. of this boke, it was proued, that the angles of a cube are ſet in the angles of a Dodecahedron. Wherefore the ſelfe ſame angles of the cube, ſhall of neceſſitie be ſet in the centres of the baſes of Icoſahedron. Wherfore the cube ſhalbe inſcribed in the Icoſahedron (by the firſt diffinition of this boke.) Wherfore in an Icoſahedron geuen, is included a cube: which was required to be done.

¶ The 12. Propoſition. The 12. Probleme.

In an Icoſahedron geuen, to inſcribe a trilater equilater pyramis.

BY the former propoſition it was manifeſt, that the angles of a cube are ſet in the centres of the baſes of the Icoſahedron. And (by the firſt of this booke) it was playne that the foure angles of a pyramis are ſet in foure angles of a cube. Wherefore it is euident, by the firſt diffinition of this booke, that a pyramis deſcribed of right lines ioyning together theſe foure centres of the baſes of the Icoſahedron, ſhalbe inſcribed in the ſame Icoſahedron. Wherefore in an Icoſahedron geuen, is inſcribed an equilater trilater pyramis: which was required to be done.

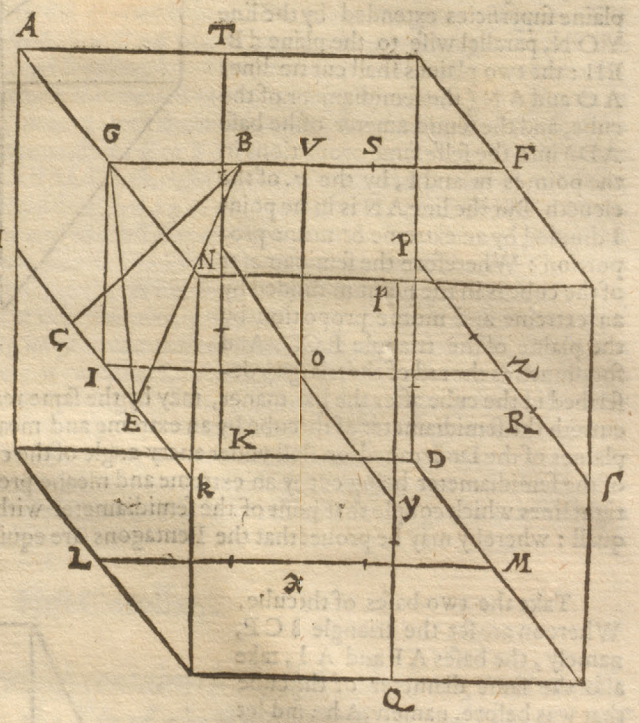
The

¶ The 13. Probleme The 13. Proposition.

In a Cube geuen, to inscribe a Dodecahedron.

TAke a Cube A D F L. And diuide euery one of the siles therof into two equall partes in the pointes T,H,K,P : G,L,M,F : and p k Q ſ. And drawe these right lines T K, G F, p Q, H k, P ſ, and L M : which lines againe diuide into two equall partes in the pointes N,V,Y,I,Z,X. And draw these right lines N Y, V X, and I Z : Now the three lines N Y, V X, and I Z, together with the diameter of the cube, shall cut the one the other into two equall partes in the centre of the cube by the 39. of the eleuenth : let that centre be the point O. And not to stand long about the demonstration, vnderstand all these right lines to be equall and parallels to the sides of the cube and to cut the one the other right angled wise, by the 29. of the first. Let their halfes, namely, F V, G V, H I, and k I, and the rest such like, be diuided by an extreme and meane proportion, by the 30. of the sixth: whose greater segmẽts let be the lines F S, G B, H C, and k E, &c. and drawe these right lines G I, G E, B C, and B E. Now forasmuch as the line G I is equall to the whole line G V, which is the halfe of the side of the cube : and the line I E is equall to the line B V, that is, to the lesse segmẽt : therfore, the squares of the lines G I and I E, are triple to the square of the line G B, by the 4. of the thirteenth : But vnto the squares of the lines G I and I E, the square of the line G E is equall, by the 47. of the first : for the angle G I E is a right angle. Wherefore the square of the line G E is triple to the square of the line G B. And forasmuch as the line F G is erected perpendicularly to the plaine A G k L, by the 4. of the eleuenth : for it is erected perpendicularly to the two lines A G and G I : therefore the angle B G E is a right angle : for the line G E is drawen in the plaine A G k L. Wherefore the line B E, containing in power the two lines B G and G E, by the 47. of the first, is in power quadruple to the line G B (for the line G E was proued to be in power triple to the same line G B) : Wherefore the line B E is in length double to the line B G, by the 20. of the sixth. But (by construction) the line C E is double to the line I E : Wherefore the halfes G B and I E, are in proportion the one to the other, as their doubles B E and C E : by the 15. of the fifth. Wherefore the line C E is the greater segment of the line B E diuided by an extreme and meane proportion. And forasmuch as the selfe same thing may be proued touching the line B C : therefore the lines B E and B C, are equall, making an Isosceles triangle. Now let vs proue that three angles of the Ientagon of the Dodecahedron are set at the pointes B, C, E : and the other two angles are set betwene the lines B C and B E.

First part of the construction.



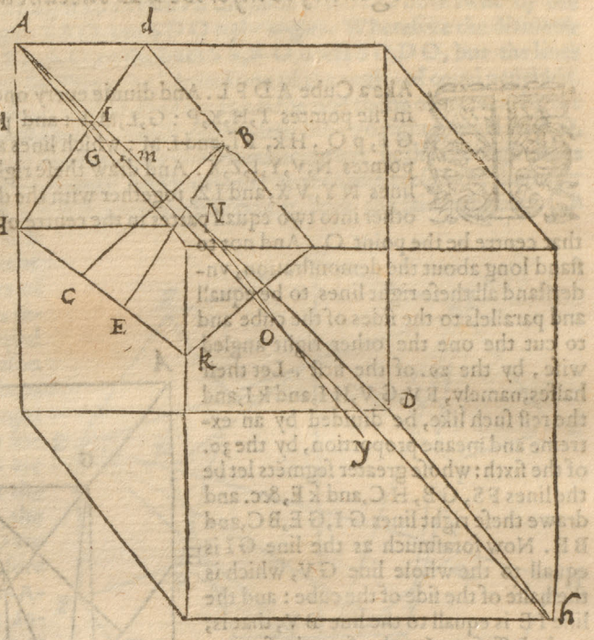
First part of the demonstration.

Forasmuch as the circle which containeth the triangle B C E circunscribeth the Pentagon whose side is the line C E, by the 11. of the fourth : Extend the plaine of the triangle B C E, by the parallel lines d B and H E, cutting the line A D, namely, the diameter of A D the base of the cube in the point I : and let it cut the line A h the diameter of the cube in the point m. And by the point I drawe in the base A D, a parallel line vnto the line A d : which let be I l. And forasmuch as from the triangle A H N is, by the parallel line l I, taken away the triangle A l I, like vnto the whole triangle A H N, by the Corollary of the 2. of the sixth : the lines A l, and l I, shall be equall. But as the line H A is to the line A d, so (by the 2. of the sixth) is the line H l to the line l I, or to the line l A, which is equall to the line l I. And the greater segment of the line H A (which is halfe the side of the cube) is, as before hath bene proued, the line A d, that is, the line G B, which is equall to the line A d (by the 33. of the first).

Second part of the construction.

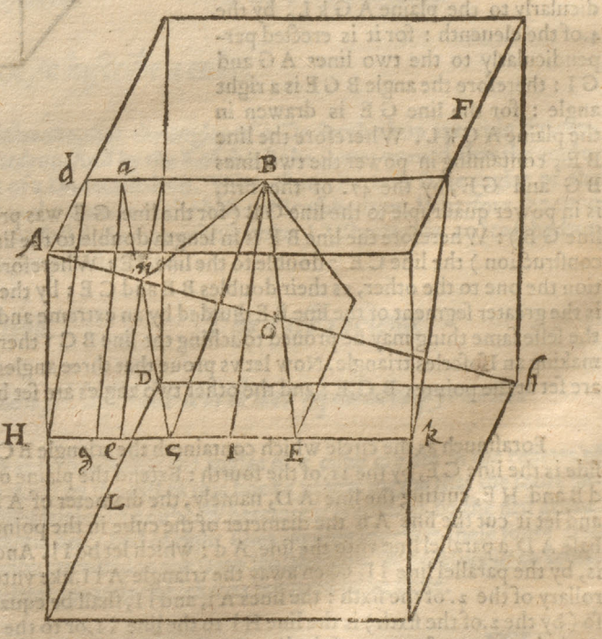
Second part of the Demonstration.

Wherefore the greater ſegment of the line H l is the line l A. And as the whole line H l is to the greater ſegment, ſo ſhall the ſame greater ſegment H l be to the leſſe ſegment l A, by the 5. of the thirtenth. Wherefore the line H A is diuided by an extreme and meane proportion in the point l. But in the triangle A H N, the line N A, which is drawen frō the centre of the baſe A D, is in the point I cut like vnto the line AH, by the parallel line l I (by the ſame ſecond of the ſixth): for the lines H N and l I, are parallels, by conſtruction. Wherefore the line N A is in the point I diuided by an extreme and meane proportion by the ſuperficies d B E H. And foraſmuch as the line Y O N which coupleth the centres of the oppoſite baſes, is a parallel to the line H E: A plaine ſuperficies extended by the line Y O N, parallel wiſe to the plaine d B E H: the two plaines ſhall cut the lines A O and A N (the ſemidiameter of the cube, and the ſemidiamerer of the baſe A D) into the ſelfe ſame proportions in the pointes m and I, by the 17. of the eleuēth. But the line A N is in the point I diuided by an extreme & meane proportion: Wherefore the ſemidiameter of the cube is in the point m diuided by an extreme and meane proportion by the plaine of the triangle B C E. And foraſmuch as the reſt of the triangles deſcribed in the cube after the like maner, may by the ſame reaſons be proued to be in a plaine which cutteth the ſemidiameter of the cube by an extreme and meane proportion: it is manifeſt that three plaines of the Dodecahedron ſhall vnder euery angle of the cube concurre in one & the ſelf ſame point of the ſemidiameter being cut by an extreme and meane proportion. Now reſteth to proue that the right lines which couple that point of the ſemidiameter with the angles of the triangle B E C, are equall: whereby may be proued that the Pentagons are equilater, and equiangle.



Third part of the conſtruction.

Take the two baſes of the cube. Whereon are ſet the triangle B C E, namely, the baſes A F and A k, take alſo the ſame diameter of the cube that was before, namely, A h: and let the ſide ſet at the poynt n, of the ſection of the diameter by an extreame & meane proportion, be the line C n or B n: and let the centre of the cube be as before the point O. And extend the line C n to the line B d, and let it concurre with it in the point a. And foraſmuch as the plaine which paſſeth by the line H C E and the centre O (cutting the cube into two equall partes) is parallel to A F the baſe of the cube by conſtruction: imagine that by the poynt n, be extended a playne ſuperficies parallel to the former parallel playnes, which ſhall cutte the ſemidiameter O A & the line C a, proportionally in the point n, by the 17. of the eleuenth: For thoſe lines doo touch the extreame parallel plaines extended by the lines H E and E O, and by the lines A d and d B. But it is proued that the line O A is diuided by an extreame and meane proportion in the



Third part of the demonſtration.

poynt

poynt n : wherefore the line C a, is alſo diuided by an extreame and meane proportion in the poynt n. Agayne foraſmuch as B C E is an Iſoſcels triangle, and it is proued that the line B I cutteth the baſe C E into two equall partes in the poynt I, the angles B I C and B I E ſhall be right angles. Imagine by the line B I and the centre O a plaine to paſſe (cutting the cube into two equall partes) parallel to the baſe A D. And vnto thoſe plaines let there be imagined an other parallel plaine paſſing by the poynt n: which let be n e: which ſhall cutte the ſemidiameter A O and the halfe ſide of the cube, namely, the line I H, like, in the pointes n and e by the 17. of the eleuenth. Wherefore the line I H is in the poynt e diuided by an extreame & meane proportiõ. Wherfore the line H e is equall to the line C I or I E: namely, ech are leſſe ſegmẽts. And foraſmuch as the line I e is to the line I C (which is equall to the line E H) as the whole is to the greater ſegment, take away from the whole line I e the greater ſegmẽt I C: there ſhall remayne the leſſe ſegment C e by the 5. of the thirteenth. Wherefore the line I e is diuided by an extreame & meane proportion in the point C. Againe vnto the ſame playnes imagine an other playne to paſſe by the point a, parallel wiſe, and let the ſame be a g. Now then (by the ſame 17. of the eleuenth) the lines C a and C g are in like ſort cut in the pointes n and e. But the line C a was in the point n cutte by an extreame and meane proportion, wherefore the line C g ſhall be cutte in the poynt e, by an extreame & meane proportion. But the line I C is to the line C e, as the greater ſegment is to the leſſe: wherfore the line C e, is to the line e g, as the greater ſegment to the leſſe: and therefore their proportion is as the whole line I C is to the greater ſegment C e, and as the greater ſegment C e is to the leſſe ſegment e g: wherefore the whole line C e g which maketh the greater ſegment and the leſſe, is equall to the whole line I C or I E. And foraſmuch as two parallel plaine ſuperficieces (namely, that which is extended by I Q B and that which is extended by the line a g) are cutte by the playne of the triangle B C E, which paſſeth by the lines a g and I B, their common ſections a g and I B ſhall be parallels (by the 16. of the eleuenth). But the angle B I E or B I C is a right angle, wherefore the angle a g C is alſo a right angle (by the 29. of the firſt) and thoſe right angles are contayned vnder equall ſides, namely, the line g C is equall to the line C I, and the line a g to the line B I, by the 33. of the firſt: wherfore the baſes C a and C B are equall, by the 4. of the firſt. But of the line C B the line C E was proued to be the greater ſegment: wherefore the ſame line C E is alſo the greater ſegment of the line C a: but c n was alſo the greater ſegment of the ſame line C a. Wherefore vnto the line C E, the line c n which is the ſide of the dodecahedron, and is ſet at the diameter, is equall. And by the ſame reaſon the reſt of the ſides, which are ſet at the diameter may be proued equall to lines equall to the line C E. Wherfore the pentagon inſcribed in the circle where in is contained the triangle B C E is, by the 11. of the fourth equiangle, and equilater. And foraſmch as two pentagons, ſet vpon euery one of the baſes of the cube doo make a dodecahedron, and ſixe baſes of the cube doo receaue twelue angles of the dodecahedron: and the 8. ſemidiameters doo in the pointes where they are cutte by an extreame and meane proportion receaue the reſt: therefore the 12. pentagon baſes contayning 20. ſolide angles doo inſcribe the dodecahedron in the cube: by the 1. diffinition of this booke. Wherefore in a cube geuen is inſcribed a dodecahedron: which was required to be done.

Firſt Corollary.

The diameter of the ſphere, which containeth the dodecahedron, containeth in power theſe two ſides, namely, the ſide of the Dodecahedron, and the ſide of the cube wherein the Dodecahedron is inſcribed. For in the firſt figure a line drawne from the centre O, to the poynt B the angle of the Dodecahedron, namely the line O B, containeth in power theſe two lines O V the halfe ſide of the cube, and V B the halfe ſide of the dodecahedron, by the 47. of the firſt. Wherefore by the 15. of the fiueth, the double of the line O B, which is the diameter of the ſphere containing the Dodecahedron, containeth in power the double of the other lines O V and V B, which are the ſides of the cube, and of the dodecahedron.

¶ Second Corollary.

The ſide of a cube diuided by an extreme and meane proportion, maketh the leſſe ſegment the ſide of the dodecahedron inſcribed in it: and the greater ſegment the ſide of the cube inſcribed in the ſame Dodecahedron: For it was before proued, that the ſide of the dodecahedron is the greater ſegment of B E the ſide of the triangle B E C: but the ſide B E (which is equall to the lines G B and S F) is the greater ſegmẽt of G F the ſide of the cube: which line B E (ſubtending the angle of the pentagon) was (by the 8. of this booke) the ſide of the cube inſcribed in the dodecahedron.

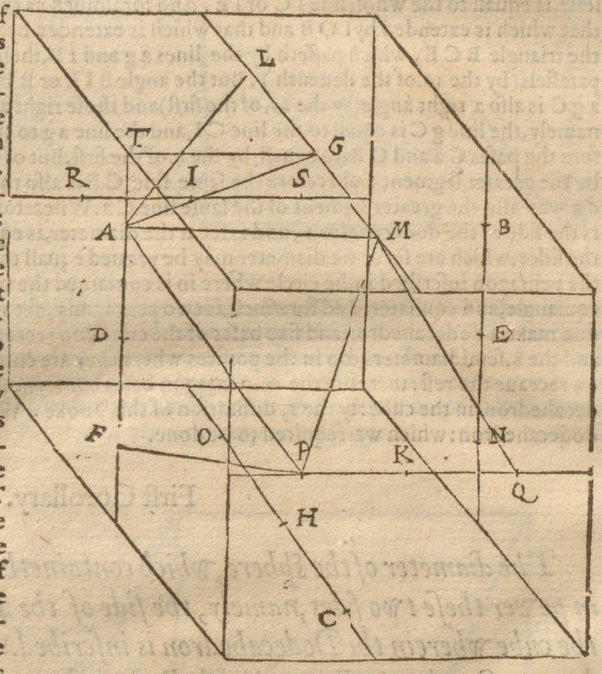
Third Corollary.

The side of a cube, is equal to the sides of a Dodecahedron inscribed in it, and circumscribed about it. For it was manifest by this proposition, that the side of a cube maketh the lesse segment, the side of a Dodecahedron inscribed in it, namely, as in the first figure the line B S the side of the Dodecahedron inscribed, is the lesse segmẽt of the line GF the side of the cube. And it was proued in the 17. of the thirtenth, that the same side of the cube subtẽdeth the angle of the pentagon of the Dodecahedron circumscribed: and therefore it maketh the greater segment the side of the Dodecahedron or of the pentagon, by the first corollary of the same. Wherefore it is equal to bothe those segments.

The 14. Probleme. *The 14. Proposition.*

In a cube geuen, to inscribe an Icosahedron.

Construction. Vppose that the cube geuen be A B C, the Centres of whose bases let be the points D, E, G, H, I, K: by whiche poyntes draw in the bases vnto the other sids parallels not touching the one the other. And deuide the lines drawn from the centres, as the line D T. &c. by an extreme and meane proportion in the poyntes A, F: L, M: N, B: P, Q: R, S: C, O: by the 30. of the sixth: and let the greater segmentes be about the cẽtres. And draw these right lines, A L, A G, A M, and T G. And forasmuch as the lines cut are parallels to the sides of the cube: they shall make right angles the one with the other by the 29. of the first: and forasmuche as they are equal: their sections shall be equal, for that the sections are like by the 2. of the fourteenth. Wherfore the line T G is equal to the line D T, for they are eche, halfe sides of the cube. Wherfore the square of the whole line TG, and of the lesse segment T A, is triple to the square of the line A D the greater segment (by the 4. of the thirtẽth). But the line A G containeth in power the lines A T & T G, for the angle A T G is a right angle. Wherefore the square of the line A G is triple to the square of the line A D. And forasmuch as the line M G L is erected perpendicularly to the plain passing by the lines A T, & which is parallel to the bases of the cube (by the corollary of the 14. of the eleuẽth) therfore the angle A G L is a right angle. But the line LG is equal to the line AD, for they are the greater segments of equal lines: Wherfore the line A G (which is in power tripe to the line A D) is in power triple to the line L G. Wherefore adding vnto the same square of the lne A G, the square of the line L G, the square of the line A L, which (by the 47. of the first) containeth in power the two lines A G and G L, shalbe quadruple to the line A D or L G. Wherefore the line A I is double to the line A D (by the 20. of the sixth:) and therfore is equal to the line A F, or to the line I M. And by the same reason may we proue that euery one of the other lines which couple the next sections of the lines cut, as the lines A M, P F, P M, M Q, and the rest are equal. Wherfore the triangles A L M, A P F, A M P, P M Q, and the rest such like, are equal, equiangle, and equilater, by the 4. and eigth of the first. And forasmuch as vpon euery one of the lines cut of the cube are set two triangles, as the triangles A L M, and B L M, there shalbe made 12. triãgles. And forasmuch as vnder euery one of the 8. angles of the cube, are subtended the other 8. triangles, as the triangle A M P. &c. of 12. and 8. triangles shall be produced 20. triangles equal and equilater cõtaining the solide of an Icosahedron, by the 25. diffinition of the eleuenth, which shalbe inscribed in the cube geuen A B C by the first diffinition of this booke. The inuention of the demonstration of this dependeth of the ground of the former. Wherfore in a cube geuen, we haue described an Icosahedron: which was required to be done.



Demonstration.

First

✻ First Corollary.

The diameter of a ſphere which containeth an Icoſahedron, containeth two ſides, namely the ſide of the Icoſahedron, and the ſide of the cube which containeth the Icoſahedron. For if we drawe the line A B, it ſhall make the angles at the poynt A right angles: for that it is a parallel to the ſides of the cube: wherfore the line which coupleth the oppoſite angles of the Icoſahedron, at the poynts F and B, cõtaineth in power the line A B (the ſide of the cube) and the line A F (the ſide of the Icoſahedron) by the 47. of the firſt. Which line F B is equal to the diameter of the ſphere, which containeth the Icoſahedron, by the demonſtration of the 16. of the thirteth.

✻ Second Corollary.

The ſix oppoſite ſides of the Icoſahedron deuided into two equal parts: their ſections are coupled by three equal right lines, cutting the one the other into two equal partes, and perpendicularly in the centre of the ſphere which containeth the Icoſahedron. For thoſe three lines are the three lines which couple the centres of the baſes of the cube, which do in ſuch ſort in the centre of the cube, cut the one the other, by the corollary of the third of this booke, and therfore are equal to the ſides of the cube. But right lines drawne from the cẽtre of the cube to the angles of the Icoſahedron, euery one of them ſhall ſubtend the halfe ſide of the cube, and the halfe ſide of the Icoſahedron (which halfe ſides containe a right angle) wherefore thoſe lines are equal. Wherby it is manifeſt that the foreſaid centre is the centre of the ſphere which containeth the Icoſahedron.

✻ Third Corollary.

The ſide of a cube deuided by an extreme and meane proportion, maketh the greater ſegment the ſide of an Icoſahedron deſcribed in it. For the half ſide of the cube maketh the halfe of the ſide of the Icoſahedron the greater ſegment: wherefore alſo the whole ſide of the cube, maketh the whole ſide of the Icoſahedron the greater ſegment by the 15. of the fifthe, for the ſections are like by the 2. of the fourteenth.

¶ Fourth Corollary.

The ſides and baſes of the Icoſahedron, which are oppoſite the one to the other, are parallels. Foraſmuch as euery one of the oppoſite ſides of the Icoſahedron, may be in the parallel lines of the cube, namely, in thoſe parallels which are oppoſite in the cube: and the triangles which are made of paralle lines, are parallels, by the 15. of the eleuenth: therfore the oppoſite triangles of the Icoſahedron, as alſo the ſides, are parallels the one to the other.

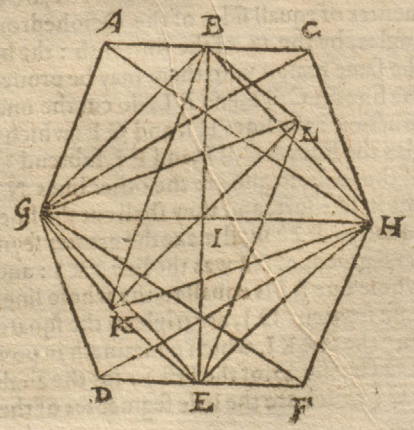
¶ *The 15. Probleme.* *The 15. Propoſition.*

In an Icoſahedron geuen, to inſcribe an Octohedron.

Conſtruction.

SVppoſe that the Icoſahedron geuen be A C D F: and by the former ſecond Corollary, let there betaké the three right lines which cut the one the other into two equall partes perpendicularly, and which couple the ſections into two equall partes of the ſides of the Icoſahedron: which let be B E, G H, and K L, cutting the one the other in the point I. And drawe theſe right lines B G, G E, E H, and H B. And foraſmuch as the angles at the point I are (by conſtruction) right angles, and are contained vnder equall lines: the baſes G B and H E ſhall make a ſquare, by the 4. of the firſt. Likewyſe vnto thoſe baſes ſhall be equall the lines drawen from the pointes K and L, to euery one of the pointes B, G, E, H: And therefore the triangles which make the pyramis B G E H K, ſhall be equall and equilater. And by the ſame reaſon ſhall the reſt of the triangles which make the other pyramis B G E H L vpon the ſame baſe B G E H, be equall and equilater. Wherefore B G E H K L ſhall be an Octohedron (by the 23. definition of the eleuenth): And ſhall be inſcribed in the Icoſahedron, by the firſt definition of this booke. Wherefore in an Icoſahedron geuen, is inſcribed an Octohedron: which was required to be done.

Demonſtration.



¶ *The*

¶ The 16. Probleme. The 16. Proposition.

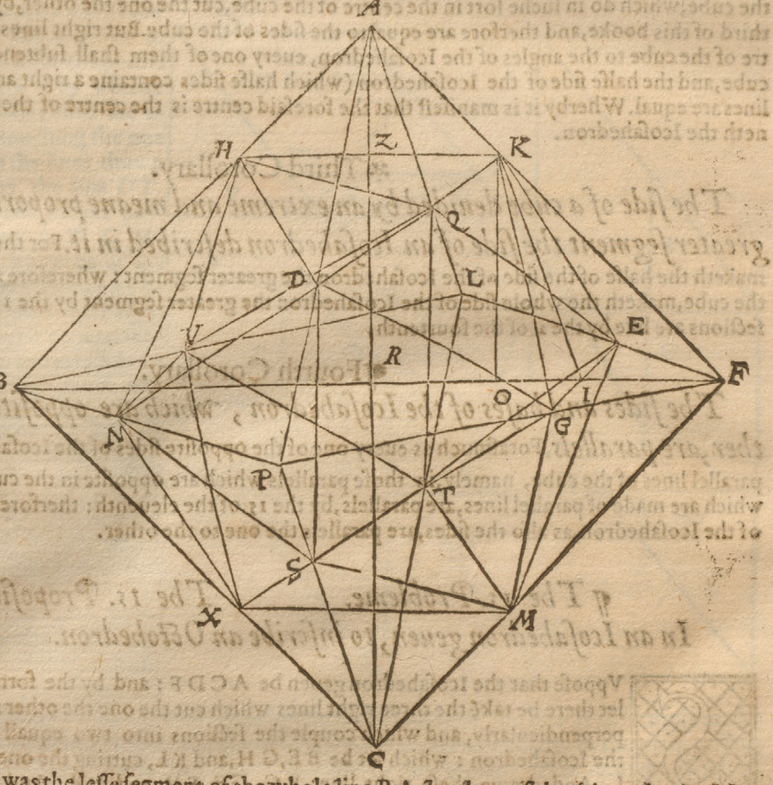
In an Octohedron geuen, to inscribe an Icosahedron.

Construction.

LEt there be taken an Octohedron, whose 6. angles, let be A,B,C,F,P,L. And draw the lines A C, B F, P L, cutting the one the other perpendicularly in the point R (by the 2. Corollary of the 14. of the thirteenth). And let euery one of the 12. sides of the Octohedron be diuided by an extreme and meane proportion, in the pointes H,X,M,K,D,S,N, G,V,E,Q, T. And let the greater segmentes be the lines B H, B X, F M, F K, A D, A Q, C S, C T, P N, P G, L V, L E: And drawe these lines H K, X M, G E, N V, D S, Q T. Now

Demonstration.

forasmuch as in the triangle A B F, the sides are cut proportionally, namely, as the line B H is to the line H A, so is the line F K to the line K A (by the 2. of the fouretenth): therefore the line H K shall be a parallel to the line B F (by the 2. of the sixth). And forasmuch as the line A C cutteth the line H K in the point Z, and the line Z K is a parallel vnto the line R F, the line R A shall be cut by an extreme and meane proportion in the point Z: by the 2. of the sixth: namely, shall be cut like vnto the line F A: and the greater segmẽt therof shall be the line Z R. Vnto the line Z K put the line R O equall, by the 3. of the first: and drawe the line K O: now then, the line K O shall be equall to the line Z R, by the 33. of the first. Draw the lines K G, K E, and K I. And forasmuch as the triangles A R F, and A Z K, are equiangle (by the 6. of the sixth) the sides A Z and Z K, shall be equall the one to the other, by the 4. of the sixth, for the sides A R and R F, are equall. Wherfore the line Z K shall be the lesse segment of the line R A. But if the greater segment R Z be diuided by an extreme & meane proportion, the greater segment therof shall be the line Z K, which was the lesse segment of the whole line R A, by the 5. of the thirteenth. And forasmuch as the two lines F E and F G, are equall to the two lines A H and A K, namely, ech are lesse segmentes of equall sides of the Octohedron, and the angles H A K and E F G are equall, namely, are right angles, by the 14. of the thirteenth: the bases H K and G F shall be equall, by the 4. of the first: And by the same reason vnto them may be proued equall the lines X M, N V, D S, and Q T. And forasmuch as the lines A C, B F, and P L, do cut the one the other into two equall parts, and perpendicularly, by construction: the lines H K and G E (which subtend angles of triangles like vnto the triangles whose angles the lines A C, B F, and P L subtend) are cut into two equall partes in the pointes Z and I, by the 4. of the sixth, so also are the other lines N V, X M, D S, Q T (which are equall vnto the lines H K & G E) cut in like sort, and they shall cut the lines A C, B F, and P L like. Wherefore the line K O (which is equall to R Z) shall make the greater segment the line R O, which is equall to the line Z K (for the greater segment of R Z was the line Z K): and therefore the line O I shall be the lesse segment, when as the whole line R I is equall to the whole line R Z. Wherefore the squares of the whole line K O, and of the lesse segment O I, are triple to the square of the greater segment R O, by the 4. of the thirteenth. Wherfore the line K I, which containeth in power the two lines K O and O I, is in power triple to the line R O (by the 47. of the first): for the angle K O I is a right angle. And forasmuch as the lines F E and F G (which are the lesse segmentes of the sides of the Octohedron) are equall: and the line F K is cõmon



mon to them both : and the angles K F G and K F E (of the triangles of the Octohedron) are equall : the baſes K G and K E ſhall (by the 4 of the firſt) be equall : and therefore the angles K I E and K I G which they ſubtend, are equall (by the 8. of the firſt): Wherefore they are right angles, by the 13. of the firſt. Wherefore the right line K E (which containeth in power the two lines K I and I E, by the 47. of the firſt) is in power quadruple to the line R O (or I E) : for the line K I is proued to be in power triple to the ſame line R O: But the line G E is double to the line I E: Wherfore the line G E is alſo in power quadruple to the line I E (by the 20. of the ſixth). Wherefore the two lines K E and G E are equall. And by the ſame reaſon, may the reſt of the lines, namely, H K, H N, N V, V X, X S, and the other lines which couple the ſections of the ſides of the Octohedron be proued equall to the ſame lines K E and G E. Wherefore the triangles deſcribed of them, namely, G E K, G K D, G D S, G S M, G M E, ſhall be equall and equilater, by the 8. of the firſt, making a ſolide angle at the point G : which is therefore the angle of an Icoſahedron, by the 16. of the thirteenth, and is ſet in the ſection G of the ſide P F. And by the ſame reaſon may be proued, that the reſt of the eleuen ſolide angles of the Icoſahedron, are ſet in the ſections of euery one of the ſides of the Octohedron, namely, in the pointes E, N, V, H, K, M, X, D, S, Q, T. Wherefore there are 12. angles of the Icoſahedron. Moreouer, foraſmuch as euery one of the baſes of the Octohedron, do eche containe triangles of the Icoſahedron, as in the pyramis A B C F P (which is the halfe of the Octohedron) the triangle F C P receaueth in the ſections of his ſides the triangle G M S : and the triangle C P B containeth the triangle N X S : and the triangle B A P contayneth the triangle H N D : and moreouer the triangle A P F containeth the triangle K D G, and the ſame may be proued in the oppoſite pyramis A B C F L : Wherefore there ſhall be eight triangles. And foraſmuch as beſides theſe triangles, to euery one of the ſolide angles of the Octohedron are ſubtended two triangles, as the triangles K E G and M E G, to the angle F : and the triangles H N V and X N V, to the angle B : alſo the triangles N D S and G D S, to the angle P : likewiſe the triangles D H K and Q H K, to the angle A : Moreouer the triangles E Q T and V Q T, to the angle L: and finally the triangles S X M and T X M, to the angle C : theſe 12. triangles being added to the 8. former triangles, ſhall produce 20. triangles equall and equilater coupled together : which ſhall make an Icoſahedron, by the 25. definition of the eleuenth : and it ſhall be inſcribed in the Octohedron geuen A B C F P L, by the firſt definition of this booke : for the 12. angles thereof are ſet in 12. like ſections of the ſides of the Octohedron. Wherefore in an Octohedron geuen, is inſcribed an Icoſahedron.

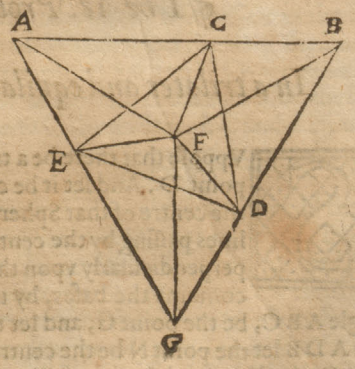
¶ Firſt Corollary.

The ſide of an equilater triangle being diuided by an extreme and meane proportion: a right line ſubtending within the triangle, the angle which is contained vnder the greater ſegment and the leſſe: is in power duple to the leſſe ſegment of the ſame ſide. For the line K E, which ſubtendeth the angle K F E of the triangle A F L, which angle K F E is contained vnder the two ſegmentes K F & F E, was proued equall to the line H K, which containeth in power the two leſſe ſegmentes H A and A K, by the 47. of the firſt, for the angle H A K is a right angle. Wherefore the line K E or H K, is in power duple to the line A K.

❊ Second Corollary.

The baſes of the Icoſahedron are concentricall (that is, haue one and the ſelfe ſame centre) with the baſes of the Octohedron which contayneth it.

For ſuppoſe that A B G be the baſe of an Octohedron contayning E C D the baſe of an Icoſahedron : and let the centre of the baſe A B G be the point F. And drawe theſe right lines F A, F B, F C, and F E. Now then the two lines F A and A E ſhall be equall to the two lines F B and B C : for they are lines drawen from the centre, and are alſo leſſe ſegmentes : and they contayne the halfes of equall angles. Wherfore (by the 4. of the firſt) the baſes F C and F E are equall : and by the ſame reaſon vnto them ſhall be equall the other line F D. Wherefore making the centre the point F : and the ſpace F E deſcribe a circle and it ſhall be circumſcribed about the triangle C E D : and ſo ſhall the point F the centre of the baſe of the Octohedron be the centre of C E D the baſe of the Icoſahedron.

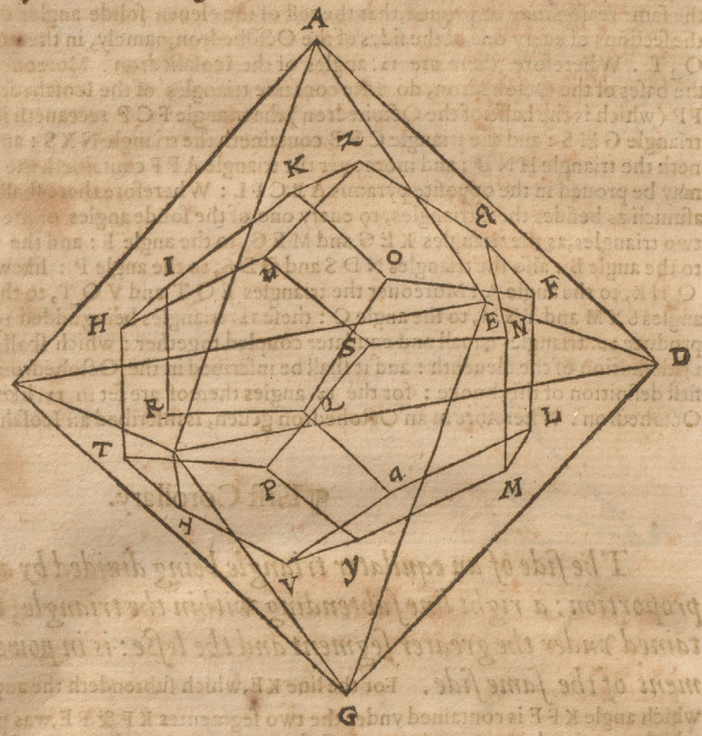


¶ The 17. Probleme. The 17. Proposition.

In an Octohedron geuen, to inscribe a Dodecahedron.

Construction. SVppose that the Octohedron geuen be A B G D E C : whose 12. sides let be cut by an extreme and meane proportion, as in the former Proposition. It was manifest that of the right lines which couple these sections, are made 20. triangles, of which 8. are concentricall with the bases of the Octohedron, by the second Corollary of the former Proposition. If therefore in euery one of the centres of the 20. triangles be inscribed (by the 5. of this booke) euery one of the 12. angles of the Dodecahedron, we shall finde, that 8. angles of the Dodecahedron are set in the 8. centres of the bases of the Octohedron: namely, these angles I, u, &, O, M, a, P, and X: and of the other 12. solide angles there are two in the centres of the two triangles which haue one side common vnder euery one of the solide angles of the Octohedron: namely, vnder the solide angle A, the two solide angles, K, Z: vnder the solide angle B, the two solide angles H, T: vnder the solide angle G, the two solide angles Y, V: vnder the solide angle D, the two solide angles F, L: vnder the solide angle E, the two solide angles S, N: vnder the solide angle C, the two solide angles Q, R: and forasmuch as in the Octohedron are sixe solide angles, vnder them shall be subtended 12. solide angles of the Dodecahedron: and so are made 20. solide angles composed of 12. equall and equilater superficiall pentagons (as it was manifest, by the 5. of this booke) which therefore containe a Dodecahedron (by the 24. definition of the eleuenth). And it is inscribed in the Octohedron (by the 1. definition of this booke): for that euery one of the bases of the Octohedron do receaue angles therof. Wherefore in an Octohedron geuen, is inscribed a Dodecahedron.

Demonstration.

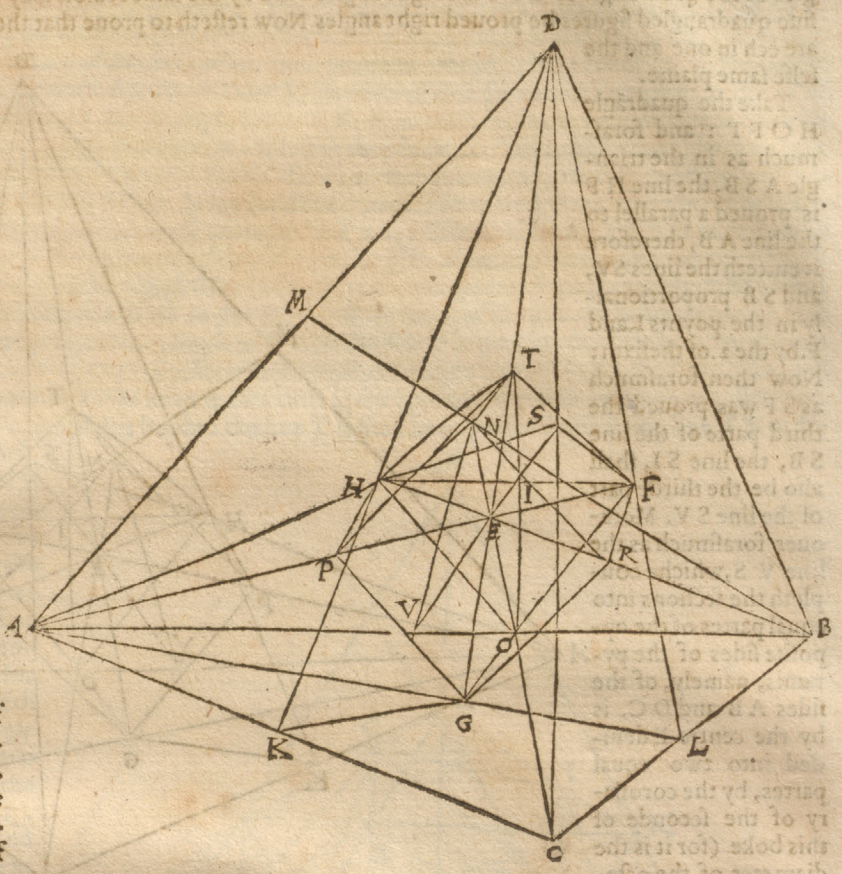


¶ The 18. Probleme. The 18. Proposition.

In a trilater and equilater Pyramis, to inscribe a Cube.

Construction. SVppose that there be a trilater equilater Pyramis, whose base let be A B C, and toppe the point D. And let it be comprehended in a Sphere, by the 13. of the thirteenth. And let the centre of that Sphere be the point E. And from the solide angles A, B, C, D, draw right lines passing by the centre E, vnto the opposite bases of the pyramis, and they shall fall perpendicularly vpon the bases, and shall also fall vpon the centres of the circles which containe the bases, by the Corollary of the 13. of the thirteenth. Let the centre of the triangle A B C, be the point G, and let the centre of the triangle A D C be the point H, and of the triangle A D B let the point N be the centre, and finally, let the point F be the centre of the other triangle D B C. And let the right lines falling vpon those cētres be D E G, B E H, C E N, & A E F. And by those centers G, H, N, F, let there be drawen from the angles to the opposite sides these right lines, A G L, D H K,

D H K, B N M, and D F L, which ſhall fall perpendicularly vpon the ſides B C, C A, A D, and C B, by the Corollary of the 12. of the thirteenth, and therefore they ſhall cut them into two equall partes in the pointes K, L, M, by the 3. of the third. Agayne let the lines which wer drawen from the ſolide angles to the oppoſite baſes be diuided into two equal partes, namely, the line D G in the point T, the line C N in the point O, the line A F in the point P, and the line B H in the point R: and drawe the lines H T, F T, H O, and F O. Now foraſmuch as the lines G K, and G L, which are drawen from the centre of one and the ſelf ſame triangle A B C to the ſides, are equall, and the lines D K and D L are equall, for they are the perpendiculars of equal & like triangles: and the line D G is common to thé. Wherefore, by the 8. of the firſt, the angles K D G & L D G, are equall. And foraſmuch as the lines H D & D F are drawen from the centre of equal circles which cõtaine the equal triangles A D C & D B C, therfore they are equal, & thé line D T is cómon to thé both, and they containe equal angles, as before hath bene proued. Wherfore the baſes H T and F T are equal by the 4. of the firſt. And by the ſame reaſon if we drawe the lines C F and C H, may we proue that the other lines H O and F O, are equal to the ſame lines H T and F T, and alſo the one to the other. Wherfore alſo after the ſame maner may be proued that the reſt of the lines, which couple the centres of the triangles and the ſections of the perpendiculars into two equal partes, as the lines N P, G R, G P, R N: N T, P H, G O, and R F, are equal. And foraſmuche as from euery one of the centres of the baſes are drawne thre right lines to the ſections into two equal parts of the perpendiculers, and there are foure centres, it followeth, that theſe equal right lines ſo drawne, are twelue in number, of which euery three and three make a ſolide angle in the foure centres of the baſes, and in the foure ſections into two equal partes of the perpendiculars: wherfore that ſolide hath 8. angles, contained vnder 12. equal ſides, which make ſixe quadrangled figures, namely, H O F T, P G R N, P H O G, G O F R, F R N T, and T N P H. Now let vs proue that thoſe quadrangled figures are rectangle.

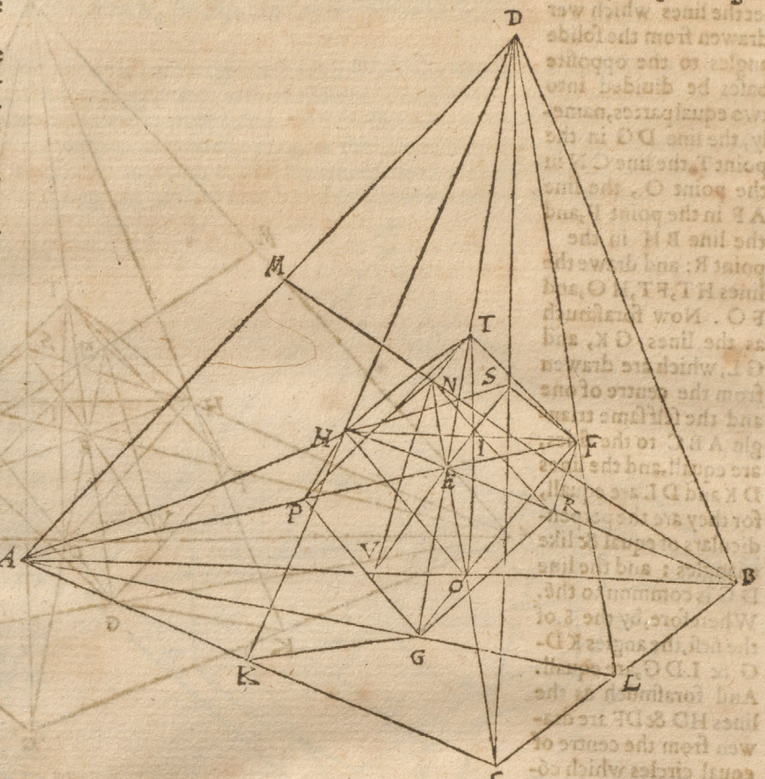


Demonſtration.

Produce in the figure the line T F to the point B.

Foraſmuch as vpon D C the common baſe of the triangles A D C and B D C falleth the perpendiculars A S and B S, which are drawne by the centres H and F: either of theſe lines H S and S F ſhalbe the third part of either of theſe lines A S and S B: for the line A H is duple to the line H S, and deuideth the baſe D C into two equal partes by the corollary of the 12. of the thirteenth. Wherefore in the triangle A B S, the ſides A S and B S are cut proportionally in the poynts H and F: and therfore the line H F is a parallel to the ſide A B, by the 2. of the ſixth. Wherfore the triangles A S B and H S F are equiangle, by the 6. of the ſixth. Wherfore the baſe H F ſhalbe the third part of the baſe A B, by the 4. of the ſixth. We may alſo proue that the line T O is the third part of the line D C, for the lines E C and E D, which are drawne from the centre of the ſphere which containeth the pyramis are equal: and the line E N, (which is drawne from the centre to the baſe) is the third part of the line E C, ſo alſo is the line G E the third part of the line E D (by the corollary of the 13. of the thirteenth) for it is the ſixth parte of the diameter of the ſphere which containeth the pyramis: And the line O N, is the half of the whole line N C wherfore the reſidue E O is the third part of the line E C, and ſo alſo is the line E T the third part of the line E D. Wherfore the line T O in the triangle D E C is a parallel to the line D C, and is a third parte of the ſame, by the former 2. and 4. of the ſixth, as the line H F was proued the third part of the line A B. But A B and D C being ſides of the pyramis are equal. Wherfore the lines H F and T O, being the third partes of equal lines, are equal, by the 15. of the fiueth. Wherfore by the 8. of the firſt the angles H T F, and T F O are equal: and by the ſame reaſon, the angles oppoſite vnto them, namely, the angles F O H

and O H T are equal the one to the other, and also are equal to the said angles H T F and T F O: but these foure angles are equal to 4. right angles by the corollary of the 32. of the first: wherefore the angles of the quadrangle H O F T are right angles. And by the same reason may the angles of the other fiue quadrangled figures be proued right angles. Now resteth to proue that the foresayde quadrangles are ech in one and the selfe same plaine.



Take the quadrāgle H O F T: and forasmuch as in the triangle A S B, the line H F is proued a parallel to the line A B, therefore it cutteth the lines S V, and S B proportionally in the poynts I. and F. by the 2. of the sixth: Now then forasmuch as S F was proued the third parte of the line S B, the line S I, shall also be the thirde part of the line S V. Moreouer forasmuch as the line V S, whiche coupleth the sections into equal partes of the opposite sides of the pyramis, namely, of the sides A B and D C, is by the centre E deuided into two equal partes, by the corollary of the seconde of this boke (for it is the diameter of the octohedron inscribed in the pyramis): therfore the line S I is two third partes of the halfe line S E. And by the same reaso, forasmuch as in the triāgle D E C the line T O is proued to be a parallel to the side D C, it shall in the selfe same triangle cut the lines C E and S E, proportionally in the poynts O and I by the same 2. of the sixth: but the line E O is proued to be a third parte of the line E C. Wherefore the lyne E I is also a third part of the line E S. Wherefore the residue I S shalbe two third partes of the whole line E S. Wherefore the point I cutteth either of the lines T O and H F. Wherefore the two lines H I F and T I O cutting the one the other, are in one and the selfe same' plaine, by the 2. of the eleuenth. And therefore the poyntes H, T, F, O are in one & the selfe same plaine. Wherfore, the rectangle figure H O F T being quadrilater and equilater, and in one and the selfe same playne, is a square, by the diffinition of a square. And by the same reason may the rest of the bases of the solide be proued to be squares equall and plaine or superficial: Now then the solide is comprehended of 6. equal squares (which are contained of 12. equal sides) which squares make 8. solide angles, of which foure are in the centres of the bases of the pyramis, and the other 4. are in the midle sections of the foure perdendiculars. Wherfore the solide H O F T P G R N, is a cube by the 21. diffinition of the eleuenth, and is inscribed in the pyramis, by the first definition of this boke. Wherfore in a trilater equilater pyramis geuen, is inscribed a cube.

¶ A Corrollary.

The line which cutteth into two equall partes the opposite sides of the Pyramis, is triple to the side of the cube inscribed in the pyramis, and passeth by the centre of the cube. For the line S E V, whose third part the line S I is, cutteth the opposite sides C D and A B into two equll partes: but the line E I (which is drawne from the centre of the cube to the base is proued to be a third part of the line E S: wherefore the side of the cube which is double to the line E I shall be a third part of the whole line V S, which is (as hath bene proued) double to the line E S.

The 19. Probleme *The 19. Proposition.*

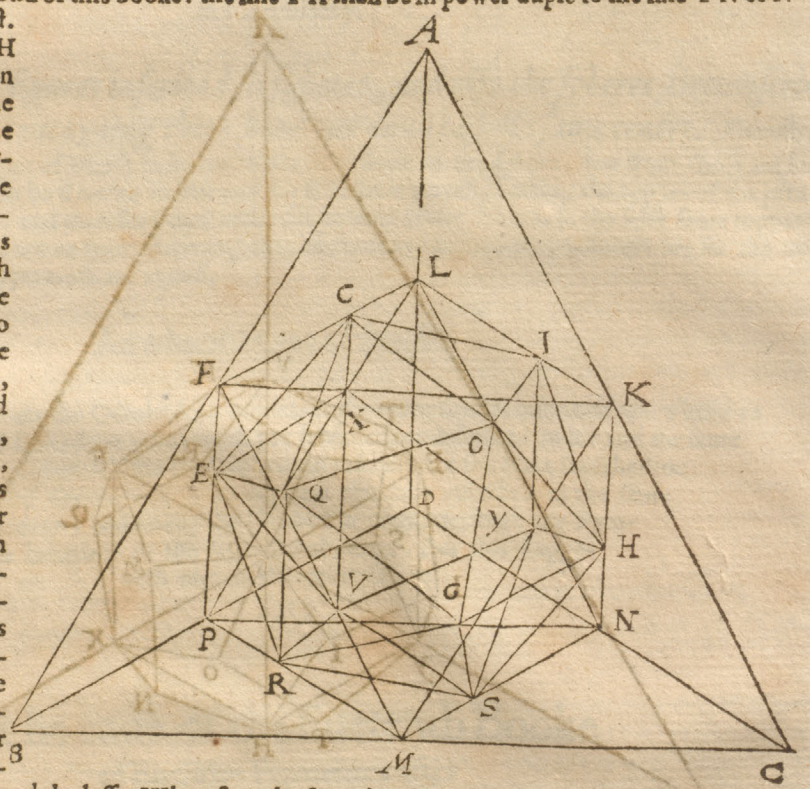
In a trilater equilater Pyramis geuen, to inscribe an Icosahedron.

Suppose

SVppoſe that the pyramis geuen, be A B G D: euery one of whoſe ſides let be diuided into two equall partes in the poyntes F, M, K, L, P, N. And in euery one of the baſes of that pyramis, deſcride the triangles L F P, P M N, N K L, and F M K: which triangles ſhall be equilater by the 4. of the firſt, for the ſides ſubtend equall angles of the pyramis, contayned vnder the halues of the ſides of the ſame pyramis: wherfore the ſides of the ſaid triangles are equall. Let thoſe ſides be diuided by an extreame and meane proportion (by the 30. of the ſixth) in the poyntes C, E, Q, R, S, T, H, I, O, V, Y, X. Now then thoſe ſides are cutte into the ſelfe ſame proportions, by the 2. of the fourteth: and therfore they make the like ſectiõs equall, by the 2. part of the ninth of the fiueth. Now I ſay, that the foreſayd poyntes doo receaue the angles of the Icoſahedron inſcribed in the pyramis A B G D. In the foreſayd triangles let there agayne be made other triangles by coupling the ſections, and let thoſe triangles be T R S, I O H, C E Q, and V X Y, which ſhall be equilater: for euery one of their ſides doo ſubtend equall angles of equilater triangles, and thoſe ſayd equall angles are contayned vnder equall ſides (namely, vnder the greater ſegment and the leſſe): and therefore the ſides which ſubtend thoſe angles are equall by the 4. of the firſt. Now let vs proue that at eche of the foreſayd poynts, as for example at T, is ſet the ſolide angle of an Icoſahedron. Foraſmuch as the triangles T R S and T Q O are equilater and equall, the 4. right lines T R, T S, T Q and T O ſhall be equall. And foraſmuch as F P N K is a ſquare cutting the pyramis A B G D into two equall partes, by the corollay of the ſecond of this booke: the line T H ſhall be in power duple to the line T N or N H by the 47. of the firſt. For the lines T N or N H are equall, for that by conſtruction they are eche leſſe ſegmentes: and the line R T or T S is in power duple to the ſame line T N or N H (by the corollary of the 16. of this booke) for it ſubtendeth the angle of the triangle contayned vnder the two ſegmentes. Wherfore the lines T H, T S, T R, T Q, and T O are equall: and ſo alſo are the lines H S, S R, R Q, Q O, and O H, which ſubtend the angles at the poynt T, equall. For the line Q R contayneth in power the two lines P Q and P R the leſſe ſegmentes, which two lines the line T H alſo contayned in power. And the reſt of the lines doo ſubtend angles (of equilater triangles) contayned vnder the greater ſegment and the leſſe. Wherefore the fiue triangles T R S, T S H, T H O, T O Q, T Q R are equilater and equall making the ſolide angle of an Icoſahedron at the poynt T, by the 16. of the thirtenth, in the ſide P N of the triangle P N M. And by the ſame reaſon in the other ſides of the 4. triangles P N M, N K L, F M K, & L F P (which are inſcribed in the baſes of the pyramis) which ſides are 12. in nũber ſhal be ſet 12. angles of the Icoſahedrõ cõtained vnder 20. equal & equilater triangles of which fowere are ſet in the 4. bales of the pyramis, namely, theſe fower triangles, T R S, H O I, C E Q, V X Y: 4. triangles are vnder 4. angles of the pyramis: that is, the fower triangles C I X, Y S H, E R V, T Q O: and vnder euery one of the fixe ſides of the pyramis are ſet two triangles, namely, vnder the ſide, of the triangles T H S and T H O: vnder the ſide D B the triangles R Q E and R Q T: vnder the ſide D A the triangles C O Q, and C O I: vnder the ſide A B, the triangles E X C and E X V: vnder the ſide B G the triangles S V R and S V Y: and vnder the ſide A G the triangles I Y H and I Y X. Wherefore the ſolide being contayned vnder 20. equilater and equall triangles ſhall be an Icoſahedron by the 23. diffinition of the eleuenth: and ſhall be inſcribed in the pyramis A B G D by the firſt diffinition of this booke, for all his angles doo at one time touch the baſes of the pyramis. Wherefore in a trilater equilater pyramis geuen, we haue inſcribed an Icoſahedron.

Conſtruction.

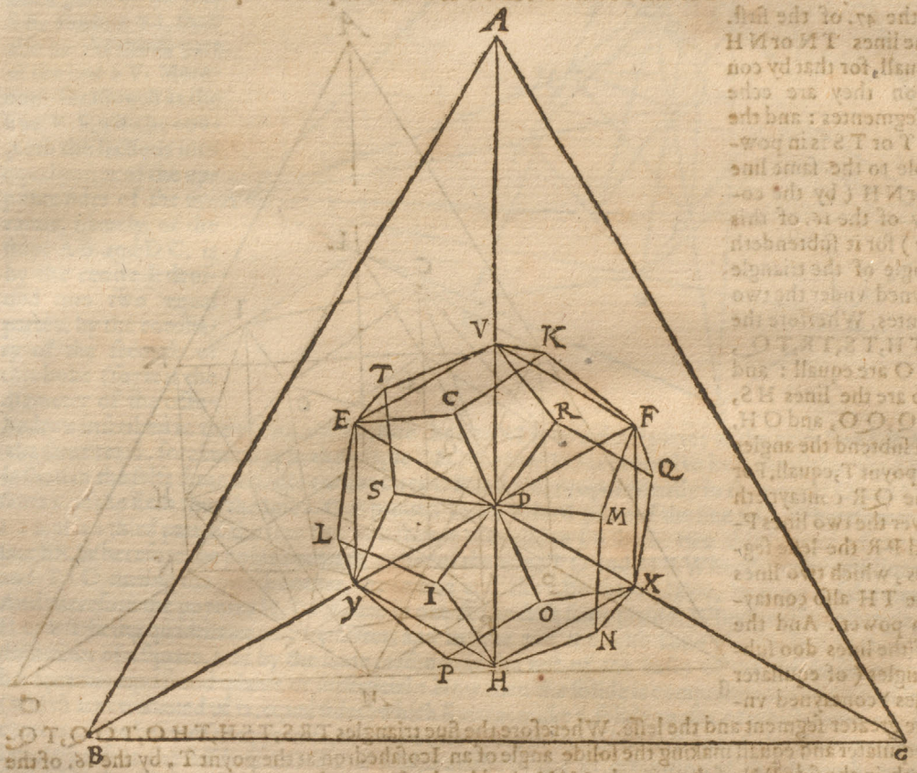
Demonſtration.



¶ *The 20. Propoſition.* *The 20. Probleme.*

In a trilater equilater Pyramis geuen, to inſcribe a dodecahedron.

SVppoſe that the pyramis geuen be A B G D, eche of whoſe ſides let be cutte into two equall partes: and draw the lines which couple the ſections, which being diuided by an extreame and meane proportion, and right lines being drawne by the ſections, ſhall receaue 20. triangles making an Icoſahedron, as in the former propoſition it was manifeſt. Now then if we take the centres of thoſe triangles, we ſhall there finde the 20. angles of the dodecahedron inſcribed in it by the 5. of this booke. And foraſmuch as 4. baſes of the foreſayd Icoſahedrō are cōcentricall with the baſes of the pyramis, as it was proued in the 2. corollary of the 6. of this boke: there ſhal be placed 4. angles of the dodecahedrō, namely, the 4. angles E, F, H, D, in the 4. centres of the baſes: and of the other 16. angles, vnder euery one of the 6. ſides of the pyramis are ſubtended two: namely, vnder the ſide A D, the angles C K: vnder the ſide B D the angles L I: vnder the ſide G D the angles M, N: vnder the ſide A B the angles T, S: vnder the ſide B G the angles P, O: and vnder the ſide A G the angles R, Q: ſo there reſt 4. angles, whoſe true place we will now appoynt. Foraſmuch as a cube contayned in one and the ſelfe ſame ſphere with a dodecahedron, is inſcribed in the ſame dodecahedron, as it was manifeſt by the 17. of the thirteenth, and 8. of this booke: it followeth that a cube and a dodecahedron circumſcribed about it, are contayned in one and the ſelfe ſame bodies, for that their angles concurre in one and the ſelfe ſame poyntes. And it was proued in the 18. of this booke, that 4. angles of the cube inſcribed in the pyramis are ſet in the middle ſections of the



perpendiculars which are drawne from the ſolide angles of the pyramis to the oppoſite baſes: wherefore the other 4. angles of the dodecahedron are alſo, as the angles of the cube, ſet in thoſe middle ſections of the perpendiculars. Namely, the angle V is ſet in the middeſt of the perpendicular A H: the angle Y in the middeſt of the perpendicular B F: the angle X in the middeſt of the perpendicular G E: and laſtly the angle D in the middeſt of the perpendicular D which is drawne from the toppe of the pyramis to the oppoſite baſe. Wherefore thoſe 4. angles of the dodecahedron may be ſayd to be directly vnder the ſolide angles of the pyramis, or they may be ſayd to be ſet at the perpendiculars. Wherefore the dodecahedron after this maner ſet, is inſcribed in the pyramis geuen (by the firſt diffinition of this booke) for that vpō euery one of the baſes of the pyramis are ſet an angle of the dodecahedrō inſcribed. Wherefore in a trilater equilater pyramis is inſcribed a dodecahedron.

The 21. Probleme. *The 21. Propoſition.*

In euery one of the regular ſolides to inſcribe a Sphere.

IN the 13. of the thirteenth and the other 4. propoſitions following, it was declared that the 5. regular ſolides, are ſo contayned in a ſphere, that right lines drawne from the centre of the ſphere or of the ſolide inſcribed, to euery one of the angles of the ſolide inſcribed, are equall. Which right lines therefore make pyramids, whoſe toppes are the centre of the ſphere, or of the ſolide, and the baſes are euery one of the baſes of thoſe ſolides. And foraſmuch as thoſe baſes are in euery ſolide equall and like the one to the other, and deſcribed in equall circles: thoſe circles ſhall cutte the ſphere: for the angles which touch the circumference of the circle, touch alſo the ſuperficies of the ſphere. Wherefore perpendiculars drawne from the centre of the ſphere to the baſes, or to the playne ſuperficieces of the equall circles, are equall, by the corollary of the aſſumpt of the 16. of the twelfth. Wherefore making the centre the centre of the ſphere which contayneth the ſolide, and the ſpace ſome one of the equall perpendiculars, deſcribe a ſphere, and it ſhall touch euery one of the baſes of that ſolide: neither ſhall the ſuperficies of the ſphere paſſe beyond thoſe baſes: when as thoſe perpendiculars are the left lines which are drawne from the centre to the baſes, by the 3. corollary of the ſame aſſumpt. Wherefore we haue in euery one of the regular bodies inſcribed a ſphere: which regular bodies are in number onely fiue, by the corollary of the 18. of the thirteenth.

This propoſition Campane hath, & is the laſt alſo in order of the 15. booke with him.

A Corollary.

The regular figures inſcribed in ſpheres, and alſo the ſpheres circumſcribed about them, or contayning them, haue one and the ſelfe ſame centre. Namely, their pyramids, the angles of whoſe baſes touch the ſuperficies of the ſphere, doo from thoſe angles cauſe equall right lines to be drawne to one and the ſelfe ſame poynt, making the toppes of the pyramids in the ſame poynt: and therefore they make the centres of the ſpheres in the ſelfe ſame toppes: when as the right lines drawne from thoſe angles to the crooked ſuperficies, wherein are ſet the angles of the baſes of the pyramids, are equall.

An aduertiſment of *Fluſſas*

Of theſe ſolides, onely the Octohedron receaueth the other ſolides inſcribed one within an other. For the Octohedron contayneth the Icoſahedron inſcribed in it: and the ſame Icoſahedron contayneth the Dodecahedron inſcribed in the ſame Icoſahedron: and the ſame dodecahedron contayneth the cube inſcribed in the ſame Octohedron, and finally the ſame cube circumſcribeth the Pyramis inſcribed in the ſayd Octohedron. But this happeneth not in the other ſolides.

(:)

The ende of the fiueteenth Booke of Euclides Elementes after *Campane* and *Fluſſas*.

¶ The sixtenth booke of the Elementes of Geometrie added by *Flussas*.

The argument of the 16. booke.

IN the former fiuetenth booke hath bene taught how to inscribe the fiue regular solides one with in an other. Now semeth to rest, to cõpare those solids so inscribed, one to an other, and to set forth their passions and proprieties: which thing, *Flussas* considering, in this sixtẽth booke added by him, hath excellently well and most conningly performed. For which vndoubtedly he hath of all them which haue a loue to the Mathematicals, deserued much prayse and commendacion: both for the great trauailes and paynes (which it is most likely) he hath taken in inuenting such straunge and wonderfull propositions with their demonstrations, in this booke contayned, as also for participating and communicating abrode the same to others. Which booke also, that the reader should want nothing conducing to the perfection of Euclides Elements: I haue with some trauaile translated, & for the worthines thereof haue added it, as a sixtenth booke to the 15. bookes of Euclide. Vouchsafe therefore gentle reader diligently to read and peyse it, for in it shall you finde not onely matter strange and delectable, but also occasion of inuention of greater things pertayning to the natures of the fiue regular solides.

¶ *The 1. Proposition.*

A Dodecahedron, and a cube inscribed in it, and a Pyramis inscribed in the same cube, are contained in one and the selfe same sphere.

For the angles of the pyramis are set in the angles of the cube wherein it is inscribed (by the first of the fiuetenth: and all the angles of the cube are set in the angles of the dodecahedron circumscribed about it (by the 8. of the fiuetenth): And all the angles of the Dodecahedron, are set in the superficies of the sphere, by the 17. of the thirteenth. Wherefore those three solides inscribed one within an other, are contained in one and the selfe same sphere, by the first diffinition of the fiuetenth. A dodecahedron therfore and a cube inscribed in it, and a pyramis inscribed in the same cube, are contained in one and the selfe same sphere.

¶ A Corollary.

These three solides likewise are set in one and the selfe same Icosahedron, or Octohedron, or Pyramis. For they are inscribed in one and the same Icosahedron, by the, 5. 11. & 12. of the fiuetenth: and they are inscribed in one and the selfe same Octohedron, by the 4. 6. and 16. of the same: lastly they are inscribed in one and the selfe same pyramis, by the first, 18. and 19. of the same. For the angles of all these solides are set in the centres of the bases of the circumscribed Icosahedron, or octohedron, or pyramis.

¶ *The 2. Proposition.*

The proportion of a Dodecahedron circumscribed about a cube, to a Dodecahedron

hedrō inscribed in the same cube, is triple to an extreme & meane propartiō.

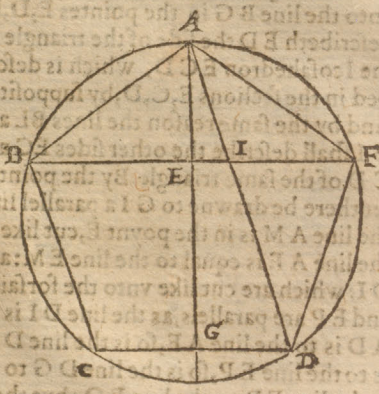
Forasmuch as in the 2. corollary of the 13. of the fiuetenth, it was proued, that the side of a Dodecahedron inscribed in a cube, is the lesse segment of the side of that cube deuided by an extreme and meane proportion: and the side of the dodecahedron circumscribed about the same cube, is the greater segment of the side of the same cube (which thing also was taught in the 13. of the fiuetenth) the side of the Dodecahedron circumscribed, shalbe to the side of the Dodecahedron inscribed, as the greater segment of a right line deuided by an extreme and meane proportion, is to the lesse segment of the same, which proportion is called an extreme and meane proportion, by the diffinition, and by the 30. of sixth. But the proportion of like solide prolihedrons, is triple to the proportion of the sides of like proportion, by the corollary of the 17. of the twelueth. Wherefore the proportion of the Dodecahedron circumscribed about the cube, is to the dodecahedron inscribed in the same cube, in triple proportion of the sides ioyned together by an extreme and meane proportion. The proportion therefore of a Dodecahedron circumscribed about a cube to a dodecahedron inscribed in the same cube, is triple to an extreme and meane proportion.

The 3. Proposition.

In euery equiangle, and equilater Pentagon, a perpendicular drawne from one of the angles to the base, is deuided by an extreme and meane proportion by a right line subtending the same angle.

Construction.

Vppose that A B C D F be an equiangle and equilater pentagon: and from one of the angles namely, from A, let there be drawne to the base C D a perpendicular A G: and let the line B F subtend the angle B A F, which line B F let the line A D cut in the poynt I. Then I say that the line B F, cutteth the line A G by an extreme and meane proportion. For forasmuche as the angles G A F and G A B are equal by the 27. of the third, and the angles A B F and A F B, are equal by the 5. of the first: therefore the angles remaining at the poynt E, of the triangles A E B and A E F are equal: for that they are the residues of two right angles by the corollary of the 32. of the first. But the angle E G C, is by construction a right angle: wherfore the lines B F & C D are parallels by the 28. of the first. Wherefore as the line D I is to the line I A, so is the line G E to the line E A, by the 2. of the sixth. But the line D A, is in the point I deuided by an extreme and meane proportion, by the 8. of the thirteenth. Wherfore the line G A is in the poynt E, deuided by an extreme and meane proportion (by the 2. of the fourteenth). Wherfore in euery equiangle and equilater pentagon, a perpendicular drawne from one of the angles to the base, is deuided by an extreme and meane proportion by a right line subtending the same angle.



Demonstration.

¶ A Corollary.

The line which subtendeth the angle of a pentagon, is a parallel to the side opposite vnto the angle. As it was manifest in the lines B F and C D.

The 4. Proposition.

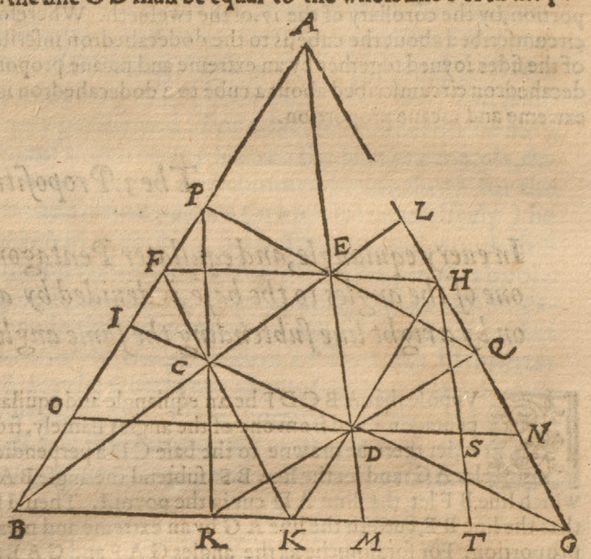
*If frō the angles of the base of a * Pyramis, be drawne to the opposite sides, right lines cutting the sayd sides by an extreme and meane proportion: they shall containe the base of the Icosahedron inscribed in the Pyramis, which base shalbe inscribed in an equilater triangle, whose angles cut the sides of the base of the Pyramis by an extreme and meane proportion.*

** By a Pyramis vnderstand a Tetrahedron throughout all this booke.*

Construction.

SVppose that A B G be the base of a pyramis, in which let be inscribed an equilater triangle F K H, which is done by deuiding the sides into two equal partes. And in this triangle let there be inscribed the base of the Icosahedrō inscribed in the pyramis: which is described by deuiding the sides FK, KH, HF, by an extreme & meane proportiō in the poynts C, D, E, by the 19. of the fiuetēth. Againe let the sides of the pyramis, namely, A B, B G, and G A be deuided by an extreme and meane proportion in the poynts I, M, L, by the 30. of the sixth. And drawe these right lines A M, B L, G I. Then I say that those lines describe the triangle CDE of the Icosahedron. For forasmuch as the lines B G and F H are parallels, by the 2. of the sixth: by the point D let the line ODN be drawne parallel to either of the lines B G & F H. Wherfore the triangle HDN shalbe like to the triangle HKG, by the corollary of the 2. of the sixth. Wherfore either of these lines D N and N H shall be equal to the line D H, the greater segment of the line K H or F H. And forasmuch as the line F O is a parallel to the line H K, and the line O D to the line F H: the line O D shall be equal to the whole line F H in the parallelogramme F O D H, by the 34. of the first. Wherefore as the whole line F H is to the greater segment F E, so shall the lines equal to them be, namely, the line O D to the line DN, by the 7. of the fifth. Wherfore the line O N is deuided by an extreme and meane proportion in the poynt D, by the 2. of the fourteenth. But the triangles A O D, A F E, and ABM, are like the one to the other, and so also are the triangles A D N, A E H, and A M G, by the corollary of the secōd of the sixth: Wherefore as F E is to E H, so is O D to D N, and B M to M G. Wherfore the line A M cutting the lines F H and O N, lyke vnto the line B G in the pointes E, D, M, describeth E D the side of the triangle of the Icosahedron E C D, which is described in the sections E, C, D, by suppositiō. And by the same reason the lines BL and G I shall describe the other sides EC and C D of the same triangle. By the point E, let there be drawne to G I a parallel line P E Q. Now forasmuch as the lines B M and F E are parallels, the line A M is in the poynt E, cut like to the line A B in the poynt F, by the 2. of the sixth. Wherefore the line A E is equal to the line E M: and vnto the line E M also are equal either of the lines G D and D I: which are cut like vnto the forsaid lines. Againe forasmuche as in the triangle A D I the lines D I and E P are parallels, as the line D I is to the line E P, so is the line A D to the line A E: but as the line A D is to the line A E, so is the line D G to the line E Q by the 2. of the sixth: wherefore as the line D I is to the line E P, so is the line D G to the line E Q: and alternately as the line D I is to the line D G, so is the line E P to the line E Q: but the lines D I and I G are equal: wherfore also the lines EP and EQ are equal. And forasmuch as the line A H is equal to the line F H, whose greater segmēt is the line HN: therfore the whole line A N, is deuided by an extreme and meane proportion in the poynt H, by the 5. of the thirtenth. But as the line A N is to the line A H, so is the line A D to the line A E, by the 2. of sixth (for the lines F H and O N are parallels:) and againe as the line A D is to the line A E, so (by the same) is the line A G to the line A Q, and the line A I to the line A P: for the lines P Q, and G I are parallels: Wherefore the lines A G and A I are deuided by an extreme and meane proportion in the points Q & P: & the line A Q shalbe the greater segmēt of the line A G or A B. And forasmuch as the whole line A G is to the greater segment A Q, as the greater segment A I is to the residue A P: the line A P shalbe the lesse segment of the whole line A B or A G. Wherfore the line P E Q (which by the poynt E passeth parallelwise to the line G I) cutteth the lines A G and B A by an extreme and meane proportion in the poynts Q and P. And by the same reason the line P R (which by the poynt C, passeth parallelwise to the line A M) shall fall vpon the sections P and R: so also shal the line RQ (which by the poynt D passeth parallelwise to the line B L) fall vpon the sections R Q. Wherefore either of the lines P E and E Q shalbe equal to the line C D, in the parallelogrammes P D, and Q C, by the 34. of the first. And forasmuch as the lines P E and E Q are equal, the lines P C, C R, R D and D Q shalbe likewise equal. Wherfore the triangle P R Q is equilater, and cutteth the sides of the base of the pyramis in the poyntes P, Q, R, by an extreme and meane proportion. And in it is inscribed the base E C D of the Icosahedron contained in the foresayd pyramis. If therefore from the angles of the base of a pyramis, be drawne to the opposite sides, right lines cutting the sayde sides by an extreme and meane proportion: they shall containe the base of the Icosahedron inscribed in the pyramis, which base shall be inscribed in an equilater triangle, whose angles cut the sides of the base of the pyramis by an extreme & meane proportion.

Demonstration.



¶A

¶ A Corollary.

The side of an Icosahedron inscribed in an Octohedron, is the greater segment of the line, which being drawen from the angle of the base of the Octohedron cutteth the opposite side by an extreame and meane proportion.

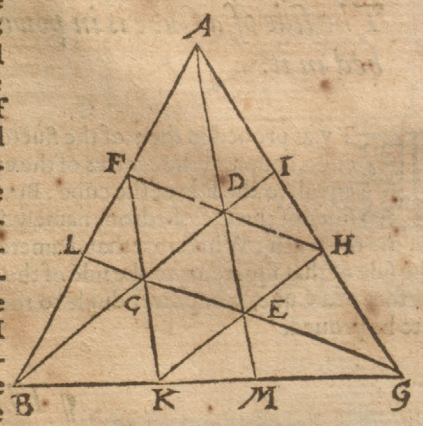
For, by the 16. of the fiuetenth, F K H is the base of the Octohedron, which containeth the base of the Icosahedron C D E: vnto which triangle F K H, the triangle H K G is equall, as hath bene proued. By the point H draw vnto the line M E a parallel line H T, cutting the line D N in the point S. Wherefore E S, D T, and E T, are parallelogrammes: and therefore the lines E H and M T are equall: and the lines E M and H T are like cut in the pointes D and S, by the 34. of the first. Wherefore the greater segment of the line H T is the line H S, which is equall to E D the side of the Icosahedron. But (by the 2. of the sixth) the line T K is cut like to the line H K by the parallel D M. And therefore (by the 2. of the fourteenth) it is diuided by an extreme and meane proportion. But the line T M is equall to the line E H. Wherefore also the line T K is equall to the line E F or D H. Wherefore the residues E H and T G are equall. For the whole lines F H and K G are equall. Wherefore K G the side of the triangle H K G is in the point T diuided by an extreme and meane proportion in the point T, by the right line H T, and the greater segment thereof is the line E D the side of the Icosahedron inscribed in the Octohedron, whose base is the triangle H K G (or the triangle F K H which is equall to the triangle H K G) by the 16. of the fiuetenth.

¶ *The 5. Proposition.*

The side of a Pyramis diuided by an extreme and meane proportion, maketh the lesse segment in power double to the side of the Icosahedron inscribed in it.

Construction.

Vppose that A B G be the base of a pyramis: and let the base of the Icosahedron inscribed in it, be C D E, described of three right lines, which being drawen from the angles of the base A B G cut the opposite sides by an extreme and meane proportion, by the former Proposition: namely, of these three lines A M, B I, and G L. Then I say, that A I the lesse segment of the side A G, is in power duple to C E the side of the Icosahedron. For, forasmuch as by the former Proposition, it was proued that the triangle C D E is inscribed in an equilater triangle, whose angles cut the sides of A B G the base of the pyramis by an extreme and meane proportion, let that triangle be F H K, cutting the line A B in the point F. Wherefore the lesse segment F A is equall to the segment A I, by the 2. of the fouretenth: (for the lines A B and A G are cut like). Moreouer the side F H of the triangle F H K is in the point D cut into two equall partes, as in the former Proposition it was proued, and F C E D also by the same is a parallelogramme: Wherefore the lines C E and F D are equall, by the 33, of the first. And forasmuch as the line F H subtendeth the angle B A G of an equilater triangle, which angle is contained vnder the greater segment A H and the lesse segment A F: therefore the line F H is in power double to the line A F or to the line A I the lesse segment, by the Corollary of the 16. of the fiuetenth. But the same line F H is in power quadruple to the line C E, by the 4. of the second: (for the line F H is double to the line C E). Wherefore the line A I being the halfe of the square of the line F H is in power duple to the line C E, to which the line F H was in power quadruple. Wherefore the side A G of the pyramis being diuided by an extreme and meane proportion, maketh the lesse segment A I in power duple to the side C E of the Icosahedron inscribed in it.



Demonstration.

¶ A Corollary.

The side of an Icosahedron inscribed in a pyramis, is a residuall line.

For the diameter of the Sphere which containeth the fiue regular bodies, being rationall, is in power sesquialtera to the side of the pyramis, by the 13. of the thirteenth: and therefore the side of the pyramis is rationall, by the definition: which side being diuided by an extreme and meane proportion, maketh

the lesse segment a residuall line, by the 6. of the thirteenth. Wherefore the side of the Icosahedron being commensurable to the same lesse segment (for the square of the side of the Icosahedron is the halfe of the square of the said lesse segment) is a residuall line, by that which was added after the 103. of the tenth booke.

¶ The 6. Proposition.

The side of a Cube containeth in power halfe the side of an equilater triangular Pyramis inscribed in the said Cube.

FOr forasmuch as the side of the pyramis inscribed in the cube subtendeth two sides of the cube which containe a right angle, by the 1. of the fiueteenth: it is manifest, by the 47. of the first, that the side of the pyramis subtending the said sides, is in power duple to the side of the cube: Wherefore also the square of the side of the cube is the halfe of the square of the side of the pyramis. The side therefore of a cube containeth in power halfe the side of an equilater triangular pyramis inscribed in the said cube.

¶ The 7. Proposition.

The side of a Pyramis is duple to the side of an Octohedron inscribed in it.

FOrasmuch as by the 2. of the fiueteenth it was proued, that the side of the Octohedron inscribed in a pyramis coupleth the midle sections of the sides of the pyramis. Wherefore the sides of the pyramis and of the Octohedron are parallels, by the Corollary of the 39. of the first: and therefore, by the Corollary of the 2. of the sixth, they subtend like triangles. Wherfore (by the 4. of the sixth) the side of the pyramis is double to the side of the Octohedron, namely, in the proportion of the sides. The side therefore of a pyramis is duple to the side of an Octohedron inscribed in it.

¶ The 8. Proposition.

The side of a Cube is in power duple to the side of an Octohedron inscribed in it.

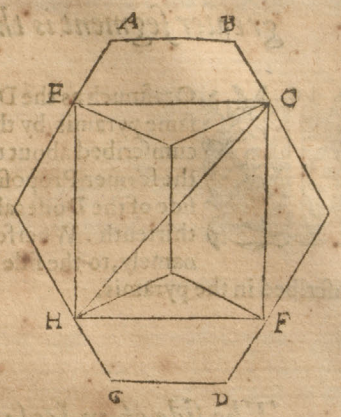
IT was proued in the 3. of the fiueteenth, that the diameter of the Octohedron inscribed in the cube, coupleth the centres of the opposite bases of the cube. Wherefore the said diameter is equall to the side of the cube. But the same is also the diameter of the square made of the sides of the Octohedron, namely, is the diameter of the Sphere which containeth it, by the 14. of the thirteenth. Wherefore that diameter being equall to the side of the cube, is in power double to the side of that square, or to the side of the Octohedron inscribed in it, by the 47. of the first. The side therefore of a Cube, is in power duple to the side of an Octohedron inscribed in it: which was required to be proued.

¶ The 9. Proposition.

The side of a Dodecahedron, is the greater segment of the line which containeth in power halfe the side of the Pyramis inscribed in the sayd Dodecahedron.

Construction. SVppose that of the Dodecahedron A B G D the side be A B: and let the base of the cube inscribed in the Dodecahedron be E C F H, by the 8. of the fiueteenth. And let the side of the pyramis inscribed in the cube be C H, by the 1. of the fiueteenth. Wherefore the same pyramis is inscribed in the Dodecahedron, by the 10. of the fiueteenth. Then I say, that A B the side of the Dodecahedron is the greater segment of the line which containeth in power halfe the line C H, which is the side of the pyramis inscribed in the Dodecahedron.

Dodecahedron. For forasmuch as E C the side of the cube being diuided by an extreme and meane proportion maketh the greater segment the line A B, the side of the Dodecahedron, by the first Corollary of the 17. of the thirteenth: (For they are contained in one and the selfe same Sphere (by the first of this booke): and the line E C the side of the cube containeth in power the halfe of the side C H, by the 6. of this booke. Wherefore A B the side of the Dodecahedron, is the greater segment of the line E C, which containeth in power the halfe of the line C H, which is the side of the Dodecahedron inscribed in the pyramis. The side therefore of a Dodecahedron, is the greater segment of the line which containeth in power halfe the side of the Pyramis inscribed in the said Dodecahedron.



Demonstration.

¶ *The 10. Proposition.*

The side of an Icosahedron, is the meane proportionall betwene the side of the Cube circumscribed about the Icosahedron, and the side of the Dodecahedron inscribed in the same Cube.

Vppose that there be a cube A B F D, in which let there be inscribed an Icosahedron C L I G O R, by the 14. of the fiueteenth. Let also the Dodecahedron inscribed in the same be E D M N P S, by the 13. of the same. Now forasmuch as C L the side of the Icosahedron is the greater segmẽt of A B the side of the cube circumscribed about it, by the 3. Corollary of the 14. of the fiueteenth: and the side E D of the Dodecahedrõ inscribed in the same cube is the lesse segmẽt of the same side A B of the cube, by the 2. Corollary of the 13. of the fiueteenth: it followeth that A B the side of the cube being diuided by an extreme and meane proportion, maketh the greater segment C L the side of the Icosahedron inscribed in it, and the lesse segment E D the side of the Dodecahedron likewise inscribed in it. Wherefore as the whole line A B the side of the cube, is to the greater segment C L the side of the Icosahedron, so is the greater segment C L the side of the Icosahedron, to the lesse segment E D the side of the Dodecahedron, by the third definition of the sixth. Wherefore the side of an Icosahedron, is the meane proportionall betwene the side of the cube circumscribed about the Icosahedron, and the side of the Dodecahedron inscribed in the same cube.

Construction.

Demonstration.

¶ *The 11. Proposition.*

The side of a Pyramis, is in power † Octodecuple to the side of the cube inscribed in it.

† *That is, as 18. to 1.*

Or, by that which was demonstrated in the 18. of the fiueteenth, the side of the pyramis is triple to the diameter of the base of the cube inscribed in it: and therefore it is in power nonecuple to the same diameter (by the 20 of the sixth). But the diamer is in power double to the side of the cube, by the 47. of the first. And the double of nonecuple maketh Octodecuple. Wherefore the side of the pyramis is in power Octodecuple to the side of the cube inscribed in it.

Demonstration.

¶ *The 12. Proposition.*

The side of a Pyramis, is in power Octodecuple to that right line, whose

greater segment is the side of the Dodecahedron inscribed in the Pyramis.

FOrasmuch as the Dodecahedron and the cube inscribed in it, are set in one and the selfe same pyramis, by the Corollary of the first of this booke: and the side of the pyramis circumscribed about the cube is in power octodecuple to the side of the cube inscribed, by the former Proposition: but the greater segment of the selfe same side of the cube, is the side of the Dodecahedron which containeth the cube, by the Corollary of the 17. of the thirteenth. Wherfore the side of the pyramis is in power octodecuple to that right line, namely, to the side of the cube, whose greater segment is the side of the Dodecahedron inscribed in the pyramis.

¶ The 13. Proposition.

The side of an Icosahedron inscribed in an Octohedron, is in power duple to the lesse segment of the side of the same Octohedron.

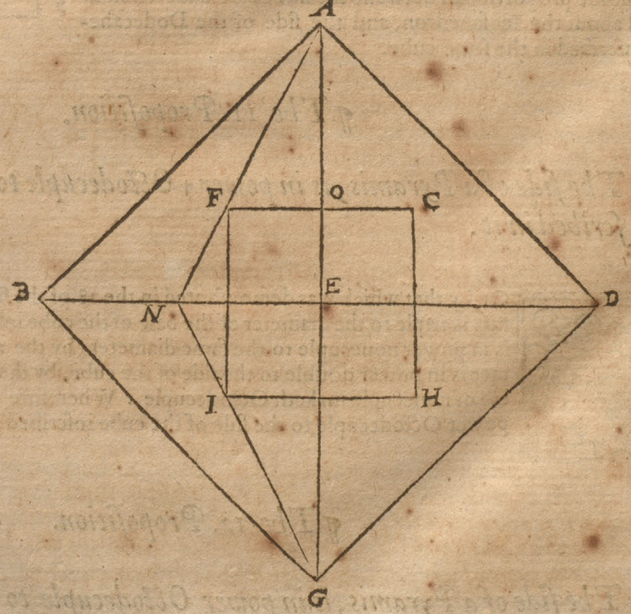
FOrasmuch as in the 17. of the fiuetenth, it was proued, that the side of an Icosahedron inscribed in a pyramis, coupleth together the two sections (which are produced by an extreme and meane proportion) of the side of the Octohedron which make a right angle: and that right angle is contained vnder the lesse segmentes of the sides of the Octohedron, and is subtended of the side of the Icosahedron inscribed: it followeth therefore, that the side of the Icosahedron which subtendeth the right angle, being in power equall to the two lines which containe the said angle, by the 47. of the first, is in power duple to euery one of the lesse segmétes of the side of the Octohedron which containe a right angle. Wherefore the side of an Icosahedron inscribed in an Octohedron, is in power duple to the lesse segment of the side of the same Octohedron.

¶ The 14. Proposition.

The sides of the Octohedron, and of the Cube inscribed in it, are in power the one to the other † in quadrupla sesquialter proportion.

† That is, as 9. to 2.

SVppose that A B G D E be an Octohedron, and let the cube inscribed in it be F C H I. Then I say, that A B the side of the Octohedron, is in power quadruple sesquialter to F I the side of the cube. Let there be drawen to B E the base of the triangle A B E a perpendicular A N: and againe let there be drawen to the same base in the triangle G B E the perpendicular G N: which A N & G N shall passe by the centres F and I: and the line A F is duple to the line F N, by the Corollary of the 12. of the thirteenth. Wherfore the line A O is duple to the line O E, by the 2. of the sixth. For the lines F O and N E are parallels. And therefore the diameter A G is triple to the line F I. Wherfore the power of A G is * noncuple to the power of F I. But the line A G is in power duple to the side A B, by the 14. of the thirteenth. Wherefore the square of the line A B, being ing the halfe of the square of the line A G, which is noncuple to the square of the line F I, is quadruple sesquialter



* That is, as 18. to 2. or 9. to 1.

alter to the square of the line F I. The sides therefore of the Octohedron and of the cube inscribed in it, are in power the one to the other, in quadruple sesquialter proportion.

¶ *The 15. Proposition.*

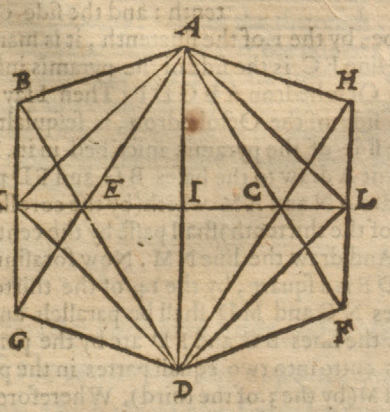
The side of the Octohedron, is in power quadruple sesquialter to that right line, whose greater segment is the side of the Dodecahedron inscribed in the same Octohedron.

FOrasmuch as in the 14. of this booke, it was proued, that the side of the Octohedron is in power quadruple sesquialter to the side of the cube inscribed in it: but the side of the cube being cut by an extreme and meane proportion, maketh the greater segment the side of the Dodecahedron circumscribed about it, by the 3. Corollary of the 13. of the fiueteenth: therefore the side of the Octohedron is in power quadruple sesquialter to that right line (namely, to the side of the cube) whose greater segment is the side of the Dodecahedron inscribed in the cube. But the Dodecahedron and the cube inscribed one within an other, are inscribed in one and the selfe same Octohedron, by the Corollary of the first of this booke. The side therefore of the Octohedron, is in power quadruple sesquialter to that right line, whose greater segment is the side of the Dodecahedron inscribed in the same Octohedron.

¶ *The 16. Proposition.*

The side of an Icosahedron, is the greater segment of that right line, which is in power duple to the side of the Octohedron inscribed in the same Icosahedron.

SVppose that there be an Icosahedron A B G D F H E C: whose side let be B G or E C: and let the Octohedron inscribed in it be A K D L: and let the side therof be A L. Then I say, that the side E C is the greater segment of that right line which is in power duple to the side A L. For forasmuch as figures inscribed and circumscribed haue one & the self same centre, by the Corollary of the 21. of the fiueteenth, let the same be the point I. Now right lines drawen by that centre to the midle sections of the opposite sides, namely, the lines A I D and K I L, do in the point I cut the one the other into two equall partes, and perpendicularly, by the Corollary of the 14. of the fiueteenth: and forasmuch as they couple the midle sections of the opposite lines B G and H F, therfore they cut them perpendicularly: wherefore also the lines B G and H F, are parallels, by the 4. Corollary of the 14. of the fiueteenth. Now then draw a line from B to H: and the sayd line B H shall be equall and parallel to the line K L, by the 33. of the first. But the line B H subtendeth two sides of the pentagon which is composed of the sides of the Icosahedron, namely, the sides B A and A H: Wherfore the line B H being cut by an extreme and meane proportion maketh the greater segment the side of the pentagon, by the 8. of the thirteenth: which side is also the side of the Icosahedron, namely, E C. And vnto the line B H the line K L, is equall: and the line K L is in power duple to A L the side of the Octohedron, by the 47. of the first: for in the square A K D L the angle K A L is a right angle. Wherefore E C the side of the Icosahedron, is the greater segment of the line B H or K L, which is in power duple to A L the side of the Octohedron inscribed in the Icosahedron. Wherefore the side of an Icosahedron, is the greater segment of that right line, which is in power duple to the side of the Octohedron inscribed in the same Icosahedron.



Draw in the figure a line from B to H.

¶ *The 17. Proposition.*

The side of a Cube is to the side of a Dodecahedron inscribed in it, in duple proportion of an extreame and meane proportion.

For

FOr it was manifest by the 2. corollary of the 13. of the fiuetenth, that the side of a cube diuided by an extreame and meane proportion, maketh the lesse segment, the side of the dodecahedron inscribed in it: but the whole is to the lesse segment in duple proportion of that in which it is to the greater, by the 10. diffinitiō of the fifth. For the whole, the greater segmēt, and the lesse, are lines in continuall proportion, by the 3. diffinition of the sixth. Wherefore the whole namely the side of the cube, is to the side of the dodecahedron inscribed in it, namely, to his lesse segment, in duple proportion of an extreame and meane proportion, namely, * of that which the whole hath to the greater segment, by the 2. of the fourteenth.

** What the duple of an extreme and meane proportion is.*

¶ The 18. Proposition.

The side of a Dodecahedron is, to the side of a Cube inscribed in it, in conuerse proportion of an extreame and meane proportion.

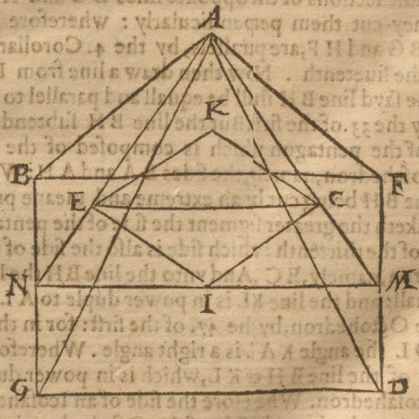
IT was proued in the 3. corollary of the 13. of the fiuetenth, that the side of a Dodecahedron circumscribed about a Cube, is the greater segment of the side of the same Cube. Wherefore the whole side of the Cube inscribed is to the greater segment, namely, to the side of the dodecahedron circumscribed, in an extreame and meane proportion: wherefore by conuersion, the greater segment, that is, the side of the dodecahedron, is to the whole, namely, to the side of the Cube inscribed, in the conuerse proportion of an extreame and meane proportion, by the 13. diffinition of the fiueth.

¶ The 19. Proposition.

The side of an Octohedron, is sesquialter to the side of a Pyramis inscribed in it.

Construction.

OCt (by the corollary of the 14. of the thirtenth) the Octohedron is cutte into two quadrilater pyramids, one of which let be A B G D F: and let the centres of the circles which contayne the 4. bases of the Octohedron be K, E, I, C. And draw these right lines K E, E I, I C, C K, and E C. Wherefore K E I C is a square, and one of the bases of the cube inscribed in the Octohedron, by the 4. of the fiuetenth. And forasmuch as the angles of a cube and of the pyramis inscribed init, are set in the centres of the bases of the Octohedron circumscribed about the cube, by the 6. of the fiuetenth: and the side of the pyramis coupleth the opposite angles of the base of the cube, by the 1. of the fiuetenth, it is manifest that the line E C is the side of the pyramis inscribed in the Octohedron A B G D F. Then I say that G D the side of the Octohedron, is sesquialter to E C the side of the pyramis inscribed in it. From the poynt A draw to the bases B G and F D perpendiculars A N and A M: which (by the corollary of the 12. of the thirteenth) shall passe by the centres E and C. And draw the line N M. Now forasmuch, as B G D F is a square, by the 14. of the thirtenth, the lines N G and M D shall be parallels and equall. For the lines B G and F D are by the perpendiculars cutte into two equall partes in the poyntes N and M (by the 3. of the third). Wherefore the lines N M and G D shall be parallels and equall, by the 33. of the first. And forasmuch as the lines A N and A M which are the perpendiculars of equall and like triangles, are cut a like in the poyntes E and C, the lines E C and N M shall be parallels, by the 2. of the sixth: and therefore by the corollary of the same, the triangles A E C, and A N M shall be like. Wherefore as the line A N is to the line A E, so is the line N M to the line E C by the 4. of the sixth. But the line A N is sesquialter to the line A E, for the line A E is duple to the line E N, by the corollary of the 12. of the thirtenth: wherefore the line N M, or the line G D which is equal vnto it, is sesquialter to the line E C. Wherefore G D the side of the Octohedron, is sesquialter to E C the side of the pyramis inscribed in it.



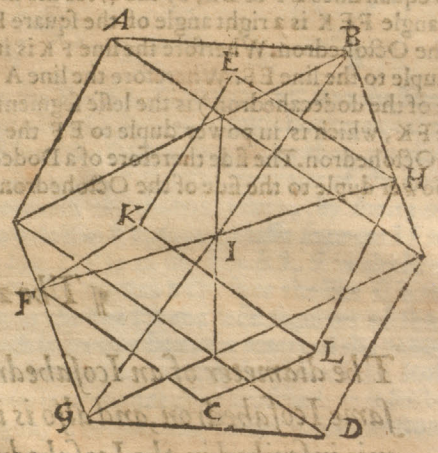
Demonstration.

¶ *The 20. Propoſition.*

If from the power of the diameter of an Icoſahedron, be taken away the power tripled of the ſide of the cube inſcribed in the Icoſahedron: the power remayning ſhall be ſeſquitertia to the power of the ſide of the Icoſahedron.

LEt there be taken an Icoſahedron A B G D: and let two baſes of the cube inſcribed in it, ioyned together be E H K L and L K F C: and let the diameter of the cube be F H and the ſide be E H, and let the diameter of the Icoſahedron be B G, and the ſide be A B. Then I ſay, that if from the power of the diameter G B, be taken away the power tripled of E H the ſide of the cube: the power remayning, ſhall be ſeſquitertia to the power of A B the ſide of the Icoſahedron. For foraſmuch as the centres of inſcribed and circumſcribed figures, are in one & the ſelfe ſame poynt, by the corollary of the 21. of the fiueteth: the diameters B G and F H ſhall in one and the ſelfe ſame poynt cutte the one the other into two equall partes: for we haue before by the ſame corollary taught, that the toppes of equall and like pyramids doo in that poynt concurre, let that poynt be the centre I. Now the angles of the cube, which are at the poyntes F and H are ſet at the centres of the baſes of the Icoſahedron, by the 11. of the fiueteenth. Wherefore the line F H ſhall be perpendicular to both the baſes of the Icoſahedrō, by the corollary of the aſſūpt of the 16. of the twelfth. Wherefore the line I B contayneth in power the two lines I H and H B, by the 47. of the firſt. But the line H B, is drawne from the centre of the circle which contayneth the baſe of the Icoſahedron, namely, the angle B is placed in the circumference, and the poynt H is the centre. Wherefore the whole line B G contayneth in power the whole lines F H and the diameter of the circle (namely, the double of the line B H) by the 15. of the fiueth. But the diameter which is double to the line H B is in power ſeſquitertia to the ſide of the equilater triangle inſcribed in the ſame circle, by the corollary of the 12. of the thirteenth. For it is in proportion to the ſide, as the ſide is to the perpendicular, by the corollary of the 8. of the ſixth. And F H the diameter of the cube, is in power triple to E H the ſide of the ſame cube, by the 15. of the thirteenth. If therefore from the power of the diameter B G, be taken away the power tripled of E H the ſide of the cube inſcribed, that is, the power of the line F H: the reſidue (namely, the power of the diameter of the circle which is duple to the line H B) ſhall be ſeſquitertia to the ſide of the triangle inſcribed in that circle: which ſelfe ſide is A B the ſide of the Icoſahedron. If therfore from the power of the diameter of an Icoſahedrō, be takē away the power tripled of the ſide of the cube inſcribed in the Icoſahedron, the power remayning ſhall be ſeſquitertia to the power of the ſide of the Icoſahedron.

Demonſtration.



A Corollary.

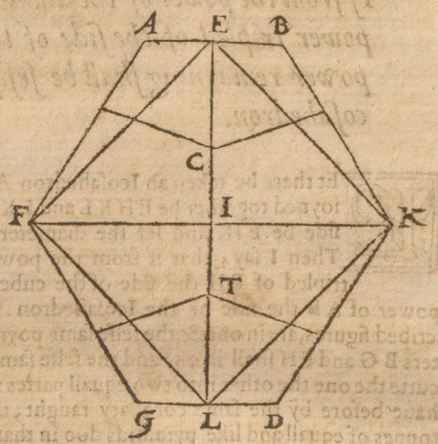
The diameter of the Icoſahedron, contayneth in power two lines, namely, the diameter of the cube inſcribed, which coupleth the centres of the oppoſite baſes, and the diameter of the circle which contayneth the baſe of the Icoſahedron.

For it was manifeſt, that B G the diameter contayneth in power the line F H which coupleth the centres, and the double of the line B H, that is, the diameter of the circle contayning the baſe wherein is the centre H.

¶ *The 21. Propoſition.*

The ſide of a Dodecahedron is the leſſe ſegment of that right line, which is in power duple to the ſide of the Octohedron inſcribed in the ſame Dodecahedron.

LEt ther be taken a Dodecahedron A B G D C T, one of whose sides let be A B. And let the Octohedron inscribed in the Dodecahedron be E F L K I: one of whose sides let be E F. Then I say that A B the side of the Dodecahedron, is the lesse segment of a certayne right line (cut by an extreame and meane proportion) which is in power duple to E F the side of the Octohedró inscribed in the Dodecahedron. Draw the diameters E L and F K of the Octohedron. Now they couple the midle sections of the opposite sides of the dodecahedron A B and G D, (by the 9. of the fiuetenth, & 3. corollary of the 17. of the thirtéth) & euery one of those diameters being diuided by an extreame and meane proportion, doo make the lesse segment, the side of the dodecahedron, by the 4. corollary of the same. Wherefore the side A B is the lesse segment of the line F K. But the line F K contayneth in power the two equall lines E F & E K, by the 47. of the first: for the angle F E K is a right angle of the square F E K L of the Octohedron. Wherfore the line F K is in power duple to the line E F. Wherefore the line A B (the side of the dodecahedron) is the lesse segment of the line F K, which is in power duple to E F the side of the Octohedron. The side therefore of a Dodecahedron is the lesse segment of that right line, which is in power duple to the side of the Octohedron inscribed in the same Dodecahedron.



¶ The 22. Proposition.

The diameter of an Icosahedron is in power sesquitertia to the side of the same Icosahedron, and also is in power sesquialter to the side of the Pyramis inscribed in the Icosahedron.

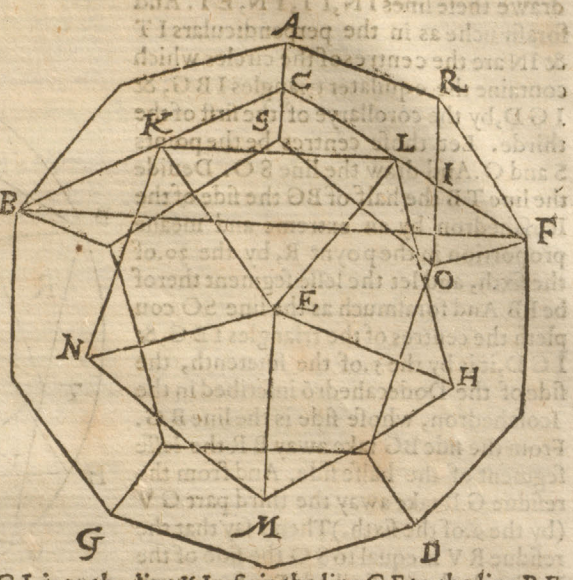
FOr forasmuch as it hath bene proued (by the 20. of this booke) that if frō the power of the diameter of the Icosahedrō be taken away the triple of the power of the side of the cube inscribed in it, there shalbe left a square sesquitertia to the square of the side of the Icosahedron: But the power of the side of the cube tripled, is the diameter of the same cube, by the 15. of the thirtéth: And the cube, & the pyramis inscribed in it are contained in one & the self same sphere, by the first of this booke, and in one & the self same Icosahedron by the corollary of the same. Wherfore one and the selfe same diameter of the cube, or of the sphere which cōtaineth the cube and the pyramis, is in power sesquialter to the side of the pyramis by the 13. of the thirteenth. Wherfore it followeth, that if from the diameter of the Icosahedron, be taken away the triple power of the side of the cube, or the sesquialter power of the side of the pyramis, which are the powers of one and the selfe same diameter, there shall be left the sesquitertia power of the side of the Icosahedron. The diameter therefore of an Icosahedron is in power sesquitertia to the side of the same Icosahedron, and also is in power sesquialter to the side of the Pyramis inscribed in the Icosahedron.

The 23. Proposition.

The side of a Dodecahedron is to the side of an Icosahedron inscribed in it, as the lesse segment of the perpendicular of the Pentagō, is to that line which is drawne from the centre to the side of the same pentagon.

Construction. LEt there be taken a Dodecahedron A B G D F S O. Whose side let be A S or S O: and let the Icosahedron inscribed in it be K L H M N E, whose side let be K L. From the two angles of the pentagons B A S and F A S of the Dodecahedron, namely, from the angles B and F, let there be drawne to the common base A S perpendicular lines B C & F C: which shal passe by the centres K & L of the sayd pentagons, by the corollary of the 10. of the thirtéth. Draw the

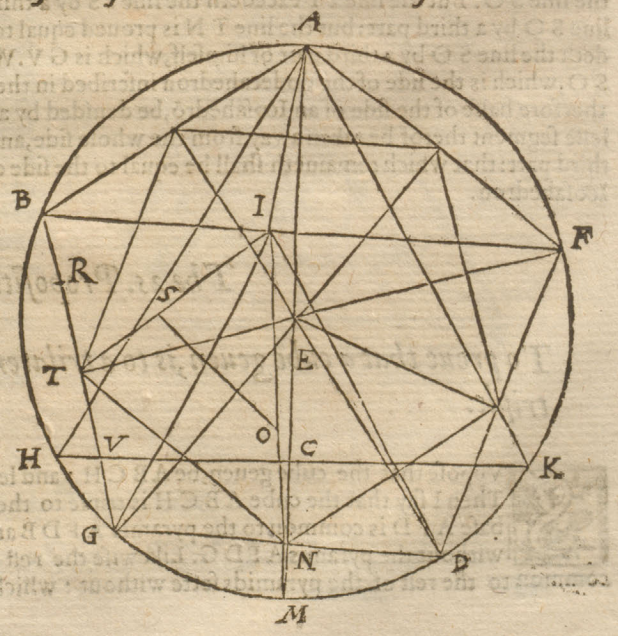
the lines B F and R O. Now forasmuche as the line R O subtendeth the angle O F R of the pentagon of the dodecahedron, it shall cut the line F C by an extreme and meane proportion, by the 3. of this booke, let it cut it in the poynt I. And forasmuche as the line K L is the side of the Icosahedron inscribed in the Dodecahedron, it coupleth the cẽtres of the bases of the dodecahedron: for the angles of the Icosahedron are set in the centres of the bases of the dodecahedron, by the 7. of the fiueteenth. Now I say that S O, the side of the dodecahedron is to K L the side of the Icosahedron, as the lesse segment I F of the perpendicular line C F, is to the line L C which is drawne from the centre L to A S the side of the pentagon. For forasmuch as in the triangle B C F the two sides C B and C F are in the centres L and K cut like proportionally, the lines B F and K L shalbe parellels, by the 2. of the sixth. Wherefore the triangles B C F, and K C L shall be equiangle, by the corollary of the same. Wherfore as the line C L is to the line K L, so is the line C F to the line B F, by the 4. of the sixth. But C F maketh the lesse segment the line I F, by the 3. of this booke, and the line B F maketh the lesse segment the line S O, namely, the side of the Dodecahedron, by the 2. corollary of the 13. of the fiueteenth For the line B F which coupleth the angles B and F of the bases of the dodecahedron, is equall to the side of the cube, which contayneth the dodecahedron, (by the. 13. of the fiueteenth). Wherefore as the whole line C F, is to the whole line B F, so is the lesse segment I F to the lesse segment S O (by the 2. of the 14). But as the line C F is to the line B F, so is the line C L proued to be to the line K L. Wherefore as the line I F is to the line S O, so is the line C L to the line K L. Wherefore alternately by the 16. of the fiueth, as the line I F the lesse segment of the perpendicular of the pentagon F A S, is to the line L C which is drawne from the centre of the pentagon, to the base, so is the line S O the side of the Dodecahedron to the line K L the side of the Icosahedron inscribed in it. The side therfore of a Dodecahedron is to the side of an Icosahedron inscribed in it, as the lesse segment of the perpendicular of the pentagon, is to that line which is drawne from the centre to the side of the same pentagon.



Demonstration.

¶ The 24. Proposition.

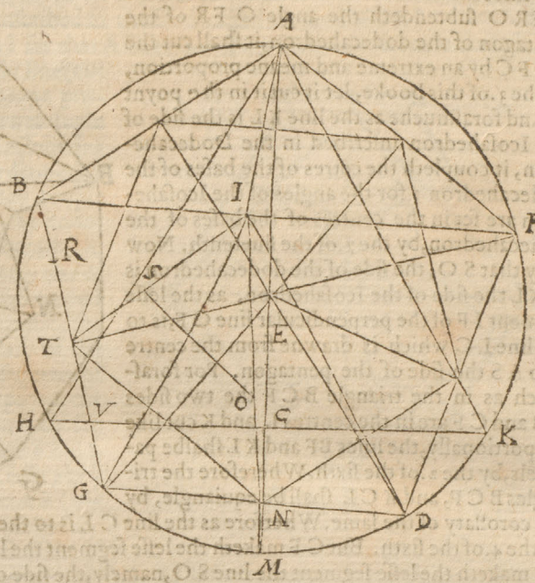
If halfe of the side of an Icosahedron be deuided by an extreme & meane proportion: and if the lesse segment thereof be taken away from the whole side, and againe from the residue be taken away the third part: that which remaineth shall be equal to the side of the Dodecahedron inscribed in the same Icosahedron.



Construction.

Vppose that A B G D F be a pentagon, containing fiue sides of the Icosahedron by the 16. of the thirtenth, and let it be inscribed in a circle, whose centre let be the point E. And vpon the sides of the pentagon, let there be reared vp triangles, making a solide angle of the Icosahedron at the poynt I, by the 16. of the thirtenth. And in the circle A B D, inscribe an equilater triangle A H K. From the centre E drawe to H K the side of the triangle, and G D the side of the pentagon, a perpendicular line, which let be E C N M. And draw these right lines E G, E D, I G and I D. And deuide the line B G into two equal parts in the poynt T. And

drawe these lines I N, I T, T N, E T. And forasmuche as in the perpendiculars I T & I N are the centres of the circles which containe the equilater triangles I B G, & I G D, by the corollarye of the first of the thirde. Let those centres be the points S and O. And draw the line S O. Deuide the line T B the half of B G the side of the Icosahedron by an extreme and meane proportion in the poynt R, by the 30. of the sixth, and let the lesse segment therof be R B. And forasmuch as the line S O coupleth the centres of the triangles I B G, & I G D, it is by the 5. of the fiueteenth, the side of the Dodecahedrō inscribed in the Icosahedron, whose side is the line B G. From the side B G take away B R the lesse segment of the halfe side. And from the residue G R take away the third part G V (by the 9. of the sixth.) Then I say that the residue R V is equal to S O the side of the Dodecahedron inscribed. For forasmuch as the perpendicular E N is in the poynt C deuided by an extreme and meane proportion, by the corollary of the first of the fourteenth, and the greater segment therof is the line E C, and vnto the line E C the line C M is equal, by the corollary of the 12. of the thirteenth: wherefore the line E C is to the line C N, as the line C M is to the same line C N, by the 7. of the fiueth. But as the line E C is to the line C N, so is the whole line E N, to the greater segment E-C, by the 3. diffinition of the sixth. Wherefore (by the 11. of the fiueth), as the whole line E N is to the greater segment E C, so is the line C M to the line C N. Wherefore the line C M, is deuided by an extreme and meane proportion in the poynt N, namely, is deuided like vnto the line E N, by the 2. of the fourteenth. Wherfore the line E M excedeth the line E N by the lesse segment of his halfe, namely, by M N. And forasmuche as E G D is the triangle of an equilater and equiangle pentagon A B G D F, and E T N is likewise the triangle of the like pentagon inscribed in the pentagon A B G D F: Therefore by the 20. of the sixth, the triangle E T N is like to the triangle E G D. Wherefore as the line E G is to the line E N, so by the 4. of the sixth, is the line G D to the line N T. Wherefore the line G D (or B G which is equal vnto it) excedeth the line N T by the lesse segment of the halfe of B G. For the line E G did in like sort excede the line E N. But that lesse segment is the line B R. Wherefore the residue R G is equal to the line T N. And forasmuch as I B G is an equilater triangle: the perpendicular S T shalbe the halfe of the line S I which is drawne from the centre, by the corollary of the 12. of the thirteenth: wherfore the line I T excedeth the line I S by his third part. And forasmuche as the line S O which coupleth the sections, is a parallel to the line T N, by the 2. of the sixth. For the equal perpendiculars I T, and I N are cut like in the poynts S & O: therfore the triangles I T N & I S O, are like by the corollary of the second of the sixth. Wherfore as the line I T is to the line I S, so by the 4. of the sixth is the line T N to the line S O. But the line I T excedeth the line I S by a third part: wherfore the line T N, excedeth the line S O by a third part: but the line T N is proued equal to the line R G. Wherfore the line R G excedeth the line S O by a third part of himself, which is G V. Wherfore the residue R V, is equal to the line S O, which is the side of the dodecahedron inscribed in the Icosahedron, whose side is the line B G. If therfore halfe of the side of an Icosahedrō, be deuided by an extreme & meane proportion: and if the lesse segment therof be taken away from the whole side, and againe from the residue be takē away the third part: that which remaineth shall be equal to the side of the dodecahedron inscribed in the same Icosahedron.



Demonstration.

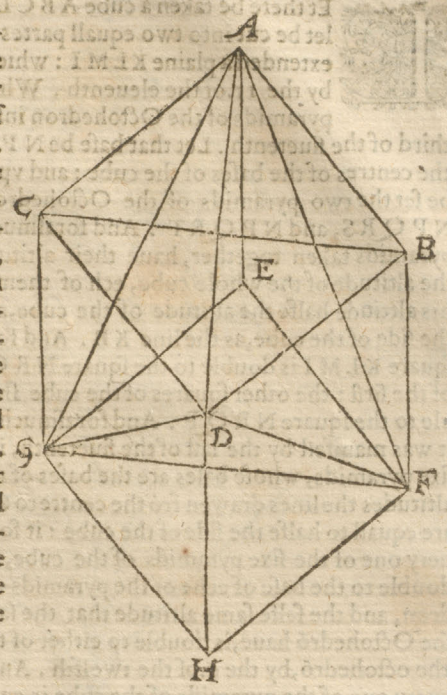
The 25. Proposition.

To proue that a cube geuen, is to a trilater equilater pyramis inscribed in it, triple.

SVppose that the cube geuen, be A B C H: and let the pyramis inscribed in it be A G D F. Then I say that the cube A B C H is triple to the pyramis A G D F. For forasmuche as the base A F D is common to the pyramis A F D B and A F D G, the pyramis A F D B shalbe set without the pyramis A F D G. Likewise the rest of the bases of the inscribed pyramis are common to the rest of the pyramids sette without: which are these: the pyramis A G D C vppon

the

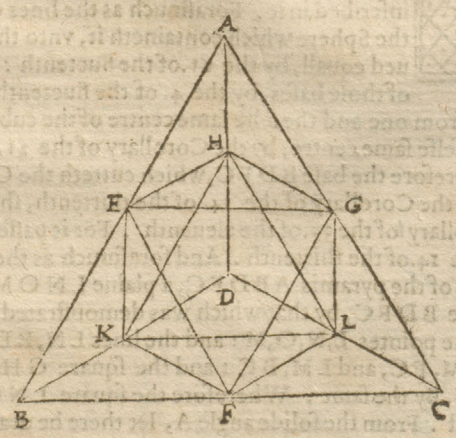
the base A G D: the pyramis A G F E vpon the base A G F: and the pyramis G D F H vpon the base G D F. Which pyramids taken without, are foure in number, equal and like the one to the other, by the 8. diffinition of the eleuéth. For euery one of them is contained vnder thre halfe squares of the cube, and one of the bases of the pyramis inscribed. Wherfore euery one of thé is cõtained vnder the halfe base of the cube, & the altitude of the cube. As the pyramis A E G F, hath to his base halfe of the square E H, namely, the triangle E G F, & hath to his altitude, the altitude of the cube, namely, the line A E. Wherfore the sayd pyramis is the sixth part of the cube. For if the cube be deuided into two prismes, by the plaine C B F G, the prisme ACBGEF, shalbe triple to the pyramis AEGF, hauing one & the selfe same base with it EGF, and one and the selfe same altitude E A, by the first corollary of the 7. of the twelueth. Wherefore the sayd outward pyramis AEGF is the sixth part of the whole cube. Wherfore also the same pyramis together wyth the other thre outwarde pyramids A F D B, A G D C, and GDFH, shal containe two third partes of the cube. Wherfore the residue, namely, the pyramis inscribed A GDF, shal contain one third part of the cube. And therefore conuersedly the cube shall be triple to it: wherefore we haue proued that a cube geué triple to a trilater & equilater pyramis inscribed in it.



¶ *The 26. Proposition.*

To proue that a trilater equilater Pyramis is duple to an Octohedron inscribed in it.

LEt there be taken a trilater Pyramis A B C D: whose fixe sides let be cut into two equall partes, in the pointes E, K, F, L, G, and H: inscribing thereby an Octohedron in the pyramis, by the 2. of the fiuetenth. Wherefore the pyramids A E G H, B E F K, C F G L, & D K H L, fall without the Octohedron inscribed, by the same second of the fiuetenth. But the outward Pyramids (namely, A E G H, and the three other) are like vnto the whole pyramis, by the 7. definition of the eleuenth. For the bases of the whole pyramis are by parallel lines drawen in them cut into like triangles, by the Corollary of the 2. of the sixth, of which the foresayd pyramids are made. Wherefore the whole pyramis is to euery one of them in treble proportion of that in which the sides of like proportion are, by the 8. of the twelfth But by construction, the proportion of the side A B to the side A E is duple. Wherefore the whole pyramis A B C D is octuple to the pyramis A E G H, and so is it to euery one of the pyramids which are equall to A E G H. For duple proportion multiplyed into it selfe twise maketh octuple. Wherefore it followeth that the 4. pyramids A E G H, B E F K, C F G L, and D K H L, taken together, make the halfe of the whole pyramis A B C D. Wherefore the residue, namely, the Octohedron E G L K H F, is the other half of the pyramis. Wherfore the pyramis is duple to the Octohedron. Wherefore we haue proued that a trilater equilater pyramis is duple to an Octohedron inscribed in it.

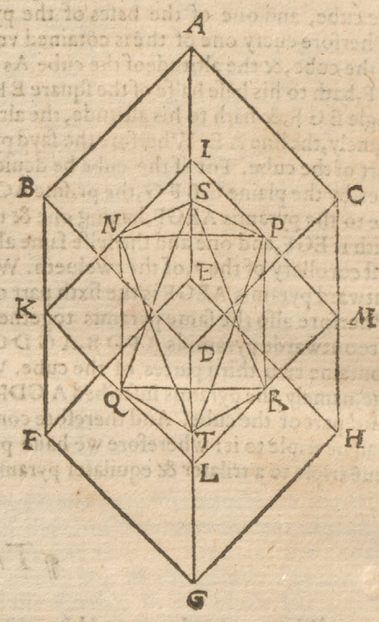


¶ *The 27. Proposition.*

To proue that a Cube is sextuple to an Octohedron inscribed in it.

LEt there be taken a cube ABCD, EFGH: whose 4. standing lines AE, BF, CH, & DG, let be cut into two equall partes in the pointes I, K, M, L: and by those pointes let there be extended a plaine KLMI: which shall be a square, and parallel to the squares BC & FH, by the 15. of the eleuenth. Wherefore in it shall be the base which is common to the two pyramids of the Octohedron inscribed in the cube, by that which was demonstrated in the third of the fiuetenth. Let that base be NPRQ, coupling the centres of the bases of the cube: and vpon that base let be set the two pyramids of the Octohedron, which let be NPQRS, and NPQRT. And forasmuch as those two pyramids taken together, haue their altitude equall with the altitude of the whole cube, ech of them a part hath to his alttiude halfe the altitude of the cube, namely, halfe of the side of the cube, as the line KB. And forasmuch as the square KLMI is double to the square NRQP, by the 47. of the first: the other squares of the cube shall also be double to the square NRQP. And forasmuch as the cube, as it was manifest by the last of the fiuetenth, is resolued into sixe pyramids, whose bases are the bases of the cube, & the altitudes the lines drawen frō the centre to the bases, which are equall to halfe the side of the cube: it followeth that euery one of the sixe pyramids of the cube, hauing his base double to the base of eche of the pyramids of the Octohedron, and the selfe same altitude that the said pyramids of the Octohedrō haue, is double to either of the pyramids of the octohedrō, by the 6. of the twelfth. And forasmuch as euery one of the pyramids of the cube is equall to the two pyramids of the Octohedron, the sixe pyramids of the cube shall be sextuple to the whole Octohedron. Wherefore it is manifest, that a cube is sextuple to an Octohedron inscribed in it.

Demonstration.

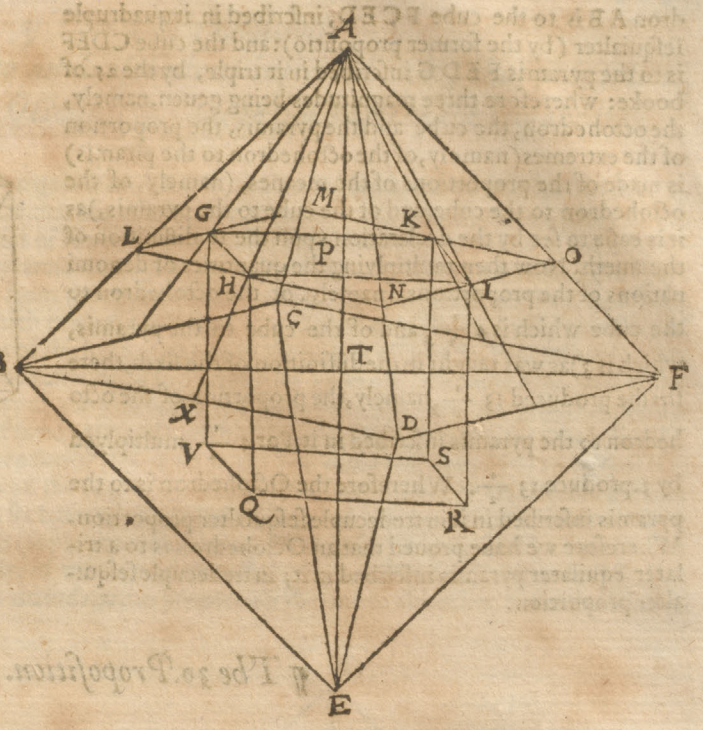


¶ The 28. Proposition.

To proue that an Octohedron is quadruple sesquialter to a Cube inscribed in it.

SVppose that the Octohedron geuen be ABCDEF: and let the cube inscribed in it be GHIK, VQRS. Then I say, that the Octohedron is quadruple sesquialter to the cube inscribed in it. Forasmuch as the lines drawen from the centre of the Octohedron, or of the Sphere which containeth it, vnto the centres of the bases of the Octohedron, are proued equall, by the 21. of the fiuetenth: and the angles of the cube are set in the centres of those bases, by the 4. of the fiuetenth: it followeth, that the selfe same right lines are drawen from one and the selfe same centre of the cube and of the Octohedron: for they haue eche one and the selfe same centre, by the Corollary of the 21. of the fiuetenth. Let that centre be the point T. Wherefore the base BDFC, which cutteth the Octohedron into two equall and quadrilater pyramids, by the Corollary of the 14. of the thirtenth, shall also cut the cube into two equall partes, by the Corollary of the 39. of the eleuenth. For it passeth by the centre T, by that which was demonstrated in the 14. of the thirtenth. And forasmuch as the base of the cube is in the 4. centres G, H, I, K, of the bases of the pyramis ABDFC, a plaine LNOM, extended by those pointes, shall be parallel to the plaine BDFC, by that which was demonstrated in the 4. of the fiuetenth, and shall cut the pyramis in the pointes L, N, O, M: and the lines LN, BD, and NO, DF, shall be parallels, so also shall the lines OM, FC, and LM, BC: and the square GHIK of the cube shall be inscribed in the square LNOM, by the same. Wherefore the square LNOM is duple to the square GHIK, by the 47. of the first. From the solide angle A, let there be drawen to the plaine superficies BDFC, a perpendicular, which let fall vppon it in the point T, and let the same perpendicular be AT, cutting the plaine LNOM in the point P. And it shall also be a perpendicular to the plaine LNOM, by the Corollary of the 14. of the eleuenth. Againe from the angle BAD of the triangle ADB, let there be drawen by the centre H of the triangle, to the base a line AHX. Wherefore the line AX is sesquialter to the line AH, by the Corollary of the 12. of the thirtenth. Wherefore the line AH is duple to the line HX. But the other lines AB, AD, AF, AC, and the perpendicular APT, are cut like vnto the line AHX, by the 17. of the eleuenth: Wherefore the line AP is double to the line PT. Wherefore the line AP is the altitude of the cube, for the line PT is the halfe thereof.

And



And forasmuch as vpon the base G H I K of the cube, and vnder the altitude A P of the same cube, is set the pyramis A G H I K: the said pyramis is the third part of the cube, by the Corollary of the 7. of the twelfth. But vnto the pyramis A G H I K the pyramis A L N O M is duple, by the 6. of the twelfth, for the base of the one is double to the base of the other. Wherefore the pyramis A L N O M is two third partes of the cube. And forasmuch as the pyramids A L N O M, and A B D F C, are like, by the 7. definition of the eleuenth: therefore they are in triple proportion of that in which the sides of like proportion A H to A X, or A L to A B, are, by the Corollary of the 8. of the twelfth. But the side A B is proued to be sesquialter to the side A L. Wherefore the pyramis A B C D F is to the pyramis A L N O M, as 27. is to 8. (that is, in sesquialter proportion tripled: for the quantitie or denomination of sesquialter proportion, namely, $1\frac{1}{2}$ multiplied into it selfe once maketh $2\frac{1}{4}$, which againe multiplyed by $1\frac{1}{2}$ maketh $3\frac{3}{8}$, that is, 27. to 8.). But of what partes the pyramis A L N O M containeth 8, of the same the cube containeth 12: namely, is sesquialter to the pyramis. Wherefore of what partes the cube containeth 12, of the same the whole Octohedron (which is double to the pyramis A B D F C) containeth 54. Which 54. hath to 12. quadruple sesquialter proportion. Wherefore the whole Octohedron is to the cube inscribed in it, in quadruple sesquialter proportion. Wherefore we haue proued that an Octohedron geuen is quadruple sesquialter to a cube inscribed in it.

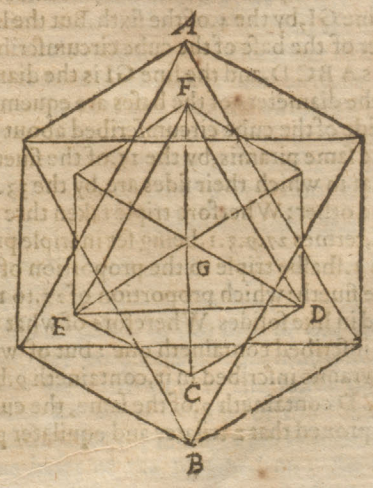
¶ A Corollary.

An Octohedron is to a cube inscribed in it, in that proportion that the squares of their sides are. For by the 14. of this booke, the side of the Octohedron is in power quadruple sesquialter to the side of the cube inscribed in it.

¶ The 29. Proposition.

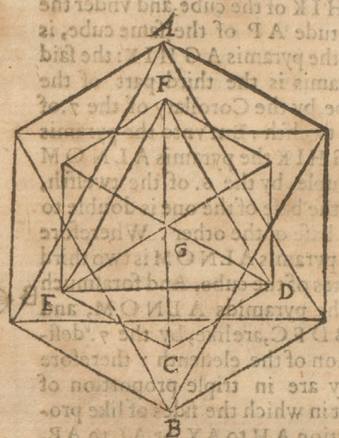
*To proue that an octohedrō geuē, is *tredecuple sesquialter to a trilater equilater pyramis inscribed in it.*

* That is as $13.\frac{1}{2}$ is to 1.



LEt the octohedron geuen, be A B: in which let there be inscribed a cube FCED, by the 4. of the fiuetenth, and in the cube let there be inscribed a pyramis FEGD, by the 1. of the fiuetenth. And forasmuche as the angles of the pyramis are (by the same first of the fiuetenth) set in the angles of the cube: and the angles of the cube are set in the centres of the bases of the Octohedron, namely, in the poyntes F, E, C, D, G by the 4. of the fiuetenth. Wherfore the angles of the pyramis, are set in the centres F, C, E, D of the octohedron. Wherefore the pyramis F E D G is inscribed in the octohedron (by the 6. of the fiuetenth.) And forasmuche as the octohedron

dron AB is to the cube FCED, inscribed in it quadruple sesquialter (by the former propositiō): and the cube CDEF is to the pyramis FEDG inscribed in it triple, by the 25. of booke: wherefore three magnitudes being geuen, namely, the octohedron, the cube and the pyramis, the proportion of the extremes (namely, of the octohedron to the piramis) is made of the proportions of the meanes, (namely, of the octohedron to the cube, and of the cube to the pyramis,) as it is easie to see by the declaration vpon the 10. diffinition of the sixeth. Now then multiplying the quantities or denominations of the proportions (namely, of the octohedron to the cube which is $4\frac{1}{2}$, and of the cube to the pyramis, which is 3) as was taught in the diffinition of the sixth, there shalbe produced $13\frac{1}{2}$, namely, the proportion of the octohedron to the pyramis inscribed in it. For $4\frac{1}{2}$, multiplyed by 3. produce $13\frac{1}{2}$. Wherefore the Octohedron is to the pyramis inscribed in it in tredecuple sesquialter proportion. Wherefore we haue proued that an Octohedron is to a trilater equilater pyramis inscribed in it, in tredecuple sesquialter proportion.



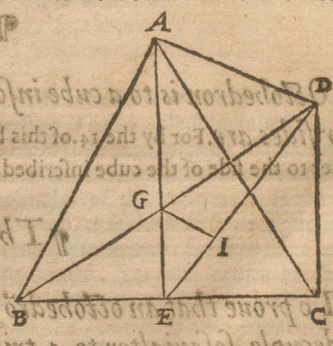
¶ The 30. Proposition.

To proue that a trilater equilater Pyramis, is noncuple to a cube inscribed in it.

Suppose that the pyramis geuen, be ABCD, whose two bases let be ABC, and DBC, and let their centres be the poynts G and I. And from the angle A, draw vnto the base BC a perpendicular AE: likewise from the angle D draw vnto the same base BC, a perpendicular DE: and they shal concurre in the section E by the 3. of the third: and in them shalbe the cētres G and I, by the corollary of the first of the third. And forasmuch as the line AD is the side of the pyramis, the same AD shall be the diameter of the base of the cube which cōtaineth the pyramis, by the 1. of the siueteth.

Demonstration.

Draw the line GI. And forasmuch as the line GI coupleth the centres of the bases of the pyramis: the saide line GI shalbe the diameter of the base of the cube inscribed in the pyramis by the 18. of the fiuetenth. And forasmuche as the line AG is double to the line GE, by the corollarye of the twelueth of the thirtenth: the whole line AE shal be triple to the line GE: and so is also the line DE to the line IE. Wherefore the lines AD and GI are parallels, by the 2. of the sixth. And therefore the triangles AED, and GEI are like, by the corollary of the same. And forasmuch as the triangles AED, and GEI are like, the line AD shalbe triple to the line GI, by the 4. of the sixth. But the line AD is the diameter of the base of the cube circumscribed about the pyramis ABCD, and the line GI is the diameter of the base of the cube inscribed in the pyramis ABCD: but the diameters of the bases are equemultiplices to the sides (namely, are in power duple). Wherfore the side of the cube circumscribed about the pyramis ABCD, is triple to the side of the cube, inscribed in the same piramis, by the 15. of the siueth: but like cubes are in triple proportion the one to the other of that in which their sides are, by the 33. of the eleuenth: and the sides are in triple proportion the one to the other: Wherfore triple taken thre times bringeth forth twenty seuencuple, which is 27. to 1: for the 4. termes 27. 9. 3. 1, being set in triple proportion: the proportion of the first to the fourth, namely, of 27. to 1. shalbe triple to the proportion of the first to the second, namely, of 27. to 9, by the 10. diffinition of the siueth: which proportion of 27. to 1. is the proportiō of the sides tripled, which proportiō also is found in like solides. Wherefore of what partes the cube circumscribed containeth 27. of the same, the cube inscribed containeth one: but of what partes the cube circumscribed, containeth 27. of the same, the pyramis inscribed in it, containeth 9. by the 25. of this booke: wherfore of what partes the pyramis ABCD containeth 9. of the same, the cube inscribed in the pyramis, containeth one. Wherefore we haue proued that a trilater and equilater pyramis, is nonecuple to a cube inscribed in it.



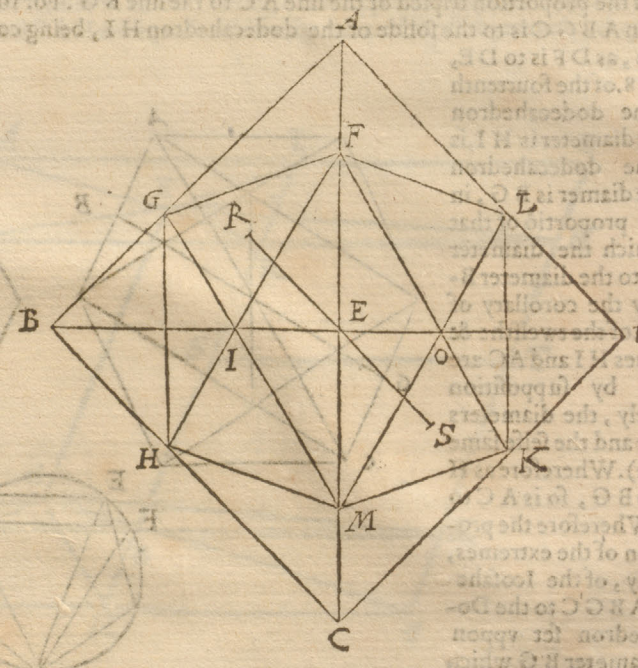
The

¶ The 31. Proposition.

An Octohedron hath to an Icosahedron inscribed in it, that proportion, which two bases of the Octohedron haue to fiue bases of the Icosahedron.

SVppose that the octohedron geuen be A B C D, and let the Icosahedron inscribed in it, be F G H M K L I O. Then I say that the octohedron is to the Icosahedron, as two bases of the octohedron, are to fiue bases of the Icosahedron. For forasmuche as the solide of the octohedron consisteth of eight pyramids, set vpon the bases of the octohedron, and hauing to theyr altitude a perpendicular line drawne from the centre to the base: let that perpendicular be E R, or E S, being drawne from the centre E (which centre is common to either of the solides, by the corollary of the 21. of the fiueteenth) to the centres of the bases, namely, to the poyntes R and S. Wherefore for that thre pyramids are equal and like, they shalbe equal to a prisme set vpon the selfe same base, and vnder the selfe same altitude, by the corollary of the seuenth of the twelueth. But vnto this prisme is double that prisme which is set vpon the self same base, and hath his altitude duple, namely, the whole line R S by the corollary of the 25. of the eleuenth: for it is equal to the two equal and like prismes whereof it is composed. Wherfore the prisme set vpō the base of the octohedron, and hauing to his altitude the line R S is equal to six pyramids, set vpon six bases of the Octohedron, and hauing to their altitude the line E R. So there remaine two pyramids (for in the octohedron are 8. bases) which shall be equal to the prisme which is set vpon the third part of the base of the octohedron, and vnder the altitude R S. For prismes vnder one and the selfe same altitude, are in proportion the one to the other, as are their bases, by the corollary of the 7 of the twelueth. Wherefore the two prismes which are set vppon the base of the octohedron, and vpon a third part therof, and vnder the altitude R S, are equal to the 8. pyramids of the Octohedron, or to the whole solide of the octohedron. And forasmuch as the Icosahedron inscribed in the octohedron, hathe his bases set in the bases of the Octohedron, by the 17. of the fiueteenth: it followeth that the pyramids set vppon the bases of the Icosahedron, & hauing to their toppes one and the selfe same centre E, are contained vnder the selfe same altitude, that the pyramids of the octohedron are cōtained vnder. namely, vnder the line E R, or E S. And therefore a prisme, set vpon the base of the Icosahedron, and hauing his altitude double to the altitude of the pyramis, namely, the whole line R S, is equal to

Demonstration.



sixe pyramids set vpon the base of the Icosahedron, and vnder the altitude E R or E S, as we haue proued in the octohedron. Wherfore the 20. pyramids, set vpon the 20. bases of the Icosahedron, are equal to thre prismes set vpon the base of the Icosahedron, and vnder the altitude R S, and moreouer to an other prisme set vppon a thirde part of the base of the Icosahedron and vnder the same altitude R S, which prisme is a thirde part of the former prisme, by the corollarye of the 7. of the twelueth: for their proportion is as the proportion of the bases. Wherfore two prismes set vpon the base of the octohedron, and a third part therof, and vnder the altitude R S, is to 4. prismes set vpon three bases of the Icosahedron, and a third part thereof, and vnder the same altitude R S, in the same proportion that the bases are, that is, as 4. third partes of the base of the Octodron (which are equal to one base, and $\frac{1}{3}$) to ten third partes of the base of the Icosahedron (which are equal to thre bases & $\frac{1}{3}$) or as two third partes of the base of the Octohedron, are to fiue thirde partes of the base of the Icosahedron. But two thirde partes of the base of the Octohedron, are to fiue thirde partes of the base of the Icosahedron, as two bases are to fiue bases (by the 15. of the fifth, for they are partes of equemultiplices:) And two prismes of the Octohedron are to 4. prismes of the Icosahedron, as the solide of the Octohedron is to the solide of the Icosahedron, when as eche are equal to eche of the solides: Wherefore (by the 11. of the fiueth) the solide of the Octohedron, is to the solide of the Icosahedron inscribed in it, as two bases of the Octohedron, are to fiue bases of the Icosahedron. An Oc-

tohedron therfore is tō an Icosahedron inscribed in it, in that proportion, that two bases of the Octohedron, are to fiue bases of the Icosahedron.

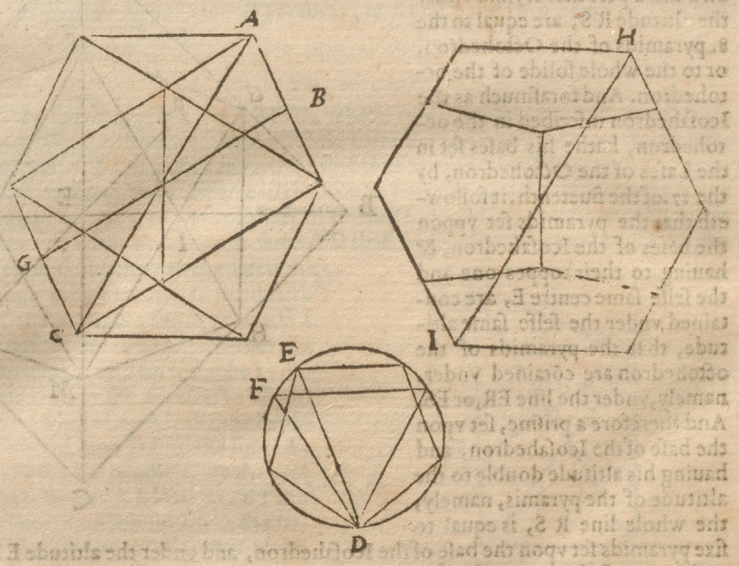
¶ The 32. Proposition.

The proportiō of the solide of an Icosahedron to the solide of a Dodecahedron inscribed in it, consisteth of the proportion of the side of the Icosahedron to the side of the Cube contayned in the same sphere, and of the proportion tripled of the diameter to the line which completh the centers of the opposite bases of the Icosahedron.

Construction.

SVppose that there be a Dodecahedron, whose diameter let be H I, and let the Icosahedron contained in the same sphere be A B G C, whose dimetient let be A C. And let the right line which coupleth the centres of the opposite bases be B G. And let the dodecahedron inscribed in the Icosahedron be that which is set vpon the diameter B G, by the 5. of the fiuetenth. And let the side of the cube be D E, and let the side of the Icosahedron be D F, both the sayd solides being described in one and the selfe same sphere. Thē I say that the proportion of the solide of the Icosahedron A B C G to the solide of the dodecahedron set vpon the diameter B G, inscribed in it, consisteth of the proportion of the line D F to the line D E, and of the proportion tripled of the line A C to the line B G. For forasmuch as the solide of the Icosahedron A B G C is to the solide of the dodecahedron H I, being contayned in one and the selfe same sphere, as D F is to D E, by the 8. of the fourteenth But the dodecahedron whose diameter is H I, is to the dodecahedron whose diamer is B G, in treble proportiō of that in which the diameter H I is to the diameter B G, by the corollary of the 17. of the twelfth: & the lines H I and A C are equall by suppoition (namely, the diameters of one and the selfesame sphere). Wherefore as H I is to B G, so is A C to B G. Wherefore the proportion of the extremes, namely, of the Icosahedron A B G C to the Dodecahedron set vpon the diameter B G which coupleth the centres, cōsisteth (by the 5. difiniti-on of the sixt) of the proportions of the meanes, namely, of the proportiō of the Icosahedron A B C G to the dodecahedron H I (which is one and the same with the proportion of D F to D E) and of the proportion of the same H I to the other dodecahedron set vpon the diameter B G, inscribed in the same Icosahedron A B G C, by the same 5. of the fiuetenth: which proportion is triple to the proportiō of the line H I (or the line A C) to G B which coupleth the centres of the opposite bases of the Icosahedron. The proportion therefore of the solide of the Icosahedron to the solide of a Dodecahedron inscribed in it, consisteth of the proportion of the side of the Icosahedron to the side of the Cube contayned in the same sphere, and of the proportion tripled of the diameter to the line which coupleth the centres of the opposite bases of the Icosahedron.

Demonstration.



¶ The 33. Proposition.

The solide of a Dodecahedron excedeth the solide of a Cube inscribed in

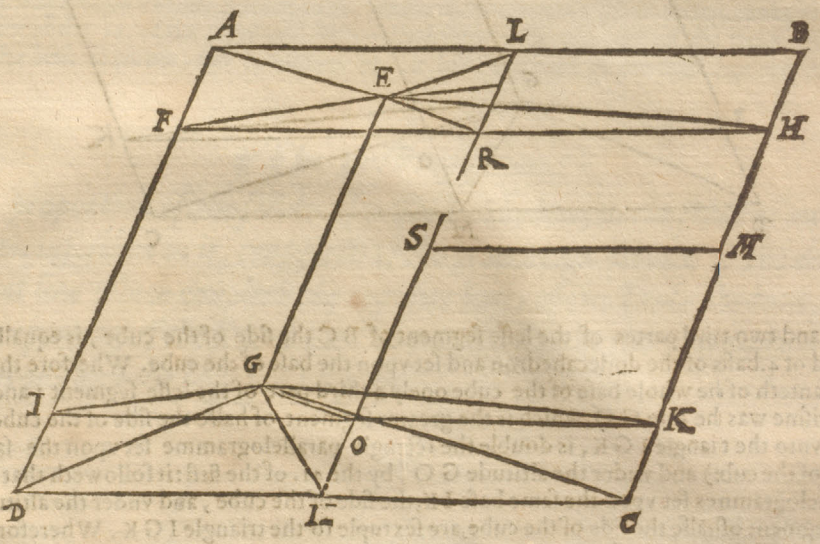
it

it, by a parallelipipedon, whoſe baſe wanteth of the baſe of the Cube by a third part of the leſſe ſegment, and whoſe altitude wanteth of the altitude of the Cube, by the leſſe ſegment of the leſſe ſegment, of halfe the ſide of the Cube.

FOraſmuch as by the 17. of the thirtenth, and 8. of the fiuetenth, it was manifeſt, that the baſe of a cube inſcribed in a dodecahedron, doth with his ſides ſubtend the angles of 4. pentagons cõcurring at one and the ſelfe ſame ſide of the dodecahedron: let that baſe of the cube be A B C D: and let the ſide wherat 4. baſes of the dodecahedron circumſcribed concurre, be E G: which ſhall contayne a ſolide A E B D G C ſet vpon the baſe A B C D. Diuide the ſides A B and D C into two equall partes in the poyntes L and N. And draw the line L N, which is a paralel to the ſide E G, as it was manifeſt by the 17. of the thirtenth. The perpendiculars alſo E R and G O vhich couple thoſe parallels, are eche equall, to halfe of the ſide E G, and eche is the greater ſegment of halfe the ſide of the cube, and therefore the whole line E G is the greater ſegment of the whole line I N the ſide of the cube (by the foreſayd 17. of the thirtenth). By the poyntes R and O, draw vnto the ſides A B and C D parallel lines F H and I K. And draw theſe right lines E F, E H, G I and G K. Now foraſmuch as the two lines F H & E R touching the one the other are parallels to the two lines I K and G O touching alſo the one the other, & not being in the ſelfe ſame playne with the two firſt lines: therfore the playne ſuperficieces E F H and G I K paſſing by thoſe lines are parallels, by the 15. of the eleuenth: which playnes do cutte the ſolide A E B D G C. Whererefore there are made fower quadrangled pyrmids ſet vpon the rectangle parallelogrames L H, L F, N K, and N I, and hauing their toppes the poynes E and G. And foraſmuch as the triangles G O K and E R H are equall and like, by the 4. of the firſt, namely, they con-

Conſtruction.

Demonſtration.

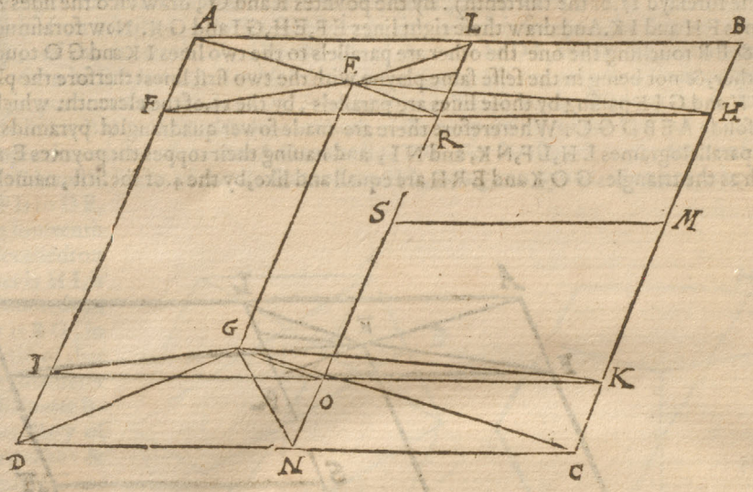


Extend in the figure a line from the poinct E to the poinct B.

tayne equall angles comprehẽded vnder equall ſides, and they are parallels by conſtruction, being ſet in the playnes G I K and E F H: the figures G K H E, O K H R, and G O R E ſhall be panllelogrammes, by the diffinition of a parallelogramme, and therefore the ſolide G O K E R H is a priſme, by the 11. diffinition of the eleuenth. And by the ſame reaſon may the ſolide G O I E R F be proued to be a priſme. And foraſmuch as vpon equall baſes N O K C, and R L B H, and vnder equall altitudes O G and R E are ſet pyramids: thoſe pyramids ſhall be equall to that pyramis which is ſet vpon the baſe C K I D (which is double to either of the baſes N O K C, and R L B H) and vnder the ſame altitude O G, by the 6. of the twelfth. And foraſmuch as the ſide G E is the greater ſegment of the line C B, the ine K H, which by the 33. of the firſt, is equall to the line G E, ſhall be the greater ſegment of the ſame line C B, by the 2. of the fourtenth. Wherefore the reſidues C K and H B ſhall make the leſſe ſegment of the whole line C B. But as the greater ſegment K H is to the two lines C K and H B the leſſe ſegment, ſo is the rectangle parallelogramme O H to the two rectangle parallelogrammes O C and H L, by the 1. of the ſixt. Wherfore the leſſe ſegment of the parallelogramme N B, ſhall be the two parallelogrammes O C and H L. Put the line K M double to the line K C and draw the line M S parallel to the line C N. Wherefore the parallelogramme O K M S is equall to the parallelogrammes O C and H L, by the 1. of the ſixth. Wherfore the pyramis ſet vpon the baſe O K M S contayneth two third partes of the priſne ſet vpon the ſelfe ſame baſe, by the 4. corollary of the 7. of the twelfth. Wherfore the priſme which is ſet vpon two third

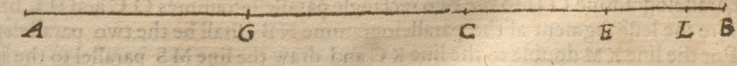
partes of the base O K M S is equall to the two pyramids N O K C G and R L B H E. For the sections of a prisme are one to the other, as the sections of the base are, by the first corollary of the 25. of the eleuenth. But the sections of the base are as the sections of the line C B or K M, by the 1. of the sixt. Wherfore the two pyramids N O K C G and R L B H E, adde vnto the prisme G O K E R H two third partes of the prisme set vpon the base O K M S. And forasmuch as the line K M is the lesse segmēt of the whole line B C (for it is equall to the two lines C K and H B), and the prisme set vpon the base O K H R is cutte like vnto the line K M, namely, in eche are taken two thirdes, as hath before bene proued: the prisme equall to the two pyramids, shall adde vnto the prisme G O K E R H, which is set vpon the greater segment K H, two thirds of the lesse segment. Wherefore in the line B C there shall remayne one third part of the lesse segment: and therefore in the rectangle parallelogramme N B which is halfe the base of the cube, there shall remayne the same third part of the lesse segment. And by the same reason may we proue that in the other pyramids O N D I G, and R L A F E, and in the prisme G O I E R F is left the selfe same excesse of the base L A N D, namely, the third part of the lesse segment. Wherefore the whole prisme contayned betwene the triangles I G K and F E H, and vnder the length of the greater

Extend in the figure a line from the point E to the point B.



segment and two third partes of the lesse segment of B C the side of the cube, is equall to the solide composed of 4. bases of the dodecahedron and set vpon the base of the cube. Wherfore the base of that prisme wanteth of the whole base of the cube onely a third part of the lesse segment: and the altitude of that prisme was the line G O, which is the greater segment of halfe the side of the cube. And forasmuch as vnto the triangle I G K, is double the rectangle parallelogramme set vpon the same base I K, (the side of the cube) and vnder the altitude G O, by the 41. of the first: it followeth that three rectangle parallelogrammes set vpon the same base I K, the side of the cube, and vnder the altitude O G the greater segment of halfe the side of the cube, are sextuple to the triangle I G K. Wherefore those three rectangle parallelogrammes doo make one rectangle parallelogramme set vpon the base I K and vnder the altitude of the line G O tripled. But by the 7. diffinition of the eleuenth, there are sixe prismes equal and like vnto the foresayd prisme, being set vpon euery one of the sixe bases of the cube: which prismes are in proportion the one to the other as their bases are by the 3. corollary of the 7. of twelfth. Wherefore the solide composed of these sixe prismes, shall want of the base A B C D the third part of the lesse segment, and taking his altitude of the foresayd rectangle parallelogramme, the sayd altitude shall be equall to three greater segmentes (one of which is G O) of halfe the side of the cube.

Now resteth to proue that these three segmentes want of the side of the cube by the lesse segment of the lesse segment of halfe the side of the cube. Suppose that A B the side of the cube be diuided into the greater segment A C, and into the lesse segment C B (by the 30. of the sixt). And diuide into two equall partes the line A C in the poynt G, and the line C B in the poynt E. And vnto the line C G put the line C L equall. Now forasmuch as the lines A G and G C are the greater segmentes of halfe the line A B, for eche of them is the halfe of the greater segment of the whole line A-B: the lines E B and E C shall be the lesse segmentes of halfe the line A B. Wherefore the whole line C L is the greater segment, and the line C-



E is

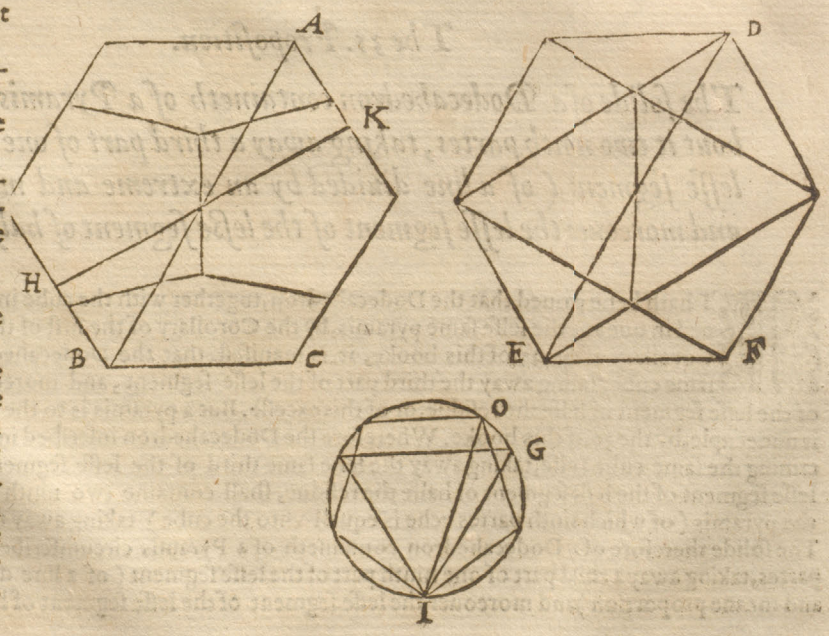
E is the lesse segment. But as the line C L is to the line C E, so is the line C I to the residue E L. Wherfore the line E L is the greater segment of the line C E, or of the line E B which is equall vnto it. Wherfore the residue L B is the lesse segment of the same E B (which is the lesse segment of halfe the side of the cube). But the lines A G, G C, and C L are three greater segmentes of the halfe of the whole line A B: which thre greater segmentes make the altitude of the foresayd solide: wherefore the altitude of the sayd solide wanteth of A B the side of the cube by the line L B, which is the lesse segment of the line B E. Which line B E agayne is the lesse segment of halfe the side A B of the cube. Wherefore the foresayd solide consisting of the sixe solides, whereby the dodecahedron exceedeth the cube inscribed in it, is set vpon a base which wanteth of the base of the cube by a third part of the lesse segment, and is vnder an altitude wanting of the side of the cube by the lesse segment of the lesse segment of halfe the side of the cube. The solide therefore of a dodecahedron exceedeth the solide of a cube inscribed in it, by a parallelipipedon, whose base wanteth of the base of the cube by a third part of the lesse segment, and whose altitude wanteth of the altitude of the cube, by the lesse segment of the lesse segment of halfe the side of the cube.

¶ A Corollary.

A Dodecahedron is double to a Cube inscribed in it, taking away the third part of the lesse segment of the cube, and moreouer the lesse segment of the lesse segment of halfe of that excesse. For if there be geuen a cube, from which is cut of a solide set vpon a third part of the lesse segment of the base, and vnder one and the same altitude with the cube: that solide taken away hath to the whole solide the proportion of the section of the base to the base, by the 32. of the eleuenth. Wherefore from the cube is taken away a third part of the lesse segment. Farther, forasmuch as the residue wanteth of the altitude of the cube, by the lesse segment of the lesse segment of halfe the altitude or side, and that residue is a parallelipipedon, if it be cut by a plaine superficies parallel to the opposite plaine superficieces, cutting the altitude of the cube by a point, it shall take away from that parallelipipedon a solide, hauing to the whole the proportion of the section to the altitude, by the 3. Corollary of the 25. of the eleuenth. Wherefore the excesse wanteth of the same cube by the third part of the lesse segment, and moreouer by the lesse segment of the lesse segment of halfe of that excesse.

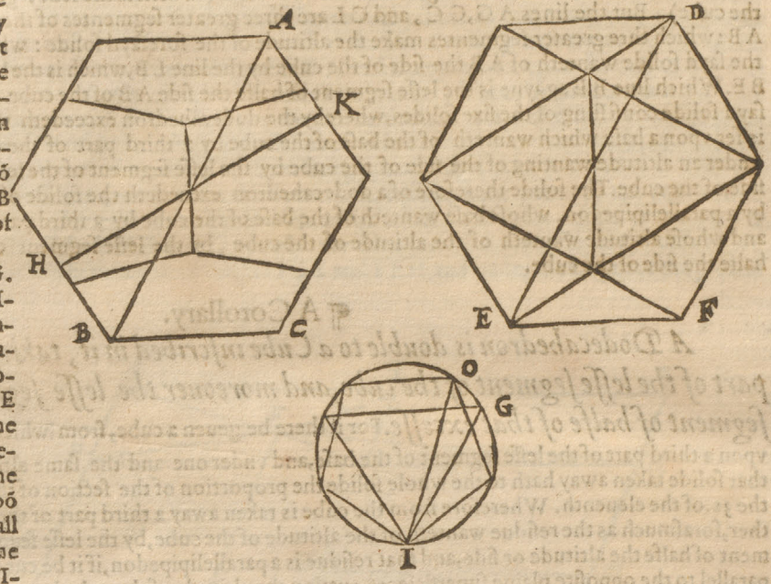
¶ The 34. Proposition.

The proportion of the solide of a Dodecahedron to the solide of an Icosahedron inscribed in it, consisteth of the proportion tripled of the diameter to that line which coupleth the opposite bases of the Dodecahedron, and of the proportion of the side of the Cube to the side of the Icosahedron inscribed in one and the selfe same Sphere.



Vppose that A H B C K be a Dodecahedron, whose diameter let be A B: and let the line which coupleth the centres of the opposite bases be K H: and let the Icosahedron inscribed in the Dodecahedron A B C, be D E F: whose diameter let be D E. Now forasmuch as one and the selfe same circle containeth the pentagon of a Dodecahedron, & the triangle of an Icosahedron described in one and the selfe same Sphere, by the 14. of the fourteenth: Let that cir-

cle be I G O. Wherfore I O is the side of the cube, and I G the side of the Icosahedron, by the same. Thē I say, that the proportion of the Dodecahedron A H B C K to the Icosahedron D E F inscribed in it, cōsisteth of the proportiō tripled of the line A B to the line K H, and of the proportion of the line I O to the line I G.



Demonstration. For forasmuch as the Icosahedron D E F is inscribed in the Dodecahedrō A B C, by suppositiō, the diameter D E shalbe equal to the line K H, by the 7. of the fiueteenth. Wherefore the Dodecahedron set vpō the diameter K H shall be inscribed in the same Sphere, wherein the Icosahedron D E F is inscribed: but the Dodecahedron A H B C K is to the Dodecahedron vpon the diameter K H in triple proportion of that in which the diameter A B is to the diameter K H, by the Corollary of the 17. of the twelfth: and the same Dodecahedron which is set vpon the diameter K H, hath to the Icosahedron D E F (which is set vpon the same diameter, or vpon a diameter equall vnto it, namely, D E) that proportion which I O the side of the cube hath to I G the side of the Icosahedron, inscribed in one & the selfe same Sphere, by the 8 of the foureteenth. Wherefore the proportion of the Dodecahedron A H B C K to the Icosahedron D E F inscribed in it, consisteth of the proportion tripled of the diameter A B to the line K H, which coupleth the centres of the opposite bases of the Dodecahedron (which proportion is that which the Dodecahedron A H B C K hath to the Dodecahedron set vpon the diameter K H) and of the proportion of I O the side of the cube to I G the side of the Icosahedron (which is the proportion of the Dodecahedron set vpon the diameter K H to the Icosahedron D E F described in one and the selfe same Sphere) by the 5. definition of the sixth. The proportion therefore of the solide of a Dodecahedron to the solide of an Icosahedron inscribed in it, consisteth of the proportion tripled of the diameter to that line which coupleth the opposite bases of the Dodecahedron, and of the proportion of the side of the cube to the side of the Icosahedron inscribed in one and the selfe same Sphere.

The 35. Proposition.

The solide of a Dodecahedron containeth of a Pyramis circumscribed about it two ninth partes, taking away a third part of one ninth part of the lesse segment (of a line diuided by an extreme and meane proportion) and moreouer the lesse segment of the lesse segment of halfe the residue.

IT hath bene proued that the Dodecahedron, together with the cube inscribed in it is contained in one and the selfe same pyramis, by the Corollary of the first of this booke. And by the Corollary of the 33. of this booke, it is manifest, that the Dodecahedron is double to the same cube, taking away the third part of the lesse segment, and moreouer the lesse segment of the lesse segment of halfe the residue, or of this excesse. But a pyramis is to the same cube inscribed in it nonecuple, by the 30. of this booke. Wherefore the Dodecahedron inscribed in the pyramis, and containing the same cube twise, taking away the selfe same third of the lesse segment, and moreouer the lesse segment of the lesse segment of halfe the residue, shall containe two ninth partes of the solide of the pyramis (of which ninth partes eche is equall vnto the cube) taking away this selfe same excesse. The solide therefore of a Dodecahedron containeth of a Pyramis circumscribed about it two ninth partes, taking away a third part of one ninth part of the lesse segment (of a line diuided by an extmere and meane proportion) and moreouer the lesse segment of the lesse segment of halfe the residue.

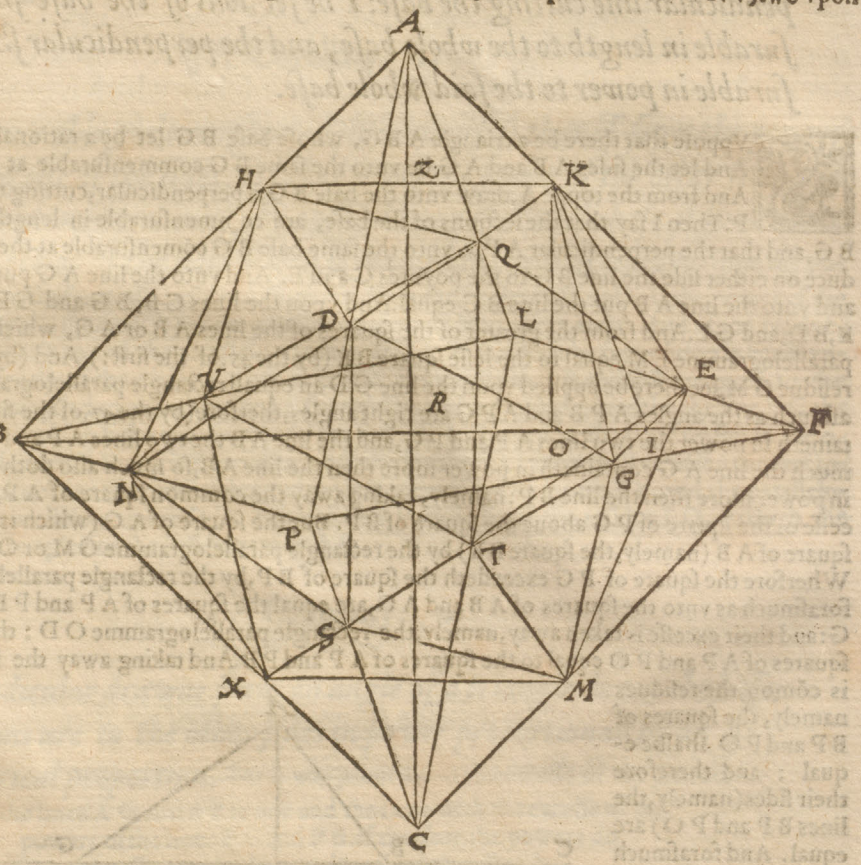
¶ The 36. Proposition.

An Octohedron exceedeth an Icosahedron inscribed in it, by a parallelipipedon set vpon the square of the side of the Icosahedron, and hauing to his altitude the line which is the greater segment of halfe the semidiameter of the Octohedron.

Construction.

Vppose that there be an Octohedron ABCFPL: in which let there be inscribed an Icosahedron HKEGMXNVDSQT, by the 16. of the fiuetenth. And draw the diameters AZRC, BROIF, and the perpendicular KO parallel to the line AZR. Then I say, that the Octohedron ABCFPL is greater then the Icosahedron inscribed in it, by a parallelipipedon set vpon the square of the side HK or GE, and hauing to his altitude the line KO or RZ: which is the greater segment of the semidiameter AR. Forasmuch as in the same 16. it hath bene proued, that the triangles KDG and KEQ are described in the bases APF and ALF of the Octohedron: therefore about the solide angle there remaine vppon the base FEG three triangles KEG, KFE, and KFG, which containe a pyramis KEFG. Vnto which pyramis shall be equall and like the opposite pyramis MEFG set vpon the same base FEG, by the 8. definition of the eleuenth. And by the same reason shall there at euery solide angle of the Octohedron remayne two pyramids equall and like: namely, two vpon the base AHK, two vpon the base DPS, and moreouer two vpon the base QLT. Now thé there shal be made twelue pyramids, set vpon a base contained of the side of the Icosahedron, and vnder two lesse segmentes of the side of the Octohedron containing a right angle, as for example the base GEF, And forasmuch as the side GE subtéding a right angle, is, by the 47. of the first, in power duple to either of the lines EF and FG, and so the side KH is in power duple to either of the sides AH and AK: and either of the lines AH, AK, or EF, FG, is in power duple to eyther of the lines AZ or ZK which cõtayne a right angle, made in the triangle or base AHK by the perpendicular AZ. Wherfore it followeth that the side GE or HK, is in power quadruple to the triangle EFG or AHK. But the pyramis KEFG, hauing his base EFG in the plaine FLBP of the Octohedron, shall haue to his altitude the perpendicular KO (by the 4. definition of the sixth) which is the greater segment of the semidiameter of the Octohedron, by the 16. of the fiuetenth. Wherfore three pyramids set vnder the same altitude and vpon equall bases, shall be equall to one prisme set vpon the same base, and vnder the same altitude, by the 1. Corollary of the 7. of the twelfth. Wherefore 4. prismes set vpon the base GEF quadrupled (which is equall to the square of the side GE) and vnder the altitude KO (or RZ the greater segment which is equall to KO) shall containe a solide equall to the twelue pyramids, which twelue pyramids make the excesse of the Octohedron aboue the Icosahedron inscribed in it. An Octohedron therefore excedeth an Icosahedron inscribed in it, by a parallelipipedon set vpon the square of the side of the Icosahedron, and hauing to his altitude the line which is the greater segment of halfe the semidiameter of the Octohedron.

Demonstration.



¶ A Corollary.

¶ A Corollary.

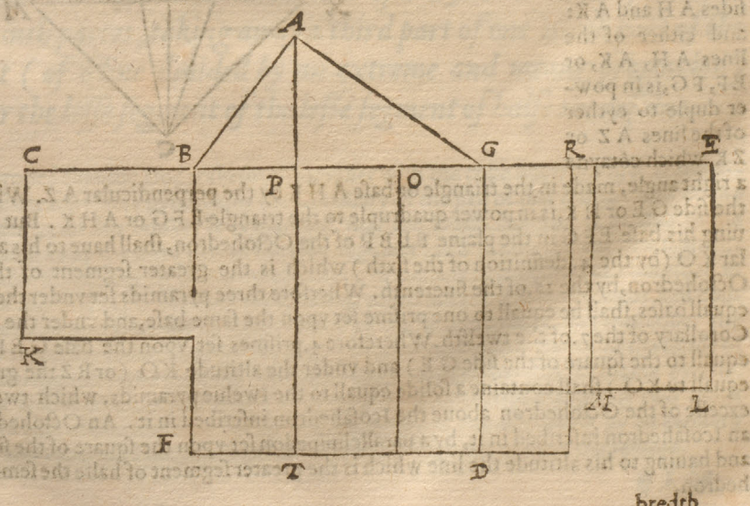
A Pyramis exceedeth the double of an Icosahedron inscribed in it, by a solide, set vpon the square of the side of the Icosahedron inscribed in it, and hauing to his altitude that whole line of which the side of the Icosahedron is the greater segmẽt. For it is manifest by the 19. of the fiuetẽth, that an octohedrõ & an Icosahedrõ inscribed in it are inscribed in one & the self same pyramis. It hath moreouer bene proued in the 16. of this boke, that a pyramis is double to an octohedrõ inscribed in it. Wherfore the two excesses of the two octohedrons (vnto which the pyramis is equal) aboue the two Icosahedrons (inscribed in the said two octohedrons) being brought into an solide, the said solide shalbe set vpon the selfe same square of the side of the Icosahedron, and shall haue to his altitude the perpendicular K O doubled: whose double coupling the opposite sides H K and X M maketh the greater segmeat the same side of the Icosahedron, by the first and second corollary of the 14. of the fiuetenth.

The 37. Proposition.

If in a triangle hauing to his base a rational line set, the sides be commensurable in power to the base, and from the toppe be drawn to the base a perpendicular line cutting the base: The sections of the base shall be commensurable in length to the whole base, and the perpendicular shall be commensurable in power to the said whole base.

Construction. Vppose that there be a triangle A B G, whose base B G let be a rational line set of purpose. And let the sides A B and A G be vnto the same B G commensurable at the least in power. And from the toppe A, draw vnto the base B G, a perpendicular, cutting the base in the point P. Then I say that the sections of the base, are commensurable in lengthe to the whole line B G, and that the perpendicular A P, is vnto the same base B G cõmensurable at the least in power. Produce on either side the line B G to the poyntes C and E. And vnto the line A G put the line G E equal, and vnto the line A B put the line B C equal. And vpon the lines C B, B G and G E describe squares B K, B D, and G L. And from the greater of the squares of the lines A B or A G, which let be G L cut of a parallelogramme E M equal to the lesse square B K (by the 45. of the first:) And (by the same) vnto the residue G M, let there be applied vpon the line G D an equall rectangle parallelogramme O D. Now forasmuch as the angles A P B and A P G are right angles, therfore (by the 47. of the first) the line A G containeth in power the two lines A P and P G, and the line A B the two lines A P and P B. Wherfore how much the line A G containeth in power more then the line A B, so much also doth the line P G contain in power more then the line B P: namely, taking away the common square of A P, there is left the excesse of the square of P G aboue the square of B P. But the square of A G (which is G L) exceedeth the square of A B (namely, the square B K) by the rectangle parallelogramme G M or O D, by construction. Wherfore the square of P G exceedeth the square of B P, by the rectangle parallelogramme O D. And forasmuch as vnto the squares of A B and A G, are equal the squares of A P and P B, and of A P and P G: and their excesse is taken away, namely, the rectangle parallelogramme O D: there shallbe left the squares of A P and P O equal to the squares of A P and P B. And taking away the square of A P which is cõmon, the residues namely, the squares of B P and P O shalbe equal: and therefore their sides (namely, the lines B P and P O) are equal. And forasmuch as the squares G L and B K are (by suppositiõ) rational, and therefore cõmẽsurable their excesse O D, shalbe commẽsurable vnto thẽ by the 15. of the tẽth. And therfore it is rationall by the 9. diffinition of the tẽth. Wherfore the rational parallelogram O D, being applied vpon the rational line G D (or B G) maketh the

First part of the demonstration.



bredth

bredth O G rational and cõmensurable in length to the whole line B G, by the 20. of the tenth. But if the whole line B G be commensurable to one of the partes O G, the lines B O, O G, and B G shall be commensurable, by the same 15. of the tenth. Wherfore also the line O G shalbe commensurable to the half of the line B O, namely, to the line P O, or P B, by the 12. of the tenth. And forasmuch as the two lines P O and O G are commensurable, the whole line P G shalbe commensurable to the line P O, or to the line P B, by the same 15. of the tenth. Wherfore either of the lines P G and P B shall be cõmensurable vnto the whole line P B, by the same. Wherefore the lines B G, P B and P G haue the one to the other that proportion which numbers haue, by the 5. of thirtenth. Wherfore the sections P B and P G of the base B G are commensurable in length to the same base, by the 6. of the tenth.

Second part of the Demonstration.

And now that the perpendicular A P is commensurable in power to the base B G, is thus proued. Forasmuch as the square of A B is by supposition, commensurable to the square of B G: and vnto the rational square of A B is commensurable the rational square of B P (by the 12. of the eleuenth) Wherfore the residue, namely, the square of P A is commensurable to the same square of B P, by the 2. part of the 15. of the eleuenth. Wherefore by the 12. of the tenth, the square of P A is commensurable to the whole square of B G. Wherefore the perpendicular A P is commensurable in power to the base B G, by the 3. diffinition of the tenth: which was required to be proued.

In demonstrating of this, we made no mention at all of the length of the sides A B and A G, but only of the length of the base B G: for that the line B G is the rational line first set: and the other lines A B and A G are supposed to be commensurable in power only to the line B G. Wherefore if that be plainely demonstrated, when the sides are commensurable in power only to the base, much more easily wil it follow, if the same sides be supposed to be commensurable both in length and in power to the base: that is, if their lengthes be expressed by the rootes of square nombers.

¶ A Corollary. 1.

By the former things demonstrated, it is manifest that if from the powers of the base, and of one of the sides, be taken away the power of the other side, and if the halfe of the power remaining, be applied vpon the whole base, it shall make the bredth that section of the base which is coupled to the first side. For from the powers of the base B G, and of one of the sides A G, that is, from the squares B D and G L, the power of the other side A B, namely the square B K (or the parallelogramme E M) is taken away. And of the residue, (namely, of the square B D, and of the parallelogramme O D, or D R, which by supposition is equal vnto O D) the halfe (namely, of the whole F R, which is P D, for the lines G R and P B are equal to the lines G O and P O) is applied to the whole line B G or G D: and maketh the bredthe the line P G the section of the base B G, which section is coupled to the first side A G. And by the same reason in the other side, if from the squares B D and B K be taken away the square G L, there shall remaine the rectangle parallelogramme F O: For the parallalelogramme E M is equal to the square B K, and the parallelogramme G M to the parallelogramme O D. Wherefore F P the halfe of the residue F O, maketh the bredth B P, which is coupled to the first side taken A B.

❋ A Corollary. 2.

If a perpendicular drawne from an angle of a triangle do cut the base: the sections are to the other sides in power proportionall by an Arithmetical proportion. For it was proued that the excesse of the powers of the lines A G and A B is one and the same with the excesse of the powers of the lines P G and P B. If therfore the powers do equally excede the one the other, they shall by an Arithmetical proportion, be proportionall.

The ende of the sixtenth Booke

of the Elementes of Geometrie added by *Flussas.*

A briefe treatiſe, added by Fluſſas, of mixt and compoſed regular ſolides.

REgular ſolides are ſayd to be compoſed and mixt, when ech of them is tranſformed into other ſolides, keeping ſtill the forme, number, and inclination of the baſes, which they before had one to the other: ſome of which yet are tranſformed into mixt ſolides, and other ſome into ſimple. Into mixt, as a Dodecahedron and an Icoſahedron: which are tranſformed or altered, if ye diuide their ſides into two equall partes, and take away the ſolide angles ſubtended of plaine ſuperficiall figures made by the lines coupling thoſe middle ſections: for the ſolide remayning after the taking away of thoſe ſolide angles, is called an Icoſidodecahedron. If ye diuide the ſides of a cube and of an Octohedron into two equall partes, and couple the ſections, the ſolide angles ſubtended of the plaine ſuperficieces made by the coupling lines, being taken away, there ſhall be left a ſolide, which is called an Exoctohedron. So that both of a Dodecahedron and alſo of an Icoſahedron, the ſolide which is made, ſhall be called an Icoſidodecahedron: and likewiſe the ſolide made of a Cube & alſo of an Octohedron, ſhall be called an Exoctohedron. But the other ſolide, namely, a Pyramis (or Tetrahedron) is tranſformed into a ſimple ſolide: for if ye diuide into two equall partes euery one of the ſides of the pyramis, triangles deſcribed of the lines which couple the ſections, and ſubtending, and taking away ſolide angles of the pyramis, are equall and like vnto the equilater triangles left in euery one of the baſes: of all which triangles is produced an Octohedron, namely, a ſimple and not a compoſed ſolide. For the Octohedron hath fower baſes, like in number, forme, and mutuall inclination with the baſes of the pyramis: and hath the other fower baſes with like ſituation oppoſite and parallel to the former. Wherefore the application of the pyramis taken twiſe, maketh a ſimple Octohedron, as the other ſolides make a mixt compound ſolide.

Icoſidodecahedron.

Exoctohedrõ.

¶ *First Definition.*

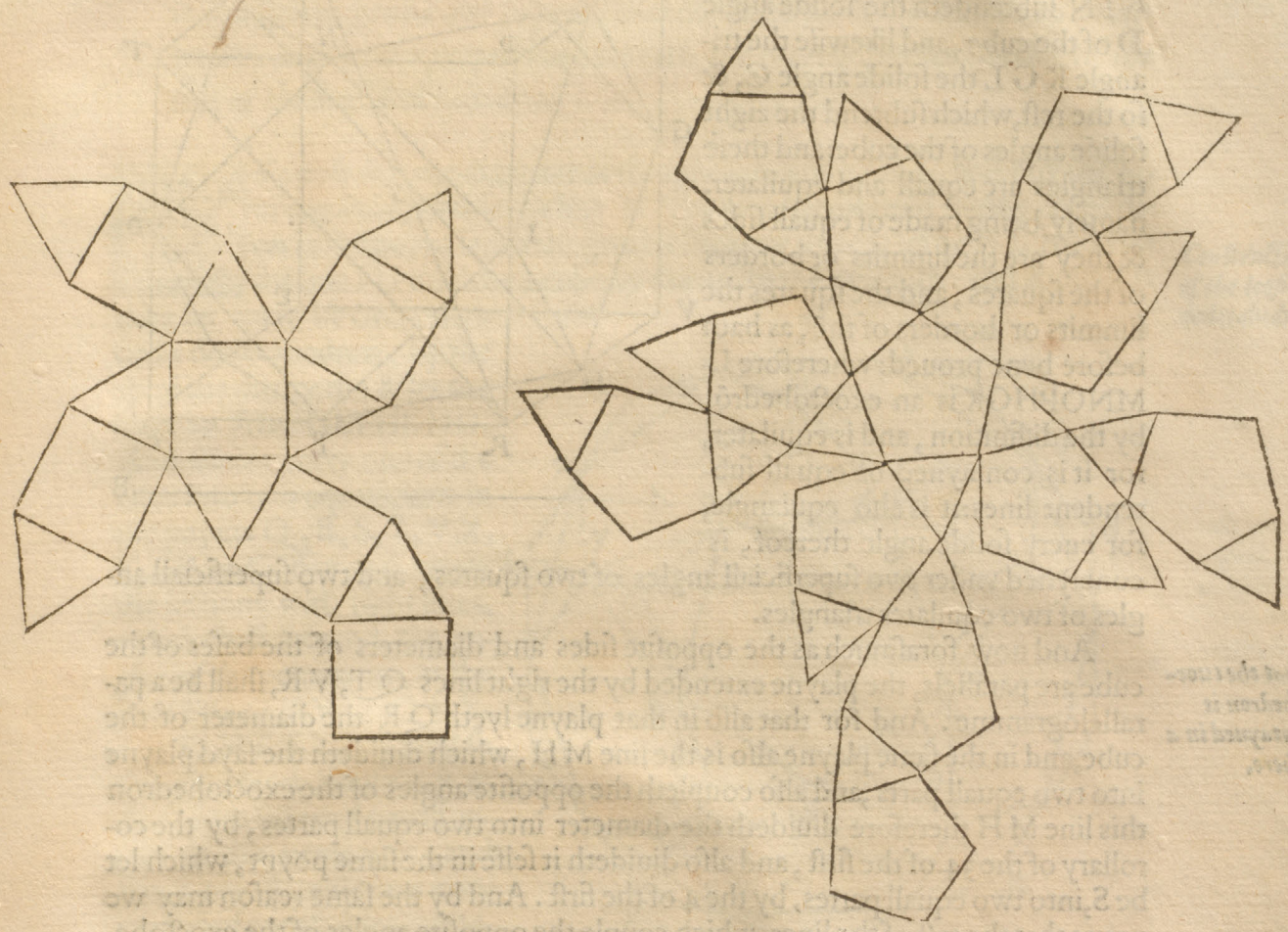
An Exoctohedron is a ſolide figure contained of ſixe equall ſquares, and eight equilater and equall triangles.

¶ *Second Definition.*

An Icoſidodecahedron is a ſolide figure, contained vnder twelue equilater equall and equiangle Pentagons, and twentie equall and equilater triangles.

For the better vnderſtanding of the two former definitions, and alſo of the two Propoſitions following, I haue here ſet two figures, whoſe formes, if ye firſt deſcribe

describe vpon pasted paper or such like matter, and then cut them and folde them accordingly, they will represent vnto you the perfect formes of an Exoctohedron and of an Icosidodecahedron.



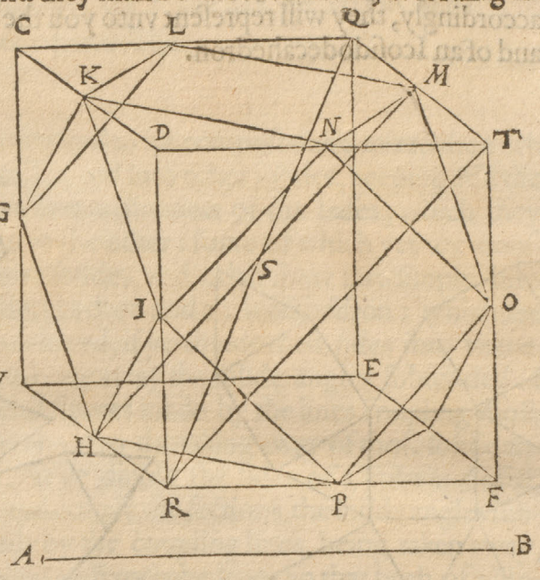
¶ *The first Probleme.*

To describe an equilater and equiangle exoctohedron, and to contayne it in a sphere geuen: and to proue that the diameter of the sphere is double to the side of the sayd exoctohedron.

Construction of the exoctohedron.

SVppose that there be a sphere geuen, whose diameter let be A B. And about the diameter A B let there be described a square by the sixth of the fourth: and vpon the square let there be described a cube by the 15. of the thirteenth: which let be C D E F Q T V R: and let the diameter thereof be Q R, and the centre S. And diuide the sides of the cube into two equall partes, in the poyntes G,H,I,K,L,M,N,O,P.&c. And couple the middle sections by the right lines I N, N O, O P, P I and such like, which subtend the angles of the squares or bases of the cube: and they are equall by the 4. first, and contalne right angles, as the angle N I P. For the angle NID which is at the base of the Isosceles triangle N D I, is the halfe of a right angle, and so likewise is the opposite angle R-I P. Wherefore the residue N I P is a right angle, and so the rest. Wherefore N I P-O is a square. And by the same reason shall the rest N M L K, K G H I &c. inscribed

in the bases of the cube, be squares: and they shall be sixe in number, according to the nũber of the bases of the cube.



Agayne forasmuch as the triangle K I N subtendeth the solide angle D of the cube, and likewise the triangle K G L the solide angle C, & so the rest, which subtend the eight solide angles of the cube: and these triangles are equall and equilater, namely, being made of equall sides & they are the limmits or borders of the squares, and the squares the limmits or borders of thẽ, as hath before bene proued: wherefore L-MNOPHGK is an exoctohedrõ, by the diffinition, and is equilater, for it is contayned of equall subtendent lines: it is also equiangle, for euery solide angle thereof, is contayned vnder two superficiall angles of two squares, and two superficiall angles of two equilater triangles.

That the exoctohedron is contayned in a sphere.

And now forasmuch as the opposite sides and diameters of the bases of the cube are parallels, the playne extended by the right lines Q T, V R, shall be a parallelogramme. And for that also in that playne lyeth Q R the diameter of the cube, and in the same playne also is the line M H, which diuideth the sayd playne into two equall parts, and also coupleth the opposite angles of the exoctohedron this line M H therefore diuideth the diameter into two equall partes, by the corollary of the 34. of the first, and also diuideth it selfe in the same poynt, which let be S, into two equall partes, by the 4 of the first. And by the same reason may we proue that the rest of the lines, which couple the opposite angles of the exoctohedron, doo in S the centre of the cube diuide the one the other into two equall parts. For euery one of the angles of the exoctohedron are set in euery one of the bases of the cube. Wherefore making the centre the poynt S, and the space S H or S M, describe a sphere, and it shall touch euery one of the angles equedistant from the poynt S.

That the exoctohedron is contayned in the sphere geuen.

And forasmuch as A B the diameter of the sphere geuen, is put equall to the diameter of the base of the cube, namely, to the line R T, and the same line R T is equall to the line M H, by the 33. of the first: which line M H coupling the opposite angles of the exoctohedron, is drawne by the centre: wherefore it is the diameter of the sphere geuen which contayneth the exoctohedron.

That the diameter of the sphere is double to the side of the exoctohedron.

Finally forasmuch as in the triangle R F T, the line P O doth cutte the sides into two equall partes, it shall cutte them proportionally with the bases, namely, as F R is to F P, so shall R T be to P O, by the 2. of the sixth. But F R is double to F P, by suppositiõ: wherefore R T, or the diameter H M, is also double to the line P O the side of the exoctohedron. Wherefore we haue described an equilater & equiangle exoctohedron, and comprehended it in a sphere geuen, and haue proued that the diameter of the sphere is double to the side of the exoctohedron.

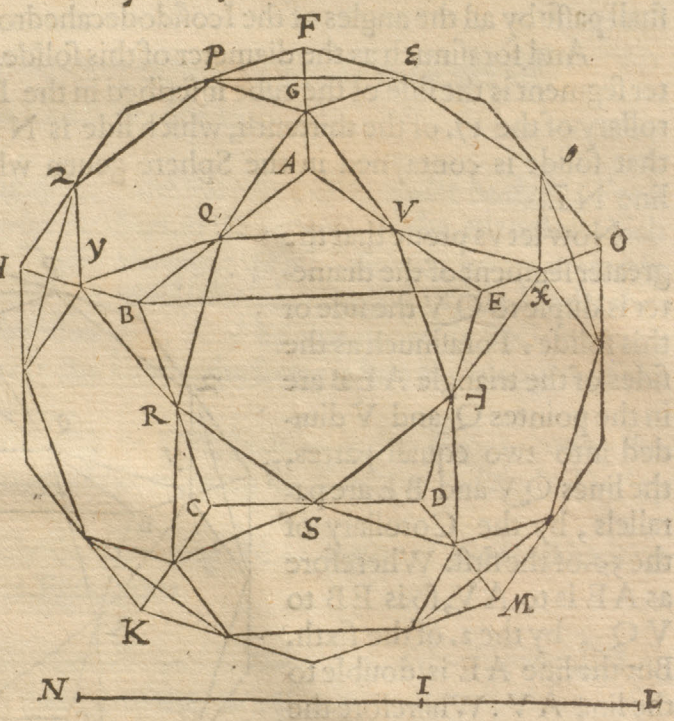
The

¶ The 2. Probleme.

To describe an equilater & equiangle Icosidodecahedron, & to cõprehend it in a sphere geuen: and to proue that the diameter being diuided by an extreame and meane proportion, maketh the greater segment double to the side of the Icosidodecahedron.

Construction of the Icosidodecahedron.

SVppose that the diameter of the sphere geuen be N L, and (by the 30. of the sixth) diuide the line N L by an extreame and meane proportion in the poynt I: and the greater segment thereof let be N I. And vpõ the line N I describe a cube by the 15. of the thirteenth: and about this cube let there be circumscribed a dodecahedron, by the 17. of the thirteenth: & let the same be ABCDEFHKMO. And diuide euery one of the sides into two equall parts in the poynts Q, R, S, T, V, X, Y, Z, P, ε, θ, G. &c. and couple the sections with right lines, which shall subtend the angles of the pentagons, as the lines P G, G V, V Q, Q Y, Y R, R Q, V T, T X, X V, and so the rest. Now forasmuch as these lines subtend equall angles of the pentagons, and those equall angles are contayned of equall sides (namely of the halues of the sides of the pentagons: therefore those subtending lines are equall, by the 4. of the first. Wherefore the triangles G Q V, Y Q R, V X T, and the rest which take away solide angles of the dodecahedron, are equilater. Agayne forasmuch as in euery pentagon are described fiue equall right lines, coupling the middle sections of the sides, as are the lines Q V, V T, T S, S R, R Q: they describe a pentagon in the playne of the pentagon of the dodecahedron: and the sayd pentagon is contayned in a circle, namely, whose centre is the centre of a pentagon of the dodecahedron. For the lines drawne from that centre to the angles of this pentagon are equall, for that they are perpendiculars vpon the bases cutte, by the 12. of the fourth. Wherefore the pentagon Q R S T V is equiangle, by the 11. of the same. And by the same reason may the rest of the pentagons described in the bases of the dodecahedron be proued equall and like. Wherefore those pentagons are 12. in number: And forasmuch as the equall and like triangles, doo subtend and take away 20. solide angles of the dodecahedron, therefore the sayd triangles shall be 20. in nũber. Wherfore we haue described an Icosidodecahedrõ by the diffinitiõ, which Icosidodecahedrõ is equilater, for that all the sides of the triangles are equal & cõmon with the pẽtagons: and it is also equiangle. For euery one of the solide angles is made of two superficiall angles of an equilater pentagon, and of two superficiall angles of an equilater triangle.



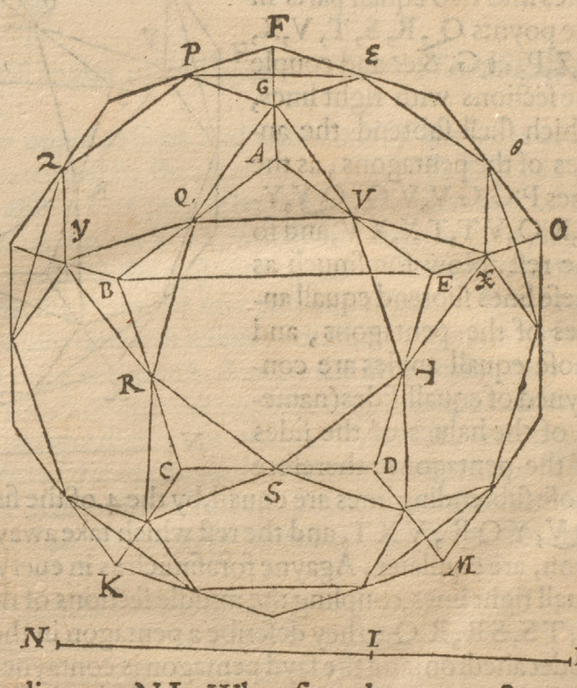
That the Icoſidodecahedron is contayned in the ſphere geuen.

Now let vs proue that it is contained in the Sphere geuen, whoſe diameter is N L. Foraſmuch as perpendiculars drawen frō the centres of the Dodecahedron, to the midle ſections of his ſides, are the halfes of the lines, which couple the oppoſite midle ſections of the ſides of the Dodecahedron, by the 3. Corollary of the 17. of the thirteenth: which lines alſo, by the ſame Corollary, do in the centre diuide the one the other into two equall partes: therefore right lines drawen from that point to the angles of the Icoſidodecahedron (which are ſet in thoſe midle ſections) are equall: which lines are 30. in number according to the number of the ſides of the Dodecahedron: for euery one of the angles of the Icoſidodecahedron are ſet in the midle ſections of euery one of the ſides of the Dodecahedron. Wherfore making the centre the centre of the Dodecahedron, and the ſpace any one of the lines drawen from the centre to the midle ſections, deſcribe a Sphere, and it ſhall paſſe by all the angles of the Icoſidodecahedron, and ſhall containe it.

And foraſmuch as the diameter of this ſolide, is that right line, whoſe greater ſegment is the ſide of the cube inſcribed in the Dodecahedron, by the 4. Corollary of the 17. of the thirteenth, which ſide is N I, by ſuppoſition. Wherefore that ſolide is contayned in the Sphere geuen whoſe diameter is put to be the line N L.

That the diameter being deuided by an extreme and meane proportion. &c.

Now let vs proue that the greater ſegment of the diameter is duple to Q V the ſide of this ſolide. Foraſmuch as the ſides of the triangle A E B are in the pointes Q and V diuided into two equall partes, the lines Q V and B E are parallels, by the Corollary of the 39. of the firſt. Wherefore as A E is to A V, ſo is E B to V Q, by the 2. of the ſixth. But the line A E is double to the line A V. Wherefore the line B E is double to the line Q V: by the 4. of the ſixth. Now the line B E is equall to N I, or to the ſide of the cube, by the 2. Corollary of the 17. of the thirteenth, which line N I is the greater ſegment of the diameter N L. Wherefore the greater ſegment of the diameter geuen, is double to the ſide of the Icoſidodecahedron inſcribed in the Sphere geuen. Wherefore we haue deſcribed an equilater and equiangle Icoſidodecahedron, and contained it in a Sphere geuen, and haue proued that the diameter thereof being diuided by an extreme and meane proportion, maketh hys greater ſegment double to the ſide of the Icoſidodecahedron.



¶ *An aduertiſment of Fluſſas.*

To the vnderſtanding of the nature of this Icoſidodecahedron, ye muſt well conceaue the paſſions and proprieties of both thoſe ſolides, of whoſe baſes it conſiſteth, namely, of the Icoſahedron and of the Dodecahedron. And although in it

the

the bases are placed oppositely, yet haue they one to the other one & the same inclination. By reason wherof there lie hidden in it the actions and passions of the other regular solides. And I would haue thought it not impertinent to the purpose to haue set forth the inscriptions and circumscriptions of this solide, if want of time had not hindred. But to the end the reader may the better attaine to the vnderstanding therof, I haue here following briefly set forth, how it may in or about euery one of the fiue regular solides be inscribed or circumscribed: by the helpe whereof he may, with small trauaile or rather none at all, so that he haue well peysed and considered the demonstrations pertayning to the foresayd fiue regular solides, demonstrate both the inscription of the sayd solides in it, and the inscription of it in the sayd solides.

¶ Of the inscriptions and circumscriptions of an Icosidodecahedron.

An Icosidodecahedron may containe the other fiue regular bodyes. For it will receaue the angles of a Dodecahedron, in the centres of the triangles which subtend the solide angles of the Dodecahedron: which solide angles are 20. in nũber, and are placed in the same order in which the solide angles of the Dodecahedron taken away or subtended by them, are. And by that reason it shall receaue a Cube and a Pyramis contayned in the Dodecahedron: when as the angles of the one are set in the angles of the other.

An Icosidodecahedron receaueth an Octohedron, in the angles cutting the sixe opposite sections of the Dodecahedron, euen as if it were a simple Dodecahedron.

And it containeth an Icosahedron, placing the 12. angles of the Icosahedron in the selfe same centres of the 12. Pentagons.

It may also by the same reason be inscribed in euery one of the fiue regular bodies: namely, in a Pyramis, if ye place 4. triangular bases concentricall with 4. bases of the Pyramis, after the same maner, that ye inscribed an Icosahedrõ in a Pyramis. So likewise may it be inscribed in an Octohedron, if ye make 8. bases thereof concentricall with the 8. bases of the Octohedron. It shall also be inscribed in a Cube, if ye place the angles which receaue the Octohedron inscribed in it, in the centres of the bases of the Cube. Moreouer, ye shall inscribe it in an Icosahedron, when the triangles compased in of the Pentagon bases, are concentricall with the triangles, which make a solide angle of the Icosahedron. Finally, it shall be inscribed in a Dodecahedron, if ye place euery one of the angles thereof in the midle sections of the sides of the Dodecahedron, according to the order of the construction thereof.

The opposite plaine superficieces also of this solide are parallels. For the opposite solide angles are subtended of parallel plaine superficieces, as well in the angles of the Dodecahedron subtended by triangles, as in the angles of the Icosahedron subtended of Pentagons, which thing may eafily be demonstrated. Moreouer in this solide are infinite properties & passions, springing of the solides wherof it is composed.

Wherefore it is manifest that a Dodecahedron & an Icosahedron, mixed, are

transformed

tranſformed into one & the ſelfe ſame ſolide of an Icoſidodecahedron. A cube alſo and an octohedrō are mixed and altered into an other ſolide, namely, into one and the ſame Exoctohedron. But a pyramis is tranſformed into a ſimple and perfect ſolide, namely into an Octohedron.

If we will frame theſe two ſolides ioyned together into one ſolide, this onely muſt we obſerue.

In the pentagon of a dodecahedron inſcribe a like pentagon, ſo that let the angles of the pentagon inſcribed be ſet in the midle ſections of the ſides of the pentagon circumſcribed, and then vpon the ſaid pentagon inſcribed, let there be ſet a ſolide angle of an Icoſahedron, and ſo obſerue the ſelfe ſame order in euery one of the baſes of the Dodecahedron: and the ſolide angles of the Icoſahedron ſet vpon theſe pentagons ſhall produce a ſolide conſiſting of the whole Dodecahedron, and of the whole Icoſahedron. In like ſort, if in euery baſe of the Icoſahedrō, the ſides being diuided into two equall partes be inſcribed an equilater triangle, and vpon euery one of thoſe equilater triangles be ſet a ſolide angle of a Dodecahedron: there ſhall be produced the ſelfe ſame ſolide conſiſting of the whole Icoſahedron, & of the whole Dodecahedron.

And after the ſame order, if in the baſes of a cube, be inſcribed ſquares ſubtending the ſolide angles of an Octohedron, or in the baſes of an Octohedron, be inſcribed equilater triangles ſubtēding the ſolide angles of a cube, there ſhall be produced a ſolide conſiſting of either of the whole ſolides, namely, of the whole cube and of the whole Octohedron.

But equilater triangles inſcribed in the baſes of a pyramis, hauing their angles ſet in the midle ſections of the ſides of the pyramis, and the ſolide angles of a pyramis ſet vpon the ſayd equilater triangles, there ſhall be produced a ſolide, conſiſting of two equal and like pyramids.

And now if in theſe ſolides thus compoſed, ye take away the ſolide angles, there ſhalbe reſtored againe the firſt compoſed ſolides: namely, the ſolide angles taken away from a Dodecahedron and an Icoſahedron compoſed into one, there ſhalbe left an Icoſidodecahedron: the ſolide angles takē away from a cube, and an octohedrō cōpoſed into one ſolide, there ſhalbe left an exoctohedrō. Moreouer the ſolide angles taken away from two pyramids compoſed into one ſolide, there ſhal be left an Octohedron.

Fluſſas after this ſetteth forth certaine paſſions and properties of the fiue ſimple regular bodies: which although he demonſtrateth not, yet are they not hard to be demonſtrated, if we wel peaſe and conceiue that, which in the former bookes hath bene taught touching thoſe ſolides.

Of the nature of a trilater and equilater Pyramis.

1. A trilater equilater Pyramis, is deuided into two equal partes, by three equal ſquares, which in the centre of the pyramis cutte the one the other into two equal partes, and perpendicularly, and whoſe angles are ſet in the midle ſections of the
2. ſides of the pyramis. From a pyramis are taken away 4. pyramids like vnto the whole, which vtterly take away the ſides of the pyramis, and that which is left

is an octohedrō inscribed in the pyramys in which all the solides inscribed in the pyramis are contained. A perpendicular drawne from the angle of the pyramis to the base,is double to the diameter of the cube inscribed in it. And a right line coupling the midle sections of the opposite sides of the pyramis,is triple to the side of the selfe same cube. The side also of the pyramis is triple to the diameter of the base of the cube. Wherefore the same side of the pyramis is in power duple to the right line which coupleth the midle sections of the opposite sides. And it is in power sesquialter to the perpendicular which is drawne from the angle to the base. Wherefore the perpendicular is in power sesquitertia to the line which coupleth the midle sections of the opposite sides. A pyramis,and an Octohedron inscribed in it,also an Icosahedron inscribed in the same Octohedron, doo containe one and the selfe same sphere.

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Of the nature of an Octohedron.

Foure perpendiculars of an Octohedron,drawne in 4.bases therof from two opposite angles of the said Octohedron, and coupled together by those 4.bases, describe a Rhombus,or diamond figure : one of whose diameters is in power duple to the other diameter.For it hath the same proportiō that the diameter of the Octohedron,hath to the side of the Octohedron. An Octohedron & an Icosahedron inscribed in it,do containe one and the selfe same sphere. The diameter of the solide of the Octohedron,is in power sesquialter to the diameter of the circle which containeth the base:and is in power triple to the right line which coupleth the cētres of the opposite bases:and is in power *duple superbipartiens tercias to the perpēdicular or side of the foresaid Rhombus:and moreouer is in lēgth triple to the line which coupleth the centres of the next bases. The angle of the inclination of the bases of the Octohedron, doth with the angle of the inclination of the bases of the pyramis,make angles equal to two right angles.

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Of the nature of a Cube.

The diameter of a cube,is in power sesquialter to the diameter of his base: and is in power triple to his side : and vnto the line which coupleth the centres of the next bases,it is in power sextuple. Moreouer the side of the cube is to the side of the Icosahedron inscribed in it, as the whole is to the greater segment : vnto the side of the Dodecahedron,it is as the whole is to the lesse segment : vnto the side of the Octohedron,it is in power duple:and vnto the side of the pyramis,it is in power subduple. Moreouer the cube is triple to the pyramis : but to the cube the Dodecahedron is in a maner duple. Wherfore the same Dodecahedron is in a maner sextuple to the sayd pyramis.

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Of the nature of an Icosahedron.

Fiue triangles of an Icosahedron,do make a solide angle, the bases of which triangles make a pentagon. If therfore from the opposite bases of the Icosahedron

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be taken the other pentagon by them deſcribed, theſe pentagons ſhall in ſuch ſort cut the diameter of the Icoſahedron which coupleth the forſaid oppoſite angles, that that part which is contained betweene the plaines of thoſe two pentagons, ſhalbe the greater ſegment: and the reſidue which is drawne from the plaine to
2. the angle ſhall be the leſſe ſegment. If the oppoſite angles of two baſes ioyned together, be coupled by a right line, the greater ſegment of that right line is the ſide
3. of the Icoſahedron. A line drawne from the centre of the Icoſahedron to the angles, is in power quintuple, to halfe that line which is takē betwene the pentagōs, or of the halfe of that line which is drawne from the centre of the circle which cō-
4. tayneth the foreſaid pentagon: which two lines are therefore equall. The ſide of the Icoſahedron contayneth in power either of them, and alſo the leſſe ſegment,
5. namely, the line which falleth from the ſolide angle to the pentagon. The diameter of the Icoſahedron contayneth in power the whole line, which coupleth the oppoſite angles of the baſes ioyned together, and the greater ſegment thereof,
6. namely, the ſide of the Icoſahedron. The diameter alſo is in power quintuple to the line which was taken betwene the pentagons, or to the line which is drawne from the centre to the circumference of the circle which containeth the pentagon
7. cōpoſed of the ſides of the Icoſahedron. The dimetient contayneth in power the right line which coupleth the centres of the oppoſite baſes of the Icoſahedron,
8. and the diameter of the circle which contayneth the baſe. Moreouer the ſayd dimetient contayneth in power the diameter of the circle, which contayneth the pentagon, and alſo the line which is drawne from the centre of the ſame circle to the circumference: That is, it is quintuple to the line drawne from the centre to
9. the circumference. The line which coupleth the centres of the oppoſite baſes, contayneth in power the line which coupleth the centres of the next baſes, and alſo the reſt of that line of which the ſide of the cube inſcribed in the Icoſahedron is
10. the greater ſegment. The line which coupleth the middle ſections of the oppoſite
11. ſides, is triple to the ſide of the dodecahedron inſcribed in it. Wherefore if the ſide of the Icoſahedron, and the greater ſegment thereof be made one line, the third part of the whole, is the ſide of the dodecahedron inſcribed in the Icoſahedron.

Of the nature of a Dodecahedron.

1. The diameter of a dodecahedron contayneth in power the ſide of the dodecahedron, and alſo that right line, vnto which the ſide of the dodecahedron is the leſſe ſegment, and the ſide of the cube inſcribed in it is the greater ſegmēt: which line is that which ſubtendeth the angle of the inclination of the baſes, contayned
2. vnder two perpendiculars of the baſes of the dodecahedron. If there be taken two baſes of the dodecahedron diſtant the one from the other by the length of one of the ſides, a right line coupling their centres, being diuided by an extreame and meane proportion, maketh the greater ſegment the right line which coupleth the
3. centres of the next baſes. If by the centres of fiue baſes ſet vppon one baſe, be drawne a playne ſuperficies, and by the centres of the baſes which are ſet vpon the oppoſite baſe be drawne alſo a playne ſuperficies, and then be drawne a right line coupling the centres of the oppoſite baſes, that right line is ſo cut, that eche of his partes ſet without the playne ſuperficies, is the greater ſegment of that part which
4. is contayned betwene the playnes. The ſide of the dodecahedron is the greater

ſegment

segment of the line which subtendeth the angle of the pentagon. A perpendicular line drawne from the centre of the dodecahedron to one of the bases, is in power quintuple to half the line which is betwene the playnes: And therfore the whole line which coupleth the centres of the opposite bases, is in power quintuple to the whole line which is betwene the sayd playnes. The line which subtēdeth the angle of the base of the dodecahedrō, together with the side of the base, are in power quintuple to the line which is drawne from the cētre of the circle, which contayneth the base, to the circumference. A section of a sphere contayning three bases of the dodecahedron taketh a third part of the diameter of the sayd sphere. The side of the dodecahedron, and the line which subtendeth the angle of the pentagon, are equall to the right line which coupleth the middle sections of the opposite sides of the dodecahedron. 5. 6. 7. 8. 9.

(:)

¶ The ende of the Elementes of Geometrie, of the most auncient Philosopher *Euclide* of Megara.

Jms: Hacon owethe this Booke /

Bought the .8. of Jme. 1594.
price 2 6.

Faultes escaped.

MArualle not (gentle reader) that faultes here following, haue escaped in the correction of this booke. For, for that the matter in it contayned is straunge to our Printers here in England, not hauing bene accustomed to Print many, or rather any bookes contayning such matter, which causeth them to be vnfurnished of a corrector skilfull in that art: I was forced, to my great trauaile and paine, to correcte the whole booke my selfe. And in deede sometimes for want of *Argus* eyes, and iuste consideration, notwithstanding my diligence in correcting, faultes escaped through me: sometimes also for lacke of diligence in the Printer to amend my corrections, faultes remayned vncorrected by his meanes. So that betwene vs both these faultes haue escaped vncorrected: which faultes yet, to say the trouth, for the most part are such, as a very young student without noting them vnto him, mought easily of him selfe finde and correcte. And this I dare boldly affirme, that not many bookes, if any, concerning this art in other tounges, Greke, Latine, or Italian, are with so fewe faultes of importance Printed, as this booke is. The triall wherof I referre to them which ha e red any bookes of this arte in other tounges, & shall happen hereafter to read this. And as touching these faultes to be corrected, I would wishe you (good reader) before you beginne to read any of the 16. bookes in this volume contained, first to amend the faultes in that booke which you will read, according as they are here signified vnto you. Where you shall finde in what booke, leafe, side, and line, both the fault escaped is, & also how it is to be corrected. And if you happen in reading to finde any more faultes not here mentioned, as peraduenture you may, for that diuers faultes were vtterly so easie and light to correcte, that I would not note them, & besides that, no one man though he be neuer so diligent and circumspecte can espie all thinges, I trust you will therefore impute no blame either vnto me or to the Printer, but gently amend and correct them, accepting our good minde, which was to haue had the booke passed to your handes vtterly without fault, as touching the Printing.

Fol.	Pag.	Line.	Faultes.	Corrected.
			Errata Lib. 1.	
1	2	41	*point B, as Campane*	*point C, as Campane*
3	1	22	*all lines drawne*	*all right lines*
3	1	28	*lines drawen to the superficies*	*right lines drawē to the circumference*
9	1	42	*lines A B and A C,*	*lines A B and B C*
15	1	35	*are equall*	*are proued equall*
20	2	28	*(by the first*	*(by the fourth*
21	1	39	*the centre C.*	*the centre E*
24	2	6	*If fower right*	*If two right*
25	2	3	*first petition*	*sixeth petition*
49	2	7	14.16.32.64. *&c.*	4.8.16.32.64. *&c.*
53	1	39	*the triangle N G*	*the triangle K G*
54	2	25	*(by the 44.*	*(by the 42.*
57	2	23	*and C B G is the*	*and C G B is the*
			In stede of Flußates through out the whole booke read Flußus.	
			Errata Lib. 2.	
60	2	29	*Gnomon F G E H*	*Gnomon A H K D*
		30	*Gnomon E H F G*	*Gnomon E C K D*
69	1	18	*the whole line*	*the whole figure*
76	2	9	*the diameter C D*	*the diameter A H F*
			Errata Lib. 3.	
82	2	36	*angle*	*equall to the angle*
92	1	*last*	*the line A C is*	*the line A F is*
			Errata Lib. 4.	
110	2	10	*C D toucheth the*	*E D toucheth the*
		12	*side of the other*	*angle of the other*
115	1	21	*and H B*	*and H E*
117	2	44	*the angle A C D*	*the angle A C B*
118	1	2	*into ten equall*	*into two equall*

Fol.	Pag.	Line.	Faultes.	Corrected.
121	1	32	*C D, and E A,*	*C D, D E, & E A,*
124	1	29	*the first*	*the third*
			Errata Lib. 5.	
126	1	43	*it maketh 22. more then 17. by 5.*	*it maketh 24. more then 17. by 7.*
129	1		*In stede of the figure of the 6. definition draw in the maget a figure like vnto this.*	

134	2	4	*As A B is to A, so is C D to C*	*As A B is to B, so is C D to D*
141	2	*last*	*But if K excede M,*	*But if H excede M,*

In

Fol.	Pag.	Line.	Faultes.	Corrected.
143	2		In the figure adde to the line N a line equall to the line C. Also diuide the line G H into three equall partes. And in the line D put out the pricke towardes one of the endes therof.	
151	1	29	vnto as is to D	vnto B as C is to D
	2	13	before this particle: [then by euersion] is left out this sentẽce [and if also the proportion of the second to the third in the first order, be greater then the proportion of the second to the third in the latter order]	
153	1		In the figure of the 7. Proposition, in stede of the lines H, G, G, which are put vnder the lines A, B, C, drawe two lines according to the measure of these two lines.	

G
H

Errata Lib. 6.

155	1	35	grãme A B G D	grãme A C G D
181	1	3	the segment	the sector
		4	the segment	the sector

Errata Lib. 7. 8. and 9.

207	2	2	17. of the fiueth	17. of the seuenth
216	1	last	measure the least	measure the last
222	1	first	the lesse shall	the last shall
224	2	23	an odde number.	an euen number

Errata Lib. 10.

241	1	28	to a number	to a square number
242	1	3	be commensurable	be incõmensurable
245	1	39	of lesse	of the lesse
253	2	36	a sencircle	a semicircle
258	1	25	lines A B and D B	lines A D and D B
258	2	16	lines A B and B G	lines A B and B C
259	1	40	H E and A F	H E and H F
261	1	21	D K is rationall D E is irrationall	D K is irrationall D E is rationall
284	2	12	E is a rationall	I is a rationall
290	1	34	C D nor the line	E D nor the line
300	1	37	lines C D and D F	lines C F and D F
302	1	36	commensurable	incommensurable
306	1	24 25	πορισμαὶ ἰῶς, that is, Acquisiuely	πορισμαὶ ικῶς, that is, Acquisitiuely

Errata Lib. 11.

314	2	33	the side B F	the side D F
320	1	12	vnder foure plaine	vnder sixe plaine
		17	vnder foure plaine	vnder sixe plaine
330	2	33	L H B M, and so	L G B M, and so
338			In the figure in the line L R A E H M N in stede of the letter R put the letter K.	
347	2	46	W, Y, d, and Q	W, Y, h, and Q

Fol.	Pag.	Line.	Faultes.	Corrected.

Errata Lib. 12.

363	2	10	B A, D A, D C,	B A, B C, D C,
368	2		In the figure in stede of the letter R put the letter Q.	
380	1			
388			In the second figure extend the line D G to the point B	
	2	19	Earthmeasures	Earthmeasurers

Errata Lib. 13.

400	1	53	in the circle F	in the circle H
406	2	17	power ble to	power treble to
407	1	48	lines K I, E M, Z N, and N T.	lines K I, E M, Z N, and N H
		last	K I and D B	K I and A B
409	2	34	line V a, which	line Z a, which
410	1	33	Y H, H W, B W, W C,	Y H, H W, B W, W C,
		34	C Z, and C B.	C Z, Z V, & V B.
412	1	16	lines A ſt	lines A &
		24	line B T is equall	line B W is equall
413	2	22	containeth sixe, of such parts the side of the pyramis cõtaineth fower	containeth in power sixe, of such partes the side of the pyramis containeth in power fower

Errata Lib. 14. Hypſ.

417	1	45	selfe same circle	selfe same Sphere
	2	43	same circle	same Sphere

Errata Lib. 14. Fluſſas.

427	2	42	in sesquitertia	in subsesquitertia
430	1	20	to the line E N	to the line A N
		51	the line F G H I M	the cube F G H I M

Errata Lib. 15.

440	1	10	line E H)	line e H)
	2	42	lines A T, and	lines A T, & T G,
444	1		In the figure in stede of the letter G put the letter T	
		54	vnder the side, of	vnder the side D G

Errata Lib. 16.

447	1		wherefore the line A I	wherfore the square of the line A I
448	2	14	in a pyramis	in an Octohedron
458	1	7	whole line P B,	whole line G B

AT LONDON

Printed by Iohn Daye, dwelling ouer Alderſgate beneath Saint Martins.

¶ Theſe Bookes are to be ſolde at his ſhop vnder the gate.

1570.

464
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Campanus, Zamberti and Clavius, the translation is geared to the increasingly large group of surveyors, navigators and military engineers required to operate the developing calculating and measuring technology of the 17th century but who had little or rudimentary Latin. This work precedes the first German translation of the first six books by ten years.

The mathematician Lucas Brunn (d. 1640) was a student of Abraham Riese (the grandson of the famous calculator) and also wrote a work on perspective. The NUC records three copies.

* Riccardi I. 1625.5; Poggendorf I.319.

112. EUCLID. Euclidis Elementorum Libri XV. Vna cum Scholiis antiquis A Federico Commandino. Pesaro, Francischini, 1572. Folio., (12), 255 (ie. 267) p.l. Bound in half-calf and early marbled paper over boards; title-page repaired at lower blank margin just touching engraved surface; some minor staining; generally clean and fresh. $2250

First edition of the most authoritative edition of Euclid published up to that time and "the base of almost all other proper translations before . . . the early nineteenth century" — *DSB* IV.450.

Federico Commandino, the editor, was "clearly the most competent mathematician of all Renaissance editors of Euclid" (*DSB*) and his version is the first that is "solidly based on a tolerably critical Greek original. It even includes, also for the first time, a rendering of numerous Greek scholia."

Commandino's translation seems to have had particular influence in the English-speaking world, serving as the basis for Brigg's *Elementorum Euclidis* (1620), Gregory's *Opera omnia* (1703), the standard pre-nineteenth century source for the Greek text, as well as texts by Simson in English at Glasgow (1756) and Horsley's London (1802).

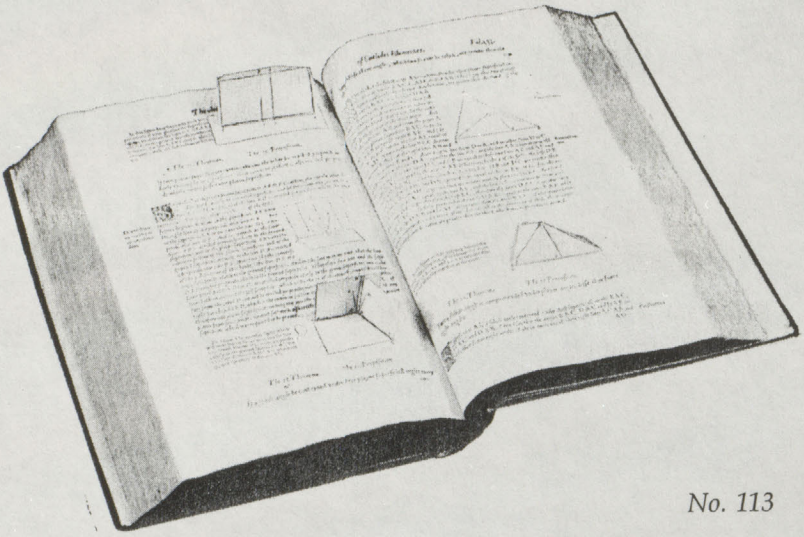
* Thomas-Stanford 18.

"ONE OF THE RICHEST SINGLE DOCUMENTS
FROM WHICH TO ANALYZE THE VARIOUS ROLES PLAYED
BY MATHEMATICS IN THE SCIENTIFIC REVOLUTION." — Hansen

113. EUCLID. The Elements of Geometrie . . . faithfully (now first) translated into the English toung by H. Billingslie. London, John Daye, 1570. Folio, (28), 464 p.l., elaborate allegorical woodcut t-p, woodcut portrait of the printer, hundreds of diagrams in text, 59 superimposed cut-outs in book 9, and 1 oversized folding plate. Bound in early speckled calf rebacked; t-p somewhat dusty & with minor handsoiling in right corner; faint waterstain in lower margin of gutter of first 100 ll., and in right margin of later quire; but overall an appealing and genuine copy $24,500

First edition of the first complete English edition of Euclid's *Elements*, "one of the most important and influential books of the Elizabethan period." — Francis Yates.

A scientific and technological landmark, this translation made the complete text of Euclid available to the middle class audience who would revolutionize navigation and numerous other technologies (surveying, horology etc.) in the following generation. "The impact of [Dee's] *Preface* upon young men of the middle class, sons of tradesmen and



No. 113

craftsmen, was very great, setting out as it did the ways in which geometry could advance technique and foster inventions." — Taylor, p. 170.

Further, Yates considers Dee's *Preface* the most influential statement of the general importance of mathematics in science, far more so than Bacon's *Advancement of Learning* published 35 years later. "Dee's . . . *Preface* had an immense influence, and was widely read and admired until well on during the 17th c. Through it . . . a school of mathematicians and scientists arose which made of the later Elizabethan age a period of great importance for scientific advance." — Yates, *op.cit.*.

Finally, the work numbers among the triumphs of 16th century English book production for its splendid allegorical t-p, the woodcut portrait of the printer (the first to appear in an English book), and above all for its 59 three-dimensional geometrical cut-outs, among the earliest attempts to reproduce geometrical solids in a printed book.

* STC 10560; Thomas-Stanford 41; Sparrow, *Milestones*, p. 2; *DSB* IV.450; Taylor, *Mathematical Practitioners*, #17, 19; Francis A. Yates, "John Dee and the Elizabethan Age," *Theatre of the World*, p. 5.

114. GARDINER, William. Tables of Logarithms for all numbers from 1 to 102100 and for the Sines and Tangents . . . with other useful and necessary tables. London, G. Smith, 1742. Sm. folio., (3) p.l., 14 pp., (113) p.l. Bound in contemporary French mottled calf, spine stamped in gold in compartments with royal monogram (a crown); t-p & early leaves lightly foxed. Generally very good. $2200

Rare. First edition of the first work on logarithms to extend calculations to the seventh figure, celebrated for the accuracy of its computations as well as the legibility and elegance of its printing. Both made the work something of a "logarithm bestseller," receiving editions in the major European languages soon after, and consulted by mathematicians and technical scientists throughout Europe.

QA31 E86713 1570

BINDING LATER + REPAIRED PREVIOUSLY, ALL EDGES SPRINKED, SEWN HEADBANDS, SEWN ON 5 RAISED SUPPORTS

CONDITION: FRONT BOARD DETACHED, BOOK BLOCK BROKEN IN THREE AT PAGE 311 (the tenth books) TO PAGE 333 THIS SECTION LOOSE PAGE EDGES OF LOOSE SECTION DAMAGED

THE FIRST ENGLISH EUCLID.

EUCLIDE. THE ELEMENTS OF GEOMETRIE of the auncient Philosopher EUCLIDE of Megara. Faithfully (now first) translated into the Englishe toung, by H. Billingsley, Citizen of London. Whereunto are annexed certaine Scholies, Annotations, and Inuentions, of the best Mathematicens, both of time past, and in this our age. Woodcut title border with symbolic figures representing the sciences, music, etc., numerous wood cut diagrams, with the complete set of 60 folding flaps in Book XI rare thus, only one of the British Museum copies has them all. Large folding chart, woodcut initials, fine woodcut printer's device with large portrait of John Daye. (Davies 262. The first authentic portrait of an English printer). Largeish wormhole through preliminary leaves, another wormhole in lower margin at end, hole through two leaves caused by ink of an early signature, some light water-staining in lower margin at end, lacks the blank EEE4, apart from these defects a very clean sound copy with good margins. Early signature, (twice), " Amos Hacon oweth this Booke. Bought the 8 of June 1594". From the library of Sion College, with book-plate and sale stamp. "Given [to Sion?] by Mr. Thomas James 1711." 4to, old calf, well rebacked, S.T.C. 10560. Cushing. E.102. Thomas Stanford, Early Editions of Euclid, No.41. John Daye, 1570. £85.

Billingsley, a scholar of St. John's, Cambridge, was afterwards apprenticed to a London haberdasher, rapidly becoming a wealthy merchant, and in 1596, Lord Mayor. He is in the long tradition, (Caxton is perhaps the finest example), of successful business men who have retained their scholarly interests. His taste for mathematics was developed under the tuition of one Whytehead who had been an Augustinian Friar. It is pleasant to record that the latter spent a secure old age in Billingsley's house. This book contains a long essay on mathematics by John Dee. "To The Unfained Lovers of truthe, and constant Studentes of Noble Scineces, IOHN DEE of London, hartily wisheth grace from heauen, and most prosperous successe in all honest attemptes and exercises." Including a defence of Science against the charge of Magic. " Ought any honest Student, and Modern Christian Philosopher be counted & called a CONIURER? Shall the folly of Idiotes, and the Mallice of the Scornfull, so much preuaile?"

together subsequent to publication.
- BM STC German, 1455-1600, p. 399.

Rare Books/Special Collections - Main Collection - By Consultation folio PA4003 L38 1526

Euclid's Geometry presented by Peletier, Jacques, Guillaume Gazeau, and Jean De Tournes

Euclidis Elementa Geometrica Demonstrationum Libri Sex. Lugduni [Lyon] : Apud Ioan. Tornæsium Et Gul. Gazeium, 1557.

Notes:

- Title page with woodcut border.

- **McGill University Library Notes:**
- With: Archimedis Syracusani philosophi ac geometrae excellentis quae quidem extant omnia / Latinitate iam olim donata. Nunc[que] primum in lucem edita. Basileae : [s.n., 1544] -- Commentarii Eutocii Ascalonitae in primum Archimedis De sphaera & cylindro from: Eutokiou Askalōnitou eis ta Archimēdous Peri sphairas [kai] cylindrou ... = Eutocii Ascalonitae in Archimedis libros De sphaera etcylindro at[que] alios quosdam commentaria. Nunc primum & Graece & Latine in lucem edita. Basileae : Ioannes Heruagius excudi fecit, an. 1544. Bound together subsequent to publication.
- Rare Books Dept.'s copy rebound.
- Adams P-587.
- Tchemerzine t. 9, p. 153.
- BM STC French, 1470-1600, p. 157.

Rare Books/Special Collections - Main Collection - By Consultation folio QA31 P485 1557

Three dimensional edition – with pop -ups

Euclid. With contributions by John Dee, John Day, John Highlord, and John Thomas Graves. *The Elements of Geometrie of the Most Auncient Philosopher Euclide of Megara.* Edited by Theon, Of Alexandria,, Thābit Ibn Qurrah Al-Ḥarrānī, and Campano, Da Novara,. Translated by Al-Ḥajjāj Ibn Yūsuf Ibn Maṭar, Adelard, Of Bath,, Bartolomeo Zamberti, and Henry Billingsley, Sir,. London: By Iohn Daye, 1570.

Notes:

- First edition of the first complete English translation.
- In the preface Dee defines the words "astronomy" and "navigation" for the first time.
- Billingsley used the Latin translations of Campanus and Zamberti for this but he also owned a copy of the Greek editio princeps edited by Grynaeus.
- **Book 11 has folding figures which demonstrate the 5 regular solids.**
- Final leaf verso has large medallion portrait of the young John Dee dated 1562.
- Woodcut emblematic illustration on t.p.
- Woodcut initials.
- Printer's tailpieces.
- Printed marginal notes.
- Many errors in foliation.
- Woodcut illustrations throughout.

J. d C. BC Euclid

Rare Books/Special Collections - Main Collection - By Consultation folio QA31 E86713 1570

A comic take on Homer's Illiad in two volumes with illustrations